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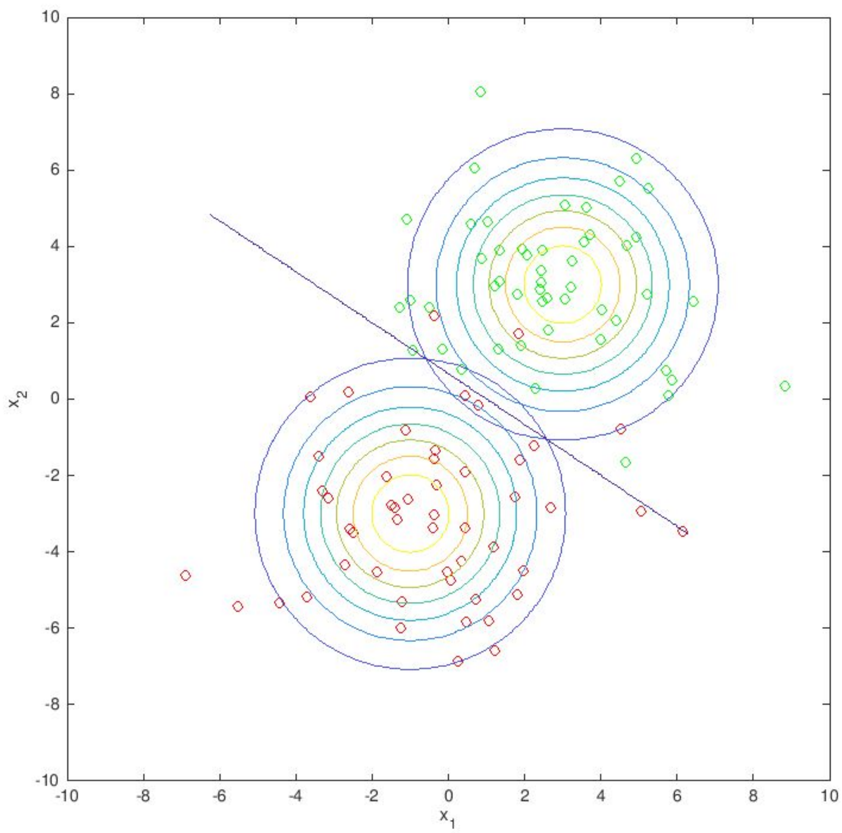
25.12.18

EE 573 Pattern Recognition – Project 1 - Discriminant Functions



1.

$\mathbf{\Sigma}_i = \sigma^2 \mathbf{I}$

$g_i(\mathbf{x}) = \ln p(\mathbf{x}|w_i) + \ln P(w_i)$

In the multivariate normal case, it becomes

$$g_i(\mathbf{x}) = -\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu}_i)^{\mathrm{T}} \mathbf{\Sigma}_i^{-1} (\mathbf{x} - \boldsymbol{\mu}_i) - \frac{d}{2} \ln 2\pi - \frac{1}{2} \ln |\mathbf{\Sigma}_i| + \ln P(w_i)$$

And after some manipulation

$$g_i(\mathbf{x}) = \mathbf{w}_i^{\mathrm{T}}\mathbf{x} + w_{i0}$$

where

$$\mathbf{w}_i = \frac{1}{\sigma^2}\boldsymbol{\mu}_i$$

$$w_{i0} = -\frac{1}{2\sigma^2}\boldsymbol{\mu}_i^{\mathrm{T}}\boldsymbol{\mu}_i + \ln P(w_i)$$

