

Consistent estimation of binary-choice panel data models with heterogeneous linear trends

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Summary This paper presents an extension of fixed effects binary choice models for panel data, to the case of heterogeneous linear trends. Two estimators are proposed: a Logit estimator based on double conditioning and a semiparametric, smoothed maximum score estimator based on double differences. We investigate small-sample properties of these estimators with a Monte Carlo simulation experiment, and compare their statistical properties with standard fixed effects procedures. An empirical application to land renting decisions of Russian households between 1996 and 2002 is proposed.

Keywords: Panel data, binary choice, semiparametric estimation, land rent.

1. INTRODUCTION

Time-varying individual effects in linear panel data models have been proposed in the literature as an alternative specification to one way, additive error-component models (Holtz-Eakin *et al.* 1988; Crépon *et al.* 1997; Ahn *et al.* 1999; Han *et al.* 2005). The basic intuition behind these models is that unobserved individual heterogeneity has a differentiated impact on the dependent variable, depending on the time period. As a consequence, the marginal effect of unobserved heterogeneity is time varying, which allows for more flexibility in modelling individual choices.

Incorporating linear heterogeneous trends as a special case of time-varying effects in a binary choice model, raises interesting questions for empirical applications. A possible motivation for such specification can be found in considering economic conditions under which there may be a tendency for all individuals or firms to move towards an equilibrium level in which a positive outcome always (or never) happens, when explanatory variables are stationary. This would be the case of a heterogeneous linear trend, either increasing or decreasing, depending on the sign of the associated individual effect. As the number of time periods increases and eventually reaches values outside of the observed sample, all units would be located in either of the two possible equilibrium regimes, depending on the value of their associated unobserved heterogeneity component. Some examples are the adoption of a new technology by firms under time-varying market conditions, the likelihood that an unemployed person finds a job given exogenous labour market shocks, and

so on (see, e.g. Bruce 2000; Comin and Hobijn 2004; Güell and Hu 2005; Kerr and Newell 2003; Steiner 2001).

A binary choice model with a linear heterogeneous trend can also be interpreted as a particular omitted-variable specification. More precisely, an omitted variable in the definition of the underlying process for binary choice may be a function of individual-specific heterogeneity, while having an 'incremental' nature. This could be the case for instance of an index of experience (or know-how) starting from an heterogeneous initial level, with an individual-specific linear growth rate. As a result, the binary outcome model would depend on an unobserved linear trend with heterogeneous slopes.

In the standard fixed effects Logit model, a sufficient statistic for the individual effect is the sum of positive outcomes, and constructing a new set of probabilities conditional on this statistic forms the basis of a conditional maximum likelihood criterion. The ability to construct a conditional version of the Logit model hence relies on the additivity of the individual effect in the index function. When the standard additive individual effect is supplemented by a second individual heterogeneity term associated with a trend term however, a simple conditioning such as the one above is not sufficient to sweep both effects. This is also true of semiparametric estimators based on first differences of the data, such as the maximum score.

We first propose a simple Logit estimator based on a double conditioning, i.e. not only on the number of positive outcomes as before, but also on a weighted sum of these outcomes. The fact that this new estimator is consistent whereas the standard fixed effects Logit is not under the heterogeneous trend assumption, can be used in applied work to test model specification by constructing a Hausman test statistic.

We then modify the semiparametric maximum score estimator for panel data (Manski 1987), to accommodate linear heterogeneous trends. A smoothed version of the estimator has been proposed by Horowitz (1992), and adapted to (unbalanced) panel data by Charlier *et al.* (1995) and Kyriazidou (1995), under the standard (single individual effect) specification. In our case, the data transformation technique required to wipe out both individual effects is similar to the double difference transformation used in linear regression models. Both the parametric fixed effects Logit and the semiparametric (smoothed) maximum score estimator are straightforward to implement in practice for applied work, and are also easily adapted to unbalanced panel data.

The rest of the paper proceeds as follows. Section 2 introduces our binary choice model with heterogeneous linear trends, and discusses available fixed effects procedures for the linear panel data regression model. Such data transformations are expected to be transposed to the case of a binary choice model for sweeping individual effects through adequate conditioning (or differencing) techniques. In Section 3, we analyse the special case of a heterogeneous trend with individual-specific slopes, and propose two simple and consistent estimators. The first one is a fixed effects Logit based on a restricted set of admissible sequences used for constructing the conditional probabilities. The second one is an extension of the semiparametric (smoothed) maximum score of Manski (1975, 1985) and Horowitz (1992), and applied to the standard one-way panel data model by Manski (1987), Charlier *et al.* (1995), and Kyriazidou (1995). To investigate the properties of our proposed Logit and semiparametric estimators, we present in Section 4 a Monte Carlo experiment and discuss the statistical properties of the estimators in terms of bias and efficiency. An empirical application to the decision of Russian households to rent land is proposed in Section 5, where we estimate a reduced-form model of land tenure using the different fixed effects estimators presented. Concluding remarks are in Section 6.

2. THE BINARY CHOICE MODEL WITH HETEROGENEOUS LINEAR TRENDS

Consider the binary panel data model with heterogeneous linear trends:

$$y_{it} = \mathbb{I}(y_{it}^* > 0), \quad i = 1, \dots, N, \quad t = 1, \dots, T, \quad (1)$$

$$y_{it}^* = x_{it}'\beta + u_{it} + \varepsilon_{it}, \quad (2)$$

$$u_{it} = \alpha_i + \eta_i t, \quad (3)$$

where $\mathbb{I}(\cdot)$ is the indicator function, y_{it} is the observed dependent variable, and α_i and η_i are two individual effects. x_{it} is a $K \times 1$ vector of explanatory variables, and ε_{it} is i.i.d. across units and time periods. We assume a balanced panel sample without loss of generality, to keep notation simple.

We are interested in estimating β (possibly up to scale) without imposing restrictions on the distribution of individual effects α_i and η_i (a fixed effects approach).

Consider first estimation procedures that are available in the linear case, i.e. when y_{it}^* is observed and takes on real values. Three special cases have been explored in the panel data literature. First, when $u_{it} = \alpha_i$ (case 1) the appropriate fixed-effects transformation of the model is either within-group (deviations from individual means) or first-difference.¹

Second, when $u_{it} = \eta_i t$ (case 2), the data generating process admits linear heterogeneous trends as a special case of the time-varying individual-effects model, which would be $u_{it} = \eta_i \theta_t$. Ahn *et al.* (2001) and Han *et al.* (2005) present consistent estimation procedures based on quasi-difference transformation of the data for the general case, where auxiliary parameters θ_t (up to a scale normalization) need to be estimated jointly with β to effectively remove unobserved heterogeneity.² The disadvantage with such a specification lies in the restriction placed on the intercept term, which is proportional to the time effect. Hence, negative individual effects would also correspond to a negative trend pattern.

Third, in the general case where α_i and η_i are two possibly correlated individual effects (case 3), a consistent estimator can be obtained by a double-difference transformation, i.e. two successive first-differences. This model has been examined by Verbeek and Knaap (1999) in a GMM framework, and extended to a general time effects structure in a dynamic panel data context by Nauges and Thomas (2003).

Let us turn back to the binary choice model (1) where we observe $y_{it} = \mathbb{I}(y_{it}^* > 0)$. Only case 1 has been considered in the literature on binary choice models for panel data, where unobserved heterogeneity (α_i) affects the level of the individual probability $\text{Prob}(y_{it} = 1 \mid x_{it}, \alpha_i) = \text{Prob}(y_{it}^* > 0 \mid x_{it}, \alpha_i)$. Nonlinear models for case 1 may include a common linear trend, but unobserved heterogeneity does not affect the time path of this probability, conditional on x_{it} . The most popular estimator is the fixed-effects Logit, based on the assumption that ε_{it} is i.i.d. Logistic across time periods and individuals, but the distribution of α_i given x_{it} is left unrestricted. Parametric

¹ The homogeneous linear trend case, when η_i is constant across units, is obtained equivalently by inserting the trend term in the vector x_{it} .

