S.H.M

Periodic Motion → (Harmonic motion)

* If a body or system repeats its motion after a fixed interval of time period, called Periodic motion or Harmonic motion.

⇒ **Oscillatory motion** →

* If a body or system repeats its motion on the same path again & again after a fixed interval of time.

⇒ **Simple Harmonic motion** →

* It is a special case of oscillatory motion in which body moves under the influence of a restoring force which always acts towards the mean position.

For S.H.M. Two conditions must be satisfied :-

i) $a \propto x$

ii) a → Always towards mean position

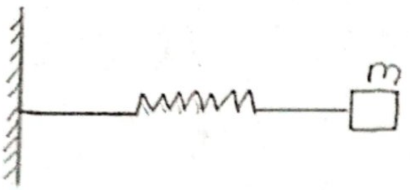
In SHM, the acceleration of the particle is directly proportional to displacement and is directed opposite to the displacement. It is represented as $x = A\cos(\omega t + \phi)$

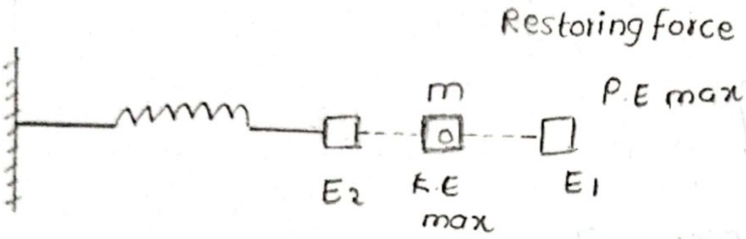
OR

$x = A\sin(\omega t + \phi)$

Scanned with OKEN Scanner

* Mechanical Oscillator *





Equation of motion :-

$F \propto -x$

$F = -sx$ [s is spring constant]

$ma = -sx$ [$F = ma$]

