ALPEN-ADRIA UNIVERSITÄT KLAGENFURT | WIEN GRAZ

[700.698] Research Seminar in Embedded Communication Systems

Massive MIMO

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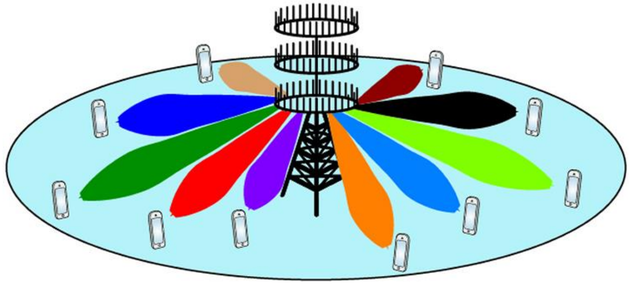
Massive MIMO - Overview

- Large-scale MIMO
- MIMO systems equipped with a large number of antennas
- Groups together antennas at the transmitter and receiver
  - better throughput
  - better spectrum efficiency
- Greater number of antennas
  - capacity gain increases
  - link reliability increases
  - system complexity increases ⇒ challenges in practical implemetation
- *How many antennas should be required to satisfy different service requirements?*



[1] Minimum Number of Antennas Required to Satisfy Outage Probability in Massive MIMO Systems

- Novel method is presented using recent non-asymptotic result.
- Exploit the statistical bounds on MIMO capacity to derive the equivalent problem of determining the number of antennas needed to satisfy outage probability constraints.
- The equivalent problem is solved by the bisection method.

System Model and Traditional Solution

System Model

- Point-to-point MIMO system
  - N transmit antennas (tends to be large)
  - K receive antennas (fixed)
- The received signal vector

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \qquad (1)$$

  - $\boldsymbol{x} \in \mathbb{C}^N$ - the transmit signal
  - $\boldsymbol{H} \in \mathbb{C}^{KxN}$ - the channel matrix
  - $\boldsymbol{n} \in \mathbb{C}^K$ - the additive white Gaussian noise with its covariance matrix $({}^1/_\rho)\boldsymbol{I}$
    - ρ - the signal-to-noise ratio (SNR)

System Model

- Assumtions:
  - perfect channel state information (CSI) available at receiver
  - power equally allocated at each transmit antenna
- Channel capacity

$$C = \log_2 det(\boldsymbol{I} + \frac{\rho}{N}\boldsymbol{H}\boldsymbol{H}^{*}) \qquad (2)$$

- Outage probability - the probability that a given target capacity R cannot be satisfied by an instantaneous realization of random channel matrix

$$P_{out}(R) = P\left(\log_2 det(\boldsymbol{I} + \frac{\rho}{N}\boldsymbol{H}\boldsymbol{H}^{*}) \leq R\right) \qquad (3)$$

  - R - given target capacity, the outage capacity at an outage probability of $P_{out}(R)$

Problem Formulation

- Determining the minimum number of antennas required to satisfy different outage probability requirements

$$N_{min} = \min[\arg\{P_{out}(R) \leq p\}] \qquad (4)$$

  - p - the outage probability requirement and
  - N_{min} - the minimum number of antennas required to satisfy the outage probability requirement

- Due to the fact:
  - MIMO capacity non-decreasing function for the number of antennas *N*

$$N_{min} = \arg\{P_{out}(R) = p\} \qquad (5)$$

Traditional Solution

- The characterization of the distribution of the MIMO capacity over the ensemble of all channel realizations needs to be known
- Authors in [2] proved that the distribution tends to be a Gaussian distribution as the number of transmit or receive antennas grows
- Channel capacity C in (2) converges to a Gaussian random variable – when N is large and K is fixed: $\mathcal{N}\left(\mu = Klog(1+\rho), \quad \sigma^2 = \frac{K\rho^2}{N(1+\rho)^2}\right)$
- Equation (5) can be written

$$N_{min} = \arg\left\{Q\left(\frac{\mu - R}{\sigma}\right) = p\right\} = \arg\left\{\mu - R = \frac{Q^{-1}(p)\rho\sqrt{K}}{(1+\rho)\sqrt{N}}\right\} \qquad (6)$$

