CS460
HW-2
Selin DINC

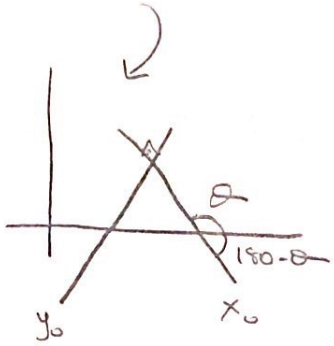
1-

1. Translate origin (x,y) to (x_0, y_0)

$$T(x_0,y_0) = \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Rotate coordinate system clock-wise $180-\theta$

$$m = \frac{y_2 - y_0}{x_2 - x_0} \qquad \tan(\theta) = m \qquad \theta = \tan^{-1} m$$



$$T_{Rot} = \begin{bmatrix} \cos(180-\theta) & \sin(180-\theta) & 0 \\ -\sin(180-\theta) & \cos(180-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = T_{Rot}\, T_{TR}$$

$$M = \begin{bmatrix} \cos(180-\theta) & \sin(180-\theta) & 0 \\ -\sin(180-\theta) & \cos(180-\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} \cos(180-\theta) & \sin(180-\theta) & \cos(180-\theta)x_0 + \sin(180-\theta)y_0 \\ -\sin(180-\theta) & \cos(180-\theta) & -\sin(180-\theta)x_0 + \cos(180-\theta)y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

2-

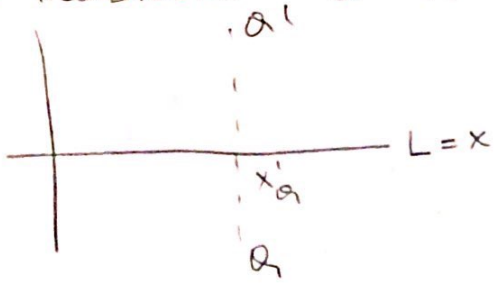
1. Rotate line L clockwise

$$R(-\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Rotate θ'θ line

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_a \\ y_a \\ 1 \end{bmatrix} = \begin{bmatrix} x_a' \\ y_a' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta\, x_a + \sin\theta\, y_a \\ -\sin\theta\, x_a + \cos\theta\, y_a \\ 1 \end{bmatrix}$$

2 - Translate x_Q' to (x_0, y_0)



$$x_{Q'} = \cos\theta x_Q + \sin\theta y_Q$$

(2)

$$T_{TR} = \begin{bmatrix} 1 & 0 & -(\cos\theta x_Q + \sin\theta y_Q) \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Reflect P to P' around y-axis

$$T_{Ref} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Translate to old position

$$T_{TRB} = \begin{bmatrix} 1 & 0 & \cos\theta x_Q + \sin\theta y_Q \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5. Rotate back (counter clockwise)

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = R(\theta)\, T_{TRB}\, T_{Ref}\, T_{TR}\, R(-\theta)$$

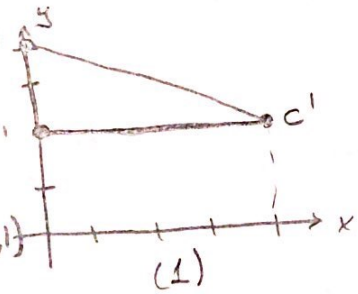
3- a) $T(-1,0)\, S(1/2, 1/4)\, R(90°)\, Rf(y,x)$ $A=(2,0)$ $B=(4,0)$ $C=(2,4)$

(1) $Rf(y,x) \rightarrow A'=(0,2)$ $B'=(0,4)$ $C'=(4,2)$

(2) $R(90°) \rightarrow (-y,x)$ $A''=(-2,0)$ $B''=(-4,0)$ $C''=(-2,4)$

(3) $S(1/2, 1/4) \rightarrow (x/h, y/k)$ $A'''=(-1,0)$ $B'''=(-2,0)$ $C'''=(-1,1)$

(4) $T(-1,0) \rightarrow A^{IV}=(-2,0)$ $B^{IV}=(-3,0)$ $C^{IV}=(-2,1)$



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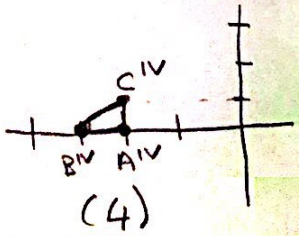

