12/08/2024

Unit - I

Matrices

Rank of a Matrix

→ By Elementary operations

→ By Minors

By Elementary operations

→ ① $R_i \longleftrightarrow R_j$ (or $C_i \longleftrightarrow C_j$)

→ ② $R_i \longleftrightarrow kR_j$ (or $C_i \longrightarrow kC_j$)
eg: → $R_1 \longrightarrow 2R_2$

→ ③ $R_i \longrightarrow R_i + kR_j$
eg:- $R_1 \longrightarrow R_1 + 2R_2$

By Minors

→ determinant of square sub matrix of matrix A of type $m \times n$

~~Defn) If A is a matrix of order $m \times n$ than this matrix is said to have a rank if~~

~~Defn~~ Acc to Elementary:

The number of non-zero rows (or columns) of matrix $A_{m\times n}$ is said to be a rank of the ~~matrix~~ matrix.

It is donated by $\rho(A)$

Eg:- $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 0 & 3 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix}$

$\boxed{\rho(A) = 2}$

Acc to Minor defn:

If A is a matrix of order $m \times n$ then it has a rank 'r' if

(i) it has atleast one non-zero minor of order r.

(ii) Every minor of order greater than $r+1$, if any are zero.

$$eg \rightarrow A = \begin{bmatrix} 1 & 6 & 3 & 1 \\ 3 & 4 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}_{3\times 4}$$

square 3×3, 2×2 & 1×1 minor.

$$\begin{vmatrix} 1 & 6 & 3 \\ 3 & 4 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0 \quad \left| \quad \right|_{3\times3} = 0 \quad \left| \quad \right|_{3\times3} = 0$$

$\rho(A) < 3$

$$\left|\cancel{8}\right| \quad \begin{vmatrix} 1 & 6 \\ 3 & 4 \end{vmatrix} \neq 0 \quad \begin{vmatrix} 3 & 4 \\ 0 & 0 \end{vmatrix} = 0$$

$$\boxed{\rho(A) = 2}$$

Ques Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & 3 \\ 2 & 4 & 2 & 6 \end{bmatrix}_{3\times 4}$$

Solⁿ: There are four three order minors.

$$\begin{vmatrix} 1 & 2 & 0 \\ 1 & 2 & 1 \\ 2 & 4 & 2 \end{vmatrix} = 0 \qquad \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 3 \\ 2 & 2 & 6 \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 4 & 2 & 6 \end{vmatrix} = 0 \qquad \begin{vmatrix} 1 & 2 & 1 \\ 1 & 2 & 3 \\ 2 & 4 & 6 \end{vmatrix} = 0$$

Since all the minors of order three are zero

$$\therefore \rho(A) \neq 3 \text{ i.e. } \rho(A) < 3$$

Minors of order 2 are

$$\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0 \qquad \begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix} = 2 \neq 0$$

$$\boxed{\rho(A) = 2}$$

__/__/__

Since we get a atleast minor which is non zero of order 2 & all the ~~non zero~~ minors of order 3 are zero then, by def^n of rank of the matrix

$$\boxed{\rho(A) = 2}$$

Ques2> Find the rank of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

Q3> Find the rank of $A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$

~~Solutions~~

~~Solut.~~

Ques 4> Find the rank of matrix with the help of elementary operation.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$
$R_3 \rightarrow R_3 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 2 & -1 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$C_2 \rightarrow C_2 - 2C_1$
$C_3 \rightarrow C_3 - 3C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$C_3 \rightarrow 2C_3 + C_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} \rightarrow R_1 \neq 0 \\ \rightarrow R_2 \neq 0 \\ \rightarrow R_3 = 0. \end{matrix}$$

Thus, rank of matrix A = no. of non-zero rows =

$$\boxed{\rho(A) = 2}$$

Ques 5) Find the rank of $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & 1 \\ 4 & 2 & 3 \end{bmatrix}$

Ques 6) Find the rank of $A = \begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$

- By Elemantry operation.

Solutions

Solⁿ2) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}_{3\times 3}$

There is one minor of order 3.

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 1\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} - 0\begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix} + 0\begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix}$$

$$= 2 \neq 0$$

Since we get atleast one minor which are non zero of order 2 ~~& all the minor~~ so the rank of matrix

$$\boxed{\rho(A) = 3}$$

Solⁿ3) $\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}_{3\times 4}$

There are four three order minors.

$$\begin{vmatrix} 1 & 3 & 4 \\ 3 & 9 & 12 \\ 1 & 3 & 4 \end{vmatrix} = 0 \qquad \begin{vmatrix} 1 & 4 & 3 \\ 3 & 12 & 3 \\ 1 & 4 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 3 & 3 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{vmatrix} = 0 \qquad \begin{vmatrix} 3 & 4 & 3 \\ 9 & 12 & 3 \\ 3 & 4 & 1 \end{vmatrix} = 0$$

Since all the minors of order three are zero.

$$\therefore \rho(A) \neq 3 \quad \text{i.e. } \rho(A) < 3$$

Minors of order 2 are

$$\begin{vmatrix} 3 & 3 \\ 3 & 9 \end{vmatrix} = 0 \qquad \begin{vmatrix} 4 & 3 \\ 12 & 3 \end{vmatrix} = -24 \neq 0$$

Since we get atleast one minor which is non zero of order 2 & all the minors of order 3 are zero then, by def^n of the rank of the matrix

$$\boxed{\rho(A) = 2}$$

Sol^n 5)
$$\begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & 1 \\ 4 & 2 & 3 \end{bmatrix}$$

$$R_3 \Rightarrow R_3 - R_2$$

$$\begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & 1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\cancel{R_3 \Rightarrow} \; R_1 \rightarrow R_1 - R_3$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 4 & 3 & 1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\cancel{R_2 \to R_2 - 2R_3}$$

$$\cancel{\begin{bmatrix} 0 & 0 & 0 \\ 4 & 5 & \end{bmatrix}}$$

$$C_3 \to C_3 + 2C_2$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 4 & 3 & \cancel{5}7 \\ 0 & -1 & 0 \end{bmatrix}$$

Thus, rank of matrix = no. of non zero rows = $\boxed{\rho(A) = 2}$

Soln 6) $\cancel{\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}}$ $\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$

$$\cancel{R_3 \to R_3 - R_2}$$

$$\cancel{\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -2 & 0 & \end{bmatrix}}$$

$$R_3 \to R_3 - \frac{1}{5}R_1$$

$$\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 0 & -8/5 & -4/5 & -4/5 \end{bmatrix}$$

$$R_3 \to R_3 + \frac{8}{5}R_2$$

$$\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & +12/5 & 4/5 \end{bmatrix} \begin{matrix} \to R_1 \neq 0 \\ \to R_2 \neq 0 \\ \to R_3 \neq 0 \end{matrix}$$

