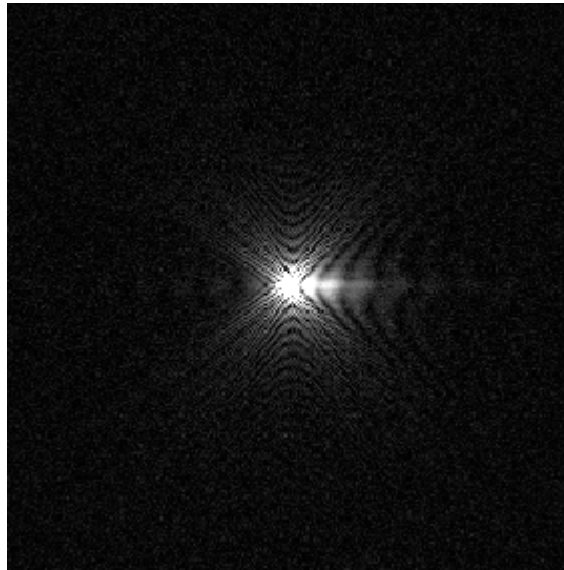


Advanced Numerical Methods in Neuroscience

k-Space!



Jürgen Finsterbusch and Selim Onat

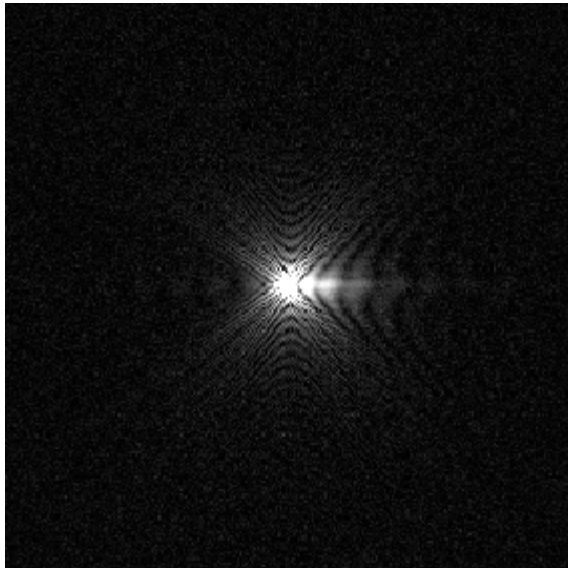
Department of Systems Neuroscience

University Medical Center Hamburg-Eppendorf

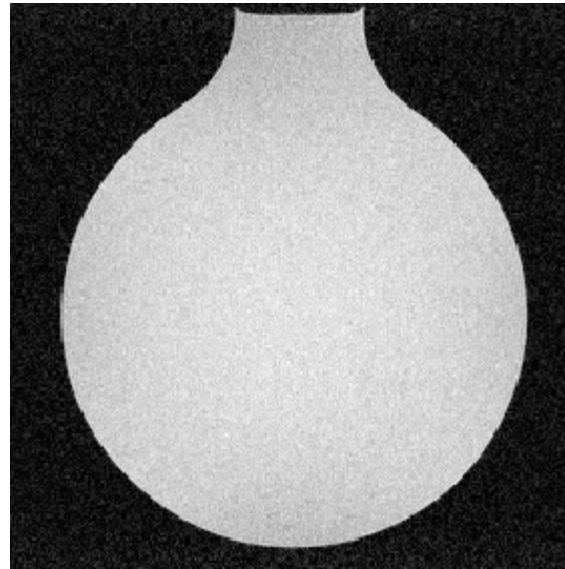
Motivation and Short Version

- MR imaging data are acquired in k-space
- k-space is the space of spatial frequencies, i.e. the Fourier space of the image space
- from the acquired (k-space) data images can be calculated with a Fourier transformation

MR data



Fourier Transformation (Image)



- some MR imaging properties and artifacts and problems / limitations of image acquisition techniques can be explained / understood when considering k-space

Kernspinnresonanz

Zutaten ...

- Atome mit einem Kernspin (magnetischen Moment):
Wasserstoff (^1H)



- starker Magnet

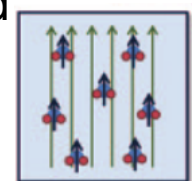
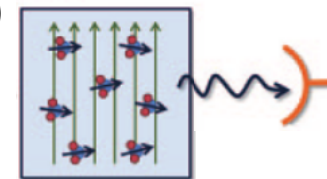
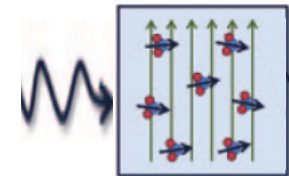
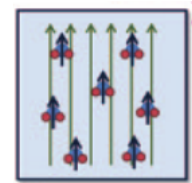
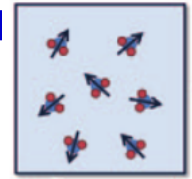


- Radiowellenantenne ("Spule")
mit Sender und Empfänger



Wie funktioniert's?

- Magnetisierung (im Magnetfeld)
 - magnetische Momente richten sich entlang des Magnetfeldes aus (ähnlich wie Kompassnadeln)
 - Wasser / wasserhaltiges Gewebe wird magnetisch
- Radiowellen: Anregung (“Hochfrequenz-/HF-Anregung”)
 - ein Radiowellenpuls “kippt” die Magnetisierung um einen bestimmten Winkel (“Kippwinkel”), z.B. um 90°
- Präzession und Induktion
 - darauf beginnt sie zu rotieren („Präzession“)
 - und induziert dabei eine Spannung in der Spule (ähnlich Dynamo)
 - diese Spannung wird gemessen und ist das MR-Signal („Echo“)
- Relaxation
 - Magnetisierung kehrt exponentiell zurück in den Ausgangszustand
 - Zeitkonstanten: T_1 (Wiederaufbau) und T_2^* (Signalabfall)



Warum “Resonanz”?

- Zusammenhang zwischen Feldstärke und Frequenz (Anregung, Signal):

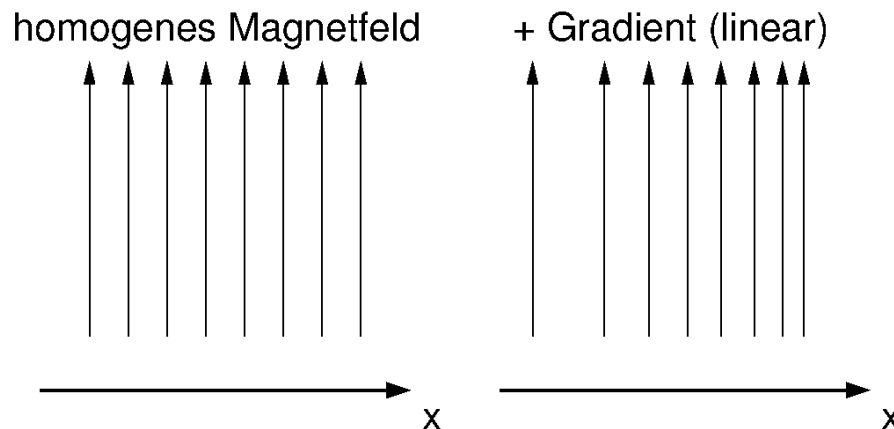
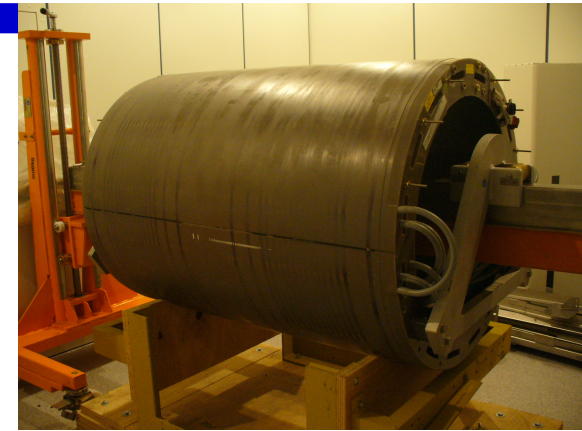
„Resonanzbedingung“

$$\omega = \gamma \cdot B$$

- ω : Frequenz
 - γ : Konstante des Atomkerns („gyromagnetisches Verhältnis“)
 - B: Stärke des Magnetfeldes
- Anregung
 - Frequenz der Radiowellen muss Resonanzbedingung erfüllen
 - Echo / Signal
 - Frequenz gegeben durch Resonanzbedingung
 - Beispiel:
 - Feldstärke 3T: MR-Frequenz für Wasser(stoff) 123MHz

