



# Context Free Language

In formal language theory, a Context Free Language is a language generated by some Context Free Grammar.

The set of all CFL is identical to the set of languages accepted by Pushdown Automata

## Context Free Grammar

Context Free Grammar is defined by 4 tuples as:

$$G = N, \Sigma, R, S$$

where

$N$  = A set of nonterminals (e.g.  $N = \{S, NP, VP, PP, Noun, Verb \dots\}$ )

$\Sigma$  = A set of terminals (e.g.  $\Sigma = \{I, you, he, eat, drink, sushi, ball, \dots\}$ )

$R$  = A set of rules  $R \subseteq \{A \rightarrow \beta \text{ with left-hand side } A \in N \text{ and right-hand side } \beta \in (N \cup \Sigma)^*\}$

$S$  = A start symbol  $S \in N$

### Example

- $DT \rightarrow \{\text{the, a}\}$
- $N \rightarrow \{\text{ball, garden, house, sushi}\}$
- $P \rightarrow \{\text{in, behind, with}\}$
- $NP \rightarrow DT\ N$
- $NP \rightarrow NP\ PP$
- $PP \rightarrow N\ PP$
  
- N: noun
- P: preposition
- NP: noun phrase
- PP: prepositional phrase

**\*\* Example\*\*** For generating a language that generates equal number of **a\*\*** and **\*\*b** in the form  $a^n b^n$ , The CFG will be defined as:

$G = \{(S,A),(a,b), (S \rightarrow aAb, A \rightarrow aAb) \in S\}$

# Example:

```
from nltk import CFG
grammar = CFG.fromstring("""
S -> NP VP
PP -> P NP
NP -> Det N | NP PP
VP -> V NP | VP PP
Det -> 'a' | 'the'
N -> 'dog' | 'cat'
V -> 'chased' | 'sat'
P -> 'on' | 'in'""")
```

grammar

```
print(grammar.start())
print(grammar.productions())
```

Output: S [S -> NP VP, PP -> P NP, NP -> Det N, NP -> NP PP, VP -> V NP, VP -> VP PP, Det -> 'a', Det -> 'the', N -> 'dog', N -> 'cat', V -> 'chased', V -> 'sat', P -> 'on', P -> 'in']

# Example:

```
import nltk
from nltk import CFG
grammar = CFG.fromstring("""
S -> NP VP
PP -> P NP
NP -> Det N | NP PP | N
VP -> V NP | VP PP
Det -> 'a' | 'the'
N -> 'dog' | 'cat' | 'Mary'
V -> 'chased' | 'sat' | 'saw'
P -> 'on' | 'in'""")

sent = 'Mary saw a dog'.split()
rd_parser = nltk.ShiftReduceParser(grammar)

for tree in rd_parser.parse(sent):
    print(tree)
```

Output: (S (NP (N Mary)) (VP (V saw) (NP (Det a) (N dog))))

# Parsing Algorithms

- Top-down vs. bottom-up:
  - Top-down: (goal driven): from the start symbol down.
  - Bottom-up: (data-driven): from the symbols up
- Naive vs. dynamic programming:
  - Naive: enumerate everything
  - Backtracing: try something, discard partial solutions
  - Dynamic programming: save partial solutions in a table
- Examples:
  - CKY: bottom-up dynamic programming
  - Earlparsing: top-down dynamic programming