Thus, the rank of matrix

$$\boxed{\rho(A) = 3}$$

Gauss Jorden's Method for inverse of the matrix working rule:

Let A be a square matrix & I be a Identity matrix

① Write a matrix A with identity matrix I

$$[A : I]$$

$$\left[\begin{array}{ccc:ccc} a_{11} & a_{12} & a_{13} & 1 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 1 & 0 \\ a_{31} & a_{32} & a_{31} & 0 & 0 & 1 \end{array}\right]$$

② Apply some row elementary operation on both side

③ Reduce the matrix A to ~~the~~ a identity matrix & I be a inverse of matrix. A i.e $[I : A^{-1}]$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & b_{11} & b_{12} & b_{13} \\ 0 & 1 & 0 & b_{21} & b_{22} & b_{23} \\ 0 & 0 & 1 & b_{31} & b_{32} & b_{33} \end{array}\right]$$

Q7 → Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & 4 & -4 \end{bmatrix}$$ with the help of Gauss Jordan's method.

Soln

$$\left[\begin{array}{ccc:ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & 4 & -4 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 - R_1$ &

$R_3 \to R_3 + 2R_1$

$$\left[\begin{array}{ccc:ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & -2 & 8 & 2 & 0 & 1 \end{array}\right]$$

~~$R_1 \to$~~

$R_1 \to R_1 - \frac{1}{2} R_2$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 6 & \frac{3}{2} & -\frac{1}{2} & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right]$$

$$R_2 \to \frac{R_2}{2}$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 6 & \frac{3}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & -3 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right]$$

$$R_3 \to R_3 + 2R_1$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 6 & \frac{3}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & -3 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right]$$

$$R_3 \to -\frac{1}{4} R_3$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 6 & \frac{3}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & -3 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{array}\right]$$

$$\cancel{R_1 \to R_1 + 2R_2}$$

$$R_1 \to R_1 - 6R_3$$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 3 & 1 & \frac{3}{2} \\ 0 & 1 & -3 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{array}\right]$$

Left over Q7 ~~a~~ will be afterwords.

Elementary Operations:

(1) interchanging the rows (or columns)

$R_i \longleftrightarrow R_j$

$C_i \longleftrightarrow C_j$

eg:- $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 3 & -4 & 0 \end{bmatrix}$

$R_2 \longrightarrow R_3$.

$\sim \begin{bmatrix} 1 & 2 & 3 \\ 3 & -4 & 0 \\ 0 & -1 & -1 \end{bmatrix}$

(2) Multiply any row or column with some non-zero scalar.

$R_i \longleftrightarrow kR_j$

$C_i \longleftrightarrow kC_j$

eg:- $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 3 & 4 & 0 \end{bmatrix}$

$R_1 \longleftrightarrow -3R_1$

$\sim \begin{bmatrix} -3 & -6 & -9 \\ 0 & -1 & -1 \\ 3 & -4 & 0 \end{bmatrix}$

$C_1 \longrightarrow 2C_1$

$$\sim \begin{bmatrix} 2 & 2 & 3 \\ 0 & -1 & -1 \\ 6 & -4 & 0 \end{bmatrix}$$

(3) Addition of two rows or columns when one of them is multiple of some non-zero scaler

$$R_i \longleftrightarrow R_i + kR_j$$
$$C_i \longrightarrow C_i + kC_j$$

eg:- $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 3 & -4 & 0 \end{bmatrix}$

$$R_3 \rightarrow R_3 - 3R_1$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 0 & -10 & -9 \end{bmatrix}$$

~~Ci~~

Sometimes, it can be written as:

~~Ri → kR~~

$$R_i \longrightarrow kR_i + R_j$$
$$R_i \longrightarrow kC_i + C_j$$

Ques) Find the rank of the matrix

(i) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -3 & 4 & 6 \\ 9 & 1 & 2 & 0 \end{bmatrix}$

(iii) $\begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & 1 \\ 4 & 2 & 3 \end{bmatrix}$

Solⁿ (i) There is one minor of order 3.

$$\begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix} + 3\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix}$$

$$= 1(2-1) - 2(3-1) + 3(3-2)$$

$$= 1 - 4 + 3$$

$$= 4 - 4$$

$= 0$, since all minors of order 3 are zero.

So, $\therefore \rho(A) \neq 3$ i.e $\rho(A) < 3$

Now, minors of order 2.

$$\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = 2 - 6 = -4 \neq 0$$

__/__/__

Since we get atleast one minor which is non zero of order 2 & all the minors of order 3 are zero then, by def^n of rank of the matrix

$$\boxed{\rho A = 2}$$

(ii) $A = \begin{bmatrix} 1 & -3 & 4 & 6 \\ 9 & 1 & 2 & 0 \end{bmatrix}_{2\times 4}$

There are ~~four~~ many minors of order 2

$\begin{vmatrix} 1 & -3 \\ 9 & 1 \end{vmatrix} = 1-(-27) = 28 \neq 0$ $\begin{vmatrix} 1 & 4 \\ 9 & 2 \end{vmatrix} \neq 0$

$\begin{vmatrix} 1 & 6 \\ 9 & 0 \end{vmatrix} \neq 0$ $\begin{vmatrix} -3 & 4 \\ 1 & 2 \end{vmatrix} \neq 0$

~~so~~ so, the ~~minor~~ rank of matrix

$$\boxed{\rho(A) = 2}$$

(iii) $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & -3 & 1 \\ 4 & 2 & 3 \end{bmatrix}$

$R_1 \Rightarrow R_1$

$$\boxed{\rho(A) = 2}$$

Ques 9) (i) $A = \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 3 & 2 & 1 & 4 \\ 7 & 6 & 2 & 5 \end{bmatrix}$