Ortskodierung: Gradientenfelder

- zusätzlich zum statischen Magnetfeld
- lineare Abhängigkeit des Magnetfeldes vom Ort, in beliebige Richtung
- Amplitude zeitlich veränderbar, kann schnell an/abgeschaltet werden
- entscheidend: Stärke des Magnetfeldes wird verändert, *nicht* die Richtung

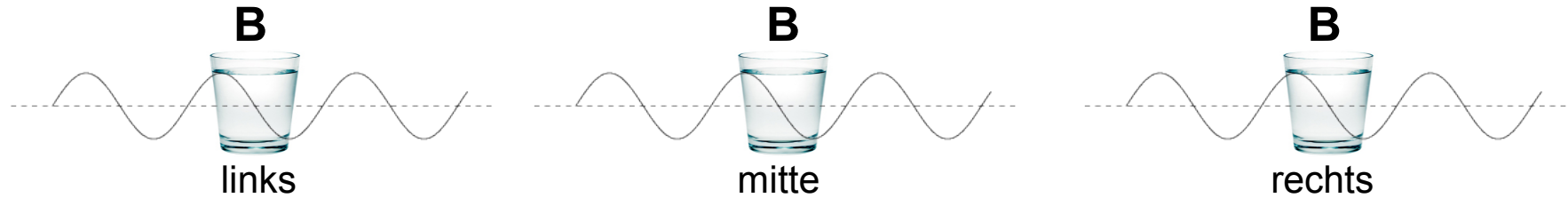


- typische Werte: $\pm 20\text{mT}$ über 0.4m , in 0.2ms
d.h. sehr klein im Vergleich zum statischen Feld
- mit Gradientenfeld: MR-Frequenz hängt linear vom Ort ab,
z.B. für x : $\omega = \gamma \cdot (B_0 + G_x \cdot x)$

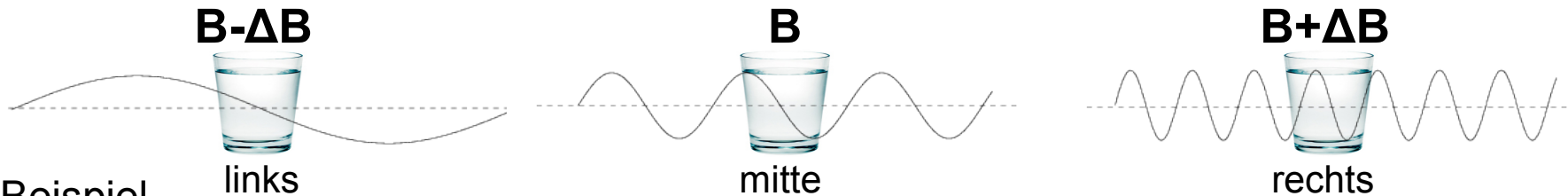
Ortskodierung: Frequenz

Signal (Echo)

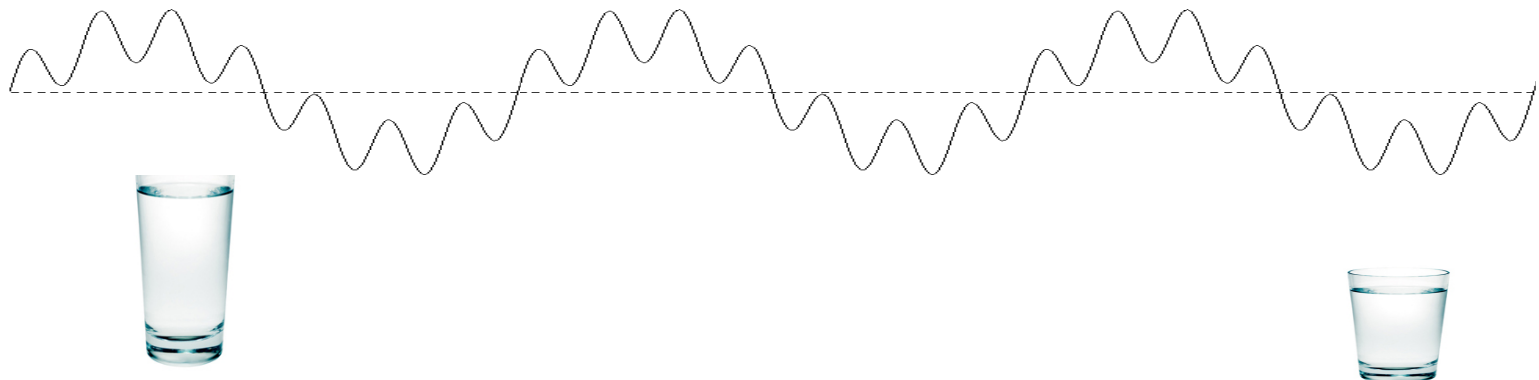
- ohne Gradientenfeld



- mit Gradientenfeld (von links nach rechts)



- Beispiel



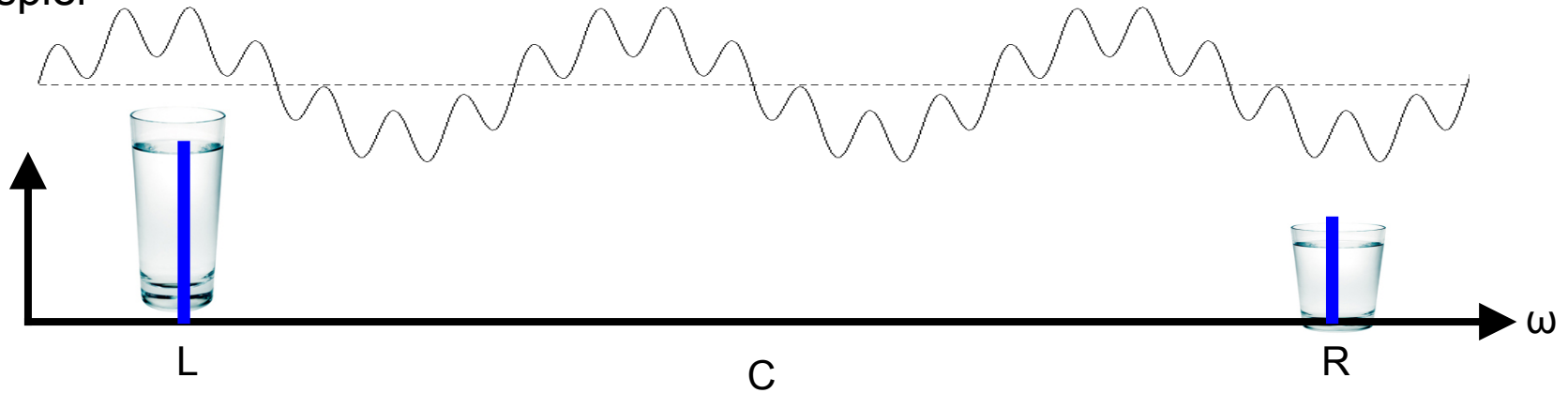
$$\omega = \gamma \cdot B$$

Ortskodierung: Frequenz

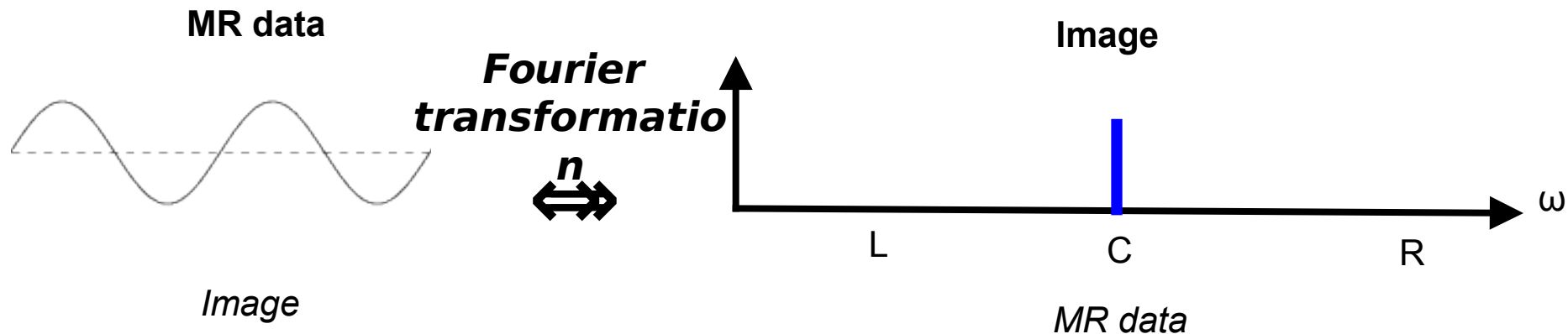
Etwas mathematischer

$$\omega = \gamma \cdot B$$

- Beispiel



- Frequenz-Analyse (Fourier-Transformation) liefert Signal-Verteilung entlang der Gradientenrichtung
- „Signal“-Raum und Bildraum sind Fourier-Paar



