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Unit - II

Wave Optics

→ **Principle of Superposition of waves →**

This principle state that the resultant displacement of a particle of the medium at / upon the two or more ~~as~~ waves. simultaneously, is the algebric sum of the displacement of the same particle due to the individual wave.

Suppose, we have y as displacement of 1st wave at any instance and y_2 is the principle of superposition $y = y_1 + y_2$. This is called the principle of superposition.

→ **Interference →**

The modification in the distribution of the intensity of the light in the region of superposition, when two or more wave of same intensity, same wave length & constant or zero phase difference are superimposed with each other.

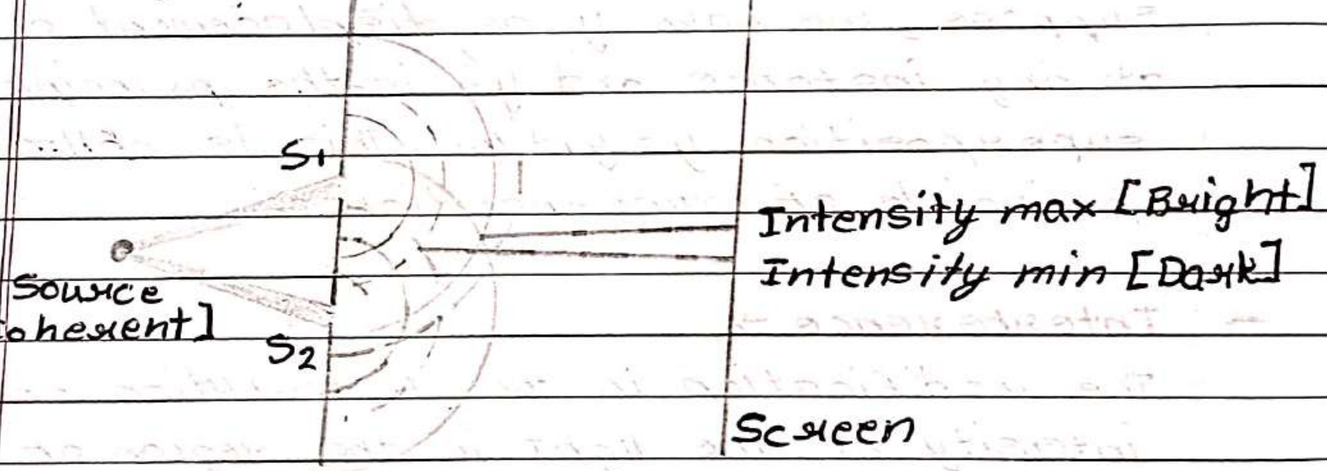
→ **sustained Interference Pattern →**

The interference pattern in which the maxima have the maximum intensity and the minima has minimum intensity is known as sustained interference pattern. To obtain this pattern following conditions must be satisfied →

- The sources must be coherent in nature.

- The two sources must be narrow.
- The two narrow sources should be very close to each other. So that, the dark & bright fringes appear separately.
- The two light source should be strong. So that we get a well defined interference pattern.
- The amplitude of two waves should be equal so that, we get good contrast b/w the dark & bright fringes.

Young's Double Slit Experiment →



Let us consider the superimposition of two sources of the same frequency. Then, the displacement at any time is represented by

$y_1 = a\sin(\omega t)$ & ~~b=~~ $y_2 = b\sin(\omega t+\phi)$

where, a & b are the amplitude ~~b/w~~ & ϕ is the phase difference b/w the two sources.

Acc. to the principle of superposition, $y = y_1 + y_2$

$$y = a\sin(\omega t) + b\sin(\omega t+\phi)$$

$$y = a\sin(\omega t) + b\sin(\omega t)\cos\phi + b\cos(\omega t)\sin\phi$$

$$y = \sin(\omega t)[a + b\cos\phi] + b\cos(\omega t)\sin\phi \quad \text{-1st}$$

Let $a + b\cos\phi = R\cos\theta \rightarrow$ 2nd

$b\sin\phi = R\sin\theta \rightarrow$ 3rd

Putting 2nd & 3rd in 1st

$y = \sin(\omega t) R\cos\theta + R\cos(\omega t)\sin\theta$

$y = R\sin(\omega t + \theta)$

Hence, the resultant wave is harmonic wave of amplitude R.

Now, squaring & adding 2 & 3

$(a + b\cos\phi)^2 + b^2\sin^2\phi = R^2\cos^2\theta + R^2\sin^2\theta$

$a^2 + b^2\cos^2\phi + 2ab\cos\phi + b^2\sin^2\phi = R^2$

$a^2 + b^2 + 2ab\cos\phi = R^2$

$R^2 = a^2 + b^2 + 2ab\cos\phi$

$R = \sqrt{a^2 + b^2 + 2ab\cos\phi}$

Since, the intensity is proportional to the square of the amplitude. Then,

$I = KR^2$

$I = K(a^2 + b^2 + 2ab\cos\phi)$

For constructive, Intensity is maximum,

$\cos\phi = \text{maximum} = 1 = \cos 2n\pi$ where $n = 0, 1, 2, 3, \ldots$

$I = K(a+b)^2$

~~For destructive interference, Intensity is minimum~~

[Δ] Path diff. $= \frac{\lambda}{2\pi} \times$ Phase diff.