Solⁿ (i) $A = \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}_{4\times4}$

$$R_4 \longrightarrow R_4 - \cancel{R}4R_2$$

$$\sim \begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 0 & -4 & -12 & 19 \end{bmatrix}$$

$$R_2 \longrightarrow \frac{1}{2} R_2$$

$$\sim \begin{bmatrix} 6 & 1 & 3 & 8 \\ 2 & 1 & 3 & -\frac{1}{2} \\ 10 & 3 & 9 & 7 \\ 0 & -4 & -12 & 19 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - 5R_2$$

$$\sim \cancel{\begin{bmatrix} 6 & 1 & 3 & 8 \\ 10 & -2 & -6 & \end{bmatrix}}$$

$$\sim \begin{bmatrix} 6 & 1 & 3 & 8 \\ 2 & 1 & 3 & -1/2 \\ 0 & -2 & -6 & 19/2 \\ 0 & -4 & -12 & 19 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - \frac{1}{3} R_1$$

$$\sim \begin{bmatrix} 6 & 1 & 3 & 8 \\ 0 & 2/3 & 2 & -19/2 \\ 0 & -2 & -6 & 19/2 \\ 0 & -4 & -12 & 19 \end{bmatrix}$$

$$R_3 \rightarrow R_3 + R_2.$$

$$\sim \begin{bmatrix} 6 & 1 & 3 & 8 \\ 0 & 2/3 & 2 & -19/2 \\ 0 & 4/3 & -4 & 0 \\ 0 & -4 & -12 & 19 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - 2R_2$$

$$\sim \begin{bmatrix} 6 & 1 & 3 & 8 \\ 0 & 2/3 & 2 & -19/2 \\ 0 & 0 & -8 & 19 \\ 0 & -4 & -12 & 19 \end{bmatrix}$$

$$R_4 \rightarrow R_4 - R_3.$$

R.W.

$7 - 5(-\frac{1}{6})$

$= 7 + \frac{5}{2}$

$\frac{14+4}{2}$

$1 - \frac{1}{3}(1)$

$= 1 - \frac{1}{3}$

$\frac{3-1}{3}$

$3 - \frac{1}{3}$

$\frac{-1}{2} - \frac{1}{3}$

$\frac{-1}{2} - \frac{8}{3}$

$\frac{-3-16}{6}$

$-2 + \frac{2}{3}$

$\frac{-6+2}{3}$

$= \frac{4}{3}$

$\frac{4}{3} - 2(\frac{2}{3})$

$\frac{4}{3} - \frac{4}{3}$

$-4 - 2(2)$

$0 - 2(\frac{-19}{2})$

$$\sim \begin{bmatrix} 6 & 1 & 3 & 8 \\ 0 & 2/3 & 2 & -19/2 \\ 0 & 0 & -8 & 19 \\ 0 & -4 & 4 & 0 \end{bmatrix}$$

$R_4 \rightarrow R_4 + 6R_2$

$$\sim \begin{bmatrix} 6 & 1 & 3 & 8 \\ 0 & 2/3 & 2 & -19/2 \\ 0 & 0 & -8 & 19 \\ 0 & 0 & 16 & 67 \end{bmatrix}$$

$$\boxed{\rho(A) = 2}$$

Ques 10) (i) $A = \begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & -2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 3 & 0 & 3 \\ 1 & -2 & -3 & -3 \\ 1 & 1 & 2 & 3 \end{bmatrix}$

(iii) $A = \begin{bmatrix} -1 & 2 & 1 & 8 \\ 2 & 1 & -1 & 0 \\ -3 & 2 & -1 & -7 \end{bmatrix}$

(iv) $A = \begin{bmatrix} 2 & -3 & 4 \\ -1 & -1 & 1 \\ 4 & -1 & 2 \end{bmatrix}$

$\frac{2}{\cancel{3}} \times \cancel{6}^{2}$

$= 4$

$-4 + \frac{\cancel{6}^{2} \times 2}{\cancel{3}}$

$4 + 6(2)$

$0 + \cancel{6}^{3}\left(\frac{19}{\cancel{2}}\right)$

$\frac{2 \cdot 4}{3}$

67

$\frac{2 \cdot 69}{3}$

57

~~Q7~~ leftrows $R_2 \to R_2 + 3R_3$.

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 3 & 1 & 3/2 \\ 0 & 1 & 0 & -5/4 & -1/4 & -7/4 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right]$$

$$[I : A^{-1}]$$

So, from ~~here~~ this matrix we can say that inverse of matrix A is

$$A^{-1} = \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

Ques 11) Find the inverse of the matrix with the help of Gauss Jordan Method?

(i) $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$ (ii) $B = \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$

(iii) $A = \begin{bmatrix} 2 & -1 & 1 \\ 4 & 3 & -1 \\ 3 & 2 & 2 \end{bmatrix}$

16/08/2024

Equivalent Matrix

Two matrices A & B are equivalent if one can be obtained from ~~fr~~ the other by applying in succession a finite number of elementary operations and it is donated by $\boxed{A \sim B}$

Row ~~A~~ Echelon Matrix:

An $m \times n$ matrix A which $A=[a_{ij}]_{m \times n}$ is called a Row echelon matrix if it holds the following conditions:

→ The first non-zero element in each row is unity which is known as a leading elements.

→ All the non-zero rows, procede the zero rows if any.

→ The no. of zero before the leading element in each row is less than the no. of such zero's in the succeeding rows.

15/08/2024

Normal form:

Defn: every non-zero matrix of order $m \times n$ can be reduced by elementary operations (row & column operations) into equivalent matrix of the following form

$$\begin{bmatrix} I_r & \vdots & O \\ \cdots & & \cdots \\ O & & O \end{bmatrix}; \quad \begin{bmatrix} I_r \\ \cdots \\ O \end{bmatrix}; \quad \begin{bmatrix} I_r : O \end{bmatrix}; \quad [I_r]$$

where, I_r is an identity matrix of order 3 and O is a null matrix (zero matrix) than it is said to be a normal form.

eg:- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ - (1).

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ - (2)

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ - (3)

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ↳ (4)

These are the four forms.

Ques1.) Change the matrix A in a normal form.

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 5 & -2 \\ 1 & 2 & 1 \end{bmatrix}$$

Soln

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$C_2 \to C_2 - 2C_1$

$$\sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$R_3 \to \frac{1}{2} R_3$

$$\sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_1 \to R_1 + R_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim [I_3]$$

$$\boxed{\rho(A) = 3}$$

Ques2:) $A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ -2 & -4 & 4 & 7 \\ 1 & 2 & 1 & 2 \end{bmatrix}$

IInd Method:

$$\underset{\text{Identity Matrix } m\times m}{P}\;\underset{m\times n}{A}\;\underset{\text{Identity Matrix } n\times n}{Q}$$

eg: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$

This matrix will reducte for 4 forms.

[P & Q are non-singular matrix]

Ques3:) Find the non-singular P & Q such that PAQ is in a normal form. Where Matrix $A = \begin{bmatrix} 2 & 2 & -6 \\ -1 & 2 & 2 \end{bmatrix}_{2\times 3}$.

Soln $A = I_{2\times 2} A I_{3\times 3}$.

