

Combinatorics Exercises:

For the following set of problems determine what part of Combinatorics we need to use and apply the appropriate formula. Have in mind that there might be more than one correct approach to some (or all) of these questions.

Problem 1:

Imagine you are working at an office and have 5 tasks labelled as “Critical” in Jira to complete by the end of the day. In how many ways can you complete said tasks before the day ends?

$$5! = 120$$

** “Jira” is a Project Management software which allows you to create tasks and label them depending on their importance. “Critical” is the highest level of importance and no task with a lower-level can be started once such a task is initiated.

Problem 2:

Imagine your company is trying to gain customers by running an online ad campaign. The idea is to focus on a certain demographic which frequently uses social media. Your campaign will run ads on Facebook, Messenger, Instagram, Twitter and Reddit. Your graphical designers have created 8 different versions of the banner you can use. Based on this information:

- a) Calculate how many different options you have for the entire campaign, assuming you want to use a different one for each platform.

$$\frac{8!}{(8-5)!} = 6720$$

- b) Calculate how many different options you have for the entire campaign, assuming you can use the same banner for some or all the platforms.

$$8^5 = 32768$$

- c) Calculate how many ways we can pick which of the 8 banners to use, assuming we use different ones for each platform.

$$\frac{8!}{5! (8-5)!} = 56$$

- d) Calculate how many ways we can pick which of the 8 banners to use, assuming we can use each one multiple times.

$$\frac{(8+5-1)!}{5! (8-1)!} = 792$$

Problem 3:

You are renovating your entire apartment and want to repaint the walls of each room. The flat consists of two bedrooms, a kitchen, a living room, a bathroom, a study and a hall, or 7 rooms in total. You have at your disposal several colors of paint: white, yellow, orange, red, purple, blue, green, grey and pink.

How many different ways can you paint the house, assuming...

7 rooms, 9 colours

- a) ...you paint all the rooms in different colours?

$$\frac{9!}{(9-7)!} = 181.440$$

- b) ...you paint the bathroom, study and hall in white?

$$9^4 = 6.561$$

- c) ...you paint the two bedrooms in identical color?

$$9^6 = 531.441$$

- d) ...you can only use grey and yellow?

$$2^7 = 128$$

Problem 4:

This year, you are helping organize your college's career fest. There are 11 companies which are participating, and you have just enough room fit all of them. How many ways can you arrange the various firms, assuming...:

- a) ... no firm has any preference where they want to be positioned?

$$11! = 39.916.800$$

- b) ... JP Morgan representatives made a deal, where they have to be located exactly in the middle?

$$10! = 3.628.800$$

- c) ... JP Morgan, Citi Bank and Morgan-Stanley must be positioned in the middle 3 spots?

$$8! * 3! = 241.920$$

- d) ... Deutsche Bank representatives cancel, so you can give the additional space to one of the other companies?

$$10! * 10 = 36.288.000$$

Problem 5:

Your best friend is organizing a birthday party for her twins – Amy and Steve - and she put you in charge of ordering the cakes. The bakery offers several types of cakes – a Cheesecake, Sacher Cake, a Chiffon Cake, a Coconut Cake and a Carrot Cake. How many different ways can you order the cakes, assuming that...

5 cakes, 2 twins

- a) ... both twins enjoy all the 5 types of cake?

$$\frac{(5+2-1)!}{2!(5-1)!} = 15$$

- b) ... Steve dislikes Coconuts?

$$5C1 * 4C1 = 20$$

Assuming that Amy can choose coconut and Steve can't (didn't say anything about sharing)

c) ... Amy loves chocolate (Sacher)?

$$1C1 * 5C1 = 5$$

d) ... each cake comes with a generic "Happy Birthday!" wish?

$$\frac{(5 + 2 - 1)!}{2! (5 - 1)!} = 15$$

e) ... each cake comes with a personalized "Happy Birthday Steve!" or "Happy Birthday Amy!" sign?

$$5^2 = 25$$

Problem 6:

You want to go to the gym between lectures every day, but you only have an hour to workout. Knowing this, you decide to do a circuit workout. You start with 5 minutes of cardio as a warm-up, then you hit two different leg exercises, followed by a chest exercise, as well as one for shoulders. After that, you continue with a bicep exercise and a triceps one, before moving to the back one. You finish the circuit with 2 abdominal exercises like a plank and some crunches. After completing the circuit several times, you end with another 10 minutes of cardio before you stretch and leave.

Now, assuming the gym has ellipticals, treadmills and stationary bikes, you have 3 options for the cardio. Additionally, you have 5 different leg exercises you can do. You have 4 choices of what to do for each of the next 3 muscle groups (chest, shoulders and bicep). For triceps you have heavy preferences towards two specific exercises, so you always do one of the two. The same can be said about the back. When it comes to the abdominal exercises, you have 4 options once again.

Taking everything into consideration, if you want to do a different workout each day, how long will it take you to run out of options?

Circuit:

- Cardio (3)
- 2x Legs (5)
- Chest (4)
- Shoulders (4)
- Biceps (4)
- Triceps (2)
- Back (2)
- 2x Abdominal (4)

Finish:

- Cardio (3)

$$3 * \frac{5!}{(5-2)!} * 4 * 4 * 4 * 2 * 2 * \frac{4!}{(4-2)!} * 3 = 552.960$$

$$\frac{552.960}{365} \sim 1515 \text{ years}$$