We know,

$v = \frac{dx}{dt}$

$a = \frac{d^2x}{dt^2}$

$m \cdot \frac{d^2x}{dt^2} = -sx$

$\frac{d^2x}{dt^2} + \frac{s}{m}x = 0$

Here, $\frac{s}{m} = \omega_0^2 \Rightarrow \omega_0 = \sqrt{\frac{s}{m}}$

ω = Angular frequency

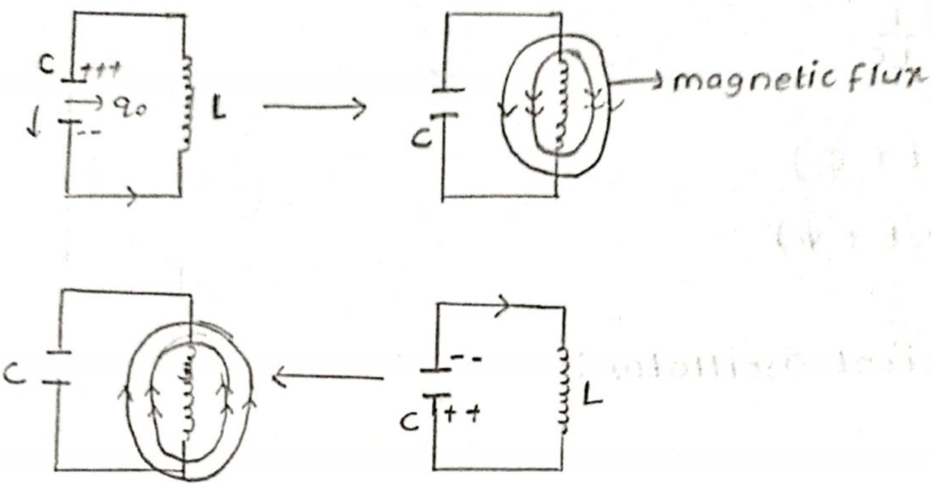
ω_0 = Angular frequency for S.H.M.O

so,

$$\boxed{\frac{d^2x}{dt^2} + \omega_0^2 x = 0}$$

Scanned with OKEN Scanner

* Electrical Oscillator :-



L , C

$$\mathcal{E}_c = \frac{q}{c} \quad , \quad \mathcal{E}_L = -L\frac{dI}{dt}$$

$$\mathcal{E}_c = \mathcal{E}_L$$

$$\frac{q}{c} = -L\frac{dI}{dt}$$

$$\frac{q}{c} + L\frac{dI}{dt} = 0$$

We know, $I = \dfrac{dq}{dt}$

$$\frac{q}{c} + L \cdot \frac{d}{dt}\left[\frac{dq}{dt}\right] = 0$$

$$L \cdot \frac{d^2q}{dt^2} + \frac{q}{c} = 0$$

$$\frac{d^2q}{dt^2} + \frac{q}{L \cdot C} = 0$$

We know, $\dfrac{1}{LC} = \omega_0^2$

Scanned with OKEN Scanner

$$\boxed{\frac{d^2q}{dt^2} + q\omega_0^2 = 0}$$

$$\omega_0^2 = \frac{1}{LC}, \quad \omega_0 = \sqrt{\frac{1}{LC}}$$

$$q = A\cos(\omega_0 t + \phi)$$

$$q = q_0 \cos(\omega_0 t + \phi)$$

* Energy in Mechanical Oscillator :-

$$x = A\cos\omega_0 t$$

$$K.E = \frac{1}{2} m v^2$$

$$v = \frac{dx}{dt} = -A\sin\omega_0 t \cdot \omega_0$$

$$K.E = \frac{1}{2} m \left[-A\omega_0 \sin\omega_0 t\right]^2$$

$$K.E = \frac{1}{2} m A^2 \omega_0^2 \sin^2\omega_0 t \quad \text{———} \quad (i)$$

P.E = workdone

$$dU_p = dw = -F.dx$$

$$dU_p = Sx.dx \quad [f = -Sx]$$

$$P.E = \int Sx\,dx$$

$$P.E = \frac{Sx^2}{2}$$

$$P.E = \frac{S}{2} A^2 \cos^2\omega_0 t \quad \text{———} \quad (ii)$$

We know,

$$\frac{S}{m} = \omega_0^2 \Rightarrow S = m\omega_0^2$$

Scanned with OKEN Scanner

Putting value of s in eq (ii)

$$P.E = \frac{m\omega_0^2 A^2 \cos^2 \omega_0 t}{2} \quad \text{——— (iii)}$$

Total Energy = K.E + P.E

$$T.E = \frac{1}{2} m\omega_0^2 A^2 [\cos^2 \omega_0 t + \sin^2 \omega_0 t]$$

$$\boxed{T.E = \frac{1}{2} m\omega_0^2 A^2}$$

* Energy in Electrical oscillator :-

$$dE_{e_c} = \frac{q}{c} \cdot dq$$

$$E_{e_c} = \frac{1}{c} \int q \cdot dq$$

$$\boxed{E_{e_c} = \frac{q^2}{2c}}$$ Energy stored in capacitor

$$Ee = \frac{1}{2} \frac{q^2}{c} \quad \text{——— (i)}$$

$$dE_L = L \cdot \frac{dI}{dt} \times dq$$

We know, $\frac{dq}{dt} = I$, $dq = I \cdot dt$

$$dE_L = L \cdot \frac{dI}{dt} \times I \cdot dt$$

$$dE_L = LIdI$$

$$E_L = L \int IdI$$

$$E_L = \frac{1}{2} LI^2 \quad \text{——— (ii)}$$

Energy in inductor

Scanned with OKEN Scanner

$q = q_0 \cos \omega_0 t$

$$I = \frac{dq}{dt} = -q_0 \sin \omega_0 t \, q_0 \omega_0$$

Now, Put value of q in eq (i)

$$E_e = \frac{1}{2C}(q_0^2 \cos^2 \omega_0 t) \text{ ———— (iii)}$$

Put value of I in eq (ii)

$$E_L = \frac{1}{2} L . q_0^2 \sin^2 \omega_0 \omega_0^2 t$$

$$E_L = \frac{1}{2} L . q_0^2 \omega_0^2 \sin^2 \omega_0 t \text{ ———— (iv)}$$

We know,

$$\frac{1}{LC} = \omega_0^2 \quad , \quad \frac{1}{C} = L\omega_0^2$$

Put this value in eq (iii)

$$E_e = \frac{1}{2} L \omega_0^2 q_0^2 \cos^2 \omega_0 t \text{ ———— (v)}$$

Total Energy = $E_L + E_e$

$$T.E = \frac{1}{2} L \omega_0^2 q_0^2 [\sin^2 \omega_0 t + \cos^2 \omega_0 t] \; [iv + v]$$

$$\boxed{T.E = \frac{1}{2} L q_0^2 \omega_0^2}$$

Scanned with OKEN Scanner

* **Forced Mechanical Oscillator**: An oscillator to which continuous excitation is provided by some external agency.

i) Transient Behaviour: When it is going to oscillate with their own frequency and external force does not effect it and it is zero then its behaviour is transient behaviour.

