COM2067/ COM267

Chapter 14: Searching and Sorting

Data Structures Using C, Second Edition

- Searching
- Sorting

INTRODUCTION TO SEARCHING

- Searching means to find whether a particular value is present in an array or not.
- If the value is present in the array, then searching is said to be successful and the searching process gives the location of that value in the array.
- However, if the value is not present in the array, the searching process displays an appropriate message and in this case searching is said to be unsuccessful.
- There are two popular methods for searching the array elements: *linear search* and *binary search*.
- The algorithm that should be used depends entirely on how the values are organized in the array.

Linear Search

- Linear search, also called as *sequential search*, is a very simple method used for searching an array for a particular value.
- It works by comparing the value to be searched with every element of the array one by one in a sequence until a match is found.
- Linear search is mostly used to search an unordered list of elements (array in which data elements are not sorted).
- For example, if an array A() is declared and initialized as, int A() = {10, 8, 2, 7, 3, 4, 9, 1, 6, 5}; and the value to be searched is VAL = 7, then searching means to find whether the value '7' is present in the array or not.
  - If yes, then it returns the position of its occurrence. Here, POS = 3 (index starting from 0).

Linear Search

- Figure 14.1 shows the algorithm for linear search.
- In Steps 1 and 2 of the algorithm, we initialize the value of POS and I.
- In Step 3, a while loop is executed that would be executed till I is less than N (total number of elements in the array).
- In Step 4, a check is made to see if a match is found between the current array element and VAL.
  - If a match is found, then the position of the array element is printed, else the value of I is incremented to match the next element with VAL.
  - However, if all the array elements have been compared with VAL and no match is found, then it means that VAL is not present in the array.

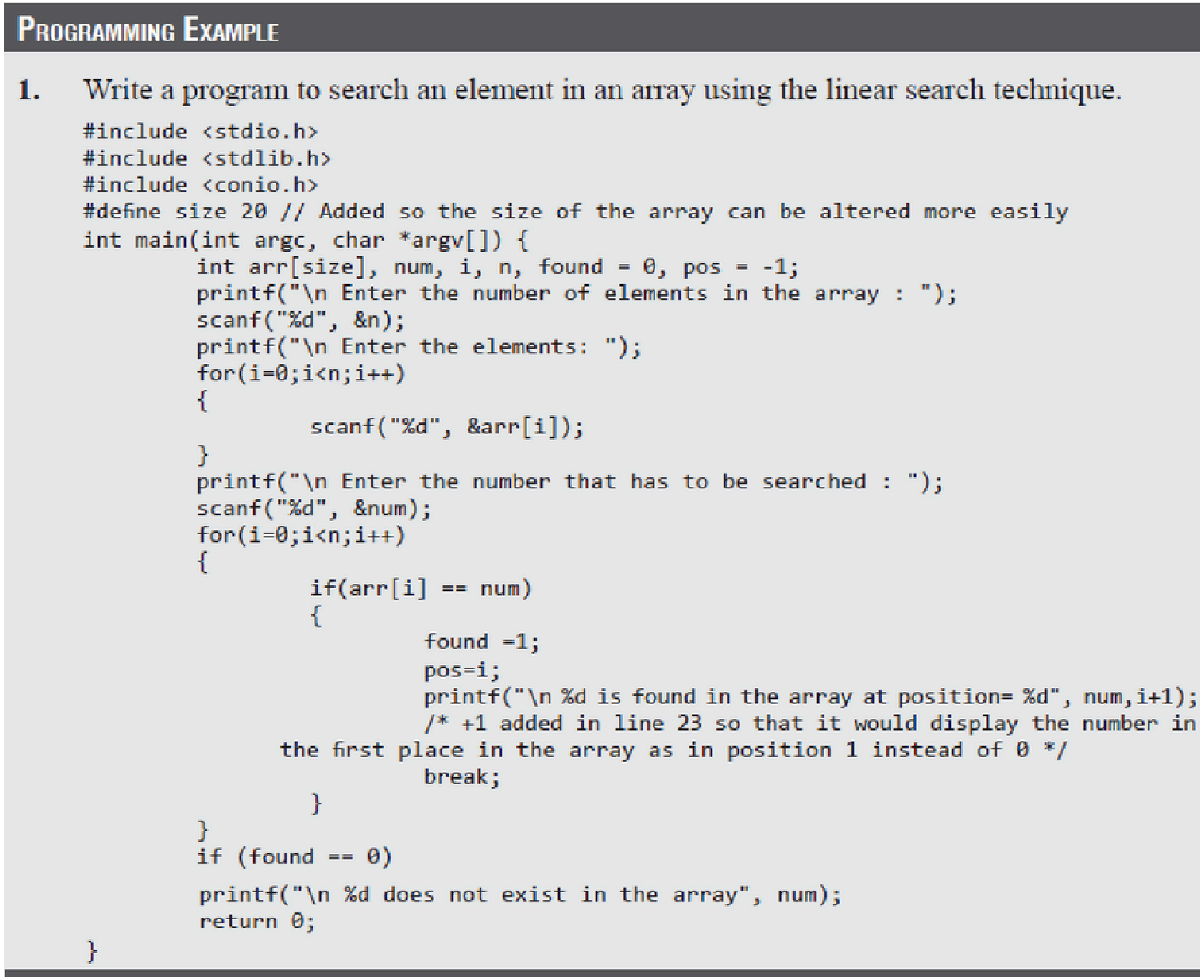
```
LINEAR_SEARCH(A, N, VAL)

Step 1: [INITIALIZE] SET POS = -1
Step 2: [INITIALIZE] SET I = 1
Step 3:     Repeat Step 4 while I<=N
Step 4:           IF A[I] = VAL
                        SET POS = I
                        PRINT POS
                        Go to Step 6
                  [END OF IF]
                   SET I = I + 1
            [END OF LOOP]
Step 5: IF POS = -1
        PRINT "VALUE IS NOT PRESENT
        IN THE ARRAY"
        [END OF IF]
Step 6: EXIT
```

Figure 14.1 Algorithm for linear search

Linear Search

- ***Complexity of Linear Search Algorithm***
- Linear search executes in O(n) time where n is the number of elements in the array.
  - Obviously, the best case of linear search is when VAL is equal to the first element of the array.
    - In this case, only one comparison will be made.
  - Likewise, the worst case will happen when either VAL is not present in the array or it is equal to the last element of the array.
    - In both the cases, n comparisons will have to be made.

Linear Search



PROGRAMMING EXAMPLE

1. Write a program to search an element in an array using the linear search technique.

```
#include <stdio.h>
#include <stdlib.h>
#include <conio.h>
#define size 20 // Added so the size of the array can be altered more easily
int main(int argc, char *argv[]) {
        int arr[size], num, i, n, found = 0, pos = -1;
        printf("\n Enter the number of elements in the array : ");
        scanf("%d", &n);
        printf("\n Enter the elements: ");
        for(i=0;i<n;i++)
        {
                scanf("%d", &arr[i]);
        }
        printf("\n Enter the number that has to be searched : ");
        scanf("%d", &num);
        for(i=0;i<n;i++)
        {
                if(arr[i] == num)
                {
                        found =1;
                        pos=i;
                        printf("\n %d is found in the array at position= %d", num,i+1);
                        /* +1 added in line 23 so that it would display the number in
              the first place in the array as in position 1 instead of 0 */
                        break;
                }
        }
        if (found == 0)
        printf("\n %d does not exist in the array", num);
        return 0;
}
```

Binary Search

- Binary search is a searching algorithm that works efficiently with a sorted list.
    - How do we find words in a dictionary? We first open the dictionary somewhere in the middle. Then, we compare the first word on that page with the desired word whose meaning we are looking for. If the desired word comes before the word on the page, we look in the first half of the dictionary, else we look in the second half. Again, we open a page in the first half of the dictionary and compare the first word on that page with the desired word and repeat the same procedure until we finally get the word.

Binary Search

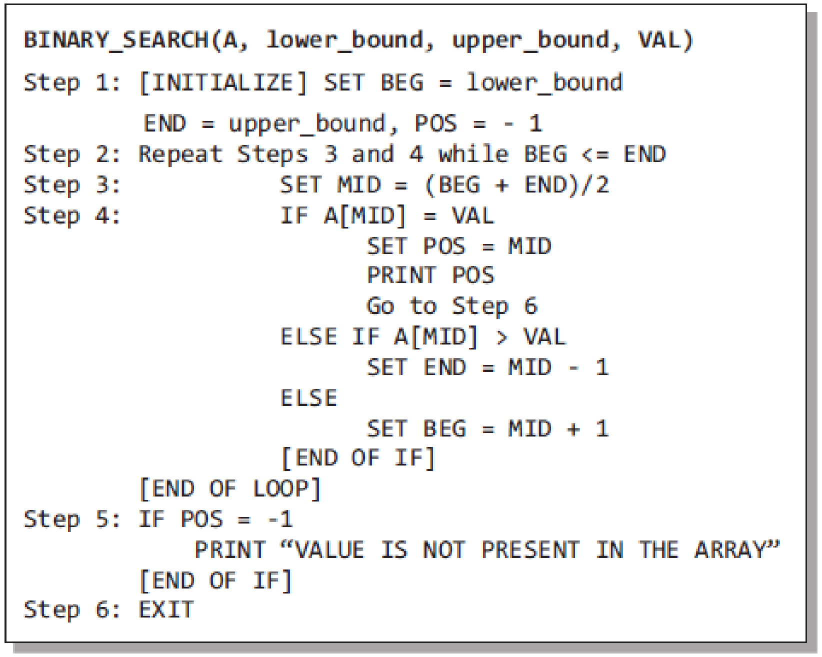
- Now, let us consider how this mechanism is applied to search for a value in a sorted array.
- Consider an array A() that is declared and initialized as
-     int A() = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10};
-     and the value to be searched is VAL = 9.
- The algorithm will proceed in the following manner.
  - BEG = 0, END = 10, MID = (0 + 10)/2 = 5
  - Now, VAL = 9 and A(MID) = A(5) = 5
  - A(5) is less than VAL, therefore, we now search for the value in the second half of the array.
  - So, we change the values of BEG and MID.
  - Now, BEG = MID + 1 = 6, END = 10, MID = (6 + 10)/2 =16/2 = 8
  - VAL = 9 and A(MID) = A(8) = 8
  - A(8) is less than VAL, therefore, we now search for the value in the second half of the segment.
  - So, again we change the values of BEG and MID.
  - Now, BEG = MID + 1 = 9, END = 10, MID = (9 + 10)/2 = 9
  - Now, VAL = 9 and A(MID) = 9.

Binary Search

- In this algorithm, we see that BEG and END are the beginning and ending positions of the segment that we are looking to search for the element.
  - MID is calculated as (BEG + END)/2.
  - Initially, BEG = lower_bound and END = upper_bound.
  - The algorithm will terminate when A(MID) = VAL.
  - When the algorithm ends, we will set POS = MID.
    - POS is the position at which the value is present in the array.
  - However, if VAL is not equal to A(MID), then the values of BEG, END, and MID will be changed depending on whether VAL is smaller or greater than A(MID). (
    - a) If VAL < A(MID), then VAL will be present in the left segment of the array. So, the value of END will be changed as END = MID – 1.
    - (b) If VAL > A(MID), then VAL will be present in the right segment of the array. So, the value of BEG will be changed as BEG = MID + 1.
  - Finally, if VAL is not present in the array, then eventually, END will be less than BEG. When this happens, the algorithm will terminate and the search will be unsuccessful.

Binary Search

- Figure 14.2 shows the algorithm for binary search.
- In Step 1, we initialize the value of variables, BEG, END, and POS.
- In Step 2, a while loop is executed until BEG is less than or equal to END.
- In Step 3, the value of MID is calculated.
- In Step 4, we check if the array value at MID is equal to VAL (item to be searched in the array).
  - If a match is found, then the value of POS is printed and the algorithm exits.
    - However, if a match is not found, and if the value of A(MID) is greater than VAL, the value of END is modified, otherwise if A(MID) is greater than VAL, then the value of BEG is altered.
- In Step 5, if the value of POS = –1, then VAL is not present in the array and an appropriate message is printed on the screen before the algorithm exits.



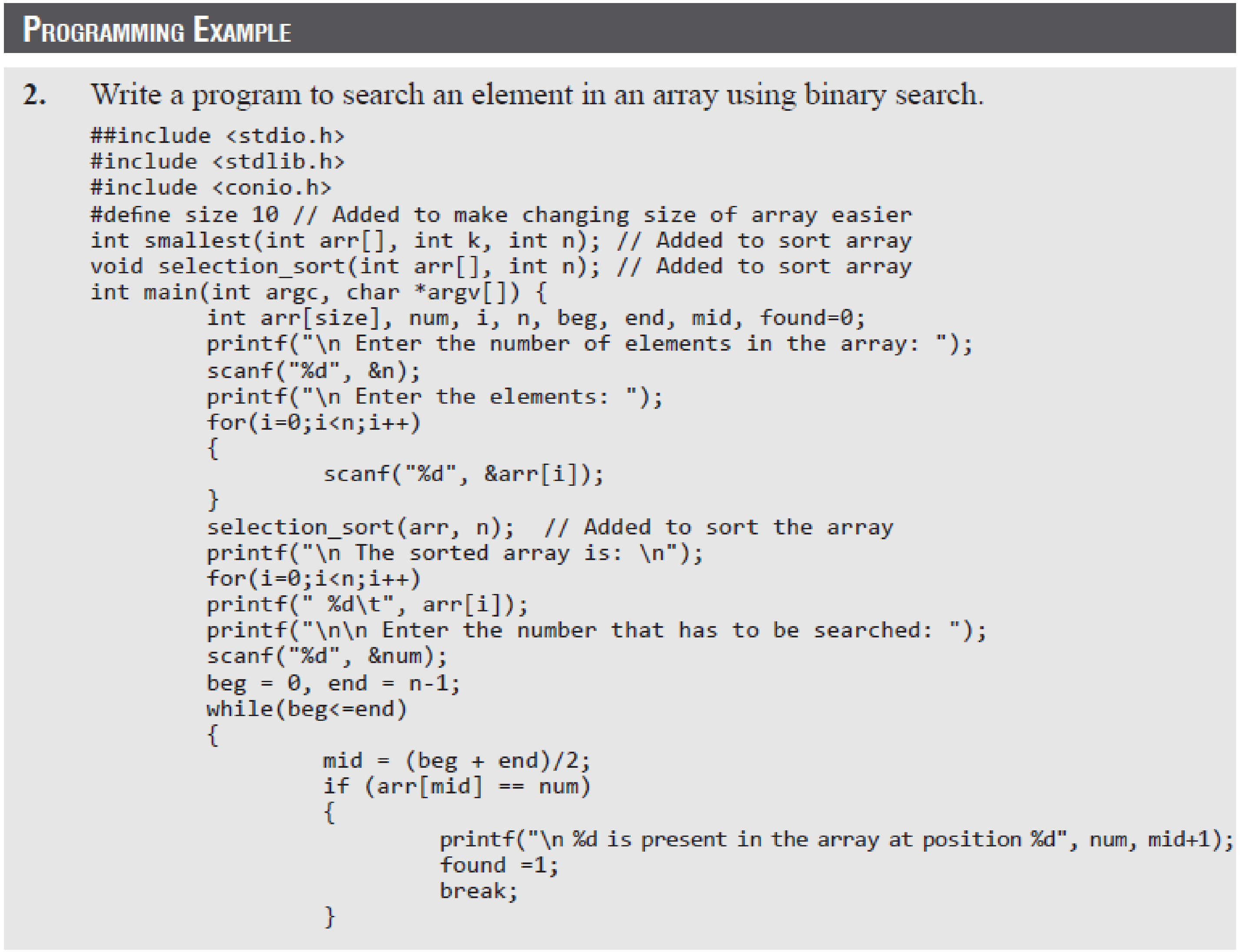
```
BINARY_SEARCH(A, lower_bound, upper_bound, VAL)
Step 1: [INITIALIZE] SET BEG = lower_bound
        END = upper_bound, POS = - 1
Step 2: Repeat Steps 3 and 4 while BEG <= END
Step 3:          SET MID = (BEG + END)/2
Step 4:          IF A[MID] = VAL
                       SET POS = MID
                       PRINT POS
                       Go to Step 6
                 ELSE IF A[MID] > VAL
                       SET END = MID - 1
                 ELSE
                       SET BEG = MID + 1
                 [END OF IF]
        [END OF LOOP]
Step 5: IF POS = -1
            PRINT "VALUE IS NOT PRESENT IN THE ARRAY"
        [END OF IF]
Step 6: EXIT
```

Figure 14.2 Algorithm for binary search

Binary Search

- ***Complexity of Binary Search Algorithm***
- The complexity of the binary search algorithm can be expressed as f(n), where n is the number of elements in the array.
- The complexity of the algorithm is calculated depending on the number of comparisons that are made.
- In the binary search algorithm, we see that with each comparison, the size of the segment where search has to be made is reduced to half.
- Thus, we can say that, in order to locate a particular value in the array, the total number of comparisons that will be made is given as
- $2^{f(n)} > n$ or $f(n) = \log_2 n$

Binary Search



PROGRAMMING EXAMPLE

2. Write a program to search an element in an array using binary search.

```
##include <stdio.h>
#include <stdlib.h>
#include <conio.h>
#define size 10 // Added to make changing size of array easier
int smallest(int arr[], int k, int n); // Added to sort array
void selection_sort(int arr[], int n); // Added to sort array
int main(int argc, char *argv[]) {
        int arr[size], num, i, n, beg, end, mid, found=0;
        printf("\n Enter the number of elements in the array: ");
        scanf("%d", &n);
        printf("\n Enter the elements: ");
        for(i=0;i<n;i++)
        {
                scanf("%d", &arr[i]);
        }
        selection_sort(arr, n);  // Added to sort the array
        printf("\n The sorted array is: \n");
        for(i=0;i<n;i++)
        printf(" %d\t", arr[i]);
        printf("\n\n Enter the number that has to be searched: ");
        scanf("%d", &num);
        beg = 0, end = n-1;
        while(beg<=end)
        {
                mid = (beg + end)/2;
                if (arr[mid] == num)
                {
                        printf("\n %d is present in the array at position %d", num, mid+1);
                        found =1;
                        break;
                }
```

Binary Search

```
                else if (arr[mid]>num)
                end = mid-1;
                else
                beg = mid+1;
        }
        if (beg > end && found == 0)
        printf("\n %d does not exist in the array", num);
        return 0;
}

int smallest(int arr[], int k, int n)
{
        int pos = k, small=arr[k], i;
        for(i=k+1;i<n;i++)
        {
                if(arr[i]< small)
                {
                        small = arr[i];
                        pos = i;
                }
        }
        return pos;
}
void selection_sort(int arr[],int n)
{
        int k, pos, temp;
        for(k=0;k<n;k++)
        {
                pos = smallest(arr, k, n);
                temp = arr[k];
                arr[k] = arr[pos];
                arr[pos] = temp;
        }
}
```

Interpolation Search

- Interpolation search is a searching technique that finds a specified value in a sorted array.
- The concept of interpolation search is similar to how we search for names in a telephone book or for keys by which a book's entries are ordered.
  - For example, when looking for a name "Bharat" in a telephone directory, we know that it will be near the extreme left, so applying a binary search technique by dividing the list in two halves each time is not a good idea.
  - We must start scanning the extreme left in the first pass itself.
- In each step of interpolation search, the remaining search space for the value to be found is calculated.
- The calculation is done based on the values at the bounds of the search space and the value to be searched.
- The value found at this estimated position is then compared with the value being searched for.
- If the two values are equal, then the search is complete.

Interpolation Search

- However, in case the values are not equal then depending on the comparison, the remaining search space is reduced to the part before or after the estimated position.
- Thus, we see that interpolation search is similar to the binary search technique.
  - However, the important difference between the two techniques is that binary search always selects the middle value of the remaining search space. It discards half of the values based on the comparison between the value found at the estimated position and the value to be searched.
  - But in interpolation search, interpolation is used to find an item near the one being searched for.

Interpolation Search

- The algorithm for interpolation search is given in Fig. 14.3.

```
INTERPOLATION_SEARCH (A, lower_bound, upper_bound, VAL)

Step 1: [INITIALIZE] SET LOW = lower_bound,
        HIGH = upper_bound, POS = -1
Step 2:    Repeat Steps 3 to 4 while LOW <= HIGH
Step 3:         SET MID = LOW + (HIGH - LOW) ×
                ((VAL - A[LOW]) / (A[HIGH] - A[LOW]))
Step 4:         IF VAL = A[MID]
                  POS = MID
                  PRINT POS
                  Go to Step 6
                 ELSE IF VAL < A[MID]
                   SET HIGH = MID - 1
                  ELSE
                   SET LOW = MID + 1
                [END OF IF]
        [END OF LOOP]
Step 5: IF POS = -1
              PRINT "VALUE IS NOT PRESENT IN THE ARRAY"
        [END OF IF]
Step 6: EXIT
```

Figure 14.3 Algorithm for interpolation search

Interpolation Search

- Figure 14.4 helps us to visualize how the search space is divided in case of binary search and interpolation search.

