

PROBLEM 2.7

Two forces are applied as shown to a hook support. Knowing that the magnitude of **P** is 35 N, determine by trigonometry (a) the required angle α if the resultant **R** of the two forces applied to the support is to be horizontal, (b) the corresponding magnitude of **R**.

SOLUTION

Using the triangle rule and law of sines:

$$(a) \quad \frac{\sin \alpha}{50 \text{ N}} = \frac{\sin 25^\circ}{35 \text{ N}}$$

$$\sin \alpha = 0.60374$$

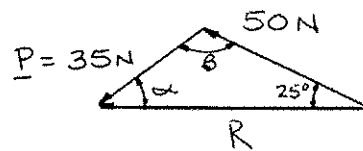
$$\alpha = 37.138^\circ$$

$$(b) \quad \alpha + \beta + 25^\circ = 180^\circ$$

$$\beta = 180^\circ - 25^\circ - 37.138^\circ$$

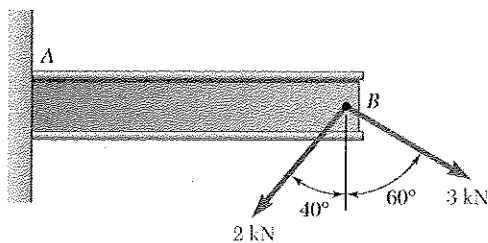
$$= 117.86^\circ$$

$$\frac{R}{\sin 117.86^\circ} = \frac{35 \text{ N}}{\sin 25^\circ}$$



$$\alpha = 37.1^\circ \quad \blacktriangleleft$$

$$R = 73.2 \text{ N} \quad \blacktriangleleft$$



PROBLEM 2.17

Solve Problem 2.4 by trigonometry.

PROBLEM 2.4 Two forces are applied at Point B of beam AB . Determine graphically the magnitude and direction of their resultant using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

Using the law of cosines:

$$R^2 = (2 \text{ kN})^2 + (3 \text{ kN})^2 - 2(2 \text{ kN})(3 \text{ kN})\cos 80^\circ$$

$$R = 3.304 \text{ kN}$$

Using the law of sines:

$$\frac{\sin \gamma}{2 \text{ kN}} = \frac{\sin 80^\circ}{3.304 \text{ kN}}$$

$$\gamma = 36.59^\circ$$

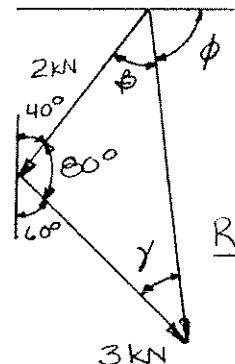
$$\beta + \gamma + 80^\circ = 180^\circ$$

$$\gamma = 180^\circ - 80^\circ - 36.59^\circ$$

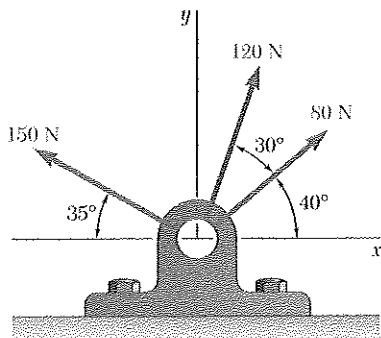
$$\gamma = 63.41^\circ$$

$$\phi = 180^\circ - \beta + 50^\circ$$

$$\phi = 66.59^\circ$$



$$R = 3.30 \text{ kN} \searrow 66.6^\circ \blacktriangleleft$$



PROBLEM 2.32

Determine the resultant of the three forces of Problem 2.24.

PROBLEM 2.24 Determine the x and y components of each of the forces shown.

