

### **PROBLEM 4.4**

For the beam and loading shown, determine (a) the reaction at A, (b) the tension in cable BC.

#### **SOLUTION**

Free-Body Diagram:

Reaction at A: (a)

$$\Sigma F_{\rm v} = 0$$
:  $A_{\rm v} = 0$ 

+) 
$$\Sigma M_B = 0$$
:  $(15 \text{ lb})(28 \text{ in.}) + (20 \text{ lb})(22 \text{ in.}) + (35 \text{ lb})(14 \text{ in.})$   
+  $(20 \text{ lb})(6 \text{ in.}) - A_y(6 \text{ in.}) = 0$ 

$$A_v = +245 \text{ lb}$$

**A** = 245 lb ◀

(b) Tension in BC

+) 
$$\Sigma M_A = 0$$
:  $(15 \text{ lb})(22 \text{ in.}) + (20 \text{ lb})(16 \text{ in.}) + (35 \text{ lb})(8 \text{ in.})$   
-  $(15 \text{ lb})(6 \text{ in.}) - F_{BC}(6 \text{ in.}) = 0$ 

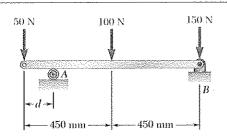
$$F_{RC} = +140.0 \, \text{lb}$$

 $F_{BC} = +140.0 \text{ lb}$   $F_{BC} = 140.0 \text{ lb}$ 

Check:

+ 
$$\Sigma F_y = 0$$
: -15 lb - 20 lb = 35 lb - 20 lb +  $A - F_{BC} = 0$   
-105 lb + 245 lb - 140.0 = 0

$$0 = 0$$
 (Checks)



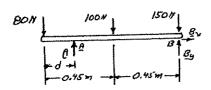
### PROBLEM 4.10



Solve Problem 4.9 if the 50-N load is replaced by an 80-N load.

**PROBLEM 4.9** The maximum allowable value of each of the reactions is 180 N. Neglecting the weight of the beam, determine the range of the distance *d* for which the beam is safe.

## SOLUTION



$$\Sigma F_x = 0: \quad B_x = 0$$
$$B = B_y$$

+)
$$\Sigma M_A = 0$$
:  $(80 \text{ N})d - (100 \text{ N})(0.45 \text{ m} - d) - (150 \text{ N})(0.9 \text{ m} - d) + B(0.9 \text{ m} - d) = 0$ 

$$80d - 45 + 100d - 135 + 150d + 0.9B - Bd = 0$$

$$d = \frac{180 \text{ N} \cdot \text{m} - 0.9B}{330 \text{N} - B} \tag{1}$$

+)
$$\Sigma M_B = 0$$
:  $(80 \text{ N})(0.9 \text{ m}) - A(0.9 \text{ m} - d) + (100 \text{ N})(0.45 \text{ m}) = 0$ 

$$d = \frac{0.9A - 117}{A} \tag{2}$$

Since  $B \le 180$  N, Eq. (1) yields.

$$d \ge (180 - 0.9 \times 180)/(330 - 180) = \frac{18}{150} = 0.12 \text{ m}$$

 $d = 120.0 \text{ mm} \triangleleft$ 

Since  $A \le 180$  N, Eq. (2) yields.

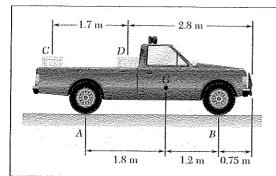
$$d \le (0.9 \times 180 - 112)/180 = \frac{45}{180} = 0.25 \text{ m}$$

 $d = 250 \,\mathrm{mm} \, \triangleleft$ 

Range:

$$120.0 \, \text{mm} \le d \le 250 \, \text{mm}$$

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## **PROBLEM 4.5**

Two crates, each of mass 350 kg, are placed as shown in the bed of a 1400-kg pickup truck. Determine the reactions at each of the two (a) rear wheels A, (b) front wheels B.

