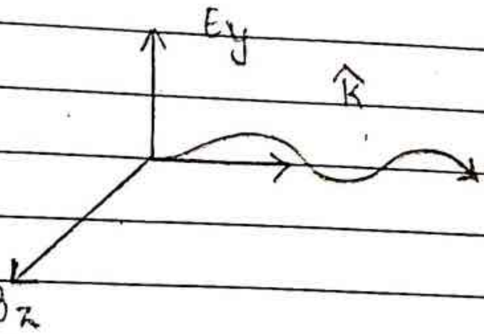
Unit-1
Electromagnetic Waves

Have electric field & Magnetic field component. Both are mutually perpendicular to each other and also perpendicular to wave propagation.

Maxwell Equation (Differential form):



1.) $\nabla . D = \rho$ (Gauss law in electrastatics)
2.) $\nabla . B = 0$ (Gauss law in Magnetostatics)
3.) $\nabla \times E = -\frac{\partial B}{\partial t}$ (Faraday's law of EMI)

4.) $\nabla \times H = J + \frac{\partial D}{\partial t}$ (Amphere Circuital law)

1.) $\nabla . D = \rho$ (Gauss law in Electrostatics)

Acc. to Gauss law,

$$\int E . ds = \frac{q}{\epsilon_0}$$

Acc to Gauss law the total electric flux through the Gaussian surface is equal to $\frac{1}{\epsilon_0}$ time the charge enclosed by the gaussian surface.

$$\boxed{\int E . ds = \frac{q}{\epsilon_0}}$$

$$\int E . \epsilon_0 \, ds = q \quad \text{—— (1)}$$

$\epsilon_0 E = D$ (Displacement Current)

$$\int D . ds = q \quad \text{—— (2)}$$

If ρ is volume charge density, then

Page :
Date: / /

$$\rho = \frac{dq}{dv}$$

$$q = \int \rho \, dv$$

Put in eq ②

Maxwell eqn in integral form $\boxed{\int D \cdot ds = \int \rho \, dv}$ —— ③

Using Gauss divergence theorm

$$\int D \cdot ds = \int \nabla \cdot D \, dv \quad \text{—— ④}$$

From eq ③ and eq ④

$$\int \nabla \cdot D \, dv = \int \rho \, dv$$

$$\boxed{\nabla \cdot D = \rho}$$

2.) $\nabla \cdot B = 0$

Acc to Gauss law in Magnetostatic, the magnetic flux over the closed surface is equal to zero.

Integral form $\boxed{\int B \cdot ds = 0}$ —— ①

Applying gauss divergence theorm

$$\int B \cdot ds = \int \nabla \cdot B \, dv \quad \text{—— ②}$$

From eqn ① & eqn ②

Page :
Date: / /

$$\int \nabla \cdot B \, dv = 0$$

$$\boxed{\nabla \cdot B = 0}$$

Maxwell this equation states that magnetic Monopole does not exist.

3.) $\nabla \times E = -\frac{\partial B}{\partial t}$

Acc. to Faraday's law of electromagnetic induction, the emf induced in the circuit is equal to the rate of change of magnetic flux.

$$\varepsilon = -\frac{\partial \phi_m}{\partial t} \quad \text{--- (1)}$$

(Emf induced in a ckt is given by) $\varepsilon = \int E \cdot dl$ --- (2)

$$\phi_m = \int B \cdot ds \quad \text{--- (3)}$$

From eqn (1), (2) and (3)

$$\boxed{\int E \cdot dl = -\int \frac{\partial B}{\partial t} \cdot ds} \longrightarrow \text{Integral form}$$

Applying Stokes theorm,

$$\int E \cdot dl = \int (\nabla \times E) ds$$

$$\int (\nabla \times E) ds = -\int \frac{dB}{dt} ds$$

$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$ This M.F relates the Electric field magnetic field.