Etwas mathematischer ...

$$\omega = \gamma \cdot B$$

B-ΔB



- bisher: G konstant links
– $k = G \cdot t$

- allgemeiner, z.B. wenn G zeitlich *nicht* konstant

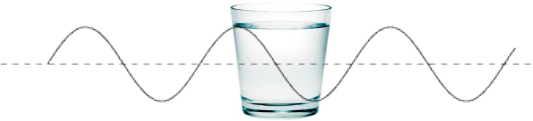
–

d.h. k ist das Zeitintegral der Gradientenamplitude

- für Schnittbild: Information in zwei Dimensionen erforderlich

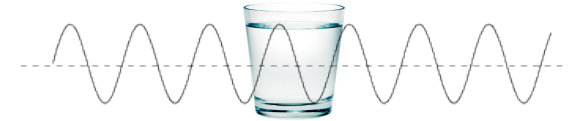
$$k(T) = \gamma \int_0^T G(t) dt$$

B

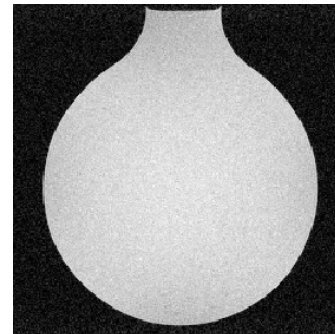
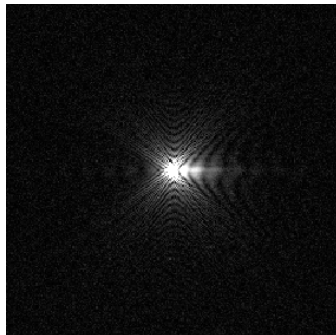


mitte

B+ΔB



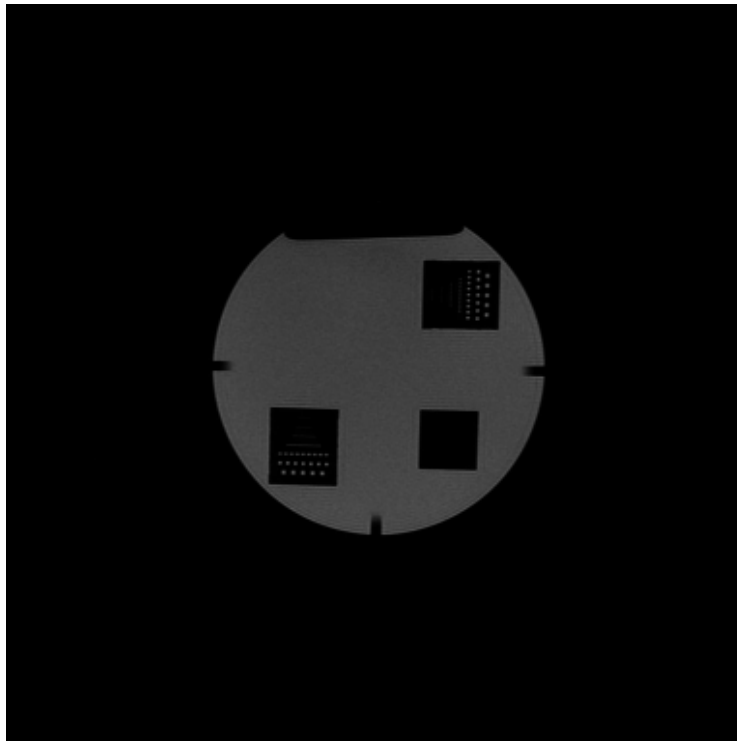
rechts



MR Image Reconstruction:

flash_1mm_384mm.dat

- read complex MR data
- make a Fourier transformation and get an MR image
- what do you see?
- what do you have to consider?



Exercise II

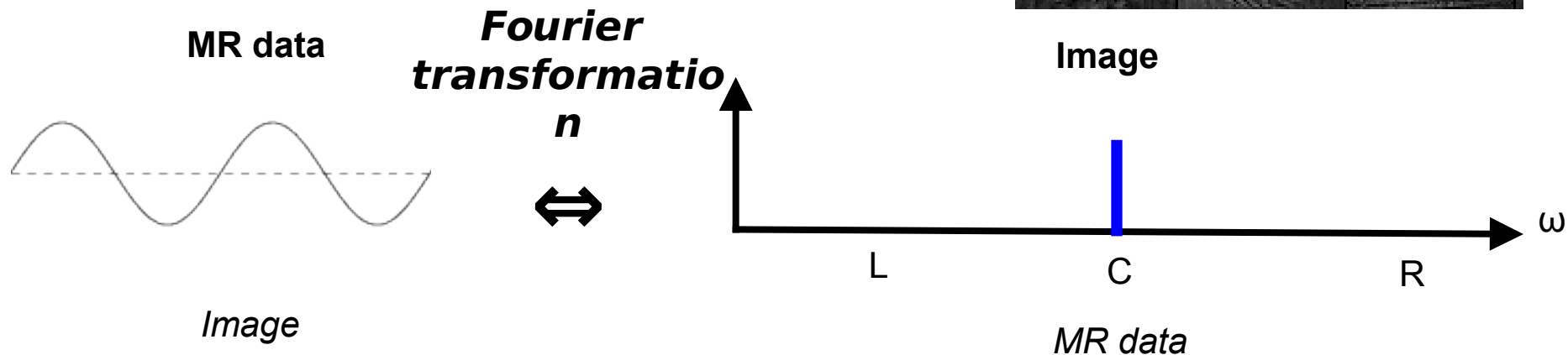
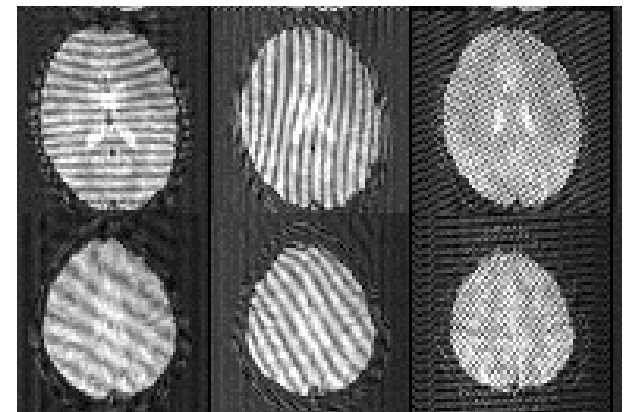
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Spike Artifact

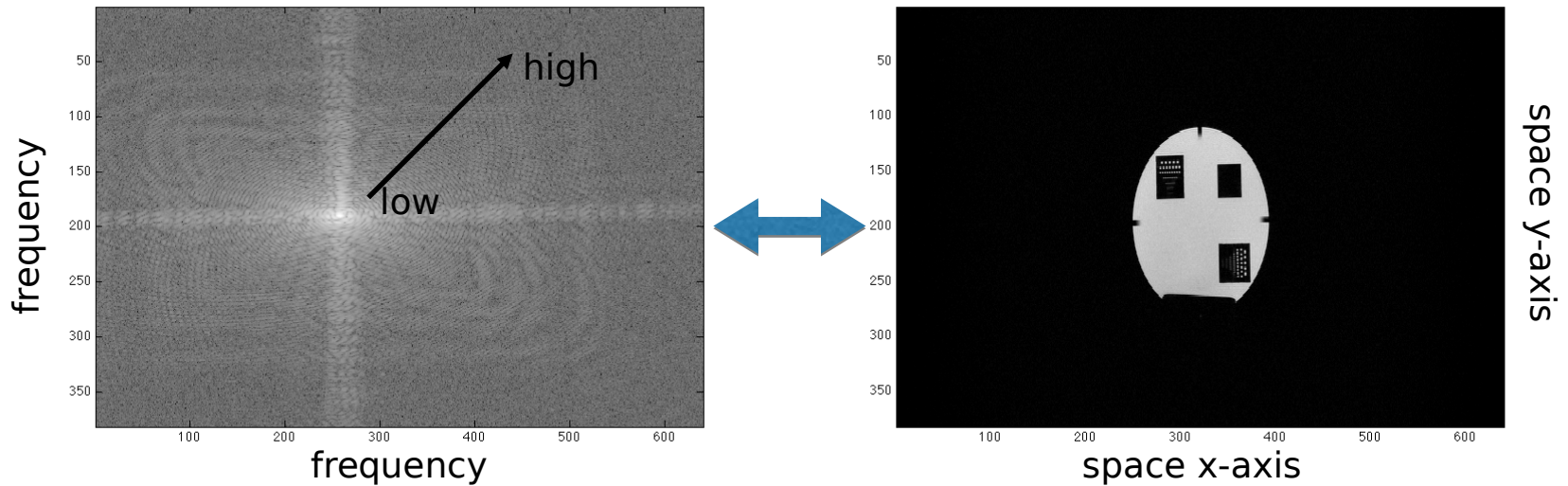
flash_1mm_384mm.dat

- read complex MR data
- add a high intensity peak somewhere in the matrix
- make a Fourier transformation and get the MR image
- vary intensity and position of peak
- what do you see?
- why?



