MECH 539: Computational Aerodynamics
Department of Mechanical Engineering, McGill University

Project # 5: Solve the Quasi One-Dimensional Euler Equations for a Supersonic Nozzle
Due 12th April, 2013

Solve the quasi-1D Euler equations for various finite-volume schemes. The nozzle geometry is given as,

$$S(x) = 1 - h\left[\sin\left(\pi x^{t_1}\right)\right]^{t_2}, \quad \text{for } \ 0 \le x \le 1$$

where $h = 0.025$ is the bump height, $t_1 = 0.8$ locates the maximum location of the bump and $t_2 = 3$ controls the width of the bump in the channel. The following are the flow conditions,

Specific heat ratio, $\gamma = 1.4$
Inlet Total Temperature, $T_t = 531.2$ R
Inlet Total Pressure, $p_t = 2117.0$ lb/ft^2
Gas Constant, $R = 1716$ ft·lb/slug·R
Inlet Mach Number, $M_{inlet} = 1.2$

Provide the following in a written report:

1. Solve the quasi one-dimensional Euler equations using a scalar dissipation scheme for the spatial discretization, and a simple Euler explicit scheme for the temporal discretization. Discretize the nozzle with 40 points. Show plots of the convergence of the density residual, pressure distribution across the channel, and Mach number distribution.

2. Evaluate the total pressure loss across the channel as a percentage of the inlet total pressure. Investigate the value of ϵ (coefficient for the scalar dissipation scheme) and its affect on the total pressure loss for a constant CFL value.

3. **Grid study.** Solve the equation for the same inlet Mach number for 10, 20, 40, 80, 160, and 320 grid points. For each grid, show the convergence of the density residual as a function of the number of iterations, pressure distribution across the channel, and Mach number distribution on the same plot. Discuss your findings by comparing the solutions. Does it require the same number of iterations to converge the answer for each grid? Explain your answer. Lastly plot on a log-log plot the percentage of the total pressure loss on the y-axis versus dx on the x-axis. On the same graph, plot a line of slope one. Does the code provide the correct order of accuracy.

4. **Spatial discretization scheme study.** Solve the equation for the Steger-Warming scheme. Repeat items 1, 2, and 3 above.

5. **Solver.** Solve the equation using Jameson's 4th. order modified Runge-Kutta. Show the convergence of the density residual as a function of the number of iterations and the convergence of the density residual as a function CPU time. Discuss your findings by comparing the solutions to the Euler explicit scheme. Ensure the rest of the scheme is identical.

[Note:]

- When required to compare different schemes, solutions on a different number of grid points, please plot the results on the same graph, and make sure that that axis are labeled and legend provided for the reader.
- When pressure plots are required, please plot, the local static to the inlet total pressure ratio.