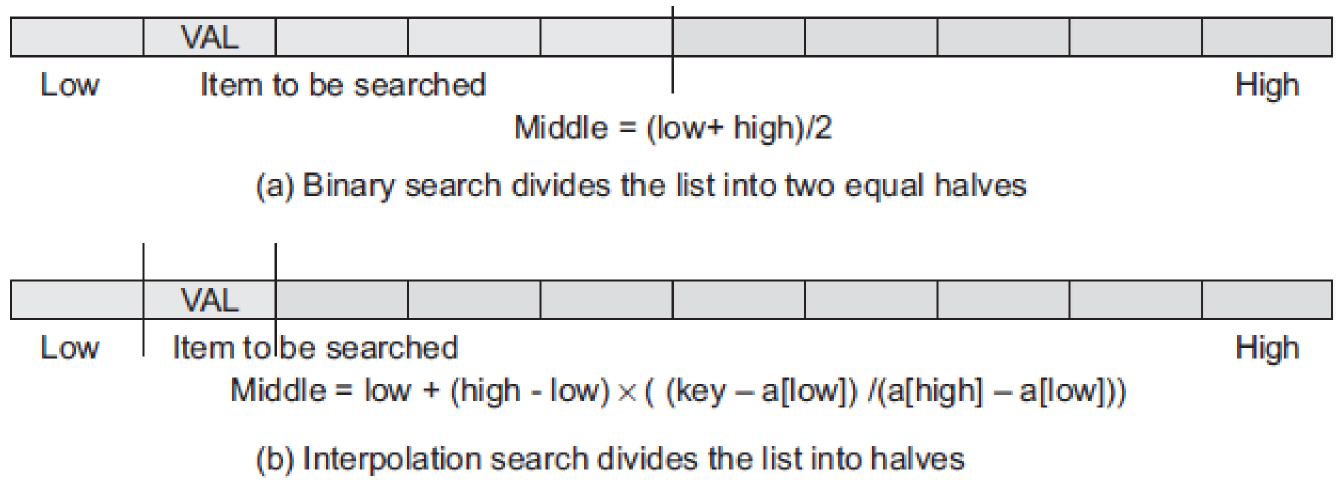
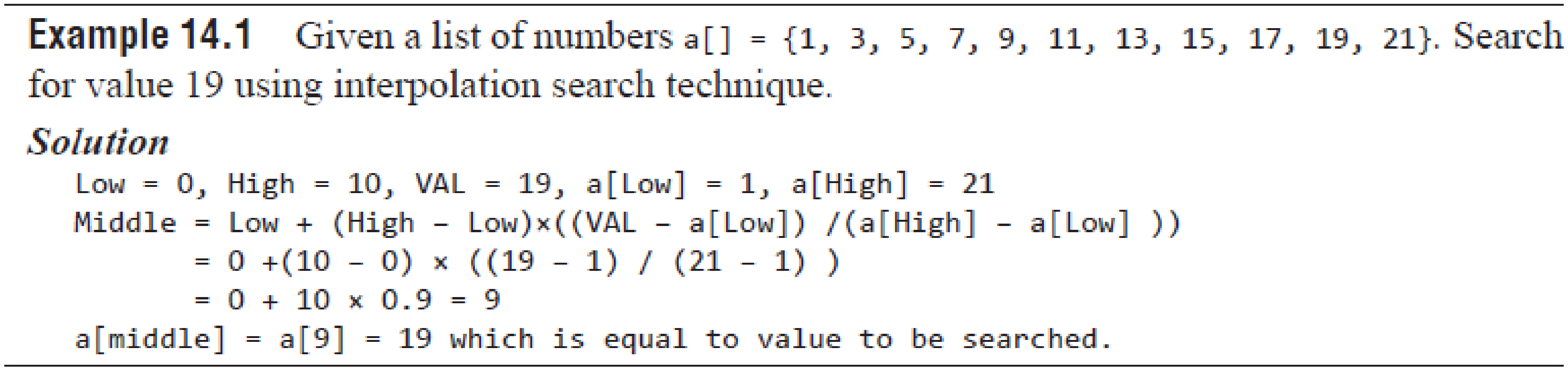

VAL
Low
Item to be searched
High
Middle = (low+ high)/2
(a) Binary search divides the list into two equal halves
VAL
Low
Item to be searched
High
Middle = low + (high - low) × ((key – a[low]) /(a[high] – a[low]))
(b) Interpolation search divides the list into halves

Figure 14.4 Difference between binary search and interpolation search

Interpolation Search

- ***Complexity of Interpolation Search Algorithm***
- When n elements of a list to be sorted are uniformly distributed (average case), interpolation search makes about log(log n) comparisons.
- However, in the worst case, that is when the elements increase exponentially, the algorithm can make up to O(n) comparisons.

Interpolation Search

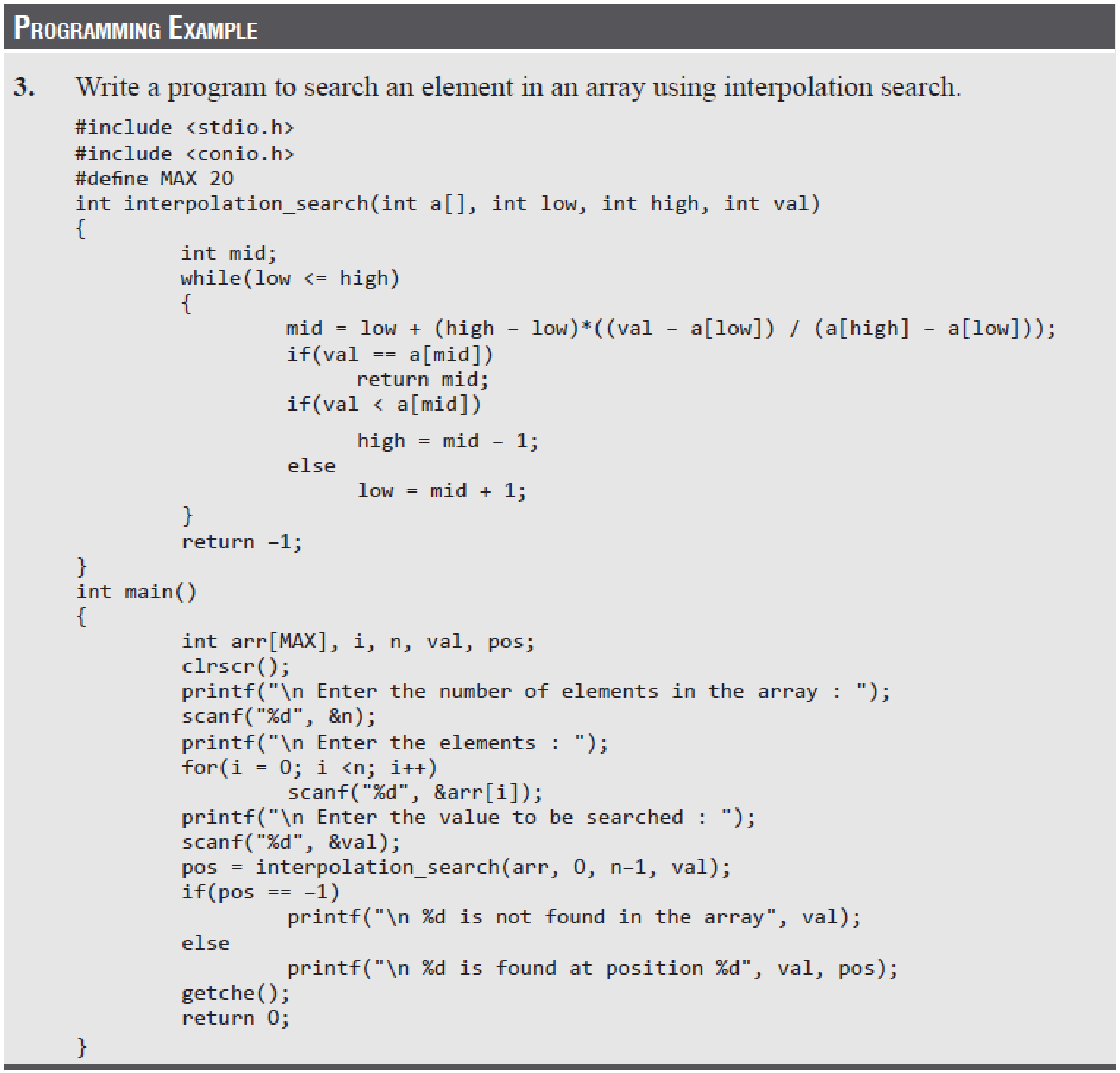


Example 14.1 Given a list of numbers `a[] = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21}`. Search for value 19 using interpolation search technique.

Solution

```
Low = 0, High = 10, VAL = 19, a[Low] = 1, a[High] = 21
Middle = Low + (High – Low)×((VAL – a[Low]) /(a[High] – a[Low] ))
       = 0 +(10 – 0) × ((19 – 1) / (21 – 1) )
       = 0 + 10 × 0.9 = 9
a[middle] = a[9] = 19 which is equal to value to be searched.
```

Interpolation Search



PROGRAMMING EXAMPLE

3. Write a program to search an element in an array using interpolation search.

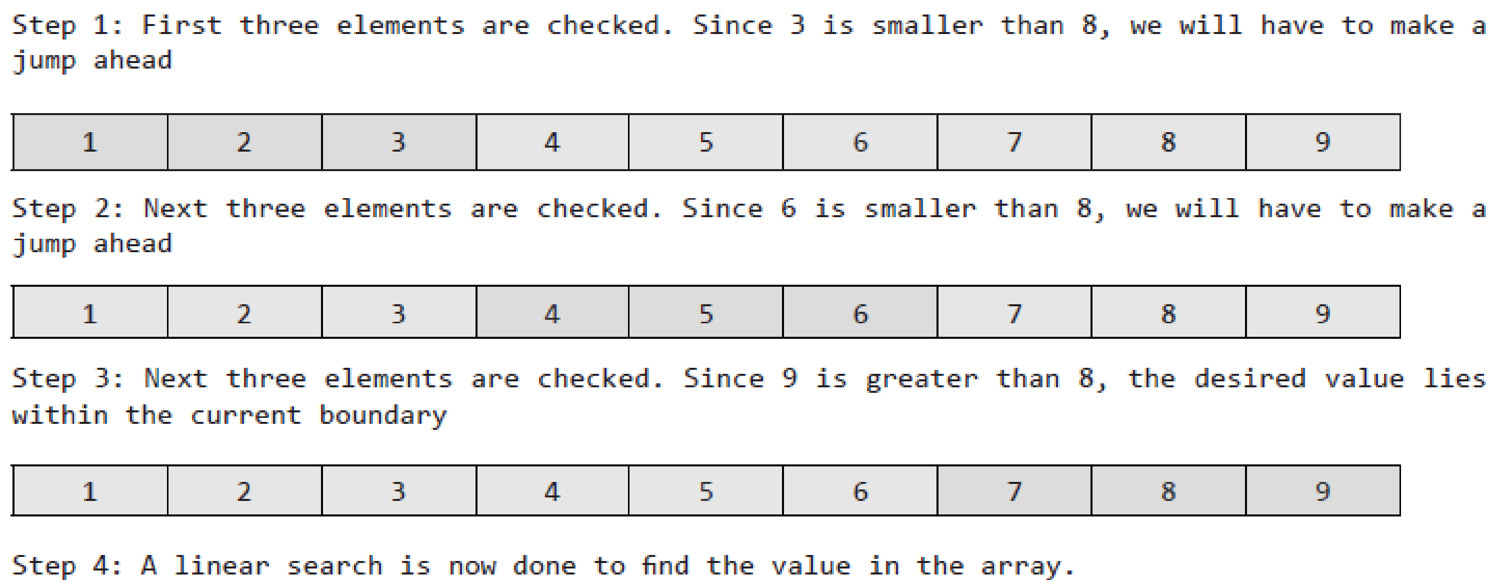
```
#include <stdio.h>
#include <conio.h>
#define MAX 20
int interpolation_search(int a[], int low, int high, int val)
{
        int mid;
        while(low <= high)
        {
                mid = low + (high - low)*((val - a[low]) / (a[high] - a[low]));
                if(val == a[mid])
                     return mid;
                if(val < a[mid])
                     high = mid - 1;
                else
                     low = mid + 1;
        }
        return -1;
}
int main()
{
        int arr[MAX], i, n, val, pos;
        clrscr();
        printf("\n Enter the number of elements in the array : ");
        scanf("%d", &n);
        printf("\n Enter the elements : ");
        for(i = 0; i <n; i++)
                scanf("%d", &arr[i]);
        printf("\n Enter the value to be searched : ");
        scanf("%d", &val);
        pos = interpolation_search(arr, 0, n-1, val);
        if(pos == -1)
                printf("\n %d is not found in the array", val);
        else
                printf("\n %d is found at position %d", val, pos);
        getche();
        return 0;
}
```

Jump Search

- When we have an already sorted list, then the other efficient algorithm to search for a value is jump search or block search.
- In jump search, it is not necessary to scan all the elements in the list to find the desired value.
- We just check an element and if it is less than the desired value, then some of the elements following it are skipped by jumping ahead.
- After moving a little forward again, the element is checked.
  - If the checked element is greater than the desired value, then we have a boundary and we are sure that the desired value lies between the previously checked element and the currently checked element.
  - However, if the checked element is less than the value being searched for, then we again make a small jump and repeat the process.
  - Once the boundary of the value is determined, a linear search is done to find the value and its position in the array.

Jump Search

- For example, consider an array
- a() = {1,2,3,4,5,6,7,8,9}.
  - The length of the array is 9.
  - If we have to find value 8 then following steps are performed using the jump search technique.



Step 1: First three elements are checked. Since 3 is smaller than 8, we will have to make a jump ahead

1	2	3	4	5	6	7	8	9

Step 2: Next three elements are checked. Since 6 is smaller than 8, we will have to make a jump ahead

1	2	3	4	5	6	7	8	9

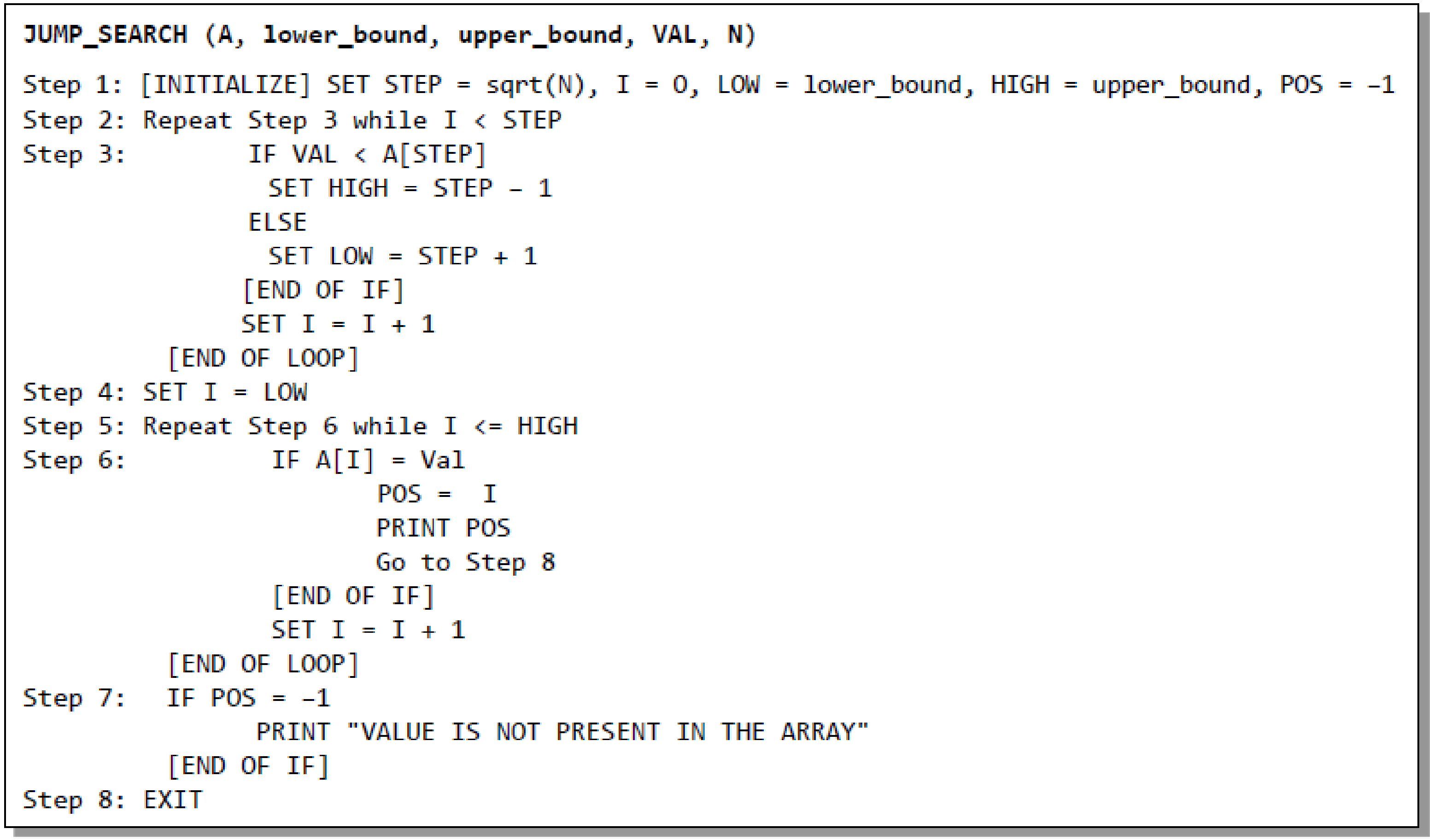
Step 3: Next three elements are checked. Since 9 is greater than 8, the desired value lies within the current boundary

1	2	3	4	5	6	7	8	9

Step 4: A linear search is now done to find the value in the array.

Jump Search

- The algorithm for jump search is given in Fig. 14.5.



```
JUMP_SEARCH (A, lower_bound, upper_bound, VAL, N)

Step 1: [INITIALIZE] SET STEP = sqrt(N), I = 0, LOW = lower_bound, HIGH = upper_bound, POS = -1
Step 2: Repeat Step 3 while I < STEP
Step 3:         IF VAL < A[STEP]
                  SET HIGH = STEP - 1
                ELSE
                  SET LOW = STEP + 1
                [END OF IF]
                SET I = I + 1
            [END OF LOOP]
Step 4: SET I = LOW
Step 5: Repeat Step 6 while I <= HIGH
Step 6:          IF A[I] = Val
                        POS =  I
                        PRINT POS
                        Go to Step 8
                 [END OF IF]
                 SET I = I + 1
            [END OF LOOP]
Step 7:     IF POS = -1
                 PRINT "VALUE IS NOT PRESENT IN THE ARRAY"
            [END OF IF]
Step 8: EXIT
```

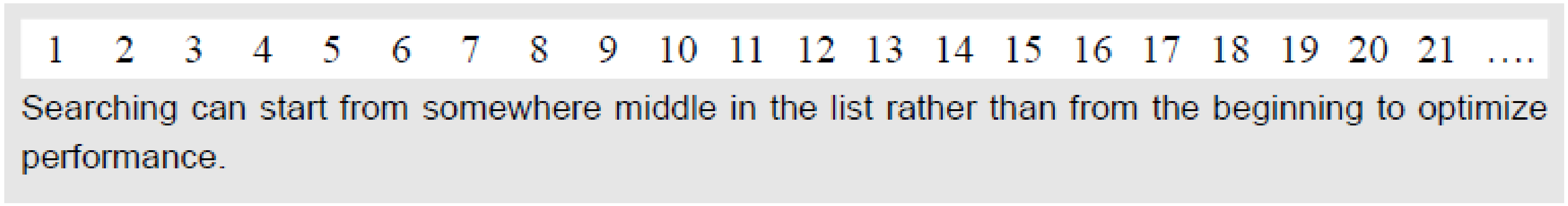
Figure 14.5 Algorithm for jump search

Jump Search

- ***Advantage of Jump Search over Linear Search***
  - Suppose we have a sorted list of 1000 elements where the elements have values 0, 1, 2, 3, 4, …, 999, then sequential search will find the value 674 in exactly 674 iterations.
  - But with jump search, the same value can be found in 44 iterations. Hence, jump search performs far better than a linear search on a sorted list of elements.
- ***Advantage of Jump Search over Binary Search***
  - No doubt, binary search is very easy to implement and has a complexity of O(log n), but in case of a list having very large number of elements, jumping to the middle of the list to make comparisons is not a good idea because if the value being searched is at the beginning of the list then one (or even more) large step(s) in the backward direction would have to be taken.
  - In such cases, jump search performs better as we have to move little backward that too only once. Hence, when jumping back is slower than jumping forward, the jump search algorithm always performs better.

