05/09/2024

Unit - 1 Chapter - 2

Non - Dispersive Longitudnal & transvers wave:

Waves is defined as the disturbance travelling in the medium due to the vibratory motion of the medium by virtue of elasticity and inertia depending on the dirn of the propagation the waves can be classified into two classes.

→ Transverse Waves.
→ Longitudnal Waves.

~~In~~ Transverse: In this wave the particle of the medium vibrate perpendicular to the direction of propogation of the waves.

Longitudnal Waves: In this wave the particle of medium vibrate parallel to the direction of propagation.

Transverse → Light wave
Longitudnal → Sound wave.

On the basis of propogation of energy the waves can be classified as

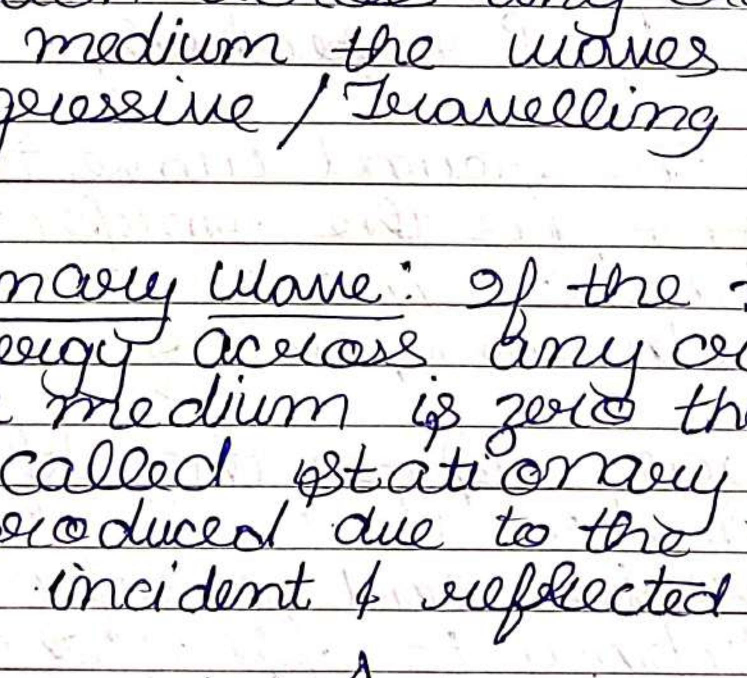
→ Broggressive waves (Travelling wave)

→ Stationary wave. (Standing wave).

~~If th~~

<u>Travelling wave</u>: If there is a continous transfer of energy in one direction across any cross section of the medium the waves are called progressive / Travelling waves.

<u>Stationary wave</u>: If the net transfer of energy across any cross section of the medium is zero than the waves are called stationary waves. The are produced due to the superposition of the incident & reflected waves.



$\boxed{\omega = 2\pi\nu}$

$\boxed{v, C = \frac{\lambda}{T}}$

ν - no. of occ. per unit cycle. $\boxed{\frac{1}{T} = \nu = f}$ $v = \lambda\nu = \lambda f$

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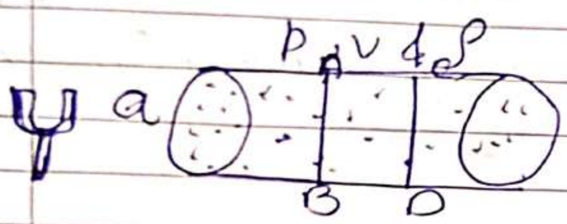
Longitudinal Waves:

The Wave Equation:



Suppose a gas is contained in a tube of area of cross section A.

Let the pressure, volume and density be p, v & ρ respectively.

When the sound wave travel through the ~~gase~~ gas the rarefraction and compression moved through the gas with the speed of the sound.

The gas apposes the deformation produced in it in the regions of compression and rarefraction and this tedency to appose the deformation is determined by the bulk modulas of the gas.