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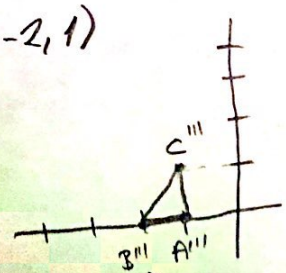

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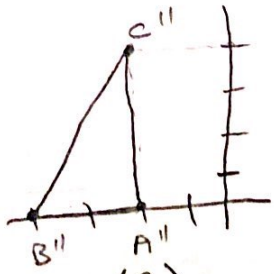


(2)

b) $R(90°)\ T(-1,0)\ S(1/2, 1/4)\ Rf(y,-x)$ $A=(2,0)$ $B=(4,0)$ $C=(2,4)$ (3)

(1) $Rf(y,-x) \rightarrow (-y,-x)$ $A'=(0,-2)$ $B'=(0,-4)$ $C'=(-4,-2)$

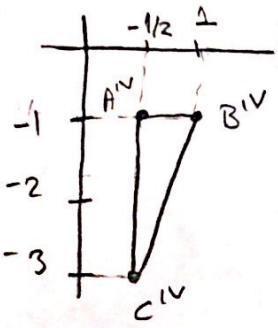
(2) $S(1/2, 1/4)$ $A''=(0,-1/2)$ $B''=(0,-1)$ $C''(-2,-1/2)$

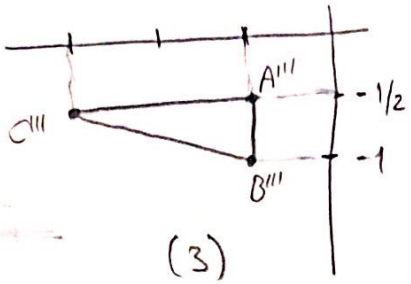
(3) $T(-1,0)$ $A'''=(-1,-1/2)$ $B'''=(-1,-1)$ $C'''(-3,-1/2)$

(4) $R(90°) \rightarrow (-y,x)$

$A^{IV}=(1/2,-1)$ $B^{IV}=(1,-1)$

$C^{IV}=(1/2,-3)$





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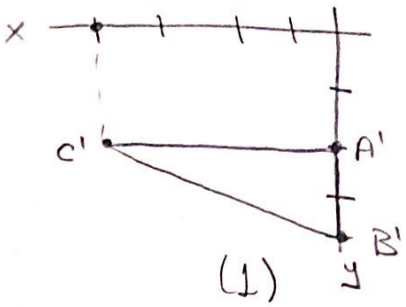

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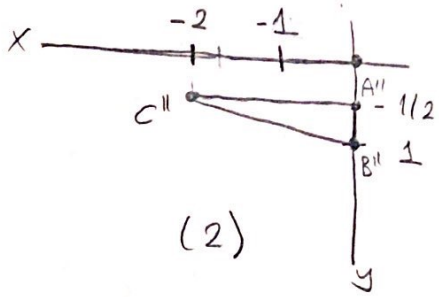


(2)

4- Viewport 1: → $x_{v_{min}}=1$ $x_{v_{max}}=3$

$y_{v_{min}}=3$ $y_{v_{max}}=6$

$$\frac{x_v - x_{v_{min}}}{x_{v_{max}} - x_{v_{min}}} = \frac{x_w - x_{w_{min}}}{x_{w_{max}} - x_{w_{min}}} \qquad \frac{y_v - y_{v_{min}}}{y_{v_{max}} - y_{v_{min}}} = \frac{y_w - y_{w_{min}}}{y_{w_{max}} - y_{w_{min}}}$$

$x_{w_{min}}=2$ $x_{w_{max}}=6$ $A=(3,5)$ $B=(4,5.5)$ $C=(5,4)$

$y_{w_{min}}=2$ $y_{w_{max}}=6$

$A(3,5)=(x_w, y_w)$

$$\frac{x_v - 1}{3-1} = \frac{3-2}{6-2} \Rightarrow x_v = 3/2$$

$(x_w, y_w) \rightarrow (3/2, 2\,1/4) = A'$

$$\frac{y_v - 3}{6-3} = \frac{5-2}{6-2} \Rightarrow y_v = 2\,1/4$$

$B(4, 5.5) \rightarrow (x_w, y_w) = B'$

$\frac{x_v - 1}{3-1} = \frac{4-2}{6-2} \Rightarrow x_v = 2$ $\quad \frac{y_v - 3}{6-3} = \frac{5.5-2}{6-2} \Rightarrow y_v = 45/8$

