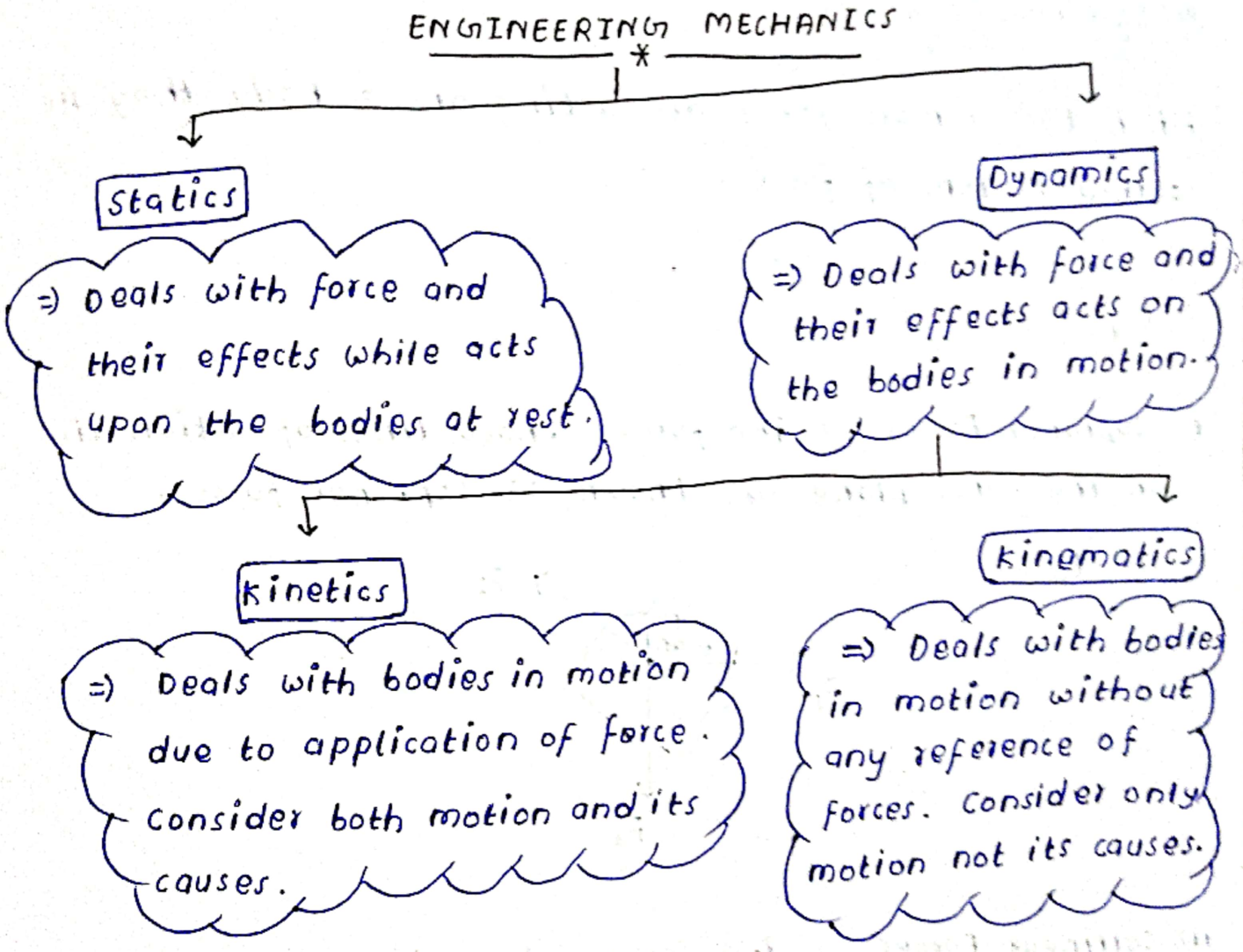
* ENGINEERING MECHANICS *

⇒ It is a subject which describes and predicts the effects of force on rigid bodies.

⇒ Deals with the study of bodies at rest or in motion when subjected to external forces.



* S.I Units → International system of measurement.

Fundamental units

1. length 2. Mass and 3. Time

S.I Units :-

1) Density → kg/m^3
2) Force → N or kgm/s^2
3) Pressure → N/m^2
4) Work done → Joule (J)
5) Power → Watt (W)

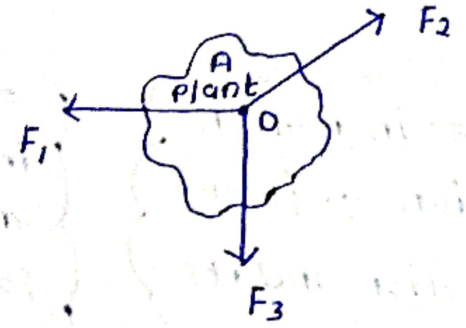
* System of forces *

When two or more force are acting on a body they are called system of forces.

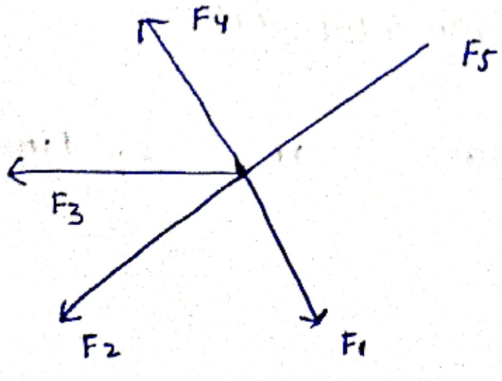
Types of forces :-

i) Coplaner Forces → The forces whose lines of action lie on the same plane are known as coplaner forces.

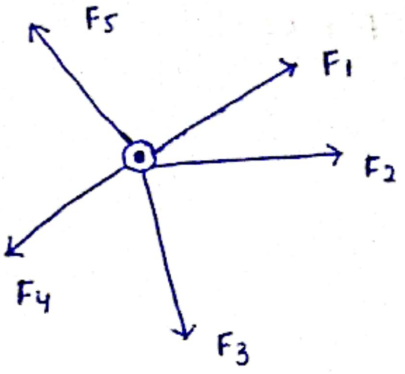
Eg :-



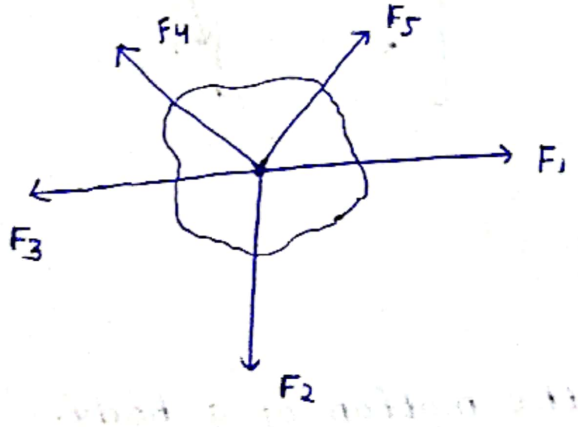
ii) Collinear Forces → The forces whose lines of action lie on the same line are known as collinear forces.



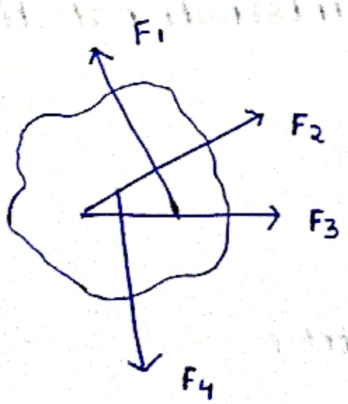
iii) Concurrent forces :- The forces which meet at one point, are known as concurrent forces.