Jump Search

- ***How to Choose the Step Length?***
  - For the jump search algorithm to work efficiently, we must define a fixed size for the step.
    - If the step size is 1, then algorithm is same as linear search.
    - Now, in order to find an appropriate step size, we must first try to figure out the relation between the size of the list (n) and the size of the step (k). Usually, k is calculated as $\sqrt{n}$.
- ***Further Optimization of Jump Search***
  - Till now, we were dealing with lists having small number of elements. But in real-world applications, lists can be very large. In such large lists searching the value from the beginning of the list may not be a good idea.
  - A better option is to start the search from the k–th element as shown in the figure below.



1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 ....

Searching can start from somewhere middle in the list rather than from the beginning to optimize performance.

Jump Search

- We can also improve the performance of jump search algorithm by repeatedly applying jump search.
- For example, if the size of the list is 1000000 (n).
  - The jump interval would then be
  - $\sqrt{n} = \sqrt{1000000} = 1000$.
  - Now, even the identified interval has 1000 elements and is again a large list. So, jump search can be applied again with a new step size of
  - $\sqrt{1000} \approx 31$.
  - Thus, every time we have a desired interval with a large number of values, the jump search algorithm can be applied again but with a smaller step.
  - However, in this case, the complexity of the algorithm will no longer be $O(\sqrt{n})$ but will approach a logarithmic value.

Jump Search

- ***Complexity of Jump Search Algorithm***
- Jump search works by jumping through the array with a step size (optimally chosen to be $\sqrt{n}$) to find the interval of the value.
- Once this interval is identified, the value is searched using the linear search technique.
- Therefore, the complexity of the jump search algorithm can be given as $O(\sqrt{n})$.

INTRODUCTION TO SORTING

- Sorting means arranging the elements of an array so that they are placed in some relevant order which may be either ascending or descending.
- That is, if A is an array, then the elements of A are arranged in a sorted order (ascending order) in such a way that

  A(0) < A(1) < A(2) < ...... < A(N).
- For example, if we have an array that is declared and initialized as

  int A() = {21, 34, 11, 9, 1, 0, 22};
- Then the sorted array (ascending order) can be given as:

  A() = {0, 1, 9, 11, 21, 22, 34;
- A sorting algorithm is defined as an algorithm that puts the elements of a list in a certain order, which can be either numerical order, lexicographical order, or any user-defined order.
- Efficient sorting algorithms are widely used to optimize the use of other algorithms like search and merge algorithms which require sorted lists to work correctly.

Bubble Sort

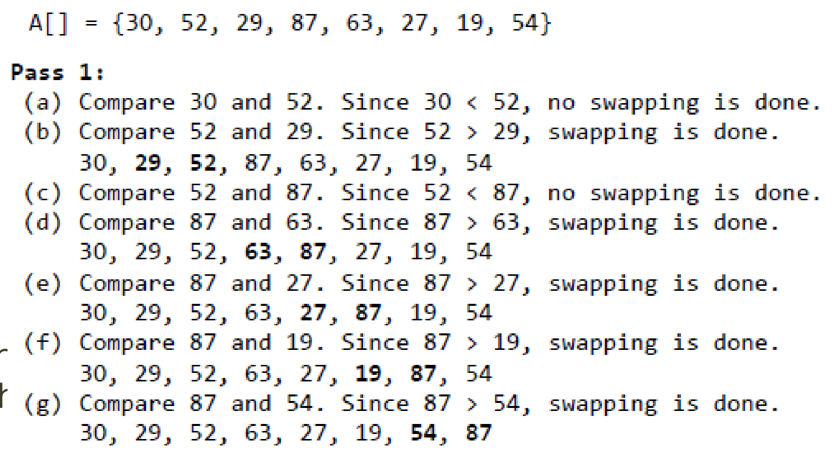
- Bubble sort is a very simple method that sorts the array elements by repeatedly moving the largest element to the highest index position of the array segment (in case of arranging elements in ascending order).
- In *bubble sorting*, consecutive adjacent pairs of elements in the array are compared with each other.
  - If the element at the lower index is greater than the element at the higher index, the two elements are interchanged so that the element is placed before the bigger one.
  - This process will continue till the list of unsorted elements exhausts.
- This procedure of sorting is called bubble sorting because elements 'bubble' to the top of the list.
- At the end of the first pass, the largest element in the list will be placed at its proper position
- **Note** If the elements are to be sorted in descending order, then in first pass the smallest element is moved to the highest index of the array.

Bubble Sort

- ***Technique***
- The basic methodology of the working of bubble sort is given as follows:
  a. In Pass 1, A(0) and A(1) are compared, then A(1) is compared with A(2), A(2) is compared with A(3), and so on. Finally, A(N–2) is compared with A(N–1). Pass 1 involves n–1 comparisons and places the biggest element at the highest index of the array.
  b. In Pass 2, A(0) and A(1) are compared, then A(1) is compared with A(2), A(2) is compared with A(3), and so on. Finally, A(N–3) is compared with A(N–2). Pass 2 involves n–2 comparisons and places the second biggest element at the second highest index of the array.
  c. In Pass 3, A(0) and A(1) are compared, then A(1) is compared with A(2), A(2) is compared with A(3), and so on. Finally, A(N–4) is compared with A(N–3). Pass 3 involves n–3 comparisons and places the third biggest element at the third highest index of the array.
  d. In Pass n–1, A(0) and A(1) are compared so that A(0)<A(1). After this step, all the elements of the array are arranged in ascending order.

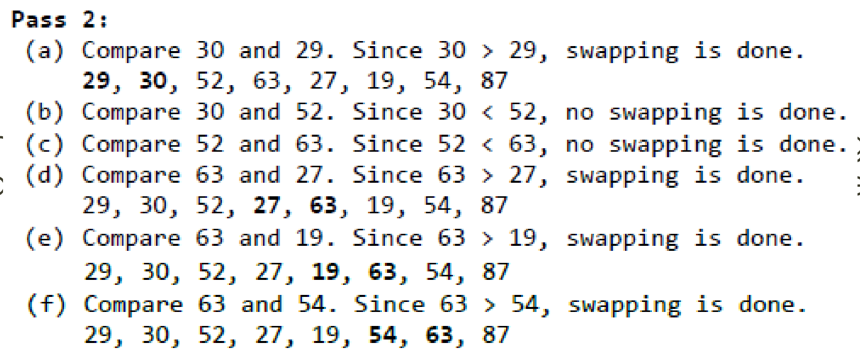
Bubble Sort

- **Example 14.2** To discuss bubble sort in detail, let us consider an array A() that has the following elements:



```
A[] = {30, 52, 29, 87, 63, 27, 19, 54}

Pass 1:
(a) Compare 30 and 52. Since 30 < 52, no swapping is done.
(b) Compare 52 and 29. Since 52 > 29, swapping is done.
    30, 29, 52, 87, 63, 27, 19, 54
(c) Compare 52 and 87. Since 52 < 87, no swapping is done.
(d) Compare 87 and 63. Since 87 > 63, swapping is done.
    30, 29, 52, 63, 87, 27, 19, 54
(e) Compare 87 and 27. Since 87 > 27, swapping is done.
    30, 29, 52, 63, 27, 87, 19, 54
(f) Compare 87 and 19. Since 87 > 19, swapping is done.
    30, 29, 52, 63, 27, 19, 87, 54
(g) Compare 87 and 54. Since 87 > 54, swapping is done.
    30, 29, 52, 63, 27, 19, 54, 87
```

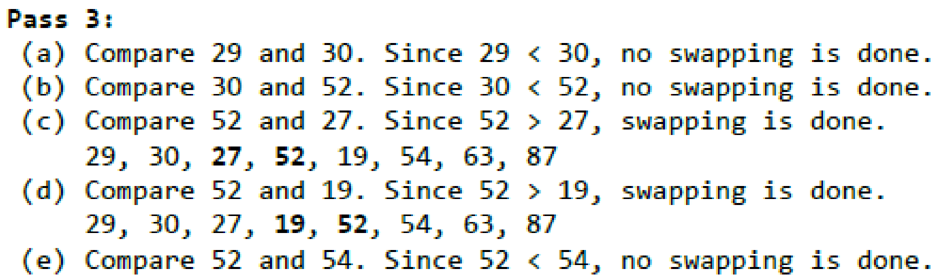
- Observe th is placed at the highest index of the array. All th



```
Pass 2:
(a) Compare 30 and 29. Since 30 > 29, swapping is done.
    29, 30, 52, 63, 27, 19, 54, 87
(b) Compare 30 and 52. Since 30 < 52, no swapping is done.
(c) Compare 52 and 63. Since 52 < 63, no swapping is done.
(d) Compare 63 and 27. Since 63 > 27, swapping is done.
    29, 30, 52, 27, 63, 19, 54, 87
(e) Compare 63 and 19. Since 63 > 19, swapping is done.
    29, 30, 52, 27, 19, 63, 54, 87
(f) Compare 63 and 54. Since 63 > 54, swapping is done.
    29, 30, 52, 27, 19, 54, 63, 87
```

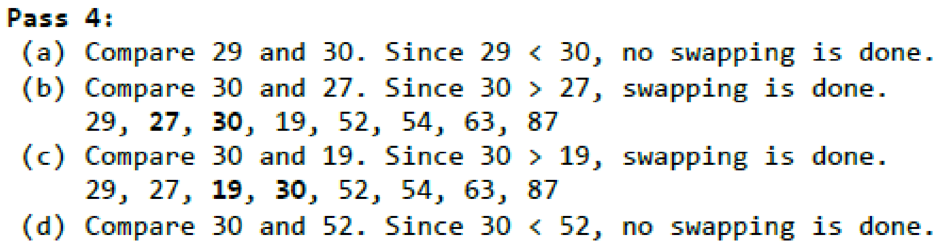
- Observe th jest element is placed at the second highest inc ed.

Bubble Sort



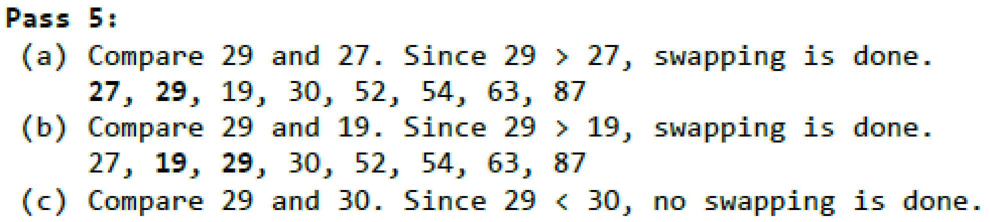
```
Pass 3:
 (a) Compare 29 and 30. Since 29 < 30, no swapping is done.
 (b) Compare 30 and 52. Since 30 < 52, no swapping is done.
 (c) Compare 52 and 27. Since 52 > 27, swapping is done.
     29, 30, 27, 52, 19, 54, 63, 87
 (d) Compare 52 and 19. Since 52 > 19, swapping is done.
     29, 30, 27, 19, 52, 54, 63, 87
 (e) Compare 52 and 54. Since 52 < 54, no swapping is done.
```

- Observe ... est element is placed at the third highest index of the array. All the other elements are still unsorted.



```
Pass 4:
 (a) Compare 29 and 30. Since 29 < 30, no swapping is done.
 (b) Compare 30 and 27. Since 30 > 27, swapping is done.
     29, 27, 30, 19, 52, 54, 63, 87
 (c) Compare 30 and 19. Since 30 > 19, swapping is done.
     29, 27, 19, 30, 52, 54, 63, 87
 (d) Compare 30 and 52. Since 30 < 52, no swapping is done.
```

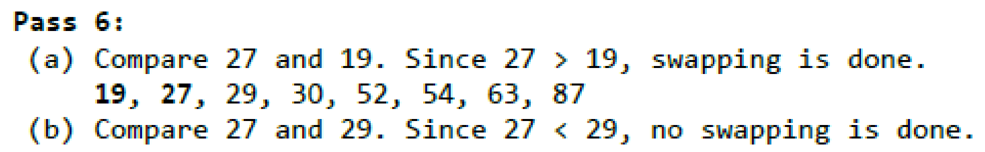
- Observe ... rgest element is placed at the fourt ... s are still unsorted.



```
Pass 5:
 (a) Compare 29 and 27. Since 29 > 27, swapping is done.
     27, 29, 19, 30, 52, 54, 63, 87
 (b) Compare 29 and 19. Since 29 > 19, swapping is done.
     27, 19, 29, 30, 52, 54, 63, 87
 (c) Compare 29 and 30. Since 29 < 30, no swapping is done.
```

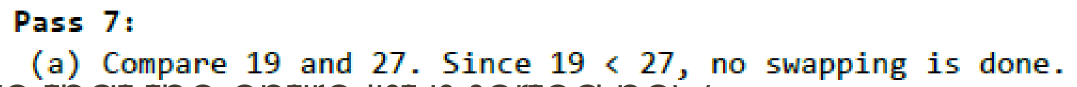
- Observe ... lement is placed at the fifth high ... unsorted.

Bubble Sort



```
Pass 6:
 (a) Compare 27 and 19. Since 27 > 19, swapping is done.
     19, 27, 29, 30, 52, 54, 63, 87
 (b) Compare 27 and 29. Since 27 < 29, no swapping is done.
```

- Observe that after the end of the sixth pass, the sixth largest element is placed at the sixth largest index of the array. All the other elements are still unsorted.

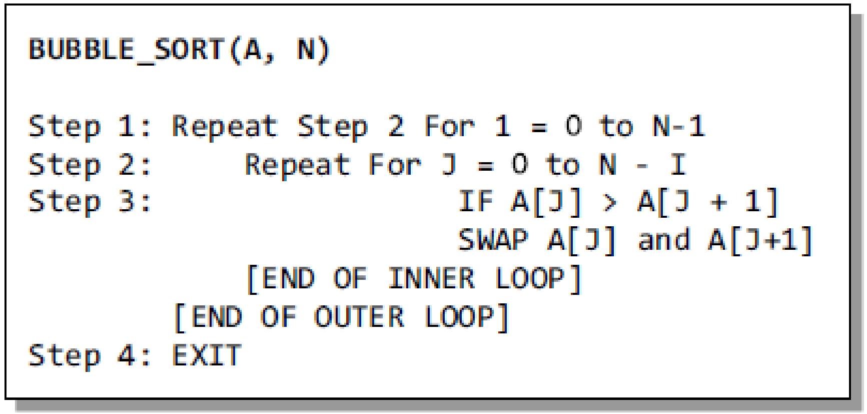


```
Pass 7:
 (a) Compare 19 and 27. Since 19 < 27, no swapping is done.
```

- Observe that the entire list is sorted now.

Bubble Sort

- Figure 14.6 shows the algorithm for bubble sort.
- In this algorithm, the outer loop is for the total number of passes which is N–1.
- The inner loop will be executed for every pass. However, the frequency of the inner loop will decrease with every pass because after every pass, one element will be in its correct position.
- Therefore, for every pass, the inner loop will be executed N–I times, where N is the number of elements in the array and I is the count of the pass.



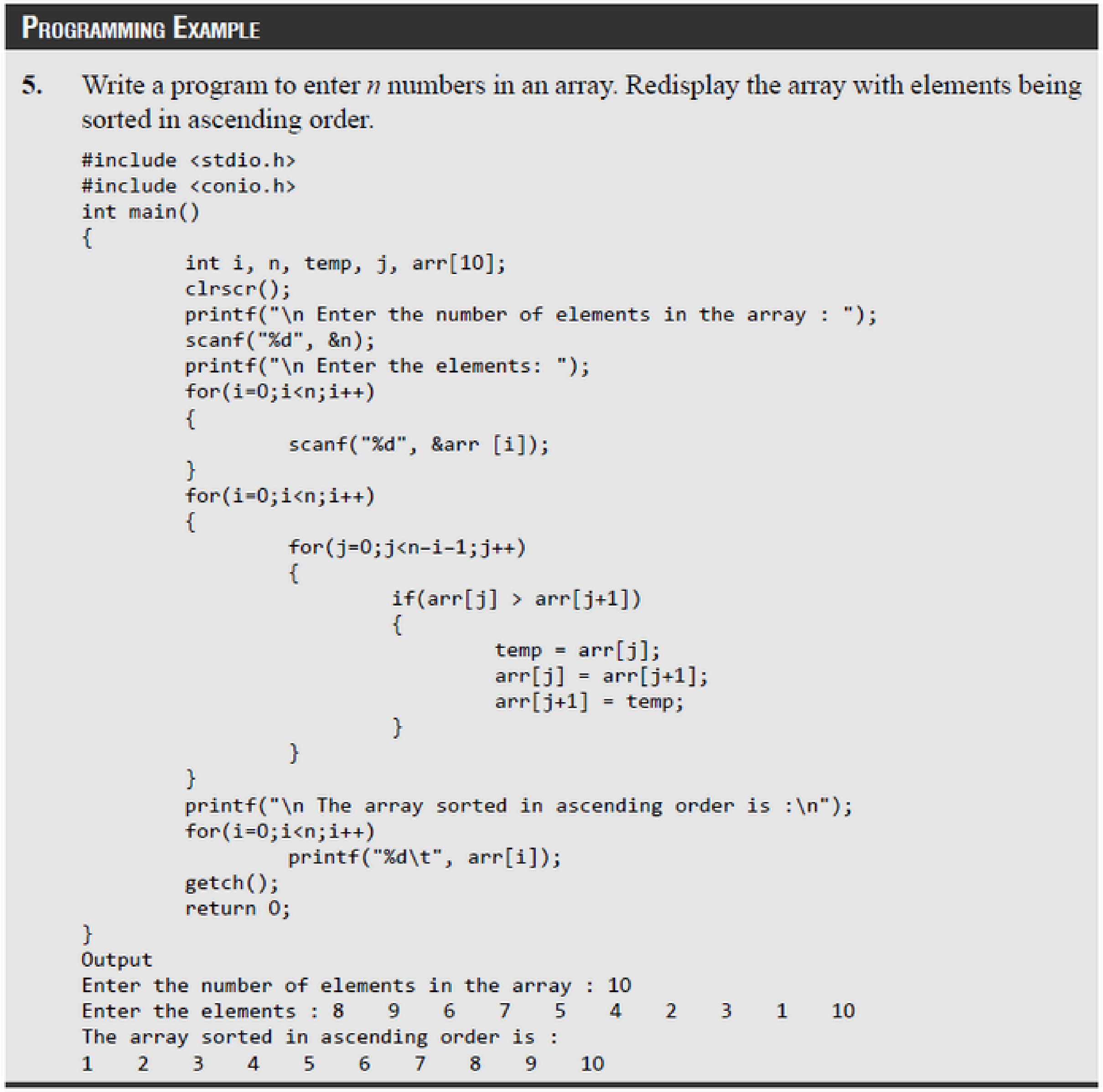
```
BUBBLE_SORT(A, N)

Step 1: Repeat Step 2 For 1 = 0 to N-1
Step 2:     Repeat For J = 0 to N - I
Step 3:                 IF A[J] > A[J + 1]
                        SWAP A[J] and A[J+1]
            [END OF INNER LOOP]
        [END OF OUTER LOOP]
Step 4: EXIT
```

Figure 14.6 Algorithm for bubble sort

Bubble Sort

- ***Complexity of Bubble Sort***
- The complexity of any sorting algorithm depends upon the number of comparisons.
- In bubble sort, we have seen that there are N–1 passes in total.
  - In the first pass, N–1 comparisons are made to place the highest element in its correct position.
  - Then, in Pass 2, there are N–2 comparisons and the second highest element is placed in its position.
  - Therefore, to compute the complexity of bubble sort, we need to calculate the total number of comparisons. It can be given as:
    - $f(n) = (n – 1) + (n – 2) + (n – 3) + ... + 3 + 2 + 1$
    - $f(n) = n (n – 1)/2$
    - $f(n) = n^2/2 + O(n) = O(n^2)$
- Therefore, the complexity of bubble sort algorithm is $O(n^2)$. It means the time required to execute bubble sort is proportional to n^2, where n is the total number of elements in the array.