Traditional Solution

- The closed-form solution of (6) is

$$\hat{N}_{min} = \left(\frac{Q^{-1}(p)\rho\sqrt{K}}{(1+\rho)(\mu - R)} \right)^2 \tag{7}$$

- In practical systems: $\mathbb{E}[C] = \mu + e$
    - e - the non-negative approximation error - vanishes at the rate $O(1/N)$ [2]

➢ The result $\hat{N}_{min}$ is smaller than the actual result N_{min}

➢ As N increases, $\hat{N}_{min}$ converges to the actual result N_{min}

Required Minimum Number of Antennas Under Outage Probability Requirement

- The statistical bounds on instantaneous MIMO capacity is first exploited to analyze the outage characterization of instantaneous capacity

Analysis of Outage Characterization

- The statistical bounds on instantaneous MIMO capacity [3]
- ***Lemma 1:*** $\boldsymbol{H}$ is a $K \times N$ MIMO channel matrix and its entries are Gaussian random variables with zero-mean and unit variance. Equal power is allocated at each transmit antenna. Then, for any $q > 0$, the following holds valid

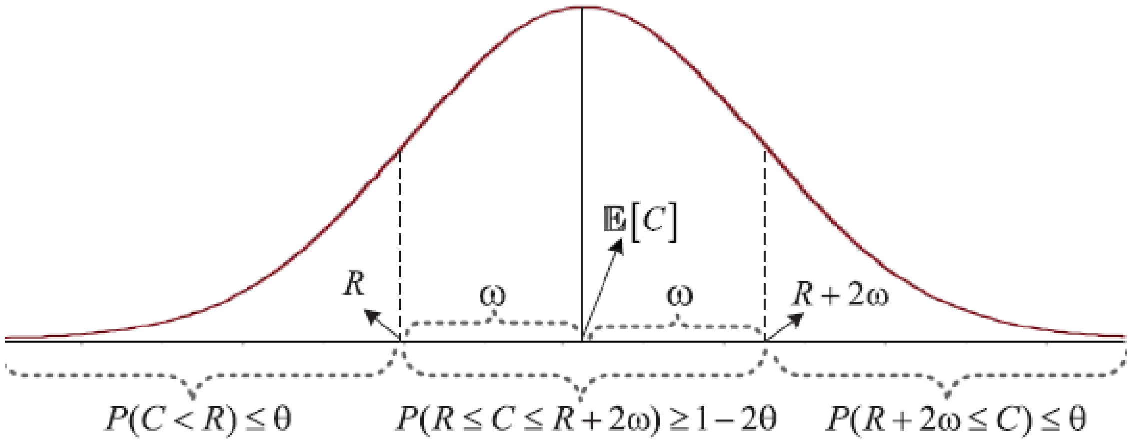
$$\mathbb{E}[C] - \frac{q\rho^{1/2}L}{N^{1/4}} < C < \mathbb{E}[C] + \frac{q\rho^{1/2}L}{N^{1/4}} \tag{8}$$

with probability at least $1 - 2\exp\left(-\frac{q^2 N^{\frac{1}{2}}}{2}\right)$, where $L = {}^{1}/_{ln2}$.

- Instantaneous capacity C falls within the symmetrical confidence interval $([\mathbb{E}[C] - \frac{q\rho^{1/2}L}{N^{1/4}}, \mathbb{E}[C] + \frac{q\rho^{1/2}L}{N^{1/4}}])$
- q-probability relation: *to study outage characterization of MIMO capacity*

Analysis of Outage Characterization

Symmetrical characteristic of Gaussian distribution



- If the outage probability is

$$P_{out}(R) = P(C < R) \leq p, \text{ from the figure}$$
$$P(R < C < R + 2\omega) \geq 1 - 2p \qquad (9)$$

  - $\omega = \mathbb{E}[C] - R$

- Upper equation (9) can be written as

$$P(\mathbb{E}[C] - \omega < C < \mathbb{E}[C] + \omega) \geq 1 - 2p \qquad (10)$$

- Outage probability requirement $P_{out}(R) \leq p$ is equivalent to (10) which has the same symmetrical structure with the result in *Lemma 1*.