Phase diff. $= \phi = 2n\pi$

$\Delta = \frac{\lambda}{\cancel{2\pi}} \times \cancel{2}n\cancel{\pi}$

$\Delta = n\lambda$

For destructive interference, Intensity minimum

$\cos\phi = \text{minimum} = -1$

$\cos\phi = \cos(2n+1)\pi$ where $n = 0, 1, 2, \ldots$

$\phi = (2n+1)\pi$

$I = K(a-b)^2$

$\Delta = \frac{\lambda}{2\cancel{\pi}} \times (2n+1)\cancel{\pi}$

$$\Delta = (2n+1)\frac{\lambda}{2}$$

Fringe width →

consider the two coherent sources S_1 & S_2 separated by the distance [d], sends the light on the screen [MN] placed at the distance of [D] from the sources. The waves reaches at point [P] on the screen after travelling the distance S_1P & S_2P. Therefore, the path diff. b/w the waves reaching at P from S_1 & S_2 is

$$\Delta = S_2P - S_1P \rightarrow A$$

Let x be the distance of the point P from center of the screen O.

From ΔS_1AP,

$$(S_1P)^2 = (S_1A)^2 + (PA)^2$$

$$(S_1P)^2 = D^2 + (x - d/2)^2$$

$$(S_1P)^2 = D^2 + x^2 + \frac{d^2}{4} - xd \rightarrow 1st$$

From ΔS_2BP,

$$(S_2P)^2 = (S_2B)^2 + (BP)^2$$

$$(S_2P)^2 = (D)^2 + (x + d/2)^2$$

$$(S_2P)^2 = D^2 + x^2 + \frac{d^2}{4} + xd \rightarrow 2nd$$

Subtract eqn 2 from 3 →

$$(S_2P)^2 - (S_1P)^2 = 2xd$$

$$(S_2P - S_1P)(S_2P + S_1P) = 2xd$$

Suppose, P is very close to O,

So,

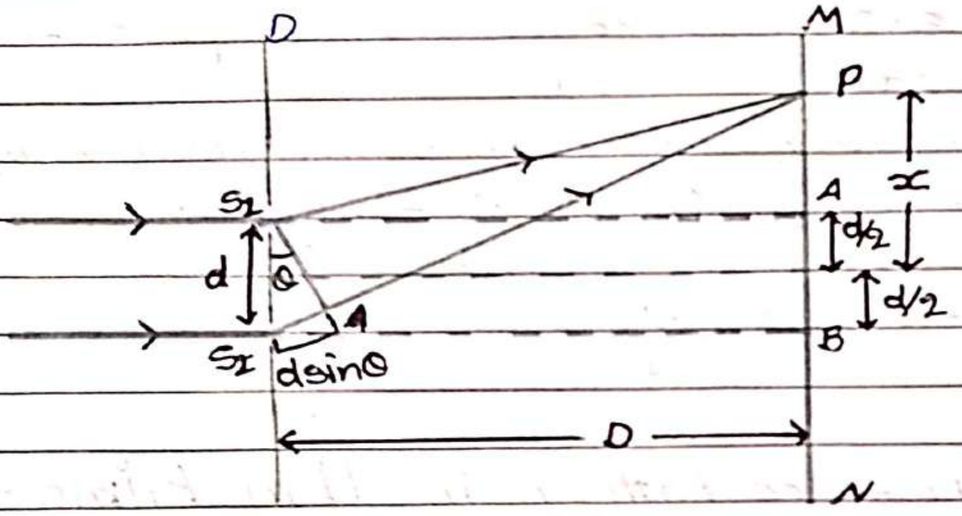
$$S_2P \approx S_1P \approx D$$

$$S_2P - S_1P = \frac{2xd}{2D}$$

$$S_2P - S_1P = \frac{xd}{D}$$

From Eq^n A,

$$\Delta = xd$$



• For position of Maxima,

$$\Delta = n\lambda$$

$$\frac{xd}{D} = n\lambda$$

$$x_n = \frac{n\lambda D}{d} \quad [\text{depends on only 'n'}]$$

$$x_1 = \frac{\lambda D}{d}, \quad x_2 = \frac{2\lambda D}{d}, \ldots, \quad x_{n-1} = \frac{(n-1)\lambda D}{d}$$

The distance b/w the two continuous bright fringes,

$$x_n - x_{n-1} = \frac{n\lambda D}{d} - \frac{(n-1)\lambda D}{d}$$

$$x_n - x_{n-1} = \frac{\cancel{n\lambda D} - \cancel{n\lambda D} + \lambda D}{d}$$

$$x_n - x_{n-1} = \frac{\lambda D}{d} = [\text{Fringe width of minima}]$$

• For position of Minima,

$$\Delta = \frac{(2n+1)\lambda}{2}$$

$$\frac{xd}{D} = \frac{(2n+1)\lambda}{2}$$

$$x_n = \frac{(2n+1)\lambda D}{2d}$$

$$x_1 = \frac{\lambda D}{2d}, x_2 = \frac{3\lambda D}{2d}, \ldots, x_{n-1} = \frac{(2n-1)\lambda D}{2d}$$

The distance b/w two consecutive dark fringes.

