09/09/2024

Unit-2

Calculas - I

Rolle's Theorem

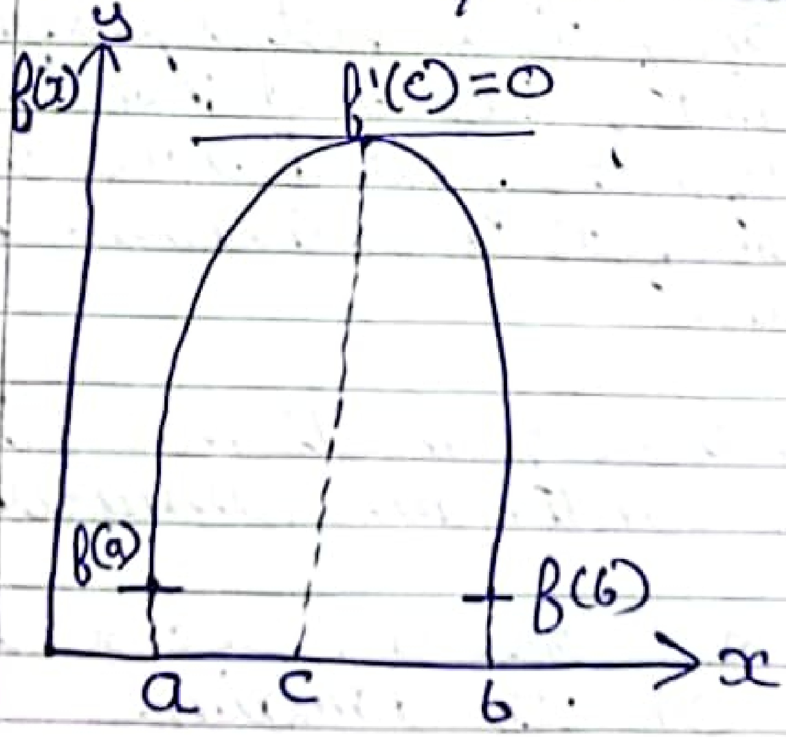
Let $f(x)$ be a function defined on ~~closed~~ $[a, b]$ such that

(a) $f(x)$ is continoues on closed interval $[a, b]$

(b) $f(x)$ is ~~deriavble~~ derivable on (a, b)

(c) $f(a) = f(b)$

Then there exists atleast one real no $c \in (a, b)$ such that $f'(c) = 0$



Remarks →

→ How to check $f(x)$ is continous on $[a,b]$ and derivable on (a,b)

1) Every derivable $f(x)$ is continoues

2) $f(x)$ is not continoues if $f(x)$ is infinite and imaginary. on $[a,b]$

$$f(x) = \frac{1}{x} \text{ on } [0,1]$$
$$= \frac{1}{0} = \infty \quad [1,2]$$
$$f(x) = \sqrt{1-x} \text{ on } [2,3]$$
$$= \sqrt{-1}$$

3) Every polynomial is continoues and derivable on every where

4) If function $f(x)$ is not a polynomial then we find $f'(x)$ and then if $f'(x)$ is finite definite, finite & real on $[a,b]$ (or (a,b)) then $f(x)$ is derivable. Hence it is continous.

5) Exponential function, $\sin x$, $\cos x$ are continous every where

6) $\tan x$, $\cot$, $\operatorname{cosec} x$, $\sec x$ are continoues on its domain.

/ /

1) Sum difference product etc. of continuous function are continuous

Q.1) Prove the Roll's theorem.

$$f(x) = x^2 - 5x + 6 \text{ on } [2, 3]$$

Ans. Since, every polynomial is continuous and derivable on everywhere so, therefore

a.) $f(x)$ is continuous on $[2, 3]$.

b.) $f(x)$ is derivable on $(2, 3)$.