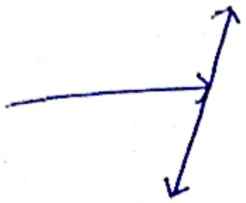
iv) Coplaner Concurrent forces :- The forces which meet at one point and their lines of action also lie on the same plane are known as Coplaner concurrent forces



v) Coplaner non-concurrent forces :- The forces which do not meet at one point but their line of action are in same plane are known as

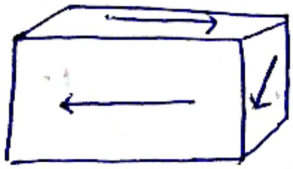


vi) Non-coplaner collinear → The force whose line of action do not lie in same line and in same plane are known as Non-coplaner collinear Forces.



vii) Non-coplaner non-Concurrent :-

The forces which do not meet at one point and their line of action do not lie on the same plane.



* Effects of a forces :-

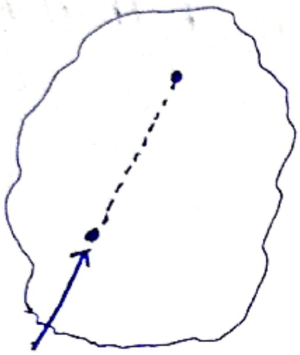
- It may change the motion of a body.
- May retard the motion of a body
- May retard the forces already acting on the body.
- May give rise to the international stresses in the body.

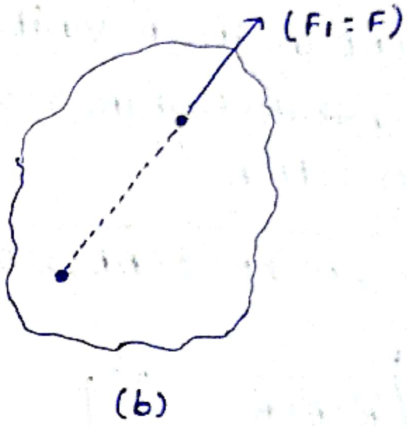
* Characteristics of forces :-

→ Nature of a force.
→ It is a vector quantity
→ Magnitude
→ Point of application.
→ line of action, and
→ direction.

** Principle of Transmissibility of forces.

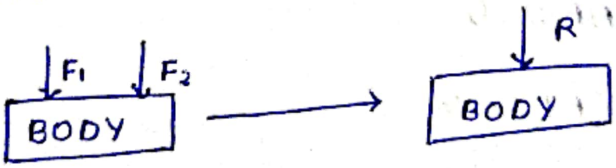
⇒ It states, "If a force acts on body at any other point on its line of action, provided this point is rigidly connected with the body."





*** Resultant Force :-

- If a number of forces are acting simultaneously on a particle, then it could be possible to replace them with a single force which produce the same effect as produced by all the given forces.
- This force is called as a resultant force.



* Methods for the resultant force.

1. Analytical method
2) Method of resolution.

i) Analytical Method :- The resultant force of a given system of forces, may be found out analytically by the following method.

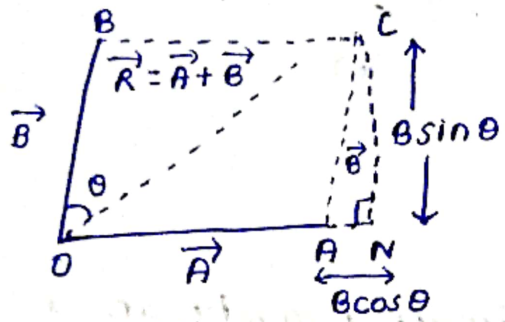
i) Parallelogram law of forces.

ii) Method of resolution.

i) Parallelogram law of forces :-

- Parallelogram law :- If two forces, acting on a body be represented in magnitude and direction by the two adjacent side of parallelogram ; their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection.

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$



In ΔOCN

$$OC^2 = ON^2 + CN^2$$
$$= (A + B\cos\theta)^2 + B^2\sin^2\theta$$
$$= A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$$
$$= A^2 + B^2 + 2AB\cos\theta$$

Hence, $R^2 = A^2 + B^2 + 2AB\cos\theta$

$$\therefore \quad R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

- case I : $\theta = 0$

$$R = A + B$$

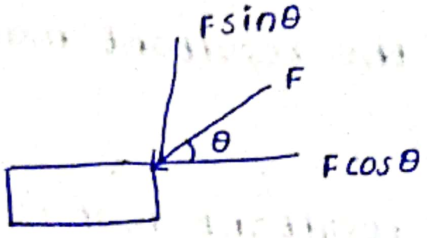
- Case - II :- $\theta = 90^\circ$

$$R = \sqrt{A^2 + B^2}$$

Limitation of law of parallelogram
i) it is applicable if there is only two concurrent forces

ii) Method of Resolution of forces :-

⇒ The process of spliting up the given forces into a number of components, without changing its effect on the body is called resolution of force.



* Principle of Resolution :-

The algebraic sum of the resolved parts of a no. of forces, in a given direction is equal to the resolve part of their resultant in the same direction.

* Steps to find Resolution for the resultant forces.

Step :- 1 ⇒ Resolve all the force horizontally and find the algebraic sum of all the Horizontal components.

i.e (ΣH)

Step :- 2 ⇒ Resolve all the force vertically and find the algebraic sum of all the vertical components.

i.e. (ΣV)

Step :- 3 ⇒ The resultant R of the given forces will be given by the equation :-

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

Step :- 4 ⇒ The resultant force will be inclined at an angle θ with the horizontal such that

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

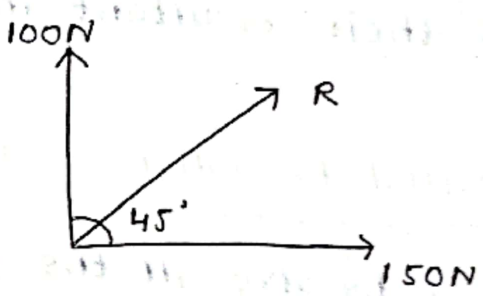
* Direction of Resultant Force *

Note :- The value of the angle θ will vary depending upon the values of ΣV and ΣH as discussed below :-

1) When ΣV is +ve, the resultant makes an angle b/w 0° to 180°. When ΣV is -ve, the resultant makes an angle between 180° and 360°.

2) When ΣH is +ve, the resultant makes an angle b/w 0° to 90° or 270° to 360°. But when ΣH is -ve, the resultant makes an angle between 90° to 270°.

Q.1 Two forces of 100N and 150N are acting on a body. Find resultant.