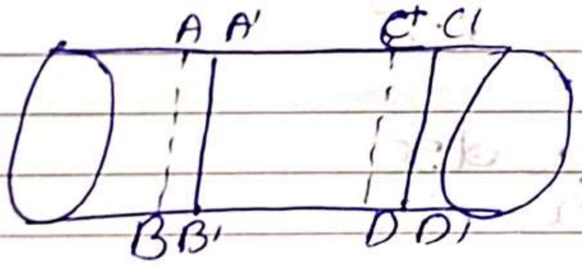
Calculation of Strain:

Consider an elementary part of the gas in the tube between AB and CD

The two planes are seperated by distance dx so the volume of the gas

$$V = a\,dx \quad - \quad (1)$$

when sound waves send ~~to the~~ through the tube by placing a tunning fork at its mouth due to the gas the position of the planes changes from AB to A'B' to C'D'



Let the displacement be represented by

$$y = y(x,t) \quad - \quad (2)$$

If $\frac{\partial y}{\partial x}$: $\frac{dy}{dx}$ be the rate of change of displacement with x than the seperation between the planes is given by

$$dy = dx + \frac{dy}{dx}\,dx \quad - \quad (3)$$

Than the volume between the plane A'B' C'D' is given by

$$V' = a\left(dx + \frac{dy}{dx}x\right) \quad \text{— (4)}$$

The change in Volume is given by

$$\Delta V = V' - V \quad \text{— (5)}$$

Putting (1) & (4) in (5)

$$\Delta V = a\left(dx + \frac{dy}{dx}dx\right) - a\,dx$$

$$= a\,dx + a\frac{dy}{dx}dx - a\,dx$$

$$= a\frac{dy}{dx}dx$$

$$S = \frac{\Delta V}{V} = a\frac{dy}{dx}dx \times \frac{1}{a\,dx}$$

$$\boxed{S = \frac{dy}{dx}} \quad \text{— (6)}$$

Calculation of Stress:

Let the normal pressure of the gas be P and in the deformed state The pressure is in the gas element a', b', c' and d'

$P-T$ $P-P$

Then the change in pressure is given by

$$\Delta p = \cancel{P} - p - \cancel{P}$$

$$\boxed{\Delta p = -p}$$

This is equal to the stress in the gas element.

Bulk Modulus (K):

$$K = \frac{\text{Stress}}{\text{Strain}}$$

$$= \frac{-p}{dy/dx}$$

$$K\frac{dy}{dx} = -p$$

$$\boxed{p = -K\frac{dy}{dx}} \quad - (7)$$

Wave Equation:

If ρ is the density of the gas element than the mass of the gas element is

~~$m = adx \cdot \rho$~~

$$m = adx \cdot \rho \quad - (8)$$

If $\frac{d^2y}{dt^2}$ is a the acceleration of the gas molecules during the propogation of longitudnal waves than force responsible for oscillating motion is given by

$$\cancel{F = \frac{md^2y}{dx^2}}$$

$$F = \frac{md^2y}{dt^2}$$

$$= \rho a\, dx \frac{d^2y}{dt^2} \quad -(9)$$

Change in the pressure at location x

$$dp = -\frac{dp}{dx} dx$$

$$F = -a\, dp$$

$$= -a \frac{dp}{dx} dx \quad -(10)$$

Put from Eqn (7)

$$= -a \frac{d}{dx}\left(-K\frac{dy}{dx}\right) dx$$

$$F = aK\frac{d^2y}{dx^2} dx \quad -(11)$$

From Eqn (9) & (11)

$$\rho a\, dx \frac{d^2y}{dt^2} = a K \frac{d^2y}{dx^2} dx$$

$$\frac{\rho}{K} \frac{d^2y}{dt^2} = \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = \frac{\rho}{K} \frac{d^2y}{dt^2}$$

$\because \sqrt{\frac{K}{\rho}}$ has the dimensions of the velocity

$$\boxed{\frac{d^2y}{dx^2} = \frac{1}{C_L^2} \frac{d^2y}{dt^2}}$$

This is the wave equation for the propogation of sound in air.

Or

This is the wave equation for the longitudnal wave.

<u>Velocity of the Sound Waves:</u>

The velocity of sound wave or longitudnal wave is given by

$$C_L = \sqrt{\frac{K}{\rho}}$$

<u>Newton Formula:</u> He assumed that the sound wave propogate under

___/___/___

the iso-thermal condition.

$$PV = \text{a constant}$$

$$P\,dV + V\,dP = \text{constant}$$
$$P\,dV = -V\,dP$$
$$P = -V\frac{dP}{dV} = \frac{-dP}{dV/V}$$

$$\boxed{K = \frac{-dP}{dV/V}}$$

$$\boxed{C_L = \sqrt{\frac{P}{\rho}}}$$

$C_L = 280\ m/s$ at normal temp. & pressure

Velocity of sound $= 320\ m/s$.