Bubble Sort



Programming Example

5. Write a program to enter *n* numbers in an array. Redisplay the array with elements being sorted in ascending order.

```
#include <stdio.h>
#include <conio.h>
int main()
{
        int i, n, temp, j, arr[10];
        clrscr();
        printf("\n Enter the number of elements in the array : ");
        scanf("%d", &n);
        printf("\n Enter the elements: ");
        for(i=0;i<n;i++)
        {
                scanf("%d", &arr [i]);
        }
        for(i=0;i<n;i++)
        {
                for(j=0;j<n-i-1;j++)
                {
                        if(arr[j] > arr[j+1])
                        {
                                temp = arr[j];
                                arr[j] = arr[j+1];
                                arr[j+1] = temp;
                        }
                }
        }
        printf("\n The array sorted in ascending order is :\n");
        for(i=0;i<n;i++)
                printf("%d\t", arr[i]);
        getch();
        return 0;
}
```

```
Output
Enter the number of elements in the array : 10
Enter the elements : 8    9    6    7    5    4    2    3    1    10
The array sorted in ascending order is :
1    2    3    4    5    6    7    8    9    10
```

Insertion Sort

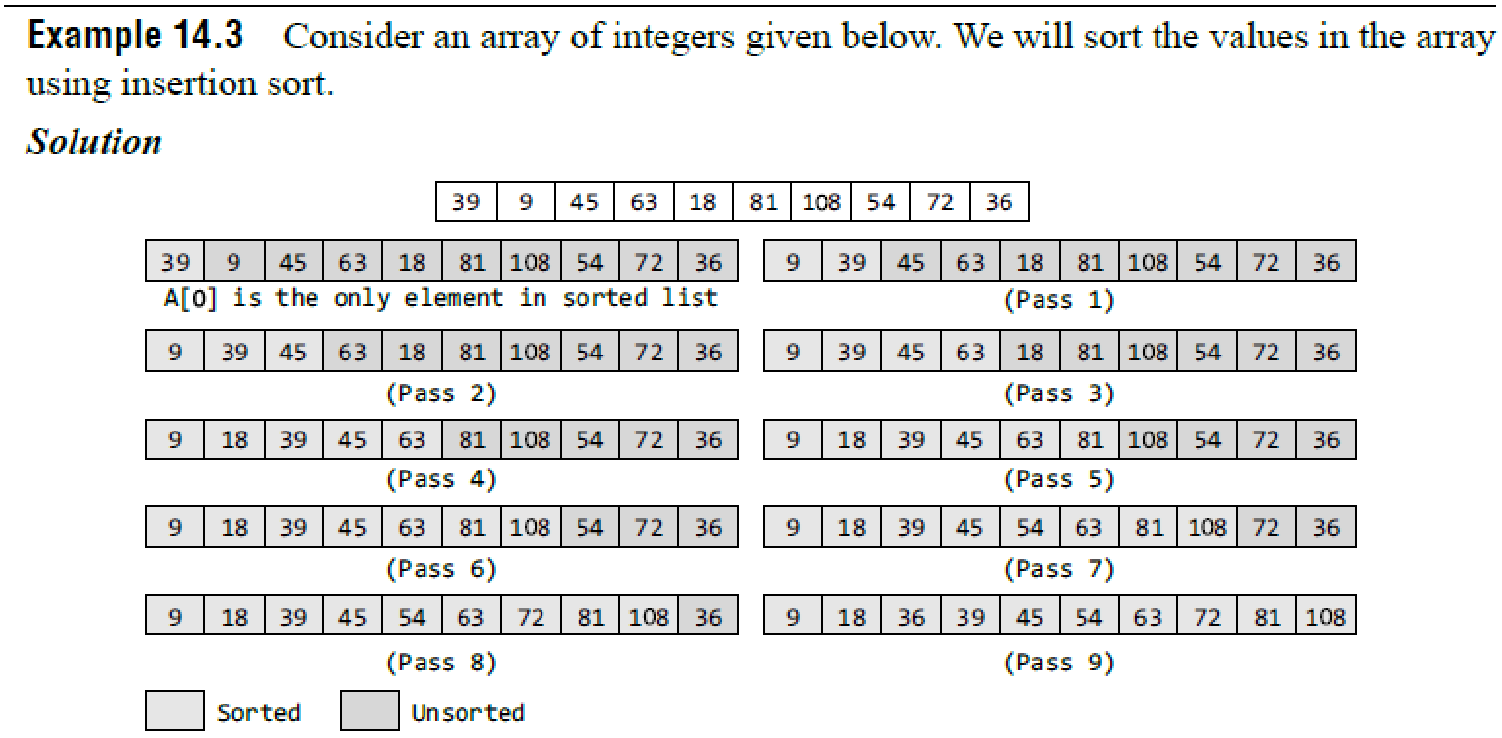
- Insertion sort is a very simple sorting algorithm in which the sorted array (or list) is built one element at a time. We all are familiar with this technique of sorting, as we usually use it for ordering a deck of cards while playing bridge.
- The main idea behind insertion sort is that it inserts each item into its proper place in the final list.
- To save memory, most implementations of the insertion sort algorithm work by moving the current data element past the already sorted values and repeatedly interchanging it with the preceding value until it is in its correct place.
- Insertion sort is less efficient as compared to other more advanced algorithms such as quick sort, heap sort, and merge sort.

Insertion Sort

- ***Technique***
- Insertion sort works as follows:
  - The array of values to be sorted is divided into two sets. One that stores sorted values and another that contains unsorted values.
  - The sorting algorithm will proceed until there are elements in the unsorted set.
  - Suppose there are n elements in the array. Initially, the element with index 0 (assuming LB = 0) is in the sorted set. Rest of the elements are in the unsorted set.
  - The first element of the unsorted partition has array index 1 (if LB = 0).
  - During each iteration of the algorithm, the first element in the unsorted set is picked up and inserted into the correct position in the sorted set.

Insertion Sort

- Initially, A(0) is the only element in the sorted set. In Pass 1, A(1) will be placed either before or after A(0), so that the array A is sorted. In Pass 2, A(2) will be placed either before A(0), in between A(0) and A(1), or after A(1). In Pass 3, A(3) will be placed in its proper place. In Pass N–1, A(N–1) will be placed in its proper place to keep the array sorted.



Example 14.3 Consider an array of integers given below. We will sort the values in the array using insertion sort.

Solution

39	9	45	63	18	81	108	54	72	36

39	9	45	63	18	81	108	54	72	36	A[0] is the only element in sorted list
9	39	45	63	18	81	108	54	72	36	(Pass 1)
9	39	45	63	18	81	108	54	72	36	(Pass 2)
9	39	45	63	18	81	108	54	72	36	(Pass 3)
9	18	39	45	63	81	108	54	72	36	(Pass 4)
9	18	39	45	63	81	108	54	72	36	(Pass 5)
9	18	39	45	63	81	108	54	72	36	(Pass 6)
9	18	39	45	54	63	81	108	72	36	(Pass 7)
9	18	39	45	54	63	72	81	108	36	(Pass 8)
9	18	36	39	45	54	63	72	81	108	(Pass 9)

▢ Sorted ▨ Unsorted

Insertion Sort

- To insert an element A(K) in a sorted list A(0), A(1), ..., A(K-1), we need to compare A(K) with A(K–1), then with A(K–2), A(K–3), and so on until we meet an element A(J) such that A(J) <= A(K). In order to insert A(K) in its correct position, we need to move elements A(K–1), A(K–2), ..., A(J) by one position and then A(K) is inserted at the (J+1)th location.
- The algorithm for insertion sort is given in Fig. 14.7.
- In the algorithm, Step 1 executes a for loop which will be repeated for each element in the array.
- In Step 2, we store the value of the Kth element in TEMP.
- In Step 3, we set the Jth index in the array.
- In Step 4, a for loop is executed that will create space for the new element from the unsorted list to be stored in the list of sorted elements.
- Finally, in Step 5, the element is stored at the (J+1)th location.

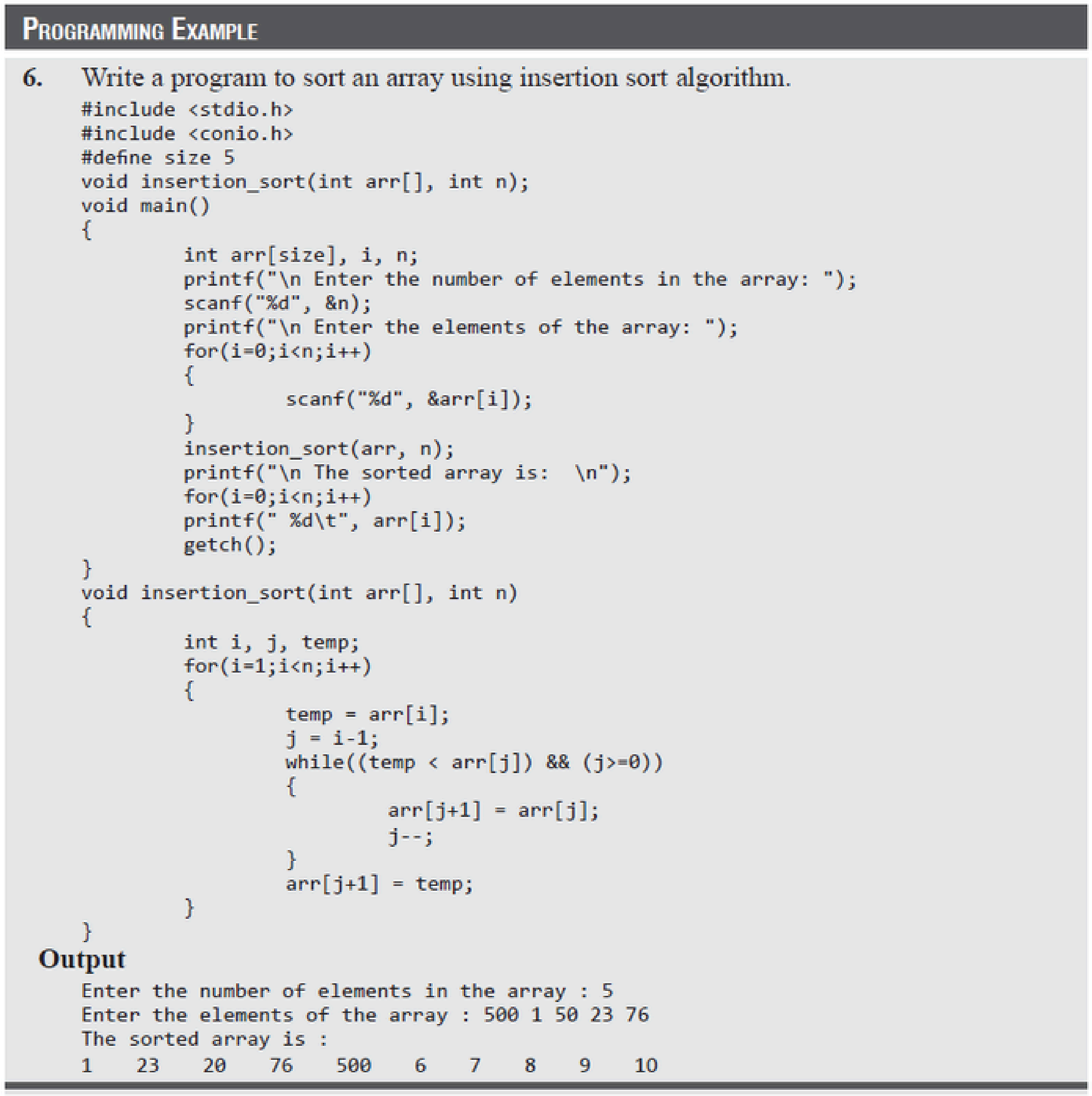
```
INSERTION-SORT (ARR, N)

Step 1: Repeat Steps 2 to 5 for K = 1 to N - 1
Step 2:      SET TEMP = ARR[K]
Step 3:      SET J = K - 1
Step 4:      Repeat while TEMP <= ARR[J]
                    SET ARR[J + 1] = ARR[J]
                    SET J = J - 1
             [END OF INNER LOOP]
Step 5:      SET ARR[J + 1] = TEMP
        [END OF LOOP]
Step 6: EXIT
```

Figure 14.7 Algorithm for insertion sort

Insertion Sort

- ***Complexity of Insertion Sort***
- For insertion sort, the best case occurs when the array is already sorted.
  - In this case, the running time of the algorithm has a linear running time (i.e., O(n)). This is because, during each iteration, the first element from the unsorted set is compared only with the last element of the sorted set of the array.
- Similarly, the worst case of the insertion sort algorithm occurs when the array is sorted in the reverse order.
  - In the worst case, the first element of the unsorted set has to be compared with almost every element in the sorted set. Furthermore, every iteration of the inner loop will have to shift the elements of the sorted set of the array before inserting the next element. Therefore, in the worst case, insertion sort has a quadratic running time (i.e., $O(n^2)$).
- Even in the average case, the insertion sort algorithm will have to make at least (K–1)/2 comparisons. Thus, the average case also has a quadratic running time.

Insertion Sort



Programming Example

6. Write a program to sort an array using insertion sort algorithm.

```
#include <stdio.h>
#include <conio.h>
#define size 5
void insertion_sort(int arr[], int n);
void main()
{
        int arr[size], i, n;
        printf("\n Enter the number of elements in the array: ");
        scanf("%d", &n);
        printf("\n Enter the elements of the array: ");
        for(i=0;i<n;i++)
        {
                scanf("%d", &arr[i]);
        }
        insertion_sort(arr, n);
        printf("\n The sorted array is:  \n");
        for(i=0;i<n;i++)
        printf(" %d\t", arr[i]);
        getch();
}
void insertion_sort(int arr[], int n)
{
        int i, j, temp;
        for(i=1;i<n;i++)
        {
                temp = arr[i];
                j = i-1;
                while((temp < arr[j]) && (j>=0))
                {
                        arr[j+1] = arr[j];
                        j--;
                }
                arr[j+1] = temp;
        }
}
```

Output

```
Enter the number of elements in the array : 5
Enter the elements of the array : 500 1 50 23 76
The sorted array is :
1    23    20    76    500    6    7    8    9    10
```

Selection Sort

- Selection sort is a sorting algorithm that has a quadratic running time complexity of $O(n^2)$, thereby making it inefficient to be used on large lists.
- Although selection sort performs worse than insertion sort algorithm, it is noted for its simplicity and also has performance advantages over more complicated algorithms in certain situations.
- Selection sort is generally used for sorting files with very large objects (records) and small keys.
- ***Technique***
- Consider an array ARR with N elements.
- Selection sort works as follows:
- First find the smallest value in the array and place it in the first position. Then, find the second smallest value in the array and place it in the second position. Repeat this procedure until the entire array is sorted. Therefore,
  - In Pass 1, find the position POS of the smallest value in the array and then swap ARR(POS) and ARR(0). Thus, ARR(0) is sorted.
  - In Pass 2, find the position POS of the smallest value in sub-array of N–1 elements. Swap ARR(POS) with ARR(1). Now, ARR(0) and ARR(1) is sorted.
  - In Pass N–1, find the position POS of the smaller of the elements ARR(N–2) and ARR(N–1). Swap ARR(POS) and ARR(N–2) so that ARR(0), ARR(1), ..., ARR(N–1) is sorted.

Selection Sort

Example 14.4 Sort the array given below using selection sort.

39	9	81	45	90	27	72	18

PASS	POS	ARR[0]	ARR[1]	ARR[2]	ARR[3]	ARR[4]	ARR[5]	ARR[6]	ARR[7]
1	1	9	39	81	45	90	27	72	18
2	7	9	18	81	45	90	27	72	39
3	5	9	18	27	45	90	81	72	39
4	7	9	18	27	39	90	81	72	45
5	7	9	18	27	39	45	81	72	90
6	6	9	18	27	39	45	72	81	90
7	6	9	18	27	39	45	72	81	90

Selection Sort

- The algorithm for selection sort is shown in Fig. 14.8.
- In the algorithm, during the Kth pass, we need to find the position POS of the smallest elements from ARR(K), ARR(K+1), ..., ARR(N).
- To find the smallest element, we use a variable SMALL to hold the smallest value in the sub-array ranging from ARR(K) to ARR(N).
- Then, swap ARR(K) with ARR(POS).
- This procedure is repeated until all the elements in the array are sorted.

```
SMALLEST (ARR, K, N, POS)

Step 1: [INITIALIZE] SET SMALL = ARR[K]
Step 2: [INITIALIZE] SET POS = K
Step 3: Repeat for J = K+1 to N-1
            IF SMALL > ARR[J]
                  SET SMALL = ARR[J]
                  SET POS = J
            [END OF IF]
        [END OF LOOP]
Step 4: RETURN POS
```

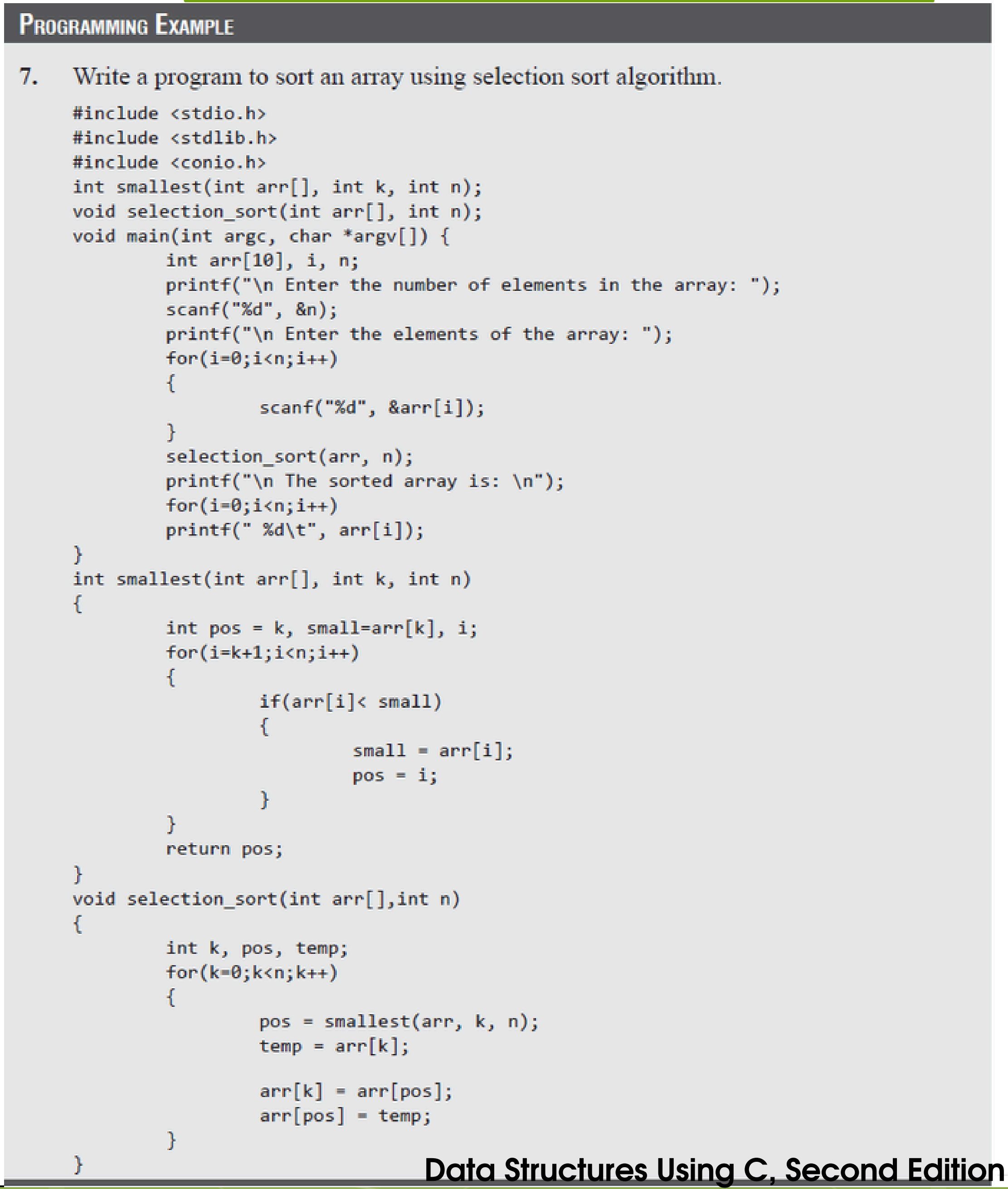
```
SELECTION SORT(ARR, N)

Step 1: Repeat Steps 2 and 3 for K = 1
        to N-1
Step 2:     CALL SMALLEST(ARR, K, N, POS)
Step 3:     SWAP A[K] with ARR[POS]
          [END OF LOOP]
Step 4: EXIT
```