² This specification has been motivated in the literature by habit formation for instance, where individual effects gradually converge towards an equilibrium level, e.g. when consumer information on a product, or producer know-how on a technology, is fully revealed (see Ravallion 2002).

identification of the fixed effects Logit has been studied intensively by Chamberlain (1992), with proofs relying on the linearity of the log odd ratios. It is well known that a sufficient statistic for α_i is $\tau_i = \sum_t y_{it}$, and maximizing the conditional log likelihood (based on all possible sequences of y_{it} 's such that the sum of positive outcomes is equal to τ_i) produces consistent estimates for β . This estimator will be referred to as the (fixed effects) Logit Type I in the rest of the paper.

Another estimator available in the literature and corresponding to case 1 is the semiparametric maximum score estimator. Under the conditional stationarity and unbounded support assumptions for the distribution of ε_{it} , strong consistency of this estimator is shown by Manski (1987) for the panel data case, following earlier work in a cross-sectional context by Manski (1975, 1985). A smoothed version of the maximum score has been proposed by Horowitz (1992) for cross-section data, later adapted by Charlier *et al.* (1995) and Kyriazidou (1995) to panel data.

An empirical issue in econometric applications with binary panel data models is then to relax the restriction implied by case 1, and allow for linear heterogeneous trends in the underlying data generating process. This requires procedures to wipe out individual effects in a non linear model, when a more general error structure such as (3) is considered. The following section presents two such estimators as straightforward extensions to the fixed effects Logit and the (smoothed) maximum score estimators.

3. TWO SIMPLE ESTIMATORS

We show in this section how consistent 'fixed effects' estimators can be obtained in the case of a linear heterogeneous trend described in equations (1)–(3). The first estimator is a straightforward extension of the (conditional) fixed effects Logit. The second estimator is an extension of the semiparametric maximum score estimator developed by Manski (1987), Charlier *et al.* (1995) and Kyriazidou (1995).

3.1. The Logit estimator

In the standard one-way specification of the literature (case 1), the fixed effects Logit estimator is based on a conditional log likelihood approach. To wipe out individual effects α_i , it suffices to construct the sample conditional log-likelihood from conditional probabilities $Prob(y_i | x_i, \tau_i) \forall i$, where $\tau_i = \sum_t y_{it}$ is a sufficient statistic for α_i , and $y_i = (y_{i1}, \dots, y_{iT})'$. These conditional probabilities are based on all possible sequences of y_{it} 's such that the sum of positive outcomes is equal to τ_i . Maximizing the conditional log likelihood produces consistent estimates for β .

When equation (3) is the true data generating process however, only α_i is properly wiped out by conditioning on τ_i . Because η_i is weighted differently according to the time period, this second individual effect is not filtered out. The conditional probability for y_i given τ_i is in this case:

$$Prob(y_i | \tau_i) \propto \frac{\exp\left(\sum_t y_{it} x'_{it} \beta + \eta_i \sum_t t y_{it}\right)}{\sum_{d \in B_i} \exp\left(\sum_t d_{it} x'_{it} \beta + \eta_i \sum_t t d_{it}\right)} \neq \frac{\exp\left(\sum_t y_{it} x'_{it} \beta\right)}{\sum_{d \in B_i} \exp\left(\sum_t d_{it} x'_{it} \beta\right)} \quad (4)$$

where $B_i = \{\{d_{it}\}_{t=1, \dots, T}, d_{it} = \{0, 1\}; \sum_{t=1}^T d_{it} = \tau_i\}$.

For example, let $T = 2$ and assume for simplicity $x_{i1} = 0$ and $x_{i2} = 1$. We have

$$Prob(y_i = (0, 1) | \tau_i = 2) = \frac{\exp(\beta + \alpha_i + 2\eta_i)}{\exp(\beta + \alpha_i + 2\eta_i) + \exp(\alpha_i + \eta_i)} = \frac{\exp(\beta + \eta_i)}{1 + \exp(\beta + \eta_i)}. \quad (5)$$

Let $N_1 = \sum_i^N \mathbf{I}(y_{i1} + y_{i2} = 1)$ and $n_1 = \sum_i^{N_1} \mathbf{I}(y_{i1} = 0, y_{i2} = 1)$. In case 1 with $\Delta u_{i2} = 0$, differentiating the conditional log likelihood with respect to β yields $\hat{\beta} = \log(n_1/(N_1 - n_1))$ and $\text{plim } \hat{\beta} = \beta$. In the heterogeneous linear trend case however, because $\Delta u_{i2} = \eta_i$, the usual fixed-effect Logit estimator is not consistent. Indeed, we have $\text{plim } \log[n_1/(N_1 - n_1)] = \beta + \log E[\exp(\alpha_i \Delta u_{i2})] \neq \beta$ unless $\Delta u_{i2} = 0, \forall i$.

Inconsistency of the fixed-effect Logit Type I being due to the presence of unobserved heterogeneity which is not fully filtered out, we need another sufficient statistic to wipe out η_i together with α_i .

From equation (4), we immediately see that, to remove η_i , we can use another sufficient statistic $\tau_i^* = \sum_{t=1}^T t y_{it}$, $\forall i$. The corresponding set of admissible sequences would be:

$$B_i^* = \left\{ d_i = \{d_{i1}, \dots, d_{iT}\}'; d_{it} = \{0, 1\}; \sum_{t=1}^T t d_{it} = \tau_i^* \right\}.$$

Therefore, to wipe out both individual effects, we can consider the following set of admissible sequences, obtained as the intersection between sets B_i and B_i^* :

$$B_i^{**} = \left\{ d_i = \{d_{i1}, \dots, d_{iT}\}; d_{it} = \{0, 1\}; \sum_{t=1}^T d_{it} = \tau_i \text{ and } \sum_{t=1}^T t d_{it} = \tau_i^* \right\}.$$

In the heterogeneous trend case, the proposed conditional Logit estimator is thus based on a restricted conditioning that explicitly accounts for the fact that positive outcomes are associated with different weights according to the position of the particular time period in the sequence. For instance, when $T = 4$, there are only 2 admissible sequences in B_i^{**} : (1, 0, 0, 1) and (0, 1, 1, 0), for which $\tau_i = 2$ and $\tau_i^* = 5$. The conditional Logit probability in this case is:

$$\begin{aligned} Prob[y_i = (0, 1, 1, 0) | x_i, \tau_i = 2, \tau_i^* = 5] &= \frac{\exp[(x_{2i} + x_{3i})'\beta]}{\exp[(x_{2i} + x_{3i})'\beta] + \exp[(x_{1i} + x_{4i})'\beta]} \\ &= \frac{\exp[(x_{2i} + x_{3i} - x_{1i} - x_{4i})'\beta]}{1 + \exp[(x_{2i} + x_{3i} - x_{1i} - x_{4i})'\beta]} = \frac{\exp[(x_{2i} - x_{1i}) - (x_{4i} - x_{3i})]'\beta]}{1 + \exp[(x_{2i} - x_{1i}) - (x_{4i} - x_{3i})]'\beta]}. \quad (6) \end{aligned}$$

The fixed effects estimator based on sequences in B_i^{**} will be referred to as the (fixed effects) Logit Type II in the rest of the paper.

It can be seen from the expression above that the conditional Logit probabilities based on B_i^{**} are functions of the covariates x_{it} in double differences. However, the double-differencing technique feasible in the linear panel data case is not applicable in nonlinear models. Hence, the equivalence between double differencing in linear regression models and double conditioning in the Logit framework is not valid in a strict sense.

Note that the number of admissible sequences in B_i^{**} is likely to be rather limited for a fixed number of time periods. Although our conditional Logit estimator will be consistent both under the heterogeneous linear trend and the standard one-way specification, it is likely to be less efficient than that based on B_i when $u_{it} = \alpha_i$ is the true process. This is because a significant amount of

information is lost with double conditioning compared to single conditioning, a similar situation to linear regression models where first or double differencing can be performed. Nevertheless, the advantage of Logit estimator Type II is, apart from its computational ease, the ability to obtain consistent estimates for β without restrictions on $E(\alpha_i, \eta_i | x_i)$, where $x_i = (x_{i1}, \dots, x_{iT})'$. Also, in empirical applications, the fact that both estimators are consistent while the standard fixed effects Logit will be more efficient, provides a guideline for implementing a standard Hausman specification test.

