Measuring Stress, Strain, and Force

Rick Sellens

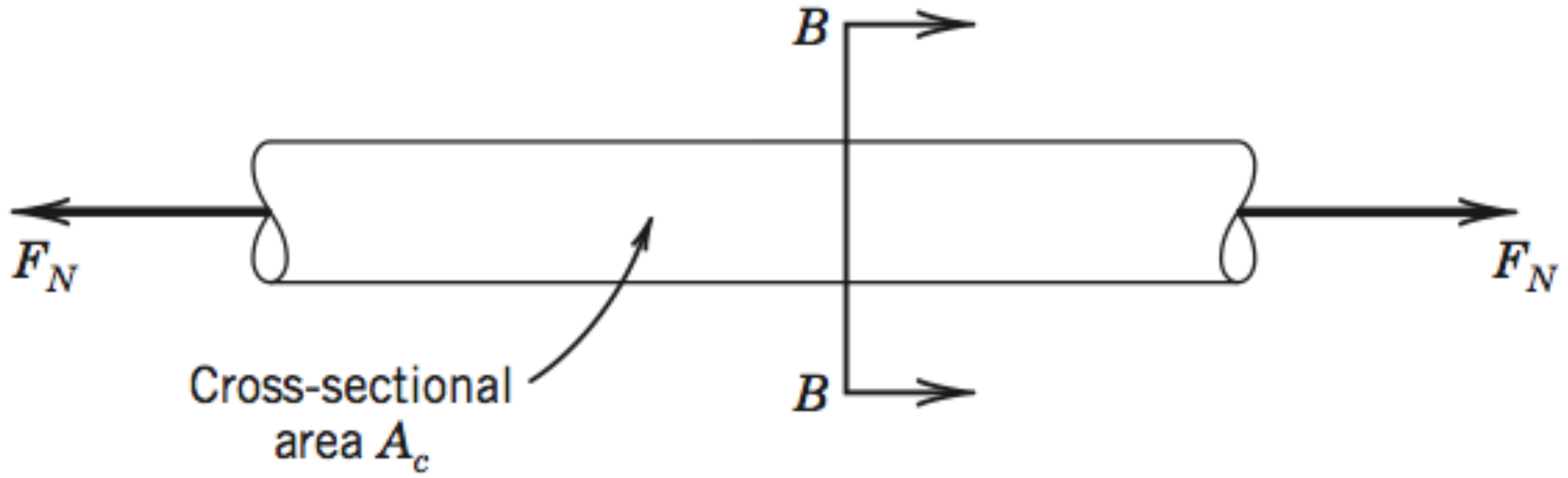
Applications

- Stress is hard to measure, but you can infer it from strain (MECH 221 Solids, MECH 270 Materials)

- Strain will tell you how mechanical systems are deforming under stress

- Force is key to simple measurements like weighing systems, and more complex dynamic loads (MECH 228 Dynamics)

- **Want something we could attach to a beam and know how much it has deformed under load**



Poisson's Ratio $$\upsilon_P = -\frac{\text{Lateral strain}}{\text{Axial strain}} = -\frac{\varepsilon_L}{\varepsilon_a}$$

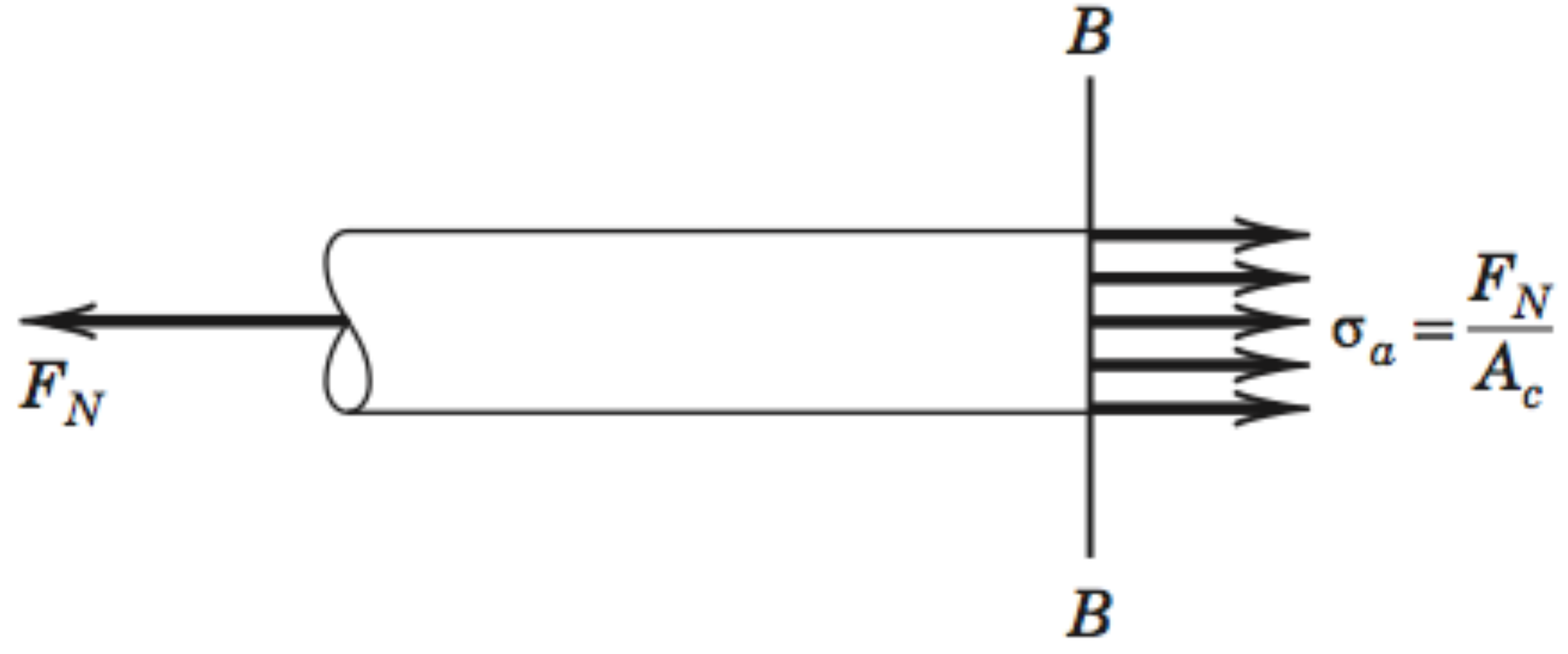


Figure 11.1 Free-body diagram illustrating internal forces for a rod in uniaxial tension.

$$\varepsilon_a = \delta L / L$$

Strain is the fractional elongation, usually in micrometres/metre or microstrain

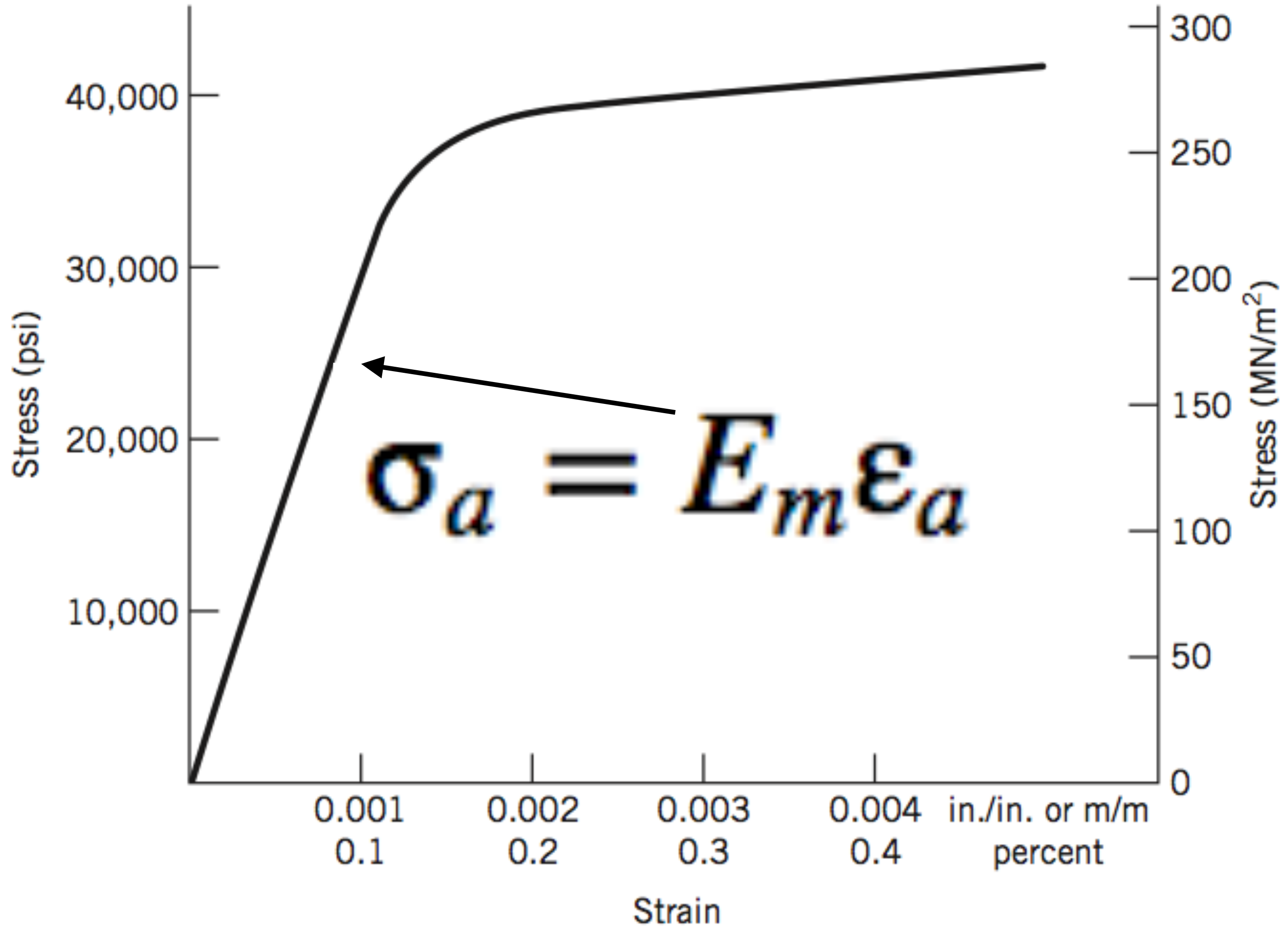
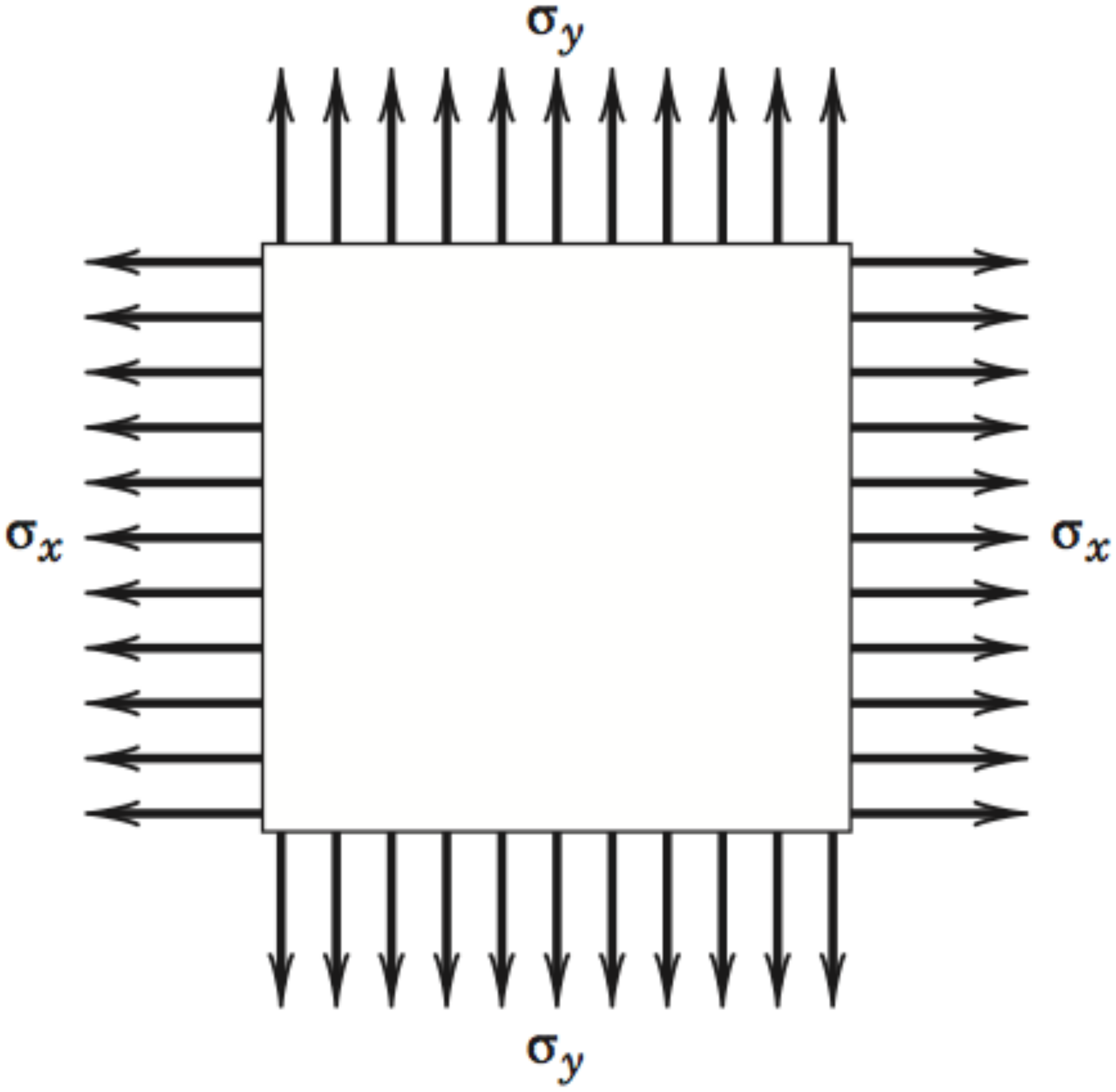