$F_1 = 100N$

$F_2 = 150N$

$\theta = 45°$

$$R = \sqrt{(100)^2 + (150)^2 + 2 \times 100 \times 150 \cos 45°}$$

$$= \sqrt{10000 + 22500 + 2 \times 15000 \times \frac{1}{\sqrt{2}}}$$

$$= 10\sqrt{100 + 225 + 150 \times \sqrt{2}}$$

$$= 10\sqrt{325 + 212.13}$$

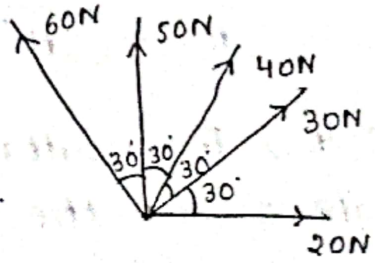
$$R = 10\sqrt{537.13}$$

$$R = 231.7\ N$$

Q2. $\Sigma H = 20 + 30 \cos 30° + 40 \cos 60° + 50 \cos 90° + 60° \cos 180°$

$$= 20 + 30 \times 0.866 + 40 \times \frac{1}{2} + 50 \times 0 + 60 \times (-0.5)$$

$$= 35.98\ N = 36.0\ N$$



$$\Sigma V = 20\sin 0^\circ + 30\sin 30^\circ + 40\sin 60^\circ + 50\sin 90^\circ + 60\sin 180^\circ$$

$$= 151.60\ N$$

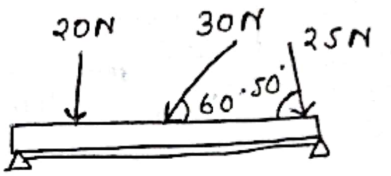
$$R = \sqrt{(36.0)^2 + (151.60)^2}$$

$$R = \sqrt{155.81}\ N$$

Q3 Find the resultant force.

$$\Sigma H = 30\cos 60^\circ - 25\cos 50^\circ$$

$$= -1.0$$



$$\Sigma V = 20 + 60\sin 30^\circ + 25\sin 50^\circ$$

$$= 62.1\ N$$

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= 62.1\ N$$

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

$$\tan\theta = \frac{62.1}{1.0} \Rightarrow \theta = \tan^{-1}(62.1) = 89.0$$

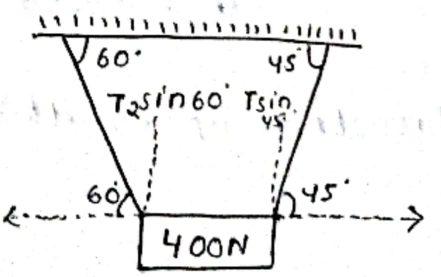
Q4 A body of weight 400N is hanging from celling by two wire AC and BD. AC making 60° and BD making 45° with ceiling. Find tension.

$$\Rightarrow \Sigma H = T_2\cos 60^\circ - T_1\cos 45^\circ = 0$$

$$T_2\cos 60^\circ = T_1\cos 45^\circ$$

$$T_2 = T_1\frac{\cos 45^\circ}{\cos 60^\circ}$$

$$T_2 = T_1\ 1.414$$



$$\Rightarrow \Sigma V = T_2\sin 60^\circ + T_1\sin 45^\circ = 400$$

$$= T_1(1.414)\sin 60^\circ + T_1\sin 45^\circ = 400$$

$$= 1.22\,T_1 + T_1(0.707) = 400$$

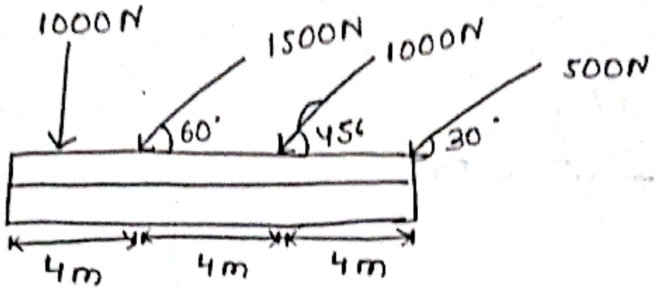
$1.927\ T_1 = 400$

$T_1 = \frac{400}{1.927} = 207.57\ N$

$T_2 = 1.414 \times T_1$

$= 293.4\ N$

Q5



$\Sigma H = 1500 \cos 60° + 1000 \cos 45° + 500 \cos 30°$

$= 1890.1\ N$

$\Sigma V = 1000 + 1500 \sin 60° + 1000 \sin 45° + 500 \sin 30°$

$= 3256.14\ N$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(1890.1)^2 + (3256.14)^2} = 3764.95\ N$

$\tan\theta = \frac{\Sigma V}{\Sigma H}$

$\theta = \tan^{-1}\left(\frac{3256.14}{1890.1}\right)$

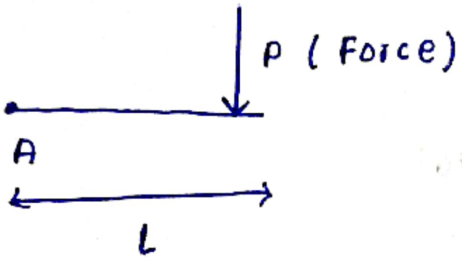
$\theta = 59.86°$

Direction of resultant force = 59.86° Ans

* Moment of force :-

- It is a turning effect produced by a force on the body on which it acts.

- The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required and the line of action of the force.

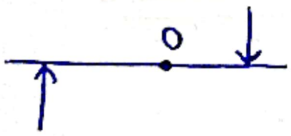


$M = P \times l$

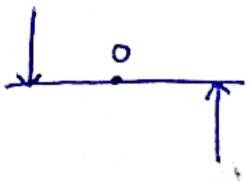
Units of Moment → N-m

* Types of Moments :-

i) Clockwise moments



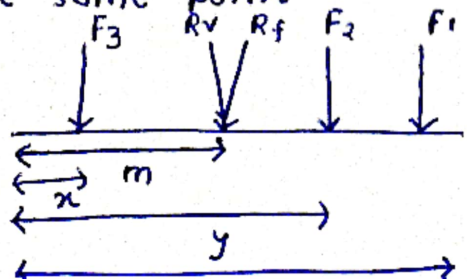
ii) Anticlockwise moments



* Law of Moments :-

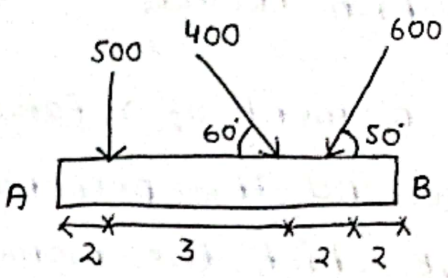
* Varignon's Principle of Moments *

- If a no. of Coplaner forces are acting simultaneously on body. The algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point



$[\quad = F_3 x + F_2 y + F_1 z\ ]$

Q 1



$\Sigma H = 600\cos 50 - 400\cos 60 = 185.67\ N$

$\Sigma V = 500 + 400\sin 60^\circ + 600\sin 50^\circ$

$= 1306.03\ N$

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= 1319.16\ N$

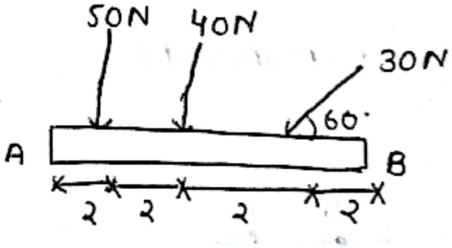
$\theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right) \Rightarrow \theta = 81.90^\circ$

$P x x = 500 \times 2 + 5 \times \sin 60^\circ \times 400 + 7 \times 600 \sin 50$

$1306.16 \times x = 1000 + 1732.05 + 3217.38$

$x = \frac{5949.43}{1306.16} = 4.554\ m$ Ans

Q2. Find the Reaction.



$R_A + R_B = 50 + 40 + 30\sin 60^\circ$

$R_A + R_B = 115.98$

Moment at point A

$R_B \times 8 = 2 \times 50 + 4 \times 40 + 6 \times 30\sin 60^\circ$

$R_B \times 8 = 100 + 160 + 180\sin 60^\circ$

$R_B \times 8 = 415.88$

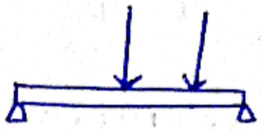
$R_B = \frac{415.88}{8} = 51.98\ N$

$R_A = 115.98 - 51.98$

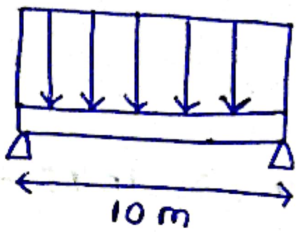
$= 64.0\ N$ Ans

* Types of load / Force :-

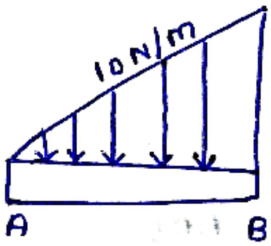
i) Point load



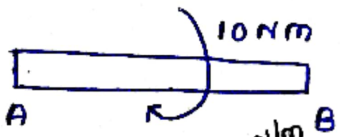
ii) Uniformly distributed load (UDL)



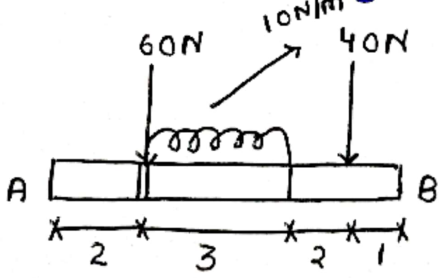
iii) Uniformly varried load (UVL)



iv) Couple.