SOLUTION

Components of the forces were determined in Problem 2.24:

Force	x Comp. (N)	y Comp. (N)
80 N	+61.3	+51.4
120 N	+41.0	+112.8
150 N	-122.9	+86.0
	$R_x = -20.6$	$R_y = +250.2$

$$\mathbf{R} = R_x \mathbf{i} + R_y \mathbf{j}$$

$$= (-20.6 \text{ N})\mathbf{i} + (250.2 \text{ N})\mathbf{j}$$

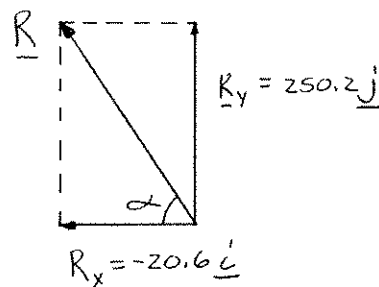
$$\tan \alpha = \frac{R_y}{R_x}$$

$$\tan \alpha = \frac{250.2 \text{ N}}{20.6 \text{ N}}$$

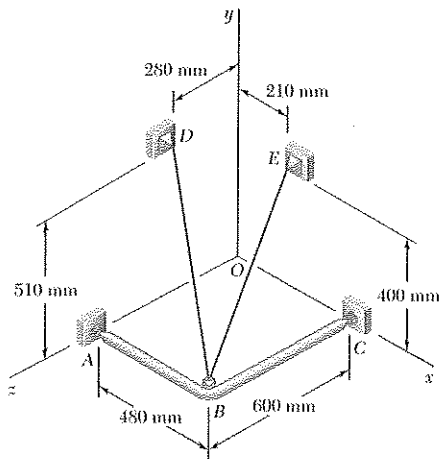
$$\tan \alpha = 12.1456$$

$$\alpha = 85.293^\circ$$

$$R = \frac{250.2 \text{ N}}{\sin 85.293^\circ}$$



$$\mathbf{R} = 251 \text{ N} \nearrow 85.3^\circ \blacktriangleleft$$



PROBLEM 2.87

A frame ABC is supported in part by cable DBE that passes through a frictionless ring at B . Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at D .

SOLUTION

$$\overrightarrow{DB} = (480 \text{ mm})\mathbf{i} - (510 \text{ mm})\mathbf{j} + (320 \text{ mm})\mathbf{k}$$

$$DB = \sqrt{(480 \text{ mm})^2 + (510 \text{ mm})^2 + (320 \text{ mm})^2}$$

$$= 770 \text{ mm}$$

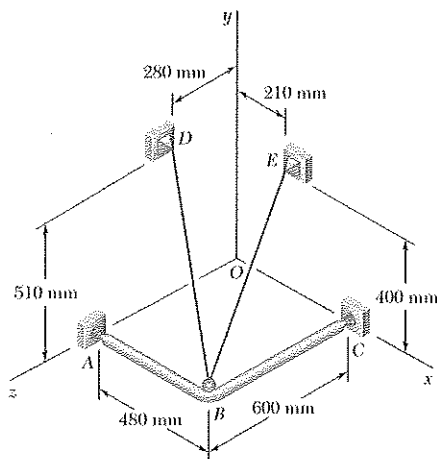
$$\mathbf{F} = F\lambda_{DB}$$

$$= F \frac{\overrightarrow{DB}}{DB}$$

$$= \frac{385 \text{ N}}{770 \text{ mm}} [(480 \text{ mm})\mathbf{i} - (510 \text{ mm})\mathbf{j} + (320 \text{ mm})\mathbf{k}]$$

$$= (240 \text{ N})\mathbf{i} - (255 \text{ N})\mathbf{j} + (160 \text{ N})\mathbf{k}$$

$$F_x = +240 \text{ N}, \quad F_y = -255 \text{ N}, \quad F_z = +160.0 \text{ N} \quad \blacktriangleleft$$



PROBLEM 2.88

For the frame and cable of Problem 2.87, determine the components of the force exerted by the cable on the support at E.

PROBLEM 2.87 A frame ABC is supported in part by cable DBE that passes through a frictionless ring at B . Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at D .