(c) $f(2) = (2)^2 - 5(2) + 6$

$= 4 - 10 + 6$

$= 0$

$f(3) = 0$

Hence, all the three condition of Roll's theorem satisfy so therefore there exists one real no. $c \in (2, 3)$ such that $f'(c) = 0$

$$f(x) = x^2 - 5x + 6$$
$$f'(x) = 2x - 5$$
$$f'(c) = 2c - 5$$

$\because f'(c) = 0$

$2c - 5 = 0$

$\boxed{c = \frac{5}{2}}$ or $\boxed{c = 2.5} \in (2, 3)$

Hence, Roll's theorem verified.

Ques) Verify the Rolle's theorem of the $f(x) = \sin 2x$ on $[0, \pi/2]$

Sol^n (a) $f(x)$ is continuous on $[0, \frac{\pi}{2}]$

(b) $f'(x) = 2\cos 2x$ = definite, finite on real

Since $\cos\theta$ always takes a value between -1 to 1.

$f(x)$ is derivable on $(0, \frac{\pi}{2})$

(c) $f(0) = 0$
$f(\frac{\pi}{2}) = 0$

On All three condition statisfy, then one real no $c \in (a,b)$ such that $f'(c) = 0$

$$f'(x) = 2\cos 2x$$
$$f'(c) = 2\cos 2c$$
$$f'(c) = 0$$
$$2\cos 2c = 0$$
$$\cos 2c = 0$$
$$\cos 2c = \cos \frac{\pi}{2}$$

$$2c = \frac{\pi}{2} \Rightarrow \boxed{c = \frac{\pi}{4}} \in (0, \frac{\pi}{2})$$

Ques.3) Verify the Rolle's theorem

(i) $f(x) = (x^2 - 4x + 3)e^{2x}$ on $[1, 3]$

(ii) $f(x) = (x-a)^m (x-b)^n$ on $[a, b]$

10/09/24

Langrange's Mean Value Theoram:

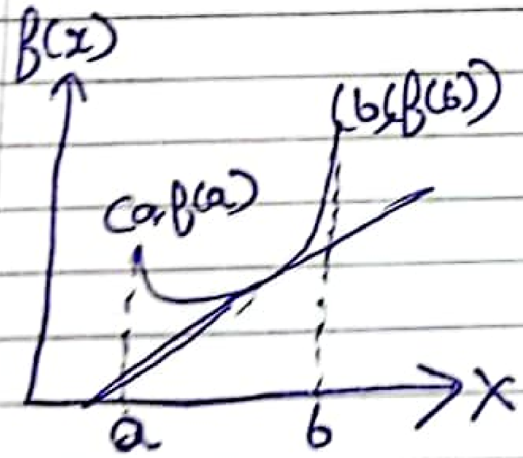
Let $f(x)$ be a function defines on $[a,b]$ such that

(a) $f(x)$ is continous on $[a,b]$

(b) $f(x)$ is derivable on (a,b)

There excist atleast one real no. $\in [a,b]$ such that

$$\boxed{\frac{f(b)-f(a)}{b-a} = f'(c)}$$



$\tan\theta$

$\frac{dy}{dx}$

$\frac{f(b)-f(a)}{b-a}$

Verify the Langrange's Mean Value Theoram when the $f(x)$ is

$$f(x) = 2x - x^2 \text{ on } [0,1]$$

Since, every polynomial are

continuous and derivable everywhere.

(a) $f(x)$ is continuous on $[0,1]$

(b) $f(x)$ is derivable on $(0,1)$

All the conditions of Langrange's Mean Value Theorem are satisfied

$\therefore$ a real no $\in (a,b)$ Such that

$$\frac{f(b)-f(a)}{b-a} = f'(c) \quad -①$$

Now,

$$f'(x) = 2-2x$$
$$f'(c) = 2-2c$$

$$f(1) = 1$$
$$f(0) = 0$$

From eq^n ①

$$\cancel{\frac{0-1}{0-1}} \quad \frac{1-0}{1-0} = 2-2c$$

$$1 = 2-2c$$
$$-2c = -1$$
$$\boxed{c = \frac{1}{2}} \in (0,1)$$

Ques2) Use Mean Value theorem to prove $1+x < e^x < 1+xe^x \ \forall \ x > 0$

Q(vii) Sol. $f(x) = e^x$ on $(0,x)$

Since $f(x)$ is continuous and derivable on interval $(0,x)$

So, All the conditions of Mean Value Theorem are satisfied

$$\exists\, c \in (a,b) \text{ s.t. } \frac{f(b)-f(a)}{b-a} = f'(c) \quad (1)$$

$f'(x) = e^x$

$f'(c) = e^c$

$f(x) = e^x$

$f(0) = e^0$

By Eqn (1)