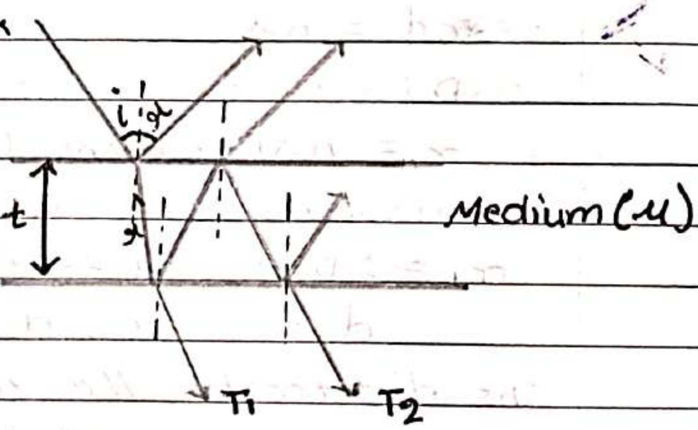
$$x_n - x_{n-1} = \frac{2n\lambda D + \lambda D - 2n\lambda D + \lambda D}{2d}$$

$$x_n - x_{n-1} = \frac{\lambda D}{d}$$

Interference Pattern in Thin Films →

- Reflected System →
  - $\Delta = 2\mu t \cos(r)$ • μ → refractive index • t → thickness

~~Transmitted System →~~
~~• $\Delta_{max} = (2n+1)\lambda/2$~~
~~• $\Delta_{min} =$~~



Maxima →
$2\mu t \cos(r) = (2n+1)\lambda/2$

Minima →
$2\mu t \cos(r) = n\lambda$

- Transmitted System →

  Minima →
  $2\mu t \cos(r) = (2n+1)\lambda/2$

  Maxima →
  $2\mu t \cos(r) = n\lambda$

Newton Rings →

When a planoconvex lens is placed over a glass plate the Newton Rings are formed. The

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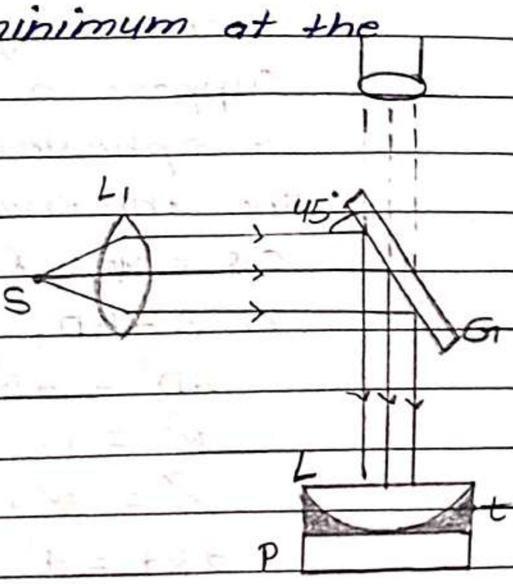
thickness of the air film is minimum at the point of contact and increases gradually from center to outwards. If this thin film is illuminated by a monochromatic light, circular interference fringes are observed by the reflected & transmitted light. The fringes are the concentric circles with center at the point of contact are known as Newton Rings.

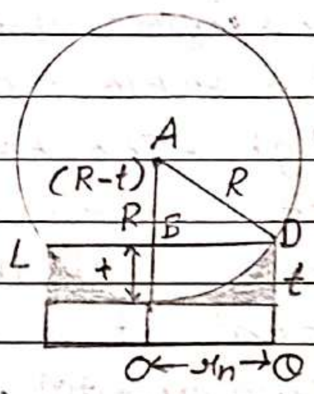


Experimantal Arrangement →

It is shown in the above fig.. In which a planoconvex lens [L] of large radius of curvature is placed on a glass plate [P]. A monochromatic light from the source [S] is made parallel by passing through lens [L1] is allowed to incident on a glass plate [G] inclined at 45° to the incident rays. After reflection from the glass plate [G] the light rays fall normally of the planoconvex lens [L]. A part of light is reflected from curved surface of planoconvex lens and transmitted light is reflected from the glass plate [P]. These two reflected rays interfere & gives the interference pattern in the form of alternate bright & dark circular rings.

Newton Rings by the Reflected System →
Suppose a planoconvex lens [L] is a part of a sphere of radius of curvature [R] and the thickness of the film enclosed at a distance $OQ = r_n$ from the point of contact O is [t].
In ΔABD,

$$AD^2 = AB^2 + BD^2$$
$$R^2 = (R-t)^2 + r_n^2$$
$$\cancel{R^2} = \cancel{R^2} + t^2 - 2Rt + r_n^2$$
$$2Rt = t^2 + r_n^2$$



Since, the thickness of the air film is very small $t^2 \approx 0$. Then,

$$2Rt = r_n^2$$
$$2t = \frac{r_n^2}{R}$$

[Wedge shape air film]

$$D_n = 2r_n$$

1st → $2t = \frac{D_n^2}{4R}$ [where D_n is the diameter of ~~nth~~ n^{th} dark ring]