$(x_w, y_w) \rightarrow (2, 45/8)$

$C(5,4) \rightarrow (x_w, y_w)$

$\frac{x_v - 1}{3-1} = \frac{5-2}{6-2} \Rightarrow x_v = 5/2$ $\quad \frac{y_v - 3}{6-3} = \frac{4-2}{6-2} \Rightarrow y_v = 9/2$

$C' = (x_w, y_w) \rightarrow (5/2, 9/2)$

Viewport 2:

$x_{v_{min}} = 4 \quad x_{v_{max}} = 7$

$y_{v_{min}} = 5 \quad y_{v_{max}} = 7$

$A(3,5) \rightarrow A''$

$\frac{x_v - 4}{7-4} = \frac{3-2}{6-2} \Rightarrow x_v = 19/4$ $\quad \frac{y_v - 5}{7-5} = \frac{5-2}{6-2} \Rightarrow y_v = 13/2$

$B(4, 5.5) \rightarrow B''$

$\frac{x_v - 4}{7-4} = \frac{4-2}{6-2} \Rightarrow x_v = 11/2$ $\quad \frac{y_v - 5}{7-5} = \frac{5.5-2}{6-2} \Rightarrow y_v = 27/4$

$C(5,4) \rightarrow C''$

$\frac{x_v - 4}{7-4} = \frac{5-2}{6-2} \Rightarrow x_v = \frac{25}{4}$ $\quad \frac{y_v - 5}{7-5} = \frac{4-2}{6-2} \Rightarrow y_v = 6$

Viewport 1 $\rightarrow A'(3/2, 21/4)$ $B'(2, 45/8)$ $C'(5/2, 9/2)$

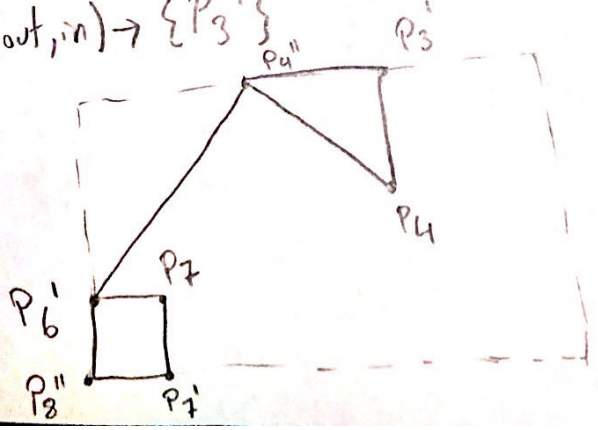
Viewport 2 $\rightarrow A'' = (19/4, 13/2)$ $B'' = (11/2, 27/4)$ $C'' = (25/4, 6)$

5-

a)

Input Edge:	→ Left Clipper	→ Input Edge:	→ Right Clipper	→ Input Edge:
{P1, P2} :	(out, out) → { }	{P2', P3}:	(in, in) → {P3}	{P3, P4}:
{P2, P3} :	(out, in) → {P2', P3}	{P3, P4}:	(in, in) → {P4}	{P4, P4'}:
{P3, P4} :	(in, in) → {P4}	{P4, P4'}:	(in, in) → {P4'}	{P4', P6'}:
{P4, P5}:	(in, out) → {P4'}	{P4', P6'}:	(in, in) → {P6'}	{P6', P7}
{P5, P6}:	(out, out) → { }	{P6', P7}:	(in, in) → {P7}	{P7, P8}
{P6, P7}:	(out, in) → {P6', P7}	{P7, P8}:	(in, in) → {P8}	{P8, P8'}
{P7, P8}:	(in, in) → {P8}	{P8, P8'}:	(in, in) → {P8'}	{P8', P2'}
{P8, P7}:	(in, out) → {P8'}	{P8', P2'}:	(in, in) → {P2'}	{P2', P3}

→ Bottom Clipper	Input Edge: →	→ Top Clipper
(in, in) → {P4}	{P4, P4'}:	(in, out) → {P4''}
(in, in) → {P4'}	{P4', P6'}:	(out, in) → {P6'} →
(in, in) → {P6'}	{P6', P7}:	(in, in) → {P7}
(in, in) → {P7}	{P7, P7'}	(in, in) → {P7'}
(in, out) → {P7'}	{P7, P8''}	(in, in) → {P8''}
(out, out) → {}	{P8'', P2'}	(in, out) → { } →
(out, in) → {P8'', P2'}	{P2', P3},	(out, out) → { }
(in, in) → {P3}	{P3', P4}	(out, in) → {P3'}