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & -6 \\ -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \to \frac{1}{2} R_1$$

$$\begin{bmatrix} 1 & 1 & -3 \\ -1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix} A \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \to R_2 + R_1$$

$$\begin{bmatrix} 1 & 1 & -3 \\ 0 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 1/2 & 1 \end{bmatrix} A \begin{bmatrix} 1/2 & 0 & 0 \\ 1/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_2 \leftrightarrow C_3$$

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 1/2 & 1 \end{bmatrix} A \begin{bmatrix} 1/2 & 0 & 0 \\ 1/2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\cancel{C_1 \to 2C_1}$$

$$\cancel{\begin{bmatrix} 2 & -3 & 1 \\ 0 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} A \begin{bmatrix} \ \end{bmatrix}}$$

$$R_2 \to \frac{1}{3} R_2$$

$$\begin{bmatrix} 1 & +3 & 1 \\ 0 & 1 & -1/3 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 1/6 & 1/3 \end{bmatrix} A \begin{bmatrix} 1/2 & 0 & 0 \\ 1/6 & 1/3 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$C_2 \to C_2 - C_1$$

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -1/3 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 \\ 1/6 & 1/6 \end{bmatrix} A \begin{bmatrix} 1/2 & -1/2 & 0 \\ 1/6 & 1/6 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_3 \longrightarrow C_3 + 3C_1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1/3 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 \\ 1/6 & 1/6 \end{bmatrix} A \begin{bmatrix} 1/2 & -1/2 & +3/2 \\ 1/6 & 1/6 & 1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C_3 \longrightarrow C_3 + \frac{1}{3} C_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 \\ 1/6 & 1/6 \end{bmatrix} A \begin{bmatrix} 1/2 & -1/2 & 4/3 \\ 1/6 & 1/6 & 2/3 \\ 0 & 0 & 1 \end{bmatrix}$$

After converting matrix A into normal form the matrix P & Q are

$$\begin{bmatrix} 1/2 & -1/2 \\ 1/6 & 1/6 \end{bmatrix} \ \& \ \begin{bmatrix} 1/2 & -1/2 & 4/3 \\ 1/6 & 1/6 & 2/3 \\ 0 & 0 & 1 \end{bmatrix}.$$

Ques4.) Find the matrices P & Q such that PAQ is in a normal form where the matrix $A = \begin{bmatrix} 3 & 1 & 2 & 1 \\ 1 & 4 & 6 & 1 \\ 2 & -3 & 1 & -2 \end{bmatrix}_{3\times 4}$

Ques 5) Find the matrix P & Q such that PAQ is in a normal form where the matrix A is

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$$

20/08/2024

Consistency of Linear System of Eqn:

Consider the m system of L.E

$$a_{11}x_1 + a_{12}x_2 - - - - + a_{1n}x_n = C_1$$
$$a_{21}x_1 + a_{22}x_2 - - - - + a_{2n}x_n = C_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 - - - + a_{mn}x_n = C_m$$

Here $x_1, x_2 - - - x_n$ are n-unknowns so from here; we can write martics

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

$$C = [A : B]$$

$$C = \left[\begin{array}{cccc:c} a_{12} & a_{12} & \cdots & a_{1n} & C_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & C_2 \\ \vdots & & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & C_m \end{array}\right]$$

→ The system of linear eqn is said to be a consistence. ~~when~~ if

$$\boxed{\rho(A) = \rho(C)}$$ Otherwise, in-consistence

→ Working Rule:

① Write the system of L.E in the form of $C = [A : B]$

② Apply some row elementary operations and convert matrix A into triangular form.

③ Check the system of L.E is consistence or not i.e rank of A = Rank of C.

④ If System of L.E is consistent than it ~~has~~ may has three types of sol^n.

(i) Unique Solution

Rank of A = Rank of C = no. of unknowns

(ii) Infinite solution

Rank of A = Rank of C < no. of n.

(iii) No Sol^n

Rank of A ≠ Rank of C

⑤ Than find the solutions that means find the values of variables (or unknowns).

Ques 1) Solve the System of L.E.

$$2x - 3y + z = 9$$
$$x + y + z = 6$$
$$x - y + z = 2$$

Sol^n $A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$

$$C = [A : B]$$

$$C = \left[\begin{array}{ccc:c} 2 & -3 & 1 & 9 \\ 1 & 1 & 1 & 6 \\ 1 & -1 & 1 & 2 \end{array}\right]$$

$$R_2 \to R_2 - \frac{1}{2} R_1$$

$$\cancel{R_3 \to R_3 - R_2}$$

$$\sim \left[\begin{array}{ccc:c} 2 & -3 & 1 & 9 \\ 0 & 5/2 & 1/2 & 3/2 \\ 1 & -1 & 1 & 2 \end{array}\right] \quad R_3 \to R_3 - \frac{1}{2} R_1$$

$$\sim \left[\begin{array}{ccc:c} 2 & -3 & 1 & 9 \\ 0 & 5/2 & 1/2 & 3/2 \\ 0 & +1/2 & 1/2 & -5/2 \end{array}\right]$$

$$R_2 \to \frac{2}{5} R_2$$

$$\sim \left[\begin{array}{ccc:c} 2 & -3 & 1 & 9 \\ 0 & 1 & 1/5 & 3/5 \\ 0 & +1/2 & 1/2 & -5/2 \end{array}\right]$$

$$R_3 \rightarrow R_3 - \frac{1}{2} R_2$$

$$\sim \cancel{\left[\begin{array}{ccc:c} 2 & -3 & 1 & 9 \\ 0 & & & \\ 0 & 1/2 & 1/2 & -5/2 \end{array}\right]}$$

$$\sim \left[\begin{array}{ccc:c} 2 & -3 & 1 & 9 \\ 0 & 1 & 1/5 & 3/5 \\ 0 & 0 & 2/5 & -14/5 \end{array}\right]$$

$$R_3 \rightarrow \frac{5}{2} R_3$$

$$\sim \left[\begin{array}{ccc:c} 2 & -3 & 1 & 9 \\ 0 & 1 & 1/5 & 3/5 \\ 0 & 0 & 1 & -7 \end{array}\right]$$

From here

$$\rho(A) = 3 \;\&\; \rho(A:B) = 3$$

so, since rank $\rho(A) = \rho(A:B)$ so system is consistent.

$$\rho(A) = \rho(A:B) = 3 = \text{no. of unknowns.}$$

and also have a unique solution

For Solutions:

$2x - 3y + z = 9$ — ①

$0x + y + \frac{z}{5} = \frac{3}{5}$ — ②

$0x + 0y + z = -7$ — ③

$\boxed{z = -7}$ — ④

Put ④ in ②.

