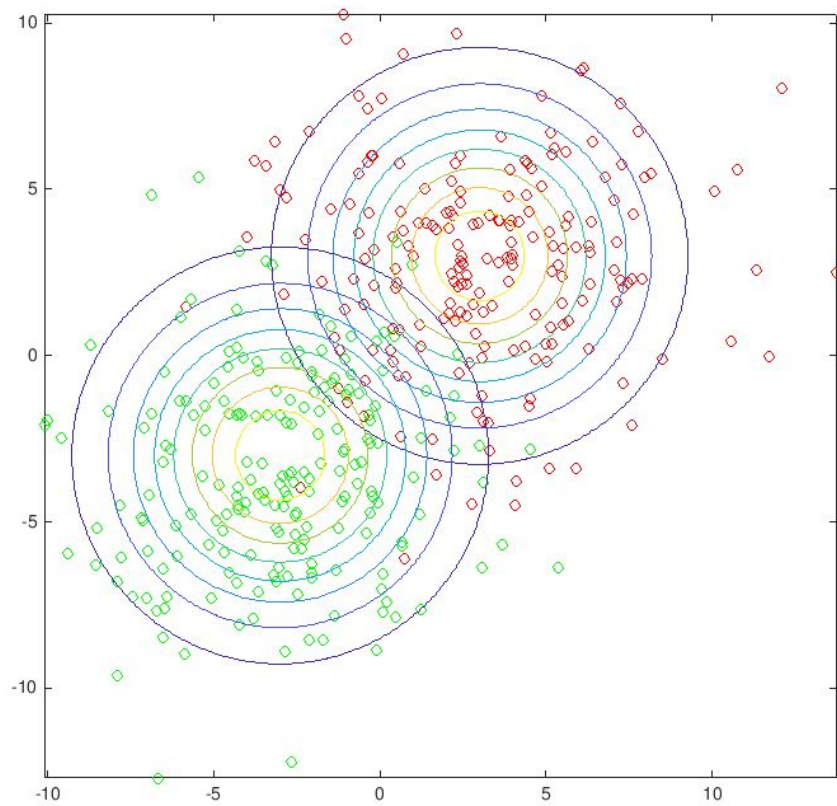


EE 573 Pattern Recognition – Project 2 - Parameter Estimation

1. Generated samples and pdf contours



2. Estimating μ_i by considering μ_i as an unknown deterministic variable is Maximum Likelihood Estimation.

$$g1(x1, x2) = -0.056 * x1^2 + 0.34 * x1 - 0.056 * x2^2 + 0.3 * x2 - 3.1$$

$$g2(x1, x2) = -0.056 * x1^2 - 0.33 * x1 - 0.056 * x2^2 - 0.39 * x2 - 3.4$$

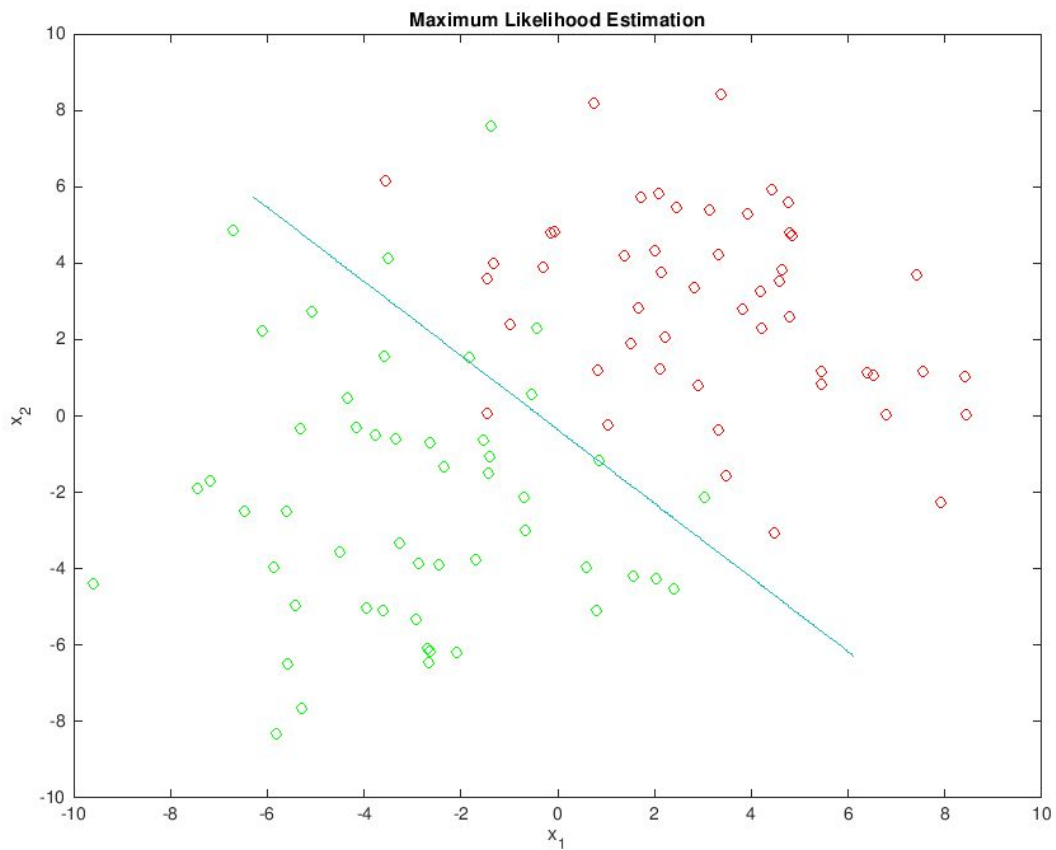
$$w = g1(x1, x2) - g2(x1, x2) = 0.67 * x1 + 0.7 * x2 + 0.25$$