Loop:

b) Polygone ⟶ Window ⟶

Polygone	Window
$\{P_1, P_2\}$: (out, out) → {}	$\{P_4', P_3'\}$: (in, in) → $\{P_3'\}$
$\{P_2, P_3\}$: (out, out) → {}	$\{P_3', P_4\}$: (in, in) → $\{P_4\}$
$\{P_3, P_4\}$: (out, in) → $\{P_3'\}$	$\{P_4, P_4'\}$: (in, in) → $\{P_4'\}$
$\{P_4, P_5\}$: (in, out) → $\{P_4'\}$	

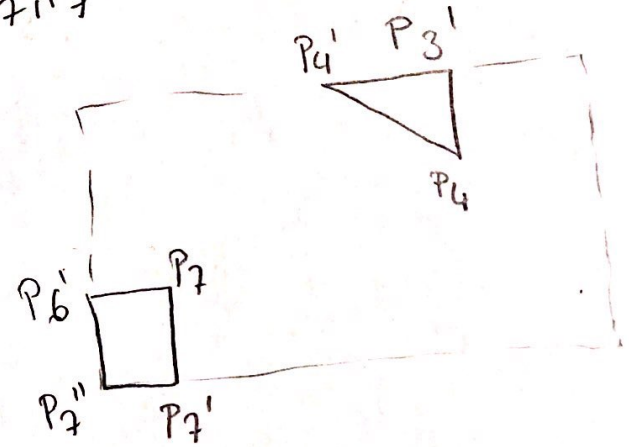
→ Polygone ⟶ Window ⟶

Polygone	Window
$\{P_4', P_5\}$: (in, out) → {}	$\{P_7', P_7''\}$: (in, in) → $\{P_7''\}$
{5, 6} : (out, out) → {}	$\{P_7'', P_6'\}$: (in, in) → $\{P_6'\}$
{6, 7} : (out, in) → $\{P_6'\}$	$\{P_6', P_7\}$: (in, in) → $\{P_7\}$
{7, 8} : (in, out) → $\{P_7'\}$	$\{P_7, P_7'\}$: (in, in) → $\{P_7'\}$

→ Polygone

$\{P_7', P_8\}$: (in, out) → {}

$\{P_8, P_1\}$: (out, out) → {}



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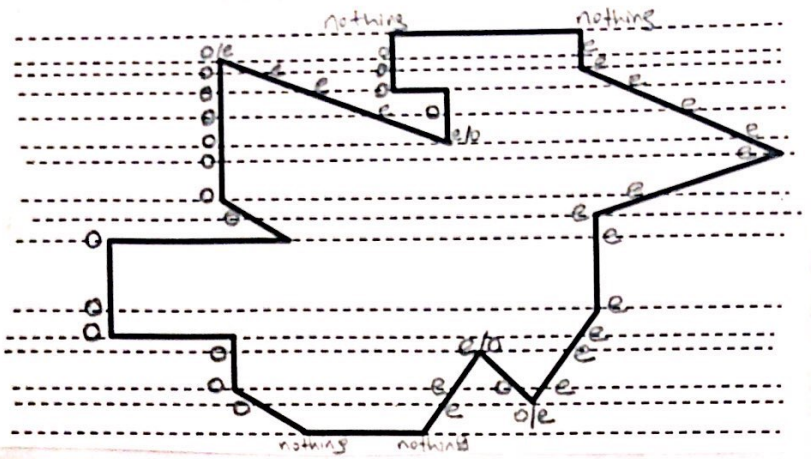

7- Subject Polygone → Clip Polygone → Subject Polygone

{P1, P2} : (out, out) → {}

{P2, P3} : (out, out) → {}

{P3, P4} : (out, in) → {3', 4}

{P4, P5} : (in, out) → {4'}

{P4', P4''} : (in, in) → {P4''}

{P4'', P6} : (in, in) → {6

{P6, P7} : (in, out) → {6'

→ Clip Polygone

{P6', P6''} : (in, in) → {P6''}

{P6'', P6'''} : (in, in) → {P6'''}

{P6''', P6''''} : (in, in) → {P6''''}

{P6'''', P3'} : (in, in) → {}

{P3', P4, P4', P4'', P6, P6', P8'', P6''', P6''''}

Turning points = {P4', P4'', P6'}

→