Advanced Numerical Methods In Neuroscience
part I:
Diving into the Convolution and Fourier Transform

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Plan

- Signal Recording Basics
- Linear Systems
  - Concept of linearity.
- Convolution and Deconvolution
  - Concept
  - Properties
  - Practical
- Fourier Transform
  - Concept
  - Windowing
  - Connection to convolution
  - time-frequency analysis?

What am I supposed to bring home from this lecture?

- What is a linear system?
- How is convolution and Fourier transform related to each other?
- What should I pay attention when I stimulate a system and record its responses ?

Our Approach

- We are not physicists. We would like to develop a practical (not necessarily formal mathematical) understanding of basic signal processing methods.
- Digital Signal Processing tools are transcendental.
    - Same for neuroscience, psychology, physics, seismology and astronomy.
    - If you have already used Photoshop, you already did digital signal processing.
- You don’t need to formally study these, self-teaching and a lot of discussions with others is also another way of learning.
- Few basics you learn here will help you to understand better what you are doing.
- Don’t assume a crystal clear structure, I hope to have a creative chaos during these two days.

Basics of Signal Acquisition

- **Signal Acquisition:** Recording a signal in an accurate manner…
  - Capture the vibrations from the source, transform them into an electric signal and assign a number to each possible amplitude.

source digital

- Basic terms:
  - Dynamic Range
  - Signal to Noise Ratio
  - Sampling Rate.
  - Nyquist Frequency.
  - Aliasing

Dynamic Range

- The ratio between smallest and largest possible values.
- Decibel, or just Bel… it is a measure of how strong is the signal compared to a baseline.

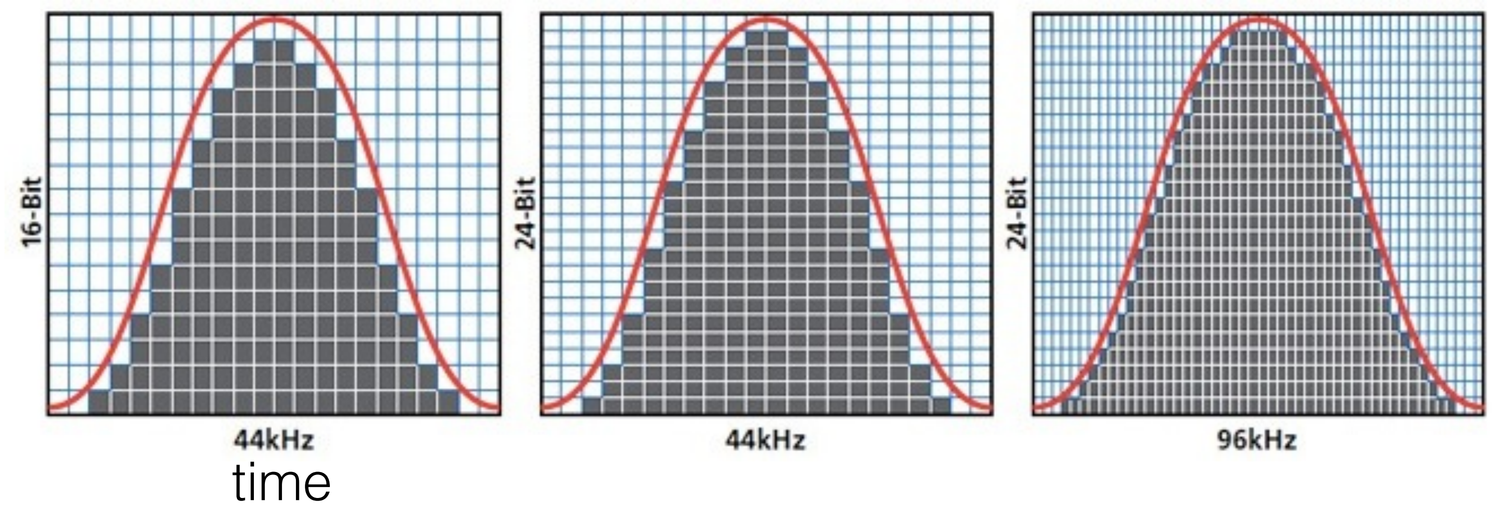
  dB = 10 x log10 (var_Signal / var_Baseline)

- We take the log because generally we talk about very large numbers.
- So a system with a 20 dB, how big is the signal from baseline?

- **Examples:**
  - **Visual system: 90 dB**
  - **Speech: 50-70 dB**
  - **Jet engine: 140 dB**

Digitalization: Sampling Rate

- Signals in nature are continuous, but our computers operate discretely.
- ADC (analog to digital conversion) or digitalization
- 2 parameters:
  - Sampling Rate
  - Bitdepth
- What is the sampling rate of CD recording?
  - why?

