

# Lecture 4: Prolog

## COMP24412: Symbolic AI

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- 1 Introduction: What is Prolog?
- 2 Prolog queries
- 3 Syntax of Prolog programs
- 4 Unification

# Outline

1 Introduction: What is Prolog?

2 Prolog queries

3 Syntax of Prolog programs

4 Unification

- Programmation en logique - “Programming in logic”
- Declarative programming language:  
describe solution, not how to get there
- Based on automated theorem proving in FOL (SLD Resolution)  
easier to reason about
- Super-set of Datalog
- Turing-complete

# History

- ~1972: Colmerauer and Roussel define language, first implementation
- 1977: Warren writes first compiler (DEC-10 Prolog)
- 1983: Warren abstract machine (WAM)
- 1995: Becomes ISO/IEC 13211-1 standard
- 2000: Latest standard so far ISO/IEC 13211-2

# Common Implementations

- Ciao Prolog
- Eclipse
- GNU Prolog
- IF Prolog
- Sicstus Prolog\*
- SWI Prolog\*
- XSB Prolog
- YAP Prolog

\* available during exercise classes

# Fields of use

- Prototyping
- Constraint Solving, Logistics
- Parsing, Natural Language Processing
- Search Problems with non-deterministic decisions

- Query Engine of IBM Watson

[https://www.theregister.co.uk/2009/04/27/ibm\\_watson\\_jeopardy?page=2](https://www.theregister.co.uk/2009/04/27/ibm_watson_jeopardy?page=2)

- Clarissa (NASA): speech guided navigations through maintenance procedures on ISS

<https://ti.arc.nasa.gov/tech/cas/user-centered-technologies/clarissa>

- ~ 1/3 of flight bookings in Europe handled by a Prolog system

<https://www.sics.se/projects/sicstus-prolog-leading-prolog-technology>



# Anatomy of a Prolog Program

- Program = Facts + Rules
- Query: “Is this fact derivable from the program?”
- Queries may contain variables
- Answer substitution:  
“Which variable assignments are necessary to derive the query?”
- Queries often have multiple answers!

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# Prolog terms

- Constant: basic object
- Variable: can be replaced by another term
- Predicate:
  - Rules define predicates
  - Never appear inside another term
  - There is no return value!

- Predefined predicates:
  - $X = Y$ : true if LHS and RHS are equal  
There is no assignment!
  - $\text{dif}(X,Y)$ : true if LHS and RHS are different (not ISO)
- Function:
  - always appears inside a predicate
  - comparable to datastructures
  - There is no return value!

# Prolog terms

```
band_song_date(rihanna, Song, date(Y,M,D))
```

- **rihanna** : Constant term  
Starts with a lower-case letter
- **'Twist and shout'**: Quoted constant term
- **Song** : Variable  
Starts with an upper-case letter
- **\_**: Anonymous variable  
We will not be informed about assignments of this variable
- **band\_song\_date/3** : Predicate (of arity 3) Starts with a lower-case letter
- **date/3**: function (of arity 3)

- Suppose we have a database of bands and their songs
- “Is Rihanna’s Diamonds in the database?”

```
?- band_song(rihanna, diamonds).  
false.
```

- “Do we know about any song by Rihanna?”

```
?- band_song(rihanna, Song).  
false.
```

- “Is there anything in the database?”

```
?- band_song(Band, Song).  
Band = beatles,  
Song = 'While_my_guitar_gently_weeps' ;  
Band = beatles,  
Song = 'Twist_and_shout' ;  
Band = beatles,  
Song = 'Love_me_do'  
% ....
```



- “There’s surely more than The Beatles?”

```
?- dif(Band, beatles), band_song(Band, Song).  
Band = 'Isley_Brothers',  
Song = 'Twist_and_shout' ;  
Band = iggy,  
Song = 'The_passenger' ;  
Band = banshees,  
Song = 'The_passenger' .  
% ....
```

- “Which versions of The Passenger are there?”

```
?- Song = 'The_passenger', band_song(Band, Song).  
Song = 'The_passenger',  
Band = iggy ;  
Song = 'The_passenger',  
Band = banshees ;  
Song = 'The_passenger',  
Band = bauhaus .
```

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# Anatomy of a query

```
?- dif(Band, beatles), band_song(Band, Song).  
Band = 'Isley Brothers',  
Song = 'Twist and shout';  
Band = iggy,  
Song = 'The passenger' a
```

- **?-** Query prompt
- **,** Conjunction of queries
- **.** End of query
- **Band = 'Isley Brothers', Song = 'Twist and shout':** Answer substitution
- **;** User input (next answer)
- **a** User input (abort)

# Turning a query into a rule

- Query

```
?- dif(Band, beatles), band_song(Band, Song).
```

- Rule

```
nobeatles_song(Band, Song) :-  
    dif(Band, beatles),  
    band_song(Band, Song).
```

# Anatomy of a rule

```
head(X,Y,Z) :-  
    goal1(X,A),  
    goal2(Y,B),  
    goal3(A,B,Z).
```

- “Derive head **if** goal1 **and** goal2 **and** goal3 are derivable.”

