MECH 539: Computational Aerodynamics
Department of Mechanical Engineering, McGill University

Final Project : Solve the Quasi One-Dimensional Euler Equations for Various Artificial Dissipation Schemes
Due 30th April, 2013

Solve the quasi-1D Euler equations for various finite-volume schemes. The nozzle geometry is given as,

$$S(x) = 1 - h\left[\sin\left(\pi x^{t_1}\right)\right]^{t_2}, \quad \text{for } 0 \leq x \leq 1$$

where $h = 0.15$ is the bump height, $t_1 = 0.8$ locates the maximum location of the bump and $t_2 = 3$ controls the width of the bump in the channel. The following are the flow conditions,

Specific heat ratio, $\gamma = 1.4$
Inlet Total Temperature, $T_t = 531.2$ R
Inlet Total Pressure, $p_t = 2117.0$ lb/ft^2
Gas Constant, $R = 1716$ ft·lb/slug·R

Provide the following in a written report:

1. Solve the quasi one-dimensional Euler equations using a scalar dissipation scheme for the spatial discretization, and a simple Euler explicit scheme for the temporal discretization if the exit static pressure, $p_{\text{exit}} = 0.8p_t$. Discretize the nozzle with 50 points. Show plots of the convergence of the density residual, pressure distribution across the channel, and Mach number distribution.

2. **Exit pressure study.** Solve the quasi one-dimensional Euler equations using the Scalar dissipation scheme for four exit static pressure to inlet total pressure ratios of 0.76, 0.72, 0.68, and 0.60 using 50 grid points. Tweek the value of ϵ for the scalar dissipation scheme until at least only two points are observed in the shock for the exit static pressure to inlet total pressure ratio of 0.76. Use the same value of ϵ value for the 0.72, 0.68, and 0.60 cases. Is the code stable at this value ϵ? If not, then what are the values for the other cases? For each exit pressure ratio, show the convergence of the density residual, pressure distribution across the channel, and Mach number distribution.

3. **Grid study.** Solve the equation for an exit pressure ratio of 0.72 for 25, 50, 100, and 200 grid points. Comment on the value of ϵ. Is there a normal shock in the channel? Is the location of the shock the same for all four grid sizes. Explain your answer. For each grid, show the convergence of the density residual as a function of the number of iterations, pressure distribution across the channel, and Mach number distribution on the same plot. Discuss your findings by comparing the solutions. Does it require the same number of iterations to converge the answer for each grid? Explain your answer.

4. **Spatial discretization scheme study.** Solve the equation for an exit pressure ratio of 0.72 for four different artificial dissipation schemes: Steger-Warming, Corrected Modified Steger-Warming, Roe, and Scalar Dissipation on a 100 point grid. For each grid, show the convergence of the density residual as a function of the number of iterations, convergence of the density residual as a function CPU time, pressure distribution across the channel, and Mach number distribution. Discuss your findings by comparing the solutions.

5. **Temporal discretization scheme study.** Solve the equation for an exit pressure ratio of 0.72 and 100 grid points for two different time stepping schemes: Euler explicit and your choice of the following: MacCormack, Crank-Nicolson, or Jameson's Runge-Kutta. For each time-stepping scheme, show the convergence of the density residual as a function of the number of iterations and the convergence of the density residual as a function CPU time. Discuss your findings by comparing the solutions.

[Note:]

- When required to compare different schemes, solutions on a different number of grid points, please plot the results on the same graph, and make sure that that axis are labeled and legend provided for the reader.

- When pressure plots are required, please plot, the local static to the inlet total pressure ratio.