4.) $\nabla \times H = J + \frac{\partial D}{\partial t}$ (Amphere Circuital law)

Acc. to Amphere Circuital law, the line integral of Magnetic field over the closed loop is equal to μ_0 times the Current linked to that loop.

$$\int B \cdot dl = \mu_0 I$$

$$\int \frac{B}{\mu_0} dl = I$$

$$\int H \, dl = I \quad \text{--- (1)}$$

If J is the current density

$$\bar{J} = \frac{dI}{ds}$$

$$I = \int J \cdot ds$$

$$\int H \cdot dl = \int J \cdot ds$$

Acc to Modified form of Amphere Circuital which is given by maxwell.. Current is given by,

$$I = \left(J + \frac{\partial D}{\partial t}\right) ds \quad \text{--- (2)}$$

From (1) & (2)

$$\boxed{\int H \, dl = \int \left(J + \frac{\partial D}{\partial t}\right) ds} \quad \text{--- Integral form}$$

Applying the stokes theorm,

$$\int (\nabla \times H) ds = \int \left(J + \frac{\partial D}{\partial t} \right) ds$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Imp
What is physical significance of Maxwell eqn ?

1st → Gauss law in Electrostatics
2nd → Gauss law in Magnetostatics
3rd → Faraday's law
4th → Amphere Circuital law

* Plane Wave Equation | Plane E.M Waves in free space | vaccum

The EM Wave Equation
We know that Maxwell's eqn's in any medium are

$$\nabla \cdot D = \rho$$

$$\nabla \cdot B = 0$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

In free space, $\rho = 0$, $\sigma = 0$, $J = 0$, $D = \epsilon_0 E$, $B = \mu_0 H$

In free space, Maxwell Equations will be

(1) $\nabla \cdot D = 0 \quad (\rho = 0)$ —— (1)

Page :
Date: / /

(2) $\nabla \cdot B = 0$ — (2)

(3) $\nabla \times E = -\frac{\partial B}{\partial t}$ — (3)

(4) $\nabla \times H = \frac{\partial D}{\partial t}$ — (4)

Taking the curl on both side of eqn - (3)

$$\nabla \times \nabla \times E = -\nabla \times \frac{\partial B}{\partial t}$$

$$\nabla \times \nabla \times E = -\frac{\partial}{\partial t}(\nabla \times B)$$

$$\nabla \times \nabla \times E = -\mu_0 \frac{\partial}{\partial t}(\nabla \times H)$$

From eqn (4)

$$\nabla \times \nabla \times E = -\mu_0 \frac{\partial}{\partial t}\left(\frac{\partial D}{\partial t}\right)$$

$$= -\mu_0 \frac{\partial}{\partial t}\left(\frac{\partial \epsilon_0 E}{\partial t}\right)$$

$$\nabla \times \nabla \times E = -\mu_0 \epsilon_0 \frac{\partial}{\partial t}\left(\frac{\partial E}{\partial t}\right) \quad — (5)$$

Using Vector triple product

$$A \times B \times C = B(A \cdot C) - C(A \cdot B)$$

$$\nabla \times \nabla \times E = \nabla(\nabla \cdot E) - E(\nabla \cdot \nabla)$$

$$= \nabla(\nabla \cdot E) - E\nabla^2$$

$$\nabla(\nabla \cdot E) - \nabla^2 E$$

$$-\nabla\left(\nabla \cdot \frac{D}{\epsilon_0}\right) - \nabla^2 E$$

Page :
Date: / /

$$= \frac{1}{\epsilon_0} \nabla(\nabla \cdot D) - \nabla^2 E$$

$$= 0 - \nabla^2 E \quad [\text{eqn} - 1]$$

Put in eqn 5

$$-\nabla^2 E = -\mu_0 \epsilon_0 \frac{\partial}{\partial t}\left(\frac{\partial E}{\partial t}\right)$$

$$-\nabla^2 E = -\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}.$$

$$\boxed{\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}} \quad — 7$$

Taking curl on both side of eqn - 4

$$\nabla \times \nabla \times H = \frac{\partial}{\partial t}(\nabla \times D)$$

$$\nabla \times \nabla \times H = \epsilon_0 \frac{\partial}{\partial t}(\nabla \times E)$$

$$\nabla \times \nabla \times H = -\epsilon_0 \frac{\partial}{\partial t}\left(\frac{\partial B}{\partial t}\right)$$

$$\nabla \times \nabla \times H = -\epsilon_0 \frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}(\mu_0 H)\right)$$

$$\nabla \times \nabla \times H = -\epsilon_0 \mu_0 \frac{\partial}{\partial t}\left(\frac{\partial}{\partial t} H\right) \quad — 6$$