It would be tempting to adapt the approach proposed above to the case of binary choice panel data model with lagged dependent variables, along the lines of Honoré and Kyriazidou (2000). However, the key behind their conditional Logit estimator is the ability to swap different time periods in the computation of probability ratios corresponding to different sequences, under the restriction that some values of covariates are the same across different time periods. As a consequence, the application of their conditioning to the model with heterogeneous linear trends is not feasible, because the conditional interchangeability condition used by Honoré and Kyriazidou (2000) does not hold any more.³

Finally, it is interesting to investigate the possible extension of the double conditioning procedure to other distributions belonging to the exponential family. Consider in particular the popular Poisson model for count data, where the mean of counts n_{it} for individual i at period t is, including our heterogeneous trend specification:

$$E(n_{it} | x_{it}, \alpha_i, \eta_i) = \lambda_{it} = \exp(x'_{it}\beta + \alpha_i + t\eta_i). \quad (7)$$

In the standard case with no heterogeneous trend, the usual fixed effect procedure consists in maximizing the log-likelihood conditional on the sum of outcomes $(\sum_t n_{it} \forall i)$, which does not depend on α_i (Hausman *et al.* 1984).⁴

With specification (7) however, the individual contribution to the conditional likelihood would still depend on η_i . If we condition instead, in a similar fashion as in the Logit model, on T sequences of counts $f_i = (f_{i1}, \dots, f_{iT})$ such that $\sum_t f_{it} = \sum_t n_{it}$ and $\sum_t t f_{it} = \sum_t t n_{it}$, it is immediate that the resulting conditional probability is free of α_i and η_i . To see this, note that

$$Pr(n_{i1}, \dots, n_{iT} | x_i, \alpha_i, \eta_i) = \prod_{t=1}^T \left[\frac{\exp(x'_{it}\beta)^{n_{it}}}{n_{it}!} \right] \exp \left\{ -e^{\alpha_i} \sum_{t=1}^T e^{x'_{it}\beta} e^{t\eta_i} + \alpha_i \sum_{t=1}^T n_{it} + \eta_i \sum_{t=1}^T t n_{it} \right\},$$

so that the suggested conditional probability for unit i would be proportional to

$$\frac{\prod_{t=1}^T [\exp(x'_{it}\beta)^{n_{it}}]}{\sum_{f_i \in C_i} \left\{ \prod_{t=1}^T [\exp(x'_{it}\beta)^{f_{it}}] \right\}} = \frac{\exp \left(\sum_{t=1}^T n_{it} x'_{it} \beta \right)}{\sum_{f_i \in C_i} \left[\exp \left(\sum_{t=1}^T f_{it} x'_{it} \beta \right) \right]},$$

$$\text{with } C_i = \left\{ f_i = \{f_{i1}, \dots, f_{iT}\}; \sum_{t=1}^T f_{it} = \sum_{t=1}^T n_{it} \text{ and } \sum_{t=1}^T t f_{it} = \sum_{t=1}^T t n_{it} \right\}.$$

³ This condition is $F_{\varepsilon_{it}|x_i, \alpha_i} = F_{\varepsilon_{is}|x_i, \alpha_i} \forall t, s$.

⁴ Note that in this standard Poisson model with panel data, the MLE of β is not affected by the incidental parameter problem, and is therefore consistent (and identical to the conditional MLE, see Blundell *et al.* 2002 and Lancaster 2002).

Inspection of models based on other distributions in the exponential family is outside the scope of the paper. We simply note that our Poisson specification (7) preserves the orthogonality property proposed by Lancaster (2002), which ensures consistency of the MLE anyway. This property would need to be verified for other models with heterogeneous trends and discrete data.

3.2. Maximum score estimation

Let us first briefly recall the principle of maximum score estimation for the standard panel data model (case 1, where $u_{it} = \alpha_i$). Consider 2 time periods t, s and 2 possible sequences $A = \{(y_{it} = 0, y_{is} = 1)\}$ and $B = \{(y_{it} = 1, y_{is} = 0)\}$. Let $F(\cdot)$ denote the unspecified distribution function of ε . Because $F(\cdot)$ is monotonically increasing, from the probability ratio

$$\frac{\text{Prob}(A | x_{is}, x_{it}, \alpha_i)}{\text{Prob}(B | x_{is}, x_{it}, \alpha_i)} = \frac{1 - F(x'_{it}\beta + \alpha_i)}{F(x'_{it}\beta + \alpha_i)} \times \frac{F(x'_{is}\beta + \alpha_i)}{1 - F(x'_{is}\beta + \alpha_i)},$$

we have that

$$\text{sgn}[\text{Prob}(A | x_{is}, x_{it}, \alpha_i) - \text{Prob}(B | x_{is}, x_{it}, \alpha_i)] = \text{sgn}[(x_{is} - x_{it})'\beta]$$

or equivalently (see the corollary to lemma 1 in Manski 1987):

$$\text{sgn}[\text{Prob}(y_{is} | x_{is}, x_{it}, \alpha_i) - \text{Prob}(y_{it} | x_{is}, x_{it}, \alpha_i)] = \text{sgn}[(x_{is} - x_{it})'\beta]$$

$$\Leftrightarrow \text{med}(y_{is} - y_{it} | x_{is}, x_{it}, y_{is} \neq y_{it}) = \text{sgn}(w'_{its}\beta),$$

where $w_{its} = x_{is} - x_{it}$. This moment condition obtains under fairly mild assumptions: conditional on x_i and α_i , the distribution of ε_{it} is stationary, and its support is unbounded (which ensures that $\forall \alpha_i, y_{is} - y_{it} \neq 0$ occurs with positive probability). Under additional assumptions on explanatory variables (Manski 1987), it is easy to show that a maximum score estimator of β up to scale obtains by maximizing the following criterion:

$$\frac{1}{N} \sum_{i=1}^N \sum_{t=2}^T \sum_{s<t}^T (y_{is} - y_{it}) \mathbf{I}(y_{it} \neq y_{is}) \text{sgn}(w'_{its}b). \quad (8)$$

A possible normalization is $\|\beta\| = 1^5$ and the vector of parameters to be estimated is therefore $\tilde{\beta} = (\beta_2, \beta_3, \dots, \beta_K)'$. Let \tilde{b} denote this estimator.

Consider now our specification (3) with heterogeneous linear trends. The conditional distribution of ε_{it} given x_i and α_i is not stationary anymore, as it depends on $\eta_i t$. Consequently, moment conditions based upon first differences as above need to be replaced by alternative moment conditions implying sequences y_i and x_i .

To see this, assume to simplify that $T = 4$, the lowest number of time periods for which there is strictly more than 1 admissible sequence. The 2 admissible sequences are $A = \{(0, 1, 1, 0)\}$ and $B = \{(1, 0, 0, 1)\}$, with probability ratio

$$\frac{\text{Prob}(A | x_i, \alpha_i, \eta_i)}{\text{Prob}(B | x_i, \alpha_i, \eta_i)} = \frac{H(x'_{i2}\beta + \alpha_i + 2\eta_i)}{H(x'_{i1}\beta + \alpha_i + \eta_i)} \times \frac{H(x'_{i3}\beta + \alpha_i + 3\eta_i)}{H(x'_{i4}\beta + \alpha_i + 4\eta_i)},$$

⁵ Or equivalently, as shown in Horowitz (1992), setting a component of β , β_1 , say, to 1 or -1 .

where $H(\cdot) = F(\cdot)/[1 - F(\cdot)]$ and $F(\cdot)$ is the distribution function of ε_{it} . Because $H(\cdot)$ is increasing, we have that

$$\text{sgn}[\text{Prob}(A | x_i, \alpha_i, \eta_i) - \text{Prob}(B | x_i, \alpha_i, \eta_i)] = \text{sgn}[(\Delta x_{i2} - \Delta x_{i4})' \beta]. \quad (9)$$