For the n^{th} dark ring →

$$2\mu t\cos(r) = n\lambda$$

For normal incidence →

$$\cos(r) = \cos 0^\circ = 1$$
$$2\mu t = n\lambda$$

From 1st

$$2t = \frac{D_n^2}{4R} \quad \& \quad 2t = \frac{n\lambda}{\mu}$$

$$\frac{D_n^2}{4R} = \frac{n\lambda}{\mu}$$

$$D_n^2 = \frac{4n\lambda R}{\mu}$$

$$D_n = 2\sqrt{\frac{n\lambda R}{\mu}}$$ [This is the diameter of n^{th} dark ring]

For air film, $\mu = 1$

$$D_n = 2\sqrt{n\lambda R}$$

For Bright Ring →

$$2\mu t\cos(r) = (2n+1)\lambda/2$$

For normal incidence, $\cos(r) = 1$

$$2\mu t = (2n+1)\lambda/2$$

$$2t = \frac{(2n+1)\lambda}{2\mu}$$

From 1st →

$$\frac{D_n^2}{2\cancel{4}R} = \frac{(2n+1)\lambda}{\cancel{2}\mu}$$

$$D_n^2 = \frac{2(2n+1)\lambda R}{\mu}$$

$$D_n = \sqrt{\frac{2R(2n+1)\lambda}{\mu}}$$ [This is dia. of n^{th} bright ring]

For air film →

$$D_n = \sqrt{2R(2n+1)\lambda}$$

For $n=0$, $D_0 = 0$, therefore, in case of the reflected system the central ring is dark and is followed by the alternative bright & the dark rings.

$$D_1 = \sqrt{\frac{4\lambda R}{\mu}}$$

$$D_2 = \sqrt{\frac{8\lambda R}{\mu}}$$

$$D_3 = \sqrt{\frac{12\lambda R}{\mu}}$$

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$$D_2 - D_1 = \sqrt{\frac{4\lambda R}{\mu}}(\sqrt{2}-1)$$

$$D_2 - D_1 = 0.414\sqrt{\frac{4\lambda R}{\mu}}$$

$$D_3 - D_2 = \sqrt{\frac{12\lambda R}{\mu}} - \sqrt{\frac{8\lambda R}{\mu}}$$

$$D_3 - D_2 = \sqrt{\frac{4\lambda R}{\mu}}(\sqrt{3}-\sqrt{2})$$

$$D_3 - D_2 = 0.318\sqrt{\frac{4\lambda R}{\mu}}$$

From the above equations

$$D_n \propto n$$

Newton Rings in the Transmitted System →

For eq^n 1st →

$$2t = \frac{D_n^2}{4R}$$

For Bright fringe OR ring →

$$2\mu t\cos(r) = n\lambda$$

$$D_n = \sqrt{\frac{4n\lambda R}{\mu}}$$

For Dark fringe OR ring →

$$2\mu t\cos(r) = (2n+1)\lambda/2$$

$$D_n = \sqrt{\frac{2R(2n+1)\lambda}{\mu}}$$

Bright ring is the center.

From above equations, it is clear that in case of reflected system, the central rings is dark whereas, in case of transmitted system, the central ring is bright. Hence, the two systems are

complementary to each other.
Diameter also depends upon wavelength [λ].

Why are Newton Rings circular in nature?
Newton rings are circular in nature due to the symmetrical nature of the thin film on both sides of the point of contact.

What is the nature of the Newton rings when white light is used?
In case of the white light, the diameter of the rings for the different colours will be different only first few coloured rings will be clear and then overlapping of the different colour ring starts and cannot be viewed.

Applications of Newton Rings →

- Determination of the wavelength of the light used →

To determine the wavelength of the light, newton rings are formed using a monochromatic source of light whose wavelength is to be determined. Measure the diameter of the n^{th} & $(n+m)^{th}$ dark ring with the help of travelling microscope.

$$D_n = \sqrt{\frac{4n\lambda R}{\mu}} \rightarrow D_n^2 = \frac{4n\lambda R}{\mu}$$

$$(D_{n+m})^2 = \frac{4(m+n)\lambda R}{\mu}$$

$$(D_{n+m})^2 - D_n^2 = \frac{4m\lambda R + \cancel{4n\lambda R} - \cancel{4n\lambda R}}{\mu}$$

$$(D_{n+m} - D_n)(D_{n+m} + D_n)^2 = \frac{4m\lambda R}{\mu}$$

$$\lambda = \frac{[(D_{n+m})^2 - D_n^2]\mu}{4mR}$$

• Determination of the Refractive Index of Liquid →

Diameter of n^{th} dark ring → $D_n^2 = \frac{4\lambda R n}{\mu_{liq.}}$

when immersed in Liquid →

$$D_{(n+m)}^2 = \frac{4\lambda R(n+m)}{\mu_{liq}}$$

Now,

$$D_{(m+n)}^2 - D_n^2 = \frac{4\lambda R(m+n)}{\mu_{liq}} - \frac{4\lambda R n}{\mu_{liq}}$$

$$D_{(m+n)}^2 - D_n^2 = \frac{4\lambda R[(m+n) - n]}{\mu_{liq}}$$

To find the refractive index of the liquid, the planoconvex lens & glass plate is immersed in a liquid whose refractive index has to be measured. The newton rings are formed using monochromatic sources of light of known wavelength. Measure the diameter of n^{th} & $(m+n)^{th}$ ring.