Using Vector triple product,

$$A \times B \times C = B(A \cdot C) - C(A \cdot B)$$

$$\nabla \times \nabla \times H = \nabla(\nabla \cdot H) - H(\nabla \cdot \nabla)$$

Date: / /

$$\nabla \times \nabla \times H = \nabla\left(\frac{\nabla \cdot B}{\mu_0}\right) - \nabla^2 H$$

$$\nabla \times \nabla \times H = -\nabla^2 H$$

Put in eqⁿ (6)

$$-\nabla^2 H = -\epsilon_0 \mu_0 \frac{\partial}{\partial t}\left(\frac{\partial H}{\partial t}\right)$$

$$\boxed{\nabla^2 H = \epsilon_0 \mu_0 \frac{\partial^2 H}{\partial t^2}} \quad \text{—— (8)}$$

Equation (7) and Equation (8) are plane wave equations or electromagnetic wave equations.

The General form of wave equation is given by

$$\boxed{\nabla^2 \psi = \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2}} \quad \text{—— (9)}$$

Comparing eq. 7, 8, 9

$$\mu_0 \epsilon_0 = \frac{1}{c^2}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{8.54 \times 10^{-12} \times 4\pi \times 10^{-7}}}$$

$$c = 3 \times 10^8 \text{ m/s}$$

This eqⁿ shows that the em waves are propagating through in the free space with the velocity of light.

Transverse Nature of E.m waves

We know that wave equation of electromagnetic waves in terms of electric vector and magnetic vector is represend as

$$\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 H = \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2}$$

The solution of above equation as

$$E(r,t) = E_0 e^{i(kr - \omega t)} \quad -(1)$$
$$H(r,t) = H_0 e^{i(kr - \omega t)} \quad -(2) \quad \left\{ K = \frac{2\pi}{\lambda} \right\} -$$

Represent the value travel

Consider the electromagnetic waves are moving in arbitary direction then e nh must satisfied maxwell eqn in the free space

$$\nabla . D = 0 \Rightarrow \nabla . \varepsilon_0 E_0 \Rightarrow \nabla . E = 0 \quad - (3)$$
$$\nabla . B = 0 \Rightarrow \nabla . \frac{H}{\mu_0} = 0 \Rightarrow \nabla . H = 0 \quad - (4)$$

$$\nabla \times E = \frac{-\partial B}{\partial t} \Rightarrow \nabla \times E = -\mu_0 \frac{\partial H}{\partial t} \quad - (5)$$

$$\nabla \times H = \frac{\partial D}{\partial t} = \nabla \times H = \varepsilon_0 \frac{\partial E}{\partial t} \quad - (6)$$

$$\nabla . E_0 e^{i(kr - \omega t)} = 0$$
$$\nabla e^{i(kr - \omega t)} . E_0 = 0 \quad - (7)$$

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$= \frac{\partial}{\partial x}$$

$$\nabla e^{i(kr-\omega t)} = \frac{\partial}{\partial r} e^{i(kr-\omega t)}$$

$$= e^{i(kr-\omega t)}$$

$$= ik\, e^{i(kr-\omega t)} \quad \text{——} \ (8)$$

$\Rightarrow$ (7) & (8)

$$ik e^{i(kr-\omega t)} \cdot E_0 = 0$$

$$ik \cdot E_0 e^{i(kr-\omega t)} = 0$$

$$ik \cdot E = 0$$

$$k \cdot E = 0$$

Similarly from eq (2) and (4)

$$\nabla \cdot H_0 e^{i(kr-\omega t)} = 0$$

$$\nabla e^{i(kr-\omega t)} \cdot H_0 = 0$$

$$ik e^{i(kr-\omega t)} \cdot H_0 = 0$$

$$ik \cdot H_0 e^{i(kr-\omega t)} = 0$$

$$ik \cdot H = 0$$

$$k \cdot H = 0$$

Hence both E and H vectors are perpendicular to the wave propa[gation]
vector

From eq (1), (2) and (5)

$$\nabla \times E_0 e^{i(kr-\omega t)} = -\mu_0 \frac{\partial H_0 e^{i(kr-\omega t)}}{\partial t}$$