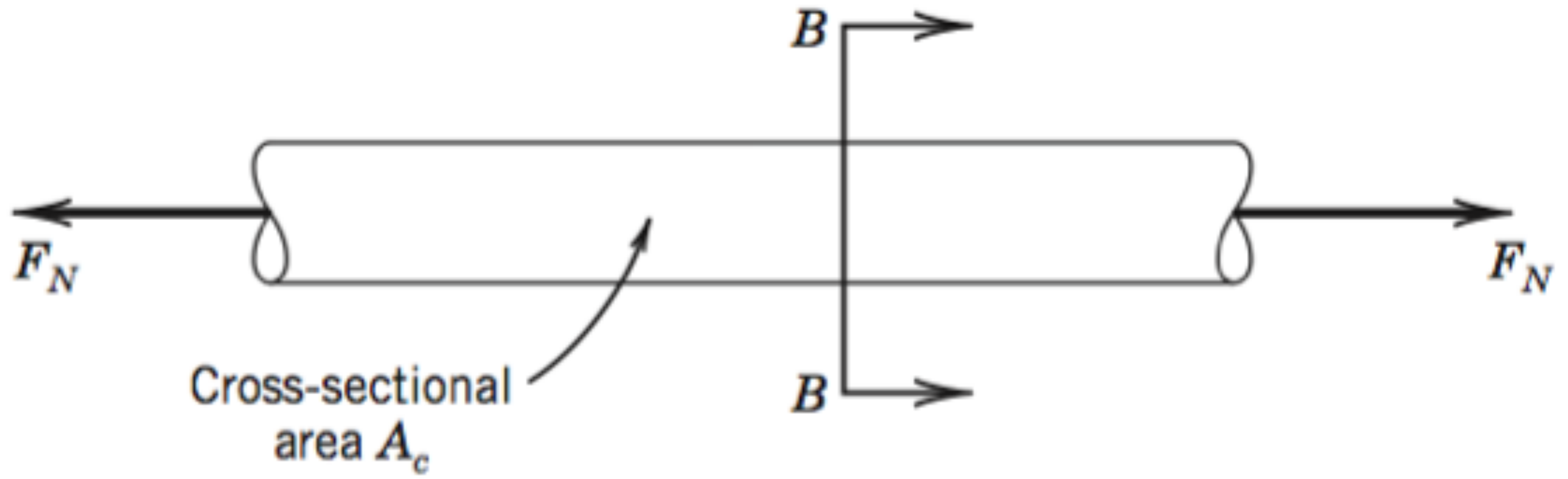
Figure 11.2 A typical stress–strain curve for mild steel.



Ideally we could make our gauge sensitive to only one direction

Figure 11.3 Biaxial state of stress.

Electrical Resistance of a Bar of Metal



$$R = \rho_e L / A_c$$

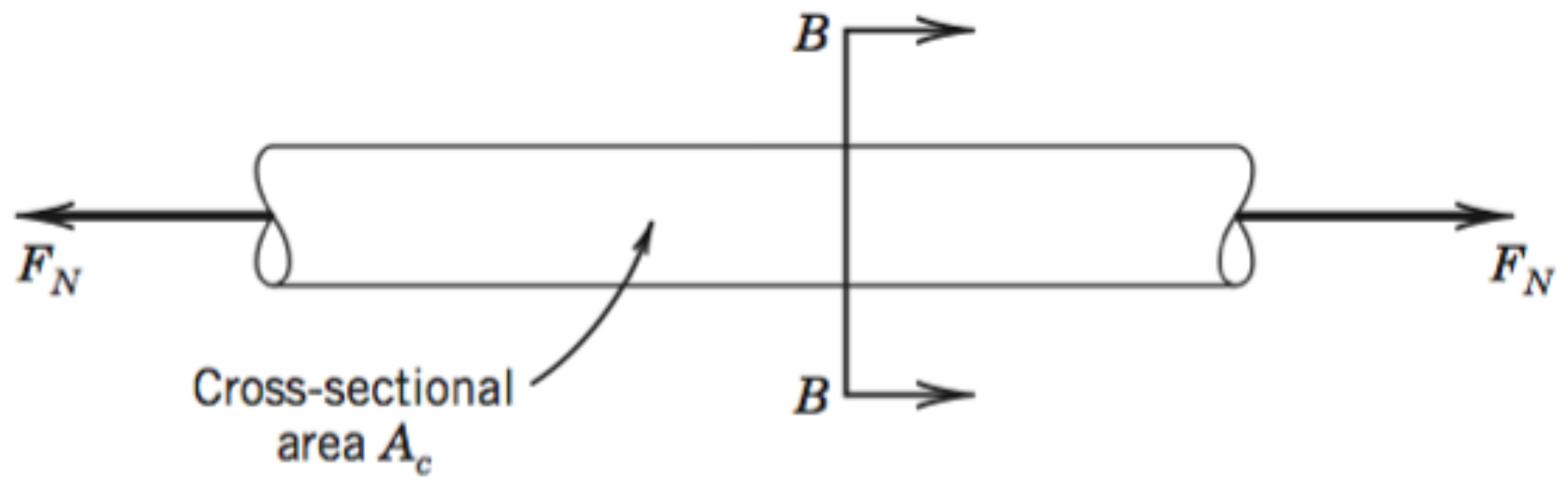
- Resistivity is the inverse of conductivity, ohms/metre, low for copper, higher for steel

- The longer the bar, the higher the resistance

- The bigger the transverse area, the lower the resistance

- More complicated for high currents, so don't go there…

Apply a Load to the Bar



$$R = \rho_e L / A_c$$

- Strain increases the length,
- decreases the diameter (Poisson's ratio),
- distorts the crystal structure, changing the resistivity (piezo resistive effects)
- all of which will increase the resistance of the bar

Change in Resistance

$$R = \rho_e L / A_c$$

$$dR = \frac{\partial R}{\partial L} dL + \frac{\partial R}{\partial A_c} dA_c + \frac{\partial R}{\partial \rho_e} d\rho_e$$

$$dR = \frac{\rho_e}{A_c} dL - \frac{\rho_e L}{A_c^2} dA_c + \frac{L}{A_c} d\rho_e$$

$$dR = \frac{A_c(\rho_e dL + L d\rho_e) - \rho_e L dA_c}{A_c^2} \qquad (11.8)$$

which may be expressed in terms of Poisson's ratio as

$$\frac{dR}{R} = \frac{dL}{L}(1 + 2v_p) + \frac{d\rho_e}{\rho_e} \qquad (11.9)$$

and resistivity is linear

Hence, the changes in resistance are caused by two basic effects: the change in geometry as the length and cross-sectional area change and the change in the value of the resistivity, ρ_e. The dependence of resistivity on mechanical strain is called piezoresistance and may be expressed in terms of a piezoresistance coefficient, π_1, defined by

