Linear Models

Introduction to R - Day 3

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A little bit of theoretical background

Linear models background

The data:

- Random sample of n data points ($Y_i, X_{i1}, X_{i2}, \ldots, X_{ip}, i = 1 \ldots n$)
  - $X_{i1}, X_{i2}, \ldots, X_{ip}$ are p **independent predictor** variables
  - Y_i is the **dependent** observation and **response variable**

Example

Data set of grassland productivity (Y_i) depending on plant species richness (X_{i1}) and the location/measurement site (X_{i1}).

Aim of linear model:

- Do the predictors (species richness, location) have a significant effect on the response (grassland productivity)?

Linear models background

The data:

- Random sample of n data points ($Y_i, X_{i1}, X_{i2}, \ldots, X_{ip}, i = 1...n$)
    - $X_{i1}, X_{i2}, \ldots, X_{ip}$ are p **independent predictor** variables
    - Y_i is the **dependent** observation and **response variable**

The model:

$$Y_i = \beta_0 + \beta_1 * X_{1,i} + \beta_2 * X_{2,i} + \epsilon$$

- Y_i value of response variable
- $\beta_0, \beta_1, \beta_2$ model coefficients
- $X_{1,i}$ value of predictor X_1
- $X_{2,i}$ value of predictor X_2
- ϵ error term

Linear models background

Linear regression

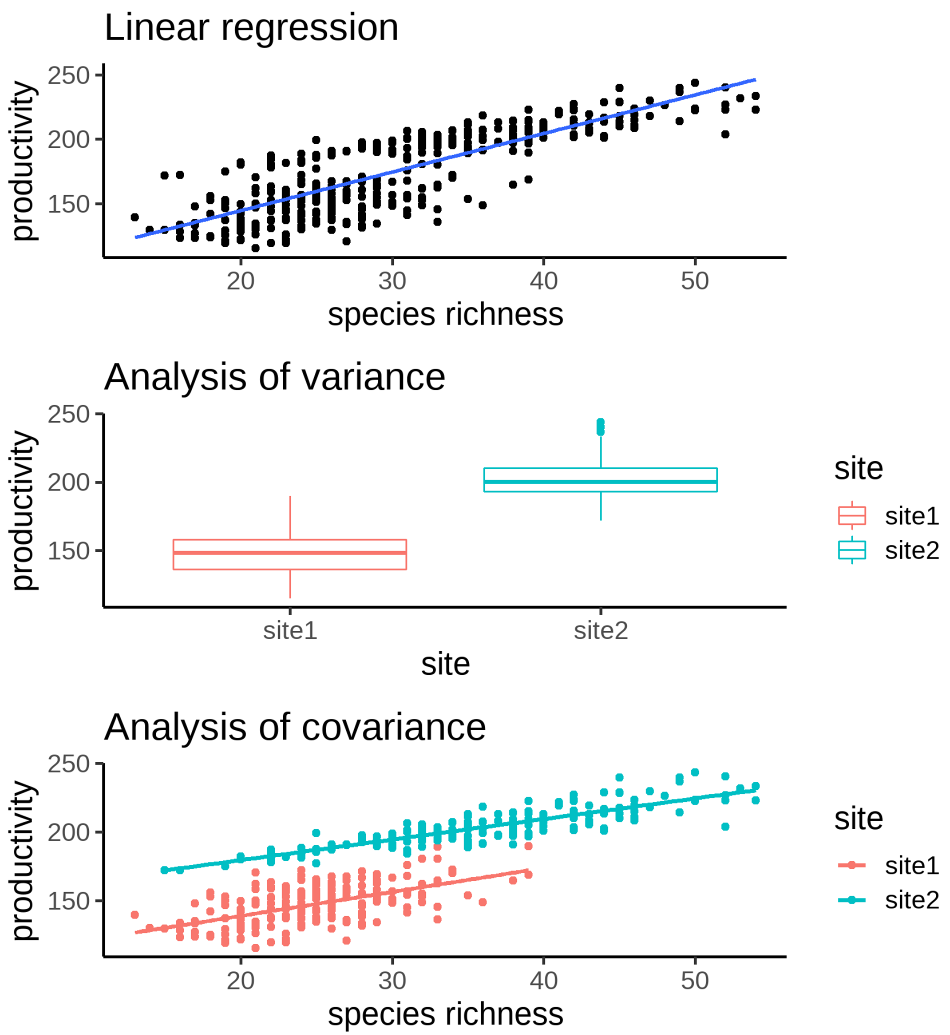
- numerical response
- numerical predictor(s)

Analysis of variance

- numerical response
- categorical predictor(s)

Analysis of covariance

- numerical response
- numerical and categorical predictor(s)



Linear models background

Linear relation with two predictors X_1 and X_2 ($i = 1...n$)

Without interaction

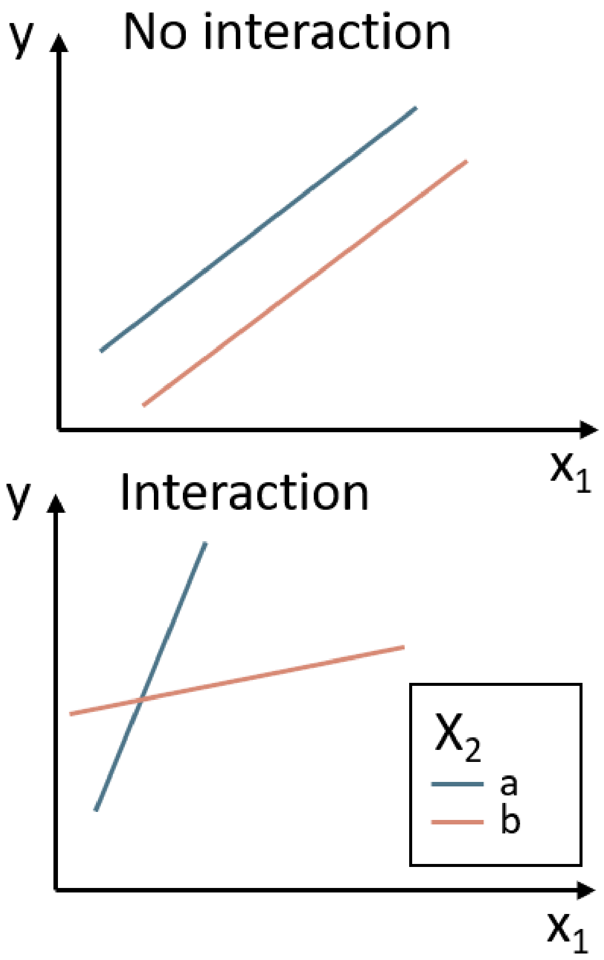
$$Y_i = \beta_0 + \beta_1 * X_{1,i} + \beta_2 * X_{2,i} + \epsilon$$

With interaction

$$Y_i = \beta_0 + \beta_1 * X_{1,i} + \beta_2 * X_{2,i} + \beta_3 * X_{1,i} * X_{2,i} + \epsilon$$

with

- Y_i value of response variable
- $\beta_0, \beta_1, \beta_2$ model coefficients
- $X_{1,i}$ value of predictor X_1
- $X_{2,i}$ value of predictor X_2
- ϵ error term