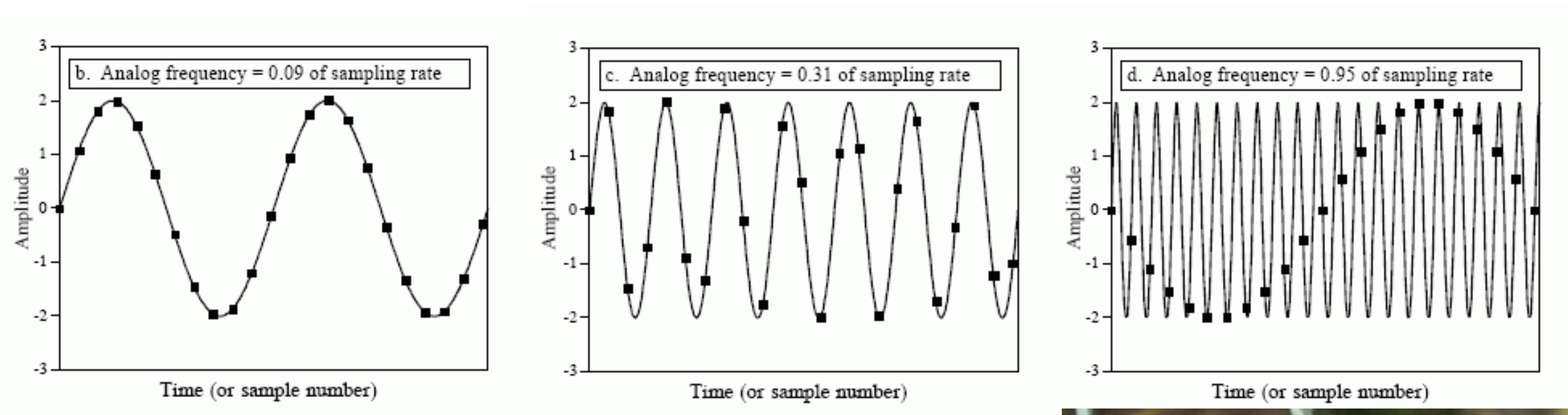


Nyquist Frequency

- So how do we connect sampling rate with the maximum recordable frequency in the data?
- Nyquist Frequency is the highest frequency one can record with given sampling rate?
- For example what is the ideal recording frequency for EEG, SCR, LFP?

Aliasing

- Aliasing happens when you record too slow or the data contains faster frequencies than your sampling rate.
- You don't want that! **We generally use low-pass filters.**
- Always record with a sampling rate that is high enough and low pass the signal that you are interested in

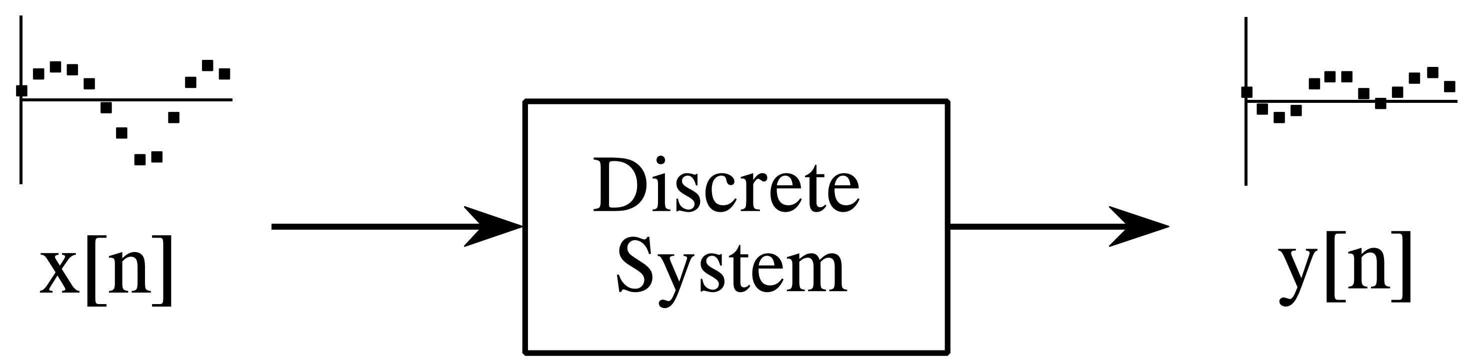


Linear Systems Theory

- We most of the time analyze a system assuming it is a linear system.
    - Neuronal Firing —> fMRI —> BOLD Response
    - Stimulus Contrast —> V1 neuron —> Spiking activity