$$\pi_1 = \frac{1}{E_m} \frac{d\rho_e/\rho_e}{dL/L} \tag{11.10}$$

With this definition, the change in resistance may be expressed

$$dR/R = dL/L\left(1 + 2\upsilon_p + \pi_1 E_m\right) \tag{11.11}$$

at least if we choose our materials carefully

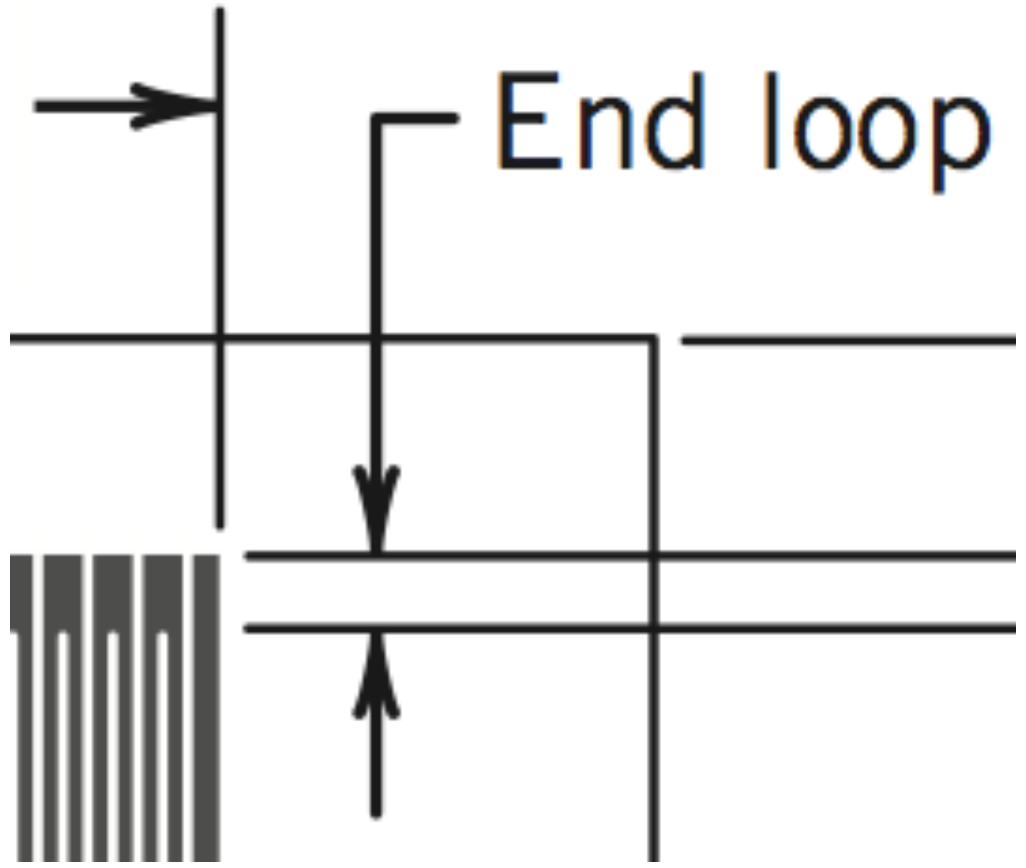
Bottom Line

$$\Delta R \propto \Delta L$$

$$\frac{\Delta R}{R} \propto \frac{\Delta L}{L} = \varepsilon_a$$

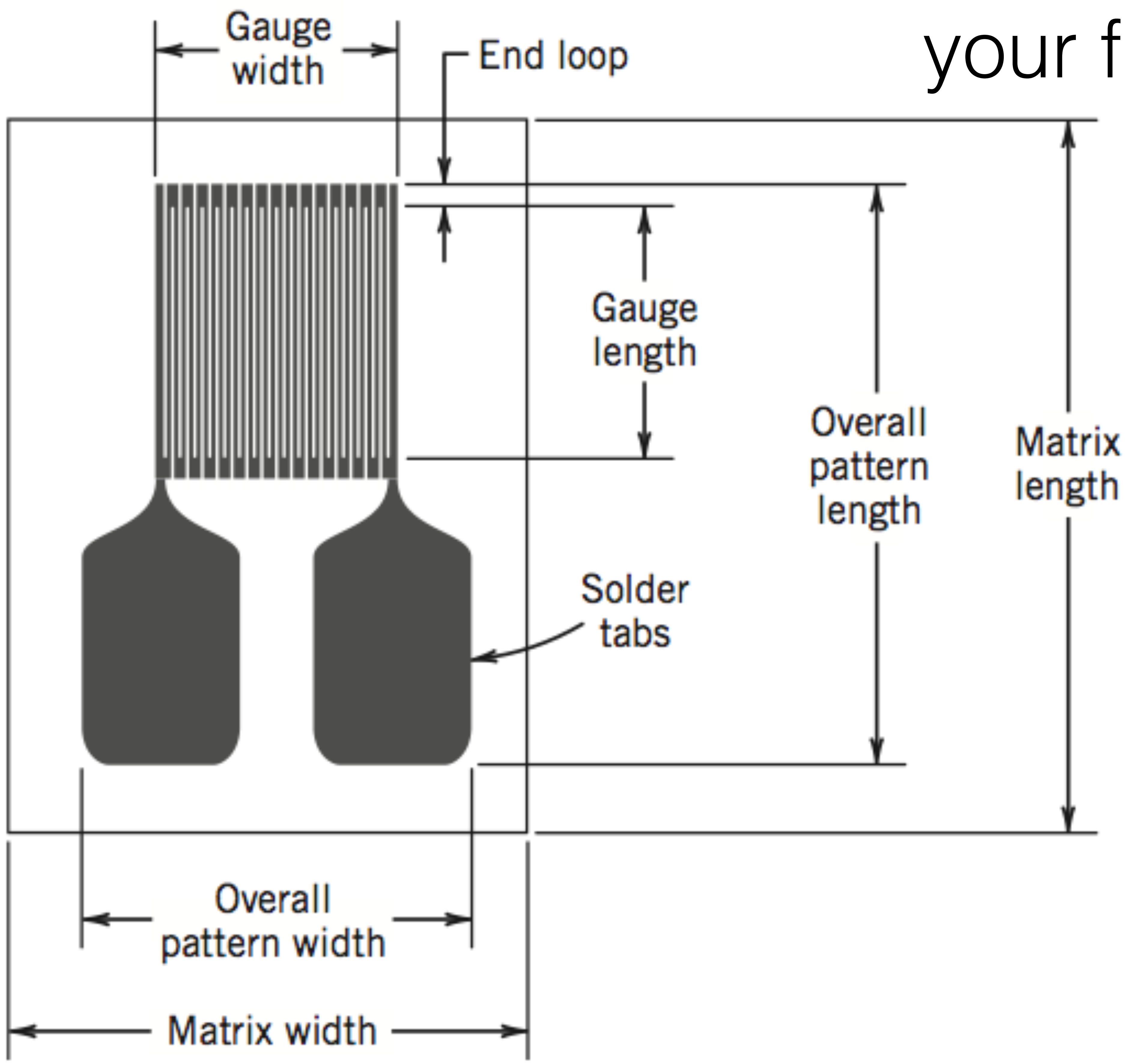
Measure small changes in R and we measure small changes in L

Foil Strain Gauge



wide, insensitive end loop oriented horizontally

smaller than your finger nail



Thin, sensitive elements oriented vertically

Figure 11.5 Construction of a typical metallic foil strain gauge. (Courtesy of Micro-Measurements, Raleigh, NC, USA.)

Foil Strain Gauges

thin conducting foil on flexible plastic carrier

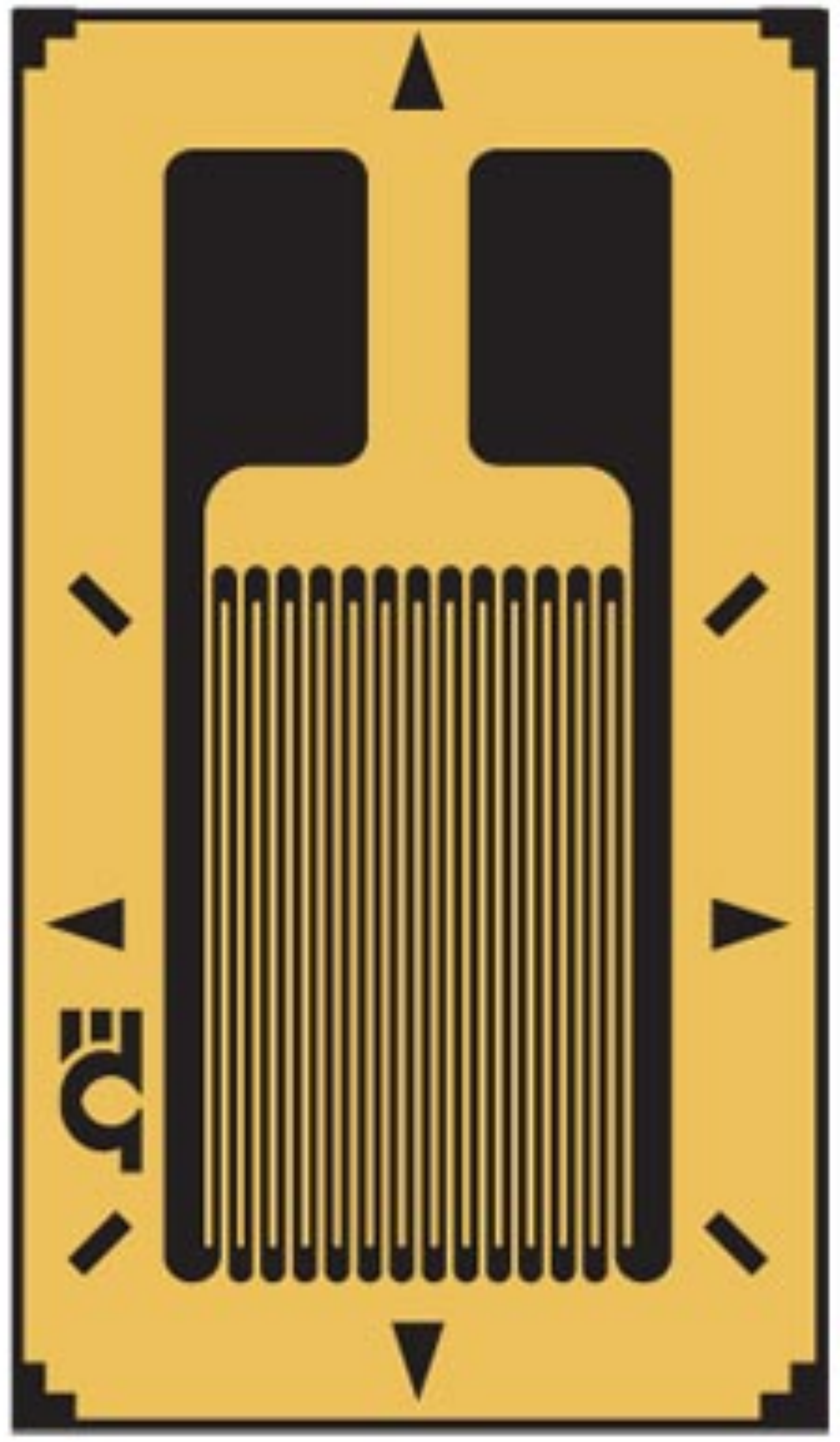
CAD206.00 KFH-3-350-C1-11L3M3R

packages of 10

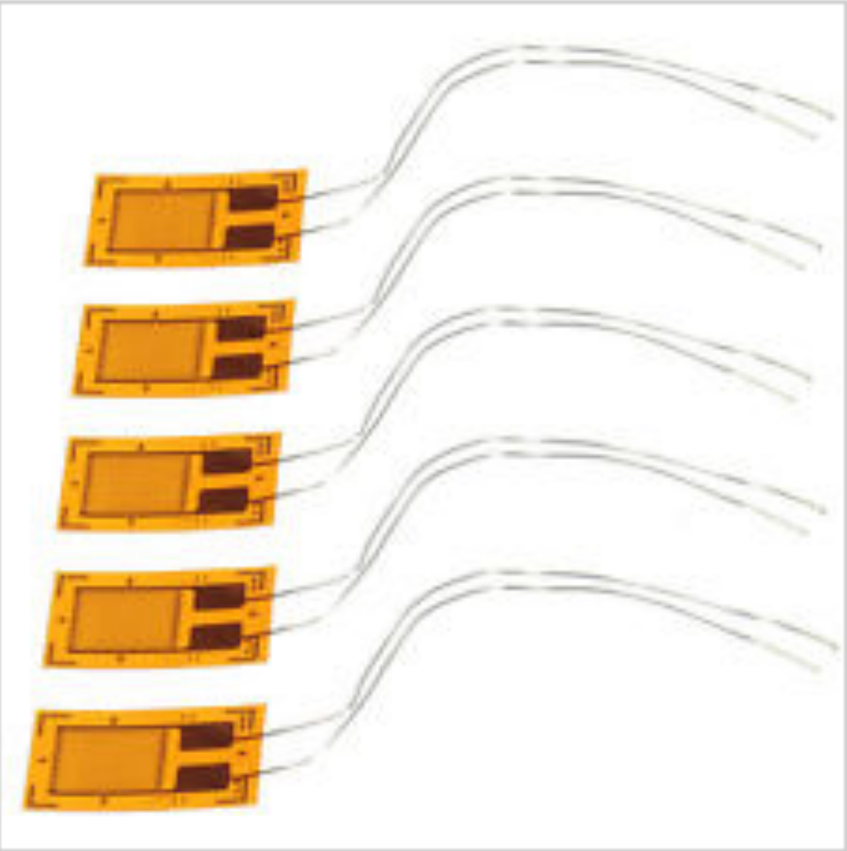
PLACE ORDER

- Pre-Wired Gages Make Installations Easy!
- No Soldering at the Measuring Point
- Linear Gages, 0.3 mm to 20 mm Grids
- XY Gages (Tee Rosettes) 0.6 to 6 mm Grids
- 0°/45°/90° Planar Rosettes 0.6 to 6 mm Grids
- Each Gage has 50 mm of PTFE Cable before Transitioning to AWG 28 Leads to Prevent Leads from Adhering During Installation
- Rugged Polyimide Carrier
- Fully Encapsulated Gages for Protection from the Environment

