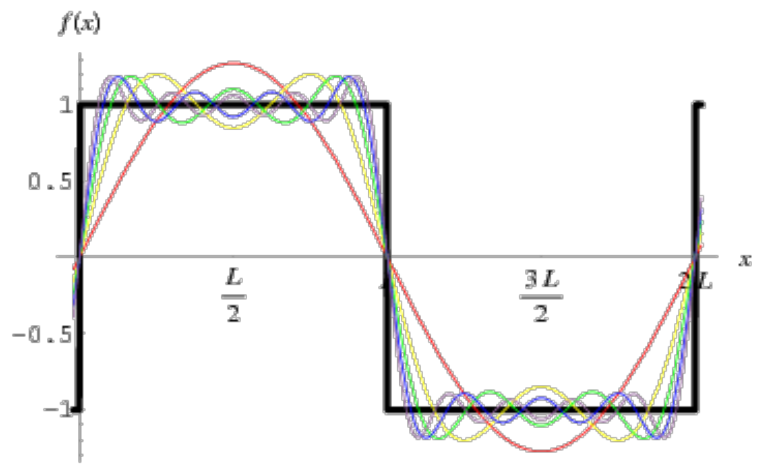
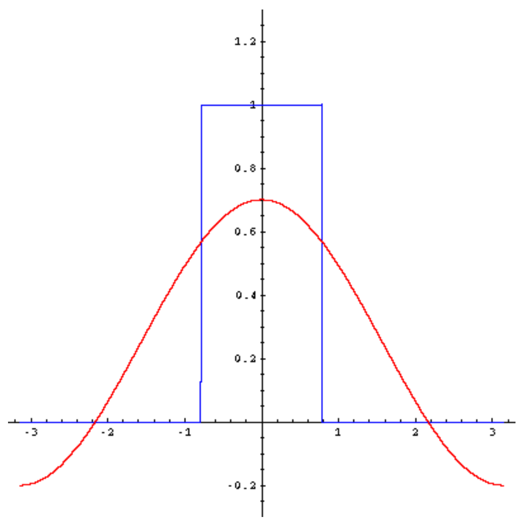
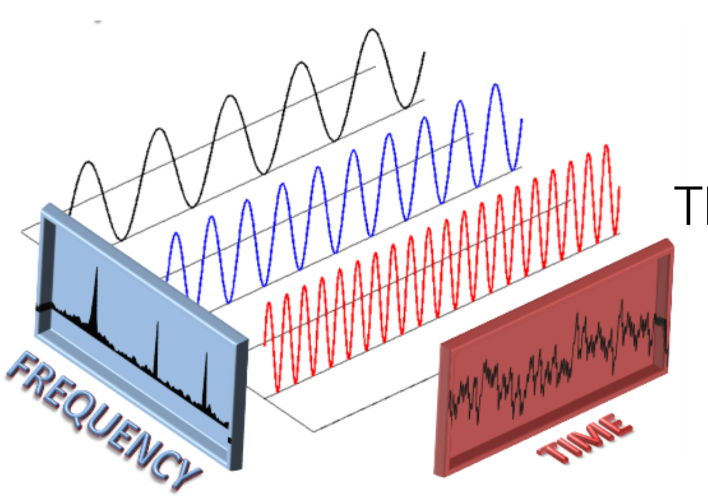
(Discrete) Fourier Decomposition

- Any signal can be expressed as the **sum of sines and cosines at different frequencies when they are appropriately scaled and shifted**.
- Fourier Transform consists of finding this amplitude and phase information.