Q1



$R_A + R_B = 60 + 3 \times 10 + 40$

$= 60 + 30 + 40 = 130\ N$

Moment at A

$R_B \times 8 = 2 \times 60 + 3.5 \times 3 \times 10 + 7 \times 40$

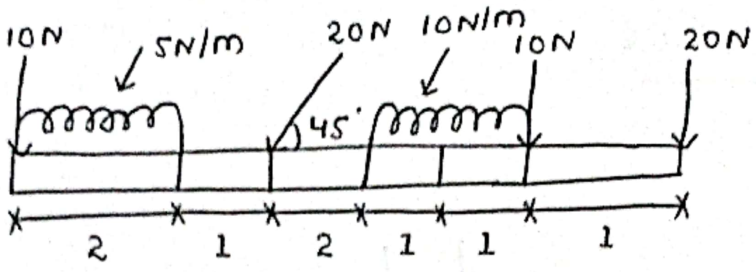
$= 120 + 105 + 280$

$= 505$

$R_B = \frac{505}{8} = 63.125\ N$

$R_A = 130 - 63.125 = 66.875\ N$ Ans

Q3



$R_A + R_B = 10\times5+2+20\sin45^\circ+10\times2+10+20$

$= 10+10+14.14+20+10+20$

$R_A + R_B = 84.14$

Moment at A

$R_B \times 4 = 20\times6+10\times5+10\times2\times4+20\sin45^\circ-5\times2\times1-10\times2$

$= 120+50+80+14.14-30$

$R_B \times 4 = 264.14-30 = 234.14$

$R_B = \frac{234.14}{4} = 58.535\ N$

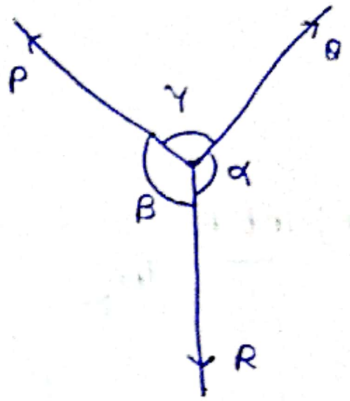
$R_A = 84.14 - 58.535 = 25.605\ N$ Ans

* Methods of the equilibrium of Coplaner Methods / Forces *

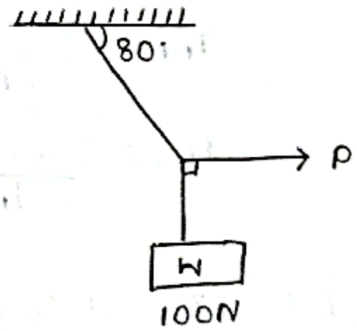
* LAMI'S THEOREM *

⇒ If three concurrent coplaner forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two.

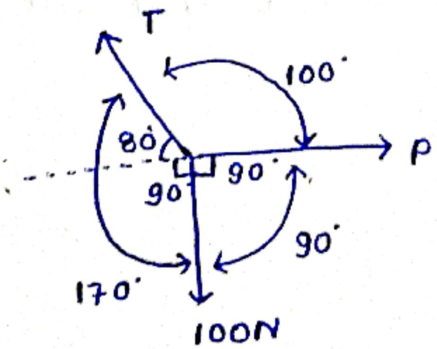
$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$



Q1 A Horizontal force P is as shown in figure keeps the weight of 100N in the equilibrium. Find the magnitude of force P and the tension in the string.



F.B.D at Pt



By Lami's Theorem :-

$$\frac{P}{\sin 170^\circ} = \frac{100}{\sin 100^\circ} = \frac{T}{\sin 90^\circ}$$

$$\frac{P}{\sin 170^\circ} = \frac{100}{\sin 100^\circ}$$

$$P = \frac{100\sin 170^\circ}{\sin 100} = 17.63\ N$$

$$\frac{T}{\sin 90^\circ} = \frac{100}{\sin 100}$$

$$T = \frac{100\sin 90^\circ}{\sin 100} = 101.54$$

Ans

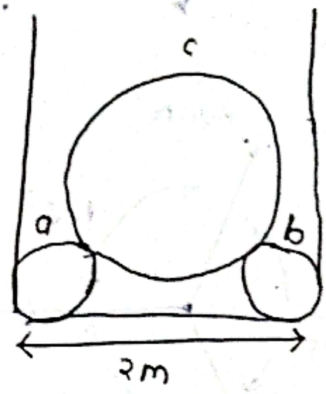
Q. Find the reaction at all point of contact

$W_a = W_b = 1000N$

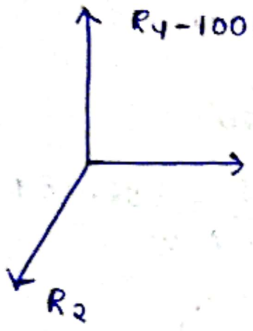
$W_c = 2000N$

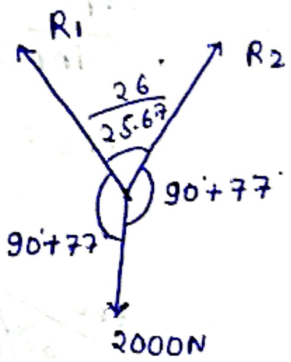
$R_a = R_b = 0.6m$

$R_c = 1.62\ m$



FBD





$$\cos\theta = \frac{0.4}{1.8}$$

$$\theta = \cos^{-1}\left(\frac{0.4}{1.8}\right) = 77$$

By Lami's Theorem,

$$\frac{R_1}{\sin 167} = \frac{2000}{\sin 26} = \frac{R_2}{\sin 167}$$

$$R_1 = \frac{2000 \sin 167}{\sin 26} = 1247.24\ N\ (1026.3041\ N)$$

$$R_2 = \frac{2000 \sin 167}{\sin 26} = 1026.3041\ N$$

$$\frac{1026}{\sin 90} = \frac{R_4 - 1000}{\sin 102.78} = \frac{R_3}{\sin 167.22}$$

$$R_4 - 1000 = 1026 \times 0.975$$

$$R_4 = 1000.35 + 1000$$

$$\therefore R_4 = 2000.35\ N$$

Q

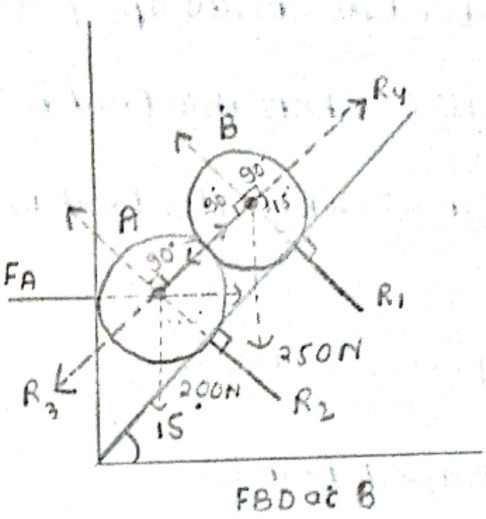
By Lami's Theorem,

$$\frac{R_1}{\sin 105^\circ} = \frac{250}{\sin 90^\circ} = \frac{R_4}{\sin 165^\circ}$$

