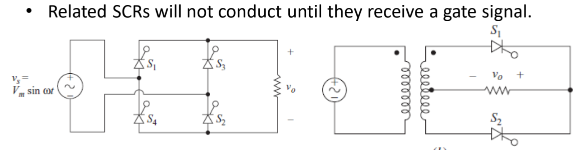
POWER ELECTRONICS

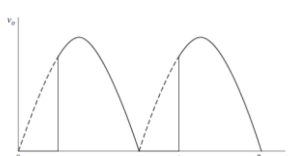
CHAPTER 5.
AC-DC Controlled Fullwave Rectifiers

Controlled Full-Wave Rectifiers

- A versatile method of controlling the output of a full-wave rectifier is to substitute controlled switches such as thyristors (SCRs) for the diodes.
- Output is controlled by adjusting the delay angle of each SCR, resulting in an output voltage that is adjustable over a limited range.
- Controlled full-wave rectifiers are shown in Fig.
- Related SCRs will not conduct until they receive a gate signal.



Controlled Full-Wave Rectifiers



- The delay angle, **α** is the angle interval between the forward biasing of the SCR and the gate signal application.
- If the delay angle is zero, the rectifiers behave exactly as uncontrolled rectifiers with diodes.
- The discussion that follows generally applies to both bridge and center-tapped rectifiers.

Controlled Full-Wave Rectifiers with Resistive Load

- The output voltage waveform for a controlled full-wave rectifier with a resistive load is shown in Fig.
- The average component of this waveform is determined from

$$\boxed{V_o = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{\pi}(1 + \cos\alpha)}$$

- Average output current is then

$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R}(1 + \cos\alpha)$$

- The power delivered to the load is a function of the input voltage, the delay angle, and the load components; P = $I^2_{rms}R$ is used to determine the power in a resistive load.

Controlled Full-Wave Rectifiers with Resistive Load

- I_{rms} is :

$$I_{\text{rms}} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}\left(\frac{V_m}{R}\sin\omega t\right)^2 d(\omega t)}$$
$$= \frac{V_m}{R}\sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}}$$

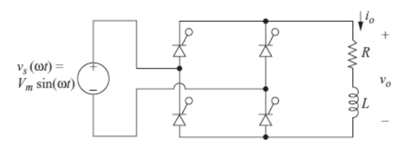
- The rms current in the source is the same as the rms current in the load.

Controlled Full-Wave Rectifiers with R/L Load

- Away to control the output of a half-wave rectifier is to use an SCR instead of a diode.
- Figure shows a basic controlled full-wave rectifier with a RL load.

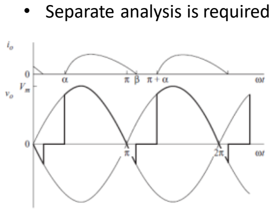
Two conditions must be met before the SCR can conduct:

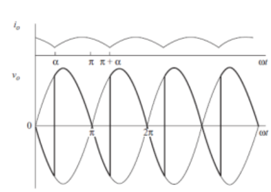
1. The SCR must be forward-biased (vSCR 0).
2. A current must be applied to the gate of the SCR.



Controlled Full-Wave Rectifiers with R/L Load

- Load current for a controlled full-wave rectifier with an RL load can be either continuous or discontinuous depending on the angle **α** and **β** at which the input signal is applied to the gate of SCR.
- Separate analysis is required for each.





Controlled Full-Wave Rectifiers with R/L Load

Analysis for discontinuous current:

- With S1 and S2 on, the load voltage is equal to the source voltage.

$$i(\omega t) = i_f(\omega t) + i_n(\omega t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-\omega t/\omega\tau}$$

The constant A is determined from the initial condition $i(\alpha) = 0$:

$$i(\alpha) = 0 = \frac{V_m}{Z}\sin(\alpha - \theta) + Ae^{-\alpha/\omega\tau}$$

$$A = \left[-\frac{V_m}{Z}\sin(\alpha - \theta)\right]e^{\alpha/\omega\tau}$$

Controlled Full-Wave Rectifiers with R/L Load

Analysis for discontinuous current:

- With S1 and S2 on, the load voltage is equal to the source voltage.

$$i(\omega t) = i_f(\omega t) + i_n(\omega t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-\omega t/\omega\tau}$$

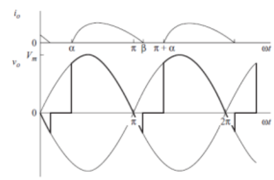
$$i_o(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)\, e^{-(\omega t - \alpha)/\omega\tau}\right] \quad \text{for } \alpha \le \omega t \le \beta$$

where $Z = \sqrt{R^2 + (\omega L)^2}$ $\quad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$ $\quad$ and $\quad \tau = \frac{L}{R}$

- Similarly, the extinction angle **β**, is defined as the angle at which the current returns to zero, as in the case of the uncontrolled rectifier. When **ωt = β** ,

$$i(\beta) = 0 = \frac{V_m}{Z}\left[\sin(\beta - \theta) - \sin(\alpha - \theta)\, e^{(\alpha - \beta)/\omega\tau}\right]$$

Controlled Full-Wave Rectifiers with R/L Load



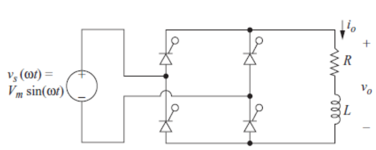
- The above current function becomes zero at **ωt = β**.
- If **β < π + α**, the current remains at zero until **ωt = π + α** when gate signals are applied to S3 and S4 which are then forward-biased and begin to conduct.
- This mode of operation is called discontinuous current.

π + α > β → discontinues current

Exercise

A controlled full-wave bridge rectifier has a source of 120 V rms at 60 Hz, R = 10Ω, L = 20 mH, and α = 60°. Determine:

a) An expression for load current,
b) The average load current, and
c) The power absorbed by the load.



Exercise

$$V_m = \frac{120}{\sqrt{2}} = 169.7 \text{ V}$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + [(377)(0.02)]^2} = 12.5\ \Omega$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left[\frac{(377)(0.02)}{10}\right] = 0.646 \text{ rad}$$

$$\omega\tau = \frac{\omega L}{R} = \frac{(377)(0.02)}{10} = 0.754 \text{ rad}$$

$$\alpha = 60° = 1.047 \text{ rad}$$

Solving $i_o(\beta) = 0$ numerically for β in below eq.:

$$i(\beta) = 0 = \frac{V_m}{Z}\left[\sin(\beta - \theta) - \sin(\alpha - \theta)\, e^{(\alpha - \beta)/\omega\tau}\right]$$

β = 3.78 rad (216°).

Exercise

a) Since **π + α = 3.14 + 1.047 = 4.19 > β = 3,78 rad,**

the current is discontinuous. The current function of the controlled fullwave rectifier for discontinues current is:

$$i_o(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)\, e^{-(\omega t - \alpha)/\omega\tau}\right] \qquad \text{for } \alpha \le \omega t \le \beta$$

$$i_o(\omega t) = 13.6 \sin(\omega t - 0.646) - 21.2 e^{-\omega t/0.754}\ \text{A} \qquad \text{for } \alpha \le \omega t \le \beta$$

b) Average load current is determined from the numerical integration of

$$I_o = \frac{1}{\pi}\int_{\alpha}^{\beta} i_o(\omega t)\, d(\omega t) = 7.05\ \text{A}$$

Exercise

c) Power absorbed by the load occurs in the resistor and is computed from $I^2_{rms}R$, where:

$$I_{\text{rms}} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\beta} i_o(\omega t)\, d(\omega t)} = 8.35\ \text{A}$$

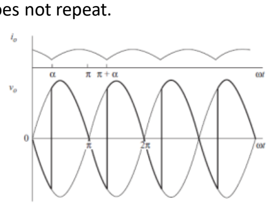
$$P = (8.35)^2(10) = 697\ \text{W}$$

Controlled Full-Wave Rectifiers

Analysis for continuous current:

- If the load current is still positive at **ωt = π + α** when gate signals are applied to S3 and S4 in the above analysis, S3 and S4 are turned on and S1 and S2 are forced off.
- Since the initial condition for current in the second half-cycle is not zero, the current function does not repeat.