Equating the discriminant functions, the boundary becomes

$$g(\mathbf{x}) \equiv g_1(\mathbf{x}) - g_2(\mathbf{x})$$

$$\mathbf{w}^{\mathrm{T}}(\mathbf{x} - \mathbf{x}_0) = 0$$

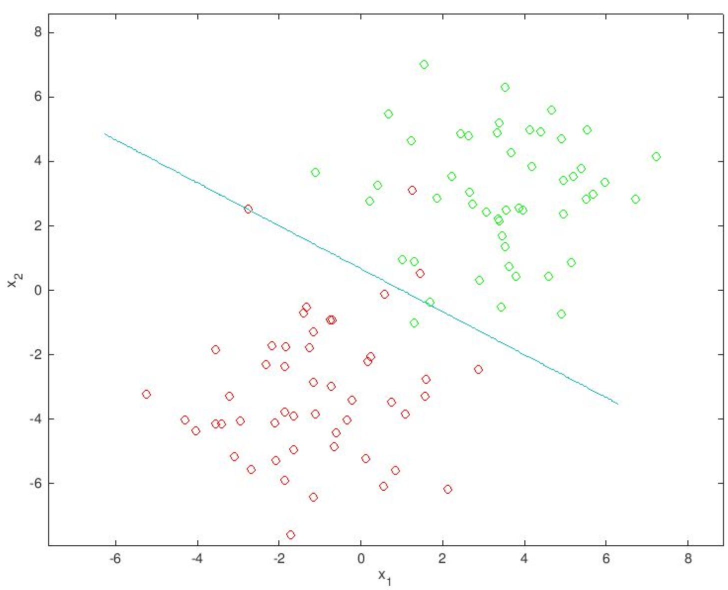
where

$$\mathbf{x}_0 = \frac{1}{2}(\boldsymbol{\mu}_j + \boldsymbol{\mu}_i) - \sigma^2 \frac{(\boldsymbol{\mu}_i - \boldsymbol{\mu}_j)}{\left\|\boldsymbol{\mu}_i - \boldsymbol{\mu}_j\right\|^2} \ln \frac{P(w_i)}{P(w_j)}$$

Assuming the prior probabilities are the same, the term on the right becomes zero.

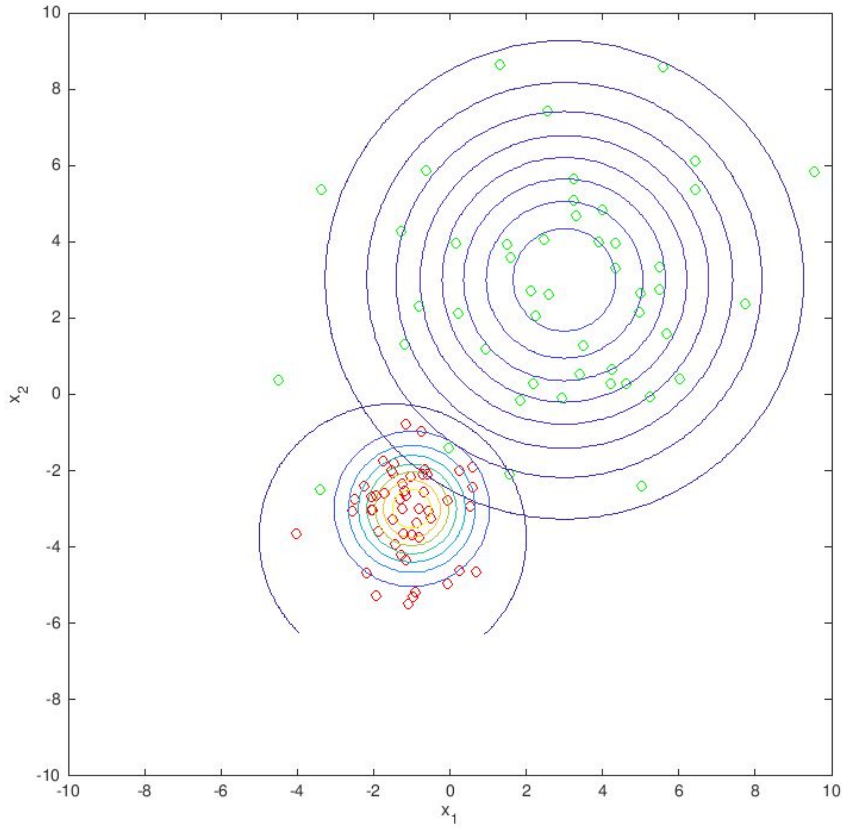
Thus the simplest decision surface is a perpendicular line to the midpoint of means.

$$g1(x1,x2) = -0.125 * x1\hat{}2 - 0.25 * x1 - 0.125 * x2\hat{}2 - 0.75 * x2 - 2.64$$
$$g2(x1,x2) = -0.125 * x1\hat{}2 + 0.75 * x1 - 0.125 * x2\hat{}2 + 0.75 * x2 - 3.64$$
$$g1 - g2 = 1.0 - 1.5 * x2 - 1.0 * x1 = 0$$



Correctly classified **97**
Misclassified **3**

2. Here the covariance matrices are different for each category. The discriminant functions can't be much simplified.



$\boldsymbol{\Sigma}_i = \text{arbitrary}$

$g_i(\mathrm{x}) = \mathrm{x}^{\mathrm{T}} \mathrm{W}_i \mathrm{x} + \mathrm{w}_i^{\mathrm{T}} \mathrm{x} + w_{i0}$