Assume that (i) $F(\varepsilon_{it} | x_i, \alpha_i, t\eta_i) = F(\varepsilon_{is} | x_i, \alpha_i, s\eta_i) \forall t, s$ (stationarity) and (ii) the support of $F(\varepsilon_{it} | x_i, \alpha_i, t\eta_i)$, $\forall t$, is unbounded. From equation (1), $E(y_{it} | x_i, \alpha_i, t\eta_i) = \text{Prob}(y_{it}^* > 0 | x_i, \alpha_i, t\eta_i) \forall t$, and therefore direct application of Manski (1987) yields, using assumptions above:

$$E[\Delta y_{i2} - \Delta y_{i4} | x_i] \lesseqgtr 0 \Leftrightarrow (\Delta x_{i2} - \Delta x_{i4})' \beta \lesseqgtr 0.$$

The heterogeneous linear trend model also admits a median regression interpretation. Since

$$\begin{aligned} & \Pr(\Delta y_{i2} - \Delta y_{i4} = 2 | x_i, \Delta y_{i2} \neq \Delta y_{i4}, \Delta y_{i2} = \pm 1, \Delta y_{i4} = \pm 1) \\ &= \frac{\Pr(\Delta y_{i2} = 1, \Delta y_{i4} = -1 | x_i)}{\Pr(R_i)}, \end{aligned}$$

$$\begin{aligned} & \Pr(\Delta y_{i2} - \Delta y_{i4} = -2 | x_i, \Delta y_{i2} \neq \Delta y_{i4}, \Delta y_{i2} = \pm 1, \Delta y_{i4} = \pm 1) \\ &= \frac{\Pr(\Delta y_{i2} = -1, \Delta y_{i4} = 1 | x_i)}{\Pr(R_i)}, \end{aligned}$$

where $\Pr(R_i) = \Pr(\Delta y_{i2} \neq \Delta y_{i4}, \Delta y_{i2} = \pm 1, \Delta y_{i4} = \pm 1)$, we have

$$\begin{aligned} & \text{med}(\Delta y_{i2} - \Delta y_{i4} | x_i, \Delta y_{i2} \neq \Delta y_{i4}, \Delta y_{i2} = \pm 1, \Delta y_{i4} = \pm 1) \\ &= 2\text{sgn}[\Pr(\Delta y_{i2} = 1, \Delta y_{i4} = -1 | x_i) \\ &\quad - \Pr(\Delta y_{i2} = -1, \Delta y_{i4} = 1 | x_i)] \\ &= 2\text{sgn}[\Pr(\Delta y_{i2} = 1 | x_i) - \Pr(\Delta y_{i4} = 1 | x_i)] = 2\text{sgn}[(\Delta x_{i2} - \Delta x_{i4})' \beta]. \end{aligned}$$

This result obtains under the stationarity assumption made above: the conditional distribution of ε_{it} given x_i , α_i and $t\eta_i$ is the same $\forall t$. Obviously, the initial assumption made by Manski (1987) in the standard case (case 1) is not valid here. A maximum score criterion for $T = 4$ is then constructed either as

$$\frac{1}{N} \sum_{i=1}^N \mathbf{I}(|\Delta y_{i2} - \Delta y_{i4}| = 2) \left[\frac{1}{2} (\Delta y_{i2} - \Delta y_{i4}) \right] \times \text{sgn}[(\Delta x_{i2} - \Delta x_{i4})' b], \quad (10)$$

or equivalently, in terms of the probability of full sequences A and B as

$$\frac{1}{N} \sum_{i=1}^N [\mathbf{I}(y_i = A) + \mathbf{I}(y_i = B) = 1] [2\mathbf{I}(y_i = A) - 1] \times \text{sgn}[(\Delta x_{i2} - \Delta x_{i4})' b]. \quad (11)$$

Because $\text{sgn}(\cdot)$ is a step function, it may be preferable (Horowitz 1992) to use instead a differentiable function (a kernel). In equation (11), $\text{sgn}(\cdot)$ would be replaced by $K((\Delta x_{i2} - \Delta x_{i4})' b / \sigma_N)$, where σ_N is a window parameter, $\sigma_N \rightarrow 0$ as $N \rightarrow \infty$, $K(\cdot)$ is a kernel function with order h , $h \geq 2$. In the standard specification (case 1), $\text{sgn}(w'_{its} \beta)$ would be replaced by $K(w'_{its} b / \sigma_N)$ in equation (8).

This yields the continuously differentiable smoothed maximum score estimator (SMS).⁶ Consistent with the notation for Logit models above, the SMS based on data in first difference (smoothed version of (8)) will be referred to as the SMS Type I, whereas the SMS based on double differences of the x_{it} 's (smoothed version of (10)) will be denoted SMS Type II.

When $T > 4$, more than just 2 admissible sequences are generally available. To generalize the criterion in equation (11) for any $T > 4$, we can consider the general form of the SMS criterion:

$$\frac{1}{N} \sum_{i=1}^N \left\{ \mathbf{I}[\text{card}(B_i^{**}) > 1] \sum_{d_i, \tilde{d}_i \in B_i^{**}} \left[[\mathbf{I}(y_i = d_i) - \mathbf{I}(y_i = \tilde{d}_i)] \times K \left(\frac{[(d_i - \tilde{d}_i)' x_i' b]}{\sigma_N} \right) \right] \right\},$$

where as before, B_i^{**} is defined as the set of all T sequences $\{d_{it}\}_{t=1, \dots, T}$ with $\tau_i = \sum_t d_{it}$ and $\sum_t t d_{it} = \tau_i^*$. The first indicator function in the criterion ensures that there are at least 2 admissible sequences for unit i , while the second is sufficient to exclude identical sequences d_i and \tilde{d}_i .

Assumptions needed to achieve consistency and asymptotic normality of the SMS estimator are easily derived from Horowitz (1992) and Charlier *et al.* (1995). Define $w_i^{(d_i, \tilde{d}_i)} = (d_i - \tilde{d}_i)' x_i$ and $\tilde{w}_i^{(d_i, \tilde{d}_i)} = (d_i - \tilde{d}_i)' \tilde{x}_i$, where \tilde{x}_i is a $T \times (K - 1)$ submatrix corresponding to the $K - 1$ last columns of x_i , and associated to the vector of parameters \tilde{b} . Let $p(z_i | \tilde{w}_i^{(d_i, \tilde{d}_i)})$ and $F(\cdot | z_i, \tilde{w}_i^{(d_i, \tilde{d}_i)})$, respectively, denote the conditional density of $z_i = (d_i - \tilde{d}_i)' x_i' \beta$ given $\tilde{w}_i^{(d_i, \tilde{d}_i)}$ and the cumulative distribution function of ε_i conditional on z_i and $\tilde{w}_i^{(d_i, \tilde{d}_i)}$, for any pair (d_i, \tilde{d}_i) , with $p^{(j)}(z_i | \tilde{w}_i^{(d_i, \tilde{d}_i)}) = \partial^j p(z_i | \tilde{w}_i^{(d_i, \tilde{d}_i)}) / \partial z_i^j$. The most important assumptions needed in our case are the following:⁷

- (1) For any pair $(d_i, \tilde{d}_i) \in B_i^{**}$, the scalar random variable $\tilde{w}_{ij}^{(d_i, \tilde{d}_i)}$, $j = 1, \dots, K - 1$ has everywhere positive Lebesgue density conditional on \tilde{w}_i and $d_i \neq \tilde{d}_i$.
- (2) $\tilde{w}_i^{(d_i, \tilde{d}_i)}$, $(\tilde{w}_i^{(d_i, \tilde{d}_i)})' \tilde{w}_i^{(d_i, \tilde{d}_i)}$ and $(\tilde{w}_i^{(d_i, \tilde{d}_i)})' \tilde{w}_i^{(d_i, \tilde{d}_i)} (\tilde{w}_i^{(d_i, \tilde{d}_i)})' \tilde{w}_i^{(d_i, \tilde{d}_i)}$ have finite first-order moments, for any pair $(d_i, \tilde{d}_i) \in B_i^{**}$.
- (3) $F_{\varepsilon_{it} | x_i, \alpha_i, t \eta_i} = F_{\varepsilon_{is} | x_i, \alpha_i, s \eta_i} \quad \forall s, t \leq T$, for all (x_i, α_i, η_i) ;
- (4) $|\beta_1| = 1$ and $\tilde{\beta} = (\beta_2, \dots, \beta_K)'$ belongs to a compact subset of R^{K-1} .