- For an RL load with continuous current, the steady-state current and voltage waveforms are generally as shown.



Controlled Full-Wave Rectifiers with R/L Load

- The current at **ωt = π + α** must be greater than zero for continuous-current operation.

$$i_o(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)\, e^{-(\omega t - \alpha)/\omega\tau}\right] \qquad \text{for } \alpha \le \omega t \le \beta$$

$$i(\pi + \alpha) \ge 0$$

$$\sin(\pi + \alpha - \theta) - \sin(\pi + \alpha - \theta)\, e^{-(\pi + \alpha - \alpha)/\omega\tau} \ge 0$$

- Solving for **α**, using $\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$\alpha \le \tan^{-1}\left(\frac{\omega L}{R}\right)$$ is found for continuous current

- Above equation or **β < π + α** can be used to check whether the load current is ***continuous*** or ***discontinuous***.

Controlled Full-Wave Rectifiers with R/L Load

- A method for determining the output voltage and current for the continuous current case is to use the ***Fourier series***.
- The Fourier series for the voltage waveform for continuous-current case is expressed in general form as:

$$v_o(\omega t) = V_o + \sum_{n=2,4,}^{\infty} V_n \cos(n\omega_0 t + \theta_n)$$

- The dc (average) value is:

$$\boxed{V_o = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{2V_m}{\pi}\cos\alpha}$$

Controlled Full-Wave Rectifiers with R/L Load

- The Fourier series for the voltage amplitudes of the ac terms for continuous-current case are calculated from

$$V_n = \sqrt{a_n^2 + b_n^2}$$

where

$$a_n = \frac{2V_m}{\pi}\left[\frac{\cos(n+1)\alpha}{n+1} - \frac{\cos(n-1)\alpha}{n-1}\right]$$

$$b_n = \frac{2V_m}{\pi}\left[\frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1}\right]$$

$$n = 2, 4, 6, \ldots$$

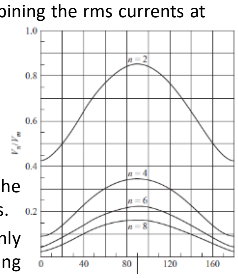
Controlled Full-Wave Rectifiers with R/L Load

- The rms current is determined by combining the rms currents at each frequency.

$$I_{\text{rms}} = \sqrt{I_o^2 + \sum_{n=2,4,6\ldots}^{\infty}\left(\frac{I_n}{\sqrt{2}}\right)^2}$$

where

$$I_o = \frac{V_o}{R} \qquad I_n = \frac{V_n}{Z_n} = \frac{V_n}{|R + jn\omega L|}$$

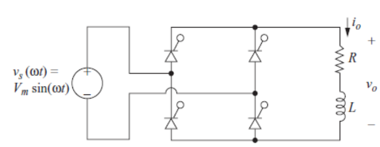


- As the harmonic number increases, the impedance for the inductance increases.
- Therefore, it is necessary to solve for only a few terms of the series for calculating the rms current.
- If the inductor is large, the ac terms will become small, and the current is essentially dc.

Exercise

A controlled full-wave bridge rectifier has a source of 120 V rms at 60 Hz, with RL load where R = 10 Ω, L = 100 mH, and α = 60°. Determine:

a) Verify that the load current is continuous.
b) Determine the dc (average) component of the current.
c) Determine the power absorbed by the load.