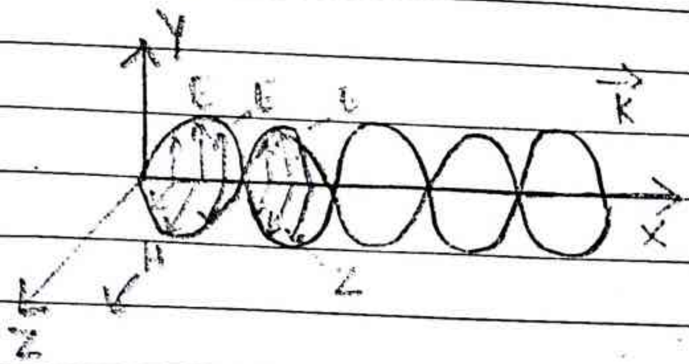
$$\nabla e^{i(kr-\omega t)} \times E_0 = -\mu_0 H_0 e^{i(kr-\omega t)} \cdot (-\omega)$$

$$ik e^{i(kr-\omega t)} \times E_0 = i \mu_0 \omega H_0 e^{i(kr-\omega t)}$$

$$ik \times E_0 e^{i(kr-\omega t)} = i\mu_0 \omega H_0 e^{i(kr-\omega t)}$$

$$K \times E = \mu_0 \omega H$$

$$A \times B = C$$



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Equation of Continuity :-

Consider a conductor carrying a charge ~~tube~~ which are moving velocity (v) if m is the mass of the particle and ~~Smal~~ n is the no. of particles per unit volume.

$$I = neAv$$

$$\frac{I}{A} = nev$$

$$J = nev$$

Consider a small area element ds then the current flowing to the element will be,

$$dI = J.ds$$

$$I = \int J.ds$$

~~Letter~~ V is the volume enclosed the surface S then the total charge within the volume V is ~~too~~ $\int \rho dv$.

$$q = \int \rho dv$$

Acc to the law of conservation of charge the rate of change of charge flowing out must be equal to the rate of decrease of charge inside it.

$$\int J ds = -\frac{dq}{dt} \quad \text{(Flow of charge)}$$

Acc to Gauss divergence theorm

$$\int J ds = -\frac{q}{dt}\int \rho dv$$

$$\int J ds = -\int \frac{d\rho}{dt} dv \quad \text{——} ①$$

$$\int J ds = \int (\nabla . J) dv$$

$$\int (\nabla J) dv = -\int \frac{d\rho}{dt} dv$$

$$\int \left(\nabla . J + \frac{\partial \rho}{\partial t}\right) dv = 0$$

$$\nabla J + \frac{\partial \rho}{\partial t} = 0$$

This equation is known as the Equation of Continuity. This is the mathematical ~~eqn~~ relation showing the law of conservation of charge

Displacement Current :-

Acc to the ampere circuital law for the steady current may be return as

$$\nabla \times H = J$$

This implies that there exist a magnetic field around the steady current

Suppose we connect a battery across ~~the~~ a capacitor through the key Kay. When the circuit is completed the current is flowing throu

Page :
Date: / /

the circuit and start charging the capacitor. ~~the~~ magnetic field is produced around the current carrying wire. After the capacitor is completely charge the flowing of current is decreased and their will be not magnetic field ~~arou~~ around the wire.
It is found that there is the magnetic field in ~~spa~~ the circuit although $J=0$ i.e., their is no flow of charge in circuit.