$y + \frac{1}{5} z = \frac{3}{5}$

$y - \frac{7}{5} = \frac{3}{5} \Rightarrow y = \frac{3}{5} + \frac{7}{5} = \cancel{\frac{10}{5}}^{2}$

$\boxed{y = 2}$ — ⑤

Put ④ & ⑤ in ①.

$2x - 3(2) + (-7) = 9$

$2x - 6 - 7 = 9$

$2x = 9 + 13$

$2x = 22$

$\boxed{x = 11}$

Ques2) Solve the Eqn

$x + y + 2z + w = 5$

$2x + 3y - z - 2w = 2$

$4x + 5y + 3z = 7$

__/__/__

Ques3) Solve the following system of Linear Equation:

$$x+2y-z-5w=4$$
$$2x-y+3z=3$$
$$x+3y-2z-7w=5.$$

Soln From the given system of Linear Equation

$$A=\begin{bmatrix}1 & 2 & -1 & -5\\ 2 & -1 & 3 & 0\\ 1 & 3 & -2 & -7\end{bmatrix} \quad B=\begin{bmatrix}4\\3\\5\end{bmatrix}$$

$$C=[A:B]$$

$$C=\left[\begin{array}{cccc:c}1 & 2 & -1 & -5 & 4\\ 2 & -1 & 3 & 0 & 3\\ 1 & 3 & -2 & -7 & 5\end{array}\right]$$

$$R_2 \to R_2 - 2R_1$$
$$R_3 \to R_3 - R_1$$

$$C \sim \left[\begin{array}{cccc:c}1 & 2 & -1 & -5 & 4\\ 0 & -5 & 5 & 10 & -5\\ 0 & 1 & -1 & -2 & 1\end{array}\right]$$

$$R_2 \to -\frac{1}{5}R_2$$

$$C \sim \left[\begin{array}{cccc:c}1 & 2 & -1 & -5 & 4\\ 0 & 1 & -1 & -2 & 1\\ 0 & 1 & -1 & -2 & 1\end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\sim \left[\begin{array}{cccc:c} 1 & 2 & -1 & -5 & 4 \\ 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\rho(A) = 2 \quad \& \quad \rho(A:B) = 2$$

Since, $\rho(A) = \rho(A:B)$
So, system of given L.E is consistent
No. of unknowns $= 4$

$$\rho(A) = \rho(A:B) < \text{no. of unknowns}$$

so it has infinite solutions

The system must have $4-2 = 2$ arbitrary constants.

Let $z = k_1$ & $w = k_2$

from the matrix $A:B$, the corresponding Eq^n are

$$x + 2y - z - 5w = 4 \quad -(1)$$
$$0x + y - z - 2w = 1 \quad -(2)$$

Put $z = k_1$ & $w = k_2$ in eq^n (2) we get

$$y - k_1 - 2k_2 = 1$$
$$\boxed{y = 1 + k_1 + 2k_2}$$

$$\boxed{x = ?}$$

_/_/_

Ques4) For what value of λ does the system of

$$\begin{bmatrix} -1 & 2 & 1 \\ 3 & -1 & 2 \\ 0 & 1 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$ has

(i) no solution
(ii) unique solution

Soln ~~A =~~ given matrix,

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 3 & -1 & 2 \\ 0 & 1 & \lambda \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$C = [A : B]$$

$$C = \left[\begin{array}{ccc:c} -1 & 2 & 1 & 1 \\ 3 & -1 & 2 & 1 \\ 0 & 1 & \lambda & 1 \end{array}\right]$$

$$R_2 \to R_2 + 3R_1$$

$$\sim \left[\begin{array}{ccc:c} -1 & 2 & 1 & 1 \\ 0 & 5 & 5 & 4 \\ 0 & 1 & \lambda & 1 \end{array}\right]$$

$$R_2 \to \frac{1}{5} R_2$$

$$\sim \left[\begin{array}{ccc:c} -1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 4/5 \\ 0 & 1 & \lambda & 1 \end{array}\right]$$

$R_3 \rightarrow R_3 - R_2$

$$\sim \left[\begin{array}{ccc:c} -1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 4/5 \\ 0 & 0 & \lambda-1 & 1/5 \end{array}\right]$$

(i) If $\lambda - 1 = 0 \Rightarrow \boxed{\lambda = 1}$

$$\left[\begin{array}{ccc:c} -1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 4/5 \\ 0 & 0 & 0 & 1/5 \end{array}\right]$$

Here,

$\rho(A) = 2$ & $\rho(A:B) = 3$.

so,

$\rho(A) \neq \rho(A:B)$

$\therefore$ ~~solutions are~~ system has no solution.

(ii) If $\lambda - 1 \neq 0$

$\lambda \neq 1$

so, here

$\rho(A) = 3$ & $\rho(A:B) = 3$.

so, $\rho(A) = \rho(A:B) = 3 =$ no. of unknowns.

$\therefore$ system has unique soln

$-x + 2y + z = 1$ — (1)

$y + z = 4/5$ — (2)

$z(\lambda - 1) = 1/5$ — (3)

$\boxed{z = 1/5(\lambda-1)}$

$x = ??$

$y = ??$

Check

Ques5) Show the system of equations are consistent and solve them.

$$x+2y-5z=-9$$
$$3x-y+2z=5$$
$$2x+3y-z=3$$
$$4x-5y+z=-3$$

Ques6) For what value of a & b the system of Eqns $x+y+5z=0$

$$x+2y+3az=b$$
$$x+3y+az=1$$

have (i) No solution.

(ii) unique solution.

(iii) infinite solution.

30/08/2021

System of Linear Homogeneous Eqⁿ:

Consider the system of linear Equation

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = 0$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = 0$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = 0$$

Where, $x_1, x_2, \ldots x_n$ are unknowns.
This type of system is known as system of linear homogeneous Eqⁿ.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

Note - In this (no solution) is not here.

This system can be written in the form of

$$AX = O$$

where,

$$A = \begin{bmatrix} a_{12} & a_{22} \cdots a_{1n} \\ a_{21} & a_{22} \cdots a_{2n} \\ a_{m1} & a_{m2} \cdots a_{mn} \end{bmatrix}_{m\times n} ; X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} ;$$

$$O = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Types of Solution:

$$AX = 0$$

- Trivial soln.
  - $\rho(A) =$ no. of unknowns
  - $x_1 = x_2 = x_3 - x_n = 0$
- non-trivial soln. (infinite soln)
  - $\rho(A) <$ no. of unknown

Trivial Solution: If the rank of A is equal to no. of unknowns than system must have trivial solution. that means the system ~~have~~ has value of $x_1 = x_2 = \cdots x_n = 0$.