SOLUTION

$$\overrightarrow{EB} = (270 \text{ mm})\mathbf{i} - (400 \text{ mm})\mathbf{j} + (600 \text{ mm})\mathbf{k}$$

$$EB = \sqrt{(270 \text{ mm})^2 + (400 \text{ mm})^2 + (600 \text{ mm})^2}$$

$$= 770 \text{ mm}$$

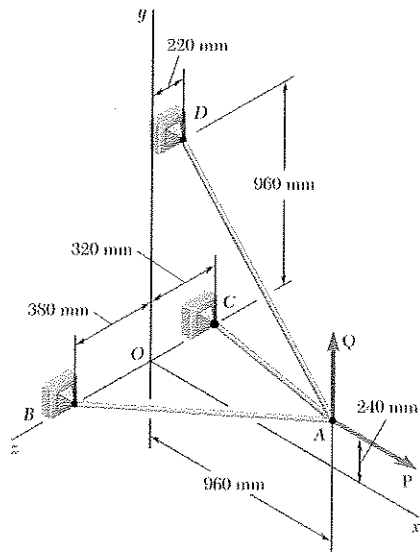
$$\mathbf{F} = F\lambda_{EB}$$

$$= F \frac{\overrightarrow{EB}}{EB}$$

$$= \frac{385 \text{ N}}{770 \text{ mm}} [(270 \text{ mm})\mathbf{i} - (400 \text{ mm})\mathbf{j} + (600 \text{ mm})\mathbf{k}]$$

$$\mathbf{F} = (135 \text{ N})\mathbf{i} - (200 \text{ N})\mathbf{j} + (300 \text{ N})\mathbf{k}$$

$$F_x = +135.0 \text{ N}, \quad F_y = -200 \text{ N}, \quad F_z = +300 \text{ N} \quad \blacktriangleleft$$



PROBLEM 2.107

Three cables are connected at A , where the forces \mathbf{P} and \mathbf{Q} are applied as shown. Knowing that $Q = 0$, find the value of P for which the tension in cable AD is 305 N.

SOLUTION

$$\Sigma \mathbf{F}_A = 0: \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{P} = 0 \quad \text{where} \quad \mathbf{P} = P\mathbf{i}$$

$$\overrightarrow{AB} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} + (380 \text{ mm})\mathbf{k} \quad AB = 1060 \text{ mm}$$

$$\overrightarrow{AC} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad AC = 1040 \text{ mm}$$

$$\overrightarrow{AD} = -(960 \text{ mm})\mathbf{i} + (720 \text{ mm})\mathbf{j} - (220 \text{ mm})\mathbf{k} \quad AD = 1220 \text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overrightarrow{AB}}{AB} = T_{AB} \left(-\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k} \right)$$

$$\mathbf{T}_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overrightarrow{AC}}{AC} = T_{AC} \left(-\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k} \right)$$

$$\begin{aligned} \mathbf{T}_{AD} = T_{AD} \lambda_{AD} &= \frac{305 \text{ N}}{1220 \text{ mm}} [(-960 \text{ mm})\mathbf{i} + (720 \text{ mm})\mathbf{j} - (220 \text{ mm})\mathbf{k}] \\ &= -(240 \text{ N})\mathbf{i} + (180 \text{ N})\mathbf{j} - (55 \text{ N})\mathbf{k} \end{aligned}$$

Substituting into $\Sigma \mathbf{F}_A = 0$, factoring \mathbf{i} , \mathbf{j} , \mathbf{k} , and setting each coefficient equal to ϕ gives:

$$\mathbf{i}: P = \frac{48}{53}T_{AB} + \frac{12}{13}T_{AC} + 240 \text{ N} \quad (1)$$

$$\mathbf{j}: \frac{12}{53}T_{AB} + \frac{3}{13}T_{AC} = 180 \text{ N} \quad (2)$$

$$\mathbf{k}: \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} = 55 \text{ N} \quad (3)$$