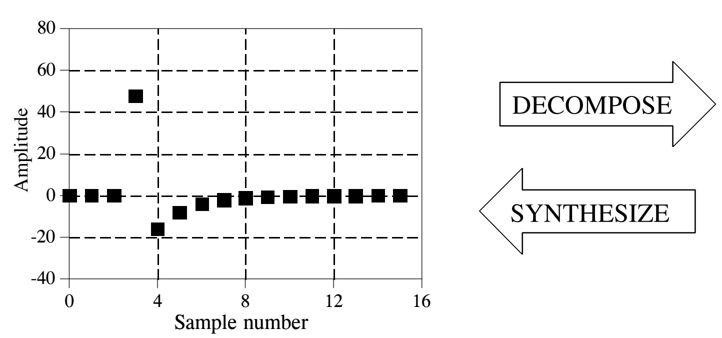


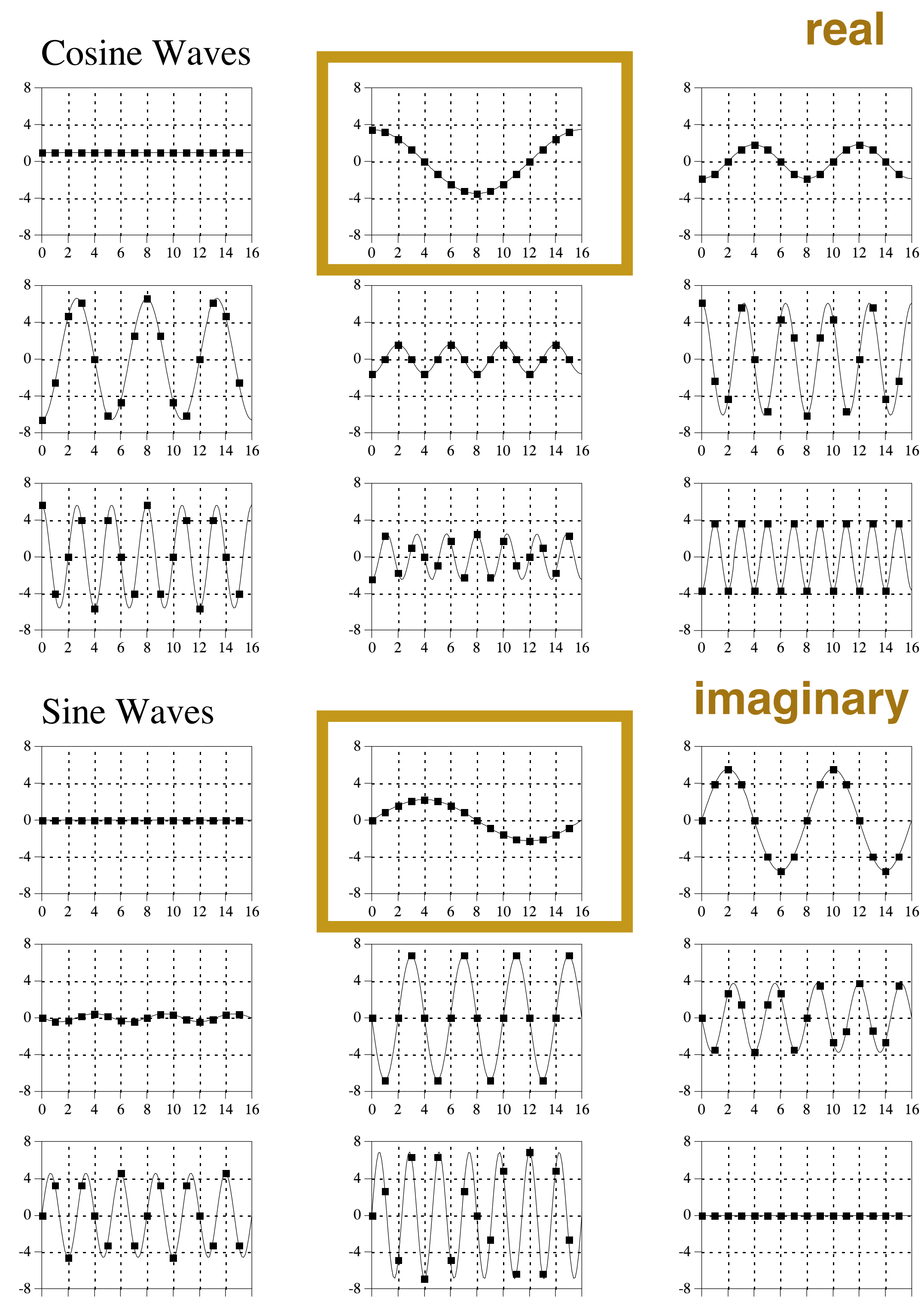


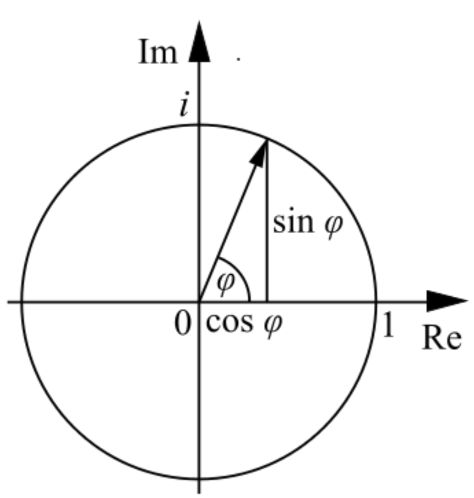
different frequencies
are orthogonal to each other.
They capture independent information.