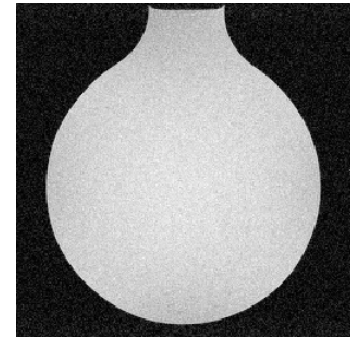
k-space \longleftrightarrow **world space**

- Data is acquired in the k-space

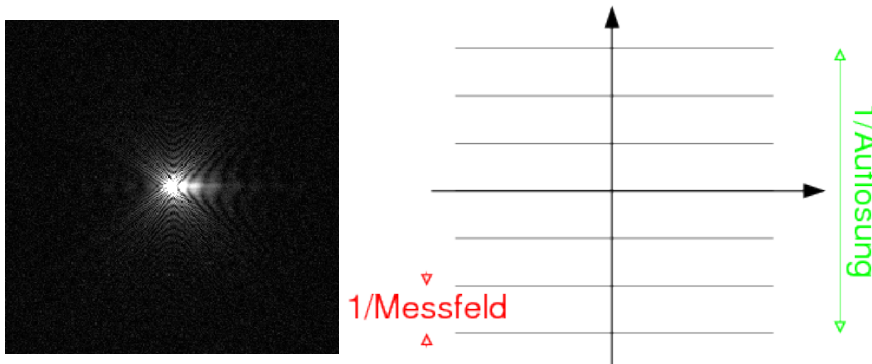


MR-Bildgebung

- Objektschnitt ist zweidimensional
- ideal: das Signal für alle Werte im k-Raum messen
- Problem
 - k-Raum unendlich groß
 - $k(T)$ beschreibt Linie im k-Raum (“Trajektorie”)
- meist: kartesische Abtastung, d.h. parallele Linien
 - abgedeckter Bereich definiert Auflösung
 - Zeilenabstand der Linien definiert Messfeld (field-of-view)



$$k(T) = \gamma \int_0^T G(t) dt$$



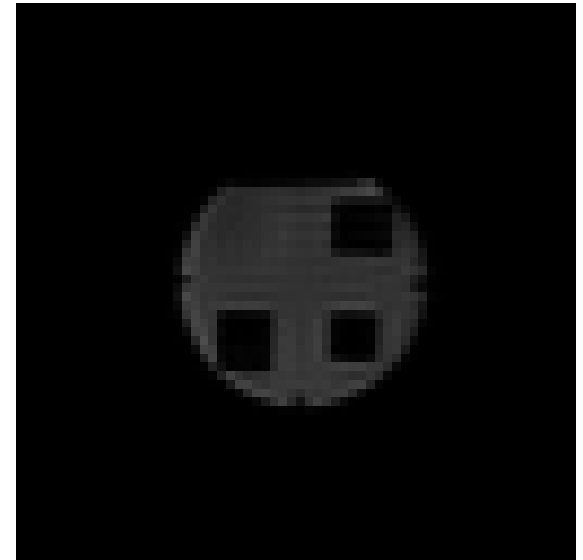
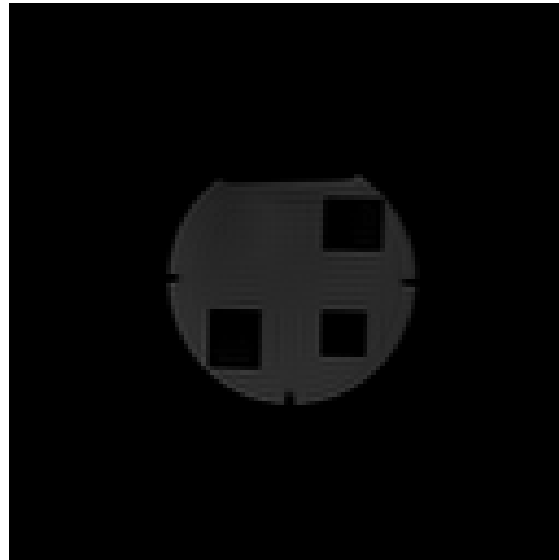
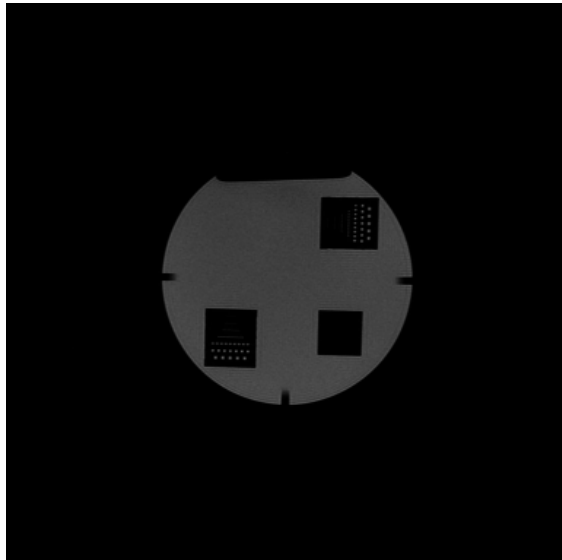
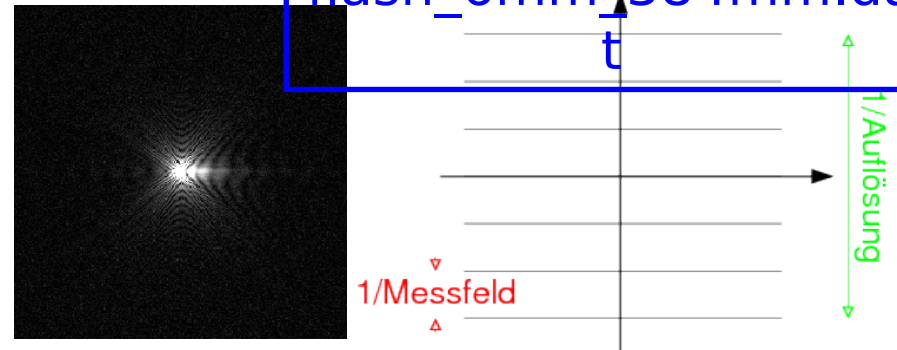
- Objekt \Leftrightarrow k-Raum \rightarrow MR-Daten \Leftrightarrow Bild

Exercise III

Resolution:

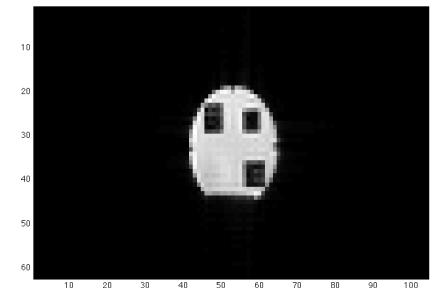
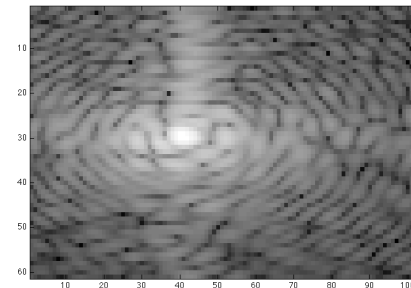
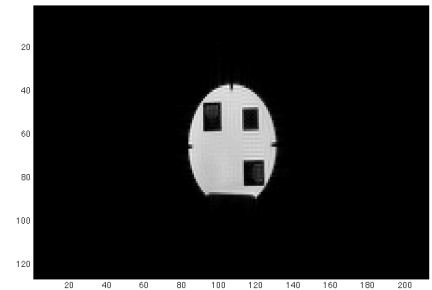
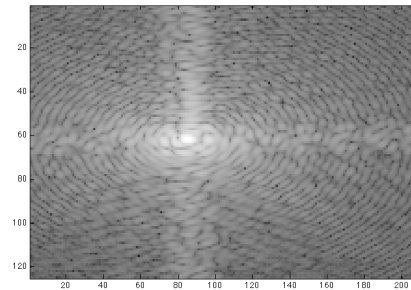
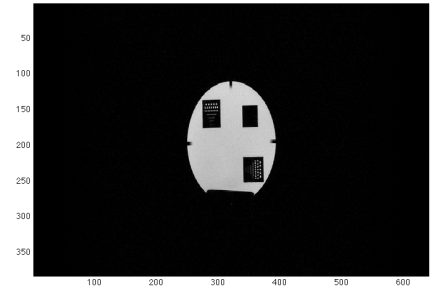
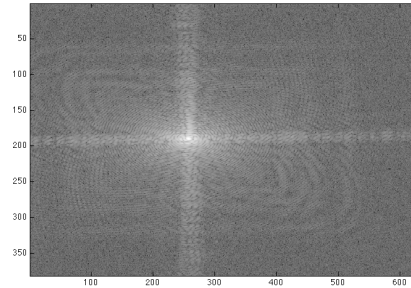
- read complex MR data
- make a Fourier transformation to get the MR images
- what is different in the images?
- what is different in the raw data?
- why are the images different?