# A simple English Grammar

$S \rightarrow NP VP$

$S \rightarrow Aux NP VP$

$S \rightarrow VP$

$NP \rightarrow Det NOM$

$NP \rightarrow ProperNoun$

$NOM \rightarrow Noun$

$NOM \rightarrow Noun NOM$

$NOM \rightarrow NOM PP$

$VP \rightarrow Verb$

$VP \rightarrow Verb NP$

$PP \rightarrow Prep NOM$

$Det \rightarrow that \mid this \mid a \mid the$

$Noun \rightarrow book \mid flight \mid meal \mid money$

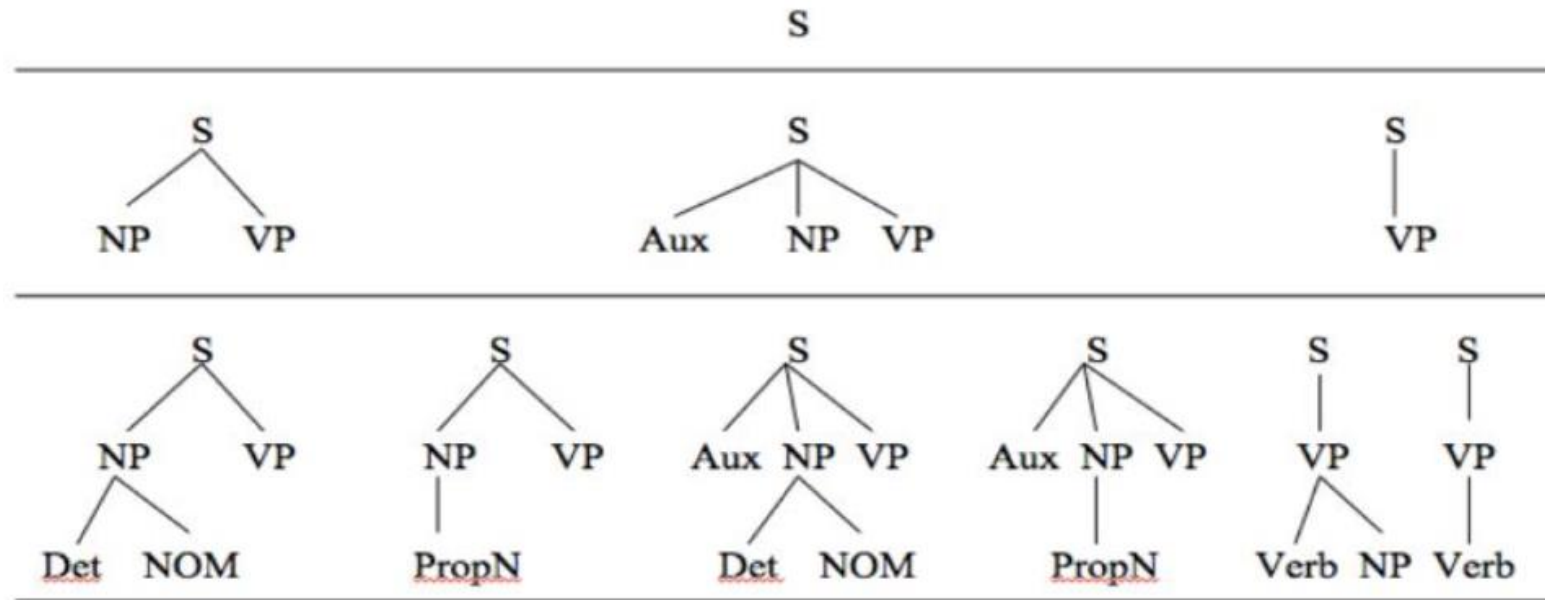
$Verb \rightarrow book \mid include \mid prefer$