Figure 14.8 Algorithm for selection sort

Selection Sort

- ***Complexity of Selection Sort***
- Selection sort is a sorting algorithm that is independent of the original order of elements in the array.
- In Pass 1, selecting the element with the smallest value calls for scanning all n elements; thus, n–1 comparisons are required in the first pass. Then, the smallest value is swapped with the element in the first position.
- In Pass 2, selecting the second smallest value requires scanning the remaining n – 1 elements and so on.
- Therefore,
    - $(n - 1) + (n - 2) + \ldots + 2 + 1$
    - $= n(n - 1) / 2 = O(n^2)$ comparisons

Selection Sort



PROGRAMMING EXAMPLE

7. Write a program to sort an array using selection sort algorithm.

```
#include <stdio.h>
#include <stdlib.h>
#include <conio.h>
int smallest(int arr[], int k, int n);
void selection_sort(int arr[], int n);
void main(int argc, char *argv[]) {
        int arr[10], i, n;
        printf("\n Enter the number of elements in the array: ");
        scanf("%d", &n);
        printf("\n Enter the elements of the array: ");
        for(i=0;i<n;i++)
        {
                scanf("%d", &arr[i]);
        }
        selection_sort(arr, n);
        printf("\n The sorted array is: \n");
        for(i=0;i<n;i++)
        printf(" %d\t", arr[i]);
}
int smallest(int arr[], int k, int n)
{
        int pos = k, small=arr[k], i;
        for(i=k+1;i<n;i++)
        {
                if(arr[i]< small)
                {
                        small = arr[i];
                        pos = i;
                }
        }
        return pos;
}
void selection_sort(int arr[],int n)
{
        int k, pos, temp;
        for(k=0;k<n;k++)
        {
                pos = smallest(arr, k, n);
                temp = arr[k];

                arr[k] = arr[pos];
                arr[pos] = temp;
        }
}
```

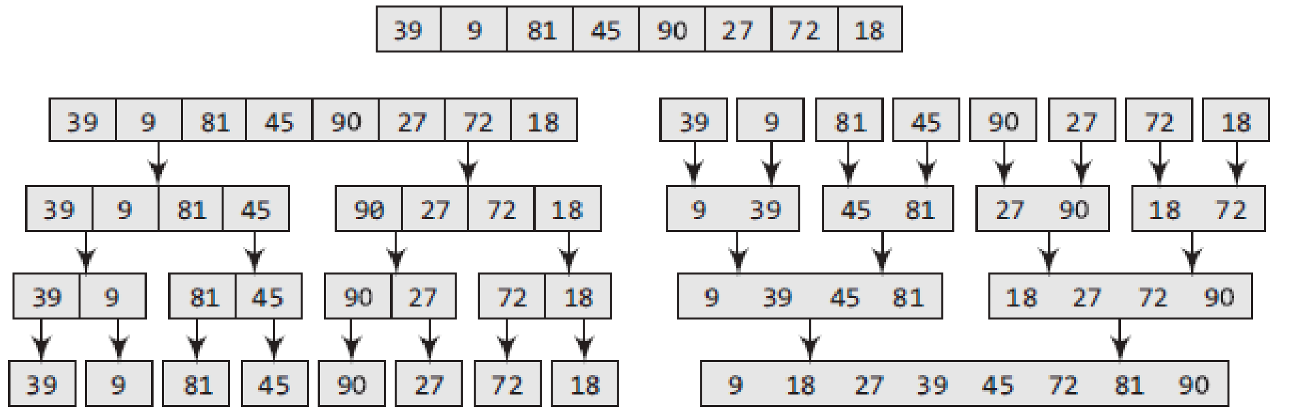
Merge Sort

- Merge sort is a sorting algorithm that uses the divide, conquer, and combine algorithmic paradigm.
- ***Divide*** means partitioning the n-element array to be sorted into two sub-arrays of n/2 elements. If A is an array containing zero or one element, then it is already sorted. However, if there are more elements in the array, divide A into two sub-arrays, A1 and A2, each containing about half of the elements of A.
- ***Conquer*** means sorting the two sub-arrays recursively using merge sort.
- ***Combine*** means merging the two sorted sub-arrays of size n/2 to produce the sorted array of n elements.

Merge Sort

Example 14.5 Sort the array given below using merge sort.

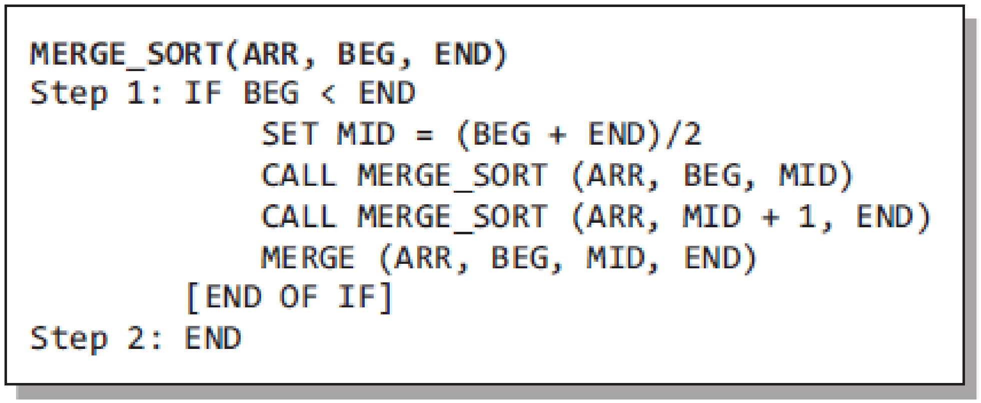
Solution



(Divide and Conquer the array) (Combine the elements to form a sorted array)

Merge Sort

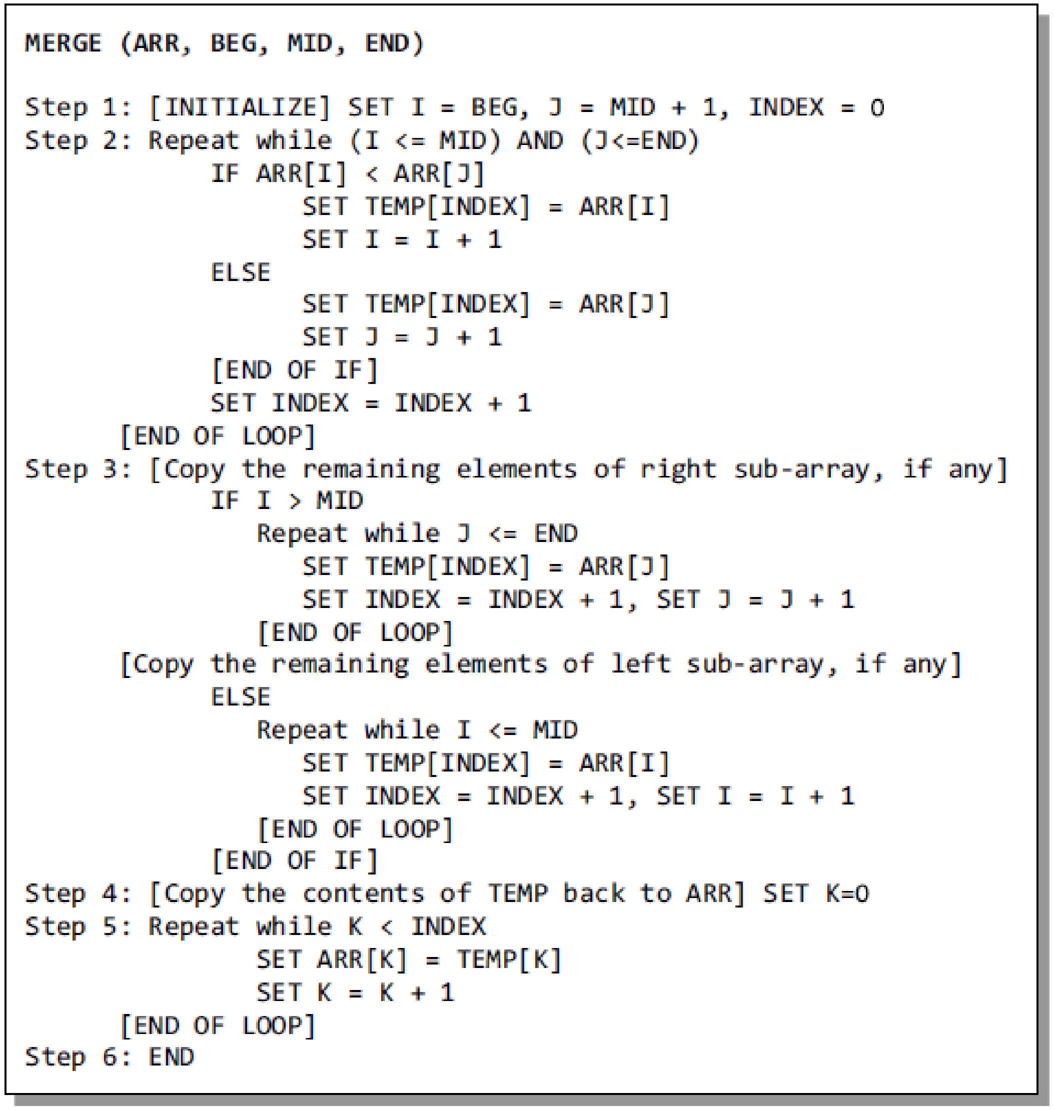
- The merge sort algorithm (Fig. 14.9) uses a function merge which combines the sub-arrays to form a sorted array.
- While the merge sort algorithm recursively divides the list into smaller lists, the merge algorithm conquers the list to sort the elements in individual lists.
- Finally, the smaller lists are merged to form one list.



```
MERGE_SORT(ARR, BEG, END)
Step 1: IF BEG < END
            SET MID = (BEG + END)/2
            CALL MERGE_SORT (ARR, BEG, MID)
            CALL MERGE_SORT (ARR, MID + 1, END)
            MERGE (ARR, BEG, MID, END)
        [END OF IF]
Step 2: END
```

Figure 14.9 Algorithm for merge sort

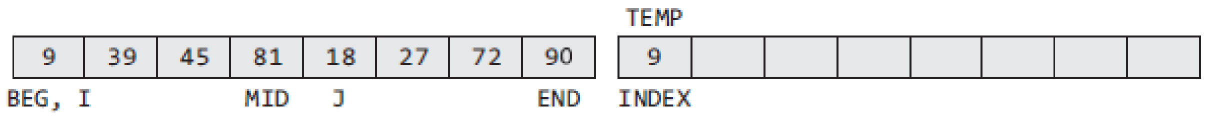
Merge Sort



```
MERGE (ARR, BEG, MID, END)

Step 1: [INITIALIZE] SET I = BEG, J = MID + 1, INDEX = 0
Step 2: Repeat while (I <= MID) AND (J<=END)
            IF ARR[I] < ARR[J]
                  SET TEMP[INDEX] = ARR[I]
                  SET I = I + 1
            ELSE
                  SET TEMP[INDEX] = ARR[J]
                  SET J = J + 1
            [END OF IF]
            SET INDEX = INDEX + 1
      [END OF LOOP]
Step 3: [Copy the remaining elements of right sub-array, if any]
            IF I > MID
               Repeat while J <= END
                  SET TEMP[INDEX] = ARR[J]
                  SET INDEX = INDEX + 1, SET J = J + 1
               [END OF LOOP]
      [Copy the remaining elements of left sub-array, if any]
            ELSE
               Repeat while I <= MID
                  SET TEMP[INDEX] = ARR[I]
                  SET INDEX = INDEX + 1, SET I = I + 1
               [END OF LOOP]
            [END OF IF]
Step 4: [Copy the contents of TEMP back to ARR] SET K=0
Step 5: Repeat while K < INDEX
               SET ARR[K] = TEMP[K]
               SET K = K + 1
      [END OF LOOP]
Step 6: END
```

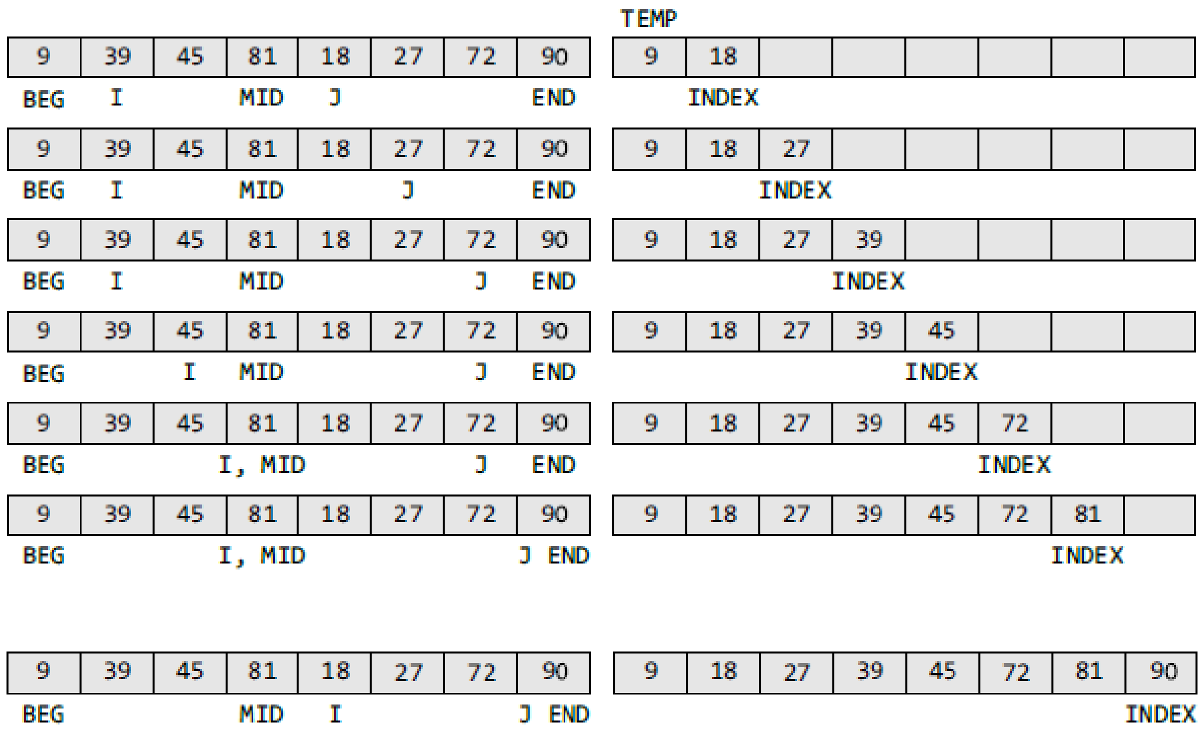
Merge Sort

- To understand the merge algorithm, consider the figure below which shows how we merge two lists to form one list.
- For ease of understanding, we have taken two sub-lists each containing four elements.
- The same concept can be utilized to merge four sub-lists containing two elements, or eight sub-lists having one element each.
- Compare



cation specified by INDEX and subsequently the value I or J is incremented.

- When I is

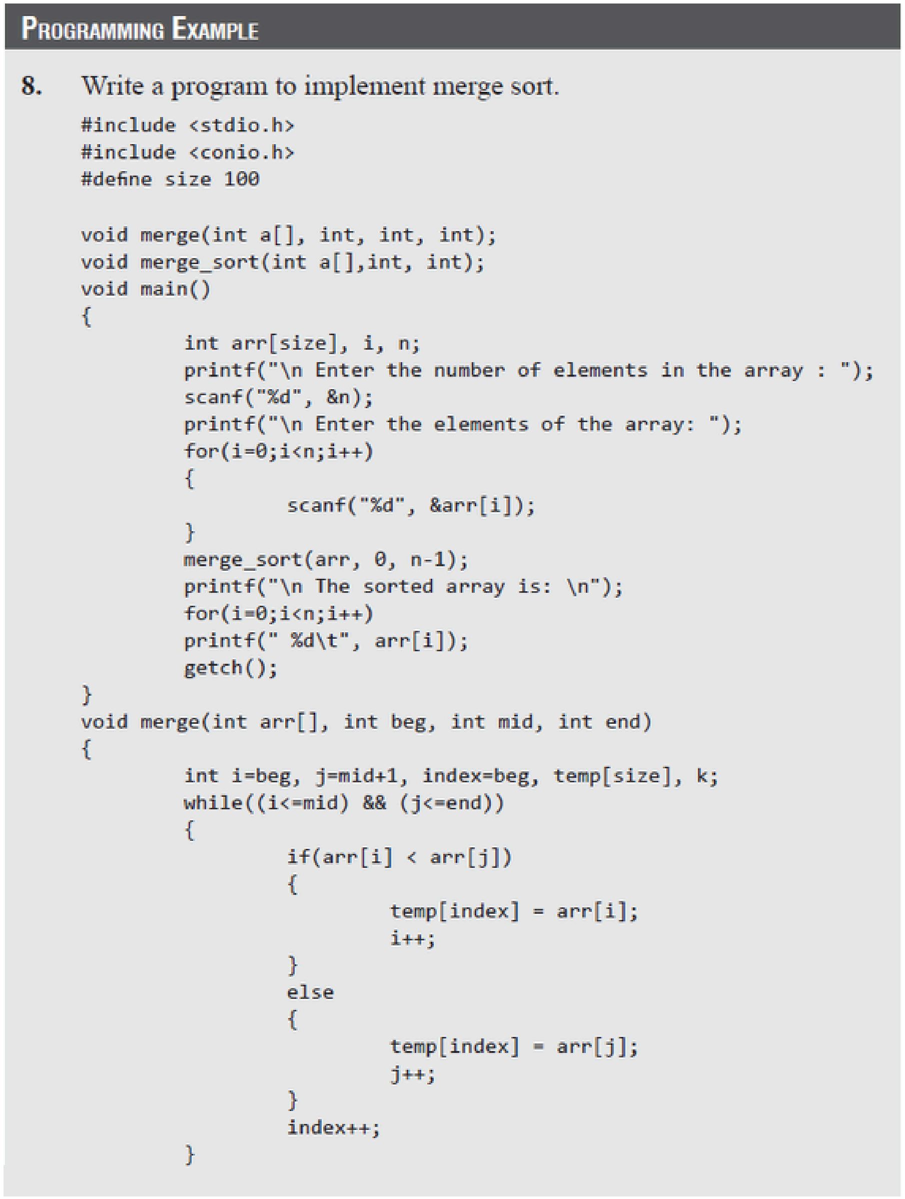


in TEMP.

Merge Sort

- ***Complexity of Merge Sort***
- The running time of merge sort in the average case and the worst case can be given as O(n log n). Although merge sort has an optimal time complexity, it needs an additional space of O(n) for the temporary array TEMP.