$$\frac{e^x - e^0}{x-0} = e^c \quad (2)$$

As, $0 < c < x$

$$e^0 < e^c < e^x$$

$$e^0 < \frac{e^x - e^0}{x-0} < e^x$$

$$e^0 < \frac{e^x - 1}{x} < e^x$$

Multiply by x.

$x < e^x - 1 < xe^x$ Add 1 on both sides

$\Rightarrow x+1 < e^x < xe^x + 1$

Q3) Verify Mean Value Theorem of $f(x) = \sin x$ on $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$

Ques4.) $f(x) = x^{1/3}$ on $[-1, 1]$

Ques5) $f(x) = |x|$ on $[-1, 1]$

Ques6.) Use mean value Theorem to prove that

$$\frac{x}{1+x} \cancel{\log} < \log(1+x) < x, \quad x>0$$

10/09/24

Taylor's Theorem with Lagrange's form of remainder:

If a function $f(x)$ is defined on $[a, a+h]$ such that

(a) $f, f', f'', \ldots, f^{n-1}$ are continuous for x on $[a, a+h]$

(b) nth derivative of a $f(x)$ exists in $(a, a+h)$ ~~then~~

Then There exist atleast one real no. θ; $0<\theta<1$

Taylor polynomial

such that

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + \cdots \frac{h^{n-1}}{(n-1)!}f^{(n-1)}(a) + \underbrace{\frac{h^n}{n!}f^n(a+\theta h)}_{\text{Remainder}}$$

13/09/2024

Taylor Series:

If $f(x)$ posses the all the derivatives than $f(a+h)$ can be expand into infinite series which is defined as

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!} f''(a) + \cdots \infty$$

Remark:

Put $h = x-a$ in above taylor series

$$f(a+x-a) = f(x) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \cdots \infty$$

this series also known as taylor's series in the power of $(x-a)$.

Ques1) Show that $\log(a+h) = \log a + \frac{h}{a} - \frac{h^2}{2a^2} + \cdots$

Soln By taylor series

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!} f''(a) + \cdots \infty \quad \text{--- (1)}$$

By the Eqn no. (1).

Let

$$f(x) = \log x \qquad f(a) = \log a$$

$$f'(x) = \frac{1}{x} \qquad f'(x) = \frac{1}{a}$$

$$f''(x) = \frac{-1}{x^2} \qquad f''(a) = \frac{-1}{a^2}$$

Substituting these values in eqn ①.

$$\log(a+h) = \log a + h\left(\frac{1}{a}\right) + \frac{h^2}{2!}\left(\frac{-1}{a^2}\right) + \cdots \infty$$

Ques 2) Expand the $\sin x$ in the power of $\left(x - \frac{\pi}{2}\right)$?

Soln The Taylor series will be

$$f\left(\frac{\pi}{2} + x - \frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right) + \left(x - \frac{\pi}{2}\right) f'\left(\frac{\pi}{2}\right) + \frac{(x-a)^2}{2!} f''\left(\frac{\pi}{2}\right) + \cdots \infty$$

Now,

$$f(x) = \sin x \Rightarrow f\left(\frac{\pi}{2}\right) = 1$$

$$f'(x) = \cos x \Rightarrow f'\left(\frac{\pi}{2}\right) = 0$$

$$f''(x) = -\sin x \Rightarrow f''\left(\frac{\pi}{2}\right) = -1$$

$$f'''(x) = -\cos x \Rightarrow f'''\left(\frac{\pi}{2}\right) = 0$$

$$f''''(x) = \sin x \Rightarrow f''''\left(\frac{\pi}{2}\right) = 1$$

Putting all these values in (1).

