

Unit - 2# Introduction to classical Mechanics :-# Field :-

Field is a region of a space in which the physical quantities are defined at all the points

There are two types of field

1) Scalar field : $\phi(r)$

The region of space in which the scalar quantities are continuous as and is defined by a single value at all the points is called scalar field.

for example : electric potential and variation of temperature

2) Vector field $\vec{A}(r)$:-

The region of space in which a vector is continuous and is defined by a single value at all the points is called vector field.

for example : Electric field, Magnetic field.

Conservative forces :-

Conservative forces are the forces which conserves the energy i.e., it follows the law of energy, if the force has a following properties then it is called Conservative force, depends only on, \vec{r}

Conservative force depends only on the initial position and the force irrespective of the path work done by force over the closed path is zero. The work done by the conservative force is irreversible.

for example : Gravitation force, electric force, elastic spring force.

* Non Conservative Forces :-

A non-conservative force does not follow the condition of conservative forces i.e., these are the forces which does not obey the law of conservation of energy.

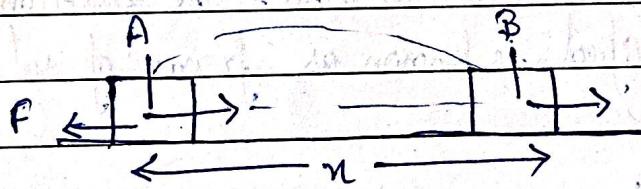
Properties :-

1.) It is path dependent.

2.) The total workdone by the non-conservative force is irreversible.

3.) for example :-

Frictional force, Air resistance, Tension in the string, the repulsive force of the raw material this is also the non-conservative.



$$F_{AB} = F_x$$

$$F_{BA} = -F_x \cos 180^\circ$$

$$F_{BA} = -F_x$$

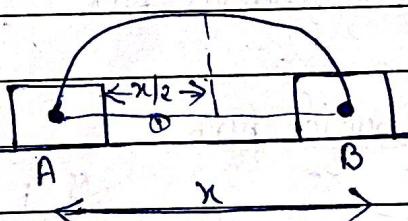
$$F_{BA} = F_x$$

$$= -F_x$$

$$F = F_{AB} + F_{BA}$$

$$= -2F_x$$

work done by the closed path, it is path dependent

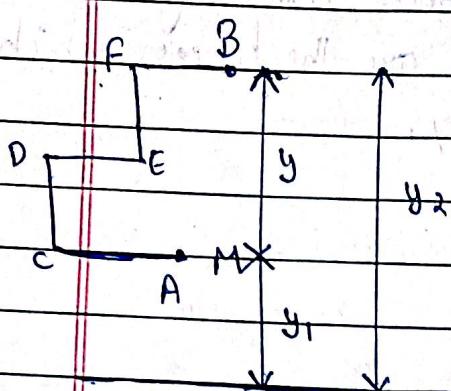


$$F_1 = F_x$$

$$= F_x$$

$$F_2 = F \cdot \pi \cdot \frac{x}{2}$$

$$F_2 = -F \pi x$$



$$y_2 - y_1 = y$$

$$W_{AB} = -mgy$$

#

Frame of Reference: -

The frames related to which the position and the motion of the body is specified is known as frame of reference.

Types of frame of reference

- 1) Inertial FOR
- 2) Non-Inertial FOR

- 3) Inertial FOR:

It is FOR in which the law of inertia and other physics laws are valid.

Any frame moving at a constant velocity related to another frame is known as inertial frame of reference:

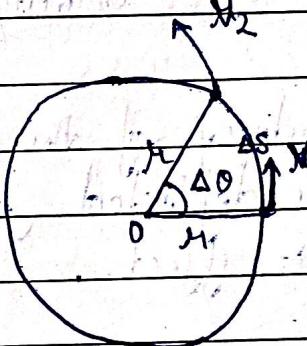
- 2) Non-Inertial FOR:

It is a FOR in which the law of inertia does not hold.

Centrifugal Acceleration:

Acceleration is the change in velocity either in which magnitude or direction or both. In the uniform circular motion the direction of the velocity changes continuously. So there is always the particle is accelerated even if its speed remain constant. Consider an object moving in a circular path at constant speed. The direction of the velocity is shown at the two points along the path. The centripetal acceleration is defined as the rate of change of velocity.

$$a_c = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t}$$



The direction of the centripetal acceleration is always towards the radius vector of the circular motion. By using the properties of singular vectors

$$\Delta v \rightarrow \Delta s \cdot \frac{v_1}{r}$$

$$\Delta v \approx \Delta s \cdot \frac{v_1}{r}$$

$$v_1 = v_2 = v$$

$$\frac{\Delta v}{\Delta t} \rightarrow \frac{\Delta s \cdot v}{\Delta t \cdot r}$$

$$a_c = \frac{v \cdot v}{r} = \frac{v^2}{r}$$

Imp

Corolis Effect:

The force which deflects the motion of the body due to rotation of the frame of reference is known as Corolis force. The acceleration ^{arisen} due to the Corolis force is known as Corolis acceleration.

$$a_{\text{Corolis}} = 2 \omega \times v$$

Where:

ω = is the angular velocity of the rotating frame

v = is the velocity of the moving object

The tendency of the object moving over the surface of the earth to be deflected from the ~~stat~~ straight line path is called Corolis effect.

- # The Corolis effect is caused by the rotation of the earth and the inertia of the mass as the earth completes one rotation per day. The Corolis effect quite small; but its effect is generally become noticeable for the motion occurring over large distances and long period of time, such as large scale movement of the air in the atmosphere or the water in the oceans. This force would be minimum for something moving as slowly as clouds in the atmosphere. This force is effective on the slowly moving bodies act over the long period of time.