Discrete Fourier Decomposition

- Any signal can be expressed as the **sum of sines and cosines at different frequencies when they are appropriately scaled and shifted**.
- Fourier Transform consists of finding this amplitude and phase information.







sine and cosine pairs can be represented as a complex number.

DFT Basis Functions

- For each frequency k, a pair of unit amplitude cosines and sines forms the DFT basis.

$$c_k[i] = \cos(2\pi k i / N)$$

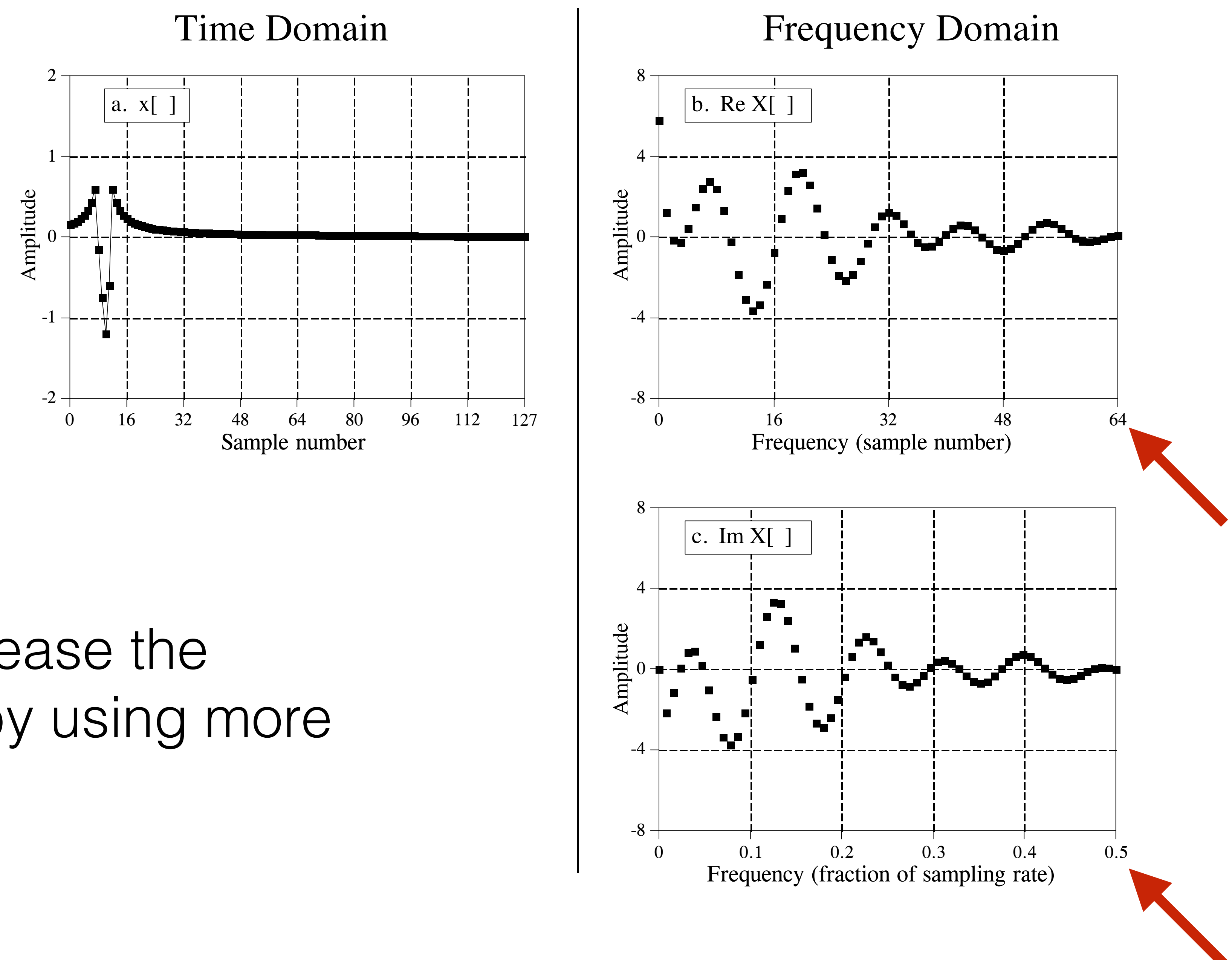
$$s_k[i] = \sin(2\pi k i / N)$$

- Synthesis Equation:

$$x[i] = \sum_{k=0}^{N/2} Re\bar{X}[k] \cos(2\pi k i / N) + \sum_{k=0}^{N/2} Im\bar{X}[k] \sin(2\pi k i / N)$$

How many basis functions?

