

## 1 Standards:

- 19 Derive and apply the relationships between the lengths, perimeters, areas, and volumes of similar figures in relation to their scale factor.
- 29 Find patterns and relationships in figures including lines, triangles, quadrilaterals, and circles, using technology and other tools. a. Construct figures, using technology and other tools, in order to make and test conjectures about their properties. b. Identify different sets of properties necessary to define and construct figures.
- 35 Discover and apply relationships in similar right triangles. a. Derive and apply the constant ratios of the sides in special right triangles ( $45^\circ$ - $45^\circ$ - $90^\circ$  and  $30^\circ$ - $60^\circ$ - $90^\circ$ ).

## 2 Discourse

This lesson should connect the two previous lessons and expand the usefulness of the trigonometric functions. So far, we have only dealt with these functions as the ratios of sides of a right triangle. To further develop these topics, let us first look at our two special triangles.

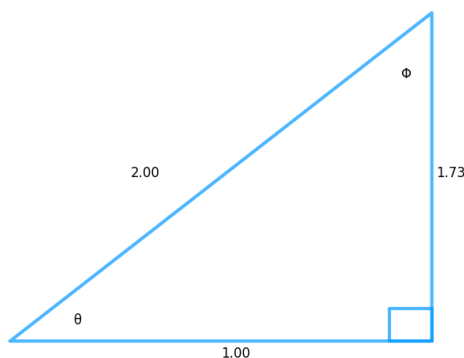


Figure 1:  $30^\circ - 60^\circ - 90^\circ$  triangle

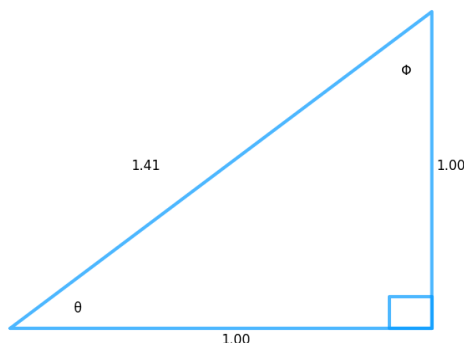


Figure 2:  $45^\circ - 45^\circ - 90^\circ$  triangle

Now find the  $\sin \theta$  and  $\cos \theta$  for Figure 1. Compare these values to the values from the scaling triangles assignment. Do they appear there as well? For Figure 2, find the following  $\sin \theta$ ,  $\cos \theta$ ,  $\sin \phi$ ,  $\cos \phi$ . Do these values look familiar?

It turns out that the values for cosine are represented as the x-coordinates of points on the unit circle and the y-coordinates of these points are the sine values. This is an amazing property of the unit circle because it allows to find the values of the trigonometric functions for any angle measure.

This [unit circle](#) from Desmos can be used to find any trigonometric value on the interval  $[-10^\circ, 360^\circ]$  and can be adjusted for practically any value.

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## 2.1 Examples

- |                     |                     |                      |
|---------------------|---------------------|----------------------|
| 1. $\sin 30^\circ$  | 5. $\cos 17^\circ$  | 9. $\cos 199^\circ$  |
| 2. $\cos 135^\circ$ | 6. $\sin 309^\circ$ | 10. $\cos -5^\circ$  |
| 3. $\sin 240^\circ$ | 7. $\cos 84^\circ$  | 11. $\sin 111^\circ$ |
| 4. $\cos 330^\circ$ | 8. $\sin 270^\circ$ |                      |

## 2.2 Extrapolate

We have seen that the unit circle expands the usefulness of the trigonometry to all angles measures, and while it has not been shown they hold for all real numbers as you will see in later courses. What would you expect to happen and why should you expect it for angles such as  $720^\circ$ ,  $390^\circ$ ,  $540^\circ$ ,  $-315^\circ$ ,  $-120^\circ$ ?

## 3 Exercises

Find the sine and cosine of each angle.

- |                 |                 |                 |
|-----------------|-----------------|-----------------|
| 1. $52^\circ$   | 15. $57^\circ$  | 29. $345^\circ$ |
| 2. $359^\circ$  | 16. $1^\circ$   | 30. $96^\circ$  |
| 3. $260^\circ$  | 17. $314^\circ$ | 31. $307^\circ$ |
| 4. $199^\circ$  | 18. $300^\circ$ | 32. $188^\circ$ |
| 5. $112^\circ$  | 19. $265^\circ$ | 33. $250^\circ$ |
| 6. $325^\circ$  | 20. $95^\circ$  | 34. $80^\circ$  |
| 7. $56^\circ$   | 21. $192^\circ$ | 35. $90^\circ$  |
| 8. $88^\circ$   | 22. $51^\circ$  | 36. $235^\circ$ |
| 9. $161^\circ$  | 23. $167^\circ$ | 37. $336^\circ$ |
| 10. $356^\circ$ | 24. $10^\circ$  | 38. $202^\circ$ |
| 11. $228^\circ$ | 25. $283^\circ$ | 39. $48^\circ$  |
| 12. $248^\circ$ | 26. $68^\circ$  | 40. $291^\circ$ |
| 13. $251^\circ$ | 27. $254^\circ$ |                 |
| 14. $331^\circ$ | 28. $49^\circ$  |                 |