flash_1mm_384mm.da
t
flash_3mm_384mm.da
t
flash_6mm_384mm.da
t





- same frequency range is acquired but with increasing steps.

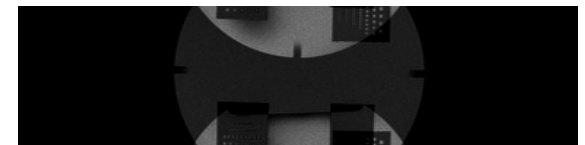
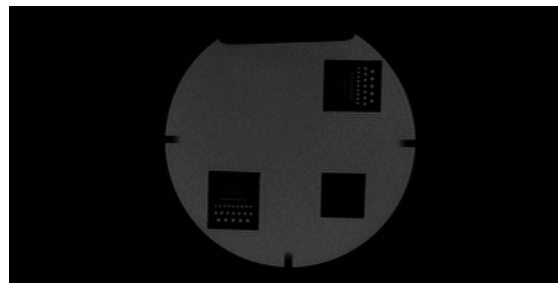
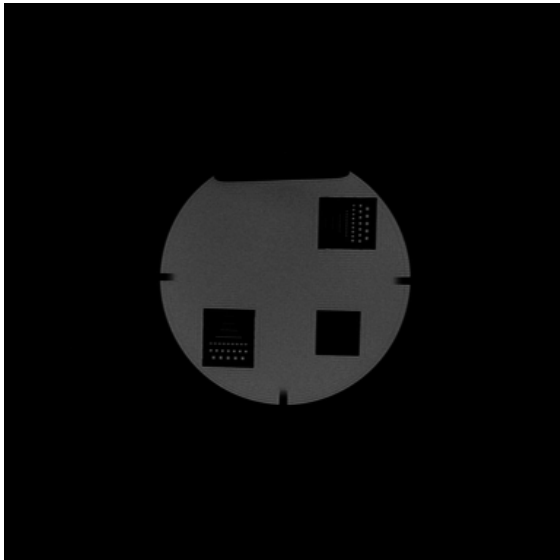
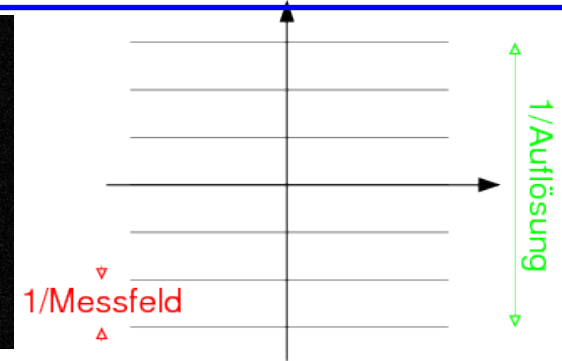
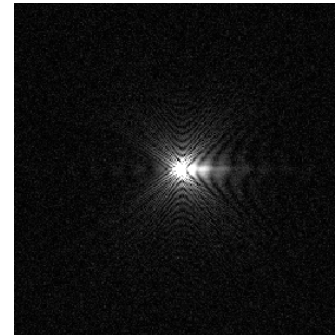


Exercise IV

Field-of-View

- read complex MR data
- make a Fourier transformation to get the MR images
- what is different in the images?
- what is different in the raw data?
- why is one of the images different?
- which data set would you prefer?

flash_1mm_384mm.dat
t
flash_1mm_192mm.dat
t
flash_1mm_96mm.dat

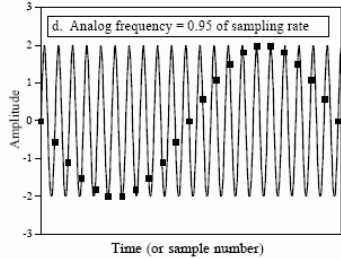


Effect of Field of View

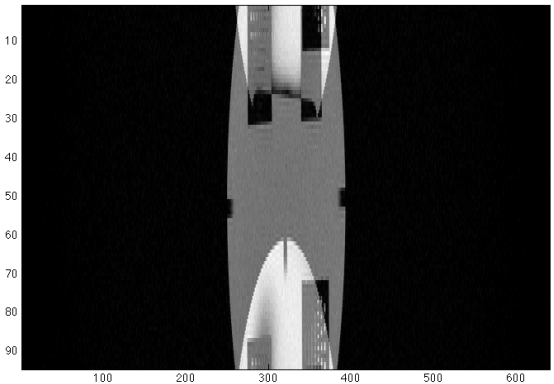
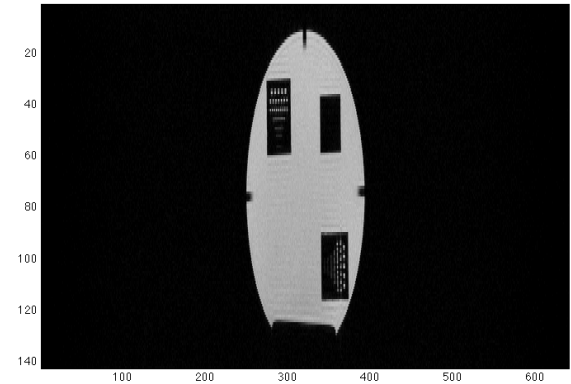
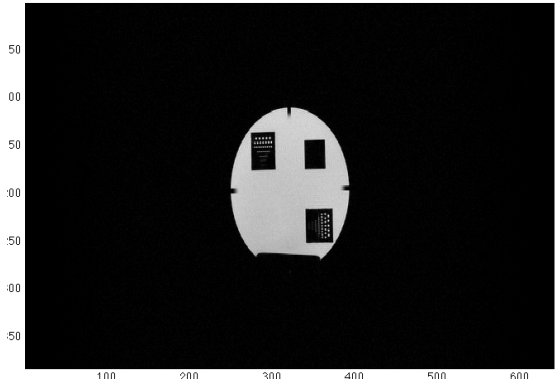
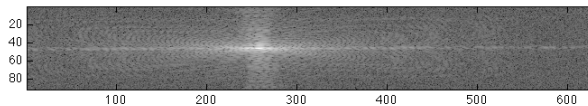
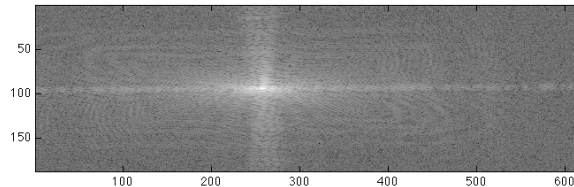
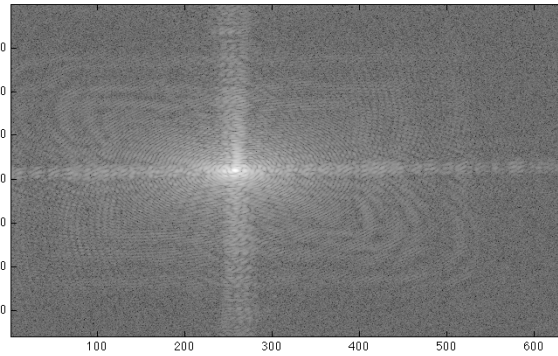


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Hamburg-Eppendorf

aliasing



Same frequency
range is covered but
at different sampling
densities / frequency
resolutions.