where

$$\mathbf{W}_i = -\frac{1}{2} \boldsymbol{\Sigma}_i^{-1}$$

$$\mathbf{w}_i = \boldsymbol{\Sigma}_i^{-1} \boldsymbol{\mu}_i$$

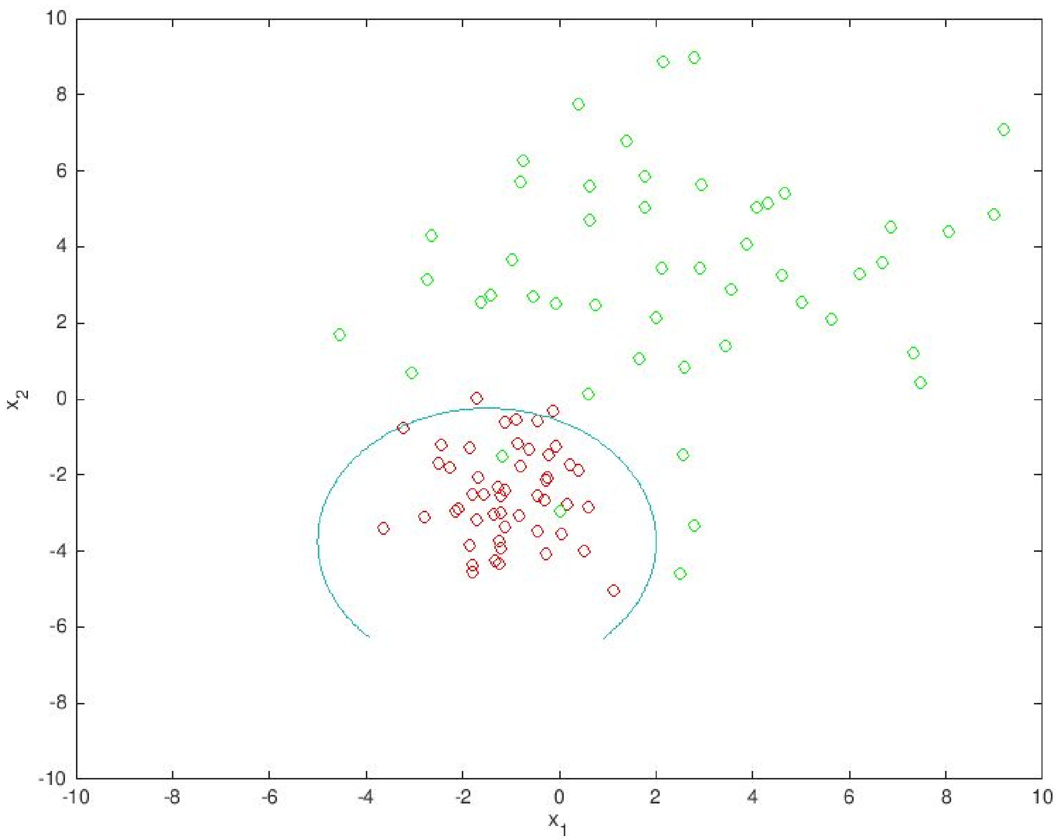
$$w_{i0} = -\frac{1}{2} \boldsymbol{\mu}_i^{\mathrm{T}} \boldsymbol{\Sigma}_i^{-1} \boldsymbol{\mu}_i - \frac{1}{2} \ln |\boldsymbol{\Sigma}_i| + \ln P(w_i)$$

Solving for given mean and variance values

$$g1(x1, x2) = -0.5 * x1\hat{}2 - 1.0 * x1 - 0.5 * x2\hat{}2 - 3.0 * x2 - 5.0$$

$$g2(x1, x2) = -0.0556 * x1\hat{}2 + 0.333 * x1 - 0.0556 * x2\hat{}2 + 0.333 * x2 - 3.2$$

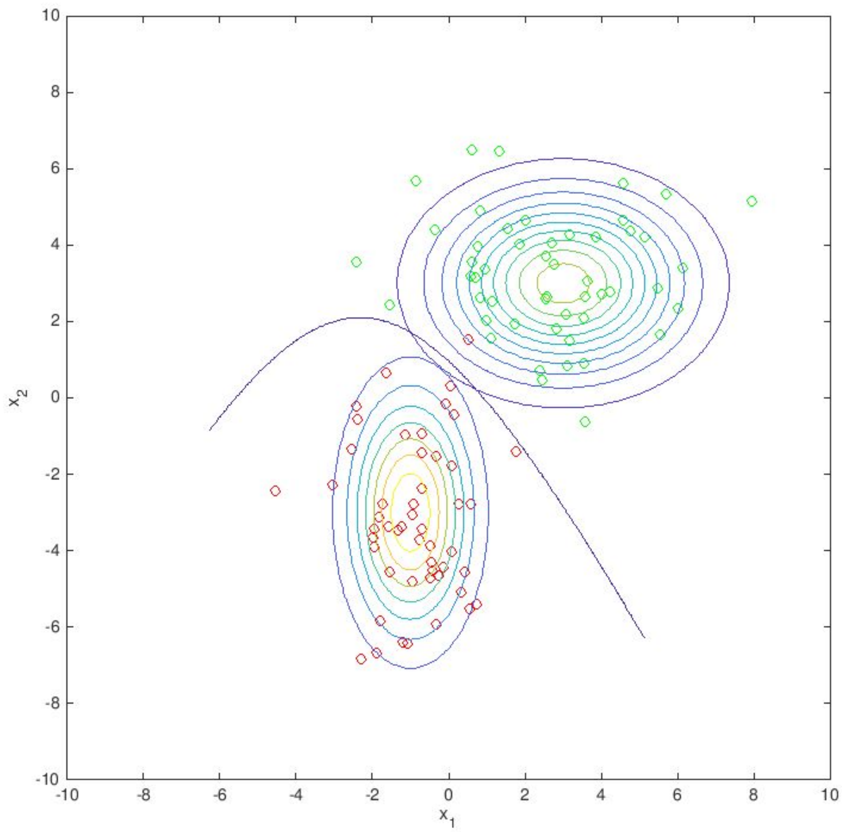
$$g1 - g2 = -0.444 * x1\hat{}2 - 1.33 * x1 - 0.444 * x2\hat{}2 - 3.33 * x2 - 1.8 = 0$$