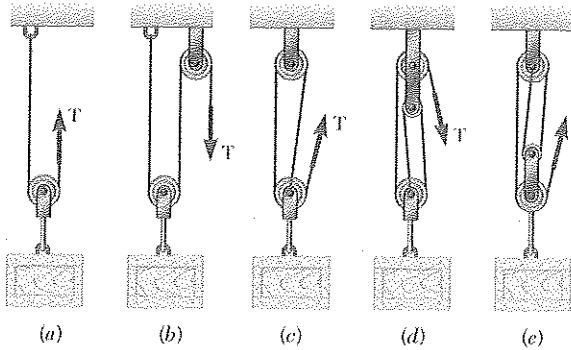
Solving the system of linear equations using conventional algorithms gives:

$$T_{AB} = 446.71 \text{ N}$$

$$T_{AC} = 341.71 \text{ N}$$

$$P = 960 \text{ N} \quad \blacktriangleleft$$

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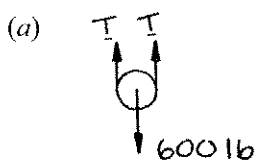


PROBLEM 2.67

A 600-lb crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (See the hint for Problem 2.65.)

SOLUTION

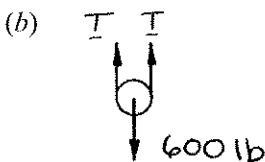
Free-Body Diagram of Pulley



$$+\uparrow \Sigma F_y = 0: 2T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{2}(600 \text{ lb})$$

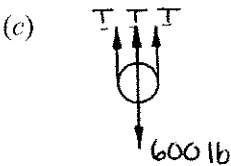
$$T = 300 \text{ lb} \quad \blacktriangleleft$$



$$+\uparrow \Sigma F_y = 0: 2T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{2}(600 \text{ lb})$$

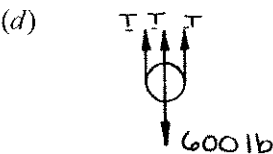
$$T = 300 \text{ lb} \quad \blacktriangleleft$$



$$+\uparrow \Sigma F_y = 0: 3T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{3}(600 \text{ lb})$$

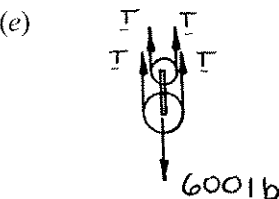
$$T = 200 \text{ lb} \quad \blacktriangleleft$$



$$+\uparrow \Sigma F_y = 0: 3T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{3}(600 \text{ lb})$$

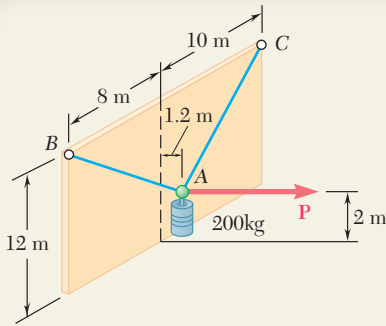
$$T = 200 \text{ lb} \quad \blacktriangleleft$$



$$+\uparrow \Sigma F_y = 0: 4T - (600 \text{ lb}) = 0$$

$$T = \frac{1}{4}(600 \text{ lb})$$

$$T = 150.0 \text{ lb} \quad \blacktriangleleft$$



SAMPLE PROBLEM 2.9

A 200-kg cylinder is hung by means of two cables AB and AC, which are attached to the top of a vertical wall. A horizontal force \mathbf{P} perpendicular to the wall holds the cylinder in the position shown. Determine the magnitude of \mathbf{P} and the tension in each cable.

SOLUTION

Free-body Diagram. Point A is chosen as a free body; this point is subjected to four forces, three of which are of unknown magnitude.