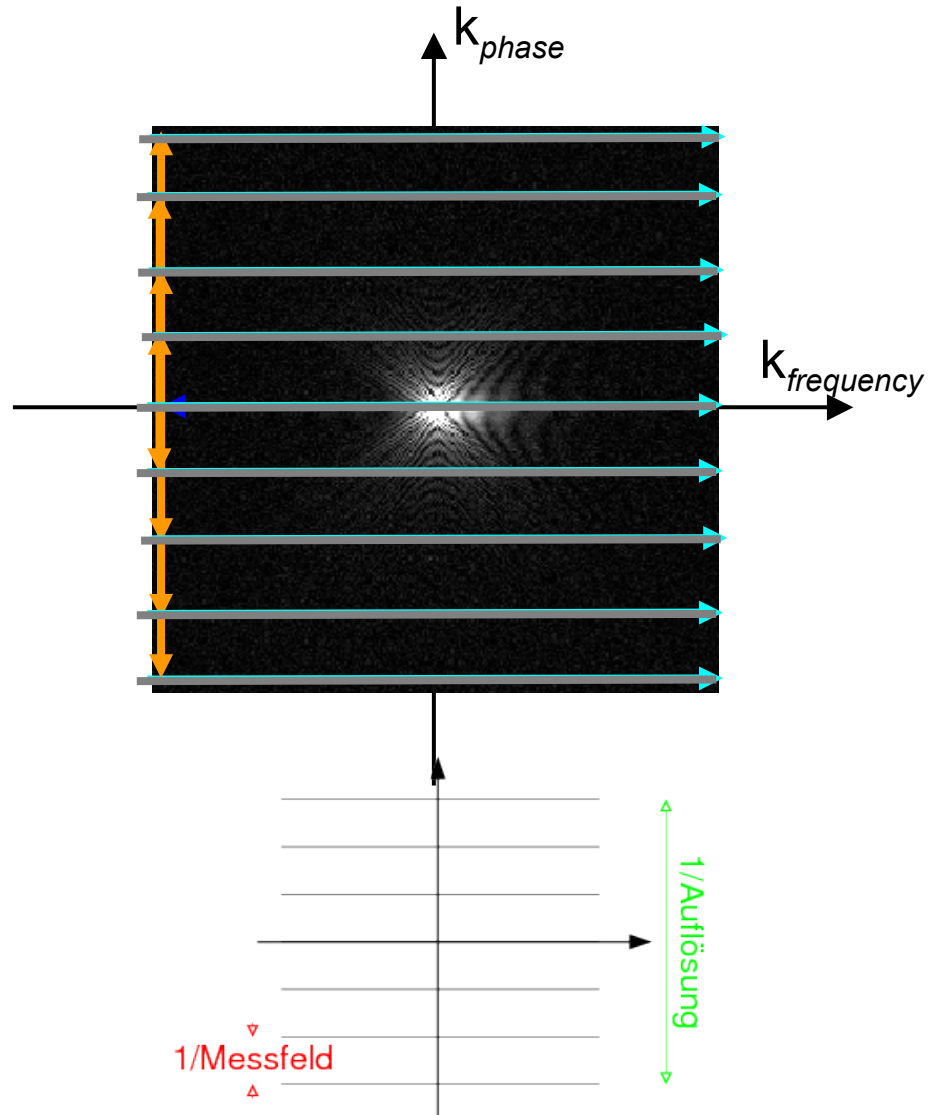
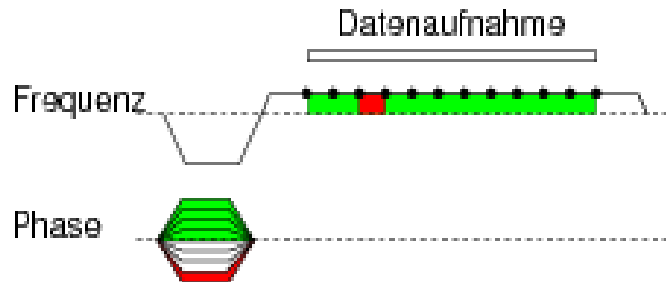
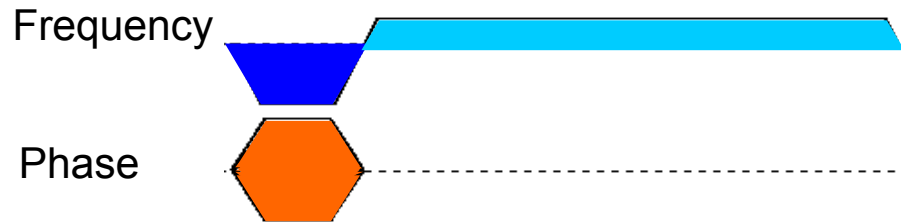


Pulse Sequence: FLASH

- pulse sequence $\Leftrightarrow k(T)$

$$k(T) = \gamma \int_0^T G(t) dt$$

– FLASH

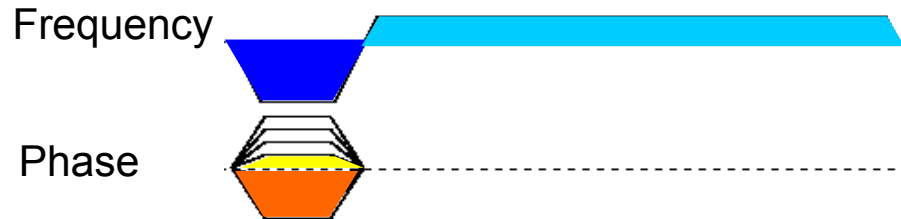


Pulse Sequence: Echo-Planar Imaging

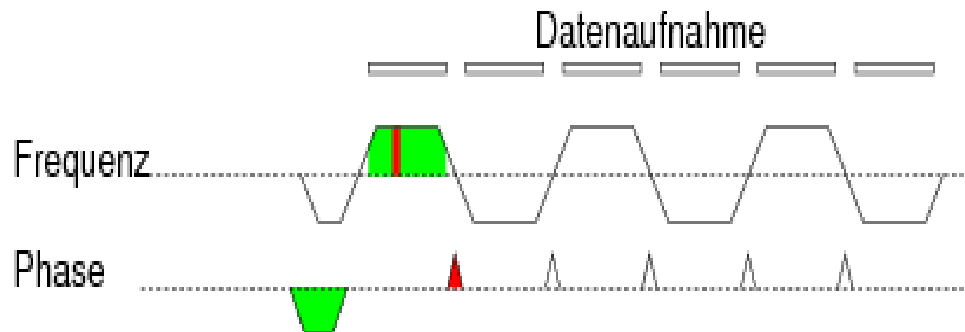
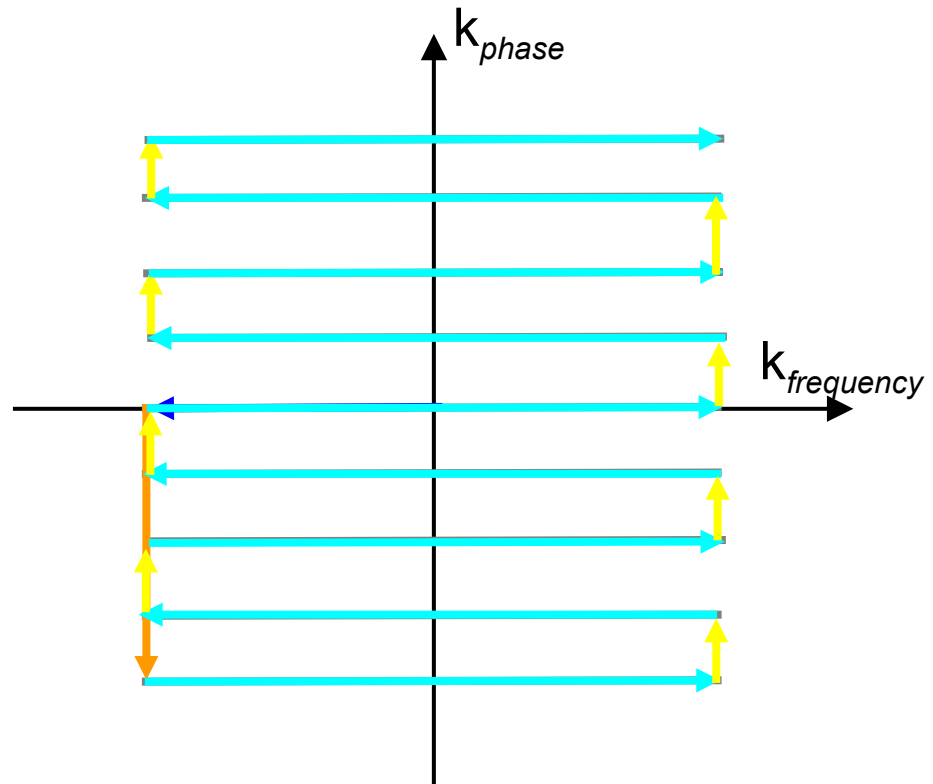
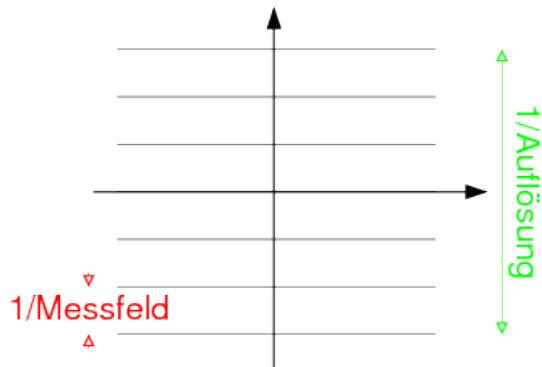
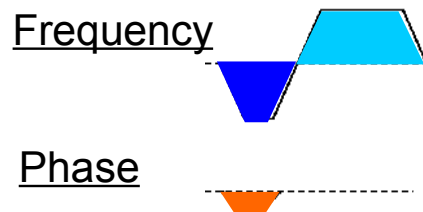