Laplace Correction: Laplace modified the velocity of sound by considering the adiabatic conditions. (Isolated system)
for adiabatic process.

$$PV^{\gamma} = \text{Constant}$$
$$\cancel{P.\gamma V}$$
$$P.\gamma V^{\gamma-1}\,dV + V^{\gamma}\,dP = 0$$
$$\gamma P V^{\gamma-1}\,dV = -V^{\gamma}\,dP$$
$$\boxed{C_L = \sqrt{\frac{\gamma P}{\rho}} = 320\ m/s}$$
$$\gamma P = -\frac{V^{\gamma}}{V^{\gamma-1}}\cdot\frac{dP}{dV}$$
$$\gamma P = -V^{\gamma-\gamma+1}\frac{dP}{dV} \Rightarrow -V\frac{dP}{dV} = \frac{dP}{dV/V}$$
$$= K$$

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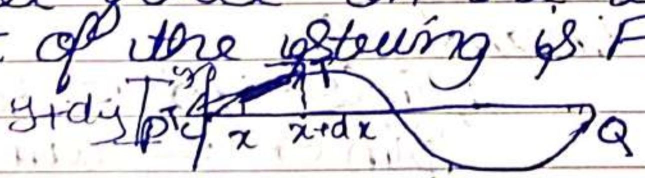
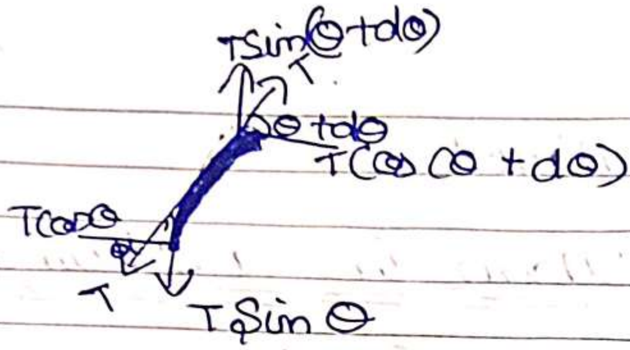
~~Transverse Wave:~~

Transverse Wave in a String :/ Wave equation for transverse waves:

A string is a cord or a wire whose length is very large as compared to its diameter which is perfectly uniform and flexible in nature when a string is streched between the two points transverse vibrations are produced in it.

Let us consider a string PQ be streched between the two points with tension T. Let us divide the string into a large no. of elements and consider the part between x and $x+dx$ in the equillibrium state the tension at the ends of the elimentary part are equal opposite it is represented by T the angle made by the tension T with the direction of elimentary parts are denoted by θ and $\theta+d\theta$ than the net force on the elimentary part of the string is F than





$$f_x = T\cos(\theta + d\theta) - T\cos\theta$$

$$f_y = T\sin(\theta + d\theta) - T\sin\theta \quad -(1)$$

Only the component ~~fy~~ Fy makes the string execute up and down motion than the net force

$$F = F_y \quad -(2)$$

If m is the mass of the elementary part of the string than according to the Newton IInd Law of motion

$$F = ma$$

$$F = m\frac{d^2y}{dt^2} \quad -(3)$$

From (1), (2) & (3)

$$F = m\frac{d^2y}{dt^2} + T\sin(\theta + d\theta) - T\sin\theta)$$

Since the angle θ is very small than

$$\sin(\theta + d\theta) \simeq \tan(\theta + d\theta)$$
$$\sin\theta \simeq \tan\theta$$

Than,

$$m\frac{d^2y}{dt^2} = T\tan(\theta+d\theta) - T\tan\theta \quad -④$$

If y co-ordinate of the lower part is y and upper part is y + dy than

$$\tan(\theta+d\theta) = \left(\frac{dy}{dx}\right)_{x+dx}$$

$$\tan\theta = \left(\frac{dy}{dx}\right)_x$$

From Eq^n ④

$$m\frac{d^2y}{dt^2} = T\left[\left(\frac{dy}{dx}\right)_{x+dx} - \left(\frac{dy}{dx}\right)_x\right] -⑤$$