Strain Gages, Accessories and Instrumentation - View related products



http://www.omega.ca/pptst_eng/KFH.html



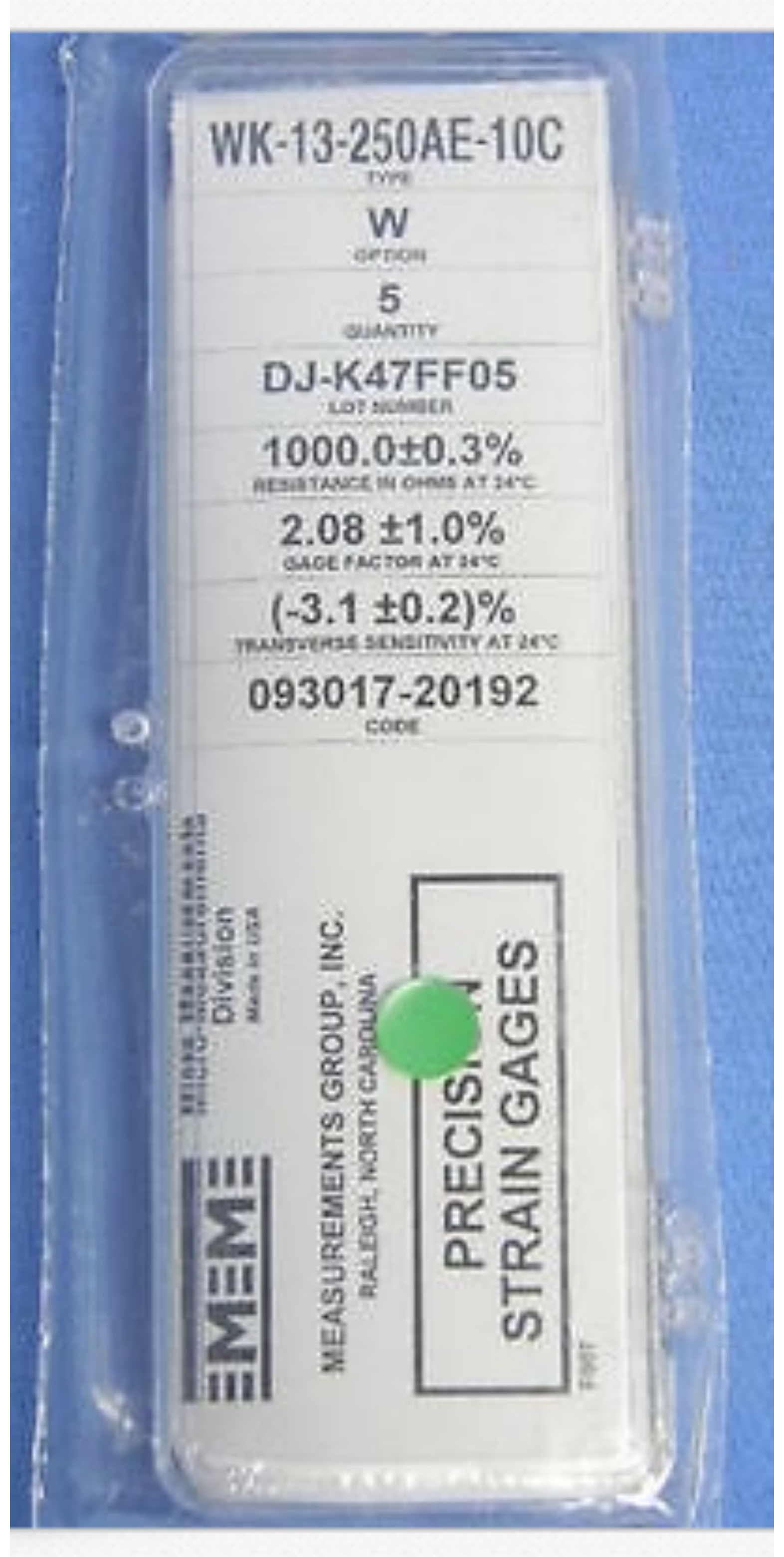
5Pcs BF350-3AA Type Constantan Metal Foil Resistance Steel Strain Gauge 350Ω

$4.44
Buy It Now

16 watching | 223 sold

5 x BF350 high-precision resistance strain gauges. BF350-3AA type constantan metal foil resistance strain gauge. Strain limit: 2.0%. Made of copper foil. Resistance: 350 / -0.1 Ω. High accuracy, good...

Gauge Factor Provided by Manufacturer

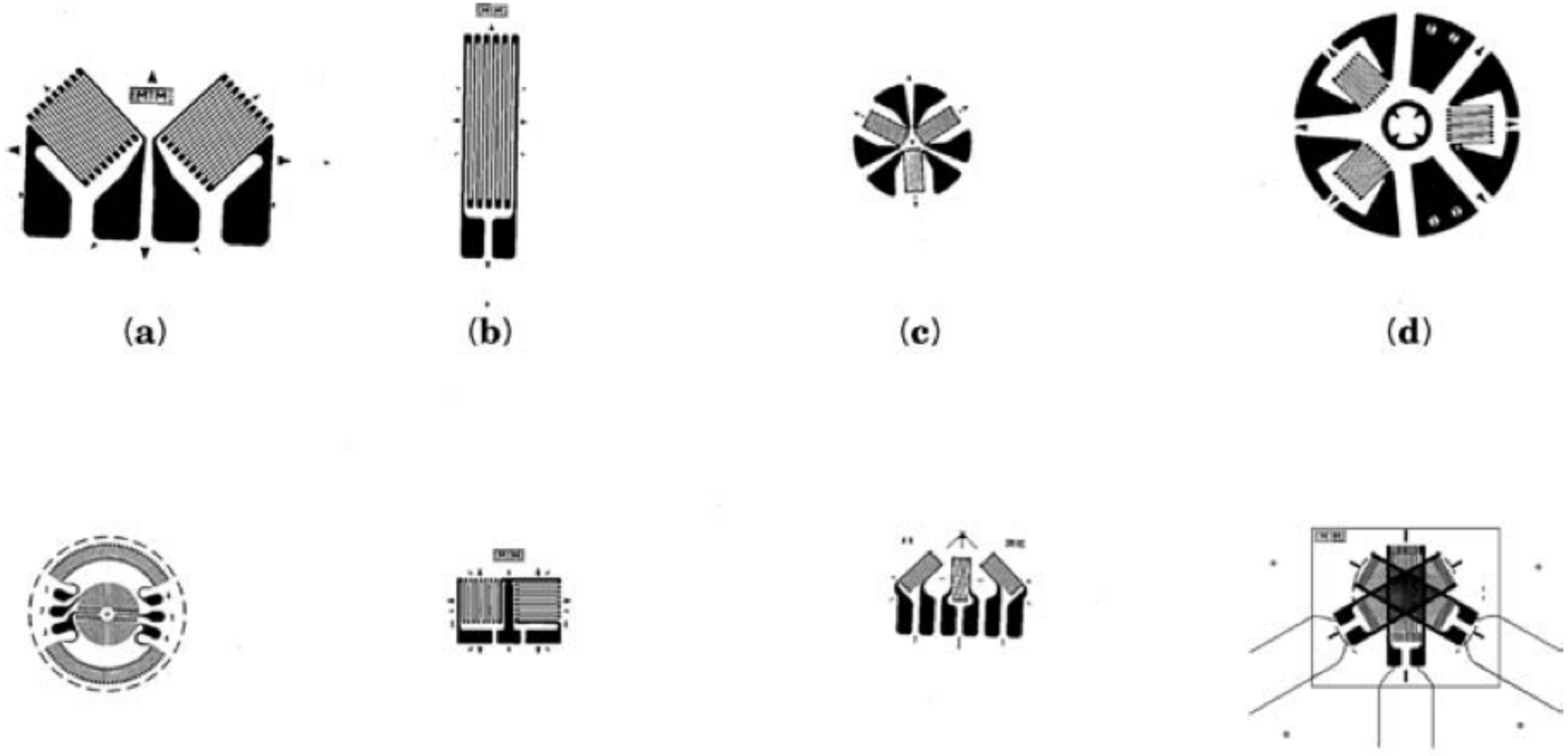


Gauge Factor

The change in resistance of a strain gauge is normally expressed in terms of an empirically determined parameter called the gauge factor (GF). For a particular strain gauge, the gauge factor is supplied by the manufacturer. The gauge factor is defined as

$$GF \equiv \frac{\delta R/R}{\delta L/L} = \frac{\delta R/R}{\varepsilon_a} \tag{11.12}$$

GF is typically ~ 2 so the resistance change is about twice as big as the change in length

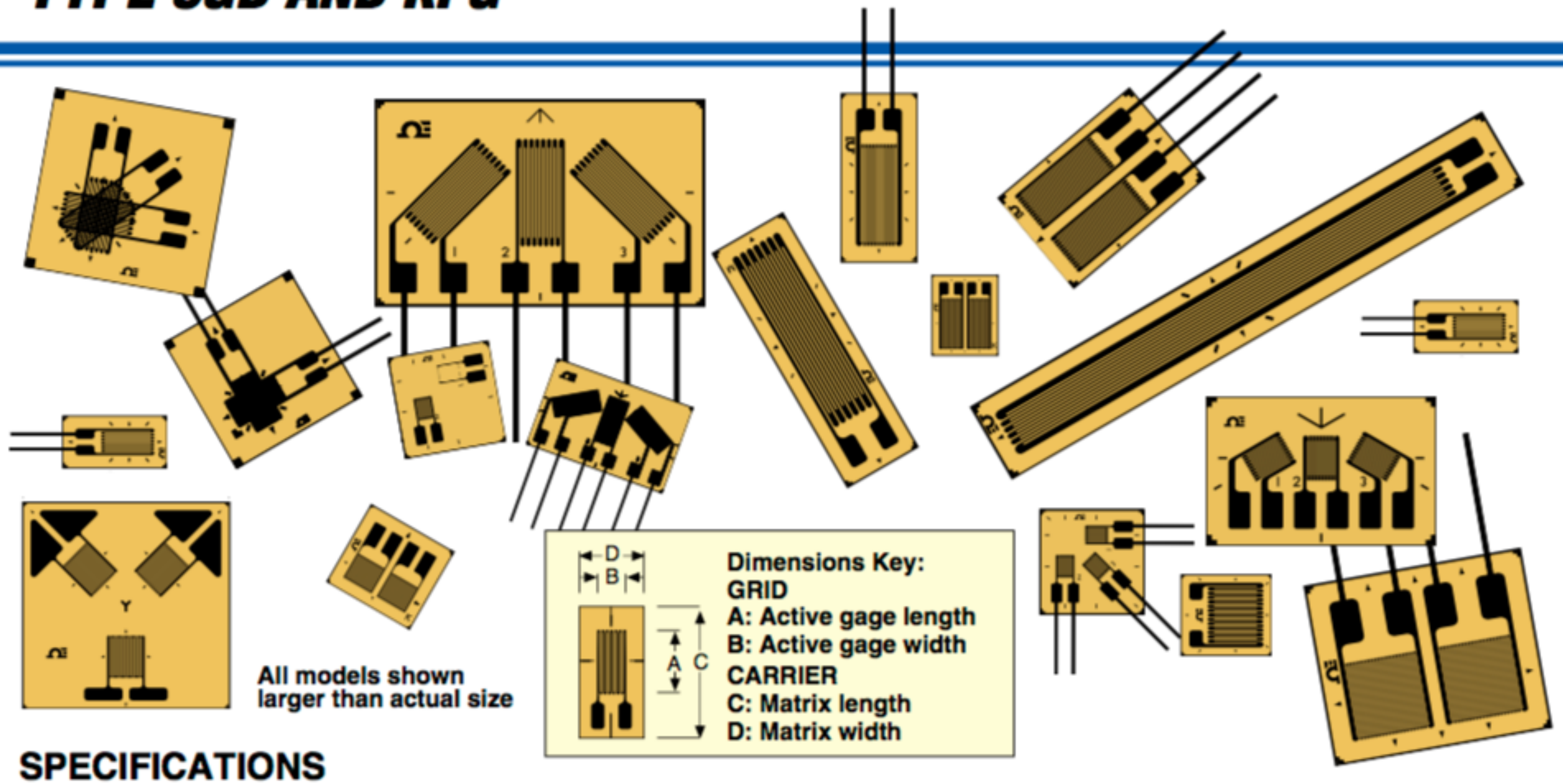


(a) (b) (c) (d)

(e) (f) (g) (h)

Figure 11.6 Strain gauge configurations. (a) Torque Rosette; (b) Linear Pattern; (c) Delta Rosette; (d) Residual Stress Pattern; (e) Diaphragm Pattern; (f) Tee Pattern; (g) Rectangular Rosette; (h) Stacked Rosette. (Courtesy of Micro-Measurements, Raleigh, NC, USA.)

