BBM 205 - Discrete Structures: Final Exam Date: 15.1.2018, Time: 9:30 - 11:30

Name:

Student ID:

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	10	22	6	8	21	10	9	14	16	10	126
Score:											

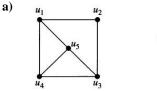
You are expected to show all your work in your answers to receive full credit from the questions.

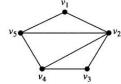
1. (10 points) Show by using induction on the number of vertices that every tree can be colored properly by 2 colors. (Information you can use: Every tree has at least two vertices with degree 1.) (Kose sayisi uzerinden tumevarim yontemiyle her agacin iki renk ile uygun boyanabilecegini gosterin. Bilgi olarak, her agacin derecesi bir olan en az iki kosesi oldugunu kullanabilirsiniz.)

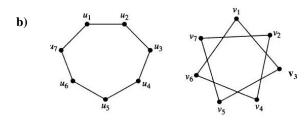
2. (a) (3 points) How many edges are there in a graph with 8 vertices each of degree 5? (Kose sayisi 8 ve her kosenin derecesinin 5 oldugu bir cizgede kac kenar vardir?) (b) (4 points) How many vertices and how many edges do these graphs have: (Asagidaki cizgelerin kose ve kenar sayisi nedir?) a)  $K_n$ , b)  $C_n$ , c)  $K_{m,n}$ , d)  $Q_n$ (c) (3 points) For which values of n are these graphs bipartite: (Asagidaki cizgeler, hangi n degerleri icin iki parcalidir?) a)  $K_n$ , b)  $C_n$ , c)  $Q_n$ (d) (3 points) For which values of m and n does the complete graph  $K_{m,n}$  have a Hamilton circuit? (Hangi m ve n degerleri icin  $K_{m,n}$  cizgesinin Hamilton dongusu vardir?) (e) (3 points) Give a list of six nonnegative integers that cannot be the degree list of a graph on six vertices. (Alti koseli bir cizgenin derece listesi olamayacak alti dogalsayidan olusan bir liste yazin.) (f) (3 points) If G is a simple graph with 15 edges and  $\bar{G}$  has 13 edges, how many vertices does G have? (Eger G cizgesi 15 kenar ve  $\bar{G}$  cizgesi 13 kenara sahipse, G'nin kac kosesi vardir?) (g) (3 points) Draw all nonisomorphic simple graphs with three vertices. (Uc koseye sahip birbirine izomorf olmayan butun cizgeleri gosterin.)

3. (6 points) Let G be a graph with n vertices and e edges. Let M be the maximum degree in G. Show that  $2e/n \leq M$ . (G cizgesi n sayida kose ve e sayida kenar icersin. Eger M, G'nin koselerinin maksimum derecesi ise,  $2e/n \leq M$  oldugunu gosterin.)

4. (8 points) Determine whether the following pairs of graphs are isomorphic or not. (If isomorphic, provide the isomorphism function, if not isomorphic, give an explanation.) (Asagida verilen cizge ciftlerinin birbirine izomorf olup olmadigini yazin. Eger izomorf ise, izomorfizma fonksiyonunu yazin. Eger izomorf degilse, sebebini aciklayin.)







- 5. In the questions below, all dice and coins are fair, unless specified differently. (Asagidaki sorularda, farkli belirtilmedikce, butun zarlar ve paralar hilesizdir.)
  - (a) (3 points) What is the probability that a die comes up six when it is rolled? (Bir zar atildiginda, alti gelme olasiligi nedir?)
  - (b) (3 points) What is the probability that a randomly selected day of the year is in April assuming a year has 365 days? (Bir yilin gunlerinden rastgele secilen bir gunun, Nisan ayindan olma olasiligi nedir?)
  - (c) (3 points) What is the probability that the sum of the numbers on two dice is even when they are rolled? (Atilan iki zar uzerindeki sayilarin toplaminin cift olma olasiligi nedir?)
  - (d) (4 points) What is the probability that when a coin is flipped six times in a row, the number of heads is at least 3? (Bir demir para alti defa sirayla havaya atildiginda, toplamda en az uc defa yazi gelme olasiligi nedir?)
  - (e) (4 points) What is the probability that n precedes 1 and n-1 precedes 2, when we randomly select a permutation of  $\{1, 2, ..., n\}$ ? ( $\{1, 2, ..., n\}$  sayi kumesinin butun permutasyonlari arasindan rastgele secilen bir permutasyonda n'in 1'den once ve n-1'in 2'den once gelme olasiligi nedir?)