$$f(x) = 1 + \left(x - \frac{\pi}{2}\right)(0) + \frac{\left(x-\frac{\pi}{2}\right)^2}{2!}(-1) + \frac{\left(x-\frac{\pi}{2}\right)^3}{3!}(0) + \frac{\left(x-\frac{\pi}{2}\right)^4}{4!}(1) + \cdots$$

$$f(x) = 1 - \frac{\left(x-\frac{\pi}{2}\right)^2}{2} + \frac{\left(x-\frac{\pi}{2}\right)^4}{24} + \cdots \infty$$

3) Expand $\log \sin x$ in the power of $(x-2)$?

→ The Taylor series will be

$$f(2+x-2) = f(2) + (x-2)f'(2) + \frac{(x-2)^2}{2!}f'' \quad \text{—(1)}$$

Now,

$f(x) = \log(\sin x) \Rightarrow f(2) = \log(\sin 2)$

$f'(x) = \frac{1}{\sin x}\cos x = \cot x \Rightarrow f'(x) = \cot(2)$

$f''(x) = -\operatorname{cosec}^2 x. \Rightarrow f''(2) = -\operatorname{cosec}^2(2).$

Putting all values in Eq^n (1).

$$\log(\sin x) = \log \sin 2 + (x-2)(\cot 2) - \frac{(x-2)^2}{2}\operatorname{cosec}^2 x$$

Ques 4) Expand $\tan^{-1} x$ in the powers of $(x-\frac{\pi}{4})$?

Soln The Taylor series will be

$$f(\tfrac{\pi}{4}+x-\tfrac{\pi}{4}) = f(\tfrac{\pi}{4}) + (x-\tfrac{\pi}{4}) f'(\tfrac{\pi}{4}) + \frac{(x-\frac{\pi}{4})^2}{2!} f''(\tfrac{\pi}{4}) + \cdots \infty$$

Now,

$$f(x) = \tan^{-1} x \Rightarrow \cancel{\tan}\ f(\tfrac{\pi}{4}) = \tan^{-1}\tfrac{\pi}{4}$$

$$f'(x) = \frac{1}{1+x^2} \Rightarrow f'(\tfrac{\pi}{4}) = \frac{1}{1+(\frac{\pi}{4})^2}$$

$$f''(x) = (1+x^2)^{-1}$$

$$= -(1+x^2) \frac{d}{dx}(1+x^2)$$

$$= -(1+x^2)\, 2x$$

$$= -2x(1+x^2) \Rightarrow f''(\tfrac{\pi}{4})$$

$$= -\frac{\pi}{2}\left(1+(\tfrac{\pi}{4})\right)^2$$

Putting in Eqn ①.

$$\tan^{-1} x = \tan^{-1}(\tfrac{\pi}{4}) + \frac{(x-\frac{\pi}{4})}{} \left(\frac{1}{1+(\frac{\pi}{4})^2}\right)$$

$$- \frac{(x-\frac{\pi}{4})^2}{2}\left(\frac{\pi}{2}\right)\left(1+(\tfrac{\pi}{4})\right)^2 \cdots \infty$$

12/4/2021

11 Maclaurin theorem:

Let $f(x)$ is defined on $[0, x]$ such that

(1) $f, f', f'' \ldots f^{n-1}$ Continuous on $[0, x]$

(2) f^{n} exist on $(0, h)$

There exist a $\theta \in (0,1)$ such that

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \ldots + \frac{x^{n-1}}{(n-1)!} f^{n-1}(0) + \frac{x^n}{n!} f^{n}(\theta x)$$

Maclaurin Series:

If $f(x)$ posses the all derivative then $f(x)$ can be expand into infinite series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \ldots + \infty$$

Ques1) Expand the series of $\sin x$ with the help of Maclaurin Series.