- Linear system describes **a specific kind of input - output relationship.**

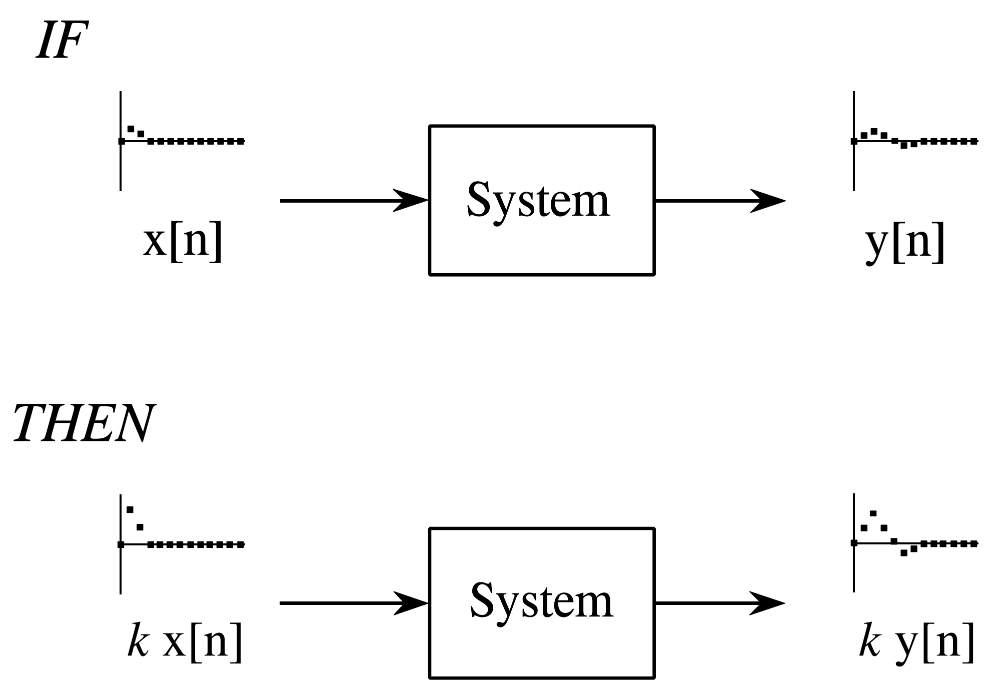
- y(t) = L[x(t)].



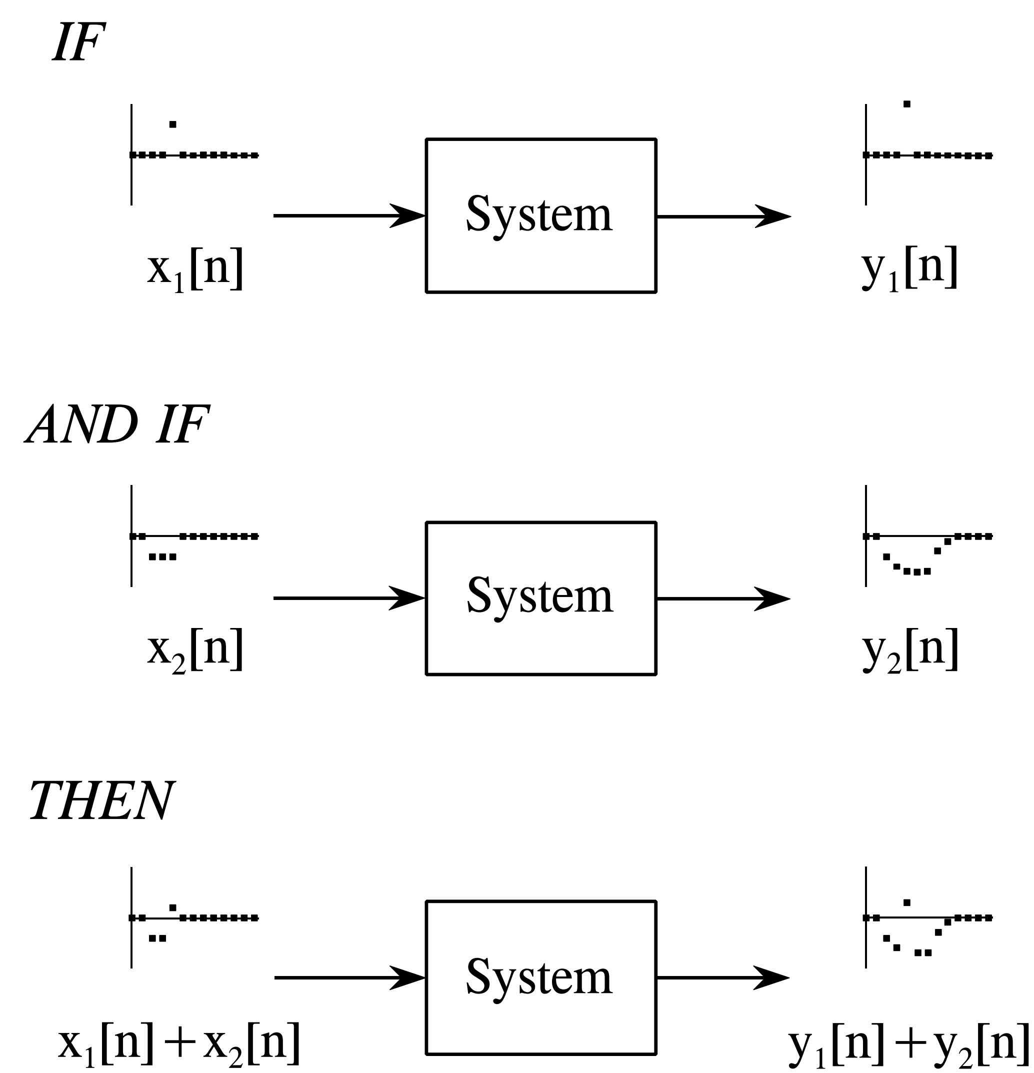
Properties of Linear Systems

- A system is called Linear if it satisfies these 3 requirements:
  - **Homogeneity**
    - if we double the input, output doubles as well. Output scales with input.
    - cy(t) = **L[**c x(t)**]** = c**L[**x(t)**]**
    - Doubling stimulus intensity, would double neuronal responses…
  - **Additivity**
    - if we add two input signals, the output is the sums of their respective outputs.
    - **L[**x(t)**]** + **L[**y(t)**]** = **L[** x(t)+y(t) **]**
    - Responses to a compound stimulus, could be predicted by a weighted sum.
    - No interaction.
  - **Shift Invariance**
    - if the input is delayed by **deltaT**, the output will also be delayed by the same interval
    - L[x(t + deltaT)] = y(t - deltaT)
    - Delaying the stimulus presentation doesn't change anything.