Following Charlier *et al.* (1995) and Horowitz (1992), the asymptotic distribution of the SMS Type II estimator is

$$N^{h\gamma} (\tilde{b} - \tilde{\beta}) \rightarrow^P N(-(\lambda^*)^{h\gamma} Q^{-1} A, (\lambda^*)^{-\gamma} Q^{-1} A Q^{-1}),$$

where $\gamma = 1/(2h + 1)$ and

$$\begin{aligned} \lambda^* &= \frac{\text{trace}(Q^{-1} \Omega Q^{-1} D_1)}{2h A' Q^{-1} \Omega Q^{-1} A}, \\ A &= -2 \sum_{i=1}^N \mathbf{I}[\text{card}(B_i^{**}) > 1] \sum_{d_i, \tilde{d}_i \in B_i^{**}} \sum_{j=1}^h \\ &\quad \times E \left\{ \frac{1}{j!(h-j)!} F^{(j)}(0 | 0, \tilde{w}_i^{(d_i, \tilde{d}_i)}) p^{(h-j)}(0 | \tilde{w}_i^{(d_i, \tilde{d}_i)}) \tilde{w}_i^{(d_i, \tilde{d}_i)} \right\} \times \left[\int \xi^h K'(\xi) d\xi \right], \end{aligned}$$

⁶ See Kyriazidou (1995) and Charlier *et al.* (1995) for a development of the SMS estimator in the case of panel data.

⁷ More technical assumptions for asymptotic normality can be found in Horowitz (1992) and Charlier *et al.* (1995).

$$D_1 = \left[\int [K'(\xi)]^2 d\xi \right] \times \sum_{i=1}^N \mathbf{I}[\text{card}(B_i^{**}) > 1] \sum_{d_i, \tilde{d}_i \in B_i^{**}} \sum_{j=1}^h E \left\{ \tilde{w}_i^{(d_i, \tilde{d}_i)} \tilde{w}_i^{(d_i, \tilde{d}_i)'} p(0 | \tilde{w}_i^{(d_i, \tilde{d}_i)}) \right\},$$

$$Q = 2 \sum_{i=1}^N \mathbf{I}[\text{card}(B_i^{**}) > 1] \sum_{d_i, \tilde{d}_i \in B_i^{**}} E \left\{ \tilde{w}_i^{(y_i, d_i)} \tilde{w}_i^{(d_i, \tilde{d}_i)'} F^{(1)}(0 | 0, \tilde{w}_i^{(d_i, \tilde{d}_i)}) p(0 | \tilde{w}_i^{(d_i, \tilde{d}_i)}) \right\},$$

Ω is an arbitrary positive definite matrix, and $\tilde{\sigma}_N = O(N^{-\delta\gamma})$, with $0 < \delta < 1$.

In empirical applications, the window parameter is set to $\sigma_N = (\lambda/N)^\gamma$, with $0 < \lambda < \infty$, and the asymptotic bias is $-\lambda^{h\gamma} Q^{-1} A$, which can be consistently estimated by $-\lambda^{h\gamma} Q_N(\tilde{b}, \sigma_N)^{-1} \hat{A}_N$, where

$$\hat{A}_N = (\sigma_N^*)^{-h} T_N(\tilde{b}, \sigma_N^*), \quad \sigma_N^* \propto N^{-\delta\gamma}, \quad Q_N(\tilde{b}, \sigma_N) = \frac{\partial^2 S_N}{\partial \tilde{b} \partial \tilde{b}'}, \quad T_N(\tilde{b}, \sigma_N) = \frac{\partial S_N}{\partial \tilde{b}}.$$

The matrix D_1 is consistently estimated by

$$\hat{D}_1 = \frac{1}{N} \sum_{i=1}^N \mathbf{I}[\text{card}(B_i^{**}) > 1] \sum_{d_i, \tilde{d}_i \in B_i^{**}} a_i(\tilde{b}, \tilde{\sigma}_N) a_i(\tilde{b}, \tilde{\sigma}_N)',$$

where

$$a_i(\tilde{b}, \tilde{\sigma}_N) = [\mathbf{I}(y_i = d_i) - \mathbf{I}(y_i = \tilde{d}_i)] \times K' \left(\frac{w_i^{(d_i, \tilde{d}_i)} b}{\tilde{\sigma}_N} \right) \frac{\tilde{w}_i^{(d_i, \tilde{d}_i)} \tilde{b}}{\tilde{\sigma}_N}.$$

When $h = 1$, the estimator converges at a rate slower than $N^{-1/3}$, but the limiting distribution is unknown. Hence, it has no advantages in this case over Manski's maximum score estimator. When $h \geq 2$ however, the rate of convergence can be made arbitrarily close to $N^{-1/2}$ but the estimator has an asymptotic bias, as shown above. Horowitz (1992) also shows that the estimator exhibits a small sample bias in \hat{A}_N , which can be partially reduced by replacing the latter by

$$\hat{A}_N^* = \hat{A}_N \times \left\{ 1 - \left[\lambda^{*-1} N \sigma_N (\sigma_N^*)^{2h} \right]^{-1/2} \right\}^{-1}.$$

4. MONTE CARLO EXPERIMENT

To investigate the small-sample behaviour of the fixed-effects Logit and semiparametric estimators presented above, we undertake a Monte Carlo simulation experiment based on various data generating process assumptions. To keep the experimental framework simple and to provide a baseline for comparison, we follow as closely as possible Honoré and Kyriazidou (2000) regarding distributional and parametric assumptions. For each sample size (N, T) , we draw values for the latent variable

$$y_{it}^* = x_{1,it} \beta_1 + x_{2,it} \beta_2 + \alpha_i + k \eta_i t + \varepsilon_{it},$$

where $k = 1$ for the heterogeneous linear trend specification (3), and $k = 0$ for the standard one-way additive effect (case 1). Explanatory variables $x_{1,it}$ and $x_{2,it}$ are both $N(0, \pi^2/3)$, and $\alpha_i = (1/T) \sum_t x_{1,it} + (1/T) \sum_t x_{2,it}$. The second heterogeneity term η_i is normal with mean 0

Table 1. Monte Carlo estimates of properties of Logit Type I and II fixed effects estimators.

			MLE fixed effects		Logit Type I		Logit Type II	
			Med(bias)	MAE	Med(bias)	MAE	Med(bias)	MAE
$N = 500$	No trend	$T = 4$	-0.4719	0.4701	0.0005	0.0491	0.0315	0.2585
		$T = 6$	-0.4352	0.4359	0.0048	0.0426	0.0065	0.1356
		$T = 8$	-0.4150	0.4166	-0.0007	0.0202	0.0053	0.0439
		$T = 10$	-0.4024	0.4017	-0.0008	0.0240	0.0018	0.0405
$N = 500$	Linear trend	$T = 4$	-0.6912	0.6896	-0.1139	0.1150	0.0775	0.3665
		$T = 6$	-0.7384	0.7373	-0.1501	0.1482	0.0183	0.2612
		$T = 8$	-0.7721	0.7717	-0.1849	0.1825	0.0114	0.1516
		$T = 10$	-0.7986	0.7981	-0.2128	0.2127	0.0015	0.0786
$N = 1,000$	No trend	$T = 4$	-0.4715	0.4721	0.0007	0.0337	0.0524	0.1902
		$T = 6$	-0.4354	0.4353	0.0003	0.0248	0.0077	0.0743
		$T = 8$	-0.4150	0.4156	-0.0007	0.0202	0.0053	0.0439
		$T = 10$	-0.4022	0.4024	0.0017	0.0167	0.0039	0.0284
$N = 1,000$	Linear trend	$T = 4$	-0.6875	0.6877	-0.1078	0.1204	0.0257	0.2540
		$T = 6$	-0.7371	0.7369	-0.1576	0.1615	0.0251	0.1282
		$T = 8$	-0.7719	0.7720	-0.1838	0.1874	0.0071	0.0812
		$T = 10$	-0.7986	0.7986	-0.2097	0.2109	0.0093	0.0571

and variance $\pi^2/3$, and is drawn conditional on α_i , with $\text{corr}(\alpha_i, \eta_i) = 0.5$. Finally, ε_{it} is i.i.d. Logistic across units and time periods, with mean 0 and variance $\pi^2/3$. In all experiments, the true value of parameters is $\beta_1 = \beta_2 = 1$. To compare the fixed effects Logit with the SMS estimator, we estimate parameter β_2 only, while normalizing β_1 to 1. The number of replications is 10,000.