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## 4 Answer Document

- |                               |                               |                               |
|-------------------------------|-------------------------------|-------------------------------|
| 1. (a) $\sin \theta =$ _____  | (b) $\cos \theta =$ _____     | 28. (a) $\sin \theta =$ _____ |
| (b) $\cos \theta =$ _____     | 15. (a) $\sin \theta =$ _____ | (b) $\cos \theta =$ _____     |
| 2. (a) $\sin \theta =$ _____  | (b) $\cos \theta =$ _____     | 29. (a) $\sin \theta =$ _____ |
| (b) $\cos \theta =$ _____     | 16. (a) $\sin \theta =$ _____ | (b) $\cos \theta =$ _____     |
| 3. (a) $\sin \theta =$ _____  | (b) $\cos \theta =$ _____     | 30. (a) $\sin \theta =$ _____ |
| (b) $\cos \theta =$ _____     | 17. (a) $\sin \theta =$ _____ | (b) $\cos \theta =$ _____     |
| 4. (a) $\sin \theta =$ _____  | (b) $\cos \theta =$ _____     | 31. (a) $\sin \theta =$ _____ |
| (b) $\cos \theta =$ _____     | 18. (a) $\sin \theta =$ _____ | (b) $\cos \theta =$ _____     |
| 5. (a) $\sin \theta =$ _____  | (b) $\cos \theta =$ _____     | 32. (a) $\sin \theta =$ _____ |
| (b) $\cos \theta =$ _____     | 19. (a) $\sin \theta =$ _____ | (b) $\cos \theta =$ _____     |
| 6. (a) $\sin \theta =$ _____  | (b) $\cos \theta =$ _____     | 33. (a) $\sin \theta =$ _____ |
| (b) $\cos \theta =$ _____     | 20. (a) $\sin \theta =$ _____ | (b) $\cos \theta =$ _____     |
| 7. (a) $\sin \theta =$ _____  | (b) $\cos \theta =$ _____     | 34. (a) $\sin \theta =$ _____ |
| (b) $\cos \theta =$ _____     | 21. (a) $\sin \theta =$ _____ | (b) $\cos \theta =$ _____     |
| 8. (a) $\sin \theta =$ _____  | (b) $\cos \theta =$ _____     | 35. (a) $\sin \theta =$ _____ |
| (b) $\cos \theta =$ _____     | 22. (a) $\sin \theta =$ _____ | (b) $\cos \theta =$ _____     |
| 9. (a) $\sin \theta =$ _____  | (b) $\cos \theta =$ _____     | 36. (a) $\sin \theta =$ _____ |
| (b) $\cos \theta =$ _____     | 23. (a) $\sin \theta =$ _____ | (b) $\cos \theta =$ _____     |
| 10. (a) $\sin \theta =$ _____ | (b) $\cos \theta =$ _____     | 37. (a) $\sin \theta =$ _____ |
| (b) $\cos \theta =$ _____     | 24. (a) $\sin \theta =$ _____ | (b) $\cos \theta =$ _____     |
| 11. (a) $\sin \theta =$ _____ | (b) $\cos \theta =$ _____     | 38. (a) $\sin \theta =$ _____ |
| (b) $\cos \theta =$ _____     | 25. (a) $\sin \theta =$ _____ | (b) $\cos \theta =$ _____     |
| 12. (a) $\sin \theta =$ _____ | (b) $\cos \theta =$ _____     | 39. (a) $\sin \theta =$ _____ |
| (b) $\cos \theta =$ _____     | 26. (a) $\sin \theta =$ _____ | (b) $\cos \theta =$ _____     |
| 13. (a) $\sin \theta =$ _____ | (b) $\cos \theta =$ _____     | 40. (a) $\sin \theta =$ _____ |
| (b) $\cos \theta =$ _____     | 27. (a) $\sin \theta =$ _____ | (b) $\cos \theta =$ _____     |
| 14. (a) $\sin \theta =$ _____ | (b) $\cos \theta =$ _____     |                               |