$J=0$

The variation in the electric field is the only source of charge which accomulate on the plates of capacitor.

Maxwell showed that the time varing electric field produces the displacement current ~~magnetic~~ field due to the variation of the electric field. which is given by

$\frac{\partial D}{\partial t}$ and modifies the ampere circuital law

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

1. Maxwell Equation Derive only two maxwell Equations.
2. What is a ~~sign~~ physical signifiance of maxwell equations.
3. Which maxwell equations explain the magnetic monopole doesnot exist.
4. Which maxwell eqn connect the electrostatic and magnetostatic.
5. what is the equation of continuity. ~~whic~~
6. Which eqn explains the law of conservation of charge
7. Explain the concept of displacement current.
8. Show that E.M. Waves moving with the velocity of light.
9. Show " " are transverse in nature.
10. What are Electromagnetic waves what are properties of electromagnetic waves.

Poynting Vector :-

The poynting vector is the cross product of magnetic field vector and electric field vector, It is represented by P. (S)

$$\vec{P} = \vec{E} \times \vec{H}$$

Dimensional formula

$$= J s^{-1} m^{-2}$$

Its signify the rate of flow of electromagnetic wave energy. The time rate of flow of per unit area of the medium.

What is poynting theorm

Consider the electromagnetic waves are travelling in the medium the energy is transfered from the source to the medium.

The poynting theorm relates the poynting vector with the workdone by the electromagnetic field in displacing the charge and the energy stored in electric field and magnetic field in the medium in which the E.M.waves are travelling. Consider a region in the medium having volume (V) enclosed by the surface (S). Let the medium is homogeneous and isotropic and has ~~~~ permeability μ and permittivity (ε) and conductivity (σ) then from the maxwell equation

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \quad \text{--- (2)}$$

Taking the dot product of H on the both side of eq^n (1)

$$H \cdot (\nabla \times E) = -H \cdot \frac{\partial B}{\partial t} \quad \text{--- (3)}$$

Page :
Date: / /

Taking the dot product of E on both side of eq

$$E(\nabla \times H) = EJ + E\cdot\frac{\partial D}{\partial t} \quad \text{--- (4)}$$

(4) − (3)

$$E.(\nabla \times H) - H(\nabla \times E) = E.J + E\frac{\partial D}{\partial t} + H\frac{\partial B}{\partial t}$$

$$A(B\times C) = C(A\times B) - B(A\times C)$$

$$\nabla(E\times H) = H(\nabla\times E) - E.(\nabla\times H)$$

$$-\nabla(E\times H) = -H(\nabla\times E) + E(\nabla\times H)$$

$$-\nabla(E\times H) = E(\nabla\times H) - H(\nabla\times E)$$

$$(5) \quad -\nabla(E\times H) = EJ + E\cdot\frac{\partial D}{\partial t} + H\frac{\partial B}{\partial t}$$

$$-\nabla(E\times H) = J.E + E\frac{\partial D}{\partial t} + H\frac{\partial B}{\partial t}$$

Integrating the above eq^n over the volume v enclosed by the surface(S)

$$-\oint \nabla.(E\times H) = J.E + E\frac{\partial D}{\partial t} + H.\frac{\partial B}{\partial t}$$

$$-\int \nabla(E\times H)dv = \int JE\,dv + \int\left(E\frac{\partial D}{\partial t} + H\frac{\partial B}{\partial t}\right)dv \quad \text{--- (6)}$$

$$\int\left(E.\frac{\partial D}{\partial t} + H.\frac{\partial B}{\partial t}\right)dv = \int\left(E.\frac{\partial \varepsilon E}{\partial t} + H.\frac{\partial \mu H}{\partial t}\right)dv$$

$$= \int\left[\frac{1}{2}\frac{\partial}{\partial t}(\varepsilon E^2) + \frac{1}{2}\frac{\partial}{\partial t}(\mu H^2)\right]dv$$