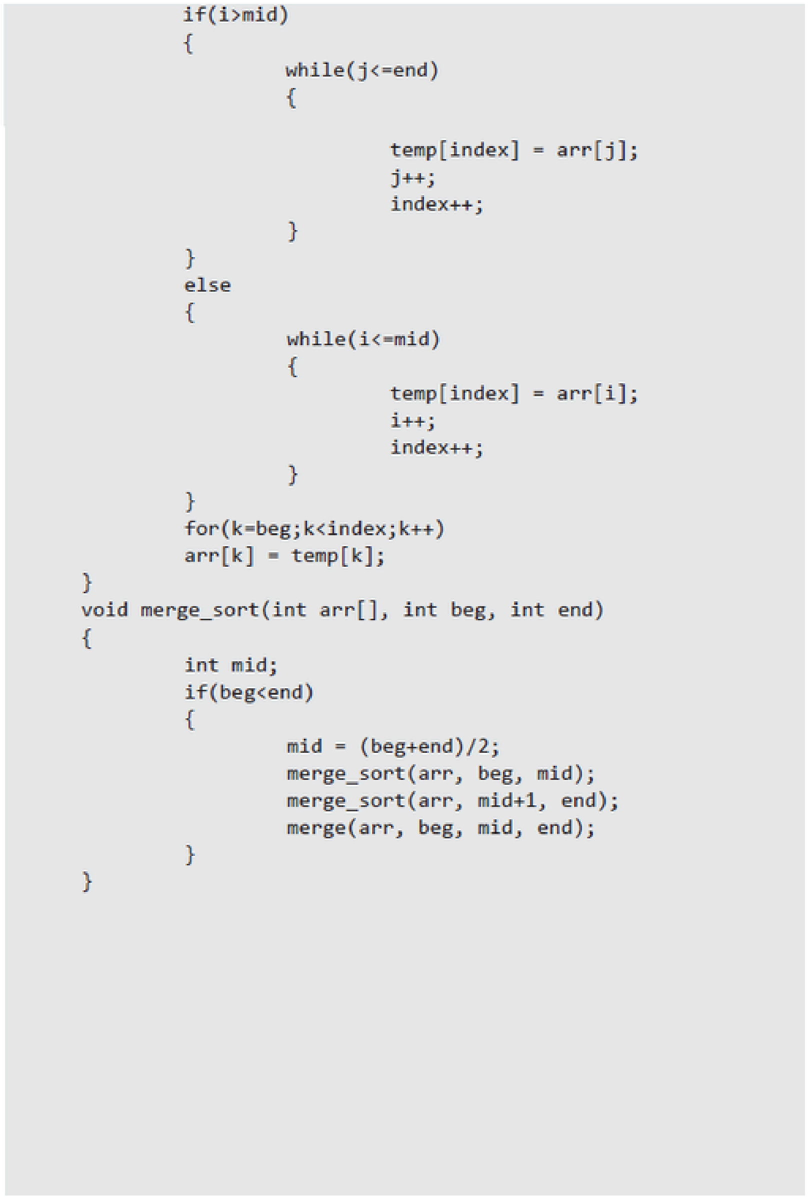
Merge Sort



Programming Example

8. Write a program to implement merge sort.

```
#include <stdio.h>
#include <conio.h>
#define size 100

void merge(int a[], int, int, int);
void merge_sort(int a[],int, int);
void main()
{
        int arr[size], i, n;
        printf("\n Enter the number of elements in the array : ");
        scanf("%d", &n);
        printf("\n Enter the elements of the array: ");
        for(i=0;i<n;i++)
        {
                scanf("%d", &arr[i]);
        }
        merge_sort(arr, 0, n-1);
        printf("\n The sorted array is: \n");
        for(i=0;i<n;i++)
        printf(" %d\t", arr[i]);
        getch();
}
void merge(int arr[], int beg, int mid, int end)
{
        int i=beg, j=mid+1, index=beg, temp[size], k;
        while((i<=mid) && (j<=end))
        {
                if(arr[i] < arr[j])
                {
                        temp[index] = arr[i];
                        i++;
                }
                else
                {
                        temp[index] = arr[j];
                        j++;
                }
                index++;
        }
```



```
        if(i>mid)
        {
                while(j<=end)
                {

                        temp[index] = arr[j];
                        j++;
                        index++;
                }
        }
        else
        {
                while(i<=mid)
                {
                        temp[index] = arr[i];
                        i++;
                        index++;
                }
        }
        for(k=beg;k<index;k++)
        arr[k] = temp[k];
}
void merge_sort(int arr[], int beg, int end)
{
        int mid;
        if(beg<end)
        {
                mid = (beg+end)/2;
                merge_sort(arr, beg, mid);
                merge_sort(arr, mid+1, end);
                merge(arr, beg, mid, end);
        }
}
```

Quick Sort

- Like merge sort, the *base case* of the recursion occurs when the array has zero or one element because in that case the array is already sorted.
- After each iteration, one element (pivot) is always in its final position.
- Hence, with every iteration, there is one less element to be sorted in the array.
- Thus, the main task is to find the pivot element, which will partition the array into two halves.

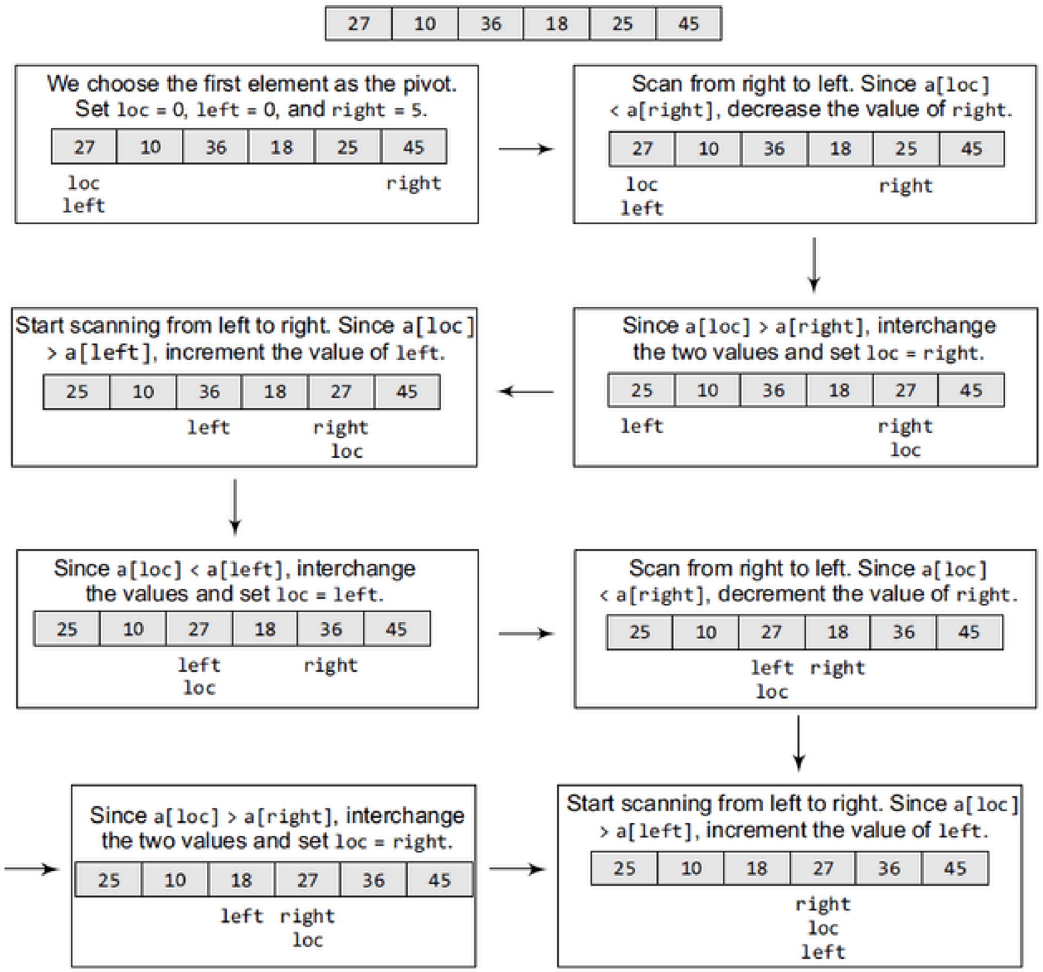
Quick Sort

- ***Technique***
- Quick sort works as follows:

1. Set the index of the first element in the array to loc and left variables. Also, set the index of the last element of the array to the right variable. That is, loc = 0, left = 0, and right = n–1 (where n in the number of elements in the array)
2. Start from the element pointed by right and scan the array from right to left, comparing each element on the way with the element pointed by the variable loc. That is, a(loc) should be less than a(right).
    a. If that is the case, then simply continue comparing until right becomes equal to loc. Once right = loc, it means the pivot has been placed in its correct position.
    b. However, if at any point, we have a(loc) > a(right), then interchange the two values and jump to Step 3.
    c. Set loc = right
3. Start from the element pointed by left and scan the array from left to right, comparing each element on the way with the element pointed by loc. That is, a(loc) should be greater than a(left).
    a. If that is the case, then simply continue comparing until left becomes equal to loc. Once left = loc, it means the pivot has been placed in its correct position.
    b. However, if at any point, we have a(loc) < a(left), then interchange the two values and jump to Step 2.
    c. Set loc = left.

Quick Sort

Example 14.6 Sort the elements given in the following array using quick sort algorithm



27	10	36	18	25	45

We choose the first element as the pivot. Set `loc = 0`, `left = 0`, and `right = 5`.

27	10	36	18	25	45
loc left					right

Scan from right to left. Since `a[loc] < a[right]`, decrease the value of `right`.

27	10	36	18	25	45
loc left				right	

Since `a[loc] > a[right]`, interchange the two values and set `loc = right`.

25	10	36	18	27	45
left				right loc	

Start scanning from left to right. Since `a[loc] > a[left]`, increment the value of `left`.

25	10	36	18	27	45
		left		right loc	

Since `a[loc] < a[left]`, interchange the values and set `loc = left`.

25	10	27	18	36	45
		left loc		right	

Scan from right to left. Since `a[loc] < a[right]`, decrement the value of `right`.

25	10	27	18	36	45
		left loc	right		

Since `a[loc] > a[right]`, interchange the two values and set `loc = right`.

25	10	18	27	36	45
		left	right loc		

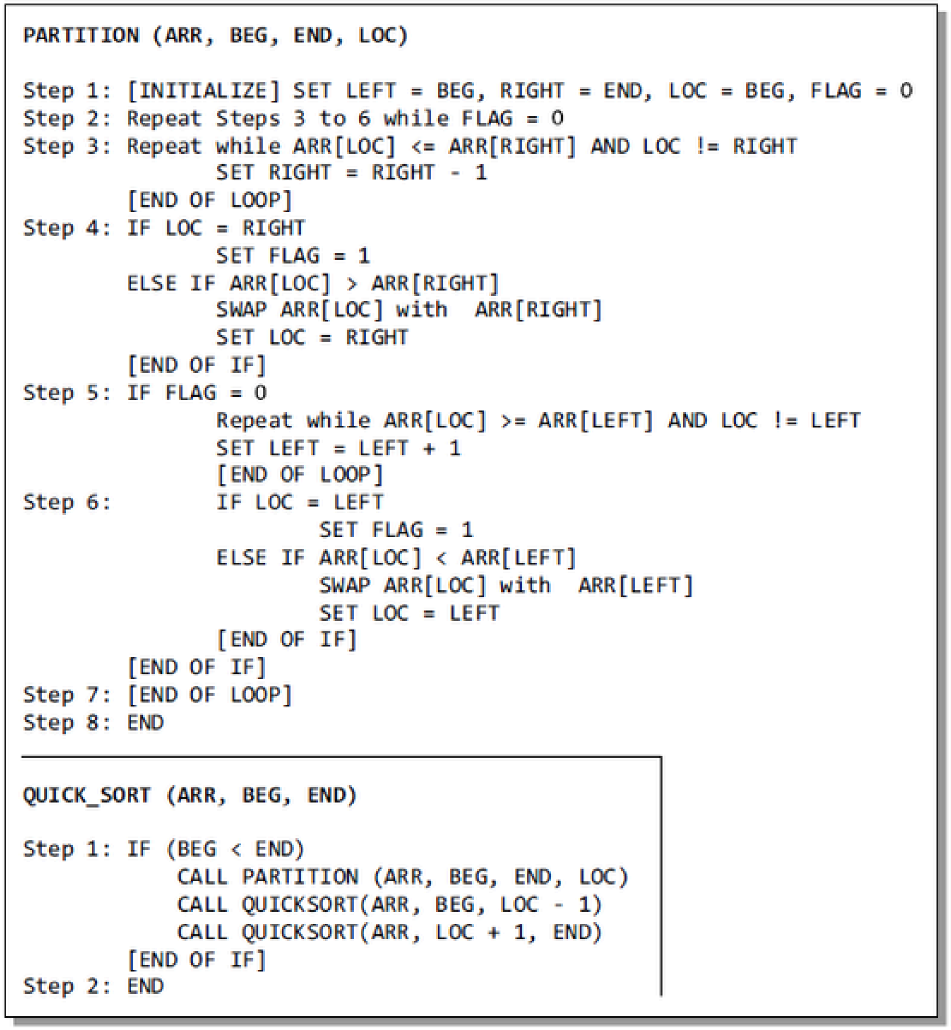
Start scanning from left to right. Since `a[loc] > a[left]`, increment the value of `left`.

25	10	18	27	36	45
			right loc left		

- Now left = loc, so the procedure terminates, as the pivot element (the first element of the array, that is, 27) is placed in its correct position. All the elements smaller than 27 are placed before it and those greater than 27 are placed after it. The left sub-array containing 25, 10, 18 and the right sub-array containing 36 and 45 are sorted in the same manner.

Quick Sort

- The quick sort algorithm (Fig. 14.10) makes use of a function Partition to divide the array into two sub-arrays.



```
PARTITION (ARR, BEG, END, LOC)

Step 1: [INITIALIZE] SET LEFT = BEG, RIGHT = END, LOC = BEG, FLAG = 0
Step 2: Repeat Steps 3 to 6 while FLAG = 0
Step 3: Repeat while ARR[LOC] <= ARR[RIGHT] AND LOC != RIGHT
                SET RIGHT = RIGHT - 1
        [END OF LOOP]
Step 4: IF LOC = RIGHT
                SET FLAG = 1
        ELSE IF ARR[LOC] > ARR[RIGHT]
                SWAP ARR[LOC] with  ARR[RIGHT]
                SET LOC = RIGHT
        [END OF IF]
Step 5: IF FLAG = 0
                Repeat while ARR[LOC] >= ARR[LEFT] AND LOC != LEFT
                SET LEFT = LEFT + 1
                [END OF LOOP]
Step 6:         IF LOC = LEFT
                        SET FLAG = 1
                ELSE IF ARR[LOC] < ARR[LEFT]
                        SWAP ARR[LOC] with  ARR[LEFT]
                        SET LOC = LEFT
                [END OF IF]
        [END OF IF]
Step 7: [END OF LOOP]
Step 8: END
```

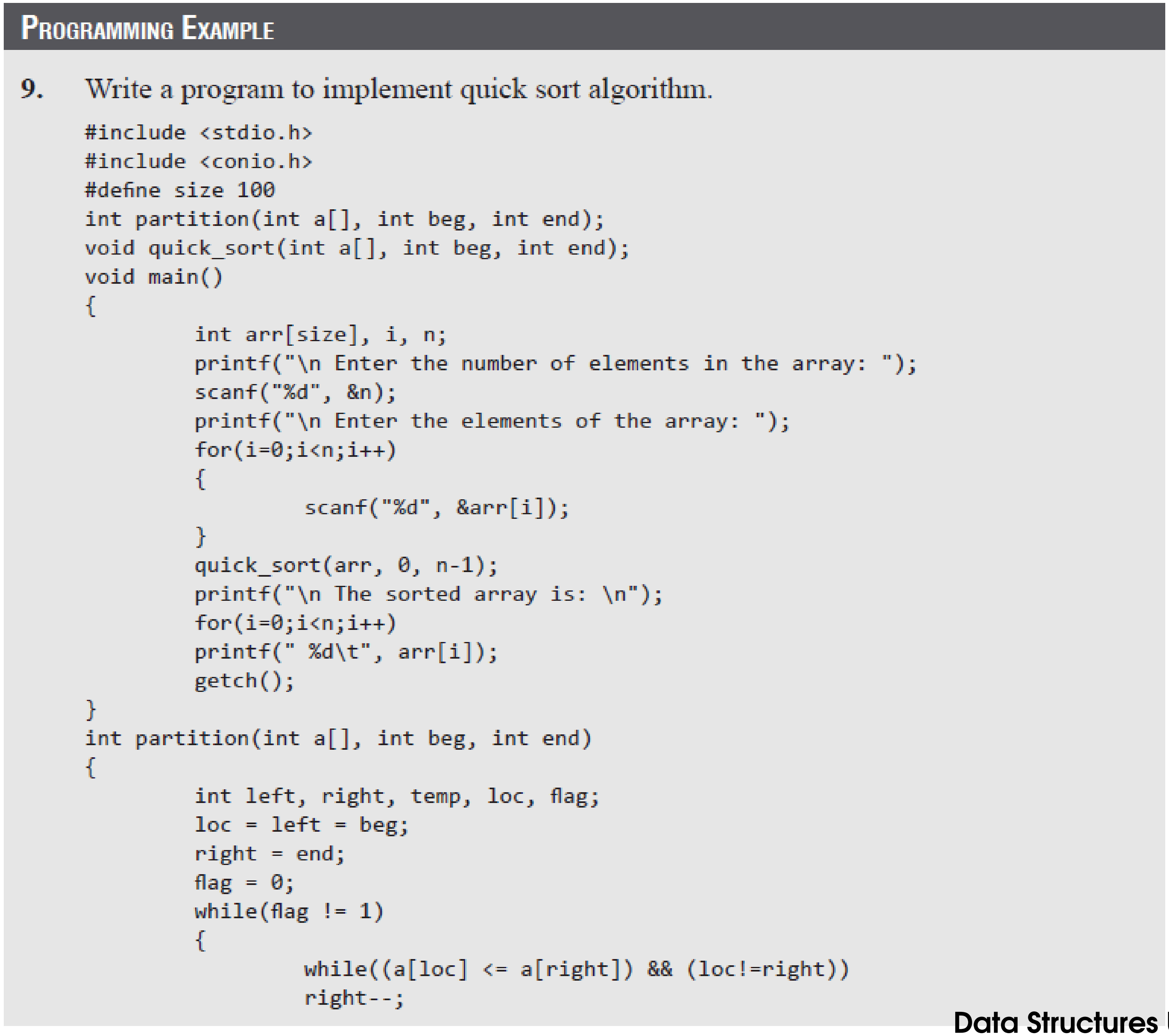
```
QUICK_SORT (ARR, BEG, END)

Step 1: IF (BEG < END)
             CALL PARTITION (ARR, BEG, END, LOC)
             CALL QUICKSORT(ARR, BEG, LOC - 1)
             CALL QUICKSORT(ARR, LOC + 1, END)
        [END OF IF]
Step 2: END
```

Figure 14.10 Algorithm for quick sort

Quick Sort

- ***Complexity of Quick Sort***
- In the average case, the running time of quick sort can be given as O(nlog n).
- The partitioning of the array which simply loops over the elements of the array once uses O(n) time.
- In the best case, every time we partition the array, we divide the list into two nearly equal pieces. That is, the recursive call processes the sub-array of half the size. At the most, only log n nested calls can be made before we reach a sub-array of size 1. It means the depth of the call tree is O(log n). And because at each level, there can only be O(n), the resultant time is given as O(n log n) time.
- Practically, the efficiency of quick sort depends on the element which is chosen as the pivot.
- Its worst-case efficiency is given as $O(n^2)$. The worst case occurs when the array is already sorted (either in ascending or descending order) and the left-most element is chosen as the pivot.
- However, many implementations randomly choose the pivot element. The randomized version of the quick sort algorithm always has an algorithmic complexity of O(n log n).

