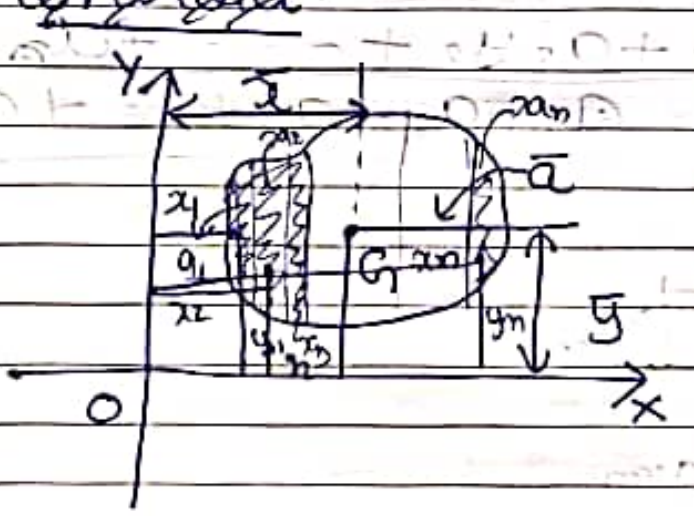
09/09/2024

Unit-2 Centre of Gravity (CG) & Moment of Inertia (MOI)

Centre of Gravity: Point at which entire mass or weight is concentrated.

Centroid:



G (x̄, ȳ)

such that

$$\bar{a} = a_1 + a_2 + a_3 + \cdots + a_n.$$

Principle of Moment:

About Y axis

Moment of Force = F × lx

" " Area = A × lx

OY axis

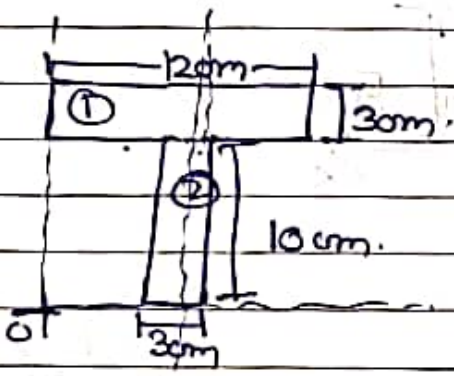
$$\bar{a}\bar{x} = a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n$$

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + \cdots + a_nx_n}{\bar{a}}$$

$$\boxed{\bar{x} = \frac{a_1x_1 + a_2x_2 + \cdots + a_nx_n}{a_1 + a_2 + a_3 + \cdots + a_n}}$$

$$\boxed{\bar{y} = \frac{a_1y_1 + a_2y_2 + \cdots + a_ny_n}{a_1 + a_2 + a_3 + \cdots + a_n}}$$

Ques →



Soln. G($\bar{x}$, $\bar{y}$)

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2}$$

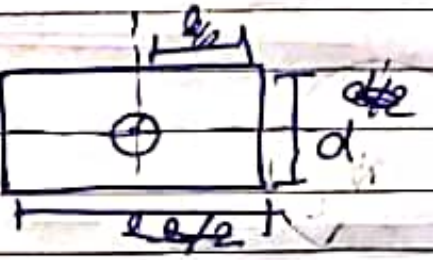
Let us consit subsection ①.