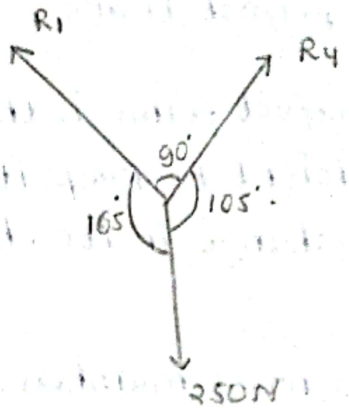
$$R_1 \sin 90^\circ = 250 \sin 105$$

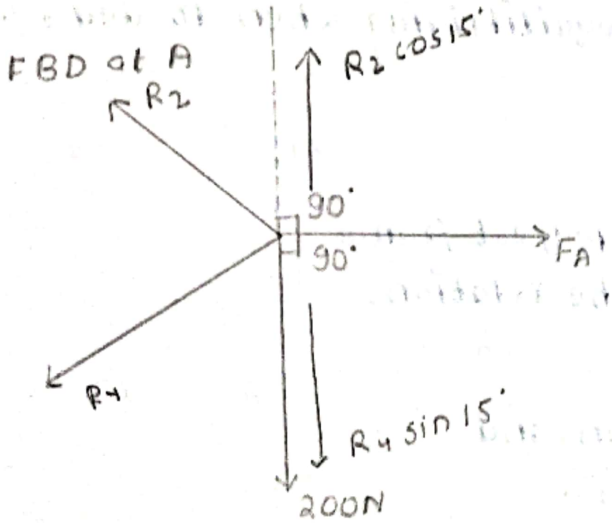
$$R_1 = \frac{250 \sin 105^\circ}{\sin 90^\circ} = 241.48\ N$$

$$R_4 = \frac{250 \sin 165^\circ}{\sin 90^\circ} = 64.704\ N$$



FBD at B





$$R_3^2 \cos 15^\circ = 200 + 64.7 \sin 15^\circ$$

$$R_3^2 \times 0.965 = 200 + 16.74$$

$$\therefore R_2 = 224.6\ N$$

V.V.I

** Frames and Trusses **

- Structure, made up of several bars, riveted or welded together.

Truss : When the load is applied only on the joints

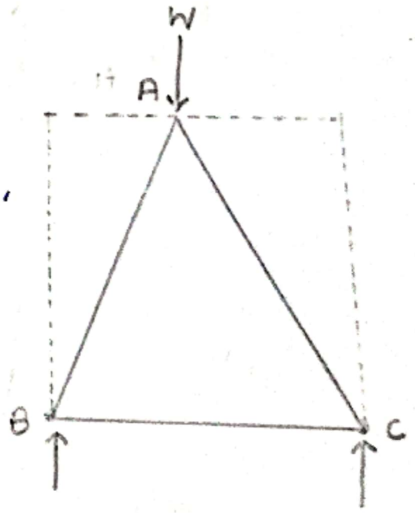
Frame: When the load is applied anywhere on the structure

* Types of frames :-

i) Perfect Frame

ii) Imperfect Frame

- A perfect frame is that, which is made up of members just sufficient to keep it in equilibrium, when loaded, without any change in its shape.



The no. of members, in a perfect frame, may also be expressed by the relation:

$n = (2j-3)$

n = No. of members, and

j = No. of joints.

- An imperfect frame is that which does not satisfy the equation:

$n = (2j-3)$

- it is a frame in which the no. of members are more or less than $(2j-3)$

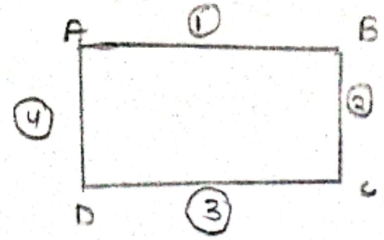
① Deficient frame. ② Redundant frame.

- Deficient frame :- A deficient frame is an imperfect frame, in which the no. of members are less than $(2j-3)$.

$n = 2j-3$

$4 = 2\times4-3 = 5$

$4 \neq 5$



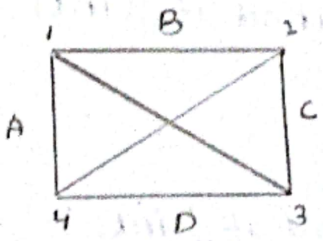
- Redundant frame :- A redundant frame is an imperfect frame, in which the no. of members are more than $(2j-3)$

$n = 2j-3$

$6 = 2 \times 4 - 3$

$6 \neq 5$

$6 > 5$

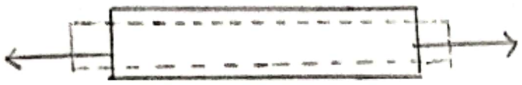


* STRESS *

- When a body is acted upon by a force, the internal force which is transmitted through the body is known as stress.

i) Tensile stress.

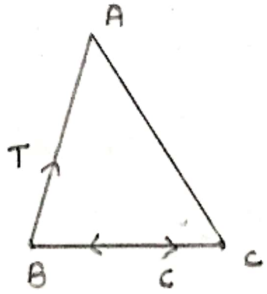
ii) Compressive stress.



a) Tensile stress

b) Compressive stress.

Stress in Truss



* Assumption for forces in the members of a perfect frame *

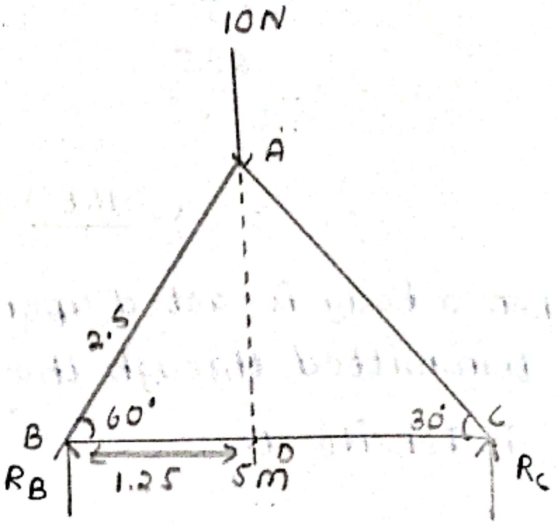
- All the members are pin-jointed.
- The frame is loaded only at the joints.
- The frame is a perfect one.
- The weight of the members, is regarded as negligible in comparison with the other external forces or loads acting on the truss.