Properties of Linear Systems



Scaling inputs with a given constant k, scales the output the same way.

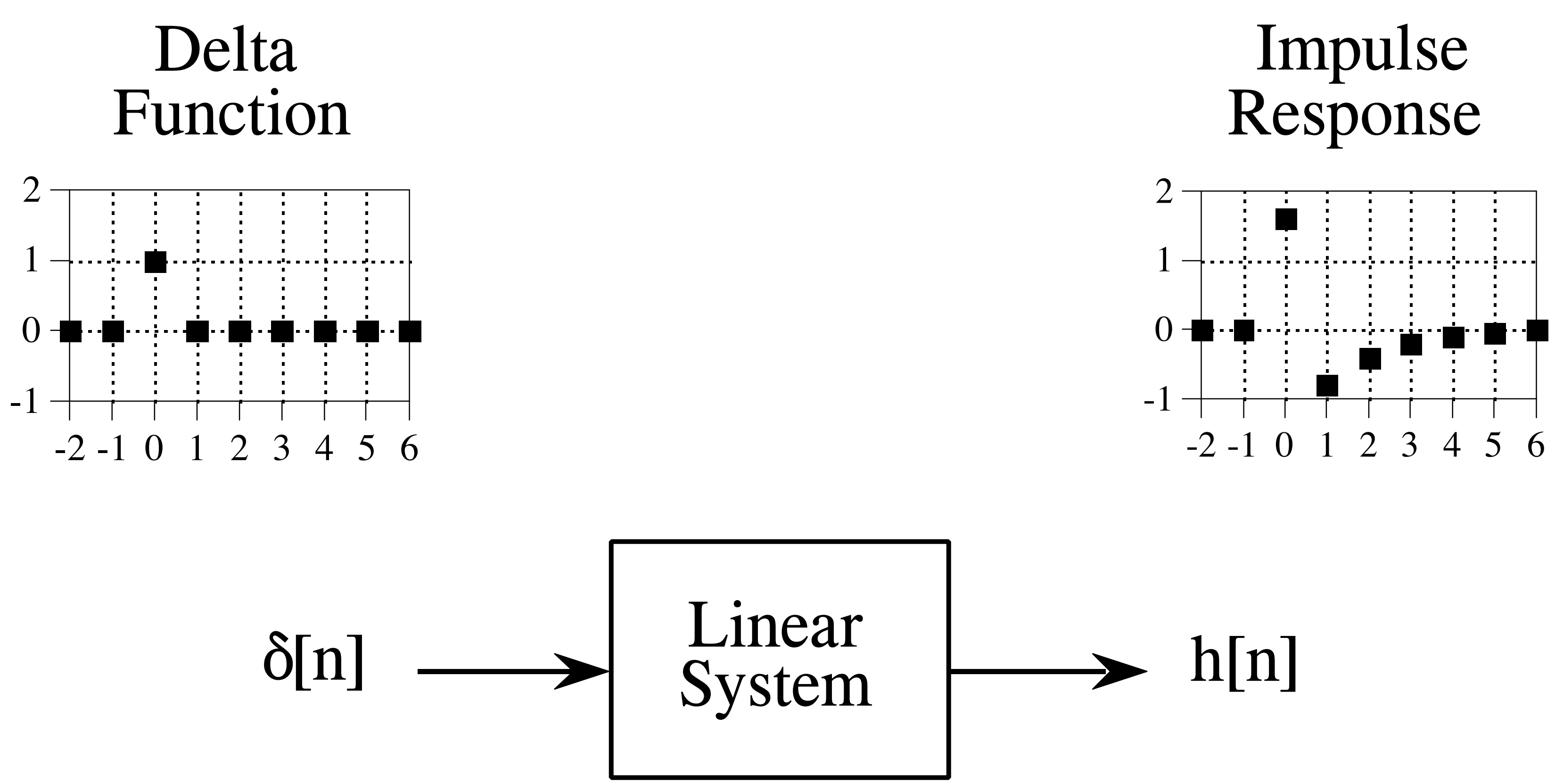


Response to sum equals the sums of individual responses.