Goodness of fit linear model

Model residuals

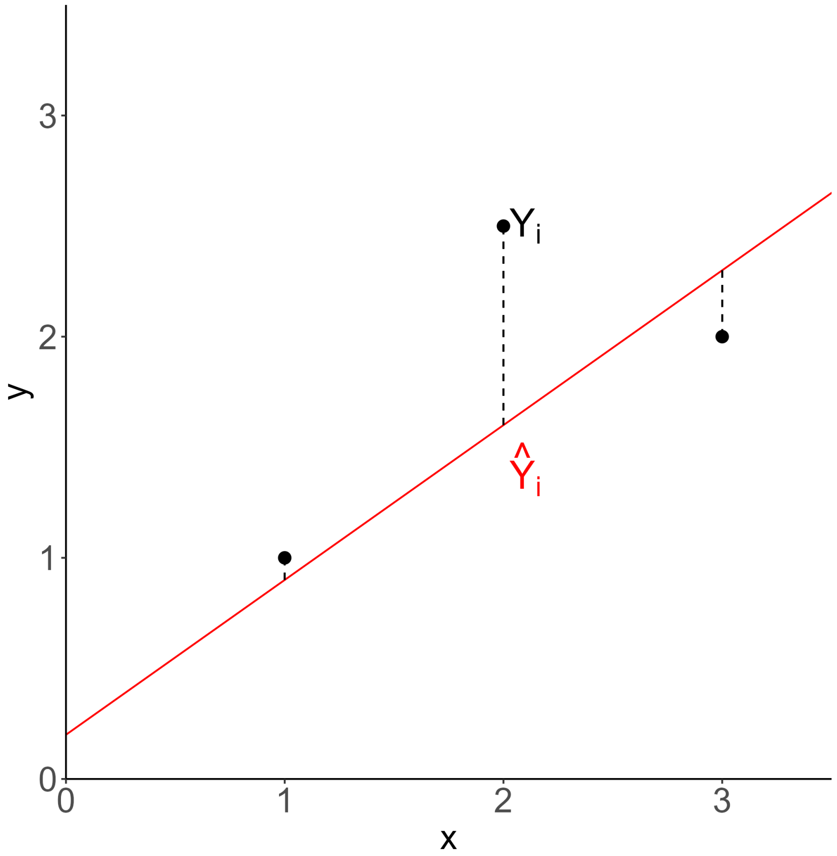
$$Y_i - \hat{Y}_i$$

Residual sum of squares (RSS)

$$RSS = \sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$$

Aim of the model fitting:

Find the model coefficients that lead to the lowest sum of squares.



Assumptions for linear models

Finding the model estimates that lead to the lowest residual sum of squares works only if:

1) Residuals are normally distributed

2) Residual variance is constant

3) No strong outliers

This has to be checked for every linear model!

If assumptions are not fullfilled, a different model approach has to be chosen.

How to fit linear models in R

The data

We use productivity data from grassland sites.
The data set is called `prod` and has 3 variables: site, productivity and richness (species richness):

```
prod
```

```
## # A tibble: 400 x 3
##   site  richness productivity
##   <fct>    <dbl>        <dbl>
## 1 site1       23         122.
## 2 site1       26         158.
## 3 site1       24         165
## 4 site1       25         165.
## # ... with 396 more rows
```

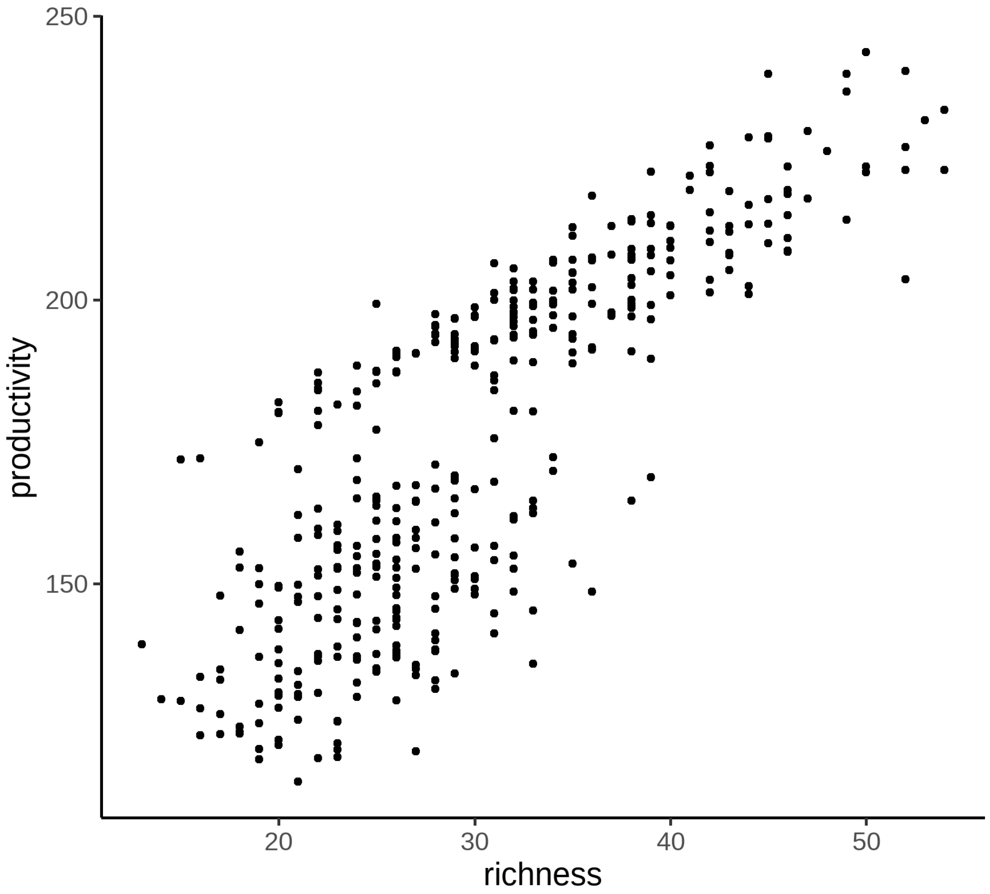
The variable site is a factor with two levels: site1 and site2

```
levels(prod$site) # get all levels of the factor
## [1] "site1" "site2"
```

Linear regression in R

Linear regression in R

H_1: Species richness has an effect on productivity



Linear regression in R

Use the `lm` function to fit a linear model in R.

The general structure of the function call is like this:

```
lm ( formula = Y ~ X, data = dat )
```

with

- Y being the response variable
- X being the predictor(s)
- dat being the name of the data

Multiple predictors can be added with `+` and `:` depending on interaction:

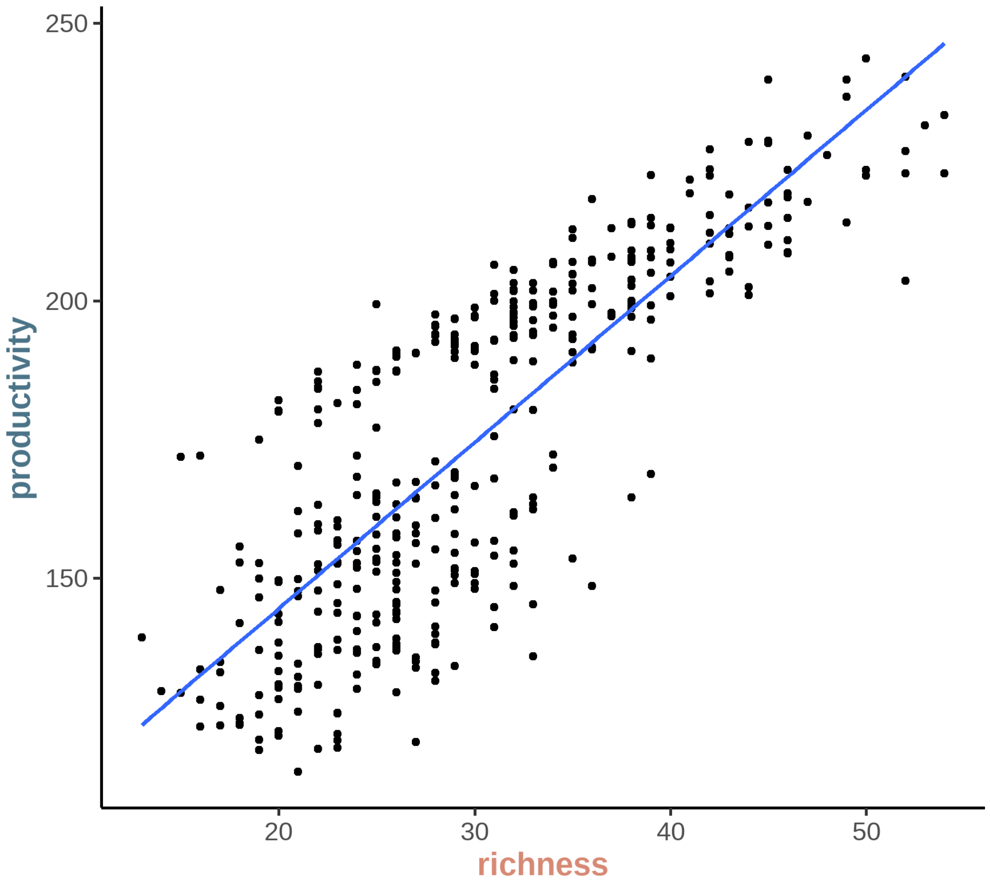
- `Y ~ X1 + X2` tests effects without interaction
- `Y ~ X1 + X2 + X1:X2` tests single effects and interaction between `X1` and `X2`

Linear regression in R

Let's fit a linear model to test our hypothesis

```
lm ( formula = Y ~ X, data = dat )
```

```
prod_lm <- lm(productivity ~ richness,
              data = prod)
```



Linear regression in R

Is the effect of richness on productivity significant?