(f) (4 points) Suppose that E and F are events such that Pr(E) = 0.7 and Pr(F) = 0.5. Show that  $Pr(E \cup F) \ge 0.7$  and  $Pr(E \cap F) \ge 0.2$ . (Diyelim ki E ve F olaylarinin olasiligi Pr(E) = 0.7 ve Pr(F) = 0.5 olsun.  $Pr(E \cup F) \ge 0.7$  ve  $Pr(E \cap F) \ge 0.2$  oldugunu gosterin.)

6.	(a)	(3 points) Write Euler's formula for planar graphs. (Duzlemsel cizgeler icin Euler formulunu yazin.)
	(b)	(7 points) Use Euler's formula to show that $K_{3,3}$ is not planar. (Euler formulumukullanarak $K_{3,3}$ 'un duzlemsel olmadigini gosterin.)
7.	a correplation (Diy topl	ose you have balls numbered $1, \ldots, n$ , where $n$ is a positive integer at least 2, inside fee mug. You pick a ball uniformly at random, look at the number on the ball ce the ball back into the coffee mug, and pick another ball uniformly at random which is a 2 olan bir pozitif $n$ sayisi icin, $1, \ldots, n$ sayilariyla numaralandirilmistr, bir bardaga doldurulmus olsun. Rastgele bir top secilip uzerindeki sayiya bakilip ekrar bardagin icine konup, tekrar baska bir top rastgele secilsin.)
	(a)	(2 points) What is the probability that the first ball is 1 and the second ball is 2? (Birinci topun 1 ve ikinci topun 2 sayisina sahip olma olasiligi nedir?)
	(b)	(3 points) What is the probability that the second ball's number is exactly one greater than the first ball's number? (Ikinci topun uzerindeki sayinin, birinci topun uzerindeki sayidan tam bir fazla olmasi olasiligi nedir?)
	(c)	(4 points) What is the probability that the second ball is strictly less than the first ball's number? (Ikinci topun uzerindeki sayinin, birinci topun uzerindeki sayidan kucuk olmasi olasiligi nedir?)

- 8. In the questions below, all dice and coins are fair, unless specified differently. (Asagidaki sorularda, farkli belirtilmedikce, butun zarlar ve paralar hilesizdir.)
  - (a) (5 points) Suppose that E,  $F_1$ ,  $F_2$ , and  $F_3$  are events from a sample space and that  $F_1$ ,  $F_2$ , and  $F_3$  are mutually disjoint and their union is S. Find  $Pr(F_2|E)$  if  $Pr(E|F_1) = 2/7$ ,  $Pr(E|F_2) = 3/8$ ,  $Pr(E|F_3) = 1/2$ ,  $Pr(F_1) = 1/6$ ,  $Pr(F_2) = 1/2$ , and  $Pr(F_3) = 1/3$ . (Diyelim ki E,  $F_1$ ,  $F_2$ , ve  $F_3$  olaylari birbirinden ayrik ve birlesimleri ornekler uzayi S olsun. Yukarida verilen olasilik degerlerini kullanarak  $Pr(F_2|E)$ 'yi bulun.)
  - (b) (5 points) What is the expected value of the sum of the numbers that appear when a pair of fair dice is rolled. Show all your work. (Iki hilesiz zar atildiginda, zarlarin uzerinde gelen degerlerin toplaminin beklenen degeri nedir? Islemlerinizin tumunu gosterin.)

(c) (4 points) When one red die and one black die are rolled, show that the probability of the events A = sum is 7 and B = red die is 1 are independent. (Bir kirmizi ve bir siyah zar atildiginda, A = toplam sayinin 7 olmasi ve B = kirmizi zarin 1 olmasi olaylarinin bagimsiz oldugunu gosterin.)

The questions in the following are to receive extra credit and you are free not to solve them.

- 9. Zeynep and Umut are flipping coins. Zeynep flips a fair coin k times and Umut flips n-k times. In total there are n coin flips. (Zeynep ve Umut para atisi yapiyorlar. Zeynep k defa hilesiz parayi atarken, Umut n-k defa atiyor. Toplamda n atis yapilliyor.)
  - (a) (6 points) Use a combinatorial proof to show that (Kombinatoryal ispatla cozun.)

$$\sum_{i=0}^{i=k} \binom{k}{k-i} \binom{n-k}{i} = \binom{n}{k}.$$

You may assume  $n-k \ge k$ .  $(n-k \ge k \text{ oldugunu varsayabilirsiniz.})$ 

(b) (6 points) Prove that Zeynep and Umut flip the same number of heads is equal to the probability that there are a total of k heads. (Zeynep ve Umut'un ayni miktarda yazi atmasinin olasiliginin, toplamda k yazi olma olasiligina esit oldugunu gosterin.)

10. (10 points) For a graph with n vertices, if the degree of each vertex is at least n/2, then G is connected. (Eger n koseli bir G cizgesinin her kosesinin derecesi en az n/2 ise, o zaman G'nin bagli bir cizge oldugunu gosterin.)