BBM 205 - Discrete Structures: Midterm 2

Date: 28.11.2017, Time: 16:00 - 17:30

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Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	10	12	8	14	8	16	8	8	8	8	100
Score:											

1. (a) (6 points) Use a **combinatorial proof** to show the following equality. (Kombinatoryal ispat yontemiyle asagidaki esitligi gosterin.)

$$\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$$

(b) (4 points) Use the equality in the first part to show that (Ilk bolumdeki esitligi kullanarak asagidakini gosterin.)

$$\binom{2n}{n} = 2\binom{2n-1}{n-1}$$

(b)	(3 points) What are the initial conditions? (Baslangic kosullari nedir?)
(c)	(3 points) How many ways can this person climb a flight of 11 stairs? (Basamak sayisi 11 oldugunda yukarida aciklandigi gibi basamaklari cikmanin kac farkli yolu vardir?)
ı rec	points) Let s_n denote the number of n -bit strings that contain the pattern 01. Find currence relation to express s_n . Justify your work. $(s_n, 01)$ altkatar olarak iceren n anli bit katarlarinin sayisi olsun. s_n i ifade eden rekursif bir iliski bulun.)
	(8 pc

4. (a) (8 points) Use extended Euclid's algorithm to calculate the multiplicative inverse of 7 mod 23. ('Extended Euclid' algoritmasini kullanarak 7'nin mod 23'de carpmaya gore tersini bulun.)

(b) (6 points) Find $x \pmod{23}$ if $7x + 20 \equiv 5 \pmod{23}$. (Verilen kosulu saglayan x'i bulun)

5. (8 points) Find the solution to the recurrence relation below with the given initial questions: (Verilen rekursif iliski ve baslangic kosullarini saglayan cozum bulun.)

$$a_n = (n+1)a_{n-1}, \qquad a_0 = 2.$$

6. (a) (4 points) Explain how the pigeonhole principle can be used to show that among 11 integers at least two must end with the same digit. (Guvercinyuvasi kuralini kullanarak her 11 tamsayi arasinda mutlaka son basamagi ayni olan iki sayi olacagini gosterin.)

(b) (4 points) Explain how the generalized pigeonhole principle can be used to show that among 91 integers, there are at least ten that end with the same digit. (Genel guvercinyuvasi kuralini kullanarak her 91 tamsayi arasinda mutlaka son basamagi ayni olan en az on sayi olacagini gosterin.)

(c) (4 points) Use generalized pigeonhole principle to find how many cards must be selected from a standard deck of 52 cards to guarantee that at least four cards of the same suit are chosen. (Genel guvercinyuvasi kuralini kullanarak 52 kartlik standart kart destesinden ayni sekilden en az dort kart cikmasi icin desteden cekilmesi gereken minimum kart sayisini bulun.)

(d) (4 points) Use binomial theorem to show that (Binom teoremini kullanarak gosterin.)

$$\sum_{k=0}^{n} 3^k \binom{n}{k} = 4^n.$$

7.	In the following	parts,	please	circle y	your f	final	answer.	(Asagida	verilen	maddelerde
	buldugunuz ceva	bi yuva	arlak ici	ne alin.	.)					

(a)	(2 points) How man	y different ways	are there to	throw 8 i	dentical	balls into	15
	distinguishable bins?	(Sekiz ozdes top	u birbirinden	farkli 15	kutuya k	koymanin l	kac
	yolu vardir?)						

(b) (2 points) A palindrome is a string whose reversal is identical to the string. How many bit strings of length n are palindromes? (Palindrom tersinden okunusu da kendisiyle ayni olan kelimedir. Uzunlugu n olan kac palindrom vardir?)

(c) (2 points) How many cards must be selected from a standard deck of 52 cards to guarantee that at least three spades are selected? (En az uc maca gelmesi icin standart 52 kartlik bir desteden en az kac kart cekmek gerekir?)

(d) (2 points) How many bit strings of length 9 over the alphabet $\{a, b, c, d\}$ have either exactly three b's or exactly five c's? (Karakter olarak $\{a, b, c, d\}$ 'nin kullanilabilecegi kelimelerden uzunlugu 9 olan ve tam tamina uc b'yi ya da tam tamina bes c'yi iceren kac kelime vardir?)

- 8. In the following parts, please circle your final answer. (Asagida verilen maddelerde, buldugunuz cevabi yuvarlak icine alin.)
 - (a) (2 points) How many different words can be written by using the letters in KIZILC-AHAMAM when all letters are used and the word starts and ends with A? (KIZILC-AHAMAM icindeki butun harfleri kullanarak ve A harfiyle baslayip biten kac kelime yazilabilir?)

(b) (2 points) How many ways are there to pair up 14 students so that every student is paired up? (Herkese eslestirilmis olacak sekilde 14 ogrenci kac farkli sekilde eslestirilebilir?)

(c) (2 points) How many solutions does $x_1+x_2+\cdots+x_k=n$ have if each x_i ($1 \le i \le k$) must be a positive integer (at least 1)? (Verilen esitligin icindeki her x_i bir pozitif tamsayi olacak sekilde kac farkli cozum vardir?)

(d) (2 points) How many ways are there to distribute 11 distinguishable balls numbered 1 through 11 among 25 distinguishable bins? (Birbirinden farkli 11 top 25 farkli kutuya kac farkli sekilde dagitilabilir?)

9. (8 points) Solve the recurrence relation

$$a_n = \sqrt{\frac{a_{n-2}}{a_{n-1}}}$$

with initial conditions $a_0=8,\,a_1=1/(2\sqrt{2}).$ (Verilen baslangic kosullari kullanılarak rekursif iliski icin cozum bulun.)

10. (8 points) Solve the recurrence relation with the given initial conditions. (Verilen baslangic kosullari kullanilarak rekursif iliski icin cozum bulun.)

$$a_n = 2a_{n-1} + 8a_{n-2}, a_0 = 4, a_1 = 10.$$