1 Standards:

19 Derive and apply the relationships between the lengths, perimeters, areas, and volumes of similar figures in relation to their scale factor.

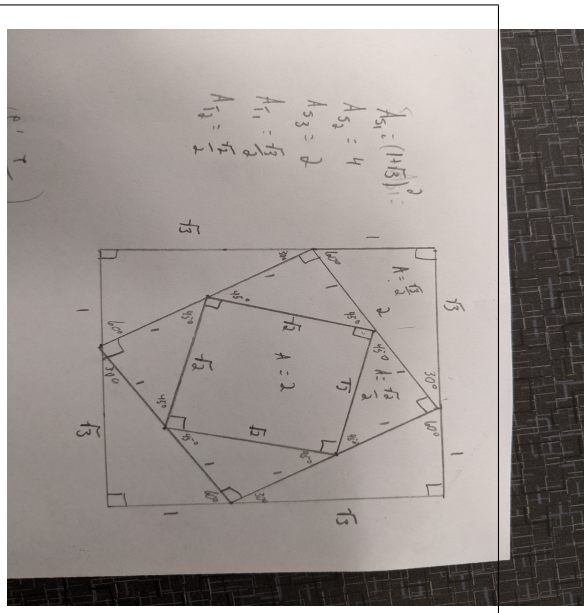
29 Find patterns and relationships in figures including lines, triangles, quadrilaterals, and circles, using technology and other tools. a. Construct figures, using technology and other tools, in order to make and test conjectures about their properties. b. Identify different sets of properties necessary to define and construct figures.

35 Discover and apply relationships in similar right triangles. a. Derive and apply the constant ratios of the sides in special right triangles (45°-45°-90° and 30°-60°-90°).

1. Consider the two special triangles, 45-45-90 and 30-60-90, in their simplified proportions, i.e. $1, 1, \sqrt{2}$ and $1, \sqrt{3}, 2$, respectively. Using these two triangles, create a series of three squares, where each successive square is contained within the previous square. Label every side length and angle and find the area of each triangle and square used in this construction.

Solution:

$$A_{s1} = (1 + \sqrt{3})^2 = 4 + 2\sqrt{3} = 7.46$$
$$A_{s2} = 2 \cdot 2 = 4$$
$$A_{s3} = 2$$
$$A_{t1} = \frac{\sqrt{3}}{2}$$
$$A_{t2} = \frac{1}{2}$$



2. Using the two special triangles, 45-45-90 and 30-60-90, create a series of three squares, where each successive square is contained within the previous square. In this problem, each triangles simplified proportions needs to be scaled by 4. Label every side length and angle and find the area of each triangle and square used in this construction.

Solution:

$$A_{s1} = (4(1 + \sqrt{3}))^2 = 16(4 + 2\sqrt{3}) = 55.43$$
$$A_{s2} = (4 \cdot 2) \cdot (4 \cdot 2) = 64$$
$$A_{s3} = 2$$
$$A_{t1} = 8\sqrt{3}$$
$$A_{t2} = 8\sqrt{2}$$

3. Form conjectures based on what you experienced in this exercise.

   (a) How did scaling the figure by 4 change the lengths of the sides of the triangles and squares?

   Solution: Scaled by a factor of 4.

   (b) How did scaling the figure by 4 change the area of the triangles and squares?

   Solution: Scaled by a factor of 4^2.

   (c) How did scaling the figure by 4 change the measure of the angles?

   Solution: No change.

   (d) What would you expect to happen to the length and area of the figures if you scaled the figures by $\frac{1}{4}$?

   Solution: Lengths: Scaled by $\frac{1}{4}$
   Area: Scaled by $(\frac{1}{4})^2$

   (e) What would you expect to happen if you scaled the figures by 10?

   Solution: Lengths: Scaled by 10
   Area: Scaled by 10^2

   (f) What would you expect to happen if you scaled the figures by a scalar k?

   Solution: Lengths: Scaled by k
   Area: Scaled by k^2