Or in other words

Does the model with richness as predictor significantly reduce the residual sum of squares?

Hypothesis testing using F-Tests

Compare the complex model with a simple model that does not contain the predictor

H_0: The error variance in the simple model is not significantly higher than in the more complex model

- H_0 accepted: simplification was justified → use simple model without predictor
- H_0 rejected: simplification reduced explanatory power → use complex model with predictor

Linear regression in R

Hypothesis testing using F-Tests in R

H_0: The error variance in the simple model is not significantly higher than in the more complex model

`drop1` deletes single terms from the model and performs an F-test:

```
drop1(prod_lm, test = "F")
## Single term deletions
##
## Model:
## productivity ~ richness
##          Df Sum of Sq    RSS    AIC F value    Pr(>F)
## <none>                132779 2326.0
## richness  1    254119 386899 2751.8  761.71 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Result: Reject H_0 → Richness increases productivity $(F_{1,398} = 761.71, p < 0.001)$

Extracting the coefficients

Look at model coefficients using the model `summary`

```
summary(prod_lm)
##
## Call:
## lm(formula = productivity ~ richness, data = prod)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -47.593 -13.683    0.029   13.970   42.295
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  84.6982     3.3987   24.92   <2e-16 ***
## richness      2.9938     0.1085   27.60   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.27 on 398 degrees of freedom
## Multiple R-squared:  0.6568,    Adjusted R-squared:  0.6559
## F-statistic: 761.7 on 1 and 398 DF,  p-value: < 2.2e-16
```

Summary table

```
summary(prod_lm)
##
## Call:
## lm(formula = productivity ~ richness, data = prod)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -47.593 -13.683    0.029   13.970   42.295
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  84.6982     3.3987   24.92   <2e-16 ***
## richness      2.9938     0.1085   27.60   <2e-16 ***
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## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
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## Residual standard error: 18.27 on 398 degrees of freedom
## Multiple R-squared:  0.6568,    Adjusted R-squared:  0.6559
## F-statistic: 761.7 on 1 and 398 DF,  p-value: < 2.2e-16
```

Calculated model

Distribution of residuals (errors)

Summary table

```
summary(prod_lm)
##
## Call:
## lm(formula = productivity ~ richness, data = prod)
##
##
##                        Median              Max
## -47.593 -13.683    0.029  13.970  42.295
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  84.6982     3.3987   24.92   <2e-16 ***
## richness      2.9938     0.1085   27.60   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
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## F-statistic: 761.7 on 1 and 398 DF,  p-value: < 2.2e-16
```

Model coefficients:
1. Intercept = 84.7
2. Slope = 3

Standard error of coefficients

Summary table

```
summary(prod_lm)
##
## Call:
## lm(formula = productivity ~ richness, data = prod)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -47.593 -13.683    0.029   13.970   42.295
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  84.6982     3.3987   24.92   <2e-16 ***
## richness      2.9938     0.1085   27.60   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.27 on 398 degrees of freedom
## Multiple R-squared:  0.6568,    Adjusted R-squared:  0.6559
## F-statistic: 761.7 on 1 and 398 DF,  p-value: < 2.2e-16
```

t-statistics: ratio of coefficients to their standard error

Summary table

```
summary(prod_lm)
##
## Call:
## lm(formula = productivity ~ richness, data = prod)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -47.593 -13.683    0.029   13.970   42.295
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  84.6982     3.3987   24.92   <2e-16 ***
## richness      2.9938     0.1085   27.60   <2e-16 ***
## ---
## Signif. co                 01 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 18.27 on 398 degrees of freedom
## Multiple R-squared:  0.6568,    Adjusted R-squared:  0.6559
## F-statistic: 761.7 on 1 and 398 DF,  p-value: < 2.2e-16
```

Coefficient of determination R^2

Test model assumptions

Test model assumptions to make sure that:

1) Residuals are normally distributed

2) Variance is constant (homogeneous)

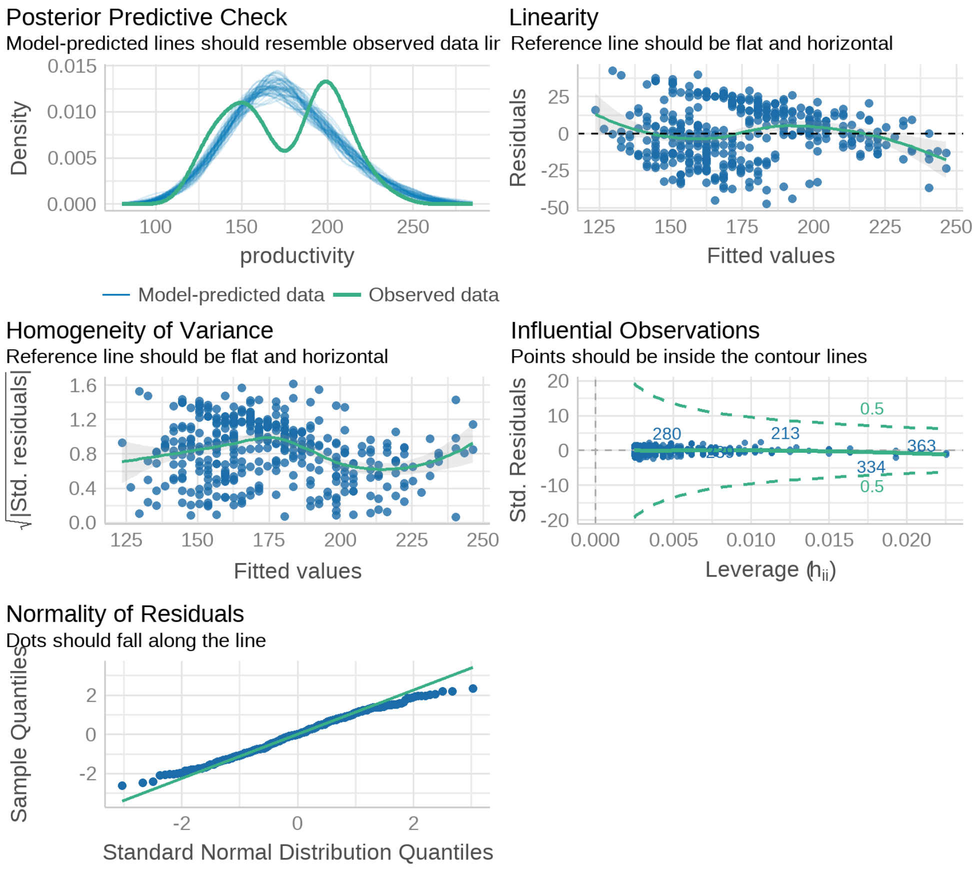
3) There are no strong outliers or very influential observations

We can check that by looking at diagnostic plots

```
# install.packages("performance")
performance::check_model(prod_lm)
```

Test model assumptions