- pulse sequence $\Leftrightarrow k(T)$

– FLASH



– echo-planar imaging

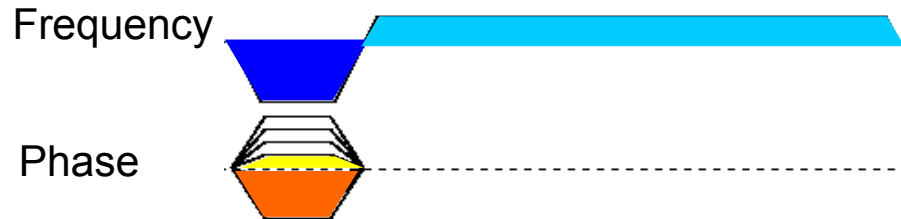


Pulse Sequence: Echo-Planar Imaging

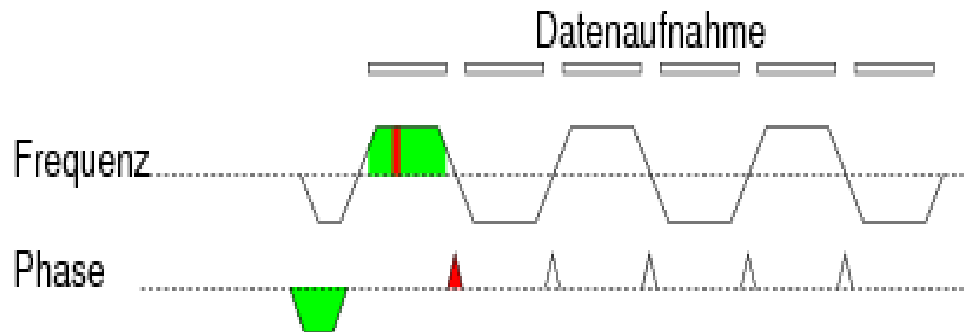
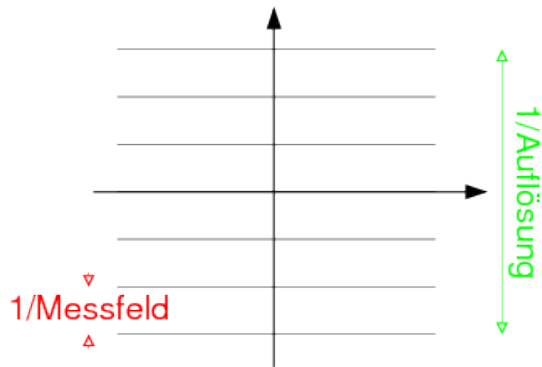
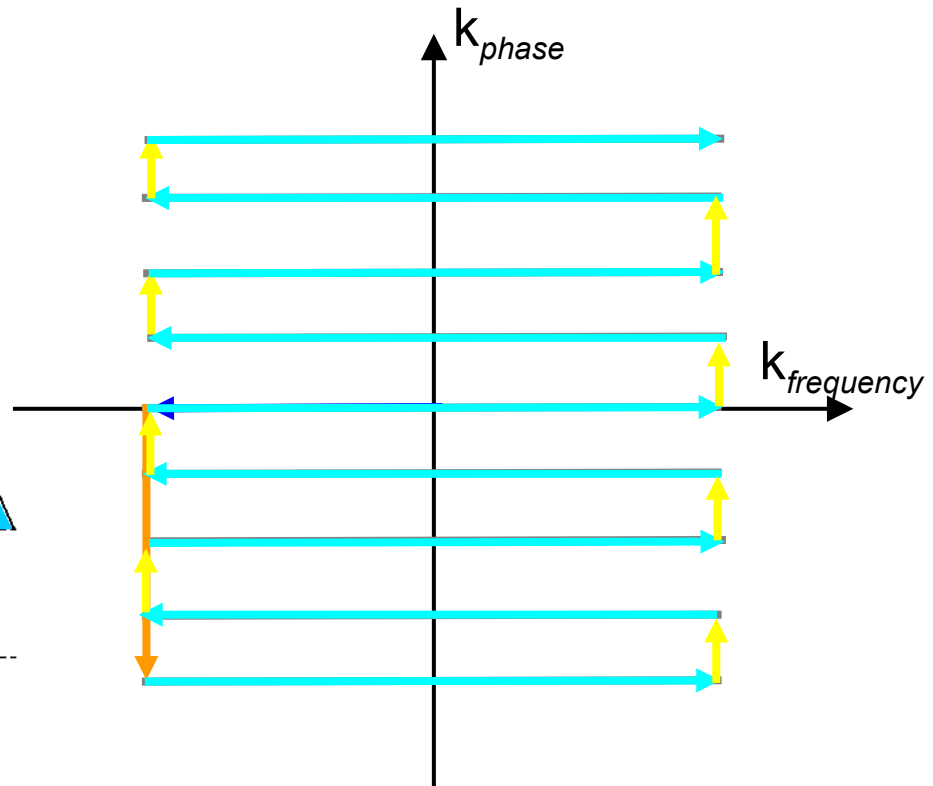
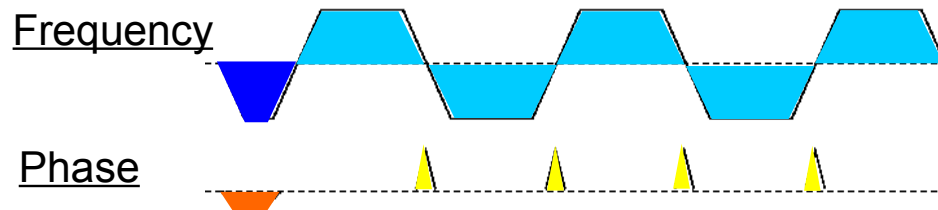


- pulse sequence $\Leftrightarrow k(T)$

– FLASH

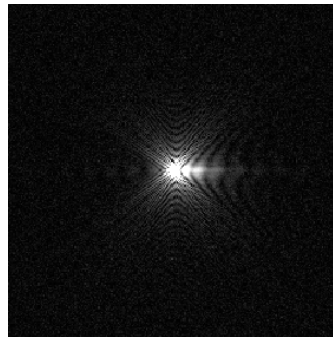


– echo-planar imaging



Properties

- all signals required for image acquired after a single RF excitation
- very fast
- sensitive to artifacts
 - odd lines: positive gradient pulse, even lines: negative gradients
 - different echo times for different lines:
relaxation, frequency offsets, geometric distortions