Introducing the unit vectors \mathbf{i} , \mathbf{j} , \mathbf{k} , we resolve each force into rectangular components.

$$\mathbf{P} = P\mathbf{i} \quad (1)$$

$$\mathbf{W} = -mg\mathbf{j} = -(200 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j} = -(1962 \text{ N})\mathbf{j}$$

In the case of \mathbf{T}_{AB} and \mathbf{T}_{AC} , it is necessary first to determine the components and magnitudes of the vectors \overrightarrow{AB} and \overrightarrow{AC} . Denoting by λ_{AB} the unit vector along AB, we write

$$\overrightarrow{AB} = -(1.2 \text{ m})\mathbf{i} + (10 \text{ m})\mathbf{j} + (8 \text{ m})\mathbf{k} \quad AB = 12.862 \text{ m}$$

$$\lambda_{AB} = \frac{\overrightarrow{AB}}{AB} = -0.09330\mathbf{i} + 0.7775\mathbf{j} + 0.6220\mathbf{k}$$

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = -0.09330T_{AB}\mathbf{i} + 0.7775T_{AB}\mathbf{j} + 0.6220T_{AB}\mathbf{k} \quad (2)$$

Denoting by λ_{AC} the unit vector along AC, we write in a similar way

$$\overrightarrow{AC} = -(1.2 \text{ m})\mathbf{i} + (10 \text{ m})\mathbf{j} - (10 \text{ m})\mathbf{k} \quad AC = 14.193 \text{ m}$$

$$\lambda_{AC} = \frac{\overrightarrow{AC}}{AC} = -0.08455\mathbf{i} + 0.7046\mathbf{j} - 0.7046\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = -0.08455T_{AC}\mathbf{i} + 0.7046T_{AC}\mathbf{j} - 0.7046T_{AC}\mathbf{k} \quad (3)$$

Equilibrium Condition. Since A is in equilibrium, we must have

$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{P} + \mathbf{W} = 0$$

or, substituting from (1), (2), (3) for the forces and factoring \mathbf{i} , \mathbf{j} , \mathbf{k} ,

$$\begin{aligned} &(-0.09330T_{AB} - 0.08455T_{AC} + P)\mathbf{i} \\ &+ (0.7775T_{AB} + 0.7046T_{AC} - 1962 \text{ N})\mathbf{j} \\ &+ (0.6220T_{AB} - 0.7046T_{AC})\mathbf{k} = 0 \end{aligned}$$

Setting the coefficients of \mathbf{i} , \mathbf{j} , \mathbf{k} equal to zero, we write three scalar equations, which express that the sums of the x , y , and z components of the forces are respectively equal to zero.

$$(\Sigma F_x = 0:) \quad -0.09330T_{AB} - 0.08455T_{AC} + P = 0$$

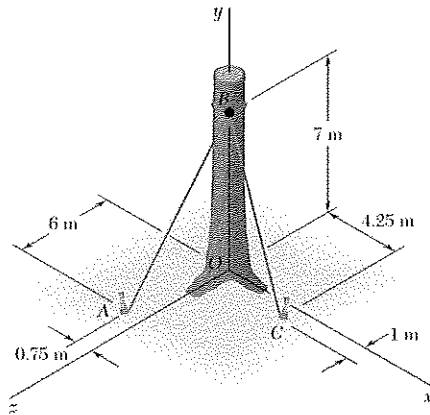
$$(\Sigma F_y = 0:) \quad +0.7775T_{AB} + 0.7046T_{AC} - 1962 \text{ N} = 0$$

$$(\Sigma F_z = 0:) \quad +0.6220T_{AB} - 0.7046T_{AC} = 0$$

Solving these equations, we obtain

$$P = 235 \text{ N} \quad T_{AB} = 1402 \text{ N} \quad T_{AC} = 1238 \text{ N} \quad \blacktriangleleft$$

PROBLEM 3.22



Before the trunk of a large tree is felled, cables AB and BC are attached as shown. Knowing that the tensions in cables AB and BC are 555 N and 660 N, respectively, determine the moment about O of the resultant force exerted on the tree by the cables at B .