#### SOLUTION

Free-Body Diagram:

$$\begin{array}{c|c}
W & 1.7m & W \\
C & D & W_t \\
\hline
C & G & \vdots \\
\hline
2A & 1.8m & 1.2m & ZB
\end{array}$$

 $W = (350 \text{ kg})(9.81 \text{ m/s}^2) = 3.434 \text{ kN}$ 

 $W_t = (1400 \text{ kg})(9.81 \text{ m/s}^2) = 13.734 \text{ kN}$ 

+)
$$\Sigma M_B = 0$$
:  $W(1.7 \text{ m} + 2.05 \text{ m}) + W(2.05 \text{ m}) + W_t(1.2 \text{ m}) - 2A(3 \text{ m}) = 0$ 

(3.434 kN)(3.75 m) + (3.434 kN)(2.05 m)

+(13.734 kN)(1.2 m) - 2A(3 m) = 0

$$A = +6.0663 \text{ kN}$$

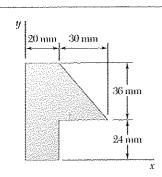
A = 6.07 kN

$$+ \sum F_y = 0$$
:  $-W - W - W_t + 2A + 2B = 0$ 

-3.434 kN - 3.434 kN - 13.734 kN + 2(6.0663 kN) + 2B = 0

$$B = +4.2347 \text{ kN}$$

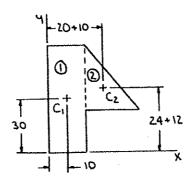
 $\mathbf{B} = 4.23 \, \mathrm{kN} \, \blacksquare$ 



Locate the centroid of the plane area shown.

# **SOLUTION**

Dimensions in mm



|   | A, mm <sup>2</sup> | $\overline{x}$ , mm | $\overline{y}$ , mm | $\overline{x}A$ , mm <sup>3</sup> | $\overline{y}A$ , mm <sup>3</sup> |
|---|--------------------|---------------------|---------------------|-----------------------------------|-----------------------------------|
| 1 | 1200               | 10                  | 30                  | 12000                             | 36000                             |
| 2 | 540                | 30                  | 36                  | 16200                             | 19440                             |
| Σ | 1740               |                     |                     | 28200                             | 55440                             |

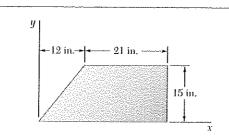
Then

$$\overline{X} = \frac{\Sigma \overline{X} A}{\Sigma A} = \frac{28200}{1740}$$

$$\overline{Y} = \frac{\Sigma \overline{y}A}{\Sigma A} = \frac{55440}{1740}$$

 $\overline{X} = 16.21 \,\mathrm{mm}$ 

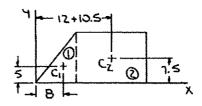
$$\overline{Y} = 31.9 \text{ mm}$$



Locate the centroid of the plane area shown.

## **SOLUTION**

Dimensions in in.



|   | A, in. <sup>2</sup>                    | $\overline{x}$ , in. | $\overline{y}$ , in. | $\overline{x}A$ , in. <sup>3</sup> | $\overline{y}A$ , in. <sup>3</sup> |
|---|--|----------------------|----------------------|------------------------------------|------------------------------------|
| 1 | $\frac{1}{2} \times 12 \times 15 = 90$ | 8                    | 5                    | 720                                | 450                                |
| 2 | 21×15 = 315                            | 22.5                 | 7.5                  | 7087.5                             | 2362.5                             |
| Σ | 405,00                                 |                      |                      | 7807.5                             | 2812.5                             |

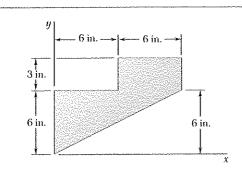
Then

$$\overline{X} = \frac{\Sigma \overline{X}A}{\Sigma A} = \frac{7807.5}{405.00}$$

$$\bar{X} = 19.28 \text{ in.} \blacktriangleleft$$

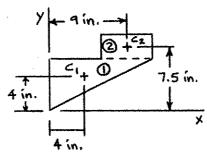
$$\overline{Y} = \frac{\Sigma \overline{y}A}{\Sigma A} = \frac{2812.5}{405.00}$$

$$\overline{Y} = 6.94 \text{ in.} \blacktriangleleft$$



Locate the centroid of the plane area shown.