Exercise

$$\alpha \le \tan^{-1}\left(\frac{\omega L}{R}\right)$$

a) This equation will be used to check if current is continuous.

$$\tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left[\frac{(377)(0.1)}{10}\right] = 75°$$

$$\alpha = 60° < 75° \qquad \therefore \text{ continuous current}$$

b) The voltage across the load is expressed in terms of the Fourier series. The dc term is:

$$V_0 = \frac{2V_m}{\pi}\cos\alpha = \frac{2\sqrt{2}(120)}{\pi}\cos(60°) = 54.0 \text{ V}$$

Exercise

c) The amplitudes of the ac terms are computed and summarized in the following table,

n	a_n	b_n	V_n	Z_n	I_n
0 (dc)	—	—	54.0	10	5.40
2	−90	−93.5	129.8	76.0	1.71
4	46.8	−18.7	50.4	151.1	0.33
6	−3.19	32.0	32.2	226.4	0.14

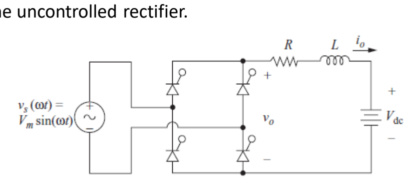
The rms current is:

$$I_{\text{rms}} = \sqrt{(5.40)^2 + \left(\frac{1.71}{\sqrt{2}}\right)^2 + \left(\frac{0.33}{\sqrt{2}}\right)^2 + \left(\frac{0.14}{\sqrt{2}}\right)^2 + \cdots} \approx 5.54 \text{ A}$$

Power is computed from $I^2_{rms}R$ as: P = (5.54) ^{2}(10) = **307 W**

Controlled Full-Wave Rectifierswith R/L and Source Load

- The controlled rectifier with a load that is a series resistance, inductance, and dc voltage as shown below is analyzed much like the uncontrolled rectifier.



- The dc source can be a battery or opposite electromotor force (emf) of a DC motor.

Controlled Full-Wave Rectifierswith R/L and Source Load

- For the controlled rectifier, the SCRs may be turned on at any time that they are forward-biased, which is at an angle

$$\alpha \ge \sin^{-1}\left(\frac{V_{dc}}{V_m}\right)$$

- For the continuous-current case, the bridge output voltage is the same as before.
- The average bridge output voltage is: $V_o = \frac{2V_m}{\pi}\cos\alpha$

$$\boxed{V_o = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{2V_m}{\pi}\cos\alpha}$$

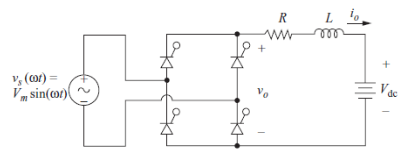
Controlled Full-Wave Rectifiers with R/L and Source Load

- The ac voltage terms can be calculated same as the controlled rectifier with an RL load with Fourier analysis.
- The average load current is: $$I_o = \frac{V_o - V_{dc}}{R}$$
- Power absorbed by the dc voltage is: $P_{dc} = I_o V_{dc}$
- Power absorbed by the resistor in the load is $I^2_{rms}R$.

Exercise

The controlled rectifier has an ac source of 240 V rms at 60 Hz, Vdc = 100 V, R = 5 Ω, and an inductor large enough to cause continuous current.

Determine the delay angle such that the power absorbed by the dc source is 1000 W.



Exercise

Power absorbed by the dc voltage is: $P_{dc} = I_o V_{dc}$

P_{dc}= 1000W=100.i_o → i_o= 10 A

- The required output voltage is determined from

$$V_o = V_{dc} + I_oR = 100 + (10)(5) = 150 \text{ V}$$

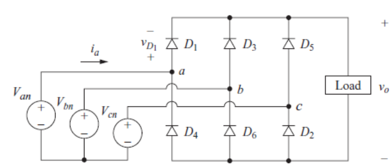
- The delay angle which will produce a 150 V dc output from the rectifier is determined from

$$V_o = \frac{2V_m}{\pi} \cos\alpha$$

$$\alpha = \cos^{-1}\left(\frac{V_o\pi}{2V_m}\right) = \cos^{-1}\left[\frac{(150)(\pi)}{2\sqrt{2}(240)}\right] = 46°$$

Three-phase Rectifiers

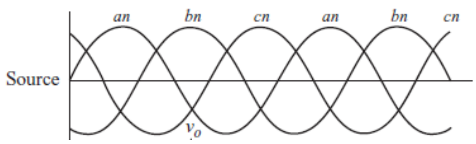
- Three-phase rectifiers are commonly used in industry to produce a dc voltage and current for large loads.
- The three-phase full-bridge rectifier is shown in Fig.