SOLUTION

We have

$$\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{F}_B$$

where

$$\mathbf{r}_{B/O} = (7 \text{ m})\mathbf{j}$$

$$\mathbf{F}_B = \mathbf{T}_{AB} + \mathbf{T}_{BC}$$

$$\begin{aligned} \mathbf{T}_{AB} &= \lambda_{BA} \mathbf{T}_{AB} \\ &= \frac{-(0.75 \text{ m})\mathbf{i} - (7 \text{ m})\mathbf{j} + (6 \text{ m})\mathbf{k}}{\sqrt{(0.75)^2 + (7)^2 + (6)^2} \text{ m}} (555 \text{ N}) \end{aligned}$$

$$\begin{aligned} \mathbf{T}_{BC} &= \lambda_{BC} \mathbf{T}_{BC} \\ &= \frac{(4.25 \text{ m})\mathbf{i} - (7 \text{ m})\mathbf{j} + (1 \text{ m})\mathbf{k}}{\sqrt{(4.25)^2 + (7)^2 + (1)^2} \text{ m}} (660 \text{ N}) \end{aligned}$$

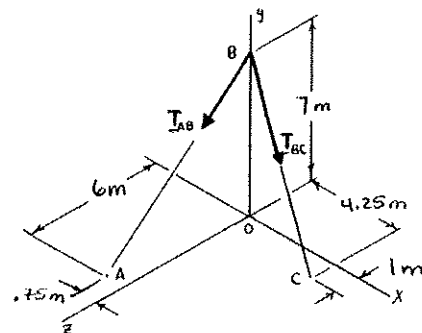
$$\begin{aligned} \mathbf{F}_B &= [-(45.00 \text{ N})\mathbf{i} - (420.0 \text{ N})\mathbf{j} + (360.0 \text{ N})\mathbf{k}] \\ &\quad + [(340.0 \text{ N})\mathbf{i} - (560.0 \text{ N})\mathbf{j} + (80.0 \text{ N})\mathbf{k}] \\ &= (295.0 \text{ N})\mathbf{i} - (980.0 \text{ N})\mathbf{j} + (440.0 \text{ N})\mathbf{k} \end{aligned}$$

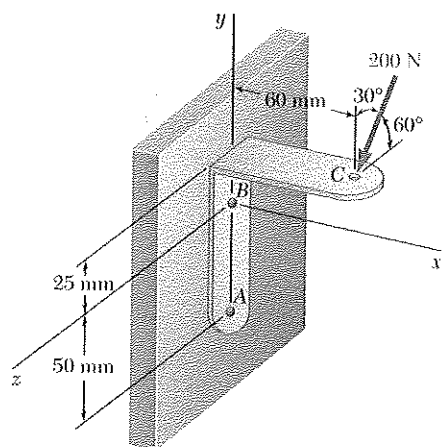
and

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 7 & 0 \\ 295 & 980 & 440 \end{vmatrix} \text{ N} \cdot \text{m}$$

$$= (3080 \text{ N} \cdot \text{m})\mathbf{i} - (2070 \text{ N} \cdot \text{m})\mathbf{k}$$

$$\text{or } \mathbf{M}_O = (3080 \text{ N} \cdot \text{m})\mathbf{i} - (2070 \text{ N} \cdot \text{m})\mathbf{k} \quad \blacktriangleleft$$





PROBLEM 3.21

A 200-N force is applied as shown to the bracket ABC . Determine the moment of the force about A .

SOLUTION

We have

$$\mathbf{M}_A = \mathbf{r}_{C/A} \times \mathbf{F}_C$$

where

$$\mathbf{r}_{C/A} = (0.06 \text{ m})\mathbf{i} + (0.075 \text{ m})\mathbf{j}$$

$$\mathbf{F}_C = -(200 \text{ N})\cos 30^\circ \mathbf{j} + (200 \text{ N})\sin 30^\circ \mathbf{k}$$