```
performance::check_model(prod_lm)
```



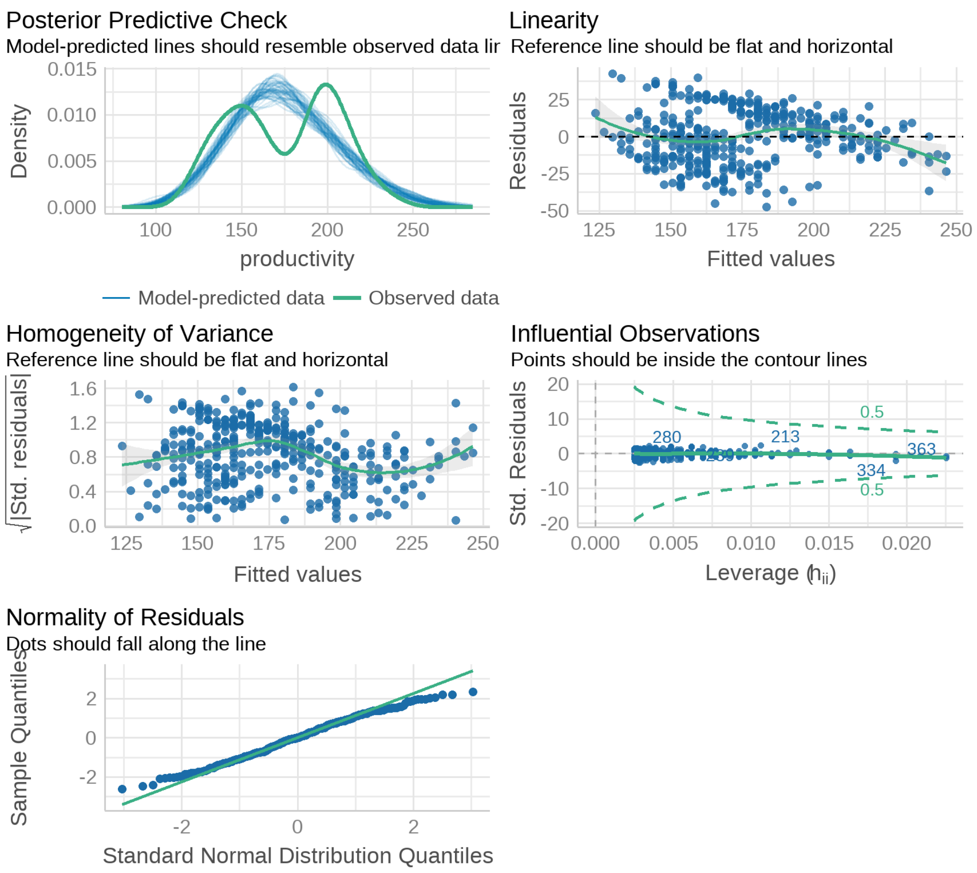
Test model assmptions

Check the assumptions

- Residuals are normally distributed ✓
- Variance is constant (homogeneous) ✓
- There are no strong outliers or very influential observations ✓

Checking diagnostic plots needs some experience

- Real life data (almost) never perfectly fit the assumptions
- Linear models are to some extent robust against violations of the assumptions



Plot the model

Option 1

Add the regression line directly as a ggplot layer

Use `geom_smooth()` to add the model directly:

```
ggplot(prod, aes(x = richness, y =
productivity)) +
 geom_point() +
geom_smooth(method = "lm", se = FALSE)
```

Plot the model

Option 2

Extract coefficients (slope + intercept) of the model and add the regression line

```
# These are the coefficients of the lm
prod_lm$coefficients
## (Intercept)    richness
##   84.698163    2.993774
```

```
intercept <- prod_lm$coefficients[1]
slope <- prod_lm$coefficients[2]
```

Plot the model

Option 2

Extract coefficients (slope + intercept) of the model and add the regression line

```
intercept <- prod_lm$coefficients[1]
slope <- prod_lm$coefficients[2]
```

Add a line defined by slope and intercept using `geom_abline()`:

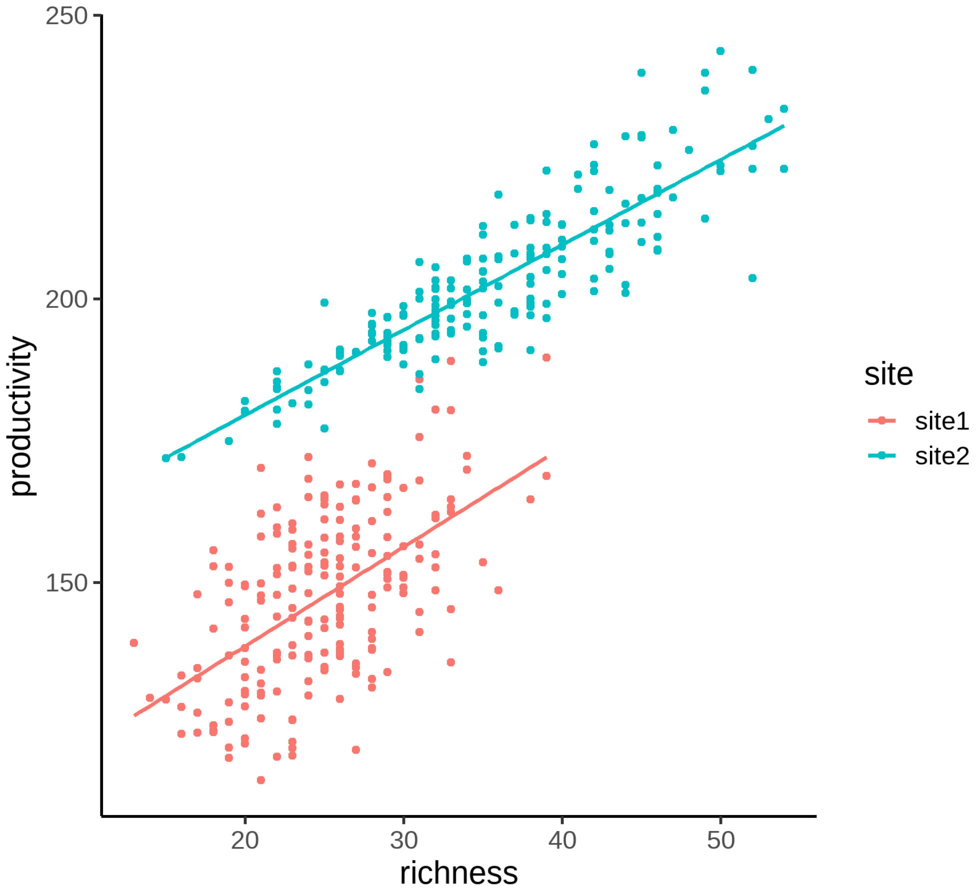
```
ggplot(prod, aes(x=richness, y=productivity))+
  geom_point() +
  geom_abline(slope = slope,
              intercept = intercept)
```

Analysis of covariance

Analysis of covariance

One categorical and one numerical predictor variable

Two possible models:



Analysis of covariance

Fit the model with `lm`

Without interaction

Species richness has an effect on productivity and there is a difference between sites.

```
prod_lm2a <- lm(productivity ~ richness + site, data = prod)
```

With interaction

Species richness has an effect on productivity and the effect differs between sites.

```
prod_lm2b <- lm(productivity ~ richness + site + richness:site, data = prod)
```

Analysis of covariance

Are the effects significant?

Without interaction

```
drop1(prod_lm2a, test = "F") # no interaction
## Single term deletions
##
## Model:
## productivity ~ richness + site
##          Df Sum of Sq    RSS    AIC F value    Pr(>F)
## <none>                 40064 1848.7
## richness  1     42908  82973 2137.9  425.18 < 2.2e-16 ***
## site      1     92715 132779 2326.0  918.72 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Both, the removal of site and richness significantly increase the model's RSS → both site and richness are significant predictors

Analysis of covariance

Are the effects significant?