- The three-phase voltage source is ***balanced*** and has phase sequence **a-b-c**.

Three-phase Rectifiers

- If the voltage sources have the same amplitude and frequency and are out of phase with each other by **120°**, the voltages are said to be ***balanced***.

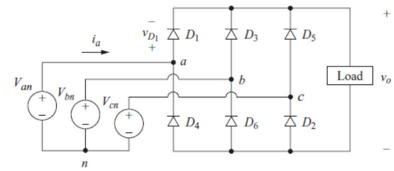


Three-phase Rectifiers

- Some basic observations about the circuit assuming the diodes are ideal are as follows:

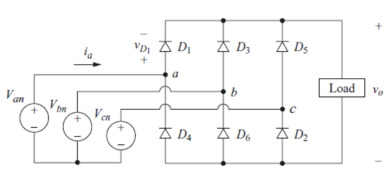
1. Kirchhoff's voltage law around any path shows that only one diode in the top half of the bridge may conduct at one time (D1, D3, or D5).

   The diode that is conducting will have its anode connected to the phase voltage that is highest at that instant.



Three-phase Rectifiers

2. Kirchhoff's voltage law also shows that only one diode in the bottom half of the bridge may conduct at one time (D2, D4, or D6).
   - The diode that is conducting will have its cathode connected to the phase voltage that is lowest at that instant.

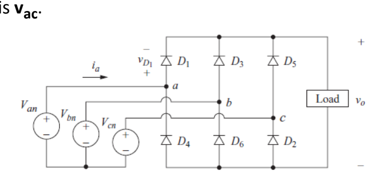


Three-phase Rectifiers

3. The output voltage across the load is one of the line-to-line voltages of the source. For example, when **D1** and **D2** are on, the output voltage is $\mathbf{v_{ac}}$.

   Furthermore, the diodes that are on are determined by which line-to-line voltage is the highest at that instant.

   For example, when $\mathbf{v_{ac}}$ is the highest line-to-line voltage, the output is $\mathbf{v_{ac}}$.



Three-phase Rectifiers

4. There are six combinations of line-to-line voltages (three phases taken two at a time).

   Considering one period of the source to be 360°, a transition of the highest line-to-line voltage must take place every 360°/6 = 60°.

   Because of the six transitions that occur for each period of the source voltage, the circuit is called a ***six-pulse rectifier***.

5. The fundamental frequency of the output voltage is **6ω**, where **ω** is the frequency of the three-phase source.

Three-phase Rectifiers

- The current in a conducting diode is the same as the load current.
- To determine the current in each phase of the source, Kirchhoff's current law is applied at nodes a, b, and c,

$$i_a = i_{D_1} - i_{D_4}$$
$$i_b = i_{D_3} - i_{D_6}$$
$$i_c = i_{D_5} - i_{D_2}$$

- Since each diode conducts one-third of the time, resulting in:

$$\boxed{\begin{aligned} I_{D,\text{avg}} &= \frac{1}{3} I_{o,\text{avg}} \\ I_{D,\text{rms}} &= \frac{1}{\sqrt{3}} I_{o,\text{rms}} \\ I_{s,\text{rms}} &= \sqrt{\frac{2}{3}} I_{o,\text{rms}} \end{aligned}}$$

Three-phase Rectifiers

- The periodic output voltage is defined as:

$$v_o(\omega t) = V_{m,L-L}\sin(\omega t) \qquad \text{for } \pi/3 \le \omega t \le 2\pi/3$$