Thus the decision rule is

w1 if $w > 0$

w2 otherwise

Which is



w1

precision	recall
0.8750	0.9800

w2

precision	recall
0.9773	0.8600

b. Considering μ_i as a random vector corresponds to the **Bayesian parameter estimation**.

with α : 2.9

α_2 : -3.1

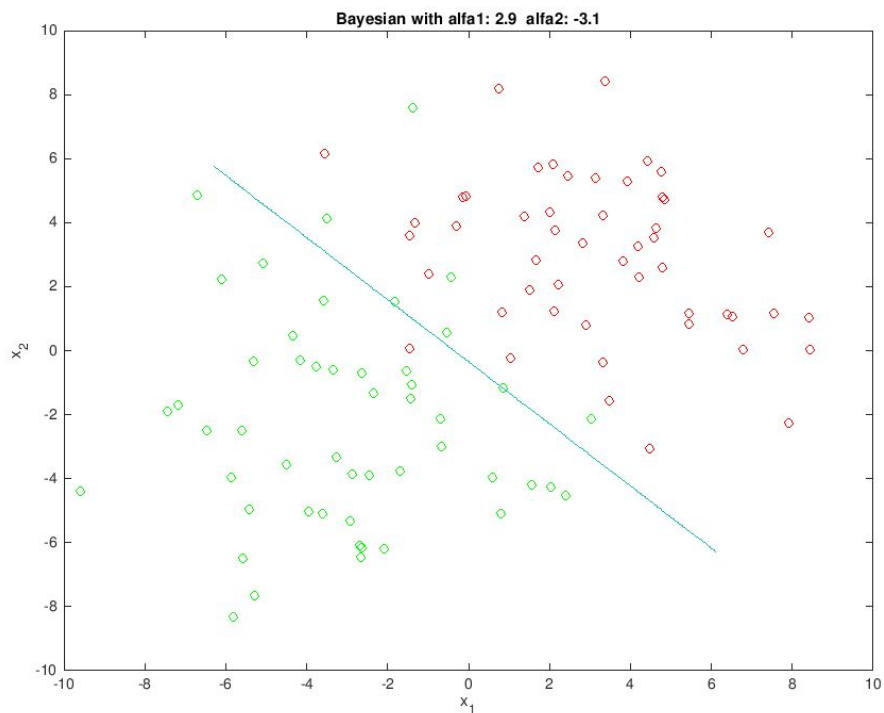
Estimate for μ_1 = 3.0680, 2.7437

Estimate for μ_2 = -2.995, -3.5068

$$g_1(x_1, x_2) = -0.0556 * x_1^2 + 0.341 * x_1 - 0.0556 * x_2^2 + 0.305 * x_2 - 3.14$$

$$g_2(x_1, x_2) = -0.0556 * x_1^2 - 0.333 * x_1 - 0.0556 * x_2^2 - 0.39 * x_2 - 3.38$$

$$w = g_1(x_1, x_2) - g_2(x_1, x_2) = 0.674 * x_1 + 0.695 * x_2 + 0.241$$



This is almost the same result with the first approach.