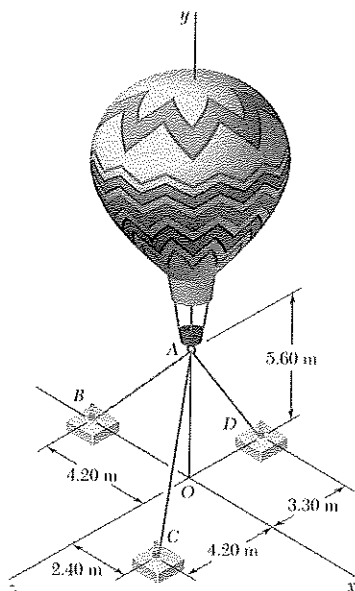
Then

$$\begin{aligned} \mathbf{M}_A &= 200 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.06 & 0.075 & 0 \\ 0 & -\cos 30^\circ & \sin 30^\circ \end{vmatrix} \\ &= 200[(0.075 \sin 30^\circ)\mathbf{i} - (0.06 \sin 30^\circ)\mathbf{j} - (0.06 \cos 30^\circ)\mathbf{k}] \end{aligned}$$

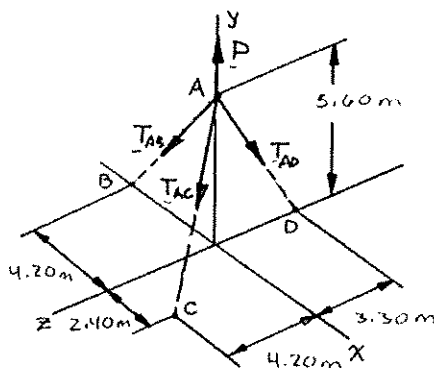
$$\text{or } \mathbf{M}_A = (7.50 \text{ N} \cdot \text{m})\mathbf{i} - (6.00 \text{ N} \cdot \text{m})\mathbf{j} - (10.39 \text{ N} \cdot \text{m})\mathbf{k} \quad \blacktriangleleft$$

PROBLEM 2.99

Three cables are used to tether a balloon as shown. Determine the vertical force P exerted by the balloon at A knowing that the tension in cable AB is 259 N.



SOLUTION



The forces applied at A are:

T_{AB} , T_{AC} , T_{AD} , and P

where $P = P\mathbf{j}$. To express the other forces in terms of the unit vectors \mathbf{i} , \mathbf{j} , \mathbf{k} , we write

$$\overrightarrow{AB} = -(4.20 \text{ m})\mathbf{i} - (5.60 \text{ m})\mathbf{j} \quad AB = 7.00 \text{ m}$$

$$\overrightarrow{AC} = (2.40 \text{ m})\mathbf{i} - (5.60 \text{ m})\mathbf{j} + (4.20 \text{ m})\mathbf{k} \quad AC = 7.40 \text{ m}$$

$$\overrightarrow{AD} = -(5.60 \text{ m})\mathbf{j} - (3.30 \text{ m})\mathbf{k} \quad AD = 6.50 \text{ m}$$

and

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB} \frac{\overrightarrow{AB}}{AB} = (-0.6\mathbf{i} - 0.8\mathbf{j})T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = T_{AC} \frac{\overrightarrow{AC}}{AC} = (0.32432 - 0.75676\mathbf{j} + 0.56757\mathbf{k})T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\lambda_{AD} = T_{AD} \frac{\overrightarrow{AD}}{AD} = (-0.86154\mathbf{j} - 0.50769\mathbf{k})T_{AD}$$

PROBLEM 2.99 (Continued)

Equilibrium condition $\Sigma F = 0: \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$

Substituting the expressions obtained for \mathbf{T}_{AB} , \mathbf{T}_{AC} , and \mathbf{T}_{AD} and factoring \mathbf{i} , \mathbf{j} , and \mathbf{k} :

$$(-0.6T_{AB} + 0.32432T_{AC})\mathbf{i} + (-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P)\mathbf{j} \\ + (0.56757T_{AC} - 0.50769T_{AD})\mathbf{k} = 0$$

Equating to zero the coefficients of \mathbf{i} , \mathbf{j} , \mathbf{k} :

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \quad (1)$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \quad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \quad (3)$$

Setting $T_{AB} = 259 \text{ N}$ in (1) and (2), and solving the resulting set of equations gives

$$T_{AC} = 479.15 \text{ N}$$

$$T_{AD} = 535.66 \text{ N}$$

$$P = 1031 \text{ N} \uparrow \blacktriangleleft$$