$$\boxed{\quad} \quad 3 = 36\text{cm}^2 \; a_1$$

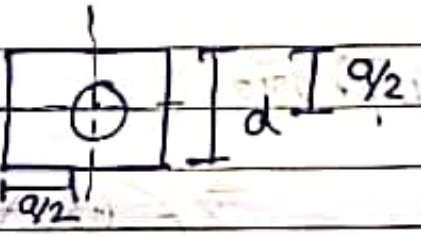
$y_1 = 11.5$

Center of Gravity of Simple Geometries

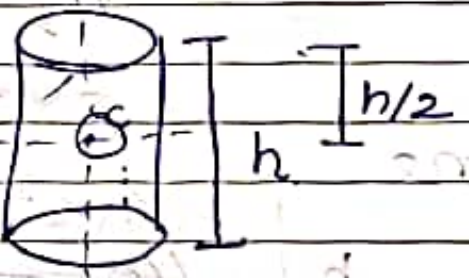
→ Rectangle =



→ Square =

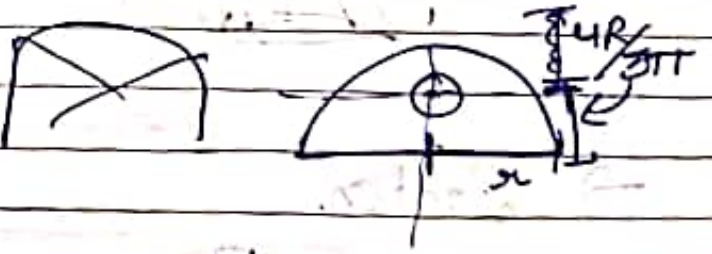


→ Cylinder =

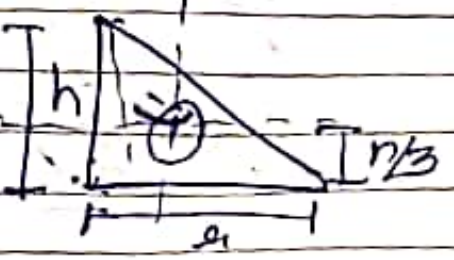


→ Circle =

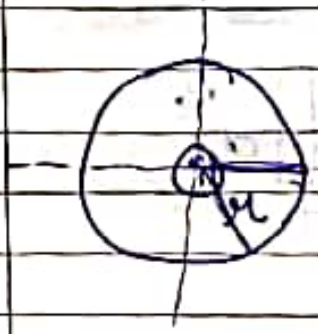
→ Semicircle =



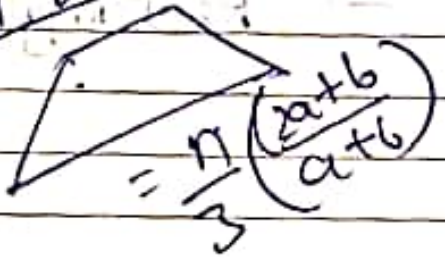
→ Right Angled Triangle



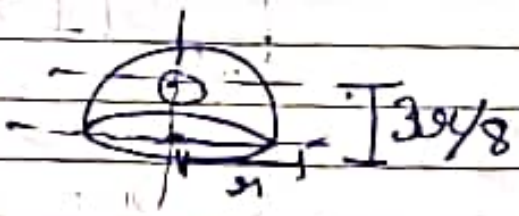
⇒ Sphere



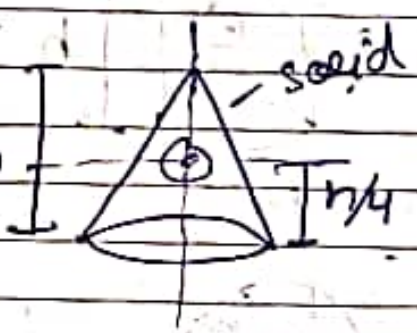
Trapezium



→ Hemisphere



→ Cone



Hollow = h/3

→ pyramid

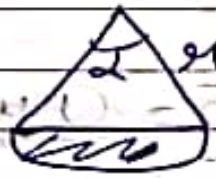
→ Sector.

Eg $CG = \frac{2 r \sin\alpha}{3\theta}$

→ Segment

CG =

→ Segment



$CG = \frac{r \sin\alpha}{\alpha}$

Remaining Ques

Let us consider subsection ②.

