1 Standards:

19 Derive and apply the relationships between the lengths, perimeters, areas, and volumes of similar figures in relation to their scale factor.

29 Find patterns and relationships in figures including lines, triangles, quadrilaterals, and circles, using technology and other tools. a. Construct figures, using technology and other tools, in order to make and test conjectures about their properties. b. Identify different sets of properties necessary to define and construct figures.

35 Discover and apply relationships in similar right triangles. a. Derive and apply the constant ratios of the sides in special right triangles (45°-45°-90° and 30°-60°-90°).

2 Discourse

From our discussion of the equation of a circle, the distance formula and the Pythagorean Theorem you should be developing a sense of the close relationship between circles and triangles. After watching the Beautiful Trigonometry video, it should be more clear that trigonometry, literally meaning the measurement of triangles, is deeply rooted in the properties of circles. Since you have now taken the two special triangles and scaled them to fit into a circle with radius 1, it is time for us to elaborate on the importance of this circle you created.

First, we need to explore a small amount of vocabulary.

- hypotenuse - ....
- leg - ....
- opposite - ....
- adjacent - ....

Using these terms, we can define two trigonometric functions, namely, sine and cosine. Let's do so.

2.1 Sine

Consider Figure 1. If we look at angle θ, then side b will be the opposite side, a is adjacent, and c is the hypotenuse. We abbreviate sine as sin and we can define it as $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$. Therefore for this triangle the $\sin\theta = \frac{b}{c}$ and the $\sin\phi = \frac{a}{c}$.

2.2 Cosine

Consider Figure 1. We abbreviate cosine as cos and we can define it as $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$. Therefore for this triangle the $\cos\theta = \frac{a}{c}$ and the $\cos\phi = \frac{b}{c}$.

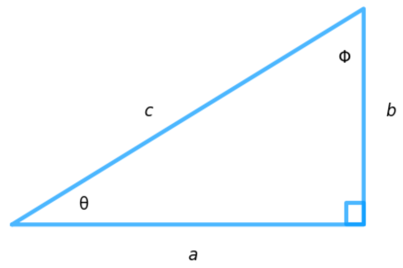


Figure 1: A right triangle

2.3 Tangent

There is a third trigonometric function, the tangent function, that can be defined based upon sine and cosine. We abbreviate tangent as tan and it can be defined as $\tan\theta = \frac{\sin\theta}{\cos\theta}$. We can manipulate this expression using substitution in order to have an equivalent, yet in many cases simpler, definition.

2.3.1 Derivation of Alternative Tangent Definition

2.4 Reciprocal Functions

The reciprocal functions are outside the scope of this course; however, the definitions will be provided for highly motivated students.

2.4.1 Cosecant

The cosecant is defined as the reciprocal of the sine function, i.e. $\csc\theta = \frac{1}{\sin\theta}$. This can be simplified to our notions of ratios of sides of triangles by substitution.

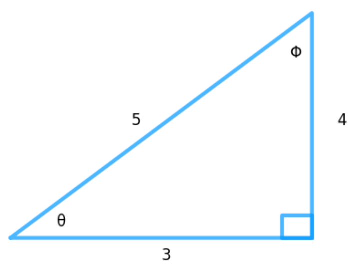
2.4.2 Secant

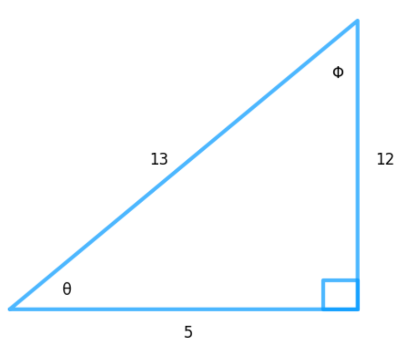
The secant is defined as the reciprocal of the cosine function, i.e. $\sec\theta = \frac{1}{\cos\theta}$. This can be simplified to our notions of ratios of sides of triangles by substitution.