Soln. The maclaurin series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \cdots \infty \quad (1)$$

$$f(x) = \sin x \Rightarrow f(0) = 0$$

$$f'(x) = \cos x \Rightarrow f'(0) = 1$$

$$f''(x) = -\sin x \Rightarrow f''(0) = 0$$

$$f'''(x) = -\cos x \Rightarrow f'''(0) = -1$$

$$f''''(x) = \cancel{\cos} x \Rightarrow f''''(0) = 1$$

Putting these in (1)

$$\sin x = 0 + x + 0 + \frac{x^3}{3!}(-1) + \cancel{x^4} + \frac{x^5}{5!}(1)$$

$$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} + \cdots \infty$$

16/09/2024

Inditerminant Form:

$\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0 \times \infty, 1^{\infty}, 0^{0}, \infty^{0}$

$\frac{0}{0}$ form:

Apply L'Hopital Rule until we get a finite value

$\frac{\infty}{\infty}$ → same as 0/0.

$\infty - \infty$ → Take LCM and then convert the problem into 0/0 form or ∞/∞ form and same as 0/0 or ∞/∞.

$0 \times \infty$ → Take one value to the denominator and convert the problem into 0/0 or ∞/∞. and same as 0/0 or ∞/∞.

$1^{\infty}, 0^{0}, \infty^{0}$ → Take log in problem and convert it into $0 \times \infty$ form and same as $0 \times \infty$.

Ques 4) Evaluate $\lim_{x \to 0} \left(\frac{1}{x} - \operatorname{Cosec} x\right)$

Solⁿ $\lim_{x \to 0} \frac{1}{x} - \frac{1}{\operatorname{Sin} x} = \infty - \infty$

$\lim_{x \to 0} \frac{\operatorname{Sin} x - x}{x \operatorname{Sin} x} = \frac{0}{0}$

Applying L' Hopital Rule

$\lim_{x \to 0} \frac{\operatorname{Cos} x - 1}{x \operatorname{Cos} x + \operatorname{Sin} x} = \frac{0}{0}$

Applying L' Hopital Rule

$\lim_{x \to 0} \frac{-\operatorname{Sin} x}{-x \operatorname{Sin} x + \operatorname{Cos} x + \operatorname{Cos} x} = \frac{-0}{0+1+1}$

$= 0$ //. finite value.

Ques 5.) Evaluate $\lim_{x \to \infty} x \cdot \tan \frac{1}{x}$

Soln $\lim_{x \to \infty} x \cdot \tan \frac{1}{x} = \infty \times 0$

$\lim_{x \to \infty} \frac{\tan 1/x}{1/x} = \frac{0}{0}$

Applying L' Hopital Rule

16-01-2021

L'Hopital Rule:

If $\lim_{x\to a} \frac{f(x)}{g(x)}$ take the form of $0/0$ or ∞/∞

then. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{f'(x)}{g'(x)}$

provided $f'(x) \neq 0$ & $g'(x) \neq 0$.

For ∞/∞

provided $\frac{f'(x)}{g'(x)}$ = exist = finite or infinite.

Ques1) Evaluate the $\lim_{x\to 0} \frac{x - \sin x}{x^3}$

Sol^n $\lim_{x\to 0} \frac{x-\sin x}{x^3} = \frac{0}{0}$

Apply L'Hopital Rule.

$\lim_{x\to 0} \frac{1-\cos x}{3x^2}$

$= \frac{1-\cos(0)}{0} = \frac{0}{0}$

Apply L'Hopital Rule

$\lim_{x\to 0} \frac{0+\sin x}{6x} = \frac{0}{0}$

Apply L' Hopit al Rule.

$$\lim_{x \to 0} \frac{\cos x}{6}$$

$$= \frac{1}{6} //.$$

Ques 2:) Evaluate $\lim_{x \to 0} \frac{x - \sin x}{e^{\sin x} - e^{x}}$

Ques 3:) Evaluate $\lim_{x \to a} \frac{\log(x-a)}{\log(e^x - e^a)}$

Sol^n $\lim_{x \to a} \frac{\log(x-a)}{\log(e^x - e^a)} = \frac{\infty}{\infty}$

Applying L' Hopit al Rule.