$A_2 = 10 \times 3$

$= 30\ cm^2$

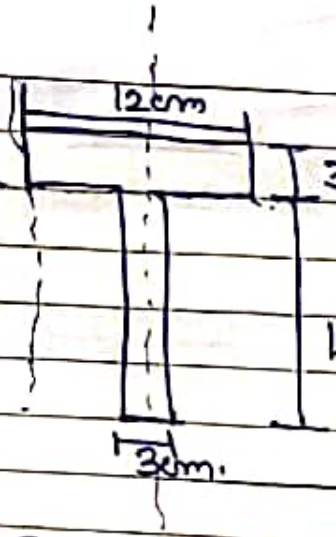
$y_2 = 5\ cm.$

$$\bar{y} = \frac{36 \times 11.5 + 30 \times 5}{11.5 + 5}$$

$$= \frac{414 + 150}{16.5} = \frac{564}{16.5}$$

$$\boxed{\bar{y} = 34.18}$$

Ques →



G $(\bar{x}, \bar{y})$ =

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

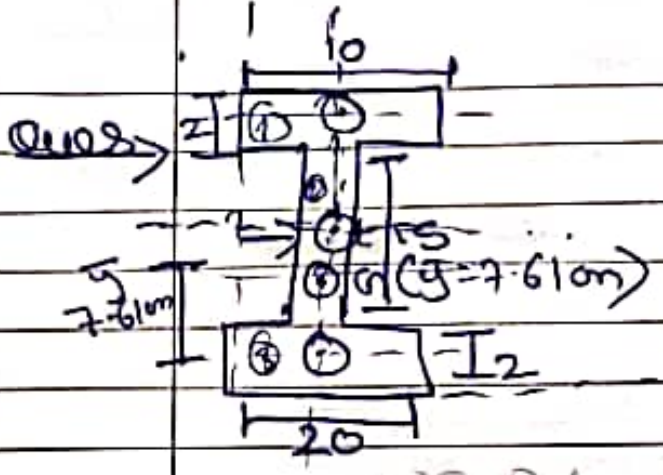
$a_1 = 12 \times 3 = 36\ cm^2$ $\quad a_2 = 10 \times 3 = 30\ cm^2$

$y_1 = 11.5\ cm$ $\quad y_2 = 5\ cm$

$$\bar{y} = \frac{36 \times 11.5 + 30 \times 5}{36 + 30}$$

$$= \frac{414 + 150}{36 + 30} = \frac{564}{96}$$

$$\bar{y} = 5.54$$



$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$a_1 = 10 \times 2$
$= 20\,cm^2$

