CS 70 Fall 2019 — Discrete Mathematics and Probability Theory — Alistair Sinclair and Yun Song — Quiz 1

1. [True or False?] Mark each of the following “True" if it is a valid logical equivalence, or “False" otherwise.

☐ $P \Rightarrow Q \equiv P \vee \neg Q$ False

☐ $P \Rightarrow Q \equiv (\neg P \Rightarrow \neg Q)$ False

☐ $P \Rightarrow Q \equiv (Q \wedge P) \vee \neg P$ True

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2. [True or False?] Let $P(x)$ be a proposition about an integer x, and suppose you want to prove the theorem $\forall x\ (P(x) \Rightarrow Q(x))$. Mark each of the following proof strategies “True" if it would be a valid way to proceed with such a proof, or “False" otherwise.

☐ Find an x such that $Q(x)$ is true or $P(x)$ is false. False

☐ Show that, for every x, if $Q(x)$ is false then $P(x)$ is false. True

☐ Assume that there exists an x such that $P(x)$ is false and $Q(x)$ is false and derive a contradiction. False

☐ Assume that there exists an x such that $P(x)$ is true and $Q(x)$ is false and derive a contradiction. True

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3. [Proof] Suppose you have a rectangular array of pebbles, where each pebble is either red or blue. Suppose that for every way of choosing one pebble from each column, there exists a red pebble among the chosen ones. Prove that there must exist an all-red column.

Proof: We give a proof by contraposition. Suppose there does not exist an all-red column. This means that, in each column, we can find a blue pebble. Therefore, if we take one blue pebble from each column, we have a way of choosing one pebble from each column without any red pebbles. This is the negation of the original hypothesis, so we are done.