$$\lim_{x \to a} \frac{1/(x-a)}{e^x/(e^x - e^a)}$$

$$\lim_{x \to a} \frac{e^x - e^a}{e^x(x-a)} = \frac{0}{0}$$

Applying L' Hopit al Rule.

$$\lim_{x \to a} \frac{e^x}{e^x + (x-a)e^x} = \frac{e^a}{e^a}$$

$$= 1 //. \text{ = finite value.}$$

Let $y = \lim_{x\to 0} (\cos x)^{\cot^2 x}$

Taking log both side

$\log y = \lim \log \left(\lim_{x\to 0} \cos x^{\cot^2 x} \right)$

$\log y = \lim_{x\to 0} \log \cot^2 x \log \cos x$

$= \infty \times 0$

$\log y = \lim_{x\to 0} \frac{\log \cos x}{\tan^2 x} = \frac{0}{0}$

Apply L'Hopital Rule.

$\log y = \lim_{x\to 0} \frac{1/\cos x \times -\sin x}{2\tan x \cdot \sec^2 x}$

$\log y = \lim_{x\to 0} \frac{-\tan x}{2\tan x \cdot \sec^2 x}$

$\log y = \lim_{x\to 0} \frac{-1}{2 \cdot \sec^2 x} = -\frac{1}{2}$

$\Rightarrow \log y = \frac{-1}{2}$

$\boxed{y = e^{-1/2}}$ $\boxed{\lim_{x\to 0} (\cos x)^{\cot^2 x} = e^{-1/2}}$

When n is +ve	$n = -ve$.
$\Gamma 6 = (6-1)! = 5!$	$\Gamma -5 = \dfrac{\Gamma -5+1}{-5}$
	$= \dfrac{\Gamma -4}{-5}$
	$= \dfrac{\Gamma -4+1}{-5 \times -4}$
	$= \dfrac{\Gamma -3}{20} = \dfrac{\Gamma -2}{-60}$
	$\dfrac{\Gamma -1}{120} = \dfrac{\Gamma 0}{-120} = \infty$

Q1.) $\Gamma \frac{5}{2} = \left(\frac{5}{2}-1\right) \Gamma \frac{5}{2}-1$

$= \frac{3}{2} \Gamma \frac{3}{2}$

$= \left(\frac{3}{2}-1\right) \Gamma \frac{3}{2}-1$

$= \frac{1}{2}\frac{3}{2} \Gamma \frac{1}{2} = \frac{3}{4}\sqrt{\pi}$

$\boxed{= \frac{3}{4}\sqrt{\pi}}$

Q2.) $\Gamma \frac{-5}{2}$

Q3.) $\Gamma \frac{-7}{2}$

Note :- $\Gamma n+1 = \int_0^\infty e^{-x} x^n \, dx$

/ /

Que. Evaluate $\lim_{x\to 0} (\operatorname{cosec} x)^{1/\log x}$

Soln. $(\operatorname{cosec} 0)^{\frac{1}{\log 0}} = \infty^0$

$$y = \lim_{x\to 0} \operatorname{cosec} x^{1/\log x}$$

$$\log y = \lim_{x\to 0} \frac{1}{\log x} \log \operatorname{cosec} x = 0 \times \infty$$

$$\log y = \lim_{x\to 0} \frac{\log \operatorname{cosec} x}{1/\log x} = \frac{\infty}{\infty}$$

Apply L'Hospital Rule.

$$\log y = \lim_{x\to 0} \frac{\frac{1}{\operatorname{cosec} x} - \operatorname{cosec} x \cot x}{\frac{1}{x}}$$

$$\log y = \lim_{x\to 0} \frac{-\cancel{x} \cdot \cot x}{1/x}$$

$$= 0$$

$$\cancel{\log y = 0}$$

$$\log y = \cancel{\lim_{x\to 0}}$$

$$\log y = \frac{-\cos x \cdot x}{\sin x}$$

$$= \frac{\pm \sin x \times \overline{\circ} \cos x)}{\cos x}$$

$$= \frac{-1}{1}$$

$$\cancel{\lim_{x\to 0} (\operatorname{cosec} x)^{1/\log x} =}$$

$$\cancel{\lim_{x\to 0} \operatorname{cosec} x^{1/\log x}}$$

$$\log y = -1$$

$$\boxed{\lim_{x\to 0} (\operatorname{cosec} x)^{1/\log x} = e^{-1}} \qquad \boxed{y = e^{-1}}$$