Analysis of Outage Characterization

- In Massive MIMO systems, due to the large number of antennas, it is intractable to obtain the ergodic capacity
- The deterministic bounds on the ergodic capacity is used to derive statistical bounds for the instantaneous capacity [3] ⇒ an approximate symmetrical confidence interval with respect to $\mathbb{E}[C]$
- Based on results from [3] the ergodic capacity $\mathbb{E}[C]$ can be bounded as follows

$$\log_2 \mathbb{E}[e^C] - \log_2 \omega \leq \mathbb{E}[C] \leq \log_2 \mathbb{E}[e^C] \tag{11}$$

  - $\omega = 1 + 2\sqrt{\frac{2\rho\pi L^2}{N}}\left(e^{\frac{2\rho L^2}{4N}} - 1\right)$, and

$$\log_2 \mathbb{E}[e^C] = \log_2 \sum_{i=0}^{K} \binom{K}{i} \frac{\left(\frac{\rho}{N}\right)^i (N)!}{(N-i)!} \tag{12}$$

Analysis of Outage Characterization

- ***Lemma 2***: H is a $K \times N$ MIMO channel matrix and its entries are Gaussian random variables with zero-mean and unit variance. Equal power is allocated at each transmit antenna. Then, for any $q > 0$, with probability at least

$$1 - 2\exp(-\frac{q^2 N^{1/2}}{2}) \quad (13)$$

we have

$$\log_2 \mathbb{E}[e^C] - \log_2 \omega - \frac{q\rho^{1/2}L}{N^{1/4}} \leq C \leq \log_2 \mathbb{E}[e^C] + \frac{q\rho^{1/2}L}{N^{1/4}} \quad (14)$$

- For convenience:

○ $|\mathbb{E}[C] - \log_2 \mathbb{E}[e^C]| = \Delta_1$

○ $|\mathbb{E}[C] - (\log_2 \mathbb{E}[e^C] - \log_2 \omega)| = \Delta_2$

Analysis of Outage Characterization

- Based on the result $\varDelta_1 \leq \varDelta_2$ [3], the probability that the instantaneous capacity C falls within the interval (14)

$$
\begin{aligned}
& p\left(\log_2 \mathbb{E}[e^C] - \log_2 \omega - \frac{q\rho^{1/2}L}{N^{1/4}} \leq C \leq \log_2 \mathbb{E}[e^C] + \frac{q\rho^{1/2}L}{N^{1/4}}\right) \\
& = p\left(\mathbb{E}[C] - \varDelta_2 - \frac{q\rho^{1/2}L}{N^{1/4}} \leq C \leq \mathbb{E}[C] + \varDelta_1 + \frac{q\rho^{1/2}L}{N^{1/4}}\right) \\
& \leq p\left(\mathbb{E}[C] - \varDelta_2 - \frac{q\rho^{1/2}L}{N^{1/4}} \leq C \leq \mathbb{E}[C] + \varDelta_2 + \frac{q\rho^{1/2}L}{N^{1/4}}\right) \\
& = p\left(\log_2 \mathbb{E}[e^C] - \log_2 \omega - \frac{q\rho^{1/2}L}{N^{1/4}} \leq C \leq \mathbb{E}[C] + \varDelta_2 + \frac{q\rho^{1/2}L}{N^{1/4}}\right) \qquad (15)
\end{aligned}
$$