Non-Trivial Solution: If the rank of A is less than the no. of unknowns than system must have a non-trivial solution (infinite solution). ~~And also~~ In this case the system has ~~no. of~~ (no. of unknown $- \rho(A)$) arbitary constant.

Working Rule

→ Derive the matrix A from the given system.

→ Perform some elementary operations so that Matrix A is reduced to a triangular form (or Echelon form).

→ Check rank of matrix A.

→ Check type of solution. (trivial or non-trivial soln).

→ Find the soln i.e. find the value of variables (unknown).

<u>Note</u> → The necessary & sufficient condition for non-trivial soln:

$$\boxed{|A| = 0}$$

Ques1.> Solve the following system of Equation

$$\cancel{x+}\quad x - y + z = 0$$
$$x + 2y + z = 0$$
$$2x + y + 3z = 0$$

<u>Soln</u> From the given system

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

$$R_2 \to R_2 - R_1$$
$$R_3 \to R_3 - 2R_1$$

$$\sim \begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$.

$$\sim \begin{bmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Which is required triangular form so,

$\rho(A) = 3 =$ no. of unknowns

so, therefore system has a trivial solution.

The corresponding Eq^ns are:

$$x - y + z = 0 \quad -(1)$$
$$0x + 3y + 0z = 0 \quad -(2)$$
$$0x + 0y + z = 0 \quad -(3)$$

From Eq^n (2) we get $\boxed{y=0}$
From Eq^n (3) we get $\boxed{z=0}$
From Eq^n (1) we get $\boxed{x=0}$

Ques2.) Solve the following system of Eq^ns:

$$x + 2y + 3z = 0$$
$$2x + 3y + 4z = 0$$
$$7x + 3y + 19z = 0.$$

Solⁿ $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 7 & 13 & 19 \end{bmatrix}$.

$R_2 - R_2 - 2R_1$

$R_3 - R_3 - 7R_1$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & -1 & -2 \end{bmatrix}.$$

$R_3 \longrightarrow R_3 - R_2$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

which is required triangular form
so,

$\rho(A) = 2 <$ no. of unknowns.

so, therefore system has a non-trivial solution. (infinite solution).

and the system Contain no. of unknown $= 3$.

$\rho(A) = 2$.

no. of arbitrary constant $= 3-2$
$= 1$

Let $\boxed{z = k}$

$x + 2y + 3z = 0$ —(1)

$0x - 1y - 2z = 0$ —(2)

$\boxed{y = -2k}$ $\boxed{x = -7k}$

Ques3) Find the value of k such that the system of Eqⁿ has non-trivial solution.

$$x + ky + 3z = 0$$
$$4x + 3y + kz = 0$$
$$2x + y + 2z = 0$$

Solⁿ
$$A = \begin{bmatrix} 1 & k & 3 \\ 4 & 3 & k \\ 2 & 1 & 2 \end{bmatrix}$$

we know, $|A| = 0$.

$$\begin{vmatrix} 1 & k & 3 \\ 4 & 3 & k \\ 2 & 1 & 2 \end{vmatrix} = 0$$

$$1\begin{vmatrix} 3 & k \\ 1 & 2 \end{vmatrix} - k\begin{vmatrix} 4 & k \\ 2 & 2 \end{vmatrix} + 3\begin{vmatrix} 4 & 3 \\ 2 & 1 \end{vmatrix} = 0$$

$$1(6-k) - k(8-2k) + 3(4-6) = 0$$
$$(6-k) - (8k - 2k^2) + (-6) = 0$$
$$\cancel{6} - k - 8k - 2k^2 - \cancel{6} = 0$$
$$-2k^2 - 9k = 0.$$
$$2k^2 + 9k = 0$$
$$k(2k+9)$$
$$\boxed{k = 0, \frac{9}{2}}$$

Ques 4) Find the value of λ if the system of Eqⁿ has non-zero solⁿ / non-trivial solⁿ.

$$x + 2y + 3z = \lambda x$$
$$3x + y + 2z = \lambda y$$
$$2x + 3y + z = \lambda z$$

Solⁿ $(1-\lambda)x + 2y + 3z = 0$

$3x + (1-\lambda)y + 2z = 0$

$2x + 3y + (1-\lambda)z = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0$$

$$1-\lambda \begin{vmatrix} 1-\lambda & 2 \\ 3 & 1-\lambda \end{vmatrix} - 2\begin{vmatrix} 3 & 2 \\ 2 & 1-\lambda \end{vmatrix} + 3\begin{vmatrix} 3 & 1-\lambda \\ 2 & 3 \end{vmatrix}$$

$1-\lambda\,[(1-\lambda)^2 - 6] - 2[3-3\lambda-4] + 3[9-(2-2$

$= 1-\lambda\,[1+\lambda^2 - 2\lambda - 6] - 2[-3\lambda - 1] + 3[9 - 2\lambda$

$=$

$$\boxed{\lambda = 6}$$

Eigen Values & Eigen Vectors:

(Characteristic value & characteristic vector)

Characteristic Matrix:

Let A be a n^{th} order square matrix & λ some scalar. Then the matrix $A-\lambda I$

$$A-\lambda I = \begin{bmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22}-\lambda & \cdots & a_{2n} \\ \vdots & \vdots & & \\ a_{n1} & a_{n2} & \cdots & a_{nn}-\lambda \end{bmatrix}$$

is called a characteristic matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{n\times n}$$

$$I = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$A-\lambda I = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} - \lambda \begin{bmatrix} \lambda & 0 & \cdots & 0 \\ 0 & \lambda & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & \lambda \end{bmatrix}$$

Characteristic Equation:

Let A be a n^{th} matrix of order matrix square matrix & some scaler λ. Then the matrix

$$|A-\lambda I| = 0$$

is called characteristic equation.

Eigen Values → The roots of characteristic equation $(|A-\lambda I| = 0)$ ~~thats~~ that is $\lambda_1, \lambda_2, \dots, \lambda_n$ are said to be eigen values of the square matrix A.
and it is also known as characteristic value or latent roots.

