Unit-3

Electrostatics

Chapter-1

Electrostatics in Vaccum

Electric Feild:

It is the space around the charge where another charge experience some force of attraction or repulsion.

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$$

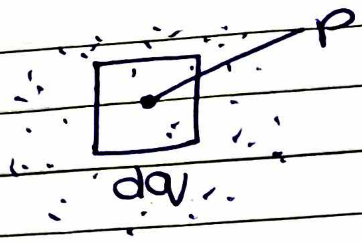
Columb's Law:

$$\boxed{\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}\hat{r}}$$

$$\vec{E} = \frac{q \cancel{q_0}}{4\pi\epsilon_0 r^2 \cancel{q_0}}\hat{r} \qquad \boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}}$$

strength of Elect feil

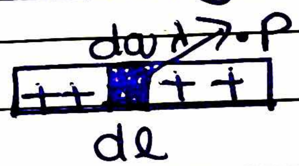
Electric Feild Due to Charge Distribution



$$d\vec{E} = \frac{dq}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0 r^2}\hat{r}\int dq} \quad -\ (1)$$

Linear Charge Density:

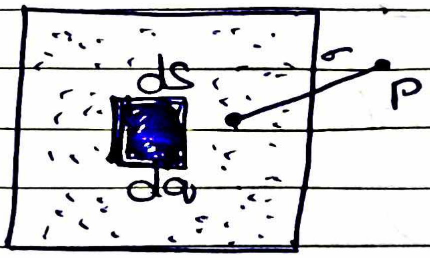


$$\lambda = \frac{dq}{dl}$$

$$dq = \lambda\, dl \ -\ (2)$$, Put (2) in (1)

$$\boxed{\vec{E_L} = \frac{1}{4\pi\epsilon_0 r^2}\hat{r}\int_l \lambda\, dl}$$

Surface Charge Density:



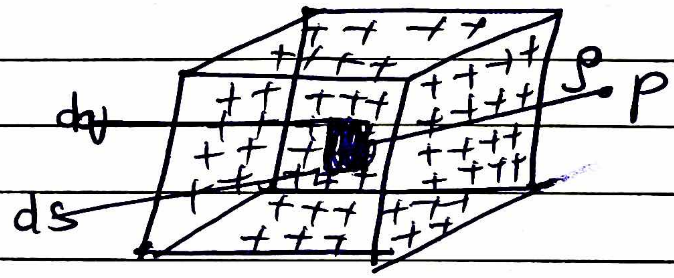
$$\sigma = \frac{dq}{ds}$$

$$dq = \sigma ds \quad -③$$

Put ③ in ①

$$\boxed{\vec{E_s} = \frac{1}{4\pi\epsilon_0} \frac{\hat{r}}{r^2} \int_S \sigma ds}$$

Volume Charge Density:



$$\rho = \frac{dq}{ds}$$

$$dq = \rho ds \quad -④$$

Put ④ in ①

$$\boxed{\vec{E}_V = \frac{1}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}\int_V \rho \, dv}$$

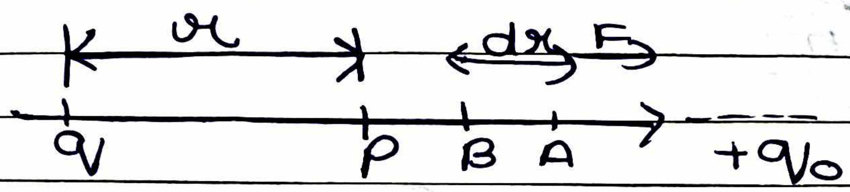
Total Electric Feild:

$$E = E_L + E_S + E_V$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}\left[\int_l \lambda \, dl + \int_S \sigma \, ds + \int_V \rho \, dv\right]}$$

Electric Potential Due to Charge Distribution:

Elec Pot Due to Point Charge:



$$dW = \int_{\infty}^{r} -F \cdot dx$$

$$= -\int_{\infty}^{r} \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2}\, dr$$

$$= -\frac{qq_0}{4\pi\epsilon_0} \int_{\infty}^{r} \frac{1}{r^2}\, dr$$

$$W = \frac{qq_0}{4\pi\epsilon_0 r} \quad \text{— (1)}$$

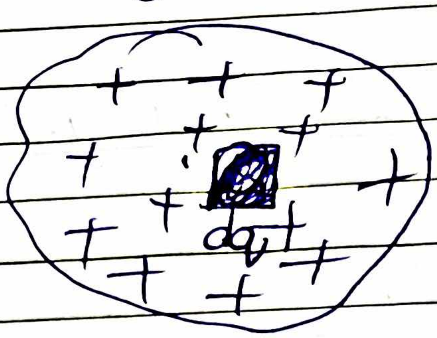
we know,

$$V = \frac{W}{q_0} \quad \text{— (2)}$$

Put (2) in (1)

$$\boxed{V = \frac{q}{4\pi\epsilon_0 r}}$$

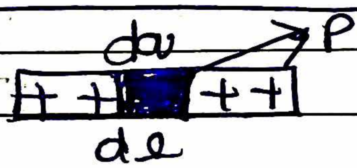
Elect potential Due to Uniform Charge Distribution:



$$dV = \frac{1}{4\pi\epsilon_0}\frac{\hat{r}}{r}\,dq$$

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{\hat{r}}{r}\int dq} \quad -(1).$$

Linear Charge Distribution:



$$\lambda = \frac{dq}{dl}$$

$$dq = \lambda\,dl \quad -(2)$$

Put (2) in (1).