- The coefficients for the sine terms are zero from symmetry, enabling the Fourier series for the output voltage to be expressed as:

$$v_o(t) = V_o + \sum_{n=6,12,18\ldots}^{\infty} V_n \cos(n\omega_0 t + \pi)$$

- The average or dc value of the output voltage is:

$$\boxed{V_0 = \frac{1}{\pi/3}\int_{\pi/3}^{2\pi/3} V_{m,L-L}\sin(\omega t)\,d(\omega t) = \frac{3V_{m,L-L}}{\pi} = 0.955\,V_{m,L-L}}$$

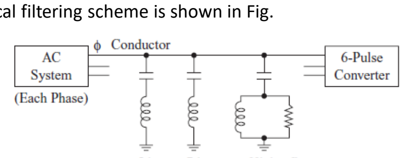
Where $\mathbf{V_{m,L-L}}$ is the peak line-to-line voltage of the three-phase source.

Three-phase Rectifiers

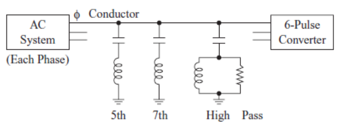
- The current will also have a dc and ac part with harmonics as:

$$i_a(t) = \frac{2\sqrt{3}}{\pi} I_o \left(\cos\omega_0 t - \frac{1}{5}\cos 5\omega_0 t + \frac{1}{7}\cos 7\omega_0 t - \frac{1}{11}\cos 11\omega_0 t + \frac{1}{13}\cos 13\omega_0 t - \cdots\right)$$

- Because these harmonic currents may present problems in the ac system, filters are frequently necessary to prevent these harmonics from entering the ac system.
- A typical filtering scheme is shown in Fig.



Three-phase Rectifiers

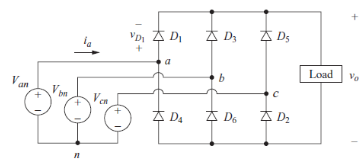


- Resonant filters are used to provide a path to ground for the fifth and seventh harmonics, which are the two lowest and are the strongest in amplitude.
- Higher-order harmonics are reduced with the high-pass filter.
- These filters prevent the harmonic currents from propagating through the ac power system.
- Filter components are chosen such that the impedance to the power system frequency is large.

Exercise

The three-phase rectifier of Fig. 4-16a has a three-phase source of 480 V rms line-to-line, and the load is a 25-Ω resistance in series with a 50-mH inductance. Determine

a) the dc level of the output voltage,
b) The dc and first ac term of the load current,
c) The average and rms current in the diodes,
d) The rms current in the source, and



Exercise

a) The dc output voltage of the bridge is:

$$V_o = \frac{3V_{m,L-L}}{\pi} = \frac{3\sqrt{2}\,(480)}{\pi} = 648 \text{ V}$$

b) The average load current is:

$$I_o = \frac{V_o}{R} = \frac{648}{25} = 25.9 \text{ A}$$

The first ac voltage term is:

$$I_6 = \frac{V_6}{Z_6} = \frac{0.0546 V_m}{\sqrt{R^2 + (6\omega L)^2}} = \frac{0.0546\sqrt{2}(480)}{\sqrt{25^2 + [6(377)(0.05)]^2}} = \frac{37.0 \text{ V}}{115.8\ \Omega} = 0.32 \text{ A}$$

$$I_{6,\text{rms}} = \frac{0.32}{\sqrt{2}} = 0.23 \text{ A}$$

This and other ac terms are much smaller than the dc term and can be neglected.

Exercise

c) The rms load current is approximately the same as average current since the ac terms are small.

$$I_{D,\text{avg}} = \frac{I_o}{3} = \frac{25.9}{3} = 8.63 \text{ A}$$

$$I_{D,\text{rms}} = \frac{I_{o,\text{rms}}}{\sqrt{3}} \approx \frac{25.9}{\sqrt{3}} = 15.0 \text{ A}$$

d) The rms source current is:

$$I_{s,\text{rms}} = \left(\sqrt{\frac{2}{3}}\right) I_{o,\text{rms}} \approx \left(\sqrt{\frac{2}{3}}\right) 25.9 = 21.2 \text{ A}$$