Five different estimation procedures are considered: (i) the MLE with fixed effects, (ii) the Type I fixed effects Logit (based on τ_i), (iii) the Type II fixed effects Logit (based on τ_i and τ_i^*), (iv) the Type I SMS (based on first difference) and (v) the Type II SMS (based on double difference). The fixed effects MLE is implemented by jointly estimating β and individual-specific intercepts. It is well known that such estimator is biased when T is fixed and N goes to infinity. All estimators are computed first under the standard one-way ($k = 0$), and then under the heterogeneous trend ($k = 1$) process. We compute the mean, standard deviation and Root Mean Square Error (RMSE) of the bias but, as these estimators are sensitive to outliers, we prefer to report the median bias and the median absolute error (MAE) of the estimator.

Table 1 presents simulation results for the MLE and fixed effects Logit estimators. As can be seen from the median bias and MAE statistics of the MLE, the bias increases with the number of units, while as expected, it tends to decrease with T when no trend is present ($k = 0$). On the other hand, the bias increases with T under specification (3). In any case, the average bias is significant, ranging from around -0.40 with no trend and $T = 10$, to -0.79 with a linear heterogeneous trend and $T = 10$.

As for Logit estimators Type I and Type II, the bias is very small with no heterogeneous linear trend, and can take positive or negative values for both estimators, although the average bias is

Table 2. Monte Carlo estimates of properties of smoothed maximum score Type I and II estimators.

				Score Type I		Score Type II	
				Med(bias)	MAE	Med(bias)	MAE
<i>h</i> = 2	<i>N</i> = 500	No trend	<i>T</i> = 6	0.1293	0.1578	0.0739	0.1260
		Linear trend	<i>T</i> = 6	0.1244	0.1926	0.0381	0.2128
	<i>N</i> = 1,000	No trend	<i>T</i> = 6	0.1235	0.1415	0.0607	0.1004
			<i>T</i> = 8	0.1213	0.1209	0.0645	0.0770
	<i>N</i> = 1,000	Linear trend	<i>T</i> = 6	0.1334	0.1672	0.0623	0.1484
			<i>T</i> = 8	0.1239	0.1477	0.0633	0.1092
	<i>N</i> = 2,000	No trend	<i>T</i> = 4	0.1285	0.1428	0.0671	0.1782
			<i>T</i> = 8	0.1150	0.1202	0.0620	0.0639
	<i>N</i> = 2,000	Linear trend	<i>T</i> = 4	0.1326	0.1540	0.0275	0.2319
			<i>T</i> = 8	0.1239	0.1367	0.0631	0.0872
	<i>N</i> = 4,000	No trend	<i>T</i> = 4	0.1203	0.1299	0.0517	0.1137
		Linear trend	<i>T</i> = 4	0.1215	0.1406	0.0677	0.1771
<i>h</i> = 4	<i>N</i> = 500	No trend	<i>T</i> = 6	0.0757	0.0779	−0.0147	0.1279
		Linear trend	<i>T</i> = 6	0.0791	0.0904	−0.0105	0.2008
	<i>N</i> = 1,000	No trend	<i>T</i> = 6	0.0770	0.0933	−0.0009	0.0908
			<i>T</i> = 8	0.0641	0.0652	−0.0038	0.0549
	<i>N</i> = 1,000	Linear trend	<i>T</i> = 6	0.0701	0.0756	−0.0169	0.1376
			<i>T</i> = 8	0.0703	0.0710	0.0071	0.1030
	<i>N</i> = 2,000	No trend	<i>T</i> = 4	0.0886	0.0888	−0.0047	0.1627
			<i>T</i> = 8	0.0645	0.0661	−0.0019	0.0401
	<i>N</i> = 2,000	Linear trend	<i>T</i> = 4	0.0777	0.0791	−0.0358	0.2387
			<i>T</i> = 8	0.0664	0.0666	0.0120	0.0735
	<i>N</i> = 4,000	No trend	<i>T</i> = 4	0.0900	0.0991	0.0079	0.1176
		Linear trend	<i>T</i> = 4	0.0842	0.1012	−0.0104	0.1632

slightly more important in the Logit Type II case. When $k = 1$, the biased nature of the Logit Type I is confirmed by the simulation experiment. This bias increases with T but tends to decrease (only slightly) with N . As far as parametric estimation efficiency is concerned, the Logit Type I performs well in terms of MAE, but the Logit II is rather inefficient. Indeed, efficiency is lower with this estimator when the linear heterogeneous trend is the true model specification. In terms of sample size, our estimator seems to require a number of time periods typically greater than 4 to provide reasonably efficient estimates. On the other hand, even with a limited number of units, 250, say, Logit Type II estimates benefit from a rapid increase in efficiency when T increases, even slightly (from $T = 6$ to $T = 8$ for instance).

Monte Carlo simulation results for the semiparametric SMS estimator are presented in Table 2. Two kernel functions are selected: a Gaussian kernel with $h = 2$ and a higher-order kernel K_4 as proposed in Horowitz (1992) when $h = 4$:

$$K_4(v) = \begin{cases} 0 & \text{if } v < -5, \\ 0.5 + (105/64)[(v/5) - (5/3)(v/5)^3 \\ + (7/5)(v/5)^5 - (3/7)(v/5)^7] & \text{if } -5 \leq v \leq 5. \\ 1 & \text{if } v > 5. \end{cases}$$

Because this estimator typically requires larger sample size, we compute the estimator for $N = 500, 1,000, 2,000$ and $4,000$. Also, as the estimator may have multiple local extrema, we perform a grid search for \tilde{h} , between -5 and 5 . For each estimation step, we use values of δ between 0.1 and 1 with an increment of 0.1 . The value of δ leading to the highest score function value is retained in the computation of the bias terms (i.e. including the small sample bias on \hat{A}_N). Interestingly, the median bias is always greater for SMS Type I than for the SMS Type II estimator, even under the one-way individual effect specification. The median bias is always greater when $h = 2$ for both semiparametric estimators, as already reported in Horowitz (1992) in a cross-sectional context. Indeed, the magnitude of the bias (around 0.07 with $h = 4$) does not seem to depend on the true data generating process. In other terms, the SMS Type I always exhibits a moderate bias, even when no heterogeneous linear trend is present.

Finally, by comparing Tables 1 and 2 for the same sample sizes, one can see that the SMS Type I estimator performs slightly better (has a lower bias) relative to the Logit Type I under the heterogeneous trend specification. As discussed above, these two estimators are not consistent under the data generating process defined in (3). This would seem to indicate that when the true data generating process for ε_{it} is Logistic (as is the case in our simulations), the SMS Type I estimator would have a lower bias than the fixed effects Logit Type I. However, a more exhaustive Monte Carlo experiment is required to explore the behaviour of these estimators under alternative distributions for the error components. This is left for future research.

5. EMPIRICAL APPLICATION

We apply in this section the binary choice panel data estimators suggested above to investigate the determinants of household decisions to rent agricultural land in Russia, in 1990s. In transition economies, the status of land ownership and land rental markets is still the subject of an ongoing debate (Deiniger 2003; Csaki *et al.* 2002). Although the market for land is definitely much wider since the beginning of the 1990s, restrictions to land ownership may still apply in cases for foreigners and major landlords. Besides, for small households with limited income, local credit market institutions are often reluctant to grant loans to families with limited experience in farming, especially when land is too limited to allow for cash crops. Still, the move towards market-driven economies, as slow as it may be for some eastern European countries, has been accompanied by a trend towards private land ownership. In Russia for instance, less than 40% of agricultural land remained in state ownership in 1998 (see Csaki *et al.* 2002).⁸ As a deep economic analysis of

⁸ The Russian Federation Law on Land Reform of November 1990 enabled private property of all land, hence giving up the state's exclusive ownership on land. The land code was finally adopted in April 1991. See Brooks and Lerman (1995) on the restructuring of traditional farms in Russia.

determinants to land tenure is outside the scope of this paper, we conduct a reduced-form analysis of those factors behind the decision of households to own or to rent land for crop.