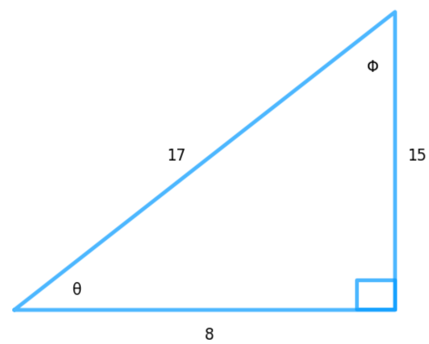
2.4.3 Cotangent

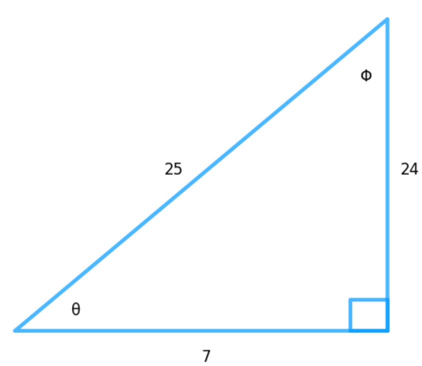
The cotangent is defined as the reciprocal of the tangent function, i.e. $\cot\theta = \frac{1}{\tan\theta}$. This can be simplified to our notions of ratios of sides of triangles by substitution.

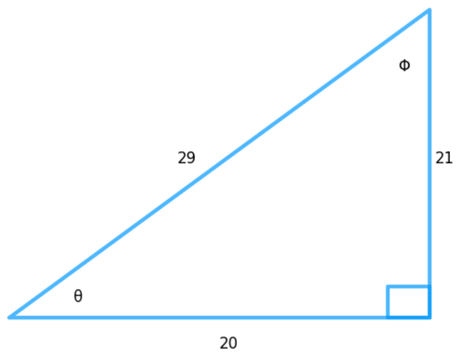
3 Problems

1. 

2. 

3. 

4. 



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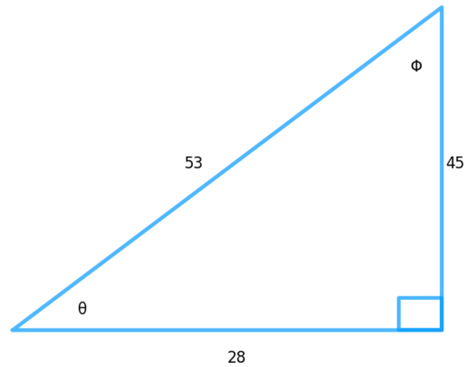

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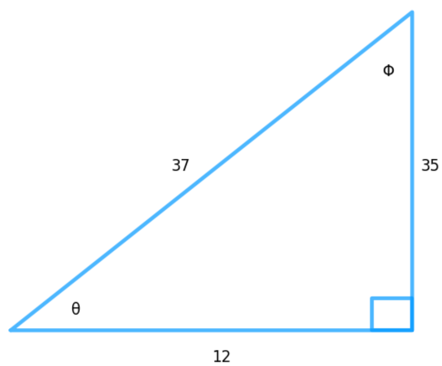

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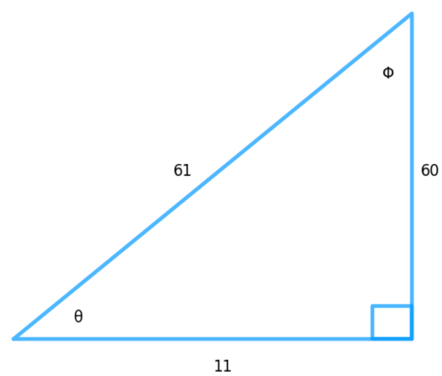