- N data points in the time domain are transformed into N/2+1 frequency points.



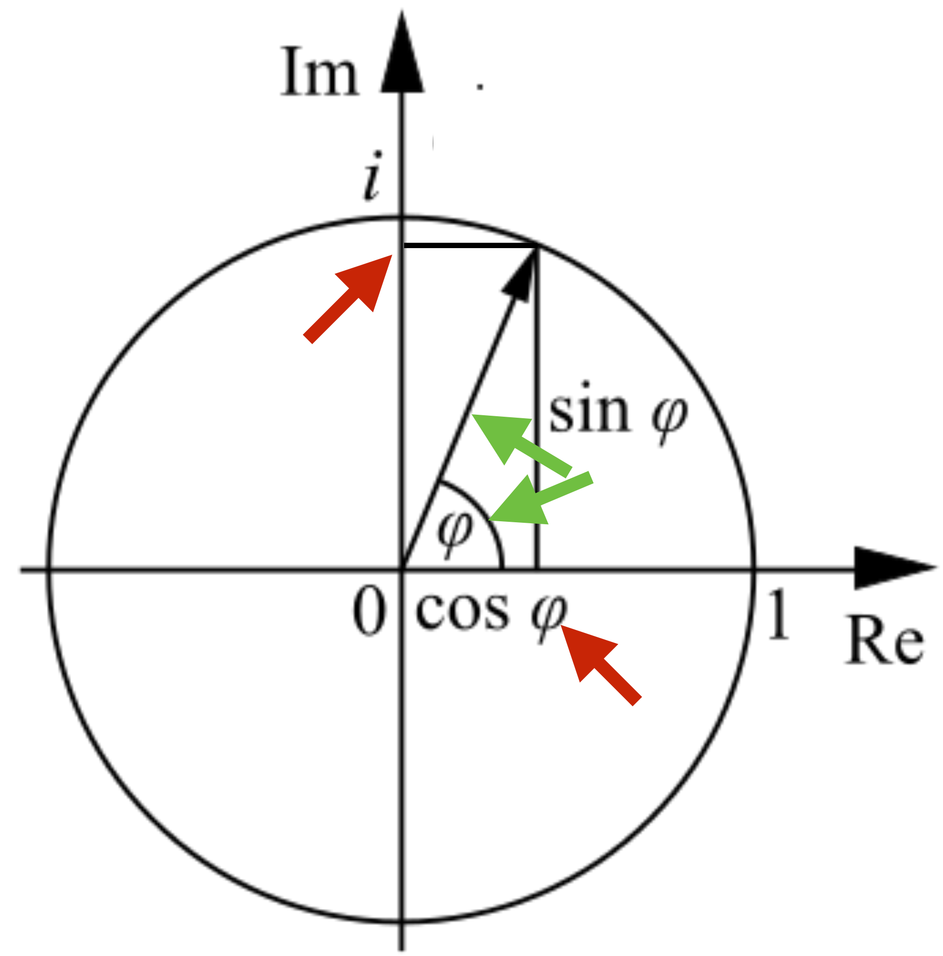
- However you can increase the frequency resolution by using more basis functions.

Two ways of thinking: Polar Notation

- **Real vs. Imag.**
  - Matlab: real(), imag()

$$A\cos(x) + B\sin(x) = M\cos(x + \theta)$$

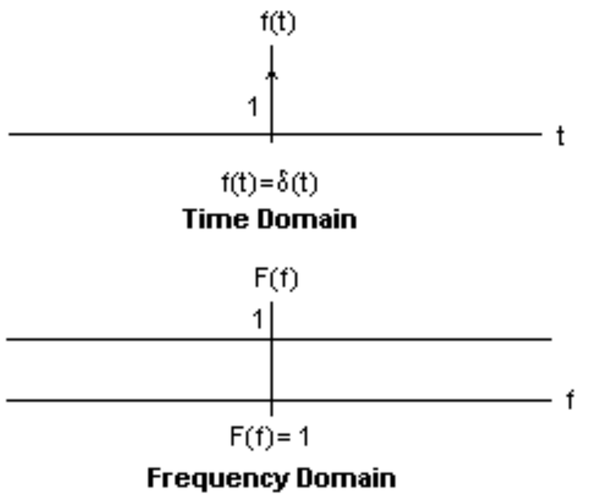
- **Magnitude vs. Phase**
  - Matlab: abs(), phase()