Correctly classified **97**

Misclassified **3**

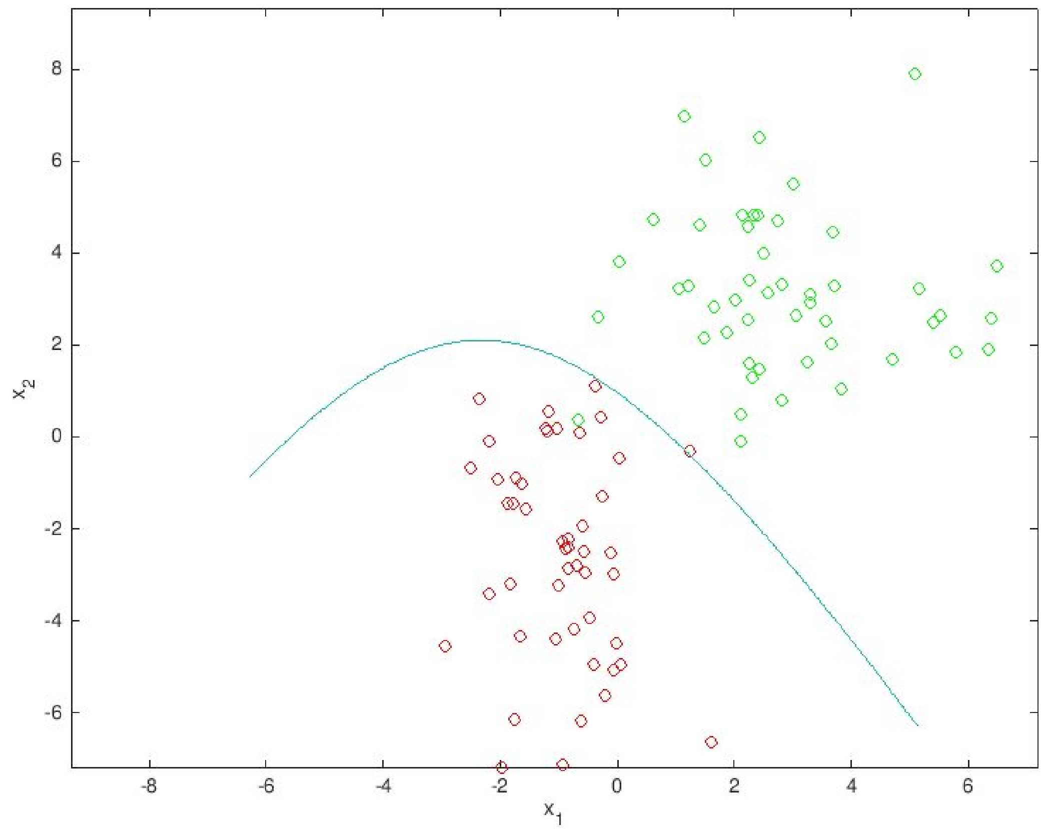
3. Covariance matrices are also arbitrary here.



$$g1(x1, x2) = -0.5 * x1\hat{}2 - 1.0 * x1 - 0.125 * x2\hat{}2 - 0.75 * x2 - 2.32$$

$$g2(x1, x2) = -0.125 * x1\hat{}2 + 0.75 * x1 - 0.222 * x2\hat{}2 + 1.33 * x2 - 4.22$$

$$g1 - g2 = -0.375 * x1\hat{}2 - 1.75 * x1 + 0.0972 * x2\hat{}2 - 2.08 * x2 + 1.91$$



Correctly classified **99**

Misclassified **1**