Analysis of Outage Characterization

- Based on the result $\Delta_1 \le \Delta_2$ [3], the probability that the instantaneous capacity C falls within the interval (14)

$$
\begin{aligned}
& p\left(\underbrace{\boxed{\log_2 \mathbb{E}[e^C] - \log_2 \omega - \frac{q\rho^{1/2}L}{N^{1/4}}}}_{\Pi_0} \le C \le \overbrace{\boxed{\log_2 \mathbb{E}[e^C] + \frac{q\rho^{1/2}L}{N^{1/4}}}}^{\Pi_1}\right) \\
& = p\left(\mathbb{E}[C] - \Delta_2 - \frac{q\rho^{1/2}L}{N^{1/4}} \le C \le \mathbb{E}[C] + \Delta_1 + \frac{q\rho^{1/2}L}{N^{1/4}}\right) \\
& \le p\left(\mathbb{E}[C] - \Delta_2 - \frac{q\rho^{1/2}L}{N^{1/4}} \le C \le \mathbb{E}[C] + \Delta_2 + \frac{q\rho^{1/2}L}{N^{1/4}}\right) \\
& = p\left(\log_2 \mathbb{E}[e^C] - \log_2 \omega - \frac{q\rho^{1/2}L}{N^{1/4}} \le C \le \overbrace{\boxed{\mathbb{E}[C] + \Delta_2 + \frac{q\rho^{1/2}L}{N^{1/4}}}}^{\Pi_2}\right)
\end{aligned}
\quad (15)
$$

Analysis of Outage Characterization

- Inequality (15) can be rewritten as
$$P(\Pi_0 \leq C \leq \Pi_1) \geq P(\Pi_0 \leq C \leq \Pi_2) \quad (16)$$
- The outage probability requirement $P_{out}(R) \leq p$ is equivalent to
$$P(\Pi_0 \leq C \leq \Pi_1) \geq 1 - 2p \quad (17)$$

with setting $\Pi_0 = R$

- Based on inequality (15), in order to guarantee (17), let
$$P(\Pi_0 \leq C \leq \Pi_2) \geq 1 - 2p \quad (18)$$

with setting $\Pi_0 = R$

Analysis of Outage Characterization

- Based on (11), the gap between Π_2 and Π_1 is evaluated

$$\begin{aligned} G &= \Pi_1 - \Pi_2 = \mathbb{E}[C] + \Delta_2 + \frac{q\rho^{1/2}L}{N^{1/4}} - \Pi_2 \\ &= 2\mathbb{E}[C] - \log_2 \mathbb{E}[e^C] + \log_2 \omega + \frac{q\rho^{1/2}L}{N^{1/4}} - \Pi_2 \\ &\leq \log_2 \mathbb{E}[e^C] + \log_2 \omega + \frac{q\rho^{1/2}L}{N^{1/4}} - \Pi_2 = \log_2 \omega \end{aligned} \quad (19)$$

- The gap between Π_2 and Π_1 vanishes at a rate no less than $O(\frac{1}{N^{3/2}})$
- When the number of antennas is large, such as in Massive MIMO, the gap is insignificant ⇒ reasonable to replace (17) with (18) as the outage probability requirement.
- In Massive MIMO systems, which have orders of magnitude more antennas (100 or more), $\widehat{N}_{min}$ is almost equal to N_{min}

Determine the Minimum Number of Antennas

- The q-probability relation is used
- We set Π_0 equal to the outage capacity requirement R

$$\Pi_0 = \log_2 \mathbb{E}[e^C] - \log_2 \omega - \frac{q\rho^{1/2}L}{N^{1/4}} = R \tag{20}$$

- The obtained factor q^*:

$$q^* = \frac{(\log_2 \mathbb{E}[e^C] - \log_2 \omega - R)N^{1/4}}{\rho^{1/2}L} \tag{21}$$

- The right of the equation is a function of the number of antennas N.
- The required minimum number of antennas from (13)