* Analytical method for the forces *

i) method of joints

* Method of joints :-

Q



$R_B + R_C = 10$

$R_C \times 5 = 10 \times 1.25$

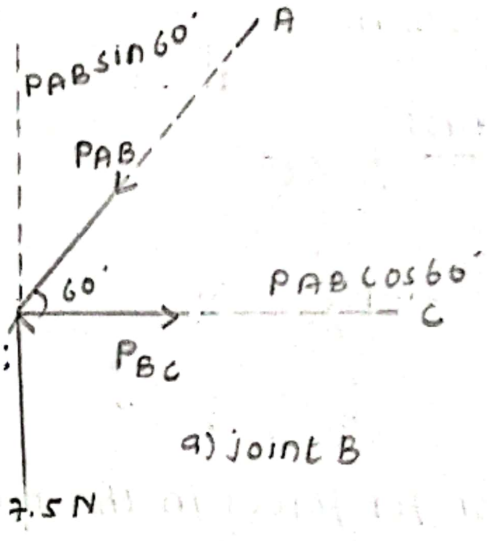
$R_C = 2.5$ KN

$R_B = 7.5$ KN

Resolving the forces vertically;

$P_{AB} \sin 60° = 7.5$

$P_{AB} = \frac{7.5}{\sin 60°} = 8.66$ KN (C)



a) joint B

Resolving the forces horizontally;

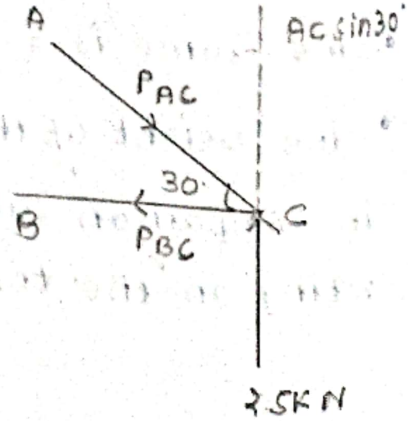
$P_{BC} = P_{AB} \cos 60°$

$= 8.66 \times 0.5 = 4.33$ KN (T)

Resolving the forces vertically;

$P_{AC} \sin 30° = 2.5$

$P_{AC} = \frac{2.5}{\sin 30°} = 5.0$ KN (C)



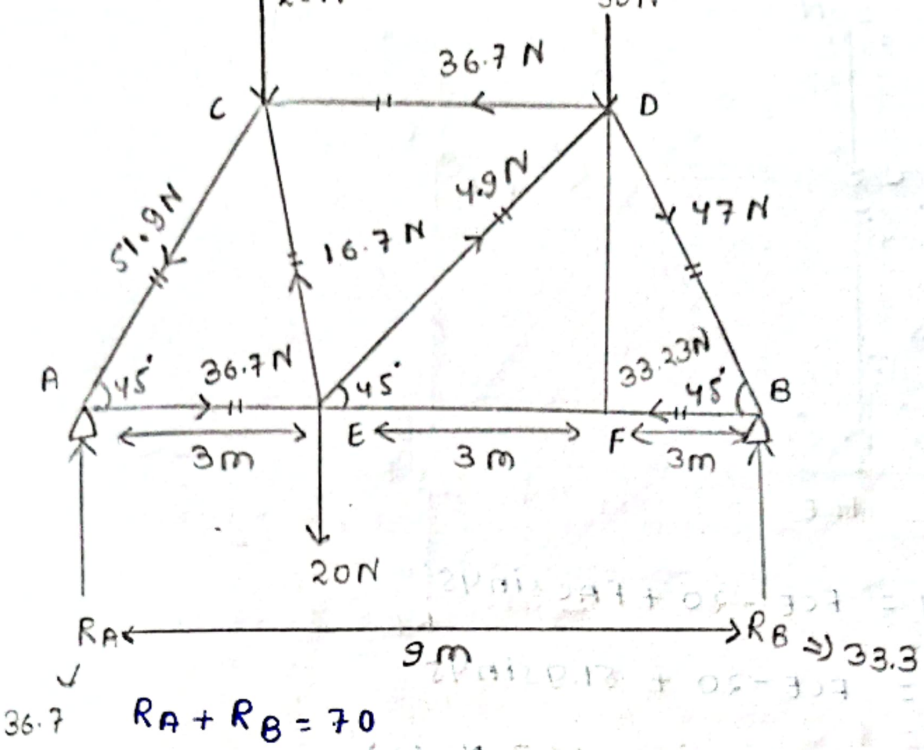
b) joint C

Resolving the forces horizontally;

$P_{BC} = P_{AC} \cos 30°$

$= 5.0 \times 0.866 = 4.3$ N (T)

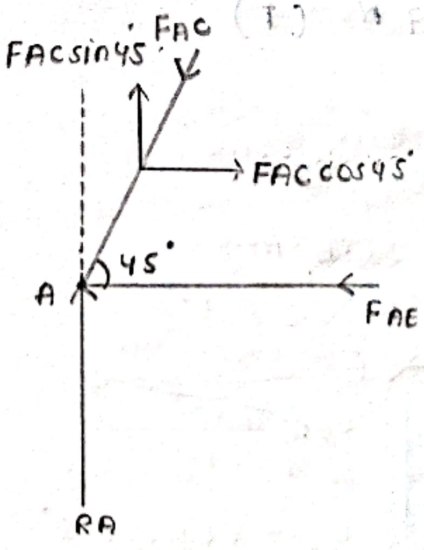
Q//



$R_A + R_B = 70$

$$R_B \times 9 = \frac{30\times 6 + 20\times 3 + 20\times 3}{9} = 33.3$$

$R_A = 70 - 33.3 = 36.7$



$\Sigma f_y = R_A + F_{AC}\sin 45°$

$0 = 36.7 + F_{AC}\sin 45°$

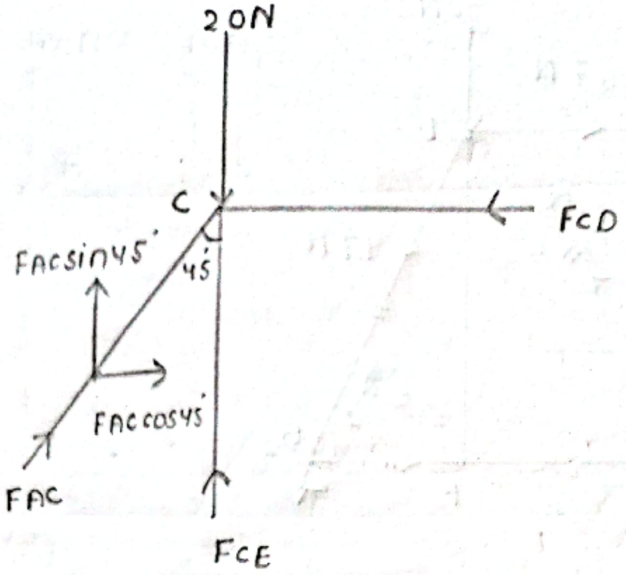
$$F_{AC} = \frac{-36.7}{\sin 45°} = -51.9\ N\ (C)$$