OR

When external forces is not applied and the oscillations are free, this behaviour is known as Transient behaviour.

ii) Steady state behaviour: When external forces is applied and the oscillations are forced oscillations. This behaviour is known as steady state behaviour.

Equation of motion:

$$F_1 = -Sx$$

$$F_2 = -r \cdot \frac{dx}{dt}$$

$$F_3 = F_0 e^{i\omega t}$$

According to Newton second law $\Rightarrow F_1 + F_2 + F_3$

$$-Sx + -r \cdot \frac{dx}{dt} + F_0 e^{i\omega t} = m\frac{d^2x}{dt^2}$$

Scanned with OKEN Scanner

Steady state Behaviour

$$m\frac{d^2x}{dt^2} + Sx + r\frac{dx}{dt} = F_0 e^{i\omega t} \text{ ——— (i)}$$

This is equation of motion forced Mechanical oscillator.

$$x = Ae^{i\omega t} \text{ ——— (ii)}$$

$$\frac{dx}{dt} = i\omega A e^{i\omega t}$$

Again. diff. w.r. to x

$$\frac{d^2x}{dt^2} = (i\omega)^2 A e^{i\omega t}$$

$$\frac{d^2x}{dt^2} = -\omega^2 A e^{i\omega t}.$$

Put the values in eq (i).

$$-m.\omega^2 A e^{i\omega t} + S.Ae^{i\omega t} + r.i\omega A e^{i\omega t} = F_0 e^{i\omega t}$$

$$-m\omega^2 A + SA + r.i\omega A = F_0$$

$$A[-m\omega^2 + S + ri\omega] = F_0$$

$$A = \frac{F_0}{-m\omega^2 + S + ri\omega}$$

$$A = \frac{F_0}{\omega\left[-m\omega + \frac{S}{\omega} + ri\right]}$$

$$A = \frac{F_0}{\omega\left[-m\omega + \frac{S}{\omega} + ri\right]} \times \frac{-i}{-i}$$

Scanned with OKEN Scanner

$$A = \frac{-F_0 i}{\omega\left[im\omega + r - \frac{s}{\omega} i\right]}$$

$$A = \frac{-F_0 i}{\omega\left[r + i\left[m\omega - \frac{s}{\omega}\right]\right]}$$

We know that,

$$r + i(m\omega - s/\omega) = Z_m$$

where Z_m = mechanical Impedence

$m\omega - \frac{s}{m}$ = reactance

$m\omega \rightarrow$ Interial reactance

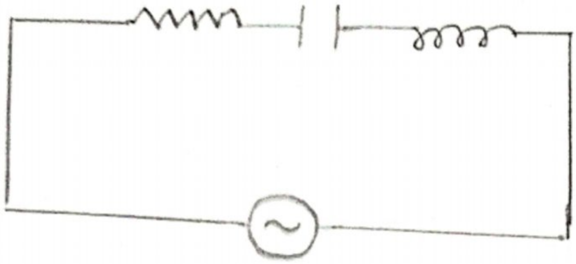
$\frac{s}{\omega} \rightarrow$ elastic reactance

So,

$$A = \frac{-F_0 i}{\omega Z_m}$$

Scanned with OKEN Scanner

* Forced Electrical Oscillator :



$\vec{E} = E_0 e^{i\omega t}$

Now,

Across resistor,

$$E_0 = R.I = R.\frac{dq}{dt}$$

$$E_c = \frac{q}{c} \quad , \quad E_L = -L\frac{d^2q}{dt^2}$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E_0 e^{i\omega t} \quad \text{———} \quad (i)$$

* Two types of behaviour :

- Transient $\longrightarrow$ No external force, free oscillation
- Steady state $\longrightarrow$ External force, damped oscillation

Now,

$$q = Ae^{i\omega t}$$

$$\frac{dq}{dt} = Ae^{i\omega t}.i\omega \Rightarrow \frac{dq}{dt} = i\omega.Ae^{i\omega t}.$$

$$\frac{d^2q}{dt^2} = (i\omega)^2 Ae^{i\omega t}$$

$$\frac{d^2q}{dt^2} = -\omega^2.Ae^{i\omega t} \quad \text{———} \quad (ii)$$

Put the values in eq (i)