There is no interaction between individual components

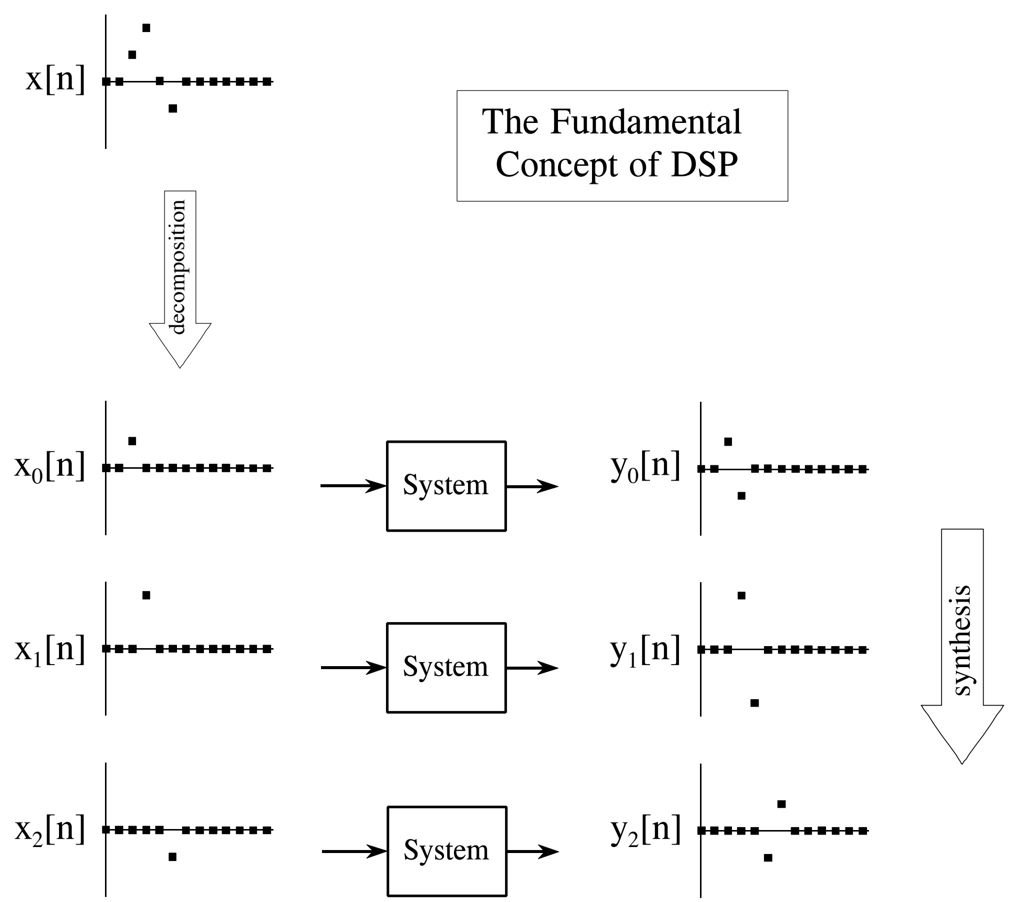
Impulse Response Function

- If the system is linear, it is fully described by its **impulse response function** (that is literary its response to an impulse).
- Because any input signal is a **superposition** of appropriately **shifted** and **scaled** impulses.
- Impulse is mathematically a delta function, ∂(t) (stick function)
    - h(t) = **L[**∂(t)**]**, where h(t) is the impulse responses function.



Impulse Response Function

- ∂(t) is important because by scaling and appropriately shifting the IR, we can model any input signal.
- Therefore the output will be a scaled and appropriately shifted version of the impulse response function specific to the system.



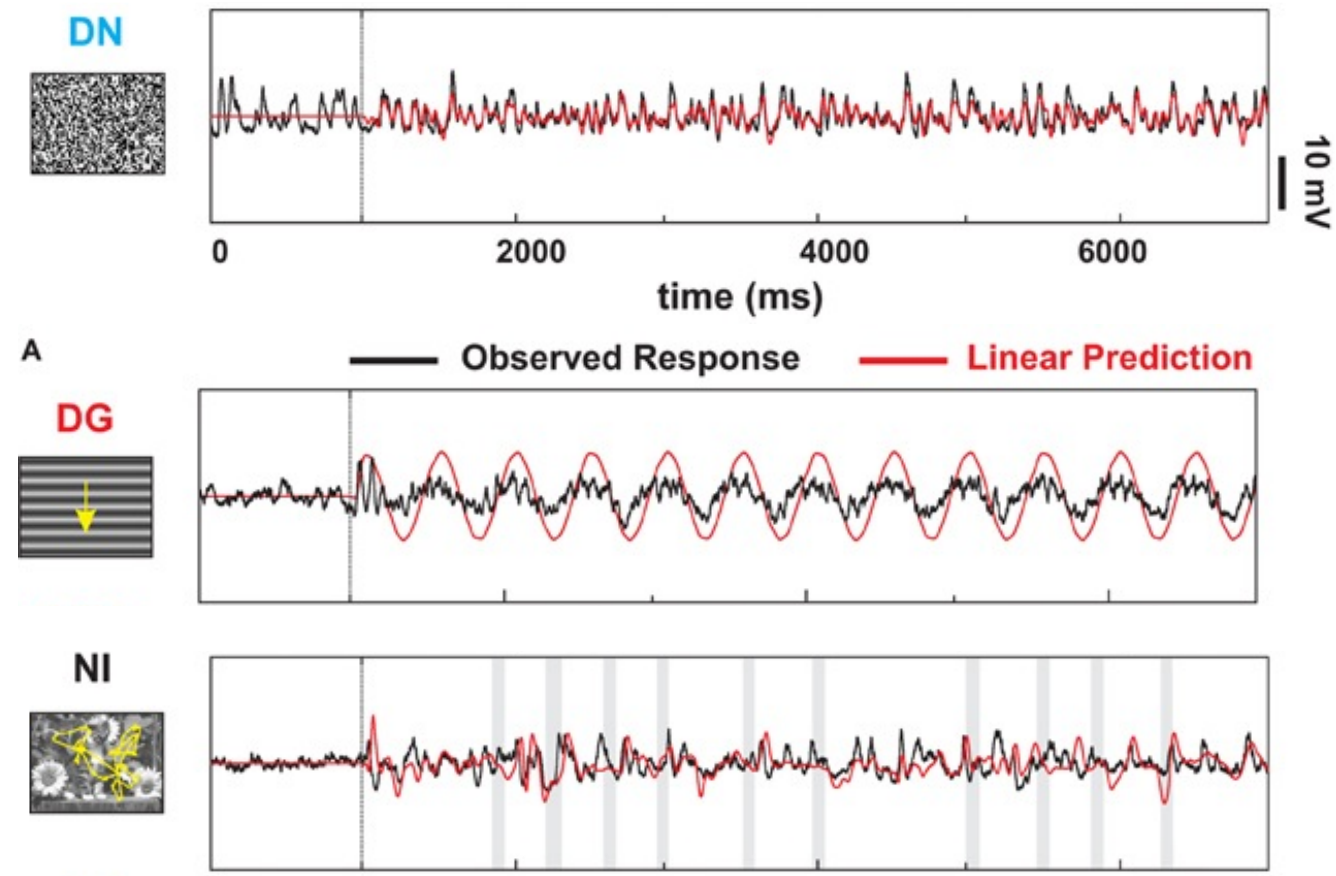
- IRF is also known as convolution kernel, filter kernel, point spread function etc.