$\Sigma f_x = -F_{AE} + F_{AC}\cos 45°$

$F_{AE} = +51.9\cos 45°$

$F_{AE} = +36.69\ N$

$= +36.7\ N\ (T)$



$$\Sigma f_y = F_{CE} - 20 + F_{AC}\sin 45^\circ$$

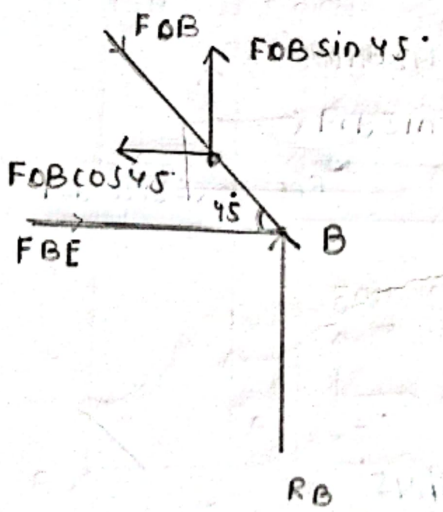
$$0 = F_{CE} - 20 + 51.9\sin 45^\circ$$

$$F_{CE} = 20 - 51.9\sin 45^\circ = -16.7\ N\ (C)$$

$$\Sigma f_x = -F_{CD} + F_{AC}\cos 45^\circ$$

$$F_{CD} = F_B\ 51.9\cos 45^\circ$$

$$F_{CD} = 36.7\ N\ (T)$$



$$\Sigma f_y = R_B + F_{DB}\sin 45^\circ$$

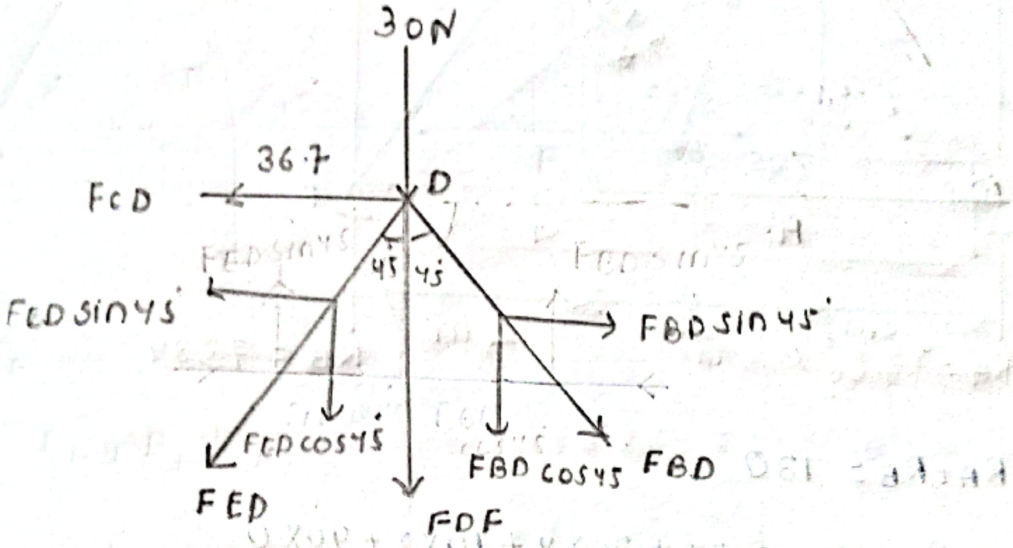
$$0 = 33.3 + F_{DB}\sin 45^\circ$$

$$-F_{DB} = \frac{3.33}{\sin 45^\circ} = -47\ N\ (C)$$

$$\Sigma F_x = F_{BF} - F_{DB}\cos 45^\circ$$

$$0 = F_{BF} - 47\cos 45^\circ$$

$$F_{BE} = 33.23\ N\ (T)$$



$$\Sigma F_x = F_{BD}\sin 45^\circ - F_{ED}\sin 45^\circ - F_{CD}$$

$$F_{ED} = \frac{47\sin 45^\circ - 36.7}{\sin 45^\circ}$$

$$= \frac{33.234 - 36.7}{\sin 45^\circ} = -\frac{3.466}{\sin 45^\circ}$$

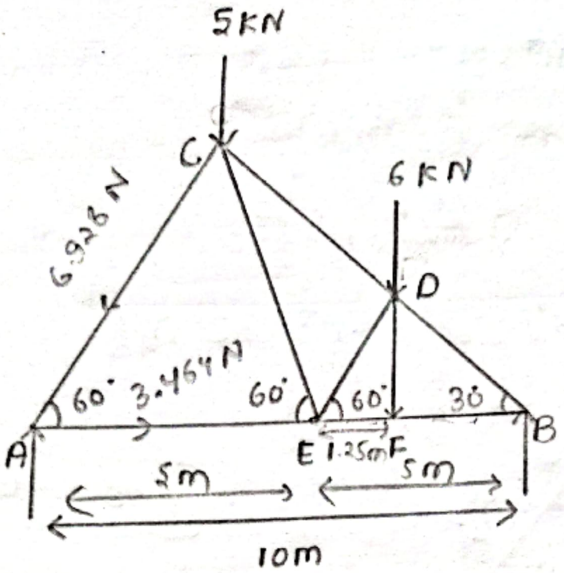
$$F_{ED} = -4.9\ N\ (C)$$

$$\Sigma F_y = -30 - F_{DF} - F_{BD}\cos 45^\circ - F_{ED}\cos 45^\circ$$

$$F_{DF} = -30 - 47\cos 45^\circ - 4.9\cos 45^\circ$$

$$F_{DF} = -30 - 33.23 - 3.464 = -66.7\ (C)$$

Q



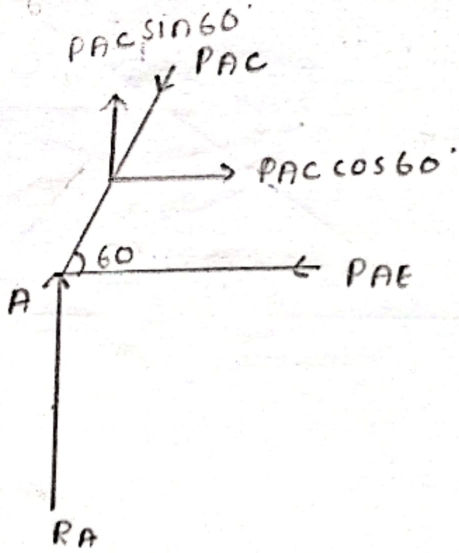
90 + 30 - 180°

$R_A + R_B = 11$

$R_B \times 10 = 5 \times 2.5 + 6(1.25 + 5) = 12.5 + 6(6.25)$

$\boxed{R_B = 5}$

$\boxed{R_A = 6}$



$\Sigma F_y = 0$

$\Sigma F_y = R_A + P_{AC} \sin 60°$

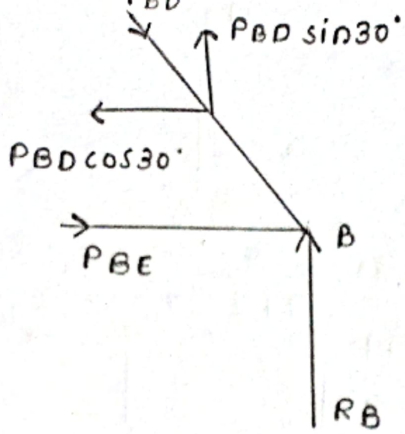
$0 = R_A + P_{AC} \sin 60°$

$-P_{AC} = \frac{6}{\sin 60°} = -6.928\ N\ (C)$

$\Sigma F_x = -P_{AE} + P_{AC} \cos 60°$

$\boxed{P_{AE} = 6.928 \cos 60°}$

$\boxed{P_{AE} = 3.464\ N\ (T)}$



$\Sigma f_y = 0$

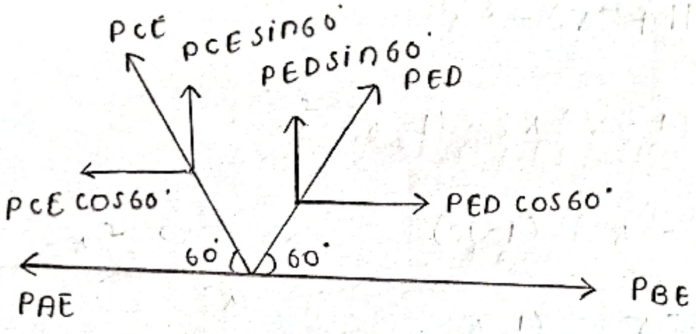
$\Sigma f_y = R_B + P_{BD} \sin 30^\circ$