Scanned with OKEN Scanner

$$-L.\omega^2 Ae^{i\omega t} + R.i\omega Ae^{i\omega t} + \frac{Ae^{i\omega t}}{c} = E_0 e^{i\omega t}$$

$$-L\omega^2 A + R.i\omega A + \frac{A}{c} = E_0$$

$$A\left[-L\omega^2 + Ri\omega + \frac{1}{c}\right] = E_0$$

$$A = \frac{E_0}{-\omega^2 L + Ri\omega + \frac{1}{c}}$$

$$A = \frac{E_0}{\omega\left[-L\omega + iR + \frac{1}{\omega c}\right]}$$

$$A = \frac{E_0}{\omega\left[iR + \frac{1}{X_c} - X_L\right]}$$

$$A = \frac{E_0}{\omega\left[iR + \left[\frac{1}{X_c} - X_L\right]\right.} \times \frac{-i}{-i}$$

$$A = \frac{-E_0 i}{\omega\left[R + i\left(X_L - \frac{1}{X_c}\right)\right]}$$

We know that,

$$Z_m = R + i\left(X_L - \frac{1}{X_c}\right), \quad Z_m = \text{Electrical Impedence.}$$

So,

$$A = \frac{-E_0 i}{\omega Z_m}$$

Scanned with OKEN Scanner

* Power absorbed by S.H Oscillator:

Power Inforced by M.O $\Rightarrow$

Power is defined as rate of doing work.

$$P = \frac{dw}{dt} \Rightarrow P = F\frac{dx}{dt} \quad \text{———— (i)}$$

$$F = F_0 \cos\omega t$$

$$x = \frac{F_0}{\omega Z_m}\sin(\omega t - \phi)$$

$$\frac{dx}{dt} = \frac{F_0 \cancel{\omega}}{\cancel{\omega} Z_m}\cos(\omega t - \phi) = \frac{F_0}{Z_m}\cos(\omega t - \phi)$$

$$P = F_0 \overset{\cos}{\wedge}\omega t \cdot \frac{F_0}{Z_m}\cos(\omega t - \phi)$$

$$P = \frac{F_0^2}{Z_m}\cos(\omega t - \phi)\cos\omega t.$$

Average power over one cycle or one time period.

$$P_{av} = \int_0^T P\,dt$$

$$P_{av} = \frac{1}{T}\int_0^T \frac{F_0^2}{Z_m}\cos(\omega t - \phi)\cos\overset{\omega t}{\phi}\,dt$$

$$P_{av} = \frac{1}{t}\int_0^T = \frac{1}{T}\frac{F_0^2}{Z_m}\int_0^T \cos(\omega t - \phi)\cos\overset{\omega t}{\phi}\,dt$$

$$P_{av} = \frac{F_0^2}{Z_m.T}\int_0^T (\cos\omega t\cos\phi + \sin\omega t\sin\phi)\cos\omega t\,dt \left\{[\cos(A-B)]\right\}$$

$$= \frac{F_0^2}{Z_m.T}\int_0^T (\cos^2\omega t \cos\phi) + (\sin\omega t\cos\omega t\sin\phi)\,dt$$

Scanned with OKEN Scanner

$$\cos 2\omega t = 2\cos^2 \omega t - 1$$

$$\frac{\cos 2\omega t + 1}{2} = \cos^2 \omega t \quad , \quad 2\sin\omega t \cos\omega t = \sin 2\omega t$$

$$\sin\omega t \cos\omega t = \frac{\sin 2\omega t}{2}$$

$$P_{av} = \frac{F_0^2}{Z_m . T} \int_0^T \left(\frac{\cos 2\omega t + 1}{2}\right)\cos\phi + \left(\frac{\sin 2\omega t}{2}\right)\sin\phi \, dt$$

$$= \frac{F_0^2}{2 Z_m . T} \int_0^T \cos 2\omega t \cos\phi \, dt + \int_0^T \cos\phi \, dt + \int_0^T \sin 2\omega t \sin\phi \, dt$$

Here,

$$\int_0^T \sin 2\omega t \, dt = \int_0^T \cos 2\omega t \, dt = 0 \quad \text{———} \quad (ii)$$

so,

$$P_{av} = \frac{F_0^2}{2 Z_m . T} \cos\phi \int_0^T \cos 2\omega t \, dt + \cos\phi \int_0^T 1 \, dt + \cancel{\cos} \sin\phi \int_0^T \sin 2\omega t \, dt$$

$$P_{av} = \frac{F_0^2}{2 Z_m . \cancel{T}} \cos\phi . \cancel{T} \quad \left\{ \text{from eq (ii)} \right\}$$

$$\boxed{P_{av} = \frac{F_0^2}{2 Z_m} \cos\phi}$$

$\cos\phi$ is called Power Factor

ϕ is the phase difference.

Scanned with OKEN Scanner