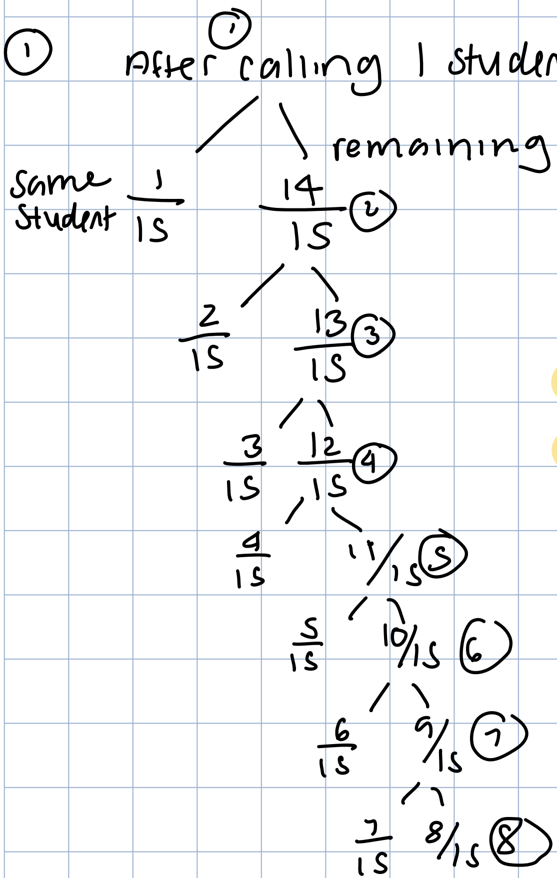
Selina Martínez

Probability



$$\overset{1}{1}\left(\overset{2}{\frac{14}{15}}\right)\left(\overset{3}{\frac{13}{15}}\right)\left(\overset{4}{\frac{12}{15}}\right)\left(\overset{5}{\frac{11}{15}}\right)\left(\overset{6}{\frac{10}{15}}\right)\left(\overset{7}{\frac{9}{15}}\right)\left(\overset{8}{\frac{8}{15}}\right)$$

$= 0.101$
$= 10.1\%$

There is a 10.1% chance that no student will have to answer more than one question

② · integer from 00000 - 99999, uniformly at random
· want integers that start with 2 odd digits + all digits are unique
· integers are even
· randomly generate 8 #s in succession

0, ~~1~~, 2, ~~3~~, 4, 5, 6, 7, 8, 9

0-99 = not possible since all even #s would have to have an even digit in the ones place

100-999 = $5 \times 4 \times 5 = 100$ possibilities
# of odd digits; # of odd digits left since digits are unique; # of even digits (since overall # has to be even)

1000 - 9999 = $5 \times 4 \times 7 \times 5 = 700$
saving an even # for last digit

10000-99999: $5 \times 4 \times 7 \times 6 \times 5 = 4200$

$100 + 700 + 4200 = 5000 \rightarrow 5000/100000 = 0.05$ chance each time

$P_5 = {}_8C_5 (0.05)^5 (0.95)^3 = 1.5 \cdot 10^{-5}$ chance $\rightarrow$ 0.0015% chance

③ · 3 six-sided, fair dice

· A = at least 2 dice show 4 or above

· B = all 3 dice show the same value

· Are A and B independent?

· If P(A) * P(B) is equal to $P(A \cap B)$, then A and B are independent.

· P(B) = 6 total ways out of 6^3 options: $\frac{6}{6^3} = \frac{1}{36}$

· P(A) = P(2 dice equal 4 or more) + P(3 dice equal 4 or more)

$$= {}_3C_2 \cdot \left(\frac{3}{6}\right)^2 \left(\frac{3}{6}\right) + {}_3C_3 \cdot \left(\frac{3}{6}\right)^3 \cdot \left(\frac{3}{6}\right) = \frac{1}{2}$$

$P(A \cap B)$ = P(all 3 dice = 4) + P(all 3 dice = 5) + P(all 3 dice = 6)

$$P(A \cap B) = \frac{1}{6^3} + \frac{1}{6^3} + \frac{1}{6^3} = \frac{1}{72}$$

$$P(A) * P(B) = \left(\frac{1}{2}\right)\left(\frac{1}{36}\right) = \frac{1}{72} = P(A \cap B)$$

∴ A and B are independent

① · all 5 cards are the same suit

· expected number of hands of poker he has to play to get a flush?

${}_{13}C_5$ → needs to get 5 cards from the same suit

4 → 4 suits possible → $4 \times {}_{13}C_5$ → # of flush possibilities

${}_{52}C_5$ → different possible hands

# of flush possibilities: $4 \times \frac{13!}{5!(13-5)!} = 4 \times 1287 = 5148$ possibilities

# of hands: $\frac{52!}{5!47!} = 2598960$ hands

probability of a flush: $\frac{5148}{2598960} = 0.00198$

$\frac{1}{0.00198}$ = 504.84 games → 505 games of poker

② · superstar plays → win 70% of the time

· superstar doesn't play → win 50% of the time

· chance SS will play next 5 games is 75%

conditional probability: $p(E|F) = \frac{p(E \cap F)}{p(F)}$

$P(\underset{E}{\text{win 4/5 games}} \mid \underset{F}{\text{superstar played}}) = \frac{{}_5C_4 \cdot 0.7^5 \cdot 0.3}{0.7} = 0.36015$

$P(\underset{E}{\text{win 4/5 games}} \mid \underset{F}{\text{superstar didn't play}}) = \frac{{}_5C_4 \cdot 0.5^5 \cdot 0.5}{0.5} = 0.15625$

$P(\underset{E}{\text{win 4/5 games}}) = (0.36015 \cdot 0.75) + (0.15625 \cdot 0.25) = 0.309175$

Bayes Theorem: $p(F|E) = \frac{p(E|F) \cdot p(F)}{p(E)}$

Probability superstar played those games?

$P(\underset{F}{\text{superstar played}} \mid \underset{E}{\text{win 4/5 games}}) = \frac{0.36015 \cdot 0.75}{0.309175} = 0.8737$

∴ There is a 87.37% chance the superstar played