$a_2 = 15 \times 2$
$= 30\,cm^2$

$a_3 = 20 \times 2$
$= 40\,cm^2$

$y_1 = 18\,cm$

$y_2 = 9.5\,cm$

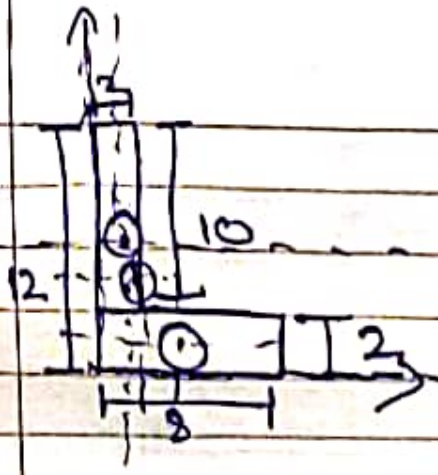
$y_3 = 1\,cm$

$$\bar{y} = \frac{20 \times 18 + 30 \times 9.5 + 40 \times 1}{20 + 30 + 40}$$

$$= \frac{360 + 285 + 40}{90}$$

$$\boxed{\bar{y} = 7.61\,cm}$$

Ques



$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2}$$

$a_1 = 1cm$ $\quad y_1 = 7cm$

$x_2 = 4cm$ $\quad y_2 = 1cm.$

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{y_1 + y_2}$$

$a_1 = 10 \times 2 = 20cm^2$

$a_2 = 8 \times 2 = 16cm^2$

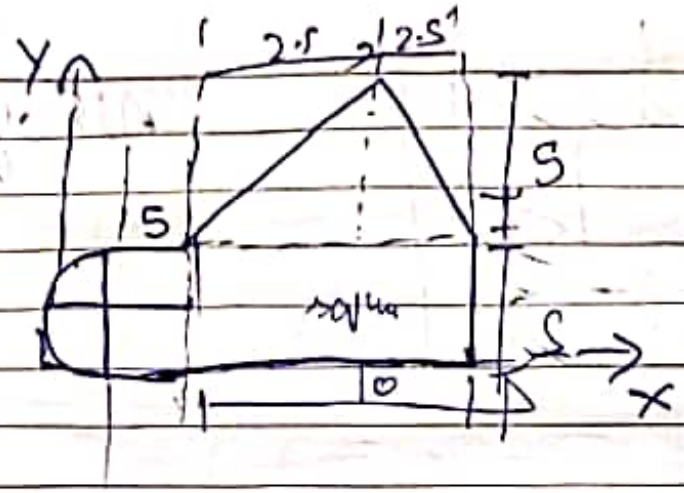
$$\bar{x} = \frac{(20 \times 1) + (16 \times 4)}{20 + 16}, \quad \bar{y} =$$

$$\bar{y} = \frac{(20 \times 7) + (16 \times 1)}{20 + 16}$$

$$= \frac{20 + 64}{36}, \quad \frac{140 + 16}{36}$$

$\boxed{\bar{x} = 2.33cm}$ $\quad \boxed{\bar{y} = 4.33cm}$

Ques →



$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$x_1 = \frac{4r}{3\pi} = \frac{4 \times 2.5}{3 \times \pi} = \frac{10}{9.42} \quad x_1 = 1.44\text{cm}$$

$x_2 = 5\text{cm}$ $x_3 = 10\text{cm}$.

$$a_1 = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \pi \times (2.5)^2 = 9.8\text{cm}^2$$

$a_2 = 10 \times 5 = 50\text{cm}^2$

$a_3 = \frac{1}{2} \times 5 \times 5 = 12.5$.

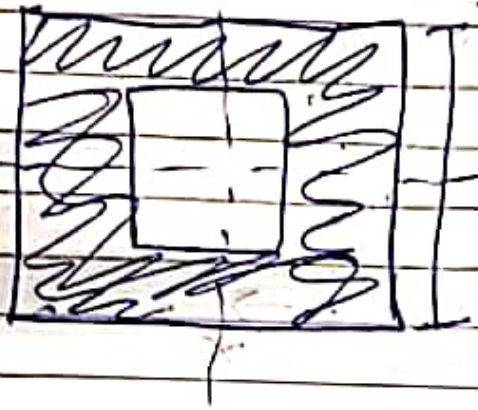
$y_1 = 2.5$, $y_2 = 2.5\text{cm}$, $y_3 = 6.6\text{cm}$

$$\bar{x}, \bar{y} = \frac{9.8 \times 1.44 + 50 \times 7.5 + 12.5 \times 10}{9.8 + 50 + 12.5}$$

$$= 41.$$

$\bar{x} = 7.11\text{cm}$, $\bar{y} = 3.2\text{cm}$.

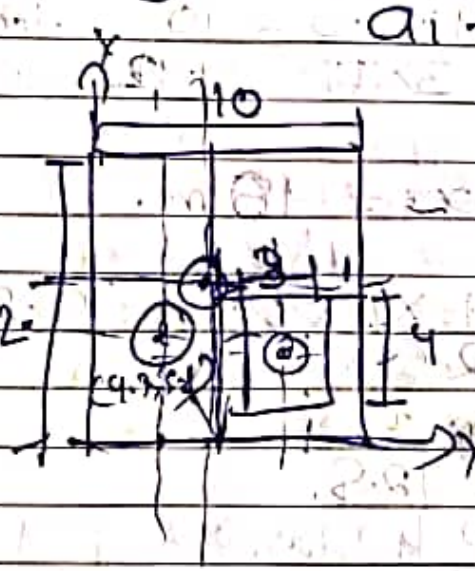
Ques →



All dimensions in cm.

$$\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2}$$

$$\bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2}$$



$a_1 = 120\ cm^2$
$x_1 = 5cm$
$y_1 = 6\ cm$

$a_2 = 12cm^2$
$x_2 = 2.5$
$y_2 = 4cm$

$$\frac{120 \times 5 + 12 \times 2.5}{120 + 12}, \frac{120 \times 6 + 12 \times 4}{120 + 12}$$

