Introduction to Algebra - Day 4

David Sellers

Spring 2025

Day 4 - Rational Exponents

Consider $x \cdot x$. Using rules we know then we know this is x^2. From this we gather that $\sqrt{x^2}$ should be x. While this is true, in more complicated cases it can be notationally confusing. For instance, $\sqrt[3]{x^{24}}$.

From here we can introduce a new notation to simplify these cases. The 3 is the index, 24 is the power and $xˆ{24} is the radicand. We can rewrite this expression as $x^{\frac{24}{3}}$ and this is called a rational exponent. We can now see that this simplifies easily to x^8 meaning that the cube root of x^{24} is x^8 since $x^8 \cdot x^8 \cdot x^8 = x^{24}$.

Applications

1. $\sqrt{p^8}$
2. $\sqrt[4]{n^{32}}$
3. $\sqrt[5]{\frac{p^{40}}{q^{15}}}$
4. $\sqrt[9]{x^{99}y^{18}}$
5. $\sqrt[3]{\frac{x^{-6}y^{12}z^{18}}{x^9y^4z^{21}}}$
6. $\sqrt[4]{16x^8}$
7. $\sqrt[3]{x^{13}}$

Simplifying radicals

These same ideas can be used to simplify radicals with constant radicands. For instance $\sqrt{8}$ is $\sqrt{4 \cdot 2}$ and by our properties $\sqrt{4}\sqrt{2} = 2\sqrt{2}$.

The process we are using here is looking for factors of the radicand that are perfect roots, factor the radicand and take the root.

Applications

1. $\sqrt{50}$
2. $\sqrt{128}$
3. $\sqrt{250}$

4. $\sqrt{108}$
5. $\sqrt[3]{256}$
6. $\sqrt[3]{250}$
7. $\sqrt[3]{16}$

« previous Home next »