$$D_{(m+n)}^2 - D_n^2 = \frac{4\lambda R(m)}{\mu_{liq}}$$

$$\mu_{liq} = \frac{4\lambda R m}{D_{(m+n)}^2 - D_n^2}$$

If the wavelength is not known

$$(D_n^2)_{air} = \frac{4\lambda R n}{\mu_{air}}$$

$$(D_n^2)_{liq} = \frac{4\lambda R n}{\mu_{liq.}}$$

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$$(D_n)^2_{air} = \frac{4\lambda R n}{\mu_{air}}$$

$$(D_n)^2_{liq} = \frac{4\lambda R n}{\mu_{liq}}$$

$$\frac{(D_n)^2_{air}}{(D_n)^2_{liq}} = \mu_{liq} \quad [\mu_{air} = 1]$$

Differaction →

1st ← size of slit (a) = 1.2 λ [wavelength of light = 10^{-6} m]

It is the phenomenon of bending of light around the corners of an obstacle or aperture in the path of light and spreading into the geometrical shadow of an obstacle. This phenomenon most prominating when the size of obstacle is comparable to the wavelength of light.

1st is also known as essential condition for differaction.

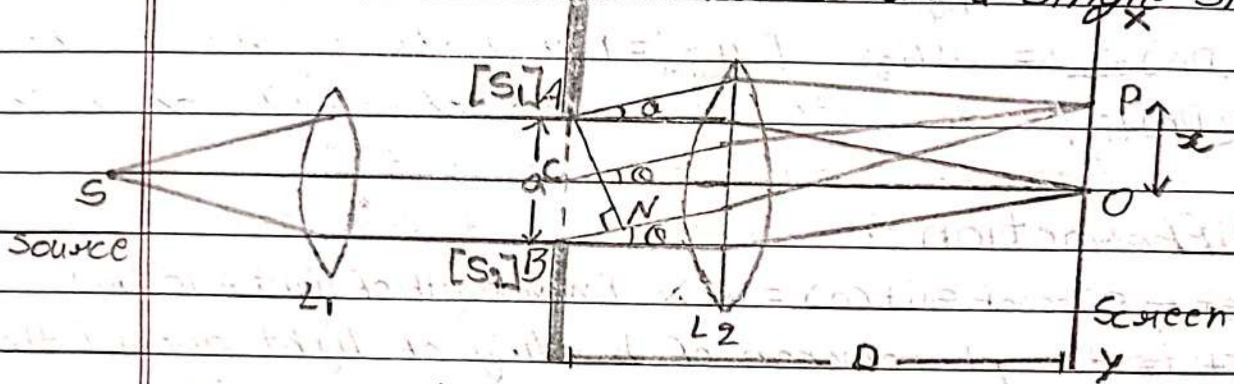
Types of Differaction →

- Fresnel's differaction
- Fraunhoffer's differaction

Fresnel differaction	Fraunhoffer's differaction
• The distance of either sources OR the screen or both is finite from the differaction aperture or obstcale.	• The distance of the source and the screen is infinite from the aperture.
• No lens is used.	• convex lens is used.
• The incident wavefront is either cylindrical or spherical.	• The incident wavefront is plane.
• It is studied with approximation.	• Study with approximations.

• The distance plays an important role.	• The angular inclination plays an important role.

Fraunhoffer Differaction From a Single slit →



The experimental setup is shown in the figure. [S] is the monochromatic source of light held at the focus of the collimating lens [L_1]. The lens [L_1] gives plane ~~at~~ wavefront ~~obtained~~ ~~on the screen placed at a distance~~ falling on the slit [AB] of width [a]. The differaction pattern is obtained on the screen placed at a distance [D] from the slit. The differaction pattern consists of central maxima and alternationg dark and weak bright bands of decreasing intensity on either side of central maxima.

Consider the secondary wavefront travelling in a direction making an angle [θ] with CO meets at P. The point P will the point of maxima OR minima will depend upon the path difference.

$\sin\theta = \frac{BN}{AB}$ [BN is path diff.]

$BN = AB\sin\theta$

$BN = a\sin\theta$

If $a\sin\theta = n\lambda$ [condition for minima]

$a\sin\theta = (2n+1)\frac{\lambda}{2}$ [condition for maxima]

If path diff. is λ then P will be the point of minima the incident wavefront can be consider to be divided into two equal halves. Then, the path diff. b/w the secondary waves from A and C reaching at point P is $\lambda/2$, and the secondary waves from B & C reaching at the point P has again the path diff. of $\lambda/2$. Hence, the destructive interference takes place at point P and P is the position of 1st secondary minima and so on. In general for n^{th} minima the path diff.

$a\sin\theta = n\lambda$ [where $n = 1, 2, 3, \ldots$]

and for n^{th} maxima, the path diff.