OMEGA® STRAIN GAGE
SPECIFICATIONS CHART
TYPE SGD AND KFG



All models shown larger than actual size

Dimensions Key:
GRID
A: Active gage length
B: Active gage width
CARRIER
C: Matrix length
D: Matrix width

SPECIFICATIONS

	SGD SERIES	KFG SERIES—PRE-WIRED
Foil Measuring Grid	Constantan foil 5 microns thick	Constantan foil 6 microns thick
Carrier	Polyimide	Kapton®
Substrate Thickness	20 microns	15 microns
Cover Thickness	25 microns	9 microns
Connection Dimensions: mm (inch)	Solder pads or ribbon leads, tinned copper flat wire 30 L x 0.1 D x 0.3 mm W (1.2 x 0.004 x 0.012")	Pre-wired, 2 or 3 leads 27 AWG strand polyvinyl insulation 1 x 2 mm (0.04 x 0.08")
Nominal Resistance	Stated in "To Order" box	120 ±0.4 or 350 Ω
Resistance Tolerance Per Package	±0.15% to ±0.5% depending on gage spec	0.3%
Gage Factor (Actual Value Printed on Each Package)	2.0 ±5%	2.10 ±10%
Gage Factor Tolerance Per Package	1.00%	1.00%

THERMAL PROPERTIES		
Reference Temperature	23°C (73°F)	23°C (73°F)
SERVICE TEMPERATURE		
Static Measurements	-75 to 200°C (-100 to 392°F)	-20 to 100°C (-4 to 212°F)
Dynamic Measurements	-75 to 200°C (-100 to 392°F)	-20 to 100°C (-4 to 212°F)
TEMPERATURE CHARACTERISTICS		
Steel (and Certain Stainless Steels)	11 ppm/°C (6.1 ppm/°F)	10.8 ppm/°C (6 ppm/°F)
Aluminum	23 ppm/°C (12.8 ppm/°F)	——
Uncompensated	±20 ppm/°C (11.1 ppm/°F)	——
Temperature Compensated Range	-5 to 120°C (5 to 248°F)	10 to 80°C (50 to 176°F)
Tolerance of Temp Compensation	2 ppm/°C (1.0 ppm/°F)	1 ppm/°C (0.5 ppm/°F)
MECHANICAL PROPERTIES		
Maximum Strain	3% or 30,000 microstrain	5% or 50,000 microstrain
Hysteresis	Negligible	Negligible
Fatigue (at ±1500 microstrain)	>10,000,000 cycles	>10,000,000 cycles
Smallest Bending Radius	3 mm (⅛")	3 mm (⅛")
Transverse Sensitivity	——	Stated on each package

How big is the resistance change?

The maximum possible load of interest in non-destructive testing is yield

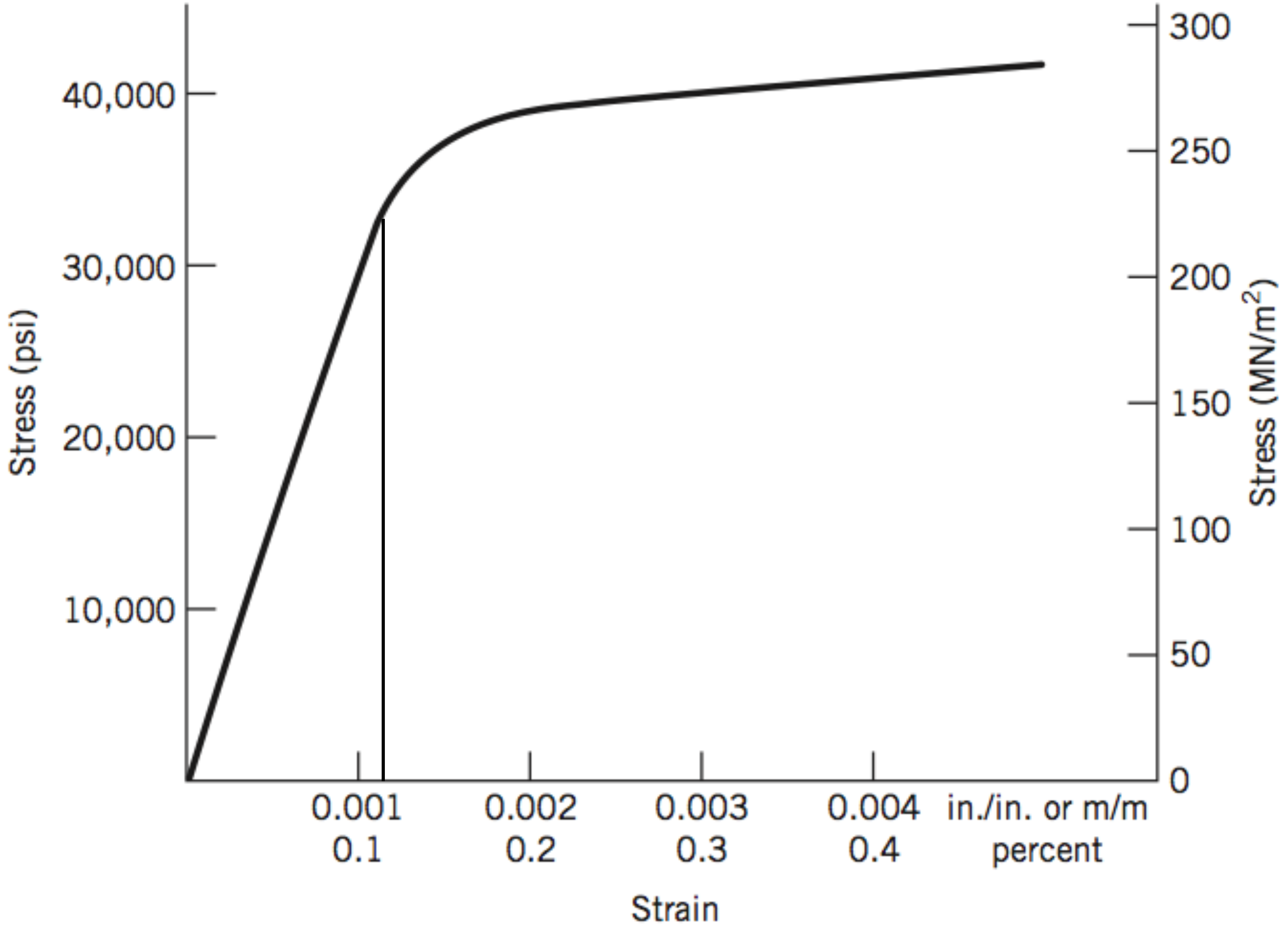


Figure 11.2 A typical stress–strain curve for mild steel.

- Yield in mild steel is happening around 0.001 mm/mm, or about 1 mm/m
- With a gauge factor of about 2, that would result in a change of about 0.002 ohms/ohm
- A 100 ohm gauge would rise to 100.2 ohms, or drop to 99.8 ohms

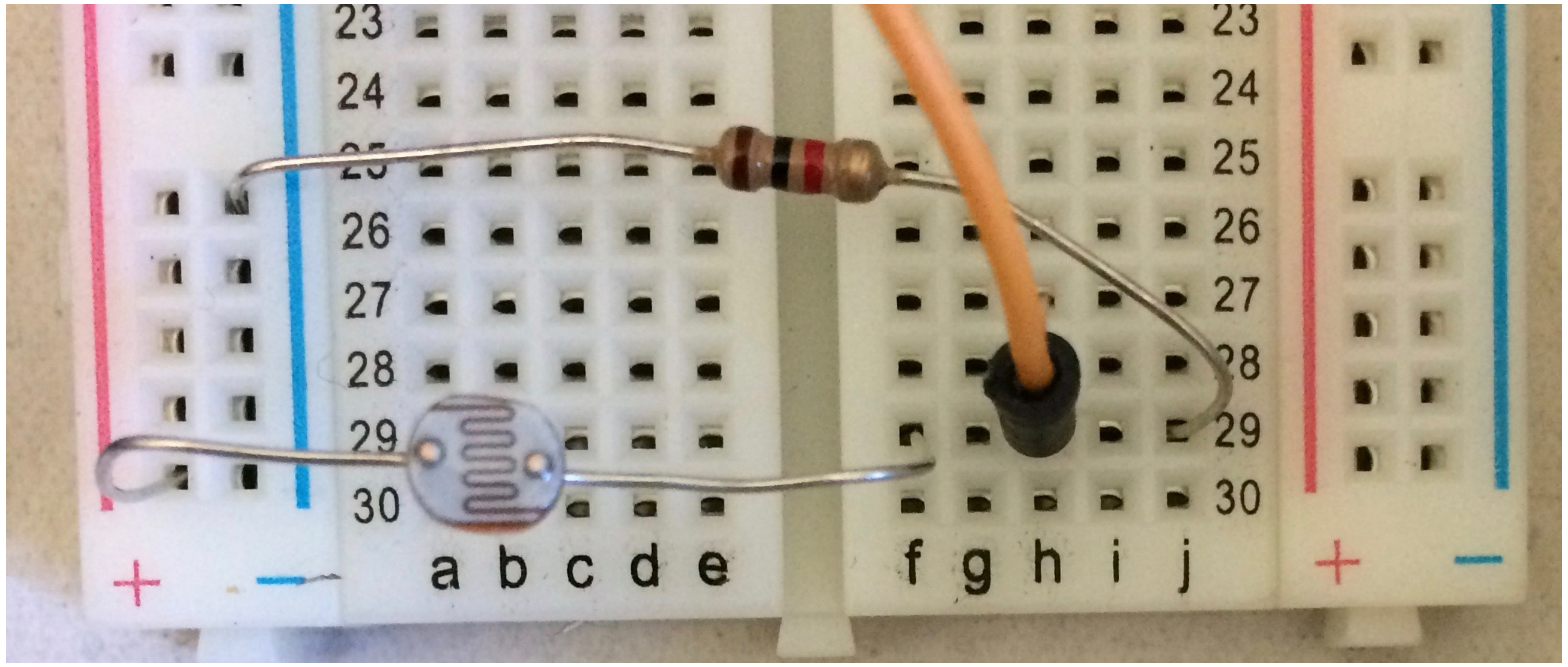
Voltage Divider for CdS Photocell or Thermistor