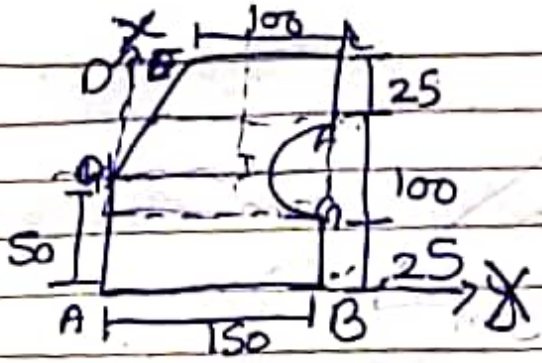
$$\frac{600 + 30}{132}, \frac{720 + 48}{132}$$

$\bar{x} = 5cm, \bar{y} = 6cm$

$$\frac{570}{108}$$

$5.2, 7.7, 6.2$

Ques



$\bar{x} = 70.9$ mm
$\bar{y} = 68.5$ mm

$G(\bar{x}, \bar{y})$

$\bar{x} = \frac{a_1x_1 - a_2x_2 - a_3x_3}{a_1 - a_2 - a_3}$

$\bar{y} = \frac{a_1y_1 - a_2y_2 - a_3y_3}{a_1 - a_2 - a_3}$

Soln

Subsection ①

$a_1 = 150 \times 150$
$= 22,500 \text{ mm}^2$
$x_1 = 75$ mm.
$y_1 = 75$ mm.

Subsection ②.

$\frac{1}{2} \times 50 \times 50$

$a_2 = 2500 \text{ mm}^2$
$x_2 = 16.6$ mm
$y_2 = 33.3$ mm. $+150 = 183.3$ mm.

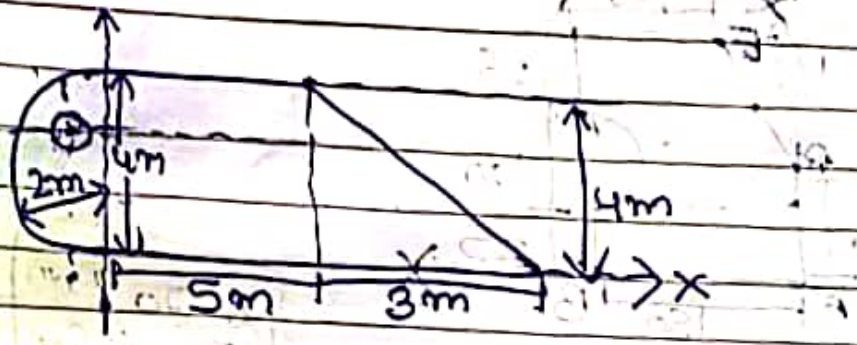
Subsection ③.

$\frac{\pi R^2}{2} = \frac{\pi \times 2500}{2}$

$\pi \times 1250 = 3926.99 \text{ mm}^2$

$a_3 = 3926.99 \text{ mm}^2$
$x_3 = \frac{4 \times 50}{3\pi} = 21.23 + 150 = 171.23$ mm
$y_3 = \frac{4 \times 25}{3\pi} = 21.23 + 25 = 46.23$ mm.

Ques →



Subsection (1) Rectangular semicircle.

$$a_1 = \frac{1}{2}\pi R^2 = \frac{1}{2}\pi 4^2$$

$$= 6.3m^2$$

$$\frac{4R}{3\pi} = \frac{4\times 2}{3\times\pi} = \frac{8}{9.42} = 0.85m$$

$$x_1 = -2m + 0.85 = -2.85m$$

$$y_1 =$$

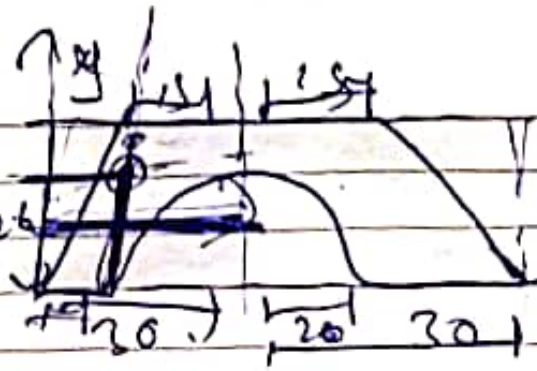
$$\frac{6.3(C-0.85) + 20\times 2.5 * 6\times 6}{6.3+20+6}$$