## **SOLUTION**



|   | A, in. <sup>2</sup>       | $\overline{x}$ , in. | $\overline{y}$ , in. | $\overline{x}A$ , in. <sup>3</sup> | $\overline{y}A$ , in. <sup>3</sup> |
|---|---------------------------|----------------------|----------------------|------------------------------------|------------------------------------|
| 1 | $\frac{1}{2}(12)(6) = 36$ | 4                    | 4                    | 144                                | 144                                |
| 2 | (6)(3) = 18               | 9                    | 7.5                  | 162                                | 135                                |
| Σ | 54                        |                      |                      | 306                                | 279                                |

Then

$$\overline{X}A = \Sigma \overline{X}A$$

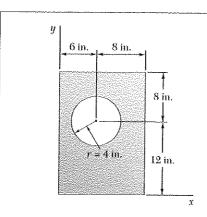
$$\overline{X}(54) = 306$$

$$\bar{X} = 5.67 \text{ in. } \blacktriangleleft$$

$$\overline{Y}A = \Sigma \overline{y}A$$

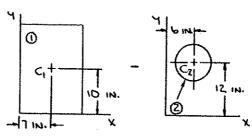
$$\overline{Y}(54) = 279$$

$$\overline{Y} = 5.17$$
 in.



Locate the centroid of the plane area shown.

## **SOLUTION**



|   | A, in. <sup>2</sup>  | $\overline{x}$ , in. | $\overline{y}$ , in. | $\overline{x}A$ , in. <sup>3</sup> | $\overline{y}A$ , in. <sup>3</sup> |
|---|----------------------|----------------------|----------------------|------------------------------------|------------------------------------|
| 1 | $14 \times 20 = 280$ | 7                    | 10                   | 1960                               | 2800                               |
| 2 | $-\pi(4)^2 = -16\pi$ | 6                    | 12                   | -301.59                            | -603.19                            |
| Σ | 229.73               |                      |                      | 1658.41                            | 2196.8                             |

Then

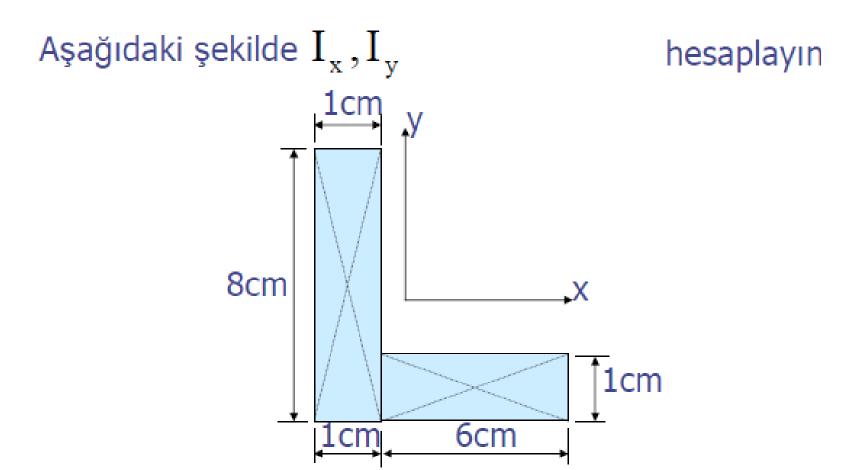
$$\overline{X} = \frac{\Sigma \overline{X}A}{\Sigma A} = \frac{1658.41}{229.73}$$