Spectrum → The set of eigen values of matrix A is said to be spectrum.

$$\{\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n\}.$$

Ques 1) Find the eigen values of

$$A = \begin{bmatrix} 5 & 4 & -4 \\ 4 & 5 & -4 \\ -1 & -1 & 2 \end{bmatrix}$$

Soln → The characteristic matrix is

$$A-\lambda I = \begin{bmatrix} 5 & 4 & -4 \\ 4 & 5 & -4 \\ -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A-\lambda I = \begin{bmatrix} 5-\lambda & 4 & -4 \\ 4 & 5-\lambda & -4 \\ -1 & -1 & 2-\lambda \end{bmatrix}$$

Now, the characteristic equation is

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 5-\lambda & 4 & -4 \\ 4 & 5-\lambda & -4 \\ -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$$5-\lambda \begin{vmatrix} 5-\lambda & -4 \\ -1 & 2-\lambda \end{vmatrix} -4 \begin{vmatrix} 4 & -4 \\ -1 & 2-\lambda \end{vmatrix} \cancel{+} -4 \begin{vmatrix} 4 & 5-\lambda \\ -1 & -1 \end{vmatrix}$$

$$\cancel{5-\lambda(\lambda^2 - 7\lambda}$$

$$5-\lambda[(5-\lambda)(2-\lambda)-4] - 4[4(2-\lambda)-4] -4[-4+(5-\lambda)=0]$$

$$\lambda^3 - 12\lambda^2 + 21\lambda - 10 = 0. \quad -(1)$$

$$\cancel{\lambda^3 + 9\lambda - 10 = 0.}$$

by inspection, we get $\boxed{\lambda = 1}$ satisfy the Eqn (1)

$\therefore$ $\lambda-1$ is the root of the eqn.

∴ By Synthethic Division

$$\begin{array}{c|cccc} 1 & 1 & -12 & 21 & -10 \\ & & 1 & -11 & 10 \\ \hline & 1 & -11 & 10 & 0 \end{array}$$

From here, the eq^n will be

$$\lambda^2 - 11\lambda + 10 = 0$$
$$\lambda^2 - 10\lambda - \lambda + 10 = 0.$$
$$\lambda(\lambda - 10) - 1(\lambda - 10) = 0$$
$$(\lambda - 10)(\lambda - 1) = 0.$$
$$\boxed{\lambda = 1, 10}$$

The Eigen values are 1, 1, 10.

Ques2) Find the eigen's values of

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ -2 & 1 & 2 \end{bmatrix}.$$

Eigen Vectors: If λ is a eigen value of matrix A, then the solutions of $(A - \lambda I X)\cancel{X} = 0$, other than $x = 0$ $(x_1, x_2, x_3 - \cdots, x_n)$ are said to be eigen vector of the matrix.

Ques 3) Find the eigen values and eigen vector of ~~the~~

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Sol^n $A - \lambda I = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$8-\lambda\,[(7-\lambda)(3-\lambda)-16] + 6[(-18+6\lambda)+8]$
$+2\,[24-(14-2\lambda)]$.

$8-\lambda\,[(21-7\lambda-3\lambda+\lambda^2)-16] + 6\,[6\lambda-10]$
$+2\,[2\lambda+10]$.

$8-\lambda\,[\lambda^2-10\lambda+5] + [36\lambda-60] +$
$[4\lambda+20]$

$$\lambda^3 - 18\lambda^2 + 45\lambda = 0.$$

$\cancel{\lambda = 0,}$ $\lambda(\lambda^2 - 18\lambda + 45) = 0$

$$\boxed{\lambda = 0, 3, 15}$$

Eigen vectors ~~are~~, non-zero sol^n of
~~$(A-\lambda I)\,X = 0$~~

$$(A - \lambda I)X = 0$$

$$\begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

For, $\lambda = 0$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}_{3\times 3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3\times 1} = 0$$

$$\cancel{\begin{bmatrix} 8x_1 - 6x_1 + 2x_3 \\ -6x_2 + 7x_2 - 4x_2 \\ 2x_1 \end{bmatrix} = 0}$$

$$\frac{x_1}{\begin{vmatrix} 7 & -4 \\ -4 & 3 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -6 & -4 \\ 2 & 3 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -6 & 7 \\ 2 & -4 \end{vmatrix}} = K$$

$$\frac{x_1}{5} = \frac{x_2}{10} = \frac{x_3}{10} = K$$

$$\frac{1}{5}\left[\frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{2}\right] = K$$

$$\frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{2} = K$$

$$\boxed{\begin{matrix} x_1 = K \\ x_2 = 2K = x_3 \end{matrix}}$$

From here, Eigen vector corresponding to

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} K \\ 2K \\ 2K \end{bmatrix} = K\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

Ques 4) Find the Eigen values and Eigen vectors of a matrix A.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$$

Soln The characteristic Eqn:

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 0 & -4-\lambda & 2 \\ 0 & 0 & 7-\lambda \end{vmatrix} = 0.$$

$(1-\lambda)\left[(-4-\lambda)(7-\lambda) - 0\right] - 2(0) + 3(0) = 0.$

$(1-\lambda)(-4-\lambda)(7-\lambda) = 0$

$\boxed{\lambda = 1, -4, +7}$

$\therefore$ The Eigen values are $1, -4, 7$

The Eigen vectors are non zero soln of $(A - \lambda I)X = 0$.

$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & -4-\lambda & 2 \\ 0 & 0 & 7-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \quad \text{--- (1)}$$

For $\lambda = 1$,

The Eqn (1) will be

$$\begin{bmatrix} 1-1 & 2 & 3 \\ 0 & -4-1 & 2 \\ 0 & 0 & 7-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 2 & 3 \\ 0 & -5 & 2 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$0x_1 + 2x_2 + 3x_3 = 0 \quad -② $$
$$0x_1 - 5x_2 + 2x_3 = 0 \quad -③ $$
$$0x_2 + 0x_3 + 6x_3 = 0. \quad -④ $$

$$\boxed{x_2 = x_3 = 0}$$

Let $x_1 = k$

∴ The Eigen vector corresponding to $\lambda = 1$,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} K \\ 0 \\ 0 \end{bmatrix} = K \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

For $\lambda = -4$.

$$\begin{bmatrix} 1-(-4) & 2 & 3 \\ 0 & -4-(-4) & 2 \\ 0 & 0 & 7-(-4) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 5 & 2 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & 11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$5x_1 + 2x_2 + 3x_3 = 0 \quad -⑤$$
$$2x_3 = 0 \quad -⑥$$
$$3x_3 = 0 \quad -⑦$$

$$\boxed{x_3 = 0}$$

Let $2x_2 = -5x_1$

$$x_2 = \frac{-5}{2} x_1$$

Let x_1 be k

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k \\ -\frac{5}{2}k \\ 0 \end{bmatrix} = k \begin{bmatrix} 1 \\ -\frac{5}{2} \\ 0 \end{bmatrix}$$

$\lambda = 7$

$$\begin{bmatrix} -6 & 2 & 3 \\ 0 & -11 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$-6x_1 + 2x_2 + 3x_3 = 0$$
$$-11x_2 + 2x_3 = 0$$
$$x_3 = \frac{11x_2}{2}$$

Let x_1 be k.

$$-6k + 2x_2 + 3\left(\frac{11}{2}\right)x_2 = 0$$

$$-6k + 2x_2 + \frac{33}{2} x_2 = 0$$

Cayley Hamilton

Every square matrix satisfies its characteristics equation

$$|A - \lambda I| = 0$$

$$\hookrightarrow \lambda^4 + \cdots = 0$$

$$\hookrightarrow A^n + \cdots = 0 \rightarrow \text{matrix}$$

Ques1.) Verify the Cayley Hamilton theoram for the matrix?