Ques6) Evaluate $\int_0^\infty x^{1/2} e^{-x^3} dx$

Ques7) Evaluate $\int_0^\infty e^{-x^2} dx$

Ques8) Evaluate $\int_0^\infty x^3 e^{-x} dx$

Ques9) Evaluate $\int_0^\infty x^6 e^{-2x} dx$

Type-3

$$\int_0^1 x^m (\log x)^n dx$$

Put

$$\log x = -t$$
$$x = e^{-t}$$

Ques10) Evaluate $\int_0^1 x (\log(x))^4 dx$

Soln Put $\log x = -t$

$$x = e^{-t}$$
$$dx = -e^{-t} dt$$

If $x = 0 \quad t = \infty$

$x = 1 \quad t = 0$

Beta, Gama Functions:

Gamma Functions:

The gamma function is defined as

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1}\,dx \quad (n > 0)$$

This function is also known as Euler's integral of second kind.

Some properties of gamma function:

1. $\Gamma n = (n-1)\,\Gamma(n-1)$
   $= (n-1)(n-2)\,\Gamma(n-2)$

2. $\Gamma n = (n-1)!$
   eg:- $\Gamma 5 = (5-1)! = 4!$

3. $\Gamma n = \dfrac{\Gamma n+1}{n}$ or.
   $\Gamma n-1 = \dfrac{\Gamma n}{n-1}$

4. $\Gamma 0 = \infty$

5. $\Gamma 1 = 1$

6. $\Gamma \frac{1}{2} = \sqrt{\pi}$

$$\int_{\infty}^{0} (e^{-t})^{4} (-t)^{4} e^{-t} (-dt)$$

$$= -\int_{\infty}^{0} e^{-4t} t^{4} e^{-t} dt.$$

$$= -\int_{\infty}^{0} e^{-5t} t^{4} dt$$

$$= \int_{0}^{\infty} e^{-5t} t^{4} dt$$

Put

$$5t = v$$

$$t = \frac{1}{5} v$$

$$dt = \frac{1}{5} dv.$$

If $t \Rightarrow 0$ $V = 0$

$t = \infty$ $V = \infty$

$$= \left(\frac{1}{5}\right)^{5} \int_{0}^{\infty} e^{-v} v^{4} dv$$

$$= \frac{1}{3125} \int_{0}^{\infty} e^{-v} v^{4} dv$$

$$= \frac{1}{3125} \Gamma(4+1)$$

$$= \frac{1}{3125} \Gamma 5$$

$$= \frac{(4!)}{3125} = \frac{24}{3125}$$

Type 2

$$\int_0^\infty x^m e^{-ax^n} dx$$

Put $ax^n = t$

Ques 5) Evaluate $\int_0^\infty x e^{-x^4} dx$

Put

$x^4 = t$

$x = t^{1/4}$

$dx = \frac{1}{4} t^{-3/4} dt$

If $x=0$ $t=0$

$x=\infty$ $t=\infty$

$$\frac{1}{4}\int_0^\infty t^{1/4} e^{-t} t^{-3/4} dt$$

$$= \frac{1}{4}\int_0^\infty e^{-t} t^{-1/2} dt.$$

$$= \frac{1}{4}\Gamma\left(\frac{-1}{2}+1\right)$$

$$= \frac{1}{4}\Gamma\left(\frac{1}{2}\right)$$

$$\boxed{= \frac{1}{4}\sqrt{\pi}}$$

~~Ques 6 (i) $\int_0^\infty x^{1/2} e^{-x^3} dx$~~

~~(ii) $\int_0^\infty x e^{-x^2} dx$~~