With interaction

```
drop1(prod_lm2b, test = "F") # interaction
## Single term deletions
##
## Model:
## productivity ~ richness + site + richness:site
##               Df Sum of Sq   RSS    AIC F value Pr(>F)
## <none>                     39850 1848.6
## richness:site  1    214.53 40064 1848.7  2.1319 0.1451
```

The removal of the interaction does not significantly increase the model's RSS → the interaction is not significant and the simpler model without interaction is just as good in predicting productivity.

Extracting model coefficients

```
summary(prod_lm2a)
##
## Call:
## lm(formula = productivity ~ richness + site, data = prod)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -30.1743  -6.2895  -0.0352   5.6955  28.9276
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 108.36550    2.02582   53.49   <2e-16 ***
## richness      1.56699    0.07599   20.62   <2e-16 ***
## sitesite2    38.78575    1.27962   30.31   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.05 on 397 degrees of freedom
## Multiple R-squared:  0.8964,    Adjusted R-squared:  0.8959
## F-statistic:  1718 on 2 and 397 DF,  p-value: < 2.2e-16
```

Extracting model coefficients

```
Call:
lm(formula = productivity ~ richness + site, data = prod)

Residuals:
     Min       1Q   Median
-30.1743  -6.2895  -0.0352

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 108.36550    2.02582   53.49   <2e-16 ***
richness      1.56699    0.07599   20.62   <2e-16 ***
sitesite2    38.78575    1.27962   30.31   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 10.05 on 397 degrees of freedom
Multiple R-squared:  0.8964,    Adjusted R-squared:  0.8959
F-statistic:  1718 on 2 and 397 DF,  p-value: < 2.2e-16
```

Intercept site1 = 108.4

Slope = 1.6

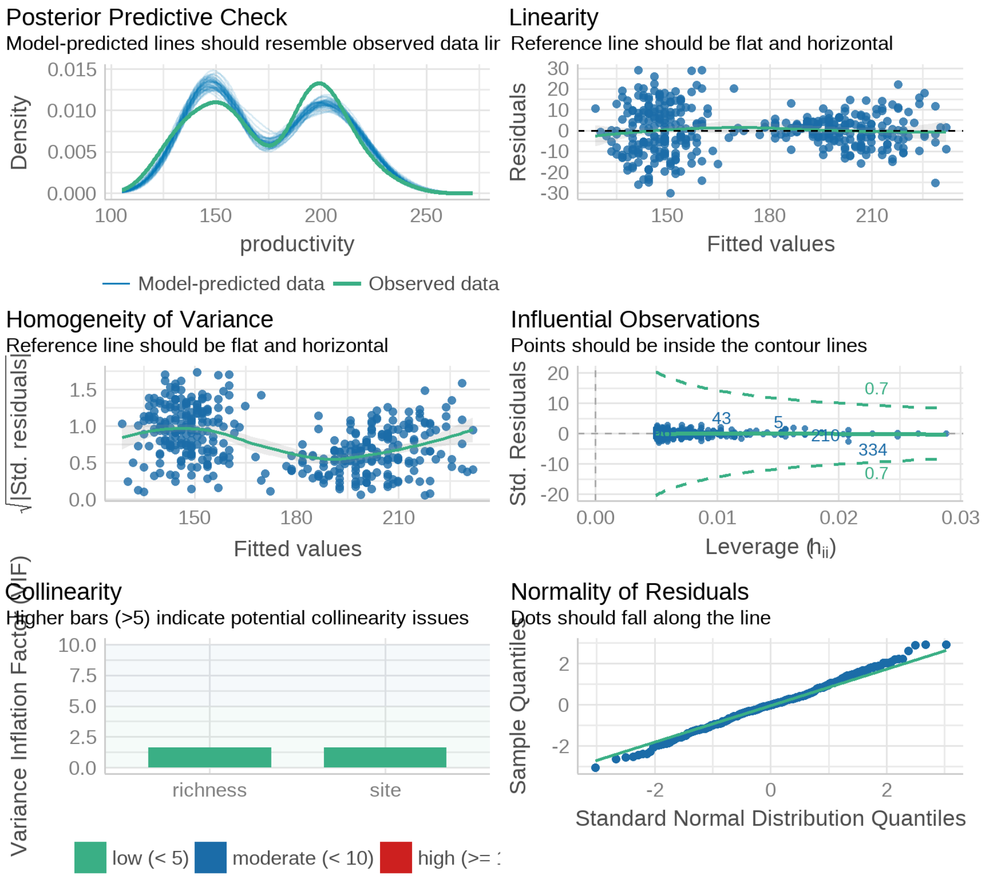
Intercept site2 = 108.4 + 38.8

Test model assumptions

Check the assumptions

- Residuals are normally distributed ✓
- Variance is constant (homogeneous) ✓
- There are no strong outliers or very influential observations ✓

```
performance::check_model(prod_lm2a)
```



Plot the model

If we want to be precise, we could not use `geom_smooth()` for this model because it plots the full model with interaction between the predictors

```
ggplot(prod, aes(x = richness,
                 y = productivity,
                 color = site)) +
  geom_point() +
  geom_smooth(method = "lm")
```

This plot is not 100% appropriate if the model you present is without interaction

Plot the model

Extract coefficients and add regression line

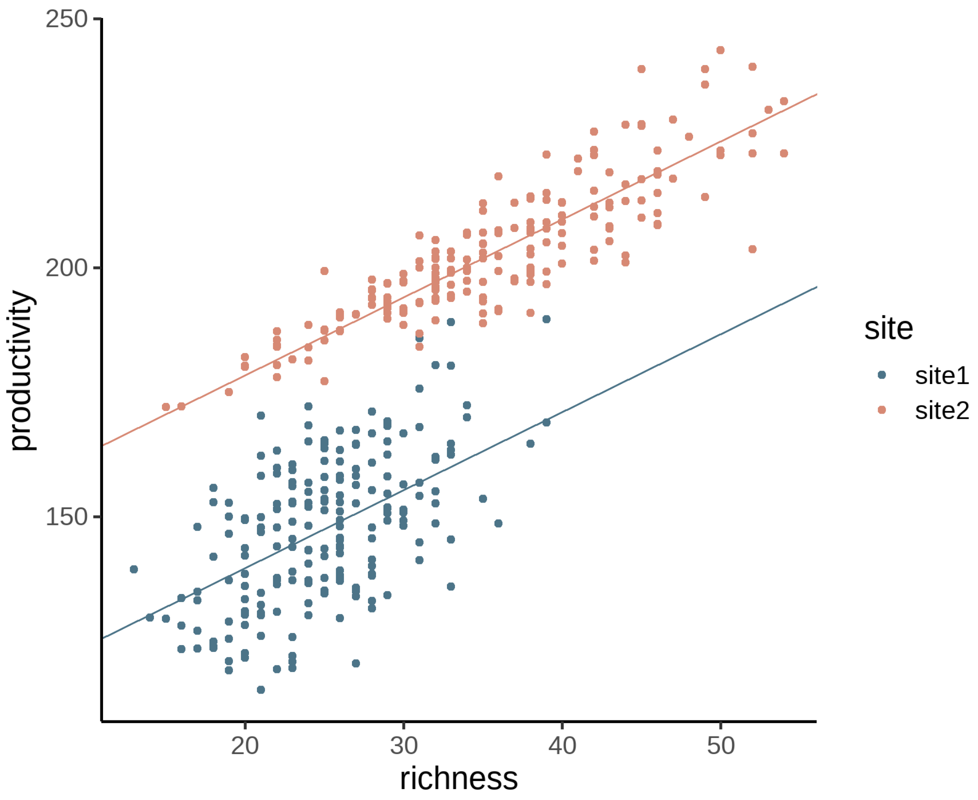
```
# these are the model coefficients
prod_lm2a$coefficients
## (Intercept)    richness   sitesite2
##  108.365497    1.566994   38.785753
```

```
slope <- prod_lm2a$coefficients[2]
intercept1 <- prod_lm2a$coefficients[1]
intercept2 <- prod_lm2a$coefficients[1] + prod_lm2a$coefficients[3]
```