$$\cancel{\frac{dx^2}{dy}} = \cancel{2x\frac{dx}{dy}} \qquad = \int\left[\frac{1}{2}\frac{\partial}{\partial t}(\varepsilon E.E) + \frac{1}{2}\frac{\partial}{\partial t}(\mu H.H)\right]dv$$

$$\cancel{\frac{1}{2}\frac{dx^2}{dy}} \qquad = \int\left[\frac{1}{2}\frac{\partial}{\partial t}(\varepsilon E.E) + \frac{1}{2}\frac{\partial}{\partial t}(H.\mu H)\right]dv$$

$$= \int\left[\frac{1}{2}\frac{\partial}{\partial t}(E.D) + \frac{1}{2}\frac{\partial}{\partial t}(H.B)\right]dv$$

Page :
Date: / /

$$= \int \left[\frac{\partial}{\partial t} \left(\frac{E.D + H.B}{2} \right) \right] dv$$

(6) $$-\int \nabla.(E \times H)\, dv = \int (J.E)\, dv + \int \frac{\partial}{\partial t} \left(\frac{E.D + H.B}{2} \right) dv \longleftarrow (7)$$

Using Gauss divergence theorm

$$\int \nabla (E \times H)\, dv = \int (E \times H)\, ds$$

(7) $$-\int (E \times H)\, ds = \int (J.E)\, dv + \int \frac{\partial}{\partial t} \left(\frac{E.D + H.B}{2} \right) dv$$

This eqn is known as Poynting theorm

$$-\int P\, ds = \int (J.E)\, dv + \int \frac{\partial}{\partial t} \left(\frac{E.D + H.B}{2} \right) dv$$

$\int (J.E)\, dv$,

This term represents the rate at which the work is done by the electromagnetic field is displacing charging in volume V.

$$\int \frac{\partial}{\partial t} \left(\frac{E.D + H.B}{2} \right) dv$$

This term represents the rate of increase of energy stored in electric and magnetic field within volume V.

Reflection and Transmission of E.M waves :-

Let an E.M waves is incident normally at the boundries of two medium of impedence Z_1 & Z_2 respectively.

The incident reflected and the transmitted Component of electric vector are E_i E_R E_t and for the magnetic vector H_i H_R H_t respectively for normal incident the direction of Incident wave is opposite to the reflective wave then $\frac{\cancel{E_r}}{H_i} = \cancel{Z_i}$ $\frac{E_i}{H_i} = Z_i$, $\frac{-E_r}{H_R} = Z_1$ — (1) , $\frac{E_t}{H_t} = Z_2$ — (2)

at the boundries of the two medium $E_i + E_r = E_t$ — (3)

$H_i + H_r = H_t$ — (4)

from eq^n (1)

$$H_i = \frac{E_i}{Z_1} , \; H_r = \frac{-E_r}{Z_1}$$

(2) $H_t = \frac{Z_2}{E_t}$

Put the value of

(4) $- \frac{E_i}{Z_i} - \frac{E_r}{Z_i} = \frac{E_t}{Z_2}$

$$\frac{E_1 - E_r}{Z_i} = \frac{E_t}{Z_r}$$

$$E_i - E_r = \frac{Z_1}{Z_2} E_t \quad \text{— (5)}$$

Adding eq^n (3) & (5)

$$E_1 + \cancel{E_r} + E_i - \cancel{E_r} = \frac{Z_1}{Z_2} E_t + E_t$$

$$2 E_i = E_t \left(\frac{Z_1}{Z_2} + 1 \right)$$

$$E_i = \frac{E_t}{2} \left(\frac{Z_1 + Z_2}{Z_2} \right)$$

Subtract (3) & (5)

$$E_i + E_r - E_i + E_r = E_t - \frac{Z_1}{Z_2}E_t$$

$$2E_r = E_t\left(1 - \frac{Z_1}{Z_2}\right)$$

$$E_r = \frac{E_t}{2}\left(\frac{Z_2 - Z_1}{Z_2}\right)$$

The coefficient of reflection is defined as the ~~component of incident w~~ ~~reflective wave~~ ratio of reflective component to incident ray.

$$T = \frac{E_t}{E_i} \quad \text{Coefficient of transmission}$$

~~Coefficient~~ $T = \frac{E_t}{E_i} = \frac{E_t}{\frac{E_t}{2}\left(\frac{Z_1 + Z_2}{Z_2}\right)}$

$$= \frac{2Z_2}{Z_1 + Z_2}$$

Coefficient of ~~reflective com~~ Transmission ratio to incident ratio.