$Aux \rightarrow does$

$Prep \rightarrow from \mid to \mid on$

$ProperNoun \rightarrow Houston \mid Washington$

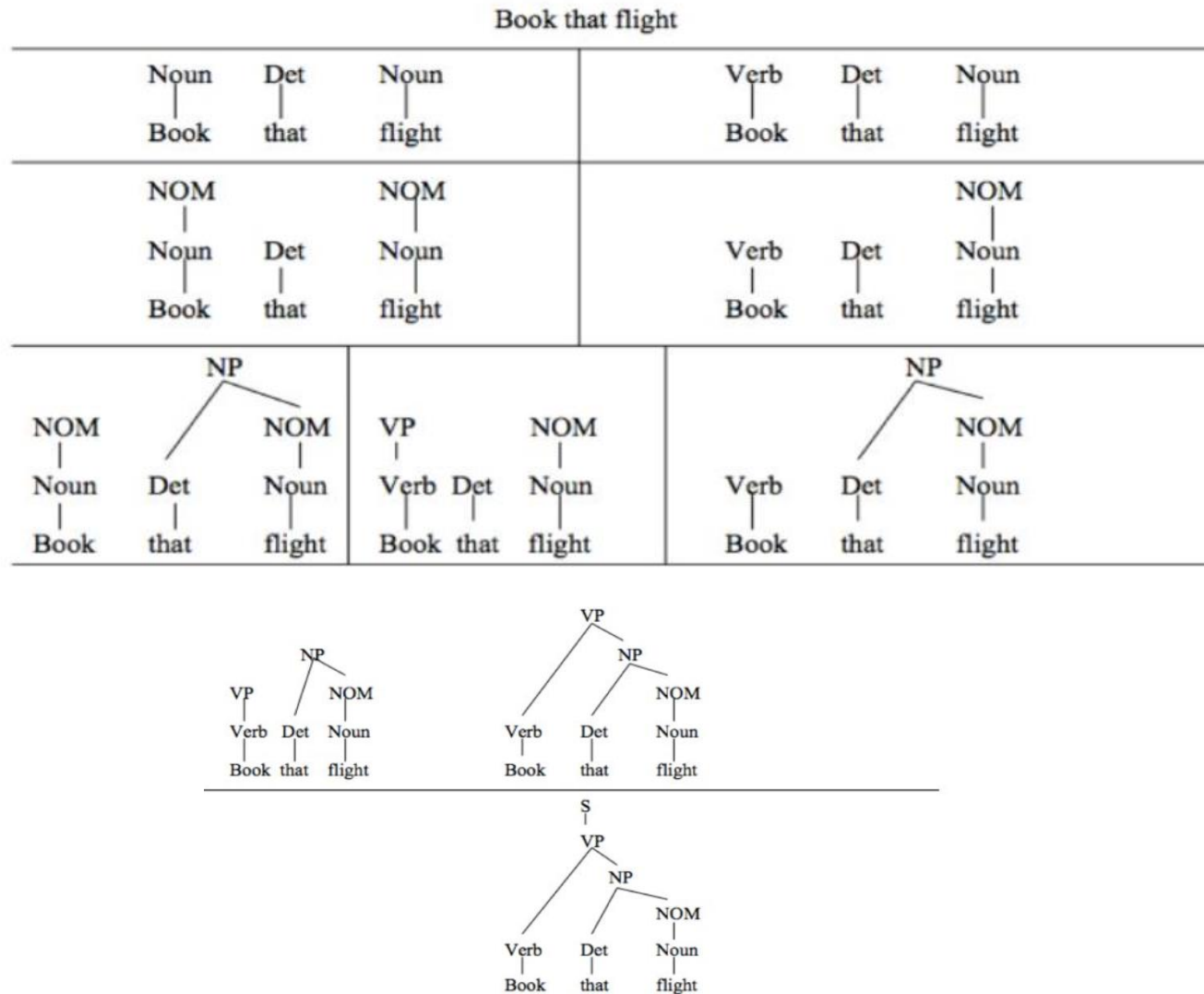
# A Top-Down Search Space



*Input:* Book that flight



# A Bottom-Up Search Space



# The CYK Algorithm

$X_{1,5}$				
$X_{1,4}$	$X_{2,5}$			
$X_{1,3}$	$X_{2,4}$	$X_{3,5}$		
$X_{1,2}$	$X_{2,3}$	$X_{3,4}$	$X_{4,5}$	
$X_{1,1}$	$X_{2,2}$	$X_{3,3}$	$X_{4,4}$	$X_{5,5}$
$w_1$	$w_2$	$w_3$	$w_4$	$w_5$

Construct a Triangular Table

# Example:

He was watching TV yesterday.

He was watching TV yesterday				
He was watching TV	was watching TV yesterday			
He was watching	was watching TV	watching TV yesterday		
He was	was watching	watching TV	TV yesterday	
He	was	watching	tv	yesterday
$w_1$	$w_2$	$w_3$	$w_4$	$w_5$

# Example:

- Fill in the CKY chart below for sentence “The rain rains down” assuming the following rules given in Table 1:


Table 1: Phrase Structure Rules

$S \rightarrow NP \ VP$	$DT \rightarrow the$
$NP \rightarrow N$	$N \rightarrow rain$
$NP \rightarrow DT \ N$	$N \rightarrow rains$
$VP \rightarrow V \ ADVP$	$V \rightarrow rain$
$VP \rightarrow V$	$V \rightarrow rains$
$ADVP \rightarrow ADV$	$ADV \rightarrow down$