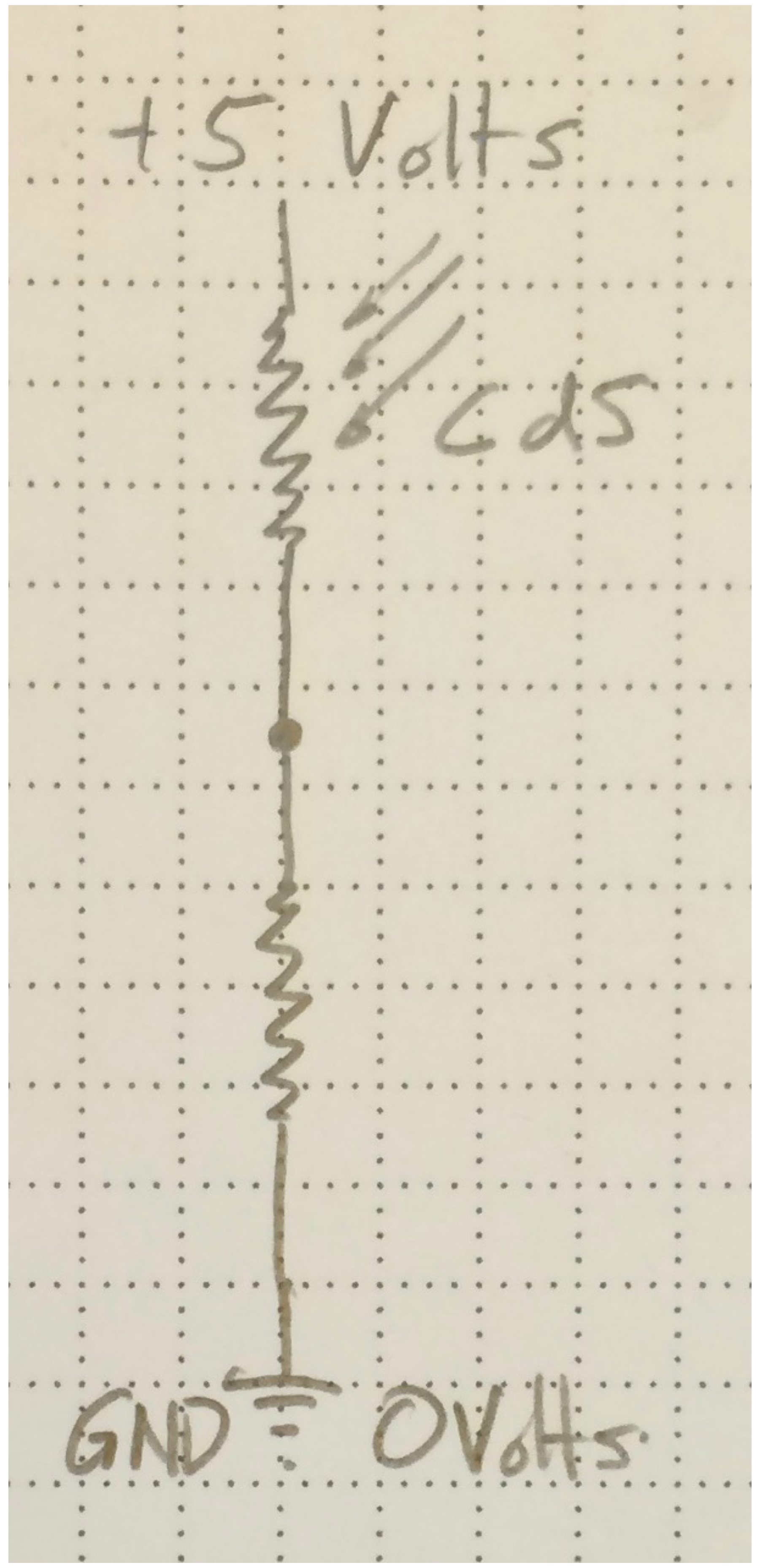
Resistance varies strongly with light

Connect as a voltage divider

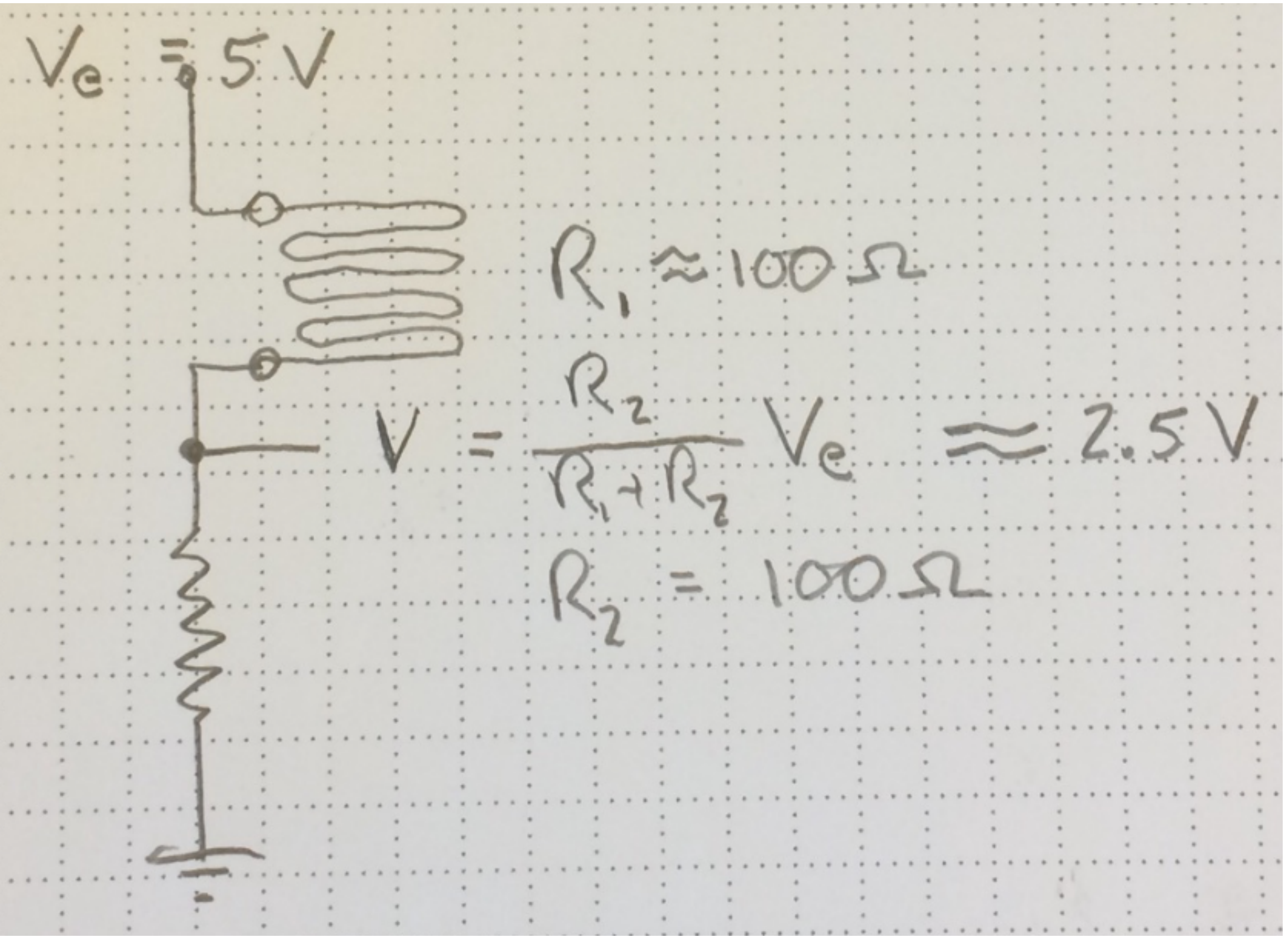
Get a large variation in voltage

This would also work for a thermistor

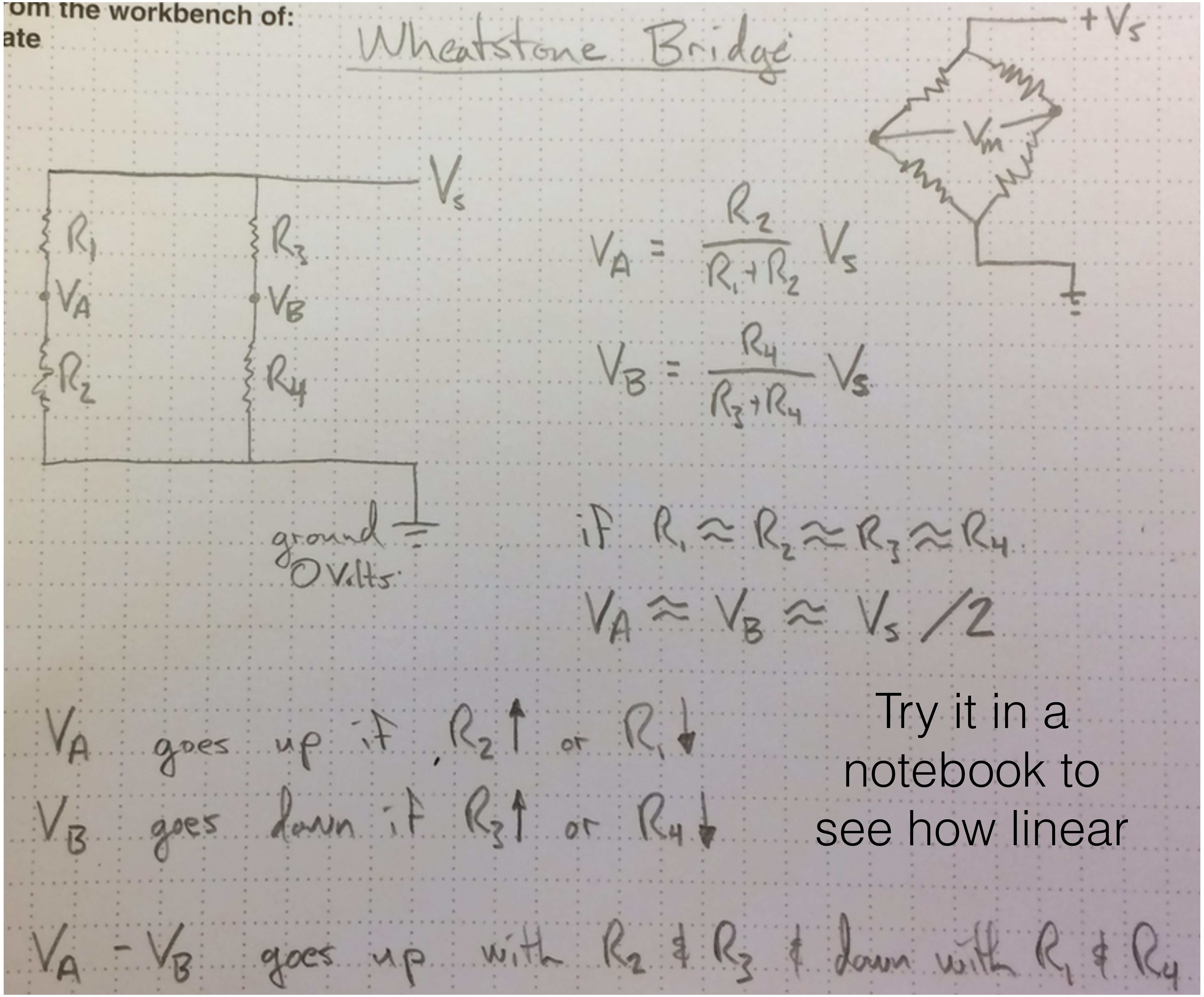




Voltage Divider too coarse for Strain Gauge

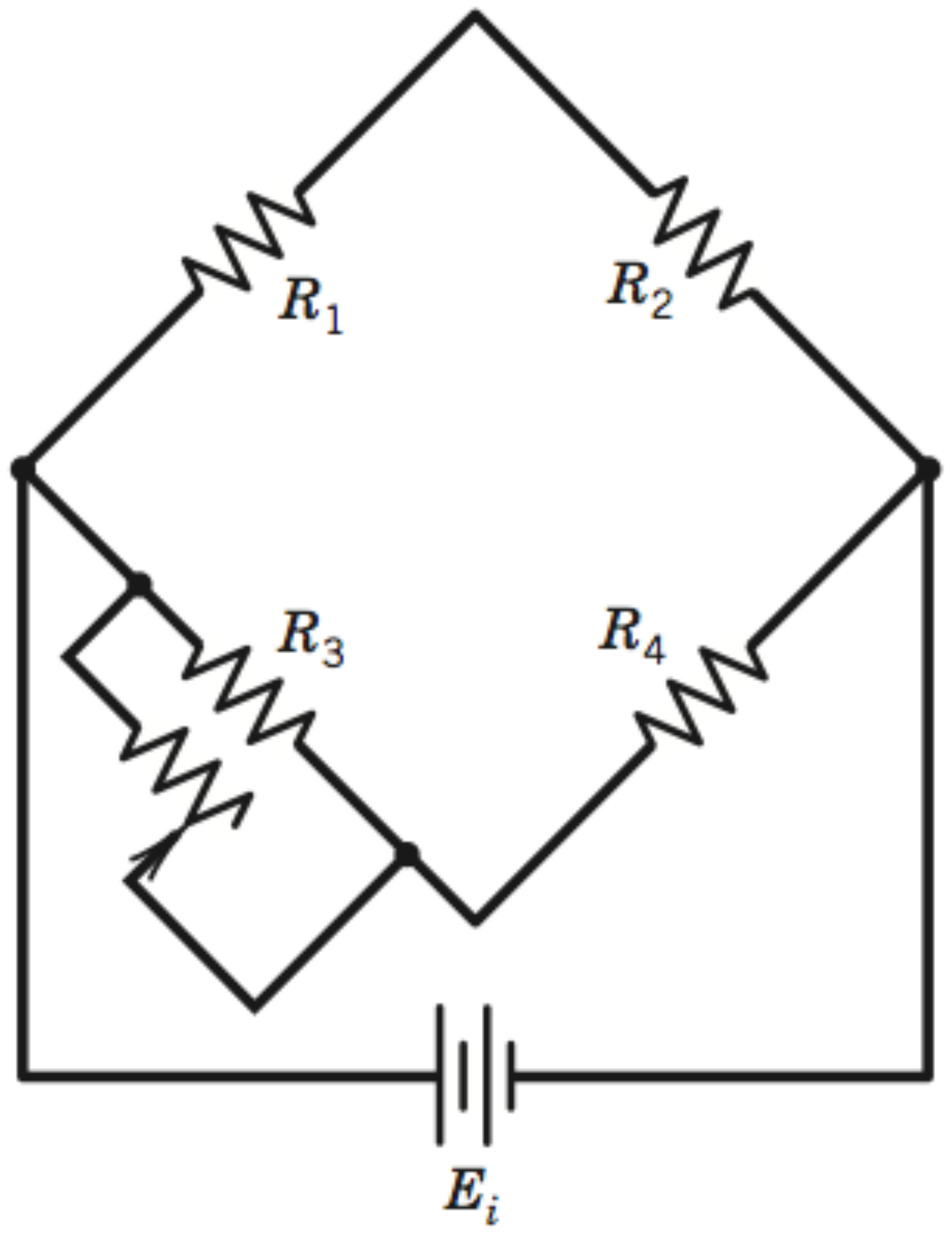


- 2.5 volts if R1 = R2
- 2.50125 volts if R1 drops to 99.9 ohms