Plot the model

Extract coefficients and add regression line

```
ggplot(prod, aes(x = richness,
                 y = productivity,
                 color = site)) +
  geom_point() +
  scale_color_manual(
    values = c("#4C7488", "#D78974")) +
  geom_abline(
    slope = slope,
    intercept = intercept1,
    color = "#4C7488"
  ) +
  geom_abline(
    slope = slope,
    intercept = intercept2,
    color = "#D78974"
  )
```



Plot the model

Option 2: Use the `predict()` function

```
# step 1: create some data to predict from
pred_data <- tidyr::expand_grid(
  richness = min(prod$richness):max(prod$richness),
  site = c("site1", "site2")
)
```

```
## # A tibble: 84 x 2
##   richness site
##      <int> <chr>
## 1       13 site1
## 2       13 site2
## 3       14 site1
## # ... with 81 more rows
```

- `tidyr::expand_grid` returns a tibble with all combinations of site and richness given as input
  - The `tidyr` package is part of the tidyverse
- The column names of the tibble that you create in this step have to correspond to the predictor names in the linear model

Plot the model

Option 2: Use the `predict()` function

```
# step 1: create some data to predict from
pred_data <- tidyr::expand_grid(
  richness = min(prod$richness):max(prod$richness),
  site = c("site1", "site2")
)
```

```
# step2: predict productivity values from pred_data
predictions <- predict(prod_lm2a, newdata = pred_data)
```

- `predict` uses the `prod_lm2a` model and the data produced with `expand_grid` to predict productivity values for each combination of site and richness.

Plot the model

Option 2: Use the `predict()` function

```
# step 1: create some data to predict from
pred_data <- tidyr::expand_grid(
  richness = min(prod$richness):max(prod$richness),
  site = c("site1", "site2")
)
```

```
# step2: predict productivity values from pred_data
predictions <- predict(prod_lm2a, newdata = pred_data)
```

```
# step 3: add predictions to the tibble
pred_data$productivity <- predictions
```

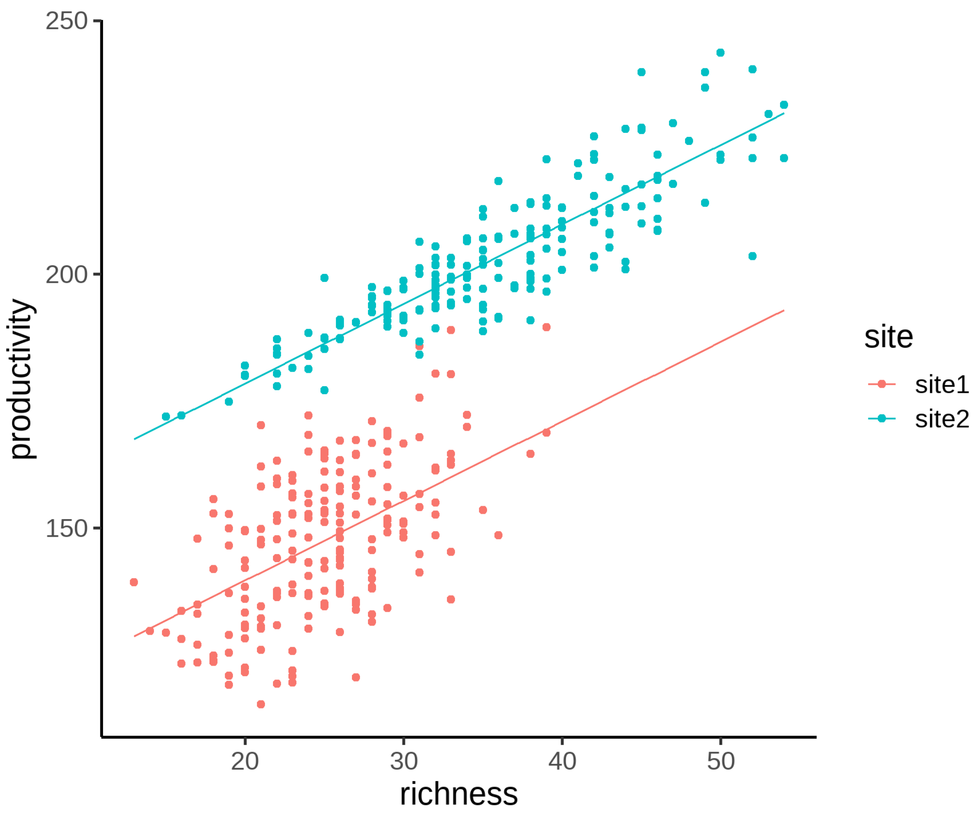
Plot the model

Option 2: Use the `predict()` function

Add a new `geom_*()` layer to the plot from the prediction data:

```
# step 4: Add predictions with geom_line
ggplot(prod, aes(x = richness,
                 y = productivity,
                 color = site)) +
  geom_point() +
  geom_line(data = pred_data)
```

- The aesthetic mapping is inherited from the top level `ggplot` call to the `geom_line`



Plot the model

Option 2: Use the `predict()` function

Summary of general workflow

- **Step 1:** Use `expand_grid` to create a tibble with values for all predictor variables (the columns have to have the exact same name as the original data)
- **Step 2:** Use `predict` to predict response variable with the model using input predictor values from tibble in step 1
- **Step 3:** Add predictions as column to the tibble from step 1
- **Step 4:** Add predictions to the plot using a new `geom_line()` layer based on the tibble from step 3

Analysis of variance (Anova)

Anova

Only categorical predictors.

Example: Data set `chickwts` about the weight of chickens fed with different diets.

```
chickwts
```

```
## # A tibble: 71 x 2
##   weight feed
##    <dbl> <fct>
## 1    179 horsebean
## 2    160 horsebean
## 3    136 horsebean
## 4    227 horsebean
## # ... with 67 more rows
```

Anova

H_1: The diet has an effect on the weight of chicken.

Fit a linear model with `lm` and test the significance of the predictor:

```
lm_chicken <- lm(weight ~ feed, data = chickwts)
drop1(lm_chicken, test = "F")
## Single term deletions
##
## Model:
## weight ~ feed
##        Df Sum of Sq    RSS    AIC F value    Pr(>F)
## <none>               195556 574.39
## feed    5    231129 426685 619.78  15.365 5.936e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

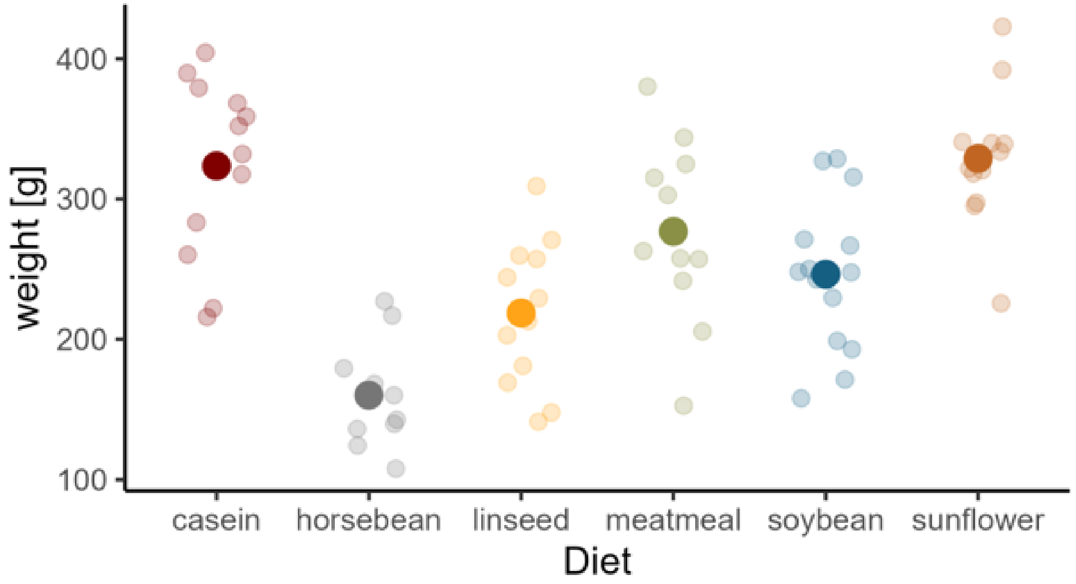
Removing the predictor `feed` from the model significantly increases the RSS → The model with `feed` explains the data better than the model without `feed`, so the effect of `feed` on the chicken weight is significant.

Extracting model coefficients