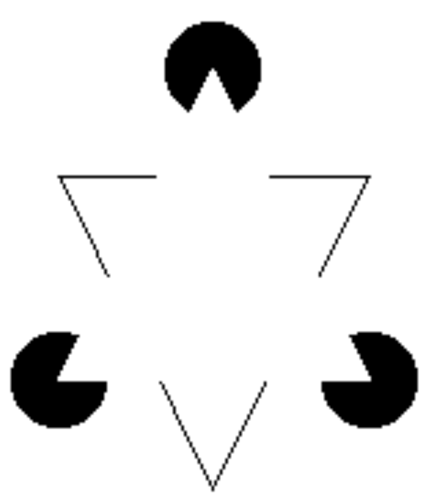
Why is this important?

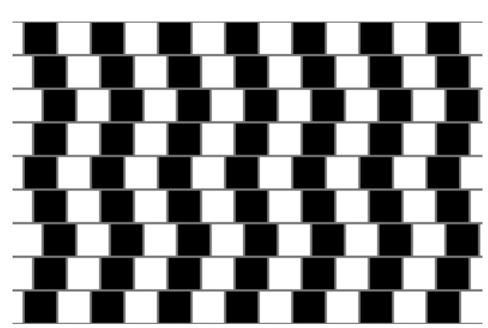
- We cannot simply test all possible combinations of input and study the system.
  - IR is the shortcut.
- So if we know the IRF, we can predict the behavior of the system to more complex situations.
- One can also analyze a nonlinear system as if it is a linear one to understand better its nonlinear behavior.
- Tools are the same whether you study seismology or fMRI psychology, neuroscience.

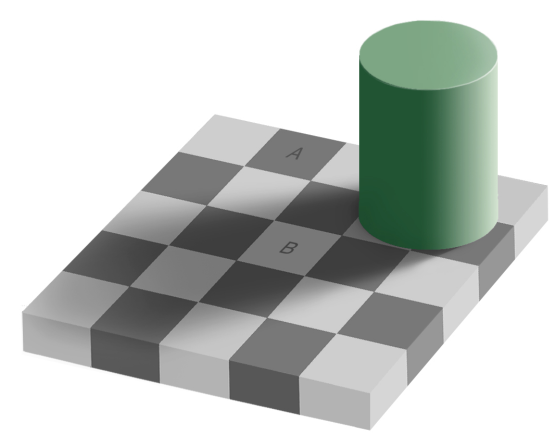
Examples of (non) linearity

- Responses of visual cortical neurons.
- Visual illusions.
- Adaptation to same input.