$$\overline{X} = 7.22 \text{ in. } \blacktriangleleft$$

$$\overline{Y} = \frac{\Sigma \overline{y}A}{\Sigma A} = \frac{2196.8}{229.73}$$

$$\overline{Y} = 9.56$$
 in.  $\blacktriangleleft$ 





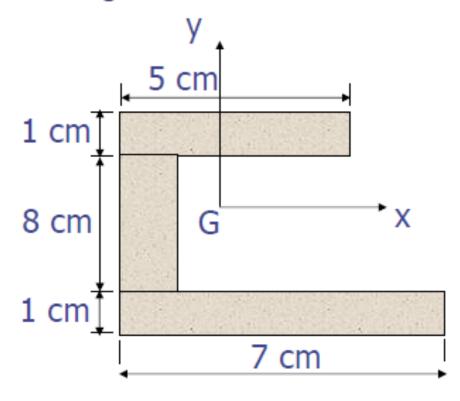
$$x_g = \frac{x_1 A_1 + x_2 A_2}{A_1 + A_2} = \frac{0.5.8 + 4.6}{8 + 6} = 2 cm$$

$$y_g = \frac{y_1 A_1 + y_2 A_2}{A_1 + A_2} = \frac{4.8 + 0.5.6}{8 + 6} = 2.5 cm$$

$$I_x = \frac{1.8^3}{12} + (4 - 2.5)^2.8 + \frac{6.1^3}{12} + (0.5 - 2.5)^2.6 = 85,16 \, \text{cm}^4$$

$$I_y = \frac{8.1^3}{12} + (0.5 - 2)^2.8 + \frac{1.6^3}{12} + (4 - 2.5)^2.6 = 50.16 \text{ cm}^4$$

Verilen profil kesitte ağırlık merkezinden geçen x, y eksen takımına göre atalet momentlerini hesaplayınız.



$$x = \frac{x_1 A_1 + x_2 A_2 + x_3 A_3}{A_1 + A_2 + A_3} = \frac{2,5.5 + 0,5.8 + 3,5.7}{20} = 2,0 \text{ cm}$$

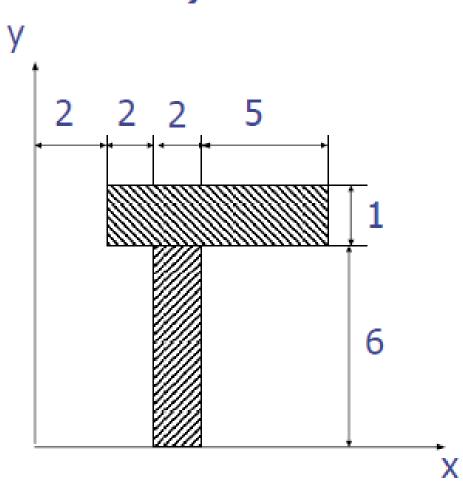
$$y = \frac{y_1 A_1 + y_2 A_2 + y_3 A_3}{A_1 + A_2 + A_3} = \frac{9,5.5 + 5.8 + 0,5.7}{20} = 4,5 \text{ cm}$$

$$\mathbf{I_x} = \frac{5.1^3}{12} + 5^2 5 + \frac{1.8^3}{12} + (0.5)^2 .8 + \frac{1^3.7}{12} + 4^2.7 = 282.6 \text{ cm}^4$$

$$I_{y} = \frac{1.5^{3}}{12} + 10,55^{2}.5 + \frac{8.1^{3}}{12} + (1,5)^{2}.8 + \frac{1.7^{3}}{12} + (1,5)^{2}.7 = 74,6 \text{ cm}^{4}$$



# Problem 1)



- a) Ağırlık merkezini bulunuz.
- b) Ağırlık merkezinder geçen eksene göre atalet momentlerini hesaplayınız.