$P_{BD} \sin 30^\circ = -R_B$

$P_{BD} = -\frac{5}{\sin 30^\circ} = -10\,N\,(C)$

$\Sigma f_x = P_{BE} - P_{BD} \cos 30^\circ$

$P_{BE} = 10 \cos 30^\circ = 8.660\,(T) \Rightarrow \boxed{P_{BE} = 8.660\,(T)}$

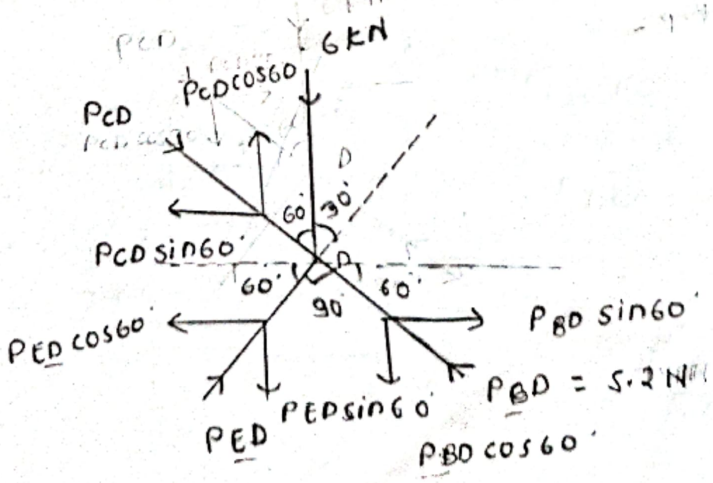


$\boxed{P_{ED} = -P_{CE}}$

$\Sigma f_x = 8.6 - 3.46 + P_{ED} \cos 60^\circ + P_{ED} \cos 60^\circ$

$0 = 5.14 + 2 P_{ED} \cos 60^\circ$

$P_{ED} = \frac{-5.14}{2 \cos 60^\circ} = -5.14N\,(C)$



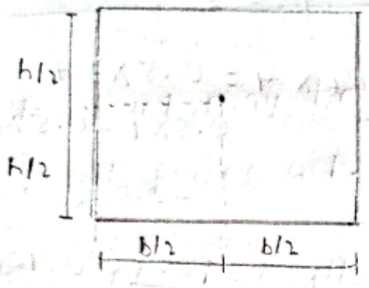
$\Sigma y = -P_{ED} \sin 60^\circ - P_{BD} \cos 60^\circ - 6 + P_{CD} \cos 60^\circ$

$-P_{CD} \cos 60^\circ = -5.2 \sin 60^\circ - 10 \cos 60^\circ - 6$

$+P_{CD} = -4.5 - 5 - 6 = \frac{+15.5}{\cos 60^\circ} = 7.75$

* CENTROID *

i) Rectangle

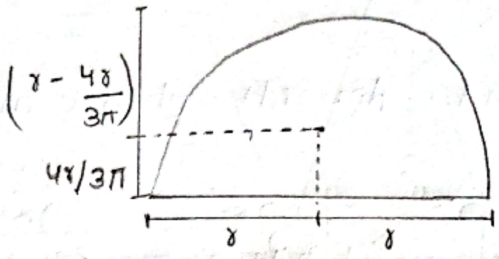


here $x = b/2$

$y = h/2$

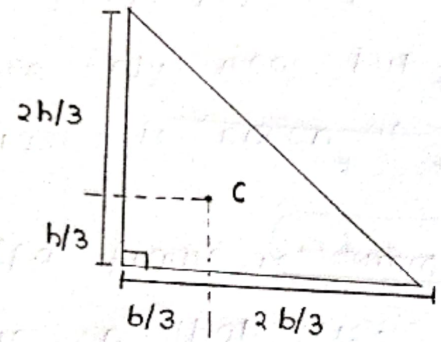
$A = b \times h$

ii) Semicircle



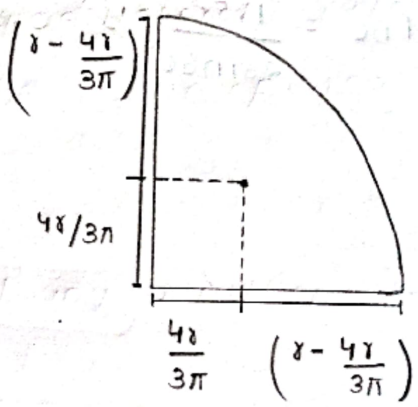
$A = \frac{\pi r^2}{2}$

iii) Right angle Triangle



$A = \frac{1}{2} \times b \times h$

iv) Quarter circle

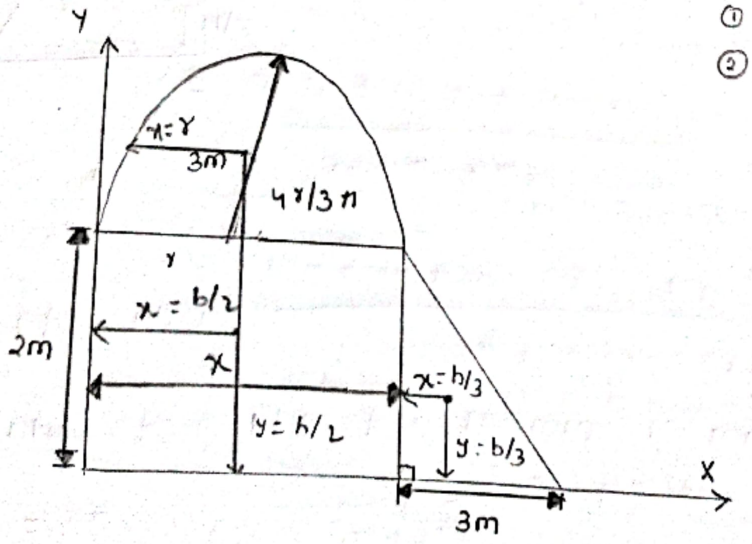


$A = \frac{\pi r^2}{4}$

Rule to find x & y :

① Locate the centre

② Draw two perpendicular along x & y axis



$C = (\bar{x}, \bar{y})$

$\frac{4 \times 3}{2 \times 3.14} = 3.273$

CENTER OF GRAVITY

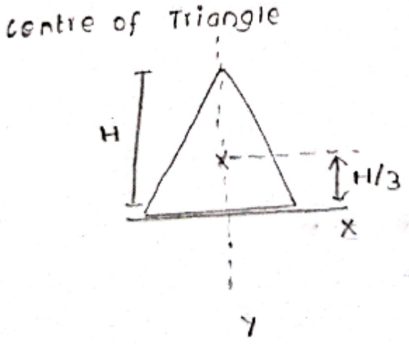
A point may be found out in a body, through which the resu of all forces act.

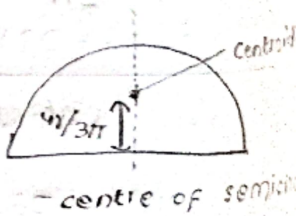
A point where all the whole weight of the body acts, irrespe of its position, is known as centre of gravity.

CENTROID

- The plane figures (like triangle, quadrilateral, circle etc.) have only areas, but no mass. The centre of area of such figures is known as centroid.

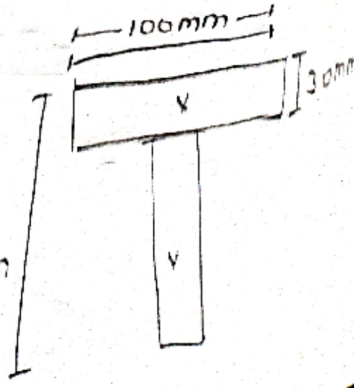
If a figure is symmetric around any axis then the centroi lies on that axis.





* Calculation of centroid :-

Find the centre of gravity of a 100mm x 150mm x 30mm T-sectio



$y_1 = 60$ $\quad y_2 = 135$

$a_1 = 120 \times 30$ $\quad a_2 = 100 \times 30$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{120 \times 30 \times 60 + 100 \times 30 \times 135}{120 \times 30 + 100 \times 30} \text{ mm}$$

$$= 94.1 \text{ mm}$$