Quick Sort



PROGRAMMING EXAMPLE

9. Write a program to implement quick sort algorithm.

```
#include <stdio.h>
#include <conio.h>
#define size 100
int partition(int a[], int beg, int end);
void quick_sort(int a[], int beg, int end);
void main()
{
        int arr[size], i, n;
        printf("\n Enter the number of elements in the array: ");
        scanf("%d", &n);
        printf("\n Enter the elements of the array: ");
        for(i=0;i<n;i++)
        {
                scanf("%d", &arr[i]);
        }
        quick_sort(arr, 0, n-1);
        printf("\n The sorted array is: \n");
        for(i=0;i<n;i++)
        printf(" %d\t", arr[i]);
        getch();
}
int partition(int a[], int beg, int end)
{
        int left, right, temp, loc, flag;
        loc = left = beg;
        right = end;
        flag = 0;
        while(flag != 1)
        {
                while((a[loc] <= a[right]) && (loc!=right))
                right--;
```

Quick Sort

```
                if(loc==right)
                flag =1;
                else if(a[loc]>a[right])
                {
                        temp = a[loc];
                        a[loc] = a[right];
                        a[right] = temp;
                        loc = right;
                }
                if(flag!=1)
                {
                        while((a[loc] >= a[left]) && (loc!=left))
                        left++;
                        if(loc==left)
                        flag =1;
                        else if(a[loc] <a[left])
                        {
                                temp = a[loc];
                                a[loc] = a[left];
                                a[left] = temp;
                                loc = left;
                        }
                }
        }
        return loc;
}
void quick_sort(int a[], int beg, int end)
{
        int loc;
        if(beg<end)
        {
                loc = partition(a, beg, end);
                quick_sort(a, beg, loc-1);
                quick_sort(a, loc+1, end);
        }
}
```

Radix Sort

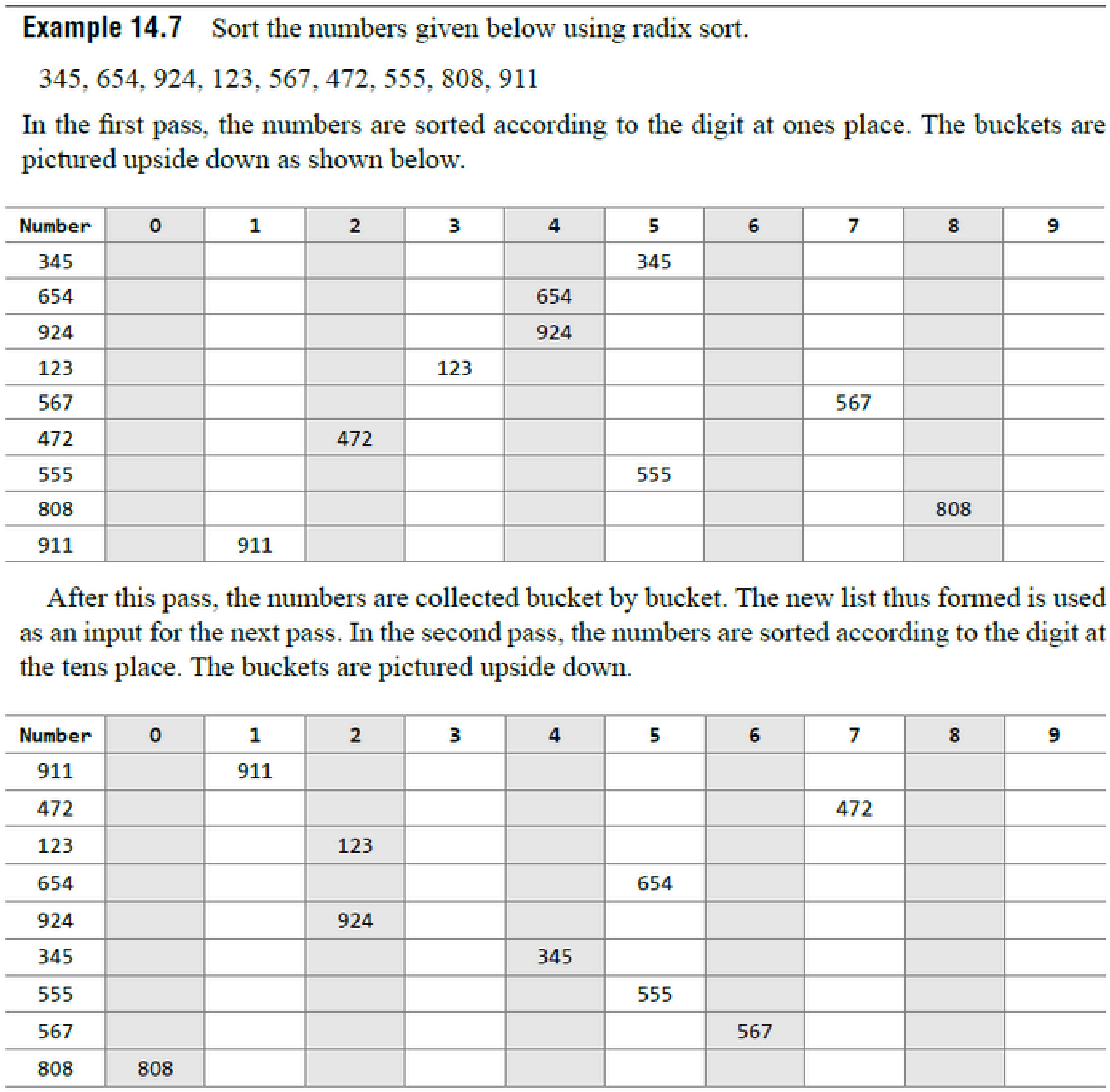
- Radix sort is a linear sorting algorithm for integers and uses the concept of sorting names in alphabetical order.
  - When we have a list of sorted names, the *radix* is 26 (or 26 buckets) because there are 26 letters in the English alphabet.
  - Observe that words are first sorted according to the first letter of the name.
  - During the second pass, names are grouped according to the second letter. After the second pass, names are sorted on the first two letters. This process is continued till the nth pass, where n is the length of the name with maximum number of letters.
- When radix sort is used on integers, sorting is done on each of the digits in the number. The sorting procedure proceeds by sorting the least significant to the most significant digit. While sorting the numbers, we have ten buckets, each for one digit (0, 1, 2, …, 9) and the number of passes will depend on the length of the number having maximum number of digits.

Radix Sort

```
Algorithm for RadixSort (ARR, N)

Step 1: Find the largest number in ARR as LARGE
Step 2: [INITIALIZE] SET NOP = Number of digits in LARGE
Step 3: SET PASS = 0
Step 4: Repeat Step 5 while PASS <= NOP-1
Step 5:          SET I = 0 and INITIALIZE buckets
Step 6:          Repeat Steps 7 to 9 while I<N-1
Step 7:                SET DIGIT  = digit at PASSth place in A[I]
Step 8:                Add A[I] to the bucket numbered DIGIT
Step 9:                INCEREMENT bucket count for bucket numbered DIGIT
                 [END OF LOOP]
Step 10:         Collect the numbers in the bucket
         [END OF LOOP]
Step 11: END
```

Figure 14.11 Algorithm for radix sort

Radix Sort



Example 14.7 Sort the numbers given below using radix sort.

345, 654, 924, 123, 567, 472, 555, 808, 911

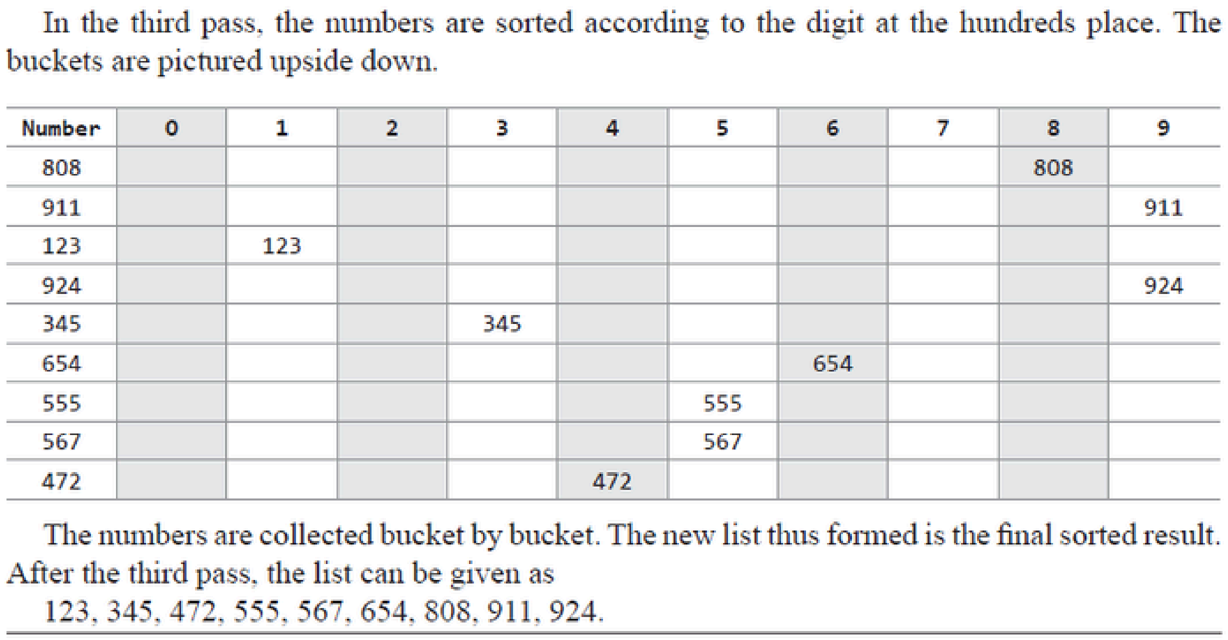
In the first pass, the numbers are sorted according to the digit at ones place. The buckets are pictured upside down as shown below.

Number	0	1	2	3	4	5	6	7	8	9
345						345				
654					654					
924					924					
123				123						
567								567		
472			472							
555						555				
808									808	
911		911								

After this pass, the numbers are collected bucket by bucket. The new list thus formed is used as an input for the next pass. In the second pass, the numbers are sorted according to the digit at the tens place. The buckets are pictured upside down.

Number	0	1	2	3	4	5	6	7	8	9
911		911								
472								472		
123			123							
654						654				
924			924							
345					345					
555						555				
567							567			
808	808									

Radix Sort



In the third pass, the numbers are sorted according to the digit at the hundreds place. The buckets are pictured upside down.

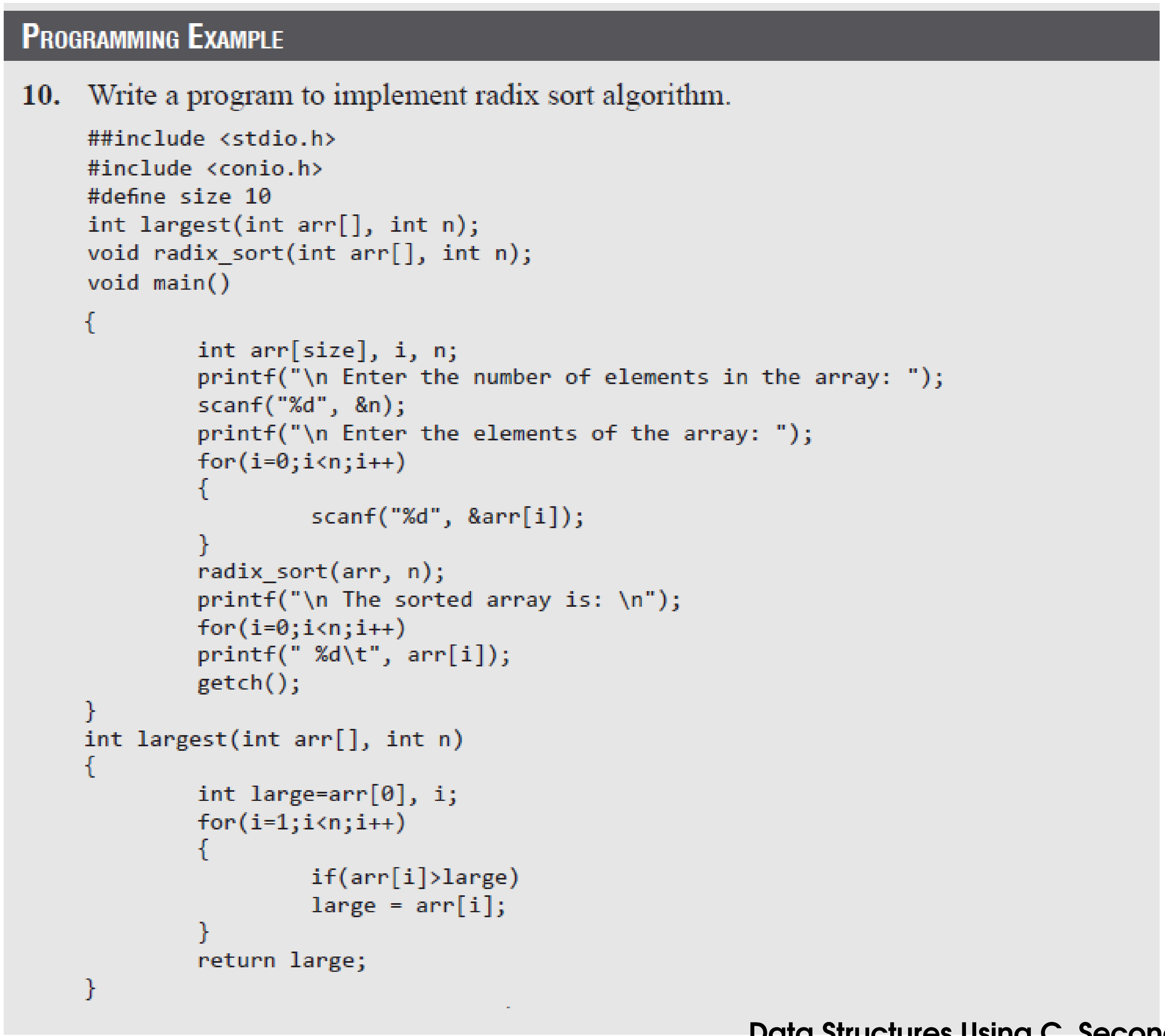
Number	0	1	2	3	4	5	6	7	8	9
808									808	
911										911
123		123								
924										924
345				345						
654							654			
555						555				
567						567				
472					472					

The numbers are collected bucket by bucket. The new list thus formed is the final sorted result. After the third pass, the list can be given as

123, 345, 472, 555, 567, 654, 808, 911, 924.

- ***Complexity of Radix Sort***
- To calculate the complexity of radix sort algorithm, assume that there are n numbers that have to be sorted and k is the number of digits in the largest number.
- In this case, the radix sort algorithm is called a total of k times.
- The inner loop is executed n times.
- Hence, the entire radix sort algorithm takes O(kn) time to execute.
- When radix sort is applied on a data set of finite size (very small set of numbers), then the algorithm runs in O(n) asymptotic time.

Radix Sort



Programming Example

10. Write a program to implement radix sort algorithm.

```
##include <stdio.h>
#include <conio.h>
#define size 10
int largest(int arr[], int n);
void radix_sort(int arr[], int n);
void main()
{
        int arr[size], i, n;
        printf("\n Enter the number of elements in the array: ");
        scanf("%d", &n);
        printf("\n Enter the elements of the array: ");
        for(i=0;i<n;i++)
        {
                scanf("%d", &arr[i]);
        }
        radix_sort(arr, n);
        printf("\n The sorted array is: \n");
        for(i=0;i<n;i++)
        printf(" %d\t", arr[i]);
        getch();
}
int largest(int arr[], int n)
{
        int large=arr[0], i;
        for(i=1;i<n;i++)
        {
                if(arr[i]>large)
                large = arr[i];
        }
        return large;
}
```

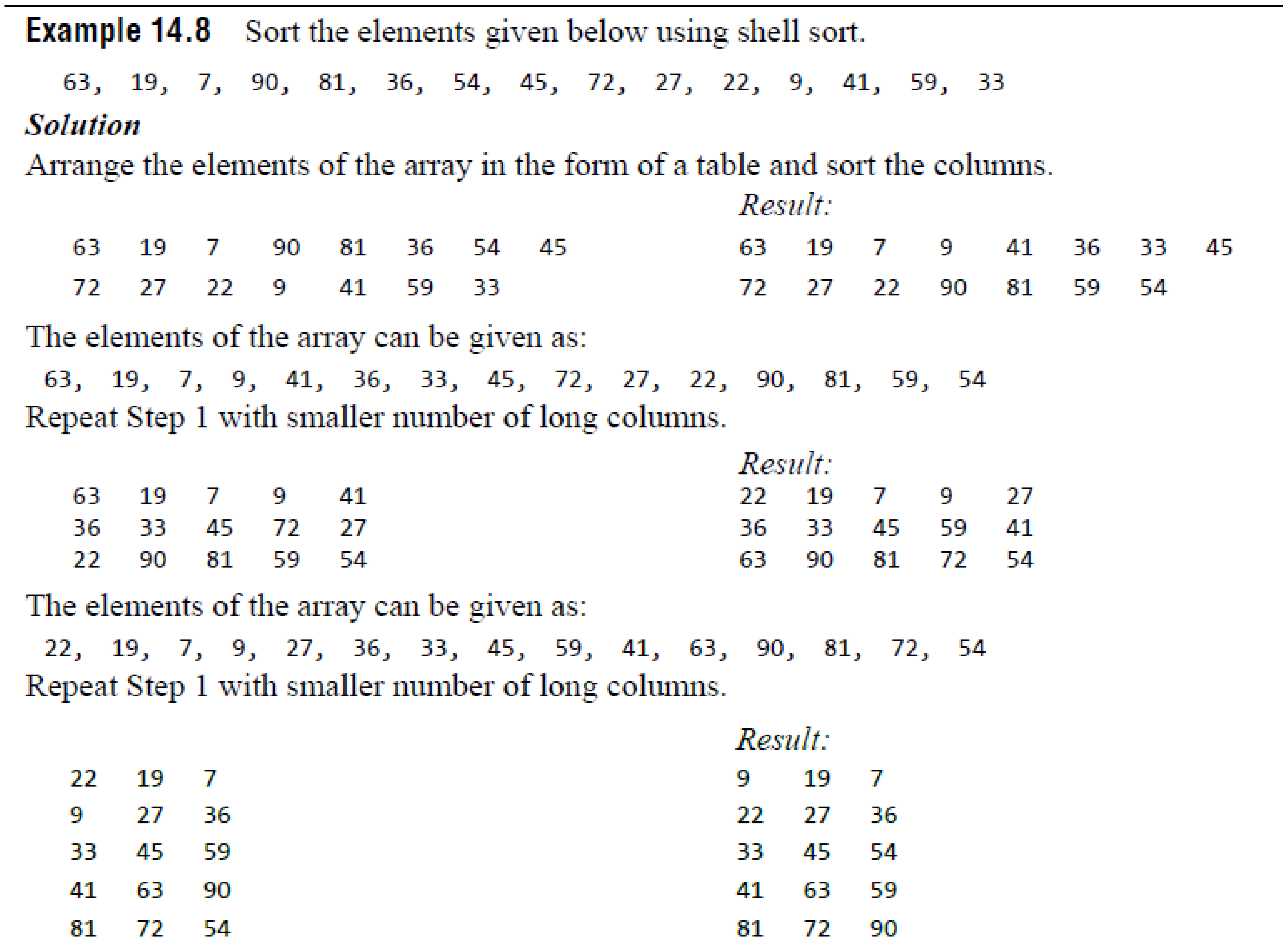
Radix Sort

```
void radix_sort(int arr[], int n)
{
        int bucket[size][size], bucket_count[size];
        int i, j, k, remainder, NOP=0, divisor=1, large, pass;
        large = largest(arr, n);
        while(large>0)
        {
                NOP++;
                large/=size;
        }
        for(pass=0;pass<NOP;pass++) // Initialize the buckets
        {
                for(i=0;i<size;i++)
                bucket_count[i]=0;
                for(i=0;i<n;i++)
                {
                        // sort the numbers according to the digit at passth place
                        remainder = (arr[i]/divisor)%size;
                        bucket[remainder][bucket_count[remainder]] = arr[i];
                        bucket_count[remainder] += 1;
                }
                // collect the numbers after PASS pass
                i=0;
                for(k=0;k<size;k++)
                {
                        for(j=0;j<bucket_count[k];j++)
                        {
                                arr[i] = bucket[k][j];
                                i++;
                        }
                }
                divisor *= size;
        }
}

}
```

Shell Sort

- Shell sort, invented by Donald Shell in 1959, is a sorting algorithm that is a generalization of insertion sort. While discussing insertion sort, we have observed two things:
  - First, insertion sort works well when the input data is 'almost sorted'.
  - Second, insertion sort is quite inefficient to use as it moves the values just one position at a time.
- Shell sort is considered an improvement over insertion sort as it compares elements separated by a gap of several positions. This enables the element to take bigger steps towards its expected position. In Shell sort, elements are sorted in multiple passes and in each pass, data are taken with smaller and smaller gap sizes. However, the last step of shell sort is a plain insertion sort. But by the time we reach the last step, the elements are already 'almost sorted', and hence it provides good performance.

Shell Sort



Example 14.8 Sort the elements given below using shell sort.

```
63,  19,  7,  90,  81,  36,  54,  45,  72,  27,  22,  9,  41,  59,  33
```

Solution

Arrange the elements of the array in the form of a table and sort the columns.

63	19	7	90	81	36	54	45
72	27	22	9	41	59	33	

Result:

63	19	7	9	41	36	33	45
72	27	22	90	81	59	54	

The elements of the array can be given as:

```
63,  19,  7,  9,  41,  36,  33,  45,  72,  27,  22,  90,  81,  59,  54
```

Repeat Step 1 with smaller number of long columns.

63	19	7	9	41
36	33	45	72	27
22	90	81	59	54

Result:

22	19	7	9	27
36	33	45	59	41
63	90	81	72	54

The elements of the array can be given as:

```
22,  19,  7,  9,  27,  36,  33,  45,  59,  41,  63,  90,  81,  72,  54
```

Repeat Step 1 with smaller number of long columns.

22	19	7
9	27	36
33	45	59
41	63	90
81	72	54

Result:

9	19	7
22	27	36
33	45	54
41	63	59
81	72	90

Shell Sort

The elements of the array can be given as:

```
9,  19,  7,  22,  27,  36,  33,  45,  54,  41,  63,  59,  81,  72,  90
```

Finally, arrange the elements of the array in a single column and sort the column.

