

Counting

① unusual \rightarrow 3 'u' letters

Since a set cannot have duplicates, only one subset exists:

$\{U, N, S, A, L\}$

From these 5 letters, $5!$, or 120 different strings can be made.

② 5-card hand with 2 pairs (ie pair of one val, pair of second val, and 5th card)

- 13 different #s: $13C_2 = \frac{13!}{2!11!}$ = choosing 2 different values to create pairs

$$\frac{13!}{2!11!} \cdot \frac{4!}{2!2!} \cdot \frac{4!}{2!2!} \times 44 = 123,552 \text{ different ways}$$

\uparrow
choose 2
cards for
first pair

\uparrow
choose
2 cards for
second pair

44 cards
left

(can't choose the 4 cards you already chose plus the other suits with those values since you would then have a 3 of a kind)

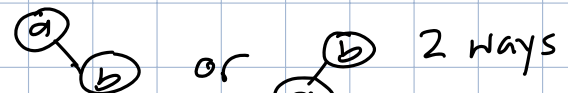
③ Address the fighting couple first. Any of the 16 songs can be played to them. $\rightarrow 16$

Now 15 songs and 6 couples left. $15C_6 : \frac{15!}{6!(15-6)!}$

$$\# \text{ of ways} = 16 \times \frac{15!}{6!(15-6)!} = 80,080 \text{ ways}$$

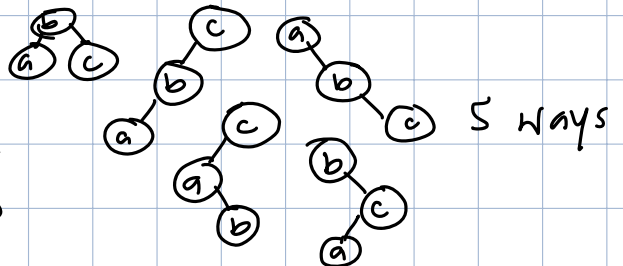
④ BST w/ 12 nodes

2 node BST:

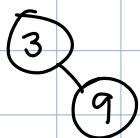


2 ways

3 node BST:



5 ways



2 values
left of 3
= 2 nodes
= 2 ways

5 values in
between 3 and 9
= 5 nodes
 $= \frac{2n!}{(n+1)!n!} = \frac{10!}{6!5!}$
= 42 ways

3 nodes right
of 9 = 3 nodes
= 5 ways

$$\text{Total ways: } (2)(42)(5) = 420 \text{ ways}$$

⑤

_____|_____|_____|_____
Nurse 1 Nurse 2 Nurse 3 Nurse 4

• 3 bars = r
• 10 stars (the patients)

$$\binom{13}{3} = {}_{13}C_3 = 286 \text{ ways}$$

If one nurse is on her break : • 2 bars = r 10 stars (the patients)

$$\binom{12}{2} = {}_{12}C_2 = 66 \text{ ways}$$

$$286 + 66 = 352 \text{ different combinations}$$