

## CS411 HW 4

1.

- a. Double encryption in RSA implies that we essentially raise  $m$  (message) to  $e_1 * e_2 = e_3$ . Calculating  $e_1 * e_2$  and finding its inverse in  $\text{mod}(\phi(n))$  is not a harder problem than finding inverse of  $e_1$  and  $e_2$ , and multiplying them thus double encryption does not increase security.
- b. The results can be seen in the console output of question1.py

2.

We know that the plaintext has 4 digits since it is a PIN. Since there are 10000 possible PINs, brute force attack becomes somewhat feasible than trying to factor  $n$  which is too large. Starting with a seed (0000), I tried to calculate ciphertext of the possible PINs one by one. The code that I have written stops when the generated ciphertext is equal to the one given in the question. Then the PIN used to generate that ciphertext is the PIN we're looking for, which is 8128. The code I've written to solve this question can be found as question2.py

3.

- a. It is given in the question that  $cp$  is congruent to  $kp^e \text{ mod } n$ , thus we can write  $cp$  as  $cp = kp^e + m*n$  for some integer  $m$ .  $p$  divides  $kp^e$  and  $p|a*n$  (since  $n$  is  $p*q$ ). We can say that  $p | cp$  and conclude that  $\text{gcd}(cp, n) = p$  since  $p$  divides both and only other factor of  $n$  is  $q$ . When  $cp$  and  $n$  is known, we can obtain  $p$  by calculating their gcd, i.e. we can factor  $n$ .
- b. I found the gcd of  $n$  and  $cp$  which is equal to  $p$ ,  $q$  is found by  $n/p$ . Run question3.py for  $p$  and  $q$  values.

4. a. If we choose  $k$  (similar to blinding in question 3) such that  $\gcd(k, n) = 1$  (it has an inverse)  
 We know  $e$ , we encrypt  $c \rightarrow c^e \pmod{n}$   
 $c' = c \cdot k^e \pmod{n}$   
 • we query the oracle for  $c'$  and get  $m' = (c')^d \pmod{n}$   

$$m' = (c')^d \pmod{n}$$

$$m' \equiv (c \cdot k^e)^d \pmod{n}$$

$$m' = \underbrace{c^d}_m \cdot k \pmod{n}$$

$$m' \equiv m \cdot k \pmod{n}$$

↳ we know  $k$ , can calculate its inverse.  
 ↳ also known

from known

$$\underline{\underline{m \equiv k^{-1} m' \pmod{n}}}$$

4.

b. I chose  $k$  to be 31, I calculated its inverse in  $\pmod{n}$  and generated the corresponding cipher then send it to oracle. I have pasted the oracle's output as oracleout then used it to find the actual message by multiplying by  $k$ 's inverse in  $\pmod{n}$ . Resulting integer can be seen as the console output of question4.py