MILP Model for Single Depot VRP with Customer and Locker Assignments

1 Mathematical Model

1.1 Decision Variables

$$x_{i,j,v} \quad \text{Binary, 1 if vehicle } v \text{ travels from node } i \text{ to } j\text{, 0 otherwise} \quad \forall i, j \in N, v \in V, i \neq j \tag{1}$$

$$y_v \quad \text{Binary, 1 if vehicle } v \text{ is used, 0 otherwise} \quad \forall v \in V \tag{2}$$

$$home_i \quad \text{Binary, 1 if customer } i \text{ is served at home, 0 otherwise} \quad \forall i \in M \tag{3}$$

$$z_{i,lk} \quad \text{Binary, 1 if customer } i \text{ is assigned to locker } lk\text{, 0 otherwise} \quad \forall i \in M, lk \in L \tag{4}$$

$$w_{lk} \quad \text{Binary, 1 if locker } lk \text{ is used, 0 otherwise} \quad \forall lk \in L \tag{5}$$

$$d_{lk} \quad \text{Continuous, demand assigned to locker } lk \quad \forall lk \in L \tag{6}$$

$$u_{i,v} \quad \text{Continuous, load of vehicle } v \text{ at node } i \quad \forall i \in M \cup L, v \in V \tag{7}$$

$$\delta_{v,i} \quad \text{Binary, 1 if vehicle } v \text{ visits node } i\text{, 0 otherwise} \quad \forall i \in M \cup L, v \in V \tag{8}$$

$$\text{Additional variables for load constraints: } load\_aux_{v,i} \quad \forall v \in V, i \in L \tag{9}$$

1.2 Objective Function

Minimize the total cost, including routing and assignment costs:

$$\min \sum_{v \in V} \left[\text{fixed cost of } v \cdot y_v + \sum_{i \in N} \sum_{j \in N} c_{i,j} \cdot x_{i,j,v} \right] + \sum_{i \in M} \sum_{lk \in L} c_{i,lk} \cdot z_{i,lk} \tag{10}$$

1.3 Constraints

- Each customer is either served at home or assigned to a locker:

$$home_i + \sum_{lk \in L} z_{i,lk} = 1, \quad \forall i \in M \tag{11}$$

- If a customer cannot be assigned to a locker, they must receive home delivery:

$$\text{If no available locker: } home_i = 1, \quad \forall i \in M \tag{12}$$

- Locker demand constraints:

$$d_{lk} = \sum_{i \in M} demand_i \cdot z_{i,lk}, \quad \forall lk \in L \tag{13}$$

- If a locker is not used, its assigned demand must be zero:

$$d_{lk} = 0, \quad \text{if no customer is assigned to locker } lk \tag{14}$$

- Locker capacity constraints:

$$d_{lk} \leq capacity_{lk} \cdot w_{lk}, \quad \forall lk \in L \tag{15}$$

- Flow conservation:

$$\sum_{i \in N} x_{i,k,v} = \sum_{j \in N} x_{k,j,v}, \quad \forall k \in M \cup L, v \in V \tag{16}$$

- MTZ subtour elimination:

$$u_{i,v} + demand_j \leq u_{j,v} + C(1x_{i,j,v}), \quad \forall i, j \in M \cup L, v \in V, i \neq j \tag{17}$$

- Customers assigned home must be visited:

$$\sum_{v \in V} \delta_{v,i} = home_i, \quad \forall i \in M \tag{18}$$

- If a locker is used, it must be visited by at least one vehicle:

$$\sum_{i \in N} x_{i,lk,v} = w_{lk}, \quad \forall lk \in L, v \in V \tag{19}$$

- Vehicle capacity constraint:

$$\sum_{i \in M} demand_i \cdot \delta_{v,i} + \sum_{i \in L} load\_aux_{v,i} \leq capacity_v, \quad \forall v \in V \tag{20}$$

- Time limit constraint:

$$\sum_{i \in N} \sum_{j \in N} \frac{c_{i,j}}{\text{speed}} \cdot x_{i,j,v} \leq max\_duration_v, \quad \forall v \in V \tag{21}$$

- Forbidden group constraints:

$$\sum_{i \in G} \sum_{j \in G} x_{i,j,v} \leq 1, \quad \forall G \in \text{forbidden groups}, v \in V \tag{22}$$