The RLMS (Russia Longitudinal Monitoring Survey) sample constructed between 1992 and 2003 by the University of North Carolina at Chapel Hill is a rich data set on Russian households, where detailed information is collected on food production, household expenditures and income. We use in the application 7 rounds of the survey, from 1996 to 2002, to keep attrition to a minimum. From the initial sample, we select the 732 households using land for farming and animal husbandry. The final sample consists in a balanced panel data set with 5,124 observations.

The dependent variable, land renting, is a dummy variable for land rent decision by the household and used for farming, including family farming. It is important to note here that households may both own and rent land at the same time. The land renting variable used in the application is only an indicator for the decision to solely rent land, or supplement land which is already owned by renting more land plots.

Explanatory variables are *LINC* (log yearly family disposable income), *LAREA* (log agricultural land), *AGSHARE* (the ratio of revenue from cropping over total income), and *SOLDSHARE* (the proportion of products that are sold in local markets over total harvest value). Land area in sotkas⁹ is computed on a yearly basis as the sum of all individual land plots used for farming, including family farming. It is important to note that this variable is not defined as total land being owned or rented, but as the land being actually used for production. As household income in the Russian Federation has experienced major shocks over the period, the choice of a consistent deflator is particularly important. In our application, income is deflated by a corrected CPI index suggested by Gibson *et al.* (2004), and obtained from the RLMS database. Variable *SOLDSHARE* is computed as a value-weighted average of the percentage of production sold for major agricultural products. To detect the presence of heterogeneous trends as opposed to a common linear trend (with η_i constant), we also include a linear trend term in the fixed effects Logit specification.

Table 3 presents descriptive statistics on the sample used. For income and land area, we report the median instead of the arithmetic mean to limit the influence of outliers. It can be seen that land renting remains fairly stable over the period 1996–2002, decreasing from 23.63% in 1996 to 20.08% in 1999, and then oscillating around 20%. As for land ownership, we report in the table the percentage of land owned in a strict sense, i.e. without additional land being rented. This proportion increases from 1996 to 1998–1999, and then remains stable around 61%. Hence, the trend towards land ownership seems to be rather limited, but this aggregate measure does not mean that conditional paths to land ownership are not important. Out of the 732 households in the full sample, 338 have experiences at least one move from ownership to land rental, or in the opposite direction. The average number of moves from ownership to land renting (respectively, from land renting to ownership) is 1.0828, or 18.05% of time periods (respectively, 1.1509, or 19.18% of time periods).

We choose to normalize the coefficient of *LINC* in the Logit and the semiparametric estimation procedures. Using land renting as the dependent variable, different versions of the model are estimated, with the coefficient of log income normalized to -1 and 1 . As preliminary results (including a random effects Probit as in Charlier *et al.* (1995)) indicate that model performance is better with this coefficient set to 1 , we only present this case in the following. The positive

⁹ 1 sotka = 1/100 hectare.

Table 3. Descriptive statistics on the sample, RLMS 1996–2002.

	1996	1997	1998	1999	2000	2001	2002
<i>Y</i> (1 for land renting)							
Mean	0.2363	0.1981	0.2090	0.2008	0.2213	0.1981	0.2049
Land ownership (only)							
Mean	0.5608	0.6145	0.6318	0.6275	0.6162	0.6173	0.6072
<i>INC</i> (monthly real household income, in 1,000 roubles)							
Median	1.8424	3.2895	2.2401	2.1813	3.8325	4.9426	5.4047
<i>AOLAND</i> (land area, in sotkas)							
Median	10.0000	9.0000	9.0000	9.0000	8.0000	9.0000	8.0000
<i>AGSHARE</i> (share of income from agriculture)							
Mean	0.0247	0.0375	0.0376	0.0309	0.0656	0.0846	0.0510
<i>SOLDSHARE</i> (share of agricultural products sold)							
Mean	0.9427	0.9573	0.9598	0.9638	0.9536	0.9396	0.9566

Notes: 5,124 observations ($N = 732$, $T = 7$). *INC* is in 1995 roubles.

role of household income in the decision to rent land can be interpreted as follows. First, for households with income mainly originating from agricultural product sales, owned farm land may have already been decided upon in an optimal way. An increase in income would then result in intensifying production on existing land plots, while occasionally renting additional land plots on a short-term basis. Hence, flexibility in managing total land area would be preferred with higher levels of income. Unfortunately, the data set does not provide us with information on the relative land price compared to land rent, which would enable us to test for such behaviour. Second, in the case of households for which farm land is used as an secondary source of income, the financial constraint to buy land is reduced when income increases. However, these households may then rely less on agricultural product sales, as their role as a safety net when income is low is reduced.

Table 4 presents parameter estimates for the 4 estimators discussed in this paper: the fixed effects Logit Type I and Type II, and the SMS estimator Type I and Type II. State dependence is significant in the decision to rent land, which means that a significant proportion of the data is lost when considering fixed effect techniques. Indeed, out of 732 households in the full sample, only 338 individual observations are effectively used for Type I estimators, and 144 individuals for Type II estimators.

For each fixed effects Logit, we augment the model with a linear trend term t . Concerning the SMS estimators presented above, we only report estimation results when $h = 4$, because estimates were more efficient than with $h = 2$, and also because of the discussion on Monte Carlo experiment results in Section 5. The same higher-order kernel $K_4(\cdot \cdot \cdot)$ was used as in the simulation experiment, with a grid search over different values of parameter δ between 0 and 1. We also report in Table 4 values of δ and λ^* obtained on the basis of final parameter estimates.

Land area contributes positively to the probability of land renting in all cases, with an associated parameter always significantly different from 0. The fact that positive changes in agricultural land area tend to increase the probability of renting land is interpreted here as independent from average, household-specific land area, because of the fixed effects context. This means that increases in

Table 4. Estimation results—Dependent variable: Land renting.

	Logit Type I		Logit Type II		SMS Type I	SMS Type II
	(a)	(b)	(c)	(d)	(e)	(f)
<i>LINC</i>	1.0000 (—)	1.0000 (—)	1.0000 (—)	1.0000 (—)	1.0000 (—)	1.0000 (—)
<i>LAREA</i>	0.7096*** (0.1278)	0.7107*** (0.1277)	0.9759*** (0.2476)	1.0595*** (0.2532)	0.4574*** (0.1361)	0.8509*** (0.2931)
<i>AGSHARE</i>	−0.5288*** (0.1045)	−0.5315*** (0.1045)	−0.1911 (0.1503)	−0.1792 (0.1502)	0.1457 (0.0979)	−0.2364 (0.2707)
<i>SOLDSHARE</i>	−0.0917 (0.1282)	−0.0855 (0.1284)	−0.3194 (0.2270)	−0.4003* (0.2291)	−0.2359 (0.1558)	−0.4247** (0.2170)
<i>t</i>	— (—)	0.0520 (0.0343)	— (—)	−0.1350** (0.0515)	— (—)	— (—)
$(1/N) \log L$	−1.5824	−1.5809	−1.6118	−1.5925	—	—
λ^*	—	—	—	—	1.7403	0.1369
δ	—	—	—	—	0.6	0.6
Score function					14.0334	0.0811
Hausman test (a) versus (c)	10.7576 (0.0313)					
Hausman test (b) versus (d)	34.2436 (0.0000)					
Hausman test (b) versus (c)	10.9310 (0.0121)					

Notes: 5,124 observations ($N = 732$, $T = 7$). Number of households effectively used: $N = 338$ and $N = 144$ for Type I and Type II estimators, respectively.