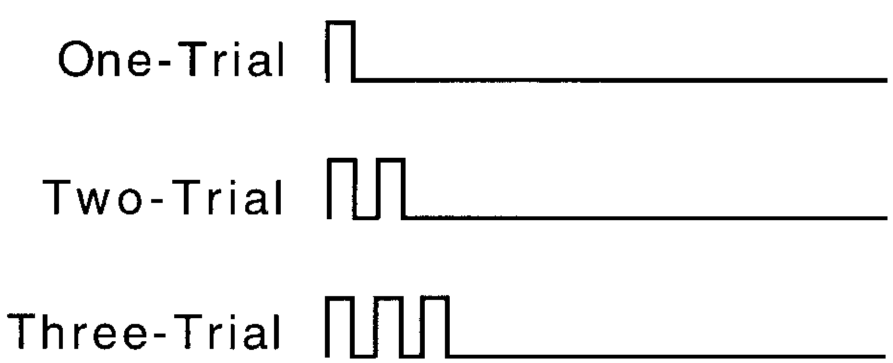


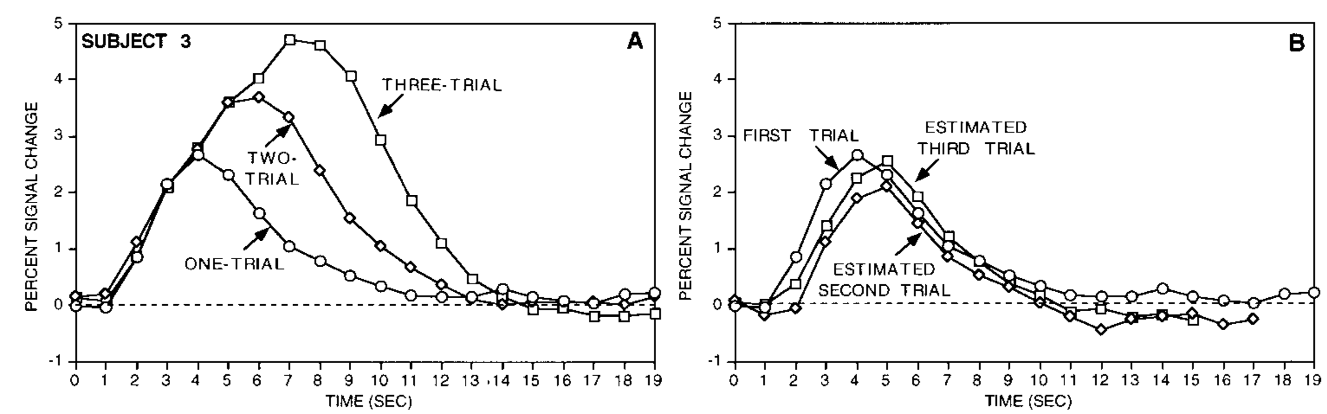




Linearity in fMRI

- Standard way of fMRI analysis assumes linearity with respect to BOLD vs. stimulus.
- BOLD responses add up linearly





- You have to be sure that we operate in the linear boundary conditions.
  - if two trials are too close in time, the linear analysis methods will not be applicable.