For non-conducting medium:

Suppose the E.M waves are incident normally from the non conducting medium (air, vaccum) to conducting medium.

$$Z_1 >>> Z_2$$

$$R = \frac{Z_2 - Z_1}{Z_1 + Z_2} = \frac{-Z_1}{Z_1} = -1$$

$$\frac{E_r}{E_i} = -1$$

$$E_r = -E_i$$

$E_i = -E_r$

$$T = \frac{2Z_2}{Z_1+Z_2} = \frac{2Z_2}{Z_1}$$

$$\frac{E_t}{E_i} = \frac{2Z_2}{Z_1}$$

$$\frac{E_t}{Z_2} = \frac{2E_i}{Z_1}$$

$$\boxed{H_t = 2H_i}$$

Momentum carried by e.m waves:

The energy per unit volume stored in electromagnetic field is given by $u = \frac{1}{2}(B.H + E.D)$

$$= \frac{1}{2}\left(B.\frac{B}{\mu_0} + E.\varepsilon_0 E\right)$$

$$= \frac{1}{2}\left(\frac{B^2}{\mu_0} + E^2\varepsilon_0\right) \text{ --- (1)}$$

$$\frac{B^2}{E^2} = \frac{1}{c^2}$$

$$c^2 = \frac{1}{\mu_0\varepsilon_0}$$

$$B^2 = \frac{E^2}{c^2} = E^2\mu_0\varepsilon_0$$

(1) $$u = \frac{1}{2}\left(\frac{E^2\mu_0\varepsilon_0}{\mu_0} + \varepsilon_0 E^2\right)$$

$$= \frac{1}{2}\left(E^2\varepsilon_0 + \varepsilon_0 E^2\right)$$

$$= \frac{1}{2}\left(2E^2\varepsilon_0\right) = E^2\varepsilon_0 \text{ --- (2)}$$

If the electric vector point towards the Y- direction

$$E(y,t) = E_0 \cos(ky - \omega t + \beta)$$

②

$$U = \varepsilon_0 E_0^2 \cos^2 (ky - \omega t + \beta)$$

$$\langle U \rangle = \langle \varepsilon_0 E_0^2 \cos^2 (ky - \omega t + \beta) \rangle$$

$$\langle \cos^2 (ky - \omega t + \phi) \rangle = \frac{1}{2}$$

$$\langle U \rangle = \frac{1}{2} \varepsilon_0 E_0^2$$

This is the energy carried by the electromagnetic wave. The E.M wa also carried by the momentum which is defined as the energy carried by the wave speed of light

$$\langle P \rangle = \frac{\langle U \rangle}{C} = \frac{\varepsilon_0 E_0^2}{2C}$$

when the e.m waves ~~those~~ falls on a surface it delivers the momentum to the surface.

$$\text{Resultant Pressure} = \frac{F}{A} = \frac{\frac{\partial P}{\partial t}}{A} = \frac{dP}{dt \cdot A}$$

A is the area of the surface on which the e.m wave incident in time dt the momentum transfer is given by

$$dP = \langle P \rangle c\, dt$$

$$\text{Resultant pressure} \Rightarrow \frac{\langle P \rangle c\, \cancel{dt}}{\cancel{dt} \cdot A}$$

Page :
Date: / /

$$= \frac{\varepsilon_0 E_0^2 \cancel{c}}{2\cancel{c}A} = \frac{\varepsilon_0 E_0^2 \cancel{c}}{2\cancel{c}A}$$

$$= \frac{1}{2}\frac{\varepsilon_0 E_0}{A}$$

[per unit area $A = 1$]

$$R.P = \frac{1}{2}\varepsilon_0 E_0^2$$