Occurrence of Cyclones:-

Air moving parallel to the ground is called wind. And the air moving up or down is called current. If the planets are non-rotating than the winds and the currents flow directly from the area of high pressure to low pressure, but due to the rotation of the planet the Corolis force acts due to which the winds and the current tend to flow right of ~~left~~ of this direction. north of the equator causing anti clockwise rotation.

around the low pressure region. This effect is responsible for the large cyclones.

In ~~southern~~^{southern} hemisphere the deflection of the air is towards left causing clockwise rotation around the low pressure region. On equator the horizontal component of the Coriolis force is zero therefore no cyclones setup at the equator.

Direction of Trade winds:-

When the earth surface near equator heat the hot air rises up and is replaced by the cool air flowing towards the equator due to Coriolis force the air gets diverted towards the right. Hence we get north west trade wind in a north hemisphere and in south hemisphere we get south east trade winds.

Foucault Pendulum:-

The Foucault Pendulum named after the French physicist Jean Foucault is the simple device conceived as an experiment to demonstrate the rotation of the earth. The first pendulum in 1851 was the first proof of the rotation of the earth. Foucault pendulum consists of a 28 kg brass coated lead ball with 67 m long wire the period of oscillation is nearly 17 seconds when the pendulum there was the sand ~~trough~~ hill one were pendulum leaves its traces.

Curl of vector field :-

If a $\vec{A}(x, y, z)$ A is differential vector field then a curl of the \vec{A} is defined as the cross product of the vector operator $\vec{\nabla}$ and vector field A. It is written as

$\text{curl } \vec{A}$	\hat{i}	\hat{j}	\hat{k}
$(\vec{\nabla} \times \vec{A})$	$\frac{\partial}{\partial x}$	$\frac{\partial}{\partial y}$	$\frac{\partial}{\partial z}$
	A_x	A_y	A_z

$$i \left(\frac{\partial A_x - \partial A_z}{\partial y} \right) + j \left(\frac{\partial A_x - \partial A_y}{\partial z} \right) + k \left(\frac{\partial A_y - \partial A_z}{\partial x} \right)$$

If the curl of a vector field is zero then such a field is called irrotational vector field.

Chapter-2

Angular Momentum :-

It is the property of the rotating body given by the product of moment of inertia and angular velocity of the rotating body i.e.,

$$\vec{L} = \vec{I} \times \vec{\omega}$$

$$= \vec{r} \times \vec{p}$$

for the system of the n particles let L.I the angular momentum of the ith particle than the total angular momentum

$$L = \sum_i^N l_i$$

$$= \sum_i^N \vec{I}_i \times \vec{\omega}_i$$

$$= \sum_i^N \vec{r}_i \times \vec{p}_i$$

Law of Conservation of angular momentum :

It states that if the absence of the external force the total angular momentum of the system is constant.

for the system of n particle the angular momentum is represented by

$$\vec{L} = \sum_i \vec{r}_i \times \vec{p}_i$$

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^n \left(\frac{dr_i}{dt} \times \vec{p}_i + r_i \times \frac{dp_i}{dt} \right)$$

$$\frac{dr_i}{dt} \times \vec{p}_i = \vec{v}_i \times m_i \vec{v}_i = 0$$

$$\frac{d\vec{p}_i}{dt} = \sum_{i=1}^n \left(0 + r_i \times \frac{dp_i}{dt} \right)$$

$$m_i \times \frac{dp_i}{dt} = \vec{r}_i \times \vec{F}_i = \vec{\tau}_i$$

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^n \vec{r}_i \times \vec{\tau}_i = \vec{\tau}_{\text{total}}$$

$$\vec{\tau}_{\text{total}} = 0 : \frac{d\vec{L}}{dt} = 0$$

If torque

$$\vec{L} = \text{constant}$$

which is the law of conservation of angular momentum

Kepler's law of Planetary Motion:-

① Law of elliptical orbits:-

each planets moves around the sun in an elliptical orbit the sun is situated at the one of the focus of the orbital ellipses.

② Law of Areal velocity:-

A planet moves around the sun in such a way that its areal velocity remains constant area $P_1 S_1 \phi_1 = \text{Area } P_2 S_2 \phi_2$

Let a planet moves from in A to B in time Δt so that its position vector changes from $r_A + \Delta r$

Let $\nabla A B$ the area swept by its radius vector then $\Delta A = \frac{1}{2} \times \vec{OA} \times \vec{OB}$

$$= \frac{1}{2} \times \vec{r} \times \vec{p}$$

$$\Delta A = \frac{1}{2} \times \vec{r} \times \vec{v} t \quad \Delta A = v A t$$

$$\Delta A = \frac{1}{2} \times \vec{r} \times \vec{v} t$$

$$\vec{v} = \frac{\vec{p}}{m}$$

$$\Delta A = \frac{1}{2} \times \vec{r} \times \vec{p} \cdot \frac{\Delta t}{m}$$

$$= \frac{1}{2m} (\vec{r} \times \vec{p}) \Delta t$$

$$\frac{\Delta A}{\Delta t} = \frac{1}{2m} (\vec{r} \times \vec{p})$$

$$\frac{\Delta A}{\Delta t} = I$$

$$\Delta t : 2\pi m$$

For isolated system

$$\frac{\Delta A}{\Delta t} = \text{constant}$$

Areal Velocity = Constant

③ law of time period :-

The planet moves around the sun in such a way that the square of its time period is proportional of cube to the semimajor axis of its elliptical orbit; i.e.,

$$T^2 \propto R^3$$