```
summary(lm_chicken)
##
## Call:
## lm(formula = weight ~ feed, data = chickwts)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -123.909  -34.413    1.571   38.170  103.091
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)    323.583     15.834  20.436  < 2e-16 ***
## feedhorsebean -163.383     23.485  -6.957 2.07e-09 ***
## feedlinseed   -104.833     22.393  -4.682 1.49e-05 ***
## feedmeatmeal   -46.674     22.896  -2.039 0.045567 *
## feedsoybean    -77.155     21.578  -3.576 0.000665 ***
## feedsunflower    5.333     22.393   0.238 0.812495
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 54.85 on 65 degrees of freedom
## Multiple R-squared:  0.5417,    Adjusted R-squared:  0.5064
## F-statistic: 15.36 on 5 and 65 DF,  p-value: 5.936e-10
```

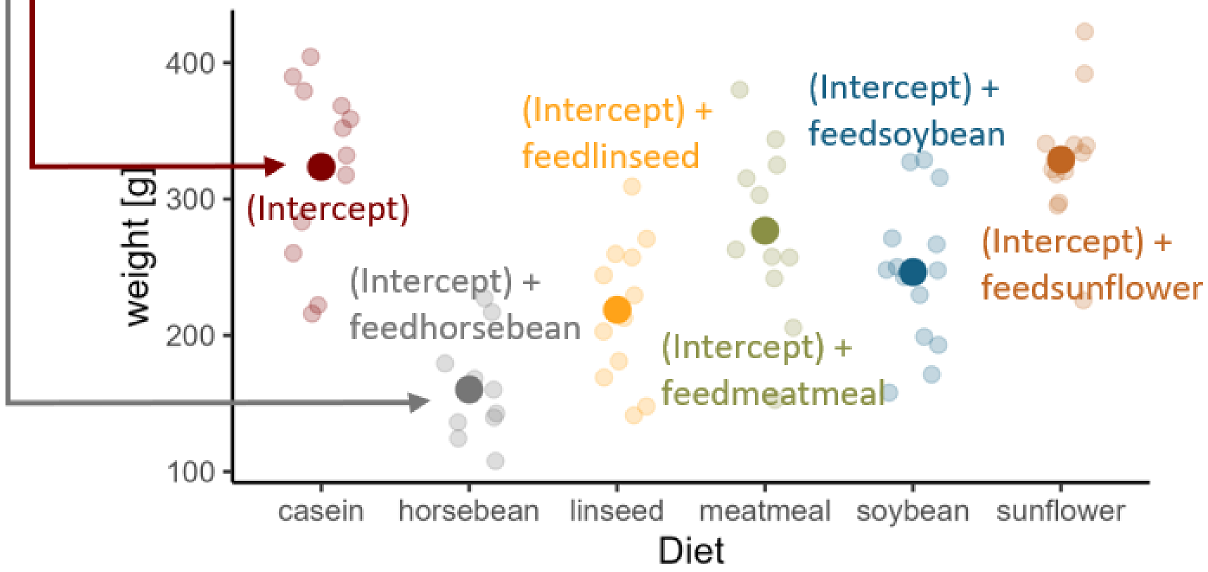
Extracting model coefficients

```
Coefficients:
               Estimate Std. Error t value Pr(>|t|)    
(Intercept)     323.583     15.834  20.436  < 2e-16 ***
feedhorsebean  -163.383     23.485  -6.957 2.07e-09 ***
feedlinseed    -104.833     22.393  -4.682 1.49e-05 ***
feedmeatmeal    -46.674     22.896  -2.039 0.045567 *  
feedsoybean     -77.155     21.578  -3.576 0.000665 ***
feedsunflower     5.333     22.393   0.238 0.812495    
---
```



Extracting model coefficients