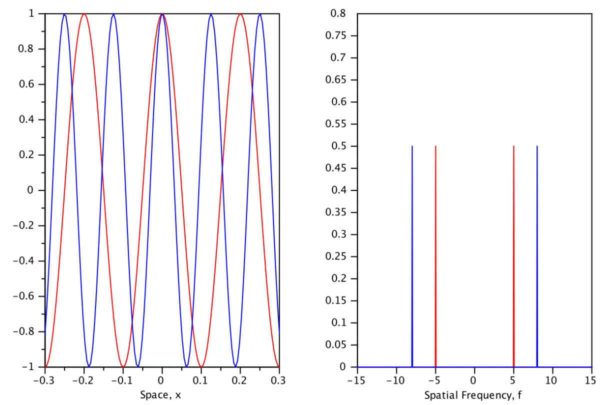


- N/2+1 cosine and sine pairs with different amplitudes
  or
  N/2+1 cosine functions with different phases and amplitudes.

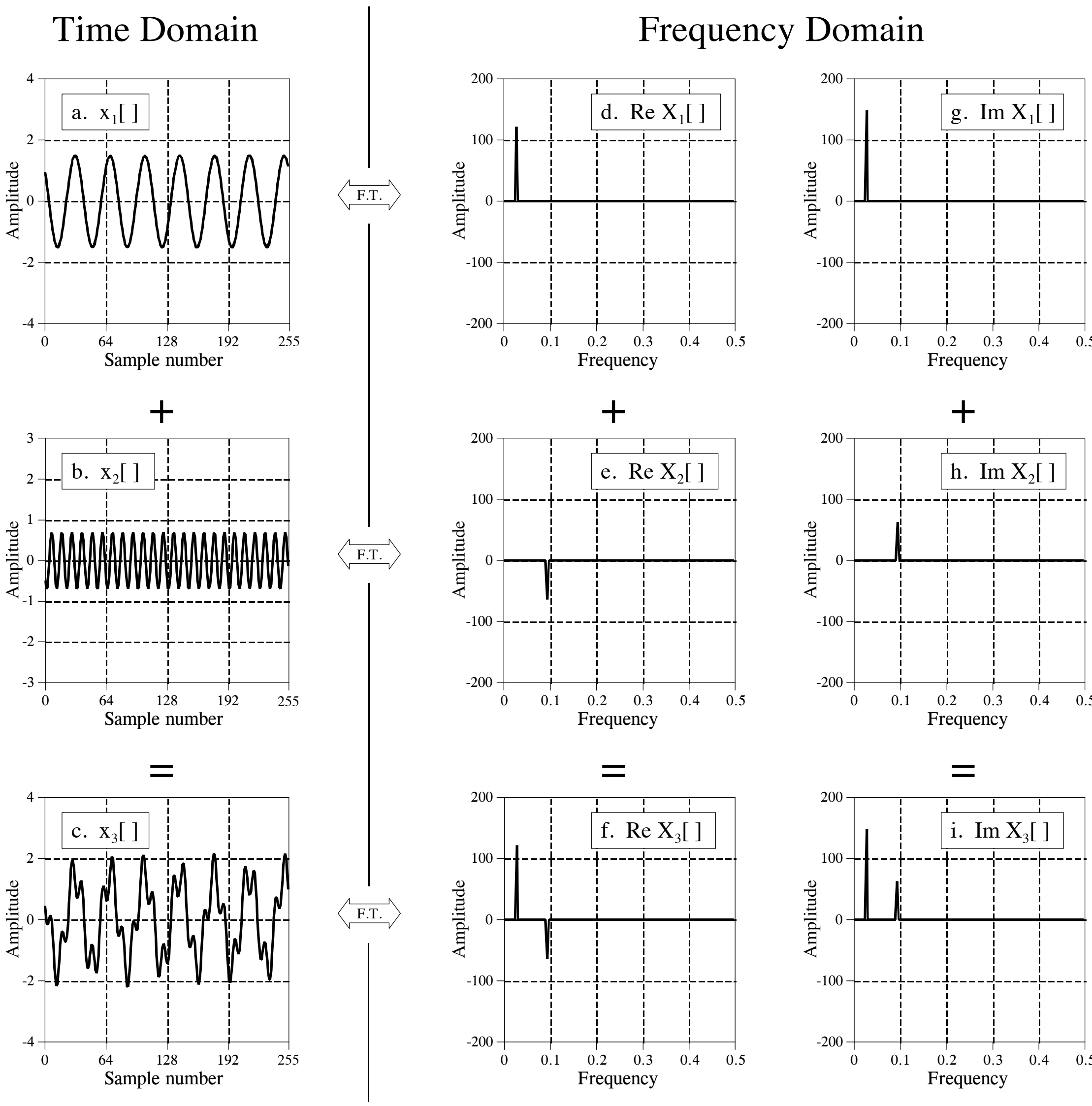
Fourier transform of an Impulse

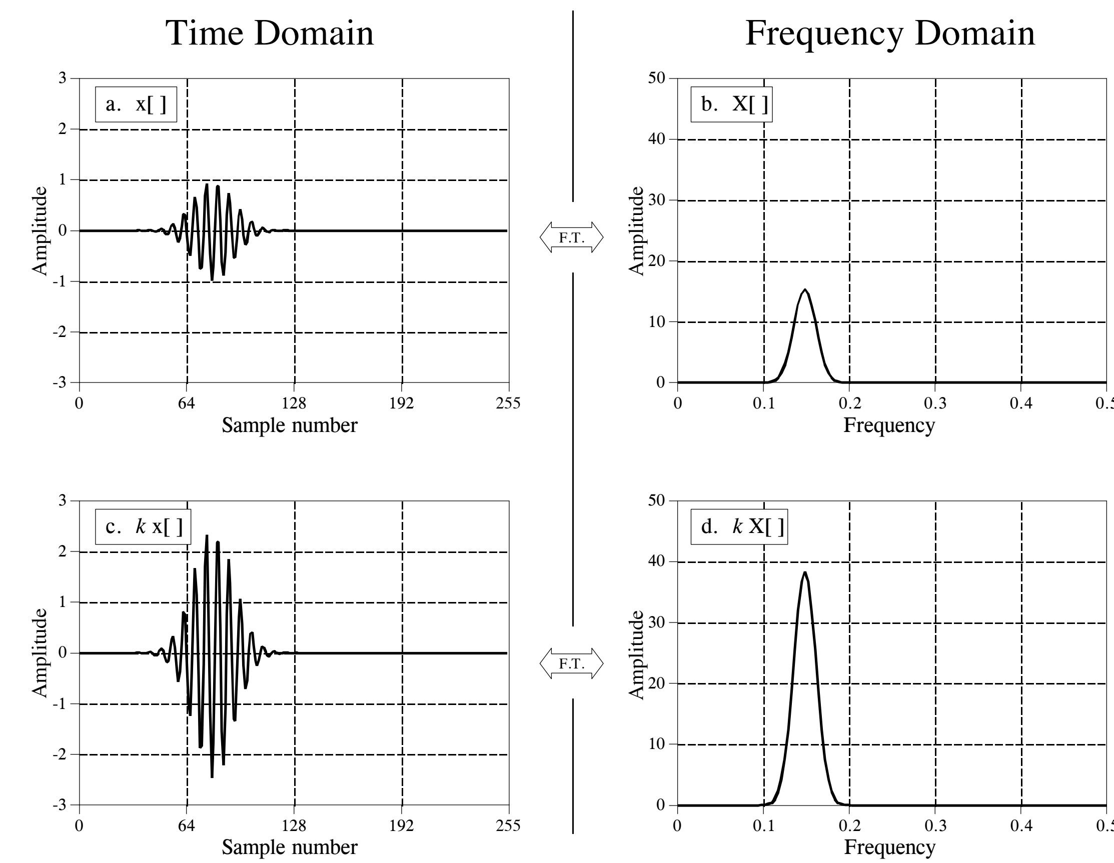
- An Impulse contains all the frequencies.
  - Basically the impulse “tests” for all the frequencies equally.
  - Now it should make more sense why a linear system is characterized by an impulse.
  - White noise because all frequencies are equally present.
- an impulse in the frequency domain?