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

Soln. From here,
the characteristic matrix

$$A - \lambda I = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \Rightarrow \begin{bmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{bmatrix}$$

The characteristic Eqn.

$$|A - \lambda I| = 0$$

$$\cancel{|1-4} \quad \begin{vmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)(1-\lambda) - 8 = 0$$

$3 - 3\lambda - \lambda + \lambda^2 - 8 = 0$

$\lambda^2 - 4\lambda + 3 - 8 = 0$

$\lambda^2 - 4\lambda - 5 = 0 \quad - (1).$

To verify cayley Hamilton theorem:
we have to prove that

$$A^2 - 4A - 5I = 0.$$

$$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1+8 & 4+12 \\ 2+6 & 8+9 \end{bmatrix} - \begin{bmatrix} 4 & 16 \\ 8 & 12 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = 0$$

$$\begin{bmatrix} 9 & 16 \\ 8 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 16 \\ 8 & 12 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = 0$$

$$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = 0$$

$$0 = 0$$

Hence proved

Ques2) ~~If~~ $A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$ compute A^{-1} ~~(where~~

~~So~~ Soln the ~~Cha~~ Characteristic Eqn:.

$$\begin{bmatrix} 0-\lambda & 0 & 1 \\ 3 & 1-\lambda & 0 \\ -2 & 1 & 4-\lambda \end{bmatrix} = A - \lambda I$$

$$|A-\lambda I| = \begin{vmatrix} 0-\lambda & 0 & 1 \\ 3 & 1-\lambda & 0 \\ -2 & 1 & 4-\lambda \end{vmatrix}$$

$$-\lambda \begin{vmatrix} 1-\lambda & 0 \\ 1 & 4-\lambda \end{vmatrix} - 0 \begin{vmatrix} 3 & 0 \\ -2 & 4-\lambda \end{vmatrix} + 1 \begin{vmatrix} 3 & 1-\lambda \\ -2 & 1 \end{vmatrix}$$

$$-\lambda[(1-\lambda)(4-\lambda)] - 0 + 1[3+2(1-\lambda)] = 0$$

$$-\lambda(4-\lambda-4\lambda+\lambda^2) + (3+2-2\lambda) = 0$$

$$-4\lambda + \lambda^2 + 4\lambda^2 - \lambda^3 + 3 + 2 - 2\lambda = 0$$

$$-\lambda^3 + 5\lambda^2 - 6\lambda + 5 = 0.$$

$$\lambda^3 - 5\lambda^2 + 6\lambda - 5 = 0$$

$$A^2 = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix} = \Big[\cancel{0+0+1 \quad 0+0}$$

$$= \begin{bmatrix} 0+0-2 & 0+0+1 & 0+0+4 \\ 0+3+0 & 0+1+0 & 3+0+0 \\ 0+3-8 & 0+1+4 & -2+0+16 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} -2 & 1 & 4 \\ 3 & 1 & 3 \\ -5 & 5 & 14 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

$$\begin{array}{r} 14 \\ \underline{4} \\ 56 \end{array}$$

$$= \begin{bmatrix} 0+3-8 & 0+1+4 & -2+0+16 \\ 0+3-6 & 0+1+3 & 3+0+12 \\ 0+15-28 & 0+5+14 & -5+0+56 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 5 & 14 \\ -3 & 4 & 15 \\ -13 & 19 & 51 \end{bmatrix}$$

$$A^3 - 5A^2 + 6A - 5I = 0$$

$$\begin{bmatrix} -5 & 5 & 14 \\ -3 & 4 & 15 \\ -13 & 19 & 51 \end{bmatrix} - 5\begin{bmatrix} -2 & 1 & 4 \\ 3 & 1 & 3 \\ -5 & 5 & 14 \end{bmatrix} + 6\begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

$$-5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} -5 & 5 & 14 \\ -3 & 4 & 15 \\ -13 & 19 & 51 \end{bmatrix} - \begin{bmatrix} -10 & 5 & 20 \\ 15 & 5 & 15 \\ -25 & 25 & 70 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 6 \\ 18 & 6 & 0 \\ -12 & 6 & 24 \end{bmatrix}$$

$$- \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ Hence proved.

$$A^3 - 5A^2 + 6A - 5I = 0 \quad (2)$$

Pre Multiply A Eqⁿ (2) by A^{-1}

$$A^{-1}A^3 - 5A^{-1}A^2 + 6A^{-1}A - 5A^{-1}I = A^{-1}0$$

$$A^2 - 5A + 6I - 5A^{-1} = 0.$$

$-5A^{-1} = A^2 + 5A - 6I$

$A^{-1} = +\frac{1}{5}\left[+A^2 \mp 5A + 6I\right]$

$\cancel{A^{-1} = \frac{1}{5}}\left(\begin{bmatrix} -2 & 1 & 4 \\ 3 & 1 & 3 \\ -5 & 5 & 14 \end{bmatrix} - 5\begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right.$

$$\begin{bmatrix} -2 & 1 & 4 \\ 3 & 1 & 3 \\ -5 & 5 & 14 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 5 \\ 15 & 5 & 0 \\ -10 & 5 & 20 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$A^{-1} = \frac{1}{5}\begin{bmatrix} 4 & 1 & -1 \\ -12 & 2 & 3 \\ 5 & 0 & 0 \end{bmatrix}$$

x——x

Ques 3

Ques1) Find the non-singular matrix PAQ is in normal form? & rank also?

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}_{3\times4}$$

Ques2) Use the Gauss Jordan Method to find the inverse of the matrix?

$$A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

Ques3) $\rho(A) = ?$ $\quad A = \begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & 4 & -3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 6 & 7 \end{bmatrix}$

Ques) Reduce to Normal form $A = \begin{bmatrix} 1 & 2 & 3 & -2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$

x———x