Not: Ölçüler cm'dir.

$$x = \frac{9x6,5 + 12x5}{9 + 12} = \frac{118,5}{21} = 5,64cm$$

$$y = \frac{9x6,5 + 12x3}{21} = 4,5cm$$

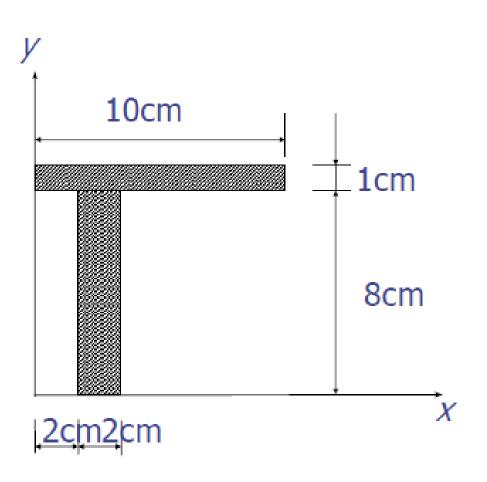
$$I_x = \frac{9x1^3}{12} + (9)(6.5 - 4.5)^2 + \frac{6^3x2}{12} + (12)(3 - 4.5)^2$$

$$I_x = 0.75 + 36 + 36 + 27 = 99.75 cm^4$$

$$I_y = \frac{1x9^3}{12} + (9)(6.5 - 5.64)^2 + \frac{2^3x6}{12} + (12)(5 - 5.64)^2$$

$$I_v = 60,75 + 6,65 + 4 + 4,91 = 76,31cm^4$$





Verilen kesitin; Ağırlık merkezinin koordinatlarını hesaplayınız. Ağırlık merkezinden geçen koordinat eksenine göre atalet momentlerini hesaplayınız. Asal atalet momentlerini hesaplayınız.

$$x$$
  $y$   $A$   $A_x$   $A_y$ 

a) 
$$\frac{1}{x} = \frac{\sum xA}{\sum A} = \frac{98}{26} = 3,77 \text{ cm}$$
  $\frac{149}{y} = \frac{\sum yA}{\sum A} = \frac{149}{26} = 5,73 \text{ cm}$ 

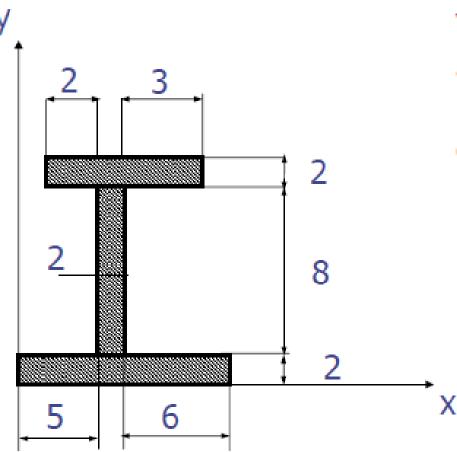
М

b)

$$I_x = \frac{10.1^3}{12} + (8,5 - 5,73)^2.10 + \frac{2.8^3}{12} + (4 - 5,73)^2.16 = 210,78 \text{ cm}^4$$

$$I_y = \frac{1.10^3}{12} + (5 - 3.77)^2.10 + \frac{8.2^3}{12} + (3 - 3.77)^2.16 = 113.28 \text{ cm}^4$$





Verilen profil kesitte;
a) Ağırlık merkezini,
b) Ağırlık merkezinden
geçen eksene göre atalet
momentini,

A X Y A<sub>X</sub> A<sub>y</sub>

1 14 6,5 11 91 154

2 16 6 6 96 96

3 26 6,5 1 169 26

T 56 356 276

$$\overline{x} = \frac{356}{56} = 6,35 \text{ cm} \quad \overline{y} = \frac{276}{56} = 4,92 \text{ cm}$$

b)
$$I_x = \frac{7.2^3}{12} + (6.08)^2 .14 + \frac{2.8^3}{12} + (1.08)^2 .16 + \frac{13.2^3}{12} + (-3.92)^2 .26 = 1034.1 \text{ cm}^4$$

$$I_y = \frac{2.7^3}{12} + (0.15)^2 .14 + \frac{8.2^3}{12} + (-0.35)^2 .16 + \frac{2.13^3}{12} + (0.15)^2 .26 = 431.1 \text{ cm}^4$$