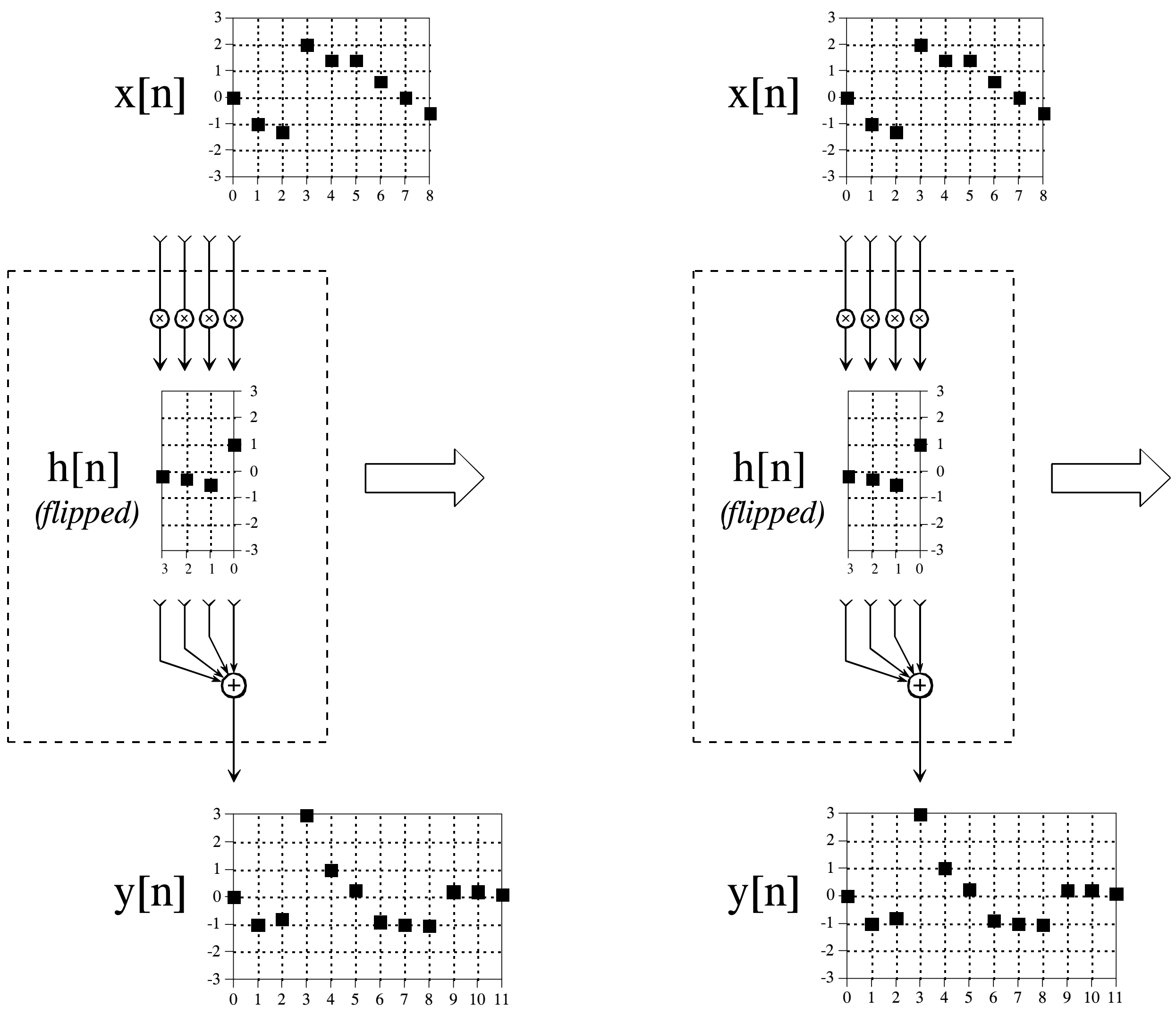
Convolution

- In linear systems, convolution is used to describe the relationship between three signals of interest: **the input signal, the impulse response, and the output signal.**

- Input -> output transformation of a linear system can be understood as a **convolution of the input signal with the impulse response** that is the characteristic of the system.

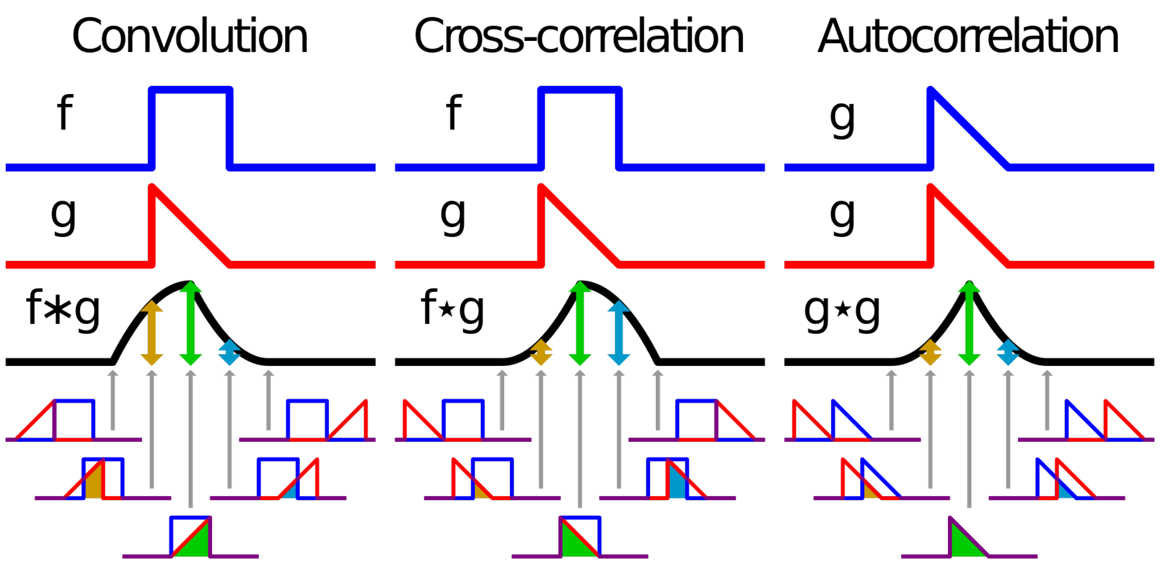
$$y[i] = \sum_{j=0}^{M-1} h[j]\, x[i-j]$$

Convolution step by step



a. Set to calculate y[0]

b. Set to calculate y[3]



Properties of Convolution

$$x * y = y * x \qquad \text{commutative}$$

$$(x * y) * z = x * (y * z) \qquad \text{associative}$$

$$(x * z) + (y * z) = (x + y) * z \qquad \text{distributive}$$

- Spatial smoothing of BOLD responses is a convolution, thus a linear operation. GLM analysis is also a linear operation. Therefore whether you smooth before or after a GLM, it doesn't really matter.

Deconvolution

- Inverse of convolution.
- This is interesting when you would like to get to the sources of a convolved signal.
  - neuronal activity from SCR responses.
  - Real-world from a blurred image.
- It is not easy. Because it involves a **division**, and division with small numbers can easily generate problems.
  - The difference between 1/0.001 or 1/0.002 is huge, if we are talking about neuronal signals.
  - Noise can badly affect deconvolution (practical session).

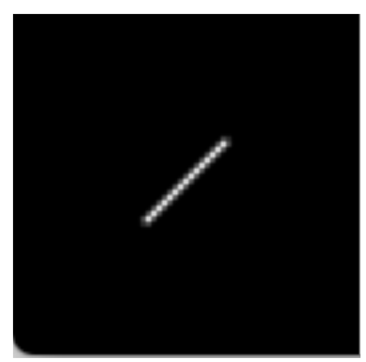
- The main drawback of the method is that $R(\omega)$ is usually a low-pass filter and therefore $1/R(\omega)$ is a high-pass filter which attains large values as the frequency increases. So the inverse of a low-pass filter is high-pass

Deconvolution example

- Deconvolution can only be used under certain conditions.

Bad photograph

Convolution kernel



Recovered Image