Using Taylor series $\left(\frac{dy}{dx}\right)_{x+dx}$ ~~$- \frac{dy}{d}$~~

$$\left[\left(\frac{dy}{dx}\right)_{x+dx} - \left(\frac{dy}{dx}\right)_x\right] = \frac{d^2y}{dx^2}dx$$

Put Eq^n ⑤

$$m\frac{d^2y}{dt^2} = T\frac{d^2y}{dx^2}dx \quad -⑥$$

If ρ is the density, ~~let the~~ of the string that is mass per unit length ~~th of th~~ than, the mass of the elementary spatial portion is

$$m = \rho\, dx$$

Put in Eqn ⑥

$$\rho\, dx \frac{d^2y}{dt^2} = T \frac{d^2y}{dx^2}\, dx$$

$$\frac{\rho}{T} \frac{d^2y}{dt^2} = \frac{d^2y}{dx^2}$$

$$\left[\because \frac{\rho}{T} = \frac{1}{C^2}\right]$$

$$\therefore \frac{1}{C^2} \frac{d^2y}{dt^2} = \frac{d^2y}{dx^2}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{1}{C^2} \frac{d^2y}{dt^2}}$$

This is the wave equation ~~for~~ from the transverse wave in a string.

<u>Harmonic Waves</u>: A wave is called harmonic during its propagation the particles of the medium execute simple harmonic oscillations. Such wave are represented by

$$\boxed{y = A \cos k(Ct - x)}$$

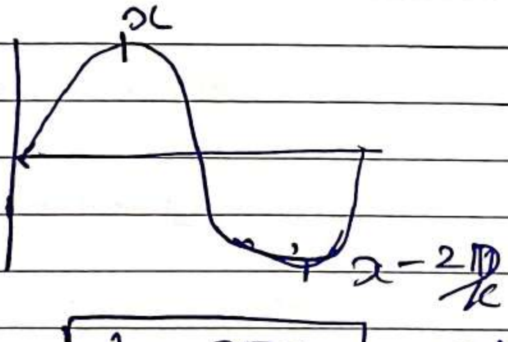
Where A is the amplitude and K is the constant called wave number. This wave can also be represented as

$$\boxed{y = Ae^{ik(ct - x)}}$$

$$y = A\cos[k(ct - x) + 2\pi]$$

$$\boxed{y = A\cos k\left[ct - \left(x - \frac{2\pi}{k}\right)\right]}$$

This shows that y repeats its value after the distance $\frac{2\pi}{k}$ that is y is same at x as well as $\left(x - \frac{2\pi}{k}\right)$. This means that in other words if there is a crest at x, than there will also be crest at $\left(x - \frac{2\pi}{k}\right)$.



$$\boxed{\lambda = \frac{2\pi}{k}}$$

If f is the frequency of oscillation than

$$c = f\lambda$$

$$\boxed{\lambda = \frac{c}{f}}$$

$$\frac{C}{f} = \frac{2\pi}{K}$$

$$\cancel{K = C}$$

$$KC = 2\pi f$$

$$KC = \frac{2\pi}{T}$$

$$\left[\because \frac{2\pi}{T} = \omega\right]$$

$$\boxed{KC = \frac{2\pi}{T} = \omega}$$

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Stationary waves.

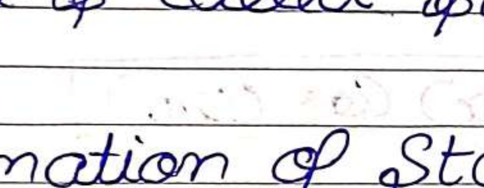
The waves in which the crest and truff do not change its location in the space is called stationary waves.

When the two proggesive waves of the same amplitude, time period and same speed are travelling in opposite direction than resultant wave & which does not appear to travel is called standing waves.