$a\sin\theta = (2n+1)\frac{\lambda}{2}$ [where $n = 0, 1, 2, 3$]

Width of the Principle Maxima →

The distance b/w first secondary minima on either side of [O] gives the width of the central maxima. If point P is the position of 1st secondary minima then,

$$a\sin\theta = \lambda$$

$$\sin\theta = \frac{\lambda}{a}$$

If θ is very small, then $\sin\theta \approx \theta$

$$\theta = \frac{\lambda}{a} \rightarrow 1st$$

$$\theta = \frac{PO}{D} = \frac{x}{D} \rightarrow 2nd$$

From 1st & 2nd, $\frac{x}{D} = \frac{\lambda}{a}$

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$x = \frac{\lambda D}{a}$

The width of principle maxima $= 2x$

$\beta = 2x = \frac{2\lambda D}{a}$

Position of maxima and minima →

Intensity distribution in differaction →

In Fraunhoffer differaction at a single slit the plane wavefront is incident on the slit [S_1S_2] can be divided into large no. of small ~~slit~~ strips. The path diff. b/w the secondary waves emitting from extreme points S_1S_2 is

$\Delta = a\sin\theta$

where, a is the width of the slit.

Let 2α be the phase diff. b/w the secondary waves from S_1S_2. The resultant amplitude due to all the strips can be obtained by polygon vector method. The amplitude of all the strips are same and small. so, the polygon converts into the arc S_1OS_2. The resultant amplitude is given by the chord [S_1S_2].

The path diff., $\Delta = a\sin\theta$

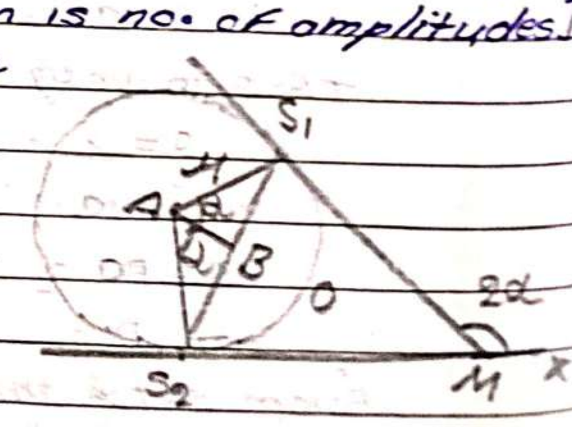
$S_1S_2 = R$

$\widehat{S_1OS_2} = nE$ [E is amplitude of each amplitude & n is no. of amplitudes]

The tangents from S_1S_2 meet at M.

So, $\angle S_1MX = 2\alpha$

Now, complete the circle with center A and radius [r].



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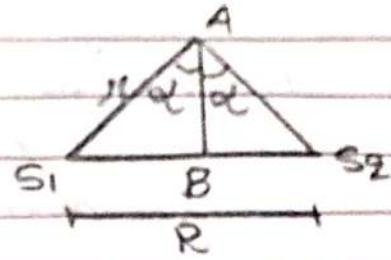
Now, draw the $\perp$ AB

From the right $\angle$ triangle.



S_1BA, $[S_1B = S_2B]$

$\sin\alpha = \frac{S_1B}{S_1A}$

$S_1B = S_1A\sin\alpha$

$S_1B = r\sin\alpha$

$S_1S_2 = 2S_1B$

$S_1S_2 = 2r\sin\alpha$

$2\alpha = \frac{\widehat{S_1OS_2}}{r}$

$2r = \frac{\widehat{S_1OS_2}}{\alpha} = \frac{nE}{\alpha}$

$S_1S_2 = \frac{nE\sin\alpha}{\alpha}$ $[S_1S_2 = R]$

$R = \frac{nE\sin\alpha}{\alpha}$

$I \propto R^2$

$I = kR^2$

$I = k\frac{n^2E^2\sin^2\alpha}{\alpha^2}$ [If $k=1$]

$I = \frac{n^2E^2\sin^2\alpha}{\alpha^2}$ [Now, $n^2E^2 = I_0$]

$I = I_0\frac{\sin^2\alpha}{\alpha^2}$

If the phase diff. is 2π and path diff. $= \lambda$

If the phase diff. is 1 then path diff. $= \lambda/2\pi$

If phase diff. is 2α then path diff. $= \lambda 2\alpha/2\pi$

$a\sin\theta = \frac{\lambda \times 2\alpha}{2\pi}$

$\alpha = \frac{a\pi\sin\theta}{\lambda}$

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• For the principle maxima, I should be maximum

$$\alpha = 0$$

$$a\pi \sin\theta = 0$$

$$\sin\theta = 0$$

$$\theta = 0^\circ \text{ [angle of differaction]}$$

Hence, for $\theta = 0^\circ$, the resultant intensity will be maximum known as Principle maxima OR central maxima

• For the minima, I should be minimum

$$\sin\alpha = 0$$

$$\alpha \neq 0^\circ$$

$$\alpha = n\pi \quad [n = 1, 2, 3, \dots]$$

$$\frac{a\cancel{\pi}\sin\theta}{\lambda} = \pm n\cancel{\pi}$$

$$a\sin\theta = \pm n\lambda$$

n=1,2, gives the position of 1st, 2nd, 3rd minima & so on.