$$N_{min} = \arg[p = \exp(-\frac{(q^*)^2 N^{1/4}}{2})] \tag{22}$$

Determine the Minimum Number of Antennas

- The form of finding the root for a equation, such as

$$N_{min} = arg[F(N) = 0] \qquad (22)$$

where $F(N) = \exp(-\frac{(q^*)^2 N^{1/4}}{2}) - p$

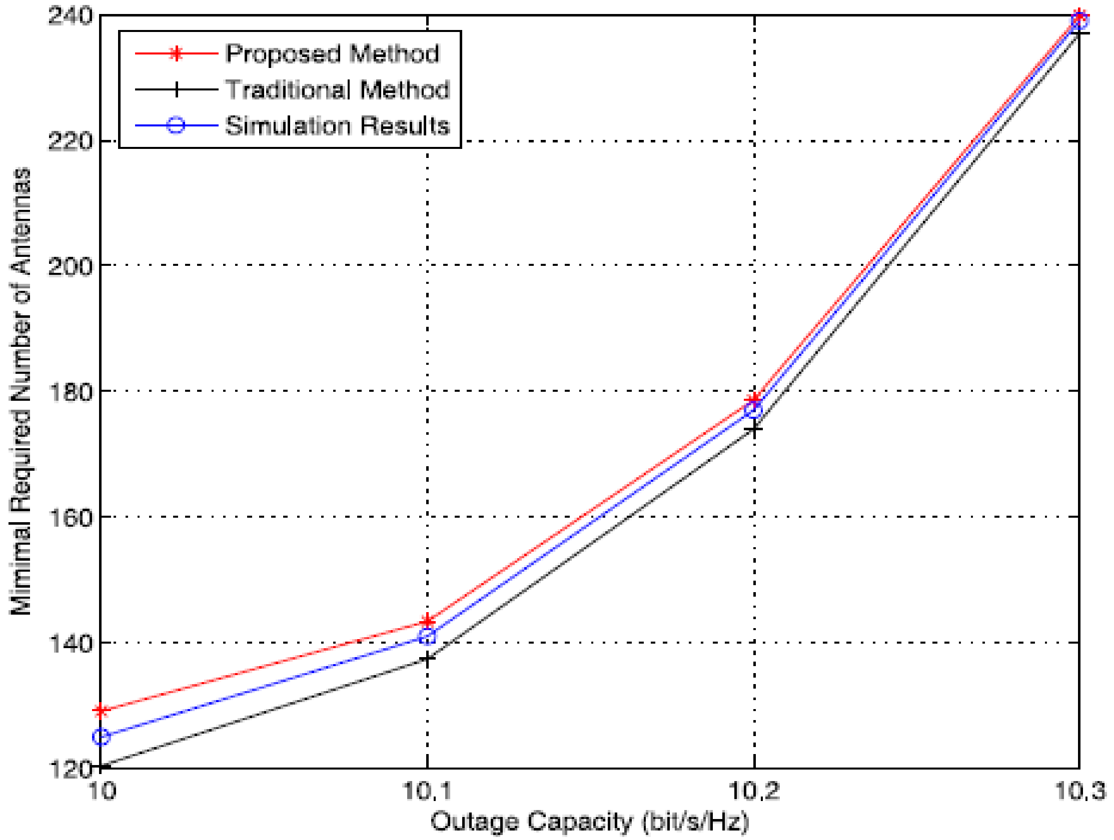
- F(N) decreases monotonically with $N \Rightarrow$ only one root for the equation $F(N) = 0$
- To solve root-finding problem (22), the bisection method was used - a numerical approximation method with high reliability and low complexity

Simulation

- The bisection method is used to estimate the root of the equivalent equation
- The iteration of the bisection method is terminated when the width of the current search interval $[b, a]$ is no more than one, $a - b \leq 1$
- The estimation error is less than one

Simulation

- Comparison of the required minimum number of antennas obtained by the proposed method with that obtained by the traditional method [4].
- Required minimum number of antennas obtained by:
  - the proposed method is greater than the simulation results,
  - the traditional method is smaller than the simulation results.
- As the required number of antennas increases: approximation error of the proposed method vanishes faster.



Conclusion

- The problem of how many antennas should be required to at least satisfy outage probability requirements
- The statistical bounds on instantaneous MIMO capacity is exploited to derive the equivalent equation of the initial outage probability requirement
- The bisection method is used to derive the only solution of the equivalent equation which is the minimum number of antennas required to satisfy a certain outage probability requirement
- The analytical and numerical results demonstrated that the proposed method is more accurate than traditional method using the Gaussian approximation

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Thank You!