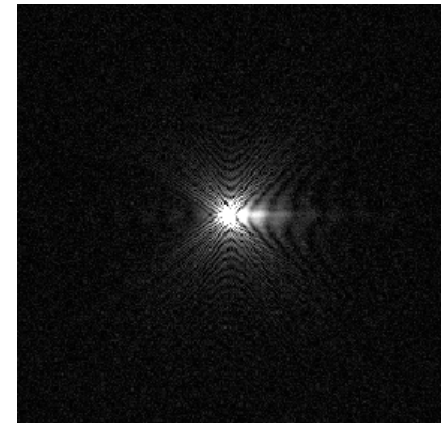
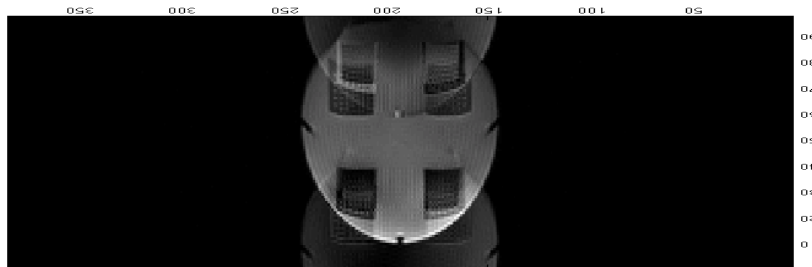
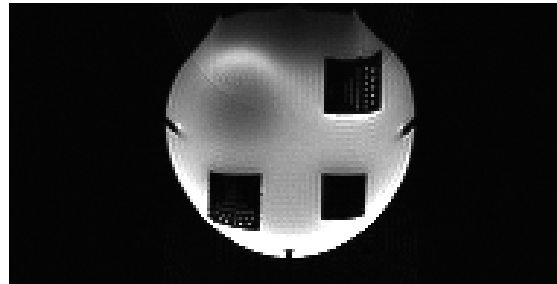
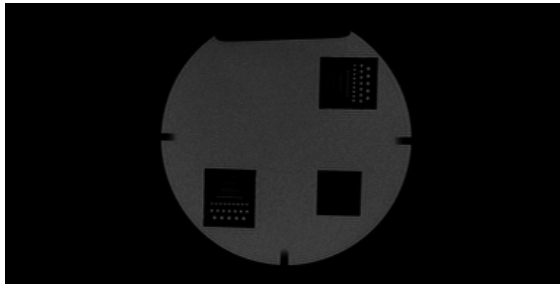


Exercise V

Echo-Planar Imaging

- read complex MR data
- make a Fourier transformation to get the MR images
- what is different between FLASH and EPI images?

flash_1mm_192mm.dat
t epi-se_2mm.dat

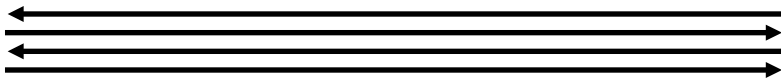
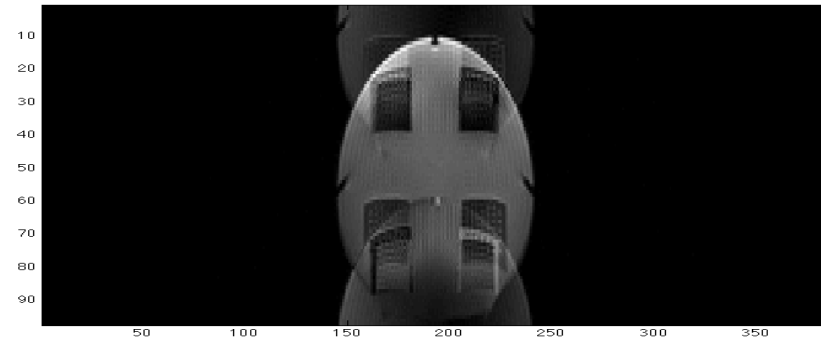
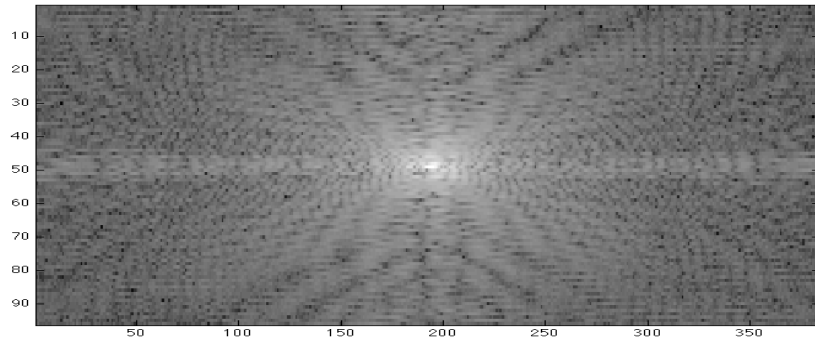


Source of Ghosting



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Hamburg-Eppendorf

- odd vs. even lines



Exercise VI

N/2 ghosting

- read complex MR data
- make a Fourier transformation to get the MR images
- add a phase offset to every 2nd line in the FLASH data
- vary the phase offset
- what happens to the images?
- what looks EPI-like?

flash_1mm_192mm.dat
t epi-se_2mm.dat

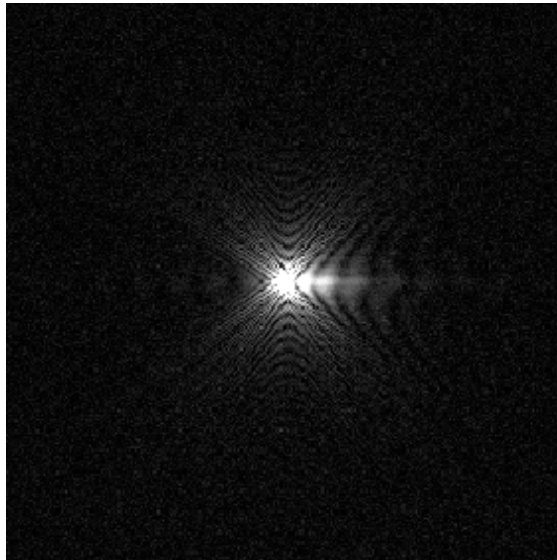
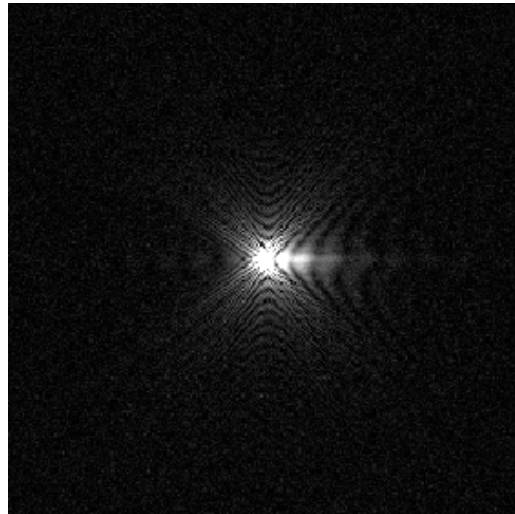


Image Blurring / Resolution

- read complex MR data
- make a Fourier transformation to get the MR images
- multiply MR data with exponential decay along line direction
- vary decay factor or exponential decay
- what happens to the images?
- why?

flash_1mm_192mm.d
t



Point Spread Function I

-

- make an empty array
- add a point somewhere
- make a Fourier transformation to get in „MR data“ in k-space
- multiply „MR data“ with exponential decay along line direction
- vary decay factor or exponential decay
- make a Fourier transformation of the modified „MR data“ to get „MR images“
- what do you see?
- why?

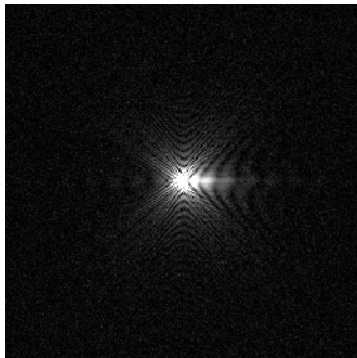
Point Spread Function II

-

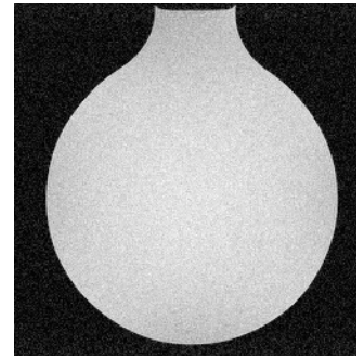
- make an empty array
- add somewhere two neighbouring points
- make a Fourier transformation to get in „MR data“ in k-space
- multiply „MR data“ with exponential decay along line direction
- vary decay factor or exponential decay
- make a Fourier transformation of the modified „MR data“ to get „MR images“
- what do you see?

- MR imaging data are acquired in k-space, the Fourier space of the image space
- MR data need to be Fourier transformed to get MR images

MR data



Fourier Transformation (Image)



- usually parallel lines are acquired, in multiple shots or a single shot
- MR data are only a subset of the objects k-space representation defining the resolution (area) and the field-of-view (sampling density)
- MR data may be disturbed / corrupted yielding artifacts, in particular for single-shot sequences
 - spikes
 - N/2 ghosting
 - image blurring
 - frequency offset shifts
 - geometric distortions
 - ...



Done.

Thanks!