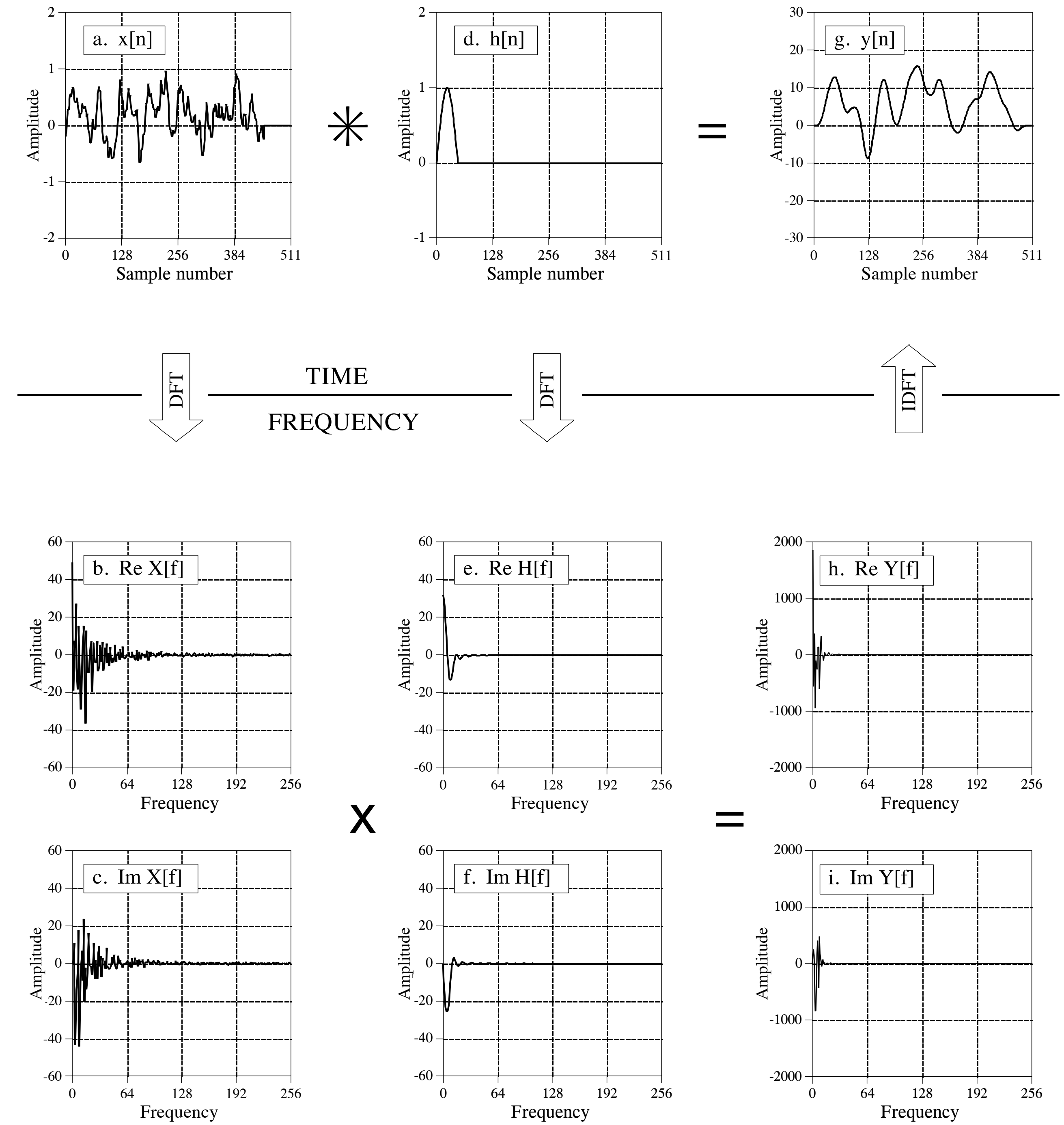
Linearity of DFT





Another view on convolution

- Convolution in the time domain is equivalent to multiplication in the Fourier domain.
- y = conv(h,x) is equal to y = $\mathbf{F^{-1}[}F(h)*F(x)\mathbf{]}$
- thanks to fft (fast fourier transform) convolution can be realized much faster.



Another view on convolution

- This is important because this means that the IRF decides which frequencies will be present in the output.