w1

precision	recall
0.8750	0.9800

w2

precision	recall
0.9773	0.8600

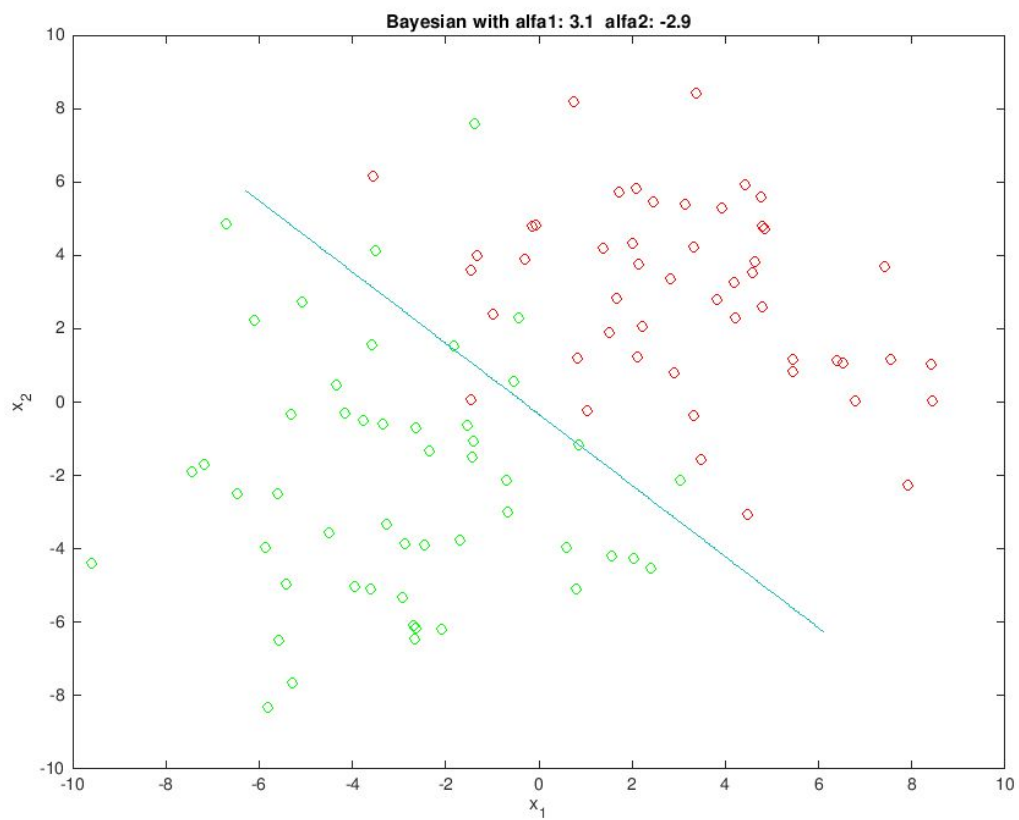
Precision and recall are exactly the same.

with $\alpha : 3.1$

$\alpha 2 : -2.9$

Estimate for $\mu 1 = 3.0767, 2.7523$

Estimate for $\mu 2 = -2.9871, -3.4982$

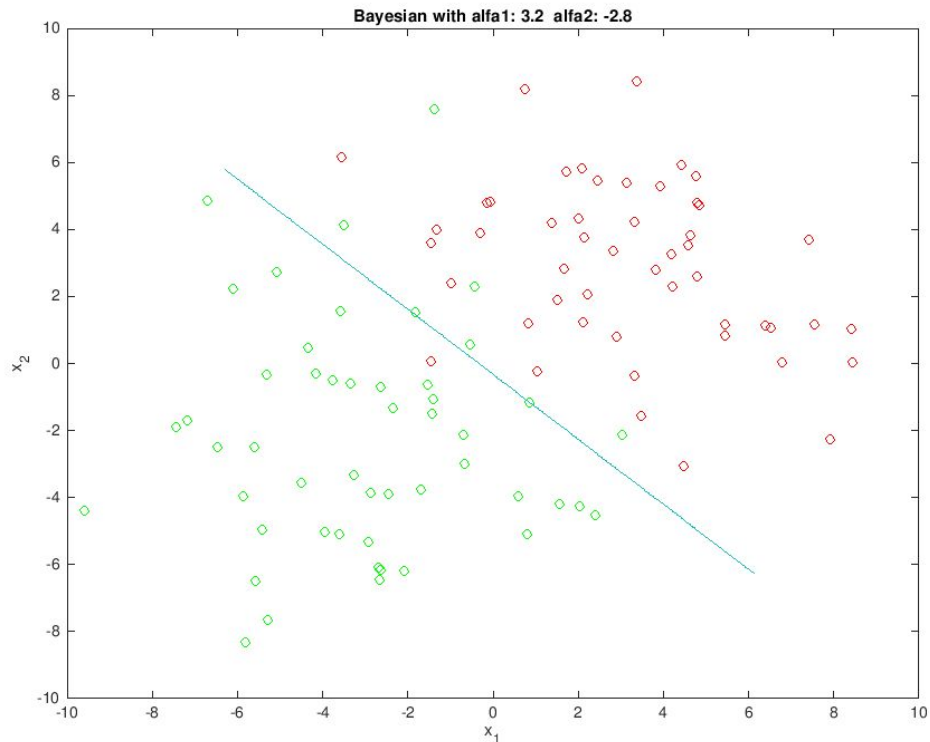


with $\alpha : 3.2$

$\alpha_2 : -2.8$

Estimate for $\mu_1 = 3.0810, 2.7566$

Estimate for $\mu_2 = -2.9827, -3.4939$



All epsilon values leads to highly similar results.

Comparison

Maximum Likelihood Estimation is easier to compute because it uses differential calculus rather than integrals. It is also easier to interpret since it returns one model $p(x|\theta)$ has the assumed parametric form in MLE.

Bayesian Estimation is harder to compute, because it needs multidimensional integration. It is also harder to interpret because it returns weighted average of models

Can give better results since use more information about the problem (prior information)