- 2.49875 volts if R1 rises to 100.1 ohms
- Hard to resolve on a 5 volt scale



A Wheatstone bridge shows up small changes in resistance as a small change in difference voltage that we can measure accurately on a low voltage scale

Circuit arrangement for shunt balance



Differential shunt balance arrangement

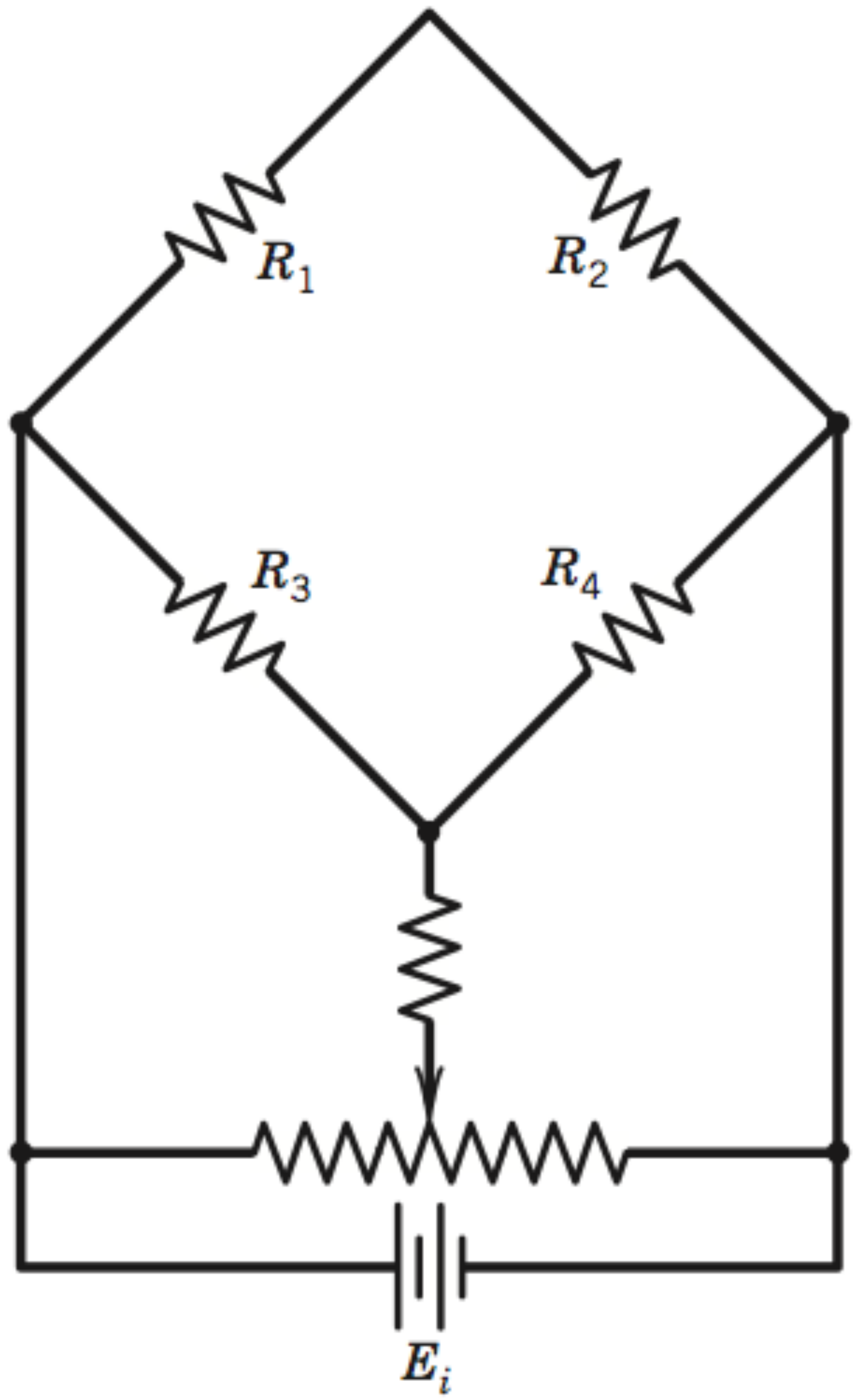


Figure 11.10 Balancing schemes for bridge circuits.

Balancing: Do we need it?