*, ** and ***, respectively, indicate parameter significance at the 10, 5 and 1 percent level.

land used are primarily due to a more active rental market, as opposed to land market. Parameter estimates associated with the different fixed effects estimators have a different order of magnitude. The Type II estimates based on double conditioning (or covariates in double differences) are always larger than their Type I counterparts (based on single conditioning, or first-differenced covariates). Turning now to parameter estimates for *AGSHARE* and *SOLDSHARE*, it can be seen that both are negative in most cases. The interpretation would be that households relying more heavily on agricultural production for their total income (as measured by *AGSHARE*) would prefer to buy land as opposed to renting it. This is consistent with the fact that the proportion of production sold to local markets is also a negative determinant of the probability of renting land. On the other hand, households with only limited income originating from agriculture will be less likely to buy land as opposed to renting it. Type II estimators being less efficient than Type I ones, parameters of *AGSHARE* are not significant for Logit and SMS Type II. Note that this is also true for the semiparametric SMS Type I estimator. For *SOLDSHARE* however, the opposite result is obtained: even though Type II parameter estimates have larger standard errors than Type I ones, estimated coefficients are greater in absolute value. As a consequence, the parameter of *SOLDSHARE* is significant in the Logit Type II and SMS Type II only. The coefficients on *AGSHARE* and *SOLDSHARE* are, respectively, under-estimated and over-estimated by the Logit Type I (absolute

value, respectively, greater and lower than the Logit Type II). The same result is observed for the SMS Type I compared to the Type II semiparametric estimator, although estimate of *AGSHARE* is not significant in either case. Finally, results do not change significantly when the trend term is introduced.

Let us now consider testing for model specification. As Logit Type II is always consistent but not efficient when the standard one-way specification is the true one, we form a Hausman test statistic based on the difference of Type I and Type II estimates. Test statistics presented in Table 4 correspond to three different null hypotheses: Type I Logit is consistent with no common trend (model (a) vs. model (c)), or with common trend (model (b) vs. (d), and model (b) vs. (c)). As all Hausman test statistics reject the null at the 5% level, we can conclude that Logit Type I estimators are not consistent. Furthermore, this result is not affected by the omission of a trend term in the equation, in which case we may not have rejected the null when comparing model (b) with model (c).

6. CONCLUSION

This paper is an attempt towards extending panel data model specifications with linear heterogeneous trends, to binary choice models. As the fixed-effects Logit is popular in applications of discrete-choice models with panel data, it seems legitimate to consider such an extension, provided that adequate economic models of individual choice match such a specification.

We propose a conditional Logit estimator based on double conditioning, which should be consistent provided the underlying latent-variable generating process admits a linear trend. Individual parameters in the linear trend function need not be known, and may be individual-specific (random coefficients). As the double conditioning uses a more limited proportion of the data, this estimator is rather inefficient, however. We then adapt the smoothed maximum score estimator procedure proposed by Horowitz (1992) and applied to panel data by Charlier *et al.* (1995) and Kyriazidou (1995). We show that the modified semiparametric estimator is in fact constructed from a double difference transformation of the data.

Small-sample properties of the Logit and smoothed maximum score estimators are then investigated with a Monte Carlo experiment. Simulation results reveal that our new conditional Logit based on double conditioning procedure performs well in terms of small-sample bias, whereas the usual Logit clearly exhibits significant small-sample bias. The same properties are found for our modified semiparametric estimator. However, a surprising result is that the smoothed maximum score estimator always exhibits a (somewhat limited) bias, even when the true data generating process does not allow for a heterogeneous linear trend.

To investigate the behaviour of our proposed estimators on real data, we present an empirical application concerning the decision of Russian households to rent agricultural land. We estimate a reduced-form model of land tenure using the different fixed effects estimators presented, using as covariates household income, land area, the ratio of agricultural revenue over total income and the proportion of production sold to local markets. The standard one-way panel specification is rejected on the basis of a Hausman specification test, even when a common trend is included in both model specifications.

The main limitations of our approach are obviously the linear trend assumption, and the relatively poor efficiency properties of the new estimators. Efficiency is, however, seen to increase rapidly with the sample size, especially when the number of time periods increases (from 4 to

10 in our Monte Carlo experiment). As for the linear trend restriction, it is the only convenient way of adapting in a straightforward manner the existing fixed effects estimators proposed in the literature. More general model specifications could allow individual effects to be multiplied by unrestricted time effects, but this is left for future research.

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REFERENCES

- Ahn, S. C., Y. H. Lee and P. Schmidt (2001). GMM estimation of linear panel data models with time-varying individual effects. *Journal of Econometrics* 101, 219–55.
- Blundell, R., R. Griffith and F. Windmeijer (2002). Individual effects and dynamics in count data models. *Journal of Econometrics* 108, 113–31.
- Brooks, K. and Z. Lerman (1995). Restructuring of traditional farms and land relations in Russia. *Agricultural Economics* 13, 11–25.
- Bruce, D. (2000). Effects of the United States tax system on transitions into self-employment. *Labour Economics* 7, 545–74.
- Chamberlain, G. (1992). Binary response models for panel data: Identification and information, Working paper Harvard University.
- Charlier, E., B. Melenberg and A. van Soest (1995). A smoothed maximum score estimator for the binary choice panel data model with an application to labour participation. *Statistica Neerlandica* 49, 324–42.
- Comin, D. and B. Hobijn (2004). Cross-country technology adoption: Making the theory face the facts. *Journal of Monetary Economics* 51, 39–83.
- Crépon, B., F. Kramarz and A. Trognon (1997). Parameters of interest, nuisance parameters and orthogonality conditions: An application to autoregressive error component models. *Journal of Econometrics* 82, 135–56.
- Csaki, C., G. Feder and Z. Lerman (2002). Land policies and evolving farm structures in transition countries. Policy Research Working paper no. 2794, World Bank, Washington, DC.
- Deiniger, K. (2003). Land policies for growth and poverty reduction. World Bank Policy Research Report, World Bank, Washington, DC.
- Gibson, J., S. Stillman and T. Le (2004). CPI bias and real living standards in Russia, Working paper 684, William Davidson Institute.
- Güell, M. and L. Hu (2006). Estimating the probability of leaving unemployment using uncompleted spells from repeated cross-section data. *Journal of Econometrics*, in press.
- Han, C., L. Orea and P. Schmidt (2005). Estimation of a panel data model with parametric temporal variation in individual effects. *Journal of Econometrics* 126, 241–67.
- Hausman, J., B. H. Hall and Z. Griliches (1984). Econometric models for count data with an application to the patents-R&D relationship. *Econometrica* 52, 909–38.
- Holtz-Eakin, D., W. Newey and H. Rosen (1988). Estimating vector autoregressions with panel data. *Econometrica* 56, 1371–95.
- Honoré, B. E. and E. Kyriazidou (2000). Panel data discrete choice models with lagged dependent variables. *Econometrica* 68, 839–74.

- Horowitz, J. L. (1992). A smoothed maximum score estimator for the binary response model. *Econometrica* 60, 505–31.
- Kerr, S. and R. Newell (2003). Policy-induced technology adoption: Evidence from the US lead phasedown. *Journal of Industrial Economics* 51, 317–43.
- Kyriazidou, E. (1995). Essays in estimation and testing of econometric models. Ph.D dissertation, Northwestern University.
- Lancaster, T. (2002). Orthogonal parameters and panel data. *Review of Economic Studies* 69, 647–66.
- Manski, C. F. (1975). Maximum score estimation of the stochastic utility model of choice. *Journal of Econometrics* 3, 205–28.
- Manski, C. F. (1985). Semi-parametric analysis of discrete response: Asymptotic properties of the maximum score estimator. *Journal of Econometrics* 27, 313–34.
- Manski, C. F. (1987). Semiparametric analysis of random effects linear models from binary panel data. *Econometrica* 55, 357–62.
- Nauges, C. and A. Thomas (2003). Consistent estimation of dynamic panel data models with time-varying individual effects. *Annales d'Economie et de Statistique* 70, 53–75.
- Ravallion, M. (2002). Externalities in rural development: Evidence from China, Working Paper 2879, World Bank Washington, DC.
- Steiner, V. (2001). Unemployment persistence in the West German labour market: Negative duration dependence or sorting? *Oxford Bulletin of Economics and Statistics* 63, 91–113.
- Verbeek, T. J. and T. Knaap (1999). Estimating a dynamic panel data model with heterogenous trends. *Annales d'Economie et de Statistique* 56-56, 331–49.