Formation of Standing waves:

Consider two transverse wave of same wavelength, amplitude and velocity of travelling in the string opposite direction the wave travelling along the x-axis is represented by

$$y_1 = A\sin(kx - \omega t)$$



$$y_2' = A\sin(kx + \omega t)$$

↓ travelling −ve x dirⁿ

By the superposition principle

$$y = y_1 + y_2$$

$$y = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$

$$\left[\because \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}\right]$$

$$y = 2A\sin\left(\frac{kx - \cancel{\omega t} + kx + \cancel{\omega t}}{2}\right)\cos\left(\frac{\cancel{kx} - \omega t - \cancel{kx} - \omega t}{2}\right)$$

$$y = 2A\sin\left(\frac{\cancel{2}kx}{\cancel{2}}\right)\cos\left(\frac{-\cancel{2}\omega t}{\cancel{2}}\right)$$

$$\boxed{y = 2A\sin(kx)\cos(\omega t)}$$

This equation represents a standing wave of amplitude

$$A_s = 2A\sin kx$$

Then, $\boxed{y = A_s\cos\omega t}$

Case 1. <u>For maximum amplitude</u>

$$A_s = \text{maximum}$$

$$2A\ \cancel{\sin}\ \sin kx = \text{max.}$$

$\sin kx = \max$

$\sin kx = \pm 1$

$$\boxed{kx = \left(n + \frac{1}{2}\right)\pi}$$

$n = 0, 1, 2, 3, \ldots$

$$K = \frac{2\pi}{\lambda}$$

$$\frac{2\pi}{\lambda} x = \left(n + \frac{1}{2}\right)\pi$$

$$\boxed{x = \frac{\lambda}{2}\left(n + \frac{1}{2}\right)}$$

~~(2) For minimum amplitude~~

$n = 0$

$x = \lambda/4$

$n = 1$

$x = 3\lambda/4$

$n = 2$

$x = \frac{5\lambda}{4}$

$$\Delta x = x_1 - x_0 = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{2\lambda}{4} = \lambda/2$$

The position where the amplitude of the standing wave θ is maximum are called antinodes. and the distance between the successive anti-nodes is Δx.

② **For minimum Amplitude:**

$$2A \sin Kx = \text{minimum}$$
$$\sin Kx = \text{minimum}$$
$$\sin Kx = 0$$
$$\boxed{Kx = n\pi}$$

Where $n = 1, 2, 3, 4 - - -$

$$K = \frac{2\pi}{\lambda}$$
$$\frac{2\pi}{\lambda} x = n\pi$$
$$\boxed{x = \frac{n\lambda}{2}}$$

$n = 0$ | $n_1 = 1$ | $n_2 = 2$
$x_0 = 0$ | $x_1 = \lambda/2$ | $x_2 = \lambda$

$n_3 = 3$ | $n = 4$
$x_3 = \frac{3\lambda}{2}$ | $x_4 = 2\lambda$

$$\Delta x = x_1 - x_0$$
$$= \frac{\lambda}{2} - 0 = \frac{\lambda}{2}$$

The position of minimum amplitude in the standing wave is called nodes

Ques. Diff b/w Dispersive and non-Dispersive wave

Dispersive wave	Non-dispersive wave
→ Here phase velocity is dependent of its frequency or wavelength	→ Here phase velocity is independent of its frequency & wavelength.
→ Waves diff f and λ has different velocity in the medium	→ wave of diff f or λ ~~have~~ travel with the same speed in it.
→ In it wave function changes	→ In it, wave function doesn't changes.
→ eg:-	→ eg:- air for sound

Ques; Diffrence between Progressive and Stationary Waves:

Progressive waves	Stationary waves
→ The amplitude of each particle on the string is same.	→ The amplitude of the particle varies from zero at the nodes to maximum at antinodes.
→ The phase of any two consecutive particles are not same	→ The phase of any two consecutive particles are same.
→ No particle is permanently at rest.	→ The particles at the nodes are permanently at rest.
→ energy is continously transported across every cross- section of the string	→ No net transportation of energy takes place across any cross section of the string.

Ques→ Diffrence between Longitudnal and transverse wave?

Soln.

Longitudnal waves	Transverse Waves
→ In this particle movement is parallel to the wave direction.	→ In this particle movement is perpendicular to wave direction.
→ It can travel through solids, liquids and gases.	→ It can typically travel through solids and on the surfaces of liquids.
→ In this compression and rarefaction is present	→ In this compression and rarefaction is absent.
eg :- sound waves	eg :- Light waves.