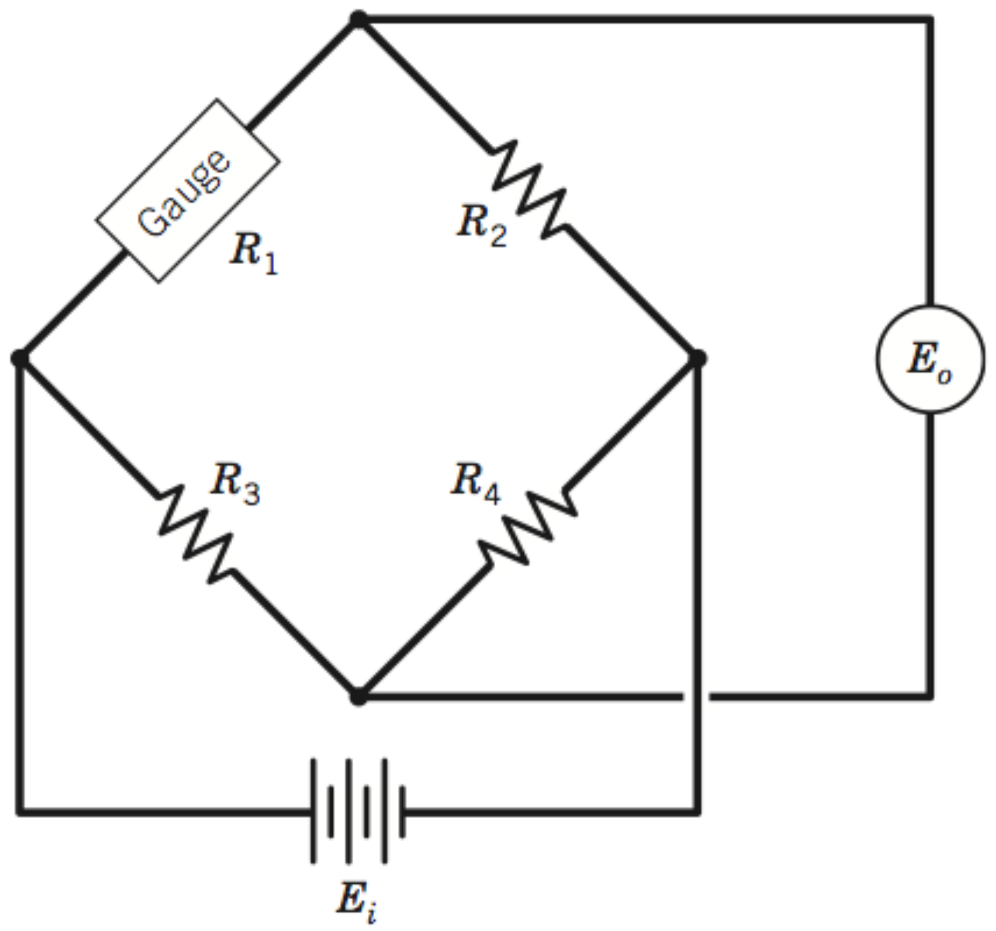
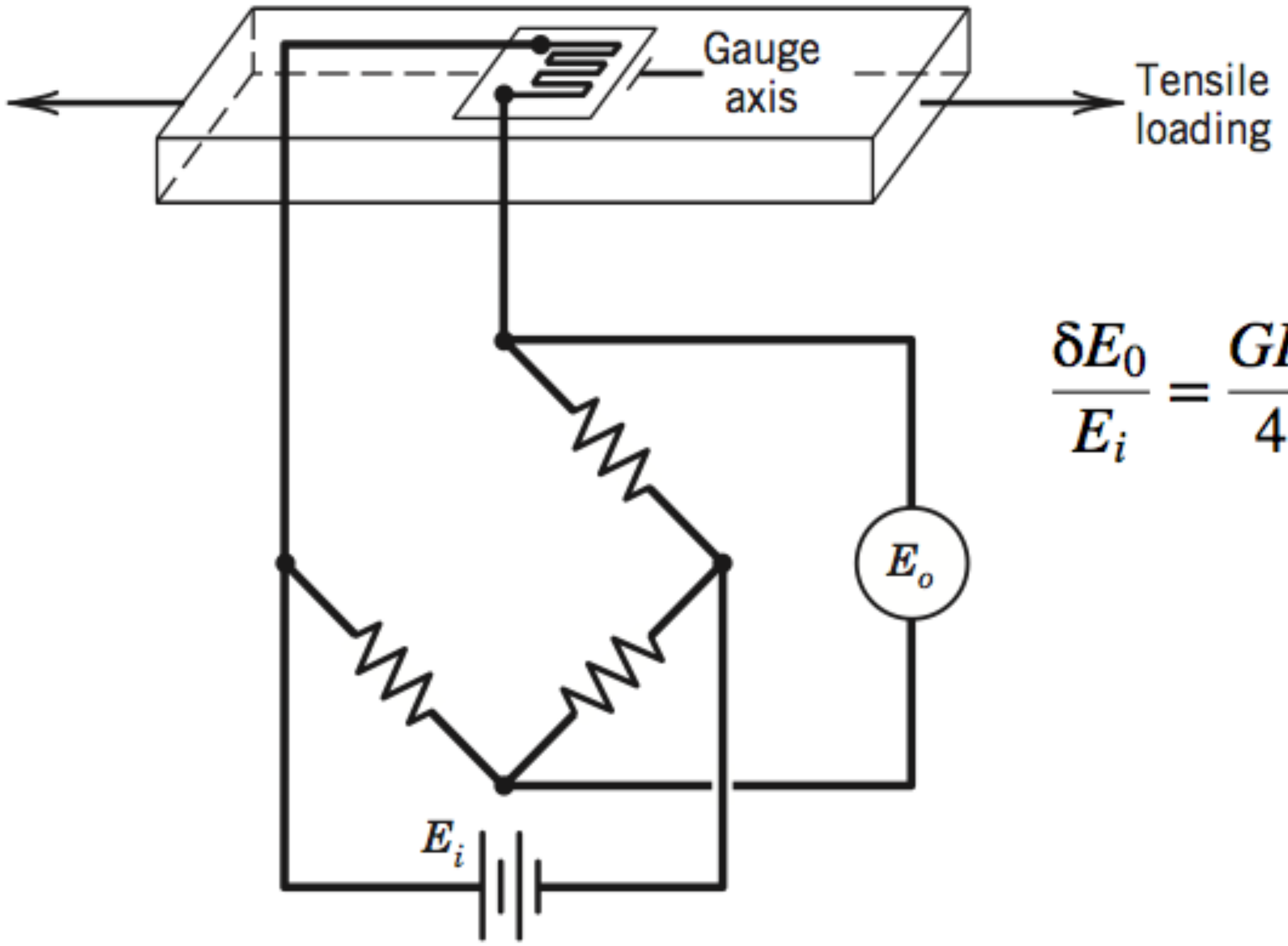


Figure 11.9 Basic strain gauge Wheatstone bridge circuit.

Add some extra resistors and you can adjust to make VA = VB so that the bridge output will be zero at rest

$$\frac{\delta E_0}{E_i} = \frac{\delta R_1/R_1}{4 + 2(\delta R_1/R_1)} \approx \frac{\delta R_1/R_1}{4} \qquad (11.15)$$

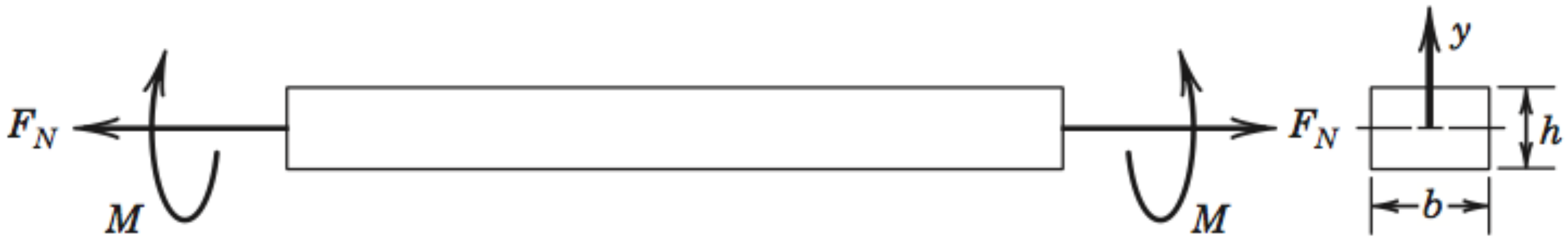
With no balancing you will have to track change in Eo



$$\frac{\delta E_0}{E_i} = \frac{GF}{4}\varepsilon_a \qquad (11.33)$$

Figure 11.11 Strain gauge circuit subject to uniaxial tension.

Eo will also change if there's bending, or thermal expansion



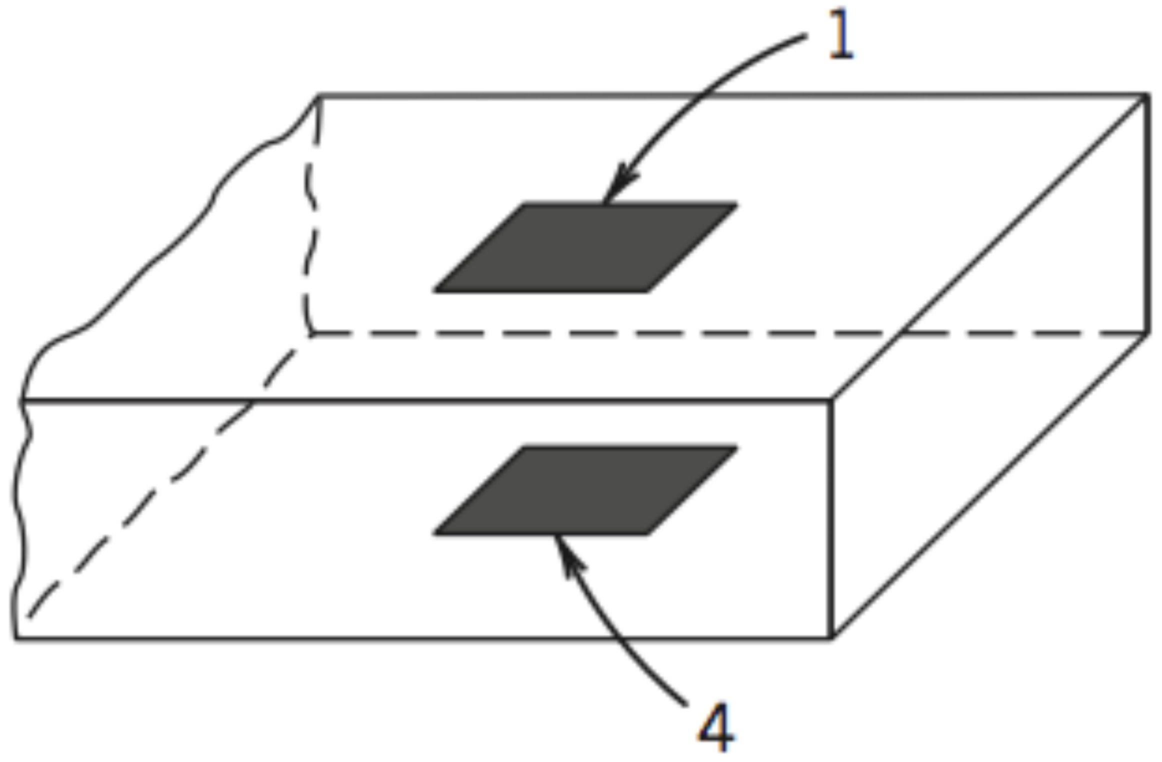
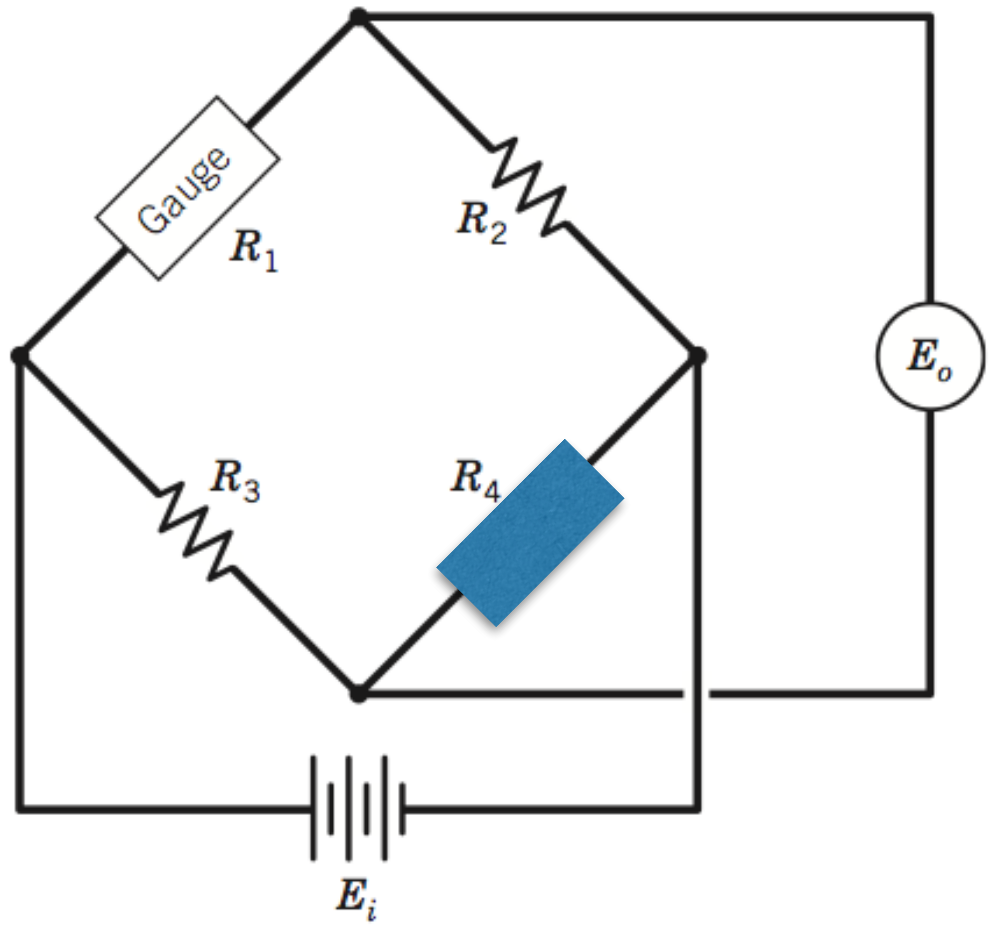


Figure 11.13 Strain gauge installation for bending compensation.

$$\frac{\delta E_0}{E_i} = \frac{GF}{4}(\varepsilon_1 + \varepsilon_4) \tag{11.31}$$

498 Chapter 11 Strain Measurement



Both gauges stretched by Force

Moment stretches one, compresses the other, effects cancel out

Figure 11.9 Basic strain gauge Wheatstone bridge circuit.