$$= \frac{5.355 * 50 + 36}{32.3} = \frac{40.64}{32.3}$$

$$= 2.5m$$

$$\bar{y} = 1.67$$

Ques →



$= \frac{2\times6}{3}\left(\frac{2(30)+60}{30+60}\right)$

$= \frac{2\times6}{3}\left(\frac{60+60}{90}\right)$

$= \frac{2\times6}{3}\times\frac{120}{90}$

$= 1.73\ m.$

$= 11.5\ cm$

$a_1 = \frac{1}{2}\times(30+60)\times 2.6$

$= \frac{1}{2}\times 90\times 2.6 = 117\ cm^2$

$\bar{x}_1 = 11.5$

$a_2 = \frac{1}{2}\pi r^2$

$= \frac{1}{2}\times\pi\times 25\times 25$

$= \frac{1}{2}\times\pi\times 625$

$= \frac{1}{2}\times 1963.5$

$= 981.75\ cm$

$\frac{4R}{3\pi} = \frac{4\times 25}{3\times\pi} = \frac{100}{9.4} = 10.6\ cm$

$\bar{x} = \frac{11.5\times 117 - 20\times 981}{117+981}$

$= \frac{20355 - 20220}{807}$

$= \frac{11,188.5}{807}$

$= 13.8$

20-6.

Ques)

$a_1 = 1 \times 1 = 1\,m^2$

$y_1 = 0.5\,m.$

$a_2 = a_3 = \frac{1}{2} \times 0.5 \times 1 = 0.25\,h\,m.$

$y_2 = y_3 = \frac{h}{3}$

$$\bar{y} = \frac{1 \times 0.5 - 0.25\,h/3 - 0.25\,h/3}{1 - 0.25 - 0.25}$$

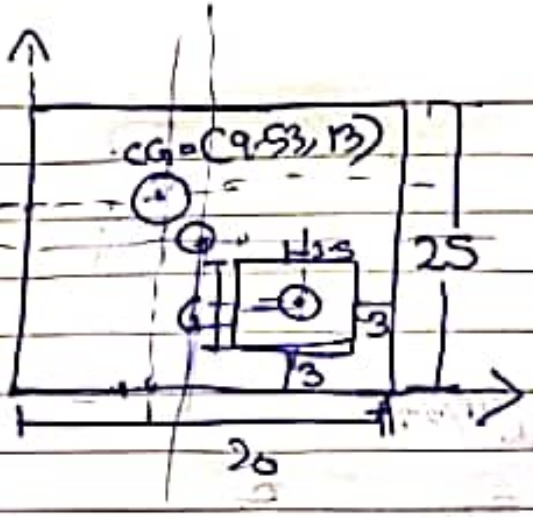
$$= \frac{0.5 - 0.5\,h/3}{1 - 0.5}$$

$$= \frac{0.5(1 - h/3)}{0.5}$$

$$\boxed{\bar{y} = 1 - \frac{h}{3}}$$

$$\boxed{\bar{y} = 0.67\,cm}$$

Ques →



All dimensions are in cm.

In subsection (1).

$$a_1 = 25 \times 20$$
$$= 500\ cm^2$$

$$x_1 = 10\ cm$$
$$y_1 = 12.5\ cm.$$

In subsection (2)