	Result:
9	7
19	9
7	19
22	22
27	27
36	33
33	36
45	41
54	45
41	54
63	59
59	63
81	72
72	81
90	90

Finally, the elements of the array can be given as:

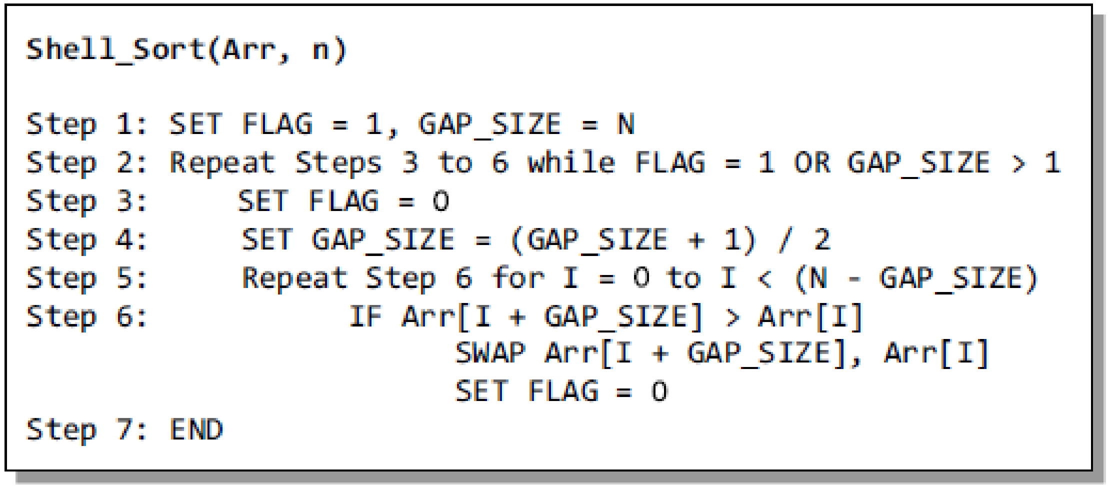
```
7,  9,  19,  22,  27,  33,  36,  41,  45,  54,  59,  63,  72,  81,  90
```

Shell Sort

- ***Technique***
- To visualize the way in which shell sort works, perform the following steps:
  - *Step 1:* Arrange the elements of the array in the form of a table and sort the columns (using insertion sort).
  - *Step 2:* Repeat Step 1, each time with smaller number of longer columns in such a way that at the end, there is only one column of data to be sorted.
- Note that we are only visualizing the elements being arranged in a table, the algorithm does its sorting in-place.

Shell Sort

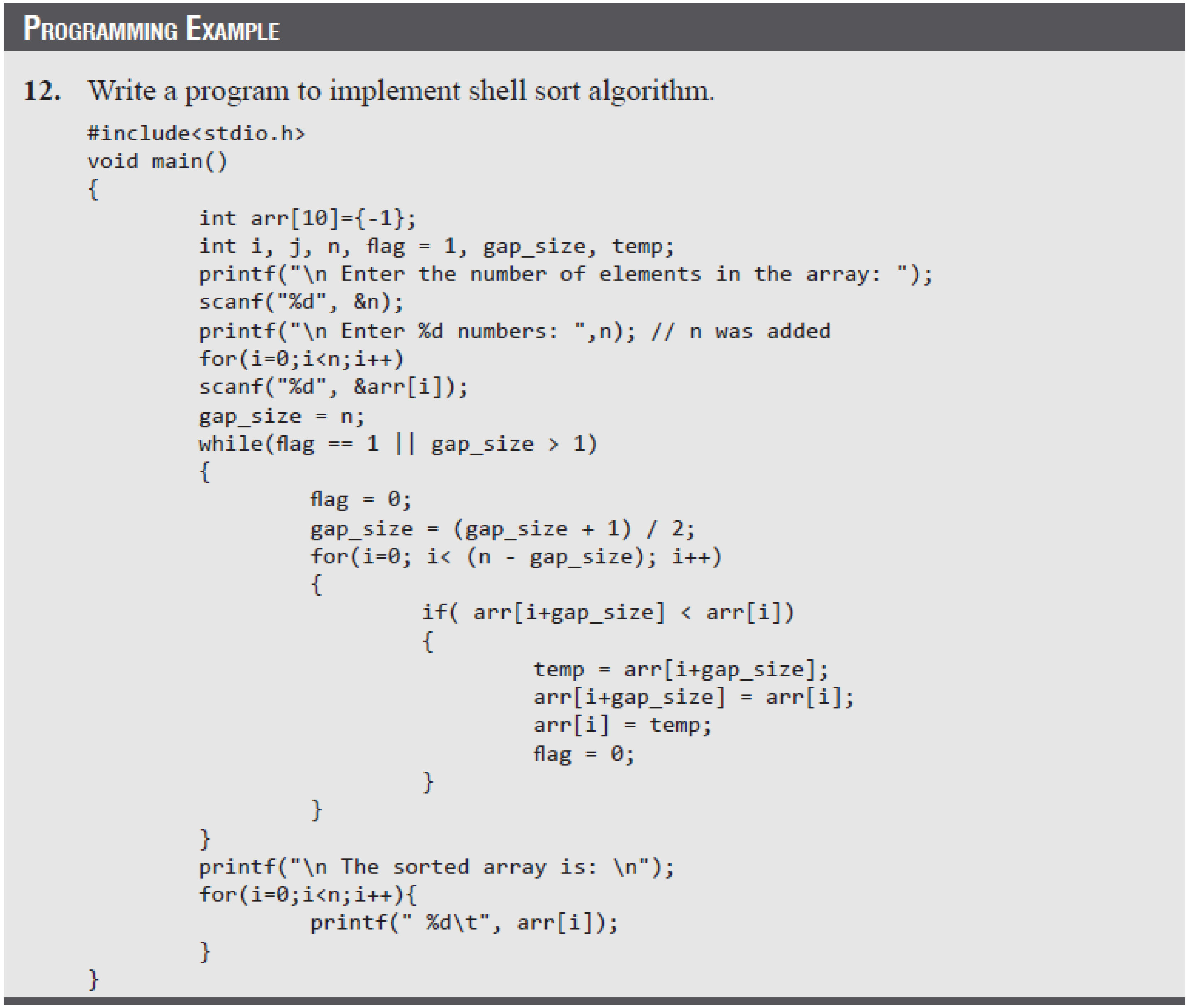
- The algorithm to sort an array of elements using shell sort is shown in Fig. 14.13.
- In the algorithm, we sort the elements of the array Arr in multiple passes.
- In each pass, we reduce the gap_size (visualize it as the number of columns) by a factor of half as done in Step 4.
- In each iteration of the for loop in Step 5, we compare the values of the array and interchange them if we have a larger value preceding the smaller one.



```
Shell_Sort(Arr, n)

Step 1: SET FLAG = 1, GAP_SIZE = N
Step 2: Repeat Steps 3 to 6 while FLAG = 1 OR GAP_SIZE > 1
Step 3:     SET FLAG = 0
Step 4:     SET GAP_SIZE = (GAP_SIZE + 1) / 2
Step 5:     Repeat Step 6 for I = 0 to I < (N - GAP_SIZE)
Step 6:             IF Arr[I + GAP_SIZE] > Arr[I]
                        SWAP Arr[I + GAP_SIZE], Arr[I]
                        SET FLAG = 0
Step 7: END
```

Figure 14.13 Algorithm for shell sort

Shell Sort



PROGRAMMING EXAMPLE

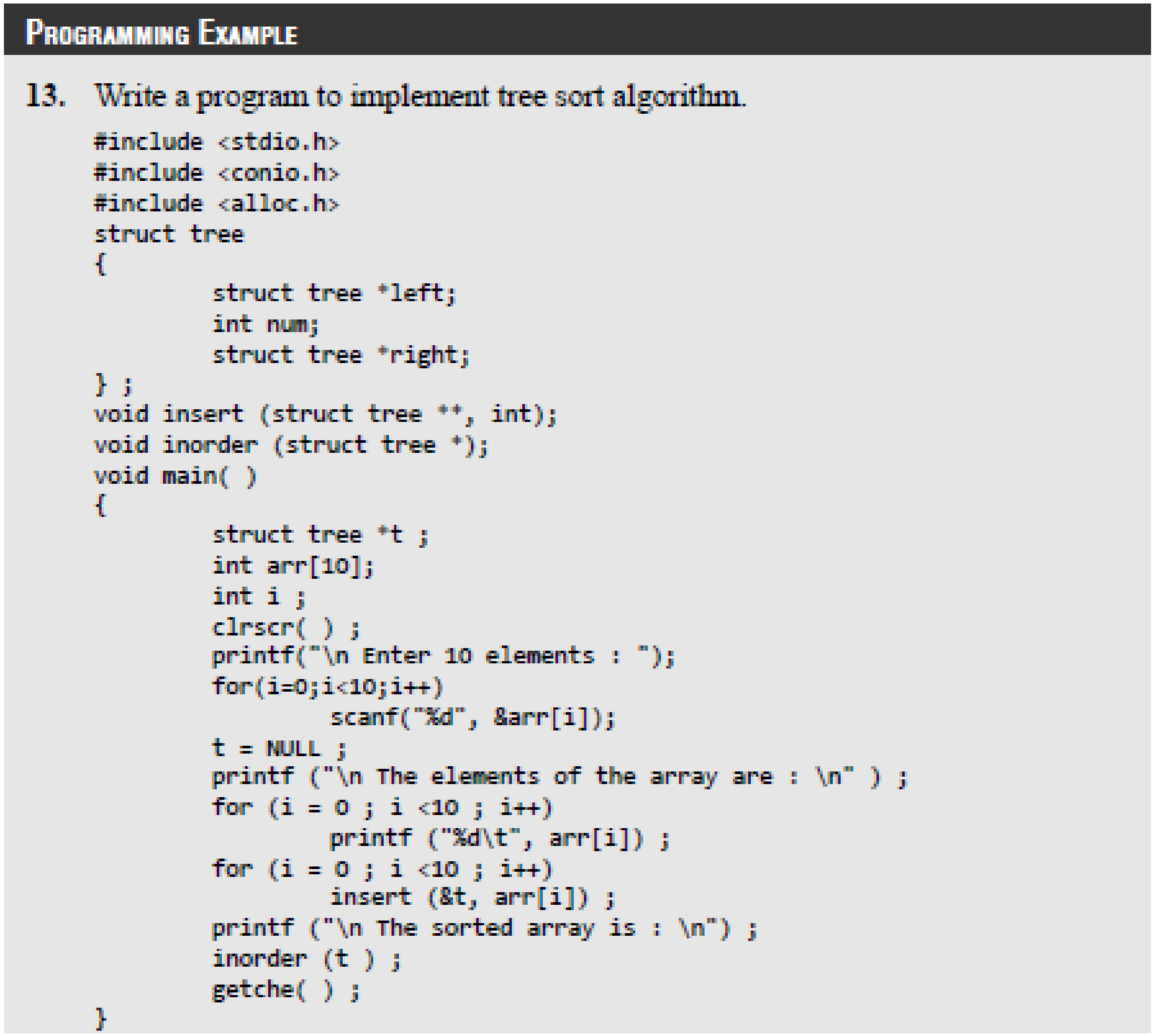
12. Write a program to implement shell sort algorithm.

```
#include<stdio.h>
void main()
{
         int arr[10]={-1};
         int i, j, n, flag = 1, gap_size, temp;
         printf("\n Enter the number of elements in the array: ");
         scanf("%d", &n);
         printf("\n Enter %d numbers: ",n); // n was added
         for(i=0;i<n;i++)
         scanf("%d", &arr[i]);
         gap_size = n;
         while(flag == 1 || gap_size > 1)
         {
                  flag = 0;
                  gap_size = (gap_size + 1) / 2;
                  for(i=0; i< (n - gap_size); i++)
                  {
                           if( arr[i+gap_size] < arr[i])
                           {
                                    temp = arr[i+gap_size];
                                    arr[i+gap_size] = arr[i];
                                    arr[i] = temp;
                                    flag = 0;
                           }
                  }
         }
         printf("\n The sorted array is: \n");
         for(i=0;i<n;i++){
                  printf(" %d\t", arr[i]);
         }
}
```

Tree Sort

- A tree sort is a sorting algorithm that sorts numbers by making use of the properties of binary search tree.
- The algorithm first builds a binary search tree using the numbers to be sorted and then does an in-order traversal so that the numbers are retrieved in a sorted order.

Tree Sort

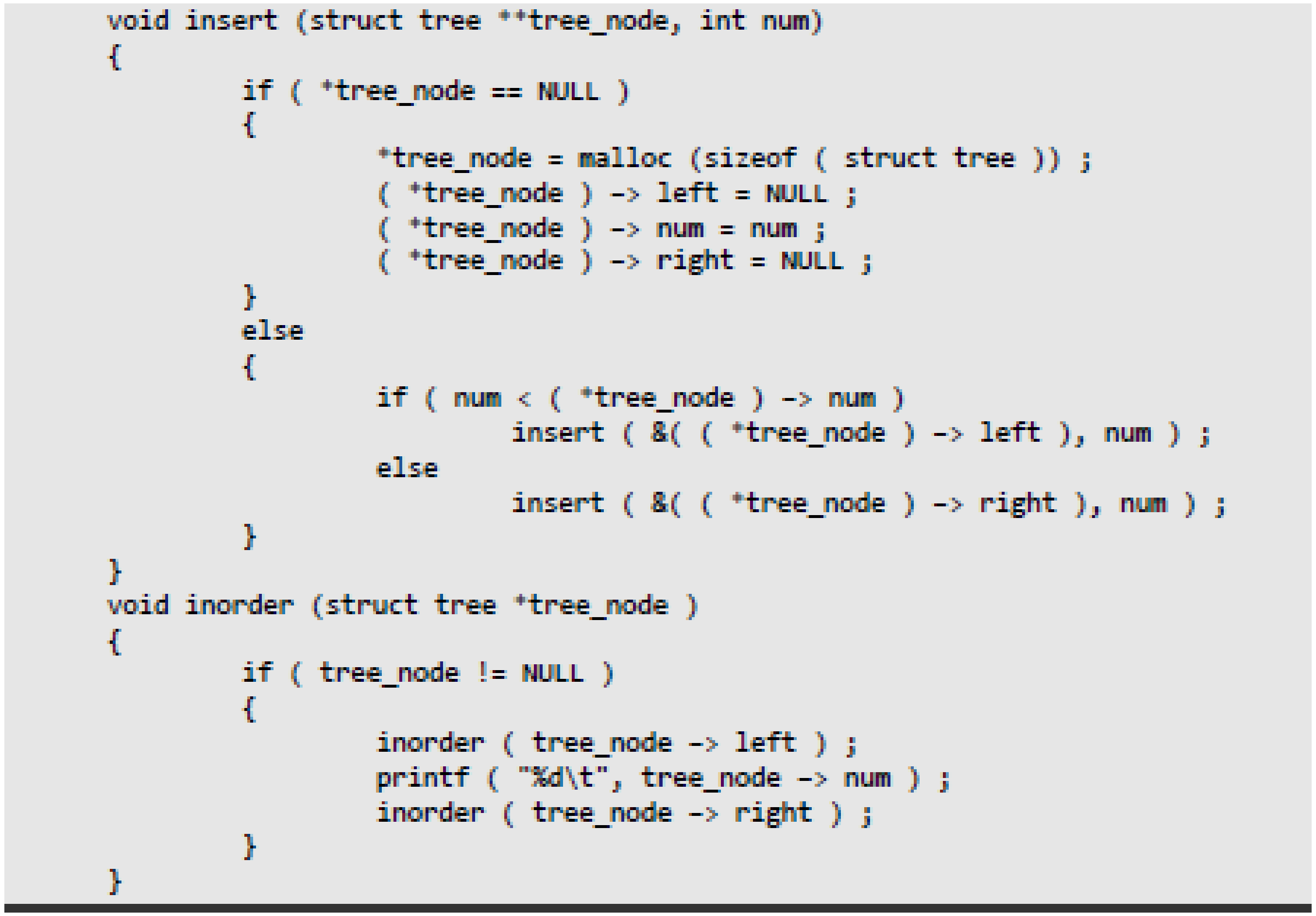


Programming Example

13. Write a program to implement tree sort algorithm.

```
#include <stdio.h>
#include <conio.h>
#include <alloc.h>
struct tree
{
        struct tree *left;
        int num;
        struct tree *right;
} ;
void insert (struct tree **, int);
void inorder (struct tree *);
void main( )
{
        struct tree *t ;
        int arr[10];
        int i ;
        clrscr( ) ;
        printf("\n Enter 10 elements : ");
        for(i=0;i<10;i++)
                scanf("%d", &arr[i]);
        t = NULL ;
        printf ("\n The elements of the array are : \n" ) ;
        for (i = 0 ; i <10 ; i++)
                printf ("%d\t", arr[i]) ;
        for (i = 0 ; i <10 ; i++)
                insert (&t, arr[i]) ;
        printf ("\n The sorted array is : \n") ;
        inorder (t ) ;
        getche( ) ;
}
```

Tree Sort

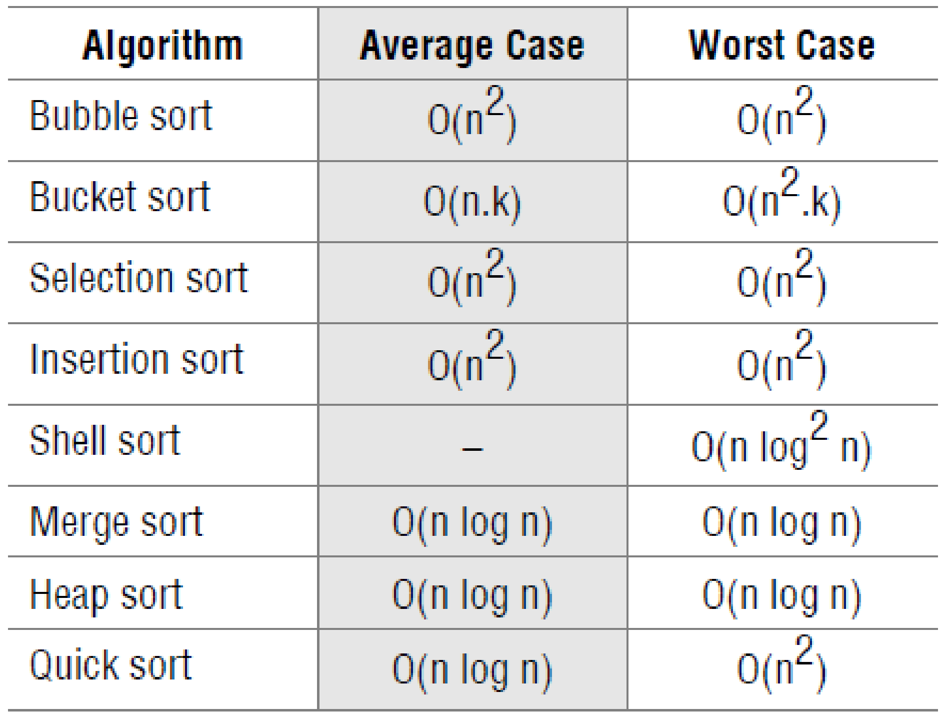


```
void insert (struct tree **tree_node, int num)
{
        if ( *tree_node == NULL )
        {
                *tree_node = malloc (sizeof ( struct tree )) ;
                ( *tree_node ) -> left = NULL ;
                ( *tree_node ) -> num = num ;
                ( *tree_node ) -> right = NULL ;
        }
        else
        {
                if ( num < ( *tree_node ) -> num )
                        insert ( &( ( *tree_node ) -> left ), num ) ;
                else
                        insert ( &( ( *tree_node ) -> right ), num ) ;
        }
}
void inorder (struct tree *tree_node )
{
        if ( tree_node != NULL )
        {
                inorder ( tree_node -> left ) ;
                printf ( "%d\t", tree_node -> num ) ;
                inorder ( tree_node -> right ) ;
        }
}
```

Comparison of Sorting Algorithms

- Table 14.1 compares the average-case and worst-case time complexities of different sorting algorithms discussed so far.

Table 14.1 Comparison of algorithms



Algorithm	Average Case	Worst Case
Bubble sort	$O(n^2)$	$O(n^2)$
Bucket sort	$O(n.k)$	$O(n^2.k)$
Selection sort	$O(n^2)$	$O(n^2)$
Insertion sort	$O(n^2)$	$O(n^2)$
Shell sort	–	$O(n \log^2 n)$
Merge sort	$O(n \log n)$	$O(n \log n)$
Heap sort	$O(n \log n)$	$O(n \log n)$
Quick sort	$O(n \log n)$	$O(n^2)$