```
Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)    323.583     15.834  20.436  < 2e-16 ***
feedhorsebean -163.383     23.485  -6.957 2.07e-09 ***
feedlinseed   -104.833     22.393  -4.682 1.49e-05 ***
feedmeatmeal   -46.674     22.896  -2.039 0.045567 *  
feedsoybean    -77.155     21.578  -3.576 0.000665 ***
feedsunflower    5.333     22.393   0.238 0.812495    
---
```

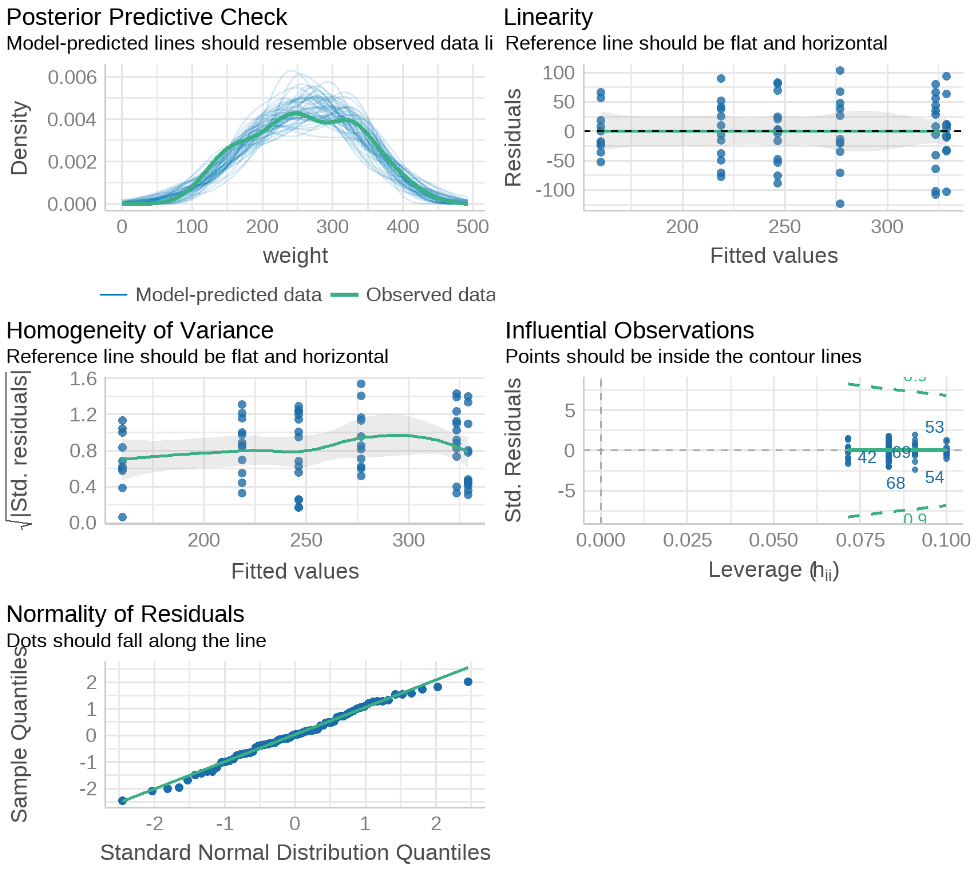


Test model assumptions

Check the assumptions

- Residuals are normally distributed ✔
- Variance is constant (homogeneous) ✔
- There are no strong outliers or very influential observations ✔

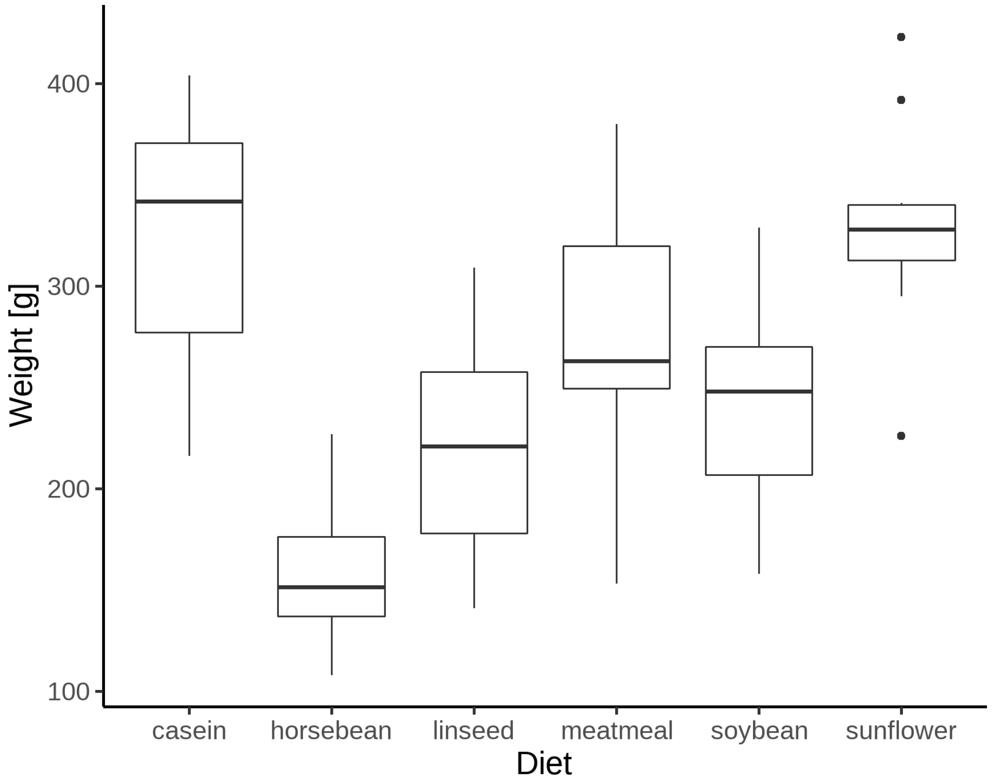
```
performance::check_model(lm_chicken)
```



Plot results

Plot anova results e.g. in a boxplot:

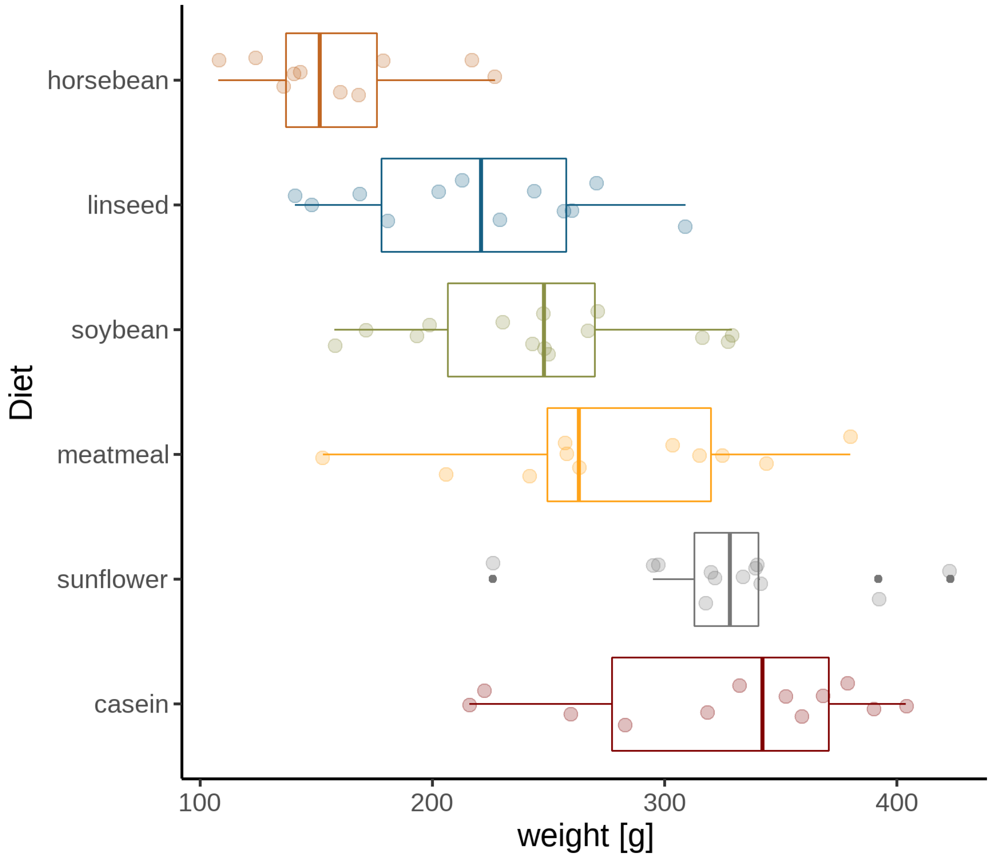
```
ggplot(chickwts, aes(feed, weight)) +
  geom_boxplot() +
  labs(x = "Diet", y= "Weight [g]")
```



Plot results

If you want a slightly nicer box plot, you could do something like this:

```
chickwts %>%
  mutate(feed = as.factor(feed)) %>%
  mutate(feed = fct_reorder(feed, -weight)) %>%
  ggplot(aes(
    x = feed,
    y = weight,
    color = feed
  )) +
  geom_boxplot() +
  geom_point(
    size = 3, alpha = 0.25,
    position = position_jitter(width = 0.2,
seed = 0)
  ) +
  coord_flip() +
  ggsci::scale_color_uchicago() +
  labs(y = "weight [g]", x = "Diet") +
  theme(legend.position = "none")
```



Linear models step by step

Step 1: Explore the data with plots

Step 2: Write down a **question/hypothesis** and think of the **model** to test it

Step 3: Fit the linear model using the `lm` function:

```
lm(formula = Y ~ X1 + X2 + X1:X2, data = dat)
```

- Use `+` for additive effects and `:` for interactions between predictors

Step 4: Check model assumptions by looking at the diagnostic plots

```
performance::check_model(mod)
```

- Normally distributed residuals
- Constant variance
- No strong outliers / influential data points

Linear models step by step

Step 5: Check significant variables by conducting F-tests with `drop1`

```
drop1(mod, test = "F")
```

- If the removal of a variable significantly increases the RSS of the model, the predictor is significant

Step 6: Check effect size e.g. in the `summary` table

```
summary(mod)
```

- Extract coefficients from the model with `mod$coefficients`

Step 7: Plot model

- Regression: regression line and scatterplot
- Analysis of covariance: regression lines and scatter plot
- Analysis of variance: boxplots, barplot, mean + sem or similar

Options for model plotting

Option 1: **`geom_smooth(method = "lm")`**

- Plots a regression line with all interactions between variables
    - only use it if this is what you want to plot

Option 2: extract coefficients and use **`geom_abline()`**

- Extract slopes and intercepts from the model
    - `mod$coefficients`
- Add a `geom_abline` layer to your plot
    - `geom_abline(slope = your_slope, intercept = your_intercept)`

Options for model plotting

Option 3: Use `predict` function

- This is the most flexible plotting option
- Step 1: Create a new tibble with data to predict from
  - Column names have to be same as predictors of linear model
  - Use e.g. `tidyr::expand_grid()` to create variable combinations
- Step 2: Predict the response for all value combinations from step 1

```
predict(mod, newdata = pred_data)
```

- Step 3: Add predicted response to tibble from step 1
- Step 4: Add a `geom_line` layer with the new data to your plot

```
ggplot(orig_dat, aes(x = some_x, y = some_y, color = some_col)) +
  geom_point() +
  geom_line(data = pred_data)
```

Now you

Task 2: Linear models with the penugin data set

Find the task description here