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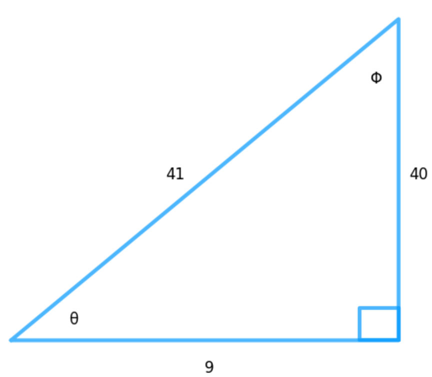

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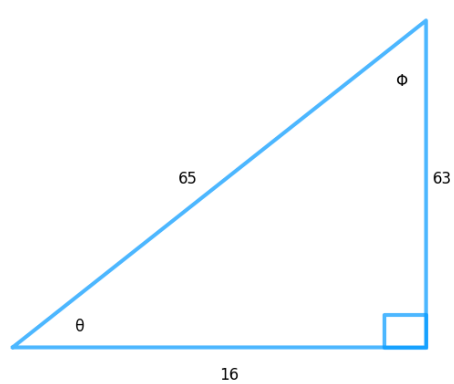

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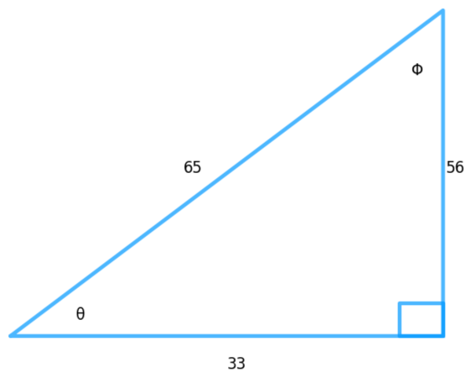
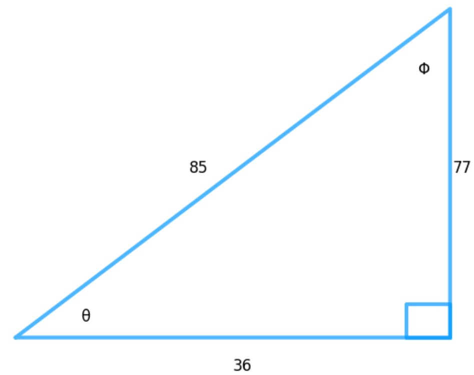
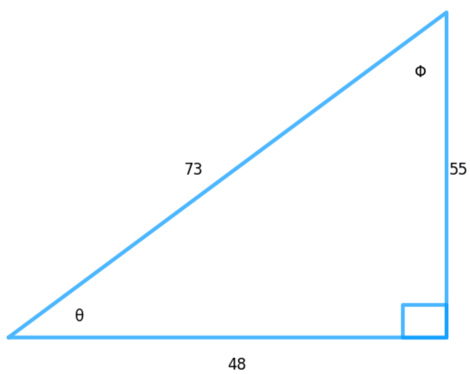
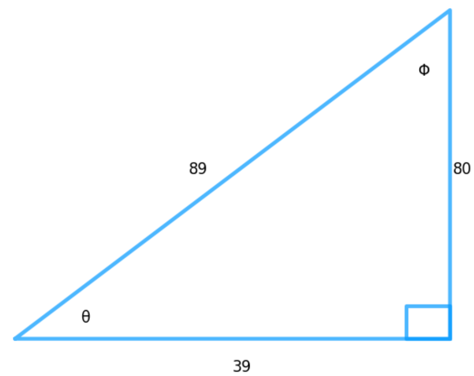
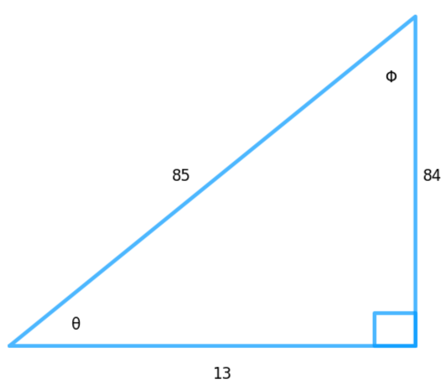
11. 

14. 

12. 

15. 

13. 

4 Answer Document

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