$$a_2 = 6 \times 6$$
$$= 36\ cm^2$$

$$x_2 = 16\ cm$$
$$y_2 = 6\ cm$$

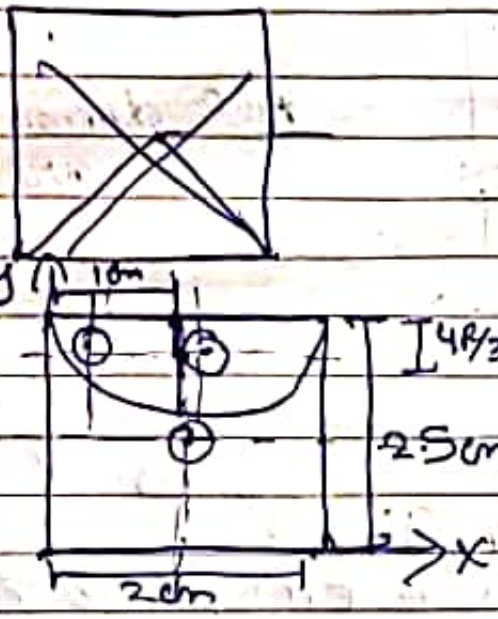
$$\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2}, \quad \bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2}$$

$$= \frac{(500 \times 10) - (36 \times 16)}{500 - 36}, \quad \frac{(500 \times 12.5) - (36 \times 6)}{500 - 36}$$

$$= \frac{5000 - 576}{464}, \quad \frac{6250 - 216}{464}$$

$$= 9.53\ cm, 13\ cm.$$

Ques



Ques

In ~~Subsection~~ 2t is symmetrical from y

So $\bar{x} = 0$

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

In subsection ①

$$a_1 = 2.5 \times 2$$
$$= 5\ cm^2$$

$\bar{x}_1 = y_1 = 1.25cm.$

In Subsection ②

$$a_2 = \frac{1}{2}\pi r^2$$
$$= \frac{1}{2}\pi (1)^2$$
$$= 1.57 cm^2$$

$$y_2 = \frac{4R}{3\pi}$$

$$= 2.5 - \frac{4R}{3\pi}$$

$$= 2.5 - \frac{4(1)}{3\pi}$$

$$= 2.5 - 0.42$$

$$= 2.08 \text{ cm}.$$

$$\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}$$

$$= \frac{(5 \times 1.25) - (1.57 \times 2.08)}{5 - 1.57}$$

$$= \frac{6.25 - 3.27}{3.43}$$

$$= \frac{2.98}{3.43}$$

$$\boxed{\bar{y} = 0.87 \text{ cm}}$$

Ques →

" It is not symmetrical.

Soln $\bar{x} = \frac{a_1x_1 + a_2x_2 - a_3x_3}{a_1 + a_2 - a_3}$

$\bar{y} = \frac{a_1y_1 + a_2y_2 - a_3y_3}{a_1 + a_2 - a_3}$

In Subsection ①.

$a_1 = \frac{1}{2} \times 6 \times h$

$= \frac{1}{2} \times 8 \times 6$

$= 24\ cm^2$

$x_1 = 4cm$ 2/6.

$y_1 = 6cm.$

In Subsection ②

$a_2 = \frac{1}{2} \pi r^2$

$= \frac{1}{2} \pi \times 8$

$= 25.13\ cm^2$

$x_2 = 4\,cm$ $\quad \bar{x} = 0.145$

$y_2 = 8 - \frac{4R}{3\pi}$ $\quad \bar{y} =$

$= 8 - \frac{4(4)}{3\pi}$

$= 8 - \frac{16}{3\pi} \Rightarrow 8 - \frac{16}{9.42}$

$= 8 - 1.70$

$= 6.3\,cm$ ~~4cm~~

Subsection (3).

$a_3 = \pi r^2 = 4\pi = 12.57\,cm^2$

$x_3 = 4\,cm.$

$y_3 = 4 - \frac{4R}{3\pi} = \frac{4 - 4(2)}{3\pi} = \frac{.8}{3\pi} = 0.84\,cm.$

$\bar{x} = \frac{(24 \times 6) + (25.13 \times 4) - (12.57 \times 4)}{24 + 25.13 - 12.57}$

$= \frac{144 + 100.52 - 50.28}{36.56}$

$= \frac{196.52}{36.52} = 5.38\,cm.$ (3.06cm)

$\bar{y} = \frac{(24 \times 6) + (25.13 \times 4.3) - (12.57 \times 0.84)}{36.56}$

$= \frac{144 + 157.88 - 87.99}{36.56}$

$= \frac{213.89}{36.56} = 5.85\,cm$