- Predicate logic formula:

$\forall X, Y, Z, A, B.$

$$\begin{aligned} & goal1(X, A) \wedge goal2(Y, B) \wedge goal3(A, B, Z) \\ & \rightarrow head(X, Y, Z) \end{aligned}$$

# Facts

A fact is always true:

```
coldplace(siberia) :-  
    true.
```

Easier to write:

```
coldplace(siberia).
```

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## Substitution:

- Maps finitely many variables to terms
- All other variables are mapped to themselves
- Apply  $\sigma = \{X=\text{car}, Y=\text{house}\}$  to  $\text{owns}(\text{lucia}, X)$  and obtain  $\text{owns}(\text{lucia}, \text{car})$
- Apply  $\sigma$  to  $\text{owns}(\text{lucia}, Z)$  and obtain  $\text{owns}(\text{lucia}, Z)$
- Substitutions can be composed:  
$$\tau = \{\text{pair}(X, Y)\}$$
$$\tau\sigma = \{\text{pair}(\text{car}, \text{house})\}$$

# Unification

```
?- X=1, X=2.  
false.
```

Why?

# Unification

```
?- X=1, X=2.  
false.
```

Why?

- assign  $X = 1$ :  
1=1, 1=2.

# Unification

```
?- X=1, X=2.  
false.
```

Why?

- assign  $X = 1$ :  
 $1=1, 1=2$ .
- assign  $X = 2$ :  
 $1=2, 2=2$ .

# Are these terms unifiable?

- `contains(X, milk) = contains(capuccino, Y)`

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- `contains(X, milk) = contains(capuccino, Y)`  
yes
- `contains(X, house) = contains(house, X)`  
yes
- `contains(X, milk) = contains(capuccino, X)`  
no
- `climate(X) = climate(Y)`  
yes



# Unification Problem

## Unification Problem

Given a set of term equalities  $s_1 = t_1, \dots, s_n = t_n$ , is there a unifying substitution  $\sigma$  such that for each equation  $s=t$ ,  $s\sigma$  and  $t\sigma$  are the same terms?

If yes, which one?

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If yes, which one?

Unifiers for the problems before:

- $X=\text{cappuccino}, Y=\text{milk}$
- $X=\text{house}, Y=X$
- not unifiable
- $X=Y$

# Unification Rules

Transformation rules:

- trivial:  $t = t, P$   
delete  $t = t$ , solve  $P$
- orient:  $t = X, P$   
move variable to LHS:  $X = t, P$
- decomposition:  $f(s1, \dots, sn) = f(t1, \dots, tn), P$   
solve  $s1 = t1, \dots, sn = tn, P$
- variable elimination:  $X = t, P$   
replace all occurrences of  $X$  in  $P$  with  $t$ , solve  $P$

# Unification Rules

Solved form:

- $P$  contains only equations  $X = a, Y = b, \dots$
- No Transformation rule can be applied

Failure cases:

- name clash (constants):  $c = d$
- name clash (functions):  $f(A, B) = g(X, Y)$
- occurs check:  $X = f(X)$   
( $X$  occurs nested inside RHS term)

# Unification: examples

- $\text{contains}(X, \text{milk}) = \text{contains}(\text{capuccino}, Y)$ 
  - Decompose:  $X = \text{capuccino}, \text{milk} = Y$
  - Orient:  $X = \text{capuccino}, Y = \text{milk}$
  - Solved!

# Unification: examples

- $\text{contains}(X, \text{house}) = \text{contains}(\text{house}, X)$ 
  - Decompose:  $X = \text{house}, \text{house} = X$
  - Eliminate  $X$ :  $\text{house} = \text{house}$
  - Remove trivial
  - Solved!

# Unification: examples

- $\text{contains}(X, \text{milk}) = \text{contains}(\text{capuccino}, X)$ 
  - Decompose:  $X = \text{capuccino}, \text{milk} = X$
  - Eliminate  $X$ :  $\text{milk} = \text{capuccino}$
  - Constant clash
  - Failure!

# Most general unifiers

Problem:  $f(X) = f(Y)$  has infinitely many unifiers:

$X=a, Y=a$

$X=b, Y=b$

$X=c, Y=c$

...

$X=f(a), Y=f(a)$

$X=g(a), Y=g(a)$

...

$X=Y$



# Most general unifiers

## More general substitutions

Let  $\sigma$  and  $\tau$  be substitutions. If there exists a non-trivial substitution  $\lambda$  such that  $\sigma\lambda = \tau$  then  $\sigma$  is *more general* than  $\tau$ .

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- $\{X=Y\}$  is more general than  $X = a$  (take  $\lambda = \{Y=a\}$ )
- $\{X=b\}$  neither more nor less general than  $\{X=a\}$

# Most general unifiers

## More general substitutions

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Applying the unification algorithm presented, we always obtain the most general unifier (up to renaming of variables).

# Summary

- Prolog is a Turing complete, logic based programming language
- Queries to Prolog program yields a sequence of answer substitutions
- Answers are found by backward chaining, trying the rules in order of appearance
- Suitable rules to apply are found via unification
- Functions allow the expression of arbitrary large terms, e.g. lists

That's all for today!