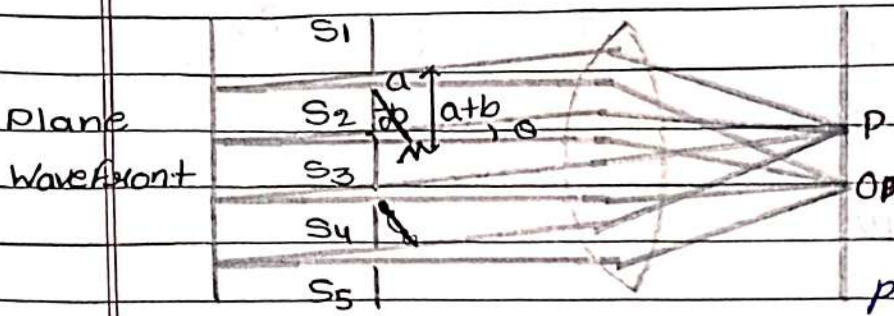
Plaine Transmission Grating →

An arrangement of very large no. of narrow slits of equal width placed side by side and separated by equal opaque portions is known as Differaction Grating.

When the wavefront ~~is~~ is incident on the grating surface the light is obstructed by opaque portion and is transmitted by the slits. Such a grating is called Plane Transmission Grating.

It is constructed by ruling equidistant parallel lines by a diamond point on the glass surface. The order of the lines on the grating is 15,000 lines per inch OR

16,000 lines per cm.
The width of the line is called Transparency and it is denoted by [a], whereas, the gap b/w two lines acts as opacity and the width of opacity is denoted by [b]. The distance [a+b] is known as grating element.



In differaction grating, N slits of [a] and seperated by [b] are placed side by side. So, the differaction pattern will be the combined differaction effect of all such slits. The resultant intensity due to single slit.

$$I = \frac{I_0 \sin^2\alpha}{\alpha^2}, \quad R = \frac{R_0 \sin\alpha}{\alpha}$$

$$\& \quad \alpha = \frac{\pi a \sin\theta}{\lambda}$$

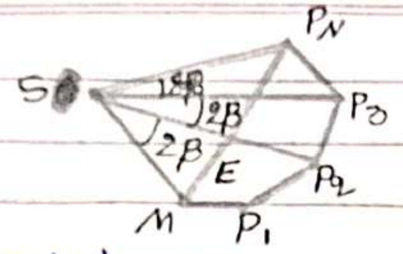
In case of differaction grating, there are N slits the secondary waves differacted in the direction other than incident light are focused on the screen at the point P.

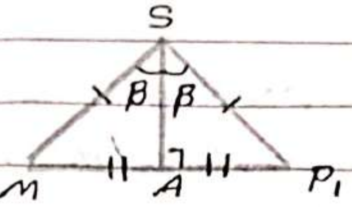
$$S_3N = (S_2S_3)\sin\theta$$
$$S_3N = (a+b)\sin\theta$$

As the path diff. is $(a+b)\sin\theta$. So, the corresponding phase diff. is 2β

$$2\beta = \frac{2\pi(a+b)\sin\theta}{\lambda}$$

Consider a polygon consists of sides $MP_1, P_1P_2, P_2P_3, \ldots, P_{N-1}P_N$. representing the N no. of equal amplitudes. The closing side MP_N gives the resultant amplitude.





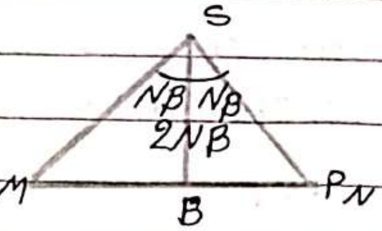
Total

$$\sin\beta = \frac{MA}{MS}$$

$$MA = MS\sin\beta$$

$$MP_1 = 2\,MA$$

$$MP_1 = 2MS\sin\beta$$



In $\Delta\, MP_NS$,

$$\sin N\beta = \frac{MB}{MS}$$

$$MB = MS\sin N\beta$$

$$MP_N = 2MB$$

$$MP_N = 2\,MS\sin N\beta$$

$$E = 2\,MS\sin N\beta \qquad [MP_N = E \text{ resultant}$$

$$E = \frac{\cancel{2}\,MP_1\sin N\beta}{\cancel{2}\sin\beta}$$

$$E = \frac{R\sin N\beta}{\sin\beta}$$

$$E = \frac{R_0\sin\alpha\sin N\beta}{\alpha\sin\beta}$$

$$I \propto E^2$$

$$I = \frac{R_0^2\sin^2\alpha\sin^2 N\beta}{\alpha^2\sin^2\beta}$$

$$I = I_0\frac{\sin^2\alpha\sin^2 N\beta}{\alpha^2\sin^2\beta} \quad [\text{Intensity due to } N \text{ slits}$$

The first factor $[I_0\sin^2\alpha/\alpha^2]$ represents the intensity ~~factor~~ pattern due to a single slit & the second factor $[\sin^2 N\beta/\sin^2\beta$ represent distribution of intensity in the diffraction pattern due to N slits.