$$\boxed{V_L = \frac{1}{4\pi\epsilon_0}\frac{\hat{r}}{r}\int_l \lambda\,dl} \quad -(3)$$

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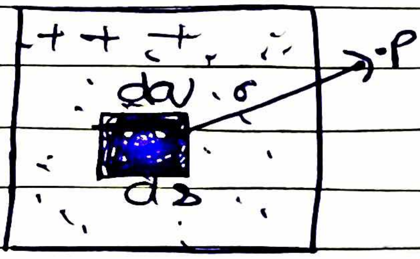
Surface Charge Distribution:



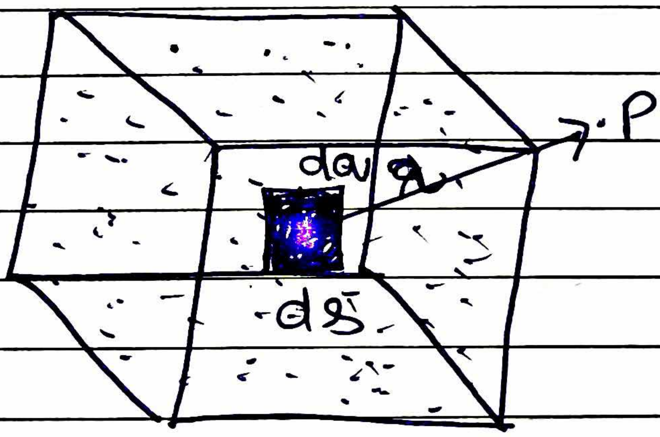
$$\sigma = \frac{dq}{ds}$$

$$dq = \sigma ds \quad — \text{④}$$

Put ④ in ①

$$\boxed{V_s = \frac{1}{4\pi\epsilon_0} \frac{\hat{r}}{r} \int_S \sigma ds} \quad — \text{⑤}$$

Volume Charge Distribution



$$\rho = \frac{dq}{dV}$$

$$dq = \rho dV \quad — \text{⑥}$$

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Put (6) in (1)

$$\boxed{V_V = \frac{1}{4\pi\epsilon_0} \frac{\hat{r}}{r} \int_V \rho \, dV} \quad -(7)$$

<u>Total Elec potential:</u>

~~$E = E_L +$~~

$$V = V_L + V_S + V_V \quad -(8)$$

Put (3), (5) & (7) in (8)

$$\boxed{V = \frac{1}{4\pi\epsilon_0} \frac{\hat{r}}{r} \left[\int_L \lambda \, dl + \int_S \sigma \, ds + \int_V \rho \, dV \right]}$$

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Divergence of Vector Feild:

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$\vec{\nabla}\cdot\vec{A}$

↓

It is known as divergence of a vector feild.

$$\vec{\nabla}\cdot\vec{A} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot$$

$$(A_x\hat{i} + A_y\hat{j} + A_z\hat{k})$$

$$\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x}(\hat{i})^2 + \frac{\partial A_y}{\partial y}(\hat{j})^2 + \frac{\partial A_z}{\partial z}(\hat{k})^2$$

$$\boxed{\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}$$

so divergence of vector feild is a scaler quantity.

$\vec{\nabla}\cdot\vec{A} = 0$

↓

$\vec{A}$ solenoidal feild.

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Divergence of Electrostatic Field:

$$\rho = \frac{dq}{dV}$$

$$dq = \rho dv \quad \text{---} (1)$$

$$q = \int_V \rho \, dv \quad \text{---} (1)$$

we know that, from the Gauss law.

$$\oint E \cdot dS = \frac{q}{\epsilon_0} \quad \text{---} (2)$$

Put (1) in (2).

$$\int E ds = \frac{1}{\epsilon_0} \int \rho dv \quad \text{---} (3)$$

By Gauss divergence theorem

$$\int_S \vec{A} ds = \int_V (\vec{\nabla} \cdot \vec{A}) dv \quad -④$$

In eqⁿ ③ Put Eqⁿ ④

$$\int_V (\nabla \cdot E) dv = \frac{1}{\epsilon_0} \int_V \rho dv$$

$$\cancel{\epsilon_0} \int (\nabla \cdot E) dv = \int \frac{\rho}{\epsilon_0} dv$$

$$\boxed{\nabla \cdot E = \frac{\rho}{\epsilon_0}}$$

Gauss theoram in diffrential form.

Also known as curl or divergence of electrostatic feild.

Poissen's & Laplace Equation:

Consider the charge distributed over the volume V.

than,

Diffrential form of Gauss theoram.

$$\nabla \cdot E = \frac{\rho}{\epsilon_0} \quad -① $$

If V is the potential feild than

$$E = -\nabla V \quad -②$$

Put ② in ①

$$\nabla \cdot (-\nabla V) = \frac{\rho}{\epsilon_0}$$

$$-\nabla(\nabla \cdot V) = \frac{\rho}{\epsilon_0}$$

$$-\nabla^2 V = \frac{\rho}{\epsilon_0}$$

∇

$$\nabla^2 V = -\frac{\rho}{\epsilon_0}$$

$$\boxed{\nabla^2 V + \frac{\rho}{\epsilon_0} = 0}$$

This is known as the Poisson's equation.

∇^2 – Laplacian operator.

<u>In Charge free space</u>:

$$\rho = 0$$

$$\boxed{\nabla^2 V = 0}$$

This is known as the Laplace equation.