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Principle maxima → For principle maxima,

$$\sin\beta = 0$$

$$\beta = \pm n\pi, \text{ where } n = 1, 2, 3, 4, \dots \text{ [0 also]}$$

$$\lim_{\beta \to \pm n\pi} \frac{\sin N\beta}{\sin\beta} = \pm N$$

The intensity of principle maxima,

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2} N^2$$

The direction of principle maxima is given by $\beta = \pm n\pi$

$$\pm \cancel{2} n \cancel{\pi} = \frac{\cancel{2\pi}}{\lambda}(a+b)\sin\theta \qquad \left[2\beta = \frac{2\pi(a+b)\sin\theta}{\lambda}\right]$$

$$\pm n\lambda = (a+b)\sin\theta \rightarrow 2nd$$

when $n = 0$, $\theta = 0$, then eq^n 2nd gives the direction of zero order principle maxima.

The direction of 1st, 2nd, 3rd order principle maxima can be obtained by putting $n = 1, 2, 3, \dots$

Minima → For minima,

$$\sin N\beta = 0$$

$$N\beta = \pm m\pi$$

$$\beta = \pm \frac{m\pi}{N}$$

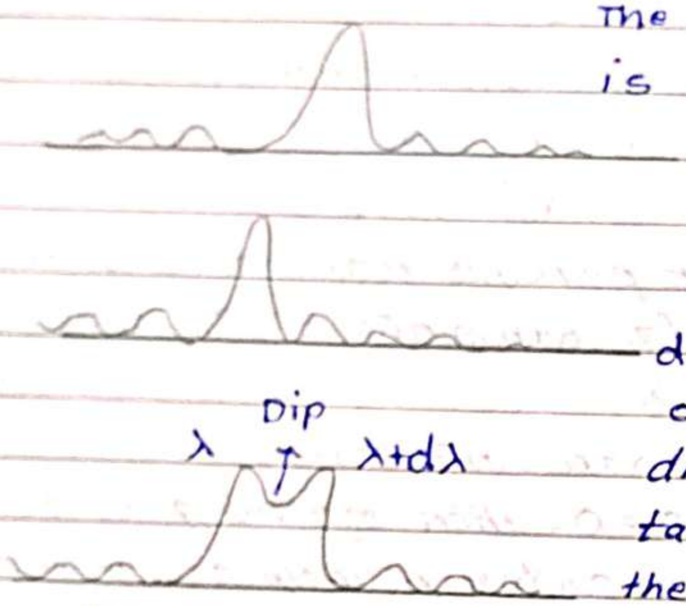
$$\pm \frac{m\cancel{\pi}}{N} = \frac{\cancel{\pi}}{\lambda}(a+b)\sin\theta$$

$$\pm m\lambda = N(a+b)\sin\theta \qquad [m \neq 0, N, 2N, \dots, nN]$$

Rayleigh criterion for limit of resolution →

According to the Rayleigh criterion for limit of resolution, the two point source or equally intense spectral lines are just resolved by an optical instrument if the central maxima of the diffraction pattern due to one falls exactly on the first minima of diffraction pattern due to other.

Consider the two wavelength [λ] & [λ+dλ], the intensity distribution due to the two wavelengths shown in the fig.



The difference in wavelength is called Limiting value. If the diff. in wavelength is less than the Limiting value. Then, the overlapping of the diffraction pattern takes place. Hence, the essential condition for the two lines to be just resolved if the principal maxima of one fall upon the first minima of the other and vice-versa.

Resolving Power of Plane Diffraction Grating →

Resolving power is defined as the ability of the grating to show two spectral lines of a spectrum as a separate one. and is measured as the ratio of the wavelength of any spectral line to the smallest possible wavelength difference.

$$\text{Resolving Power} = \frac{\lambda}{d\lambda}$$

Consider the two spectral lines of wavelength [λ] & [λ+dλ] incident on the transmission grating. Let the principle maxima for [λ] is formed in direction θ.

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$(a+b)\sin\theta = n\lambda \rightarrow$ 1st

If the first minima is formed in the direction $[\theta+d\theta]$ then,

$N(a+b)\sin(\theta+d\theta) = (Nn+1)\lambda \rightarrow$ 2nd

According to Rayleigh, the two spectral lines are just resolved if the n^{th} maxima of wavelength $[\lambda+d\lambda]$ fall on the first minima of wavelength $[\lambda]$. Hence, the n^{th} principle maxima of wavelength $[\lambda+d\lambda]$ must be formed in the direction $[\theta+d\theta]$

$(a+b)\sin(\theta+d\theta) = n(\lambda+d\lambda) \rightarrow$ 3rd

From eq^n 2nd & 3rd,

$$\frac{N\cancel{(a+b)\sin(\theta+d\theta)}}{\cancel{(a+b)\sin(\theta+d\theta)}} = \frac{(Nn+1)\lambda}{n(\lambda+d\lambda)}$$

$$N = \frac{(Nn+1)\lambda}{n(\lambda+d\lambda)}$$

$$\cancel{Nn\lambda} + Nnd\lambda = \cancel{Nn\lambda} + \lambda$$

$$Nnd\lambda = \lambda$$

$$\frac{\lambda}{d\lambda} = Nn \qquad \left[\frac{\lambda}{d\lambda} = \text{Resolving power}\right]$$

The resolving power $= Nn$.

The resolving power is equal to the product of the principle maxima & the total no. of lines on the grating.

From eq^n 1 $\rightarrow$

$$n = \frac{(a+b)\sin\theta}{\lambda}$$

$$\frac{\lambda}{d\lambda} = \frac{N(a+b)\sin\theta}{\lambda}$$