

Angular momentum localisation

The Minkowski metric tensor.

$$\eta_{\textcolor{red}{ab}} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

(1)

The metric perturbation.

$$h_{\textcolor{red}{ab}} \rightarrow \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\text{Cos}\big[\frac{\omega \textcolor{teal}{ct}}{c}-k\,z\big]\,f[x,y] & f[x,y]\,\text{Sin}\big[\frac{\omega \textcolor{teal}{ct}}{c}-k\,z\big] & -\frac{\text{Cos}\big[\frac{\omega \textcolor{teal}{ct}}{c}-k\,z\big]\,f^{(0,1)}[x,y]+\text{Sin}\big[\frac{\omega \textcolor{teal}{ct}}{c}-k\,z\big]\,f^{(1,0)}[x,y]}{k} \\ 0 & f[x,y]\,\text{Sin}\big[\frac{\omega \textcolor{teal}{ct}}{c}-k\,z\big] & \text{Cos}\big[\frac{\omega \textcolor{teal}{ct}}{c}-k\,z\big]\,f[x,y] & \frac{\text{Sin}\big[\frac{\omega \textcolor{teal}{ct}}{c}-k\,z\big]\,f^{(0,1)}[x,y]-\text{Cos}\big[\frac{\omega \textcolor{teal}{ct}}{c}-k\,z\big]\,f^{(1,0)}[x,y]}{k} \\ 0 & -\frac{\text{Cos}\big[\frac{\omega \textcolor{teal}{ct}}{c}-k\,z\big]\,f^{(0,1)}[x,y]+\text{Sin}\big[\frac{\omega \textcolor{teal}{ct}}{c}-k\,z\big]\,f^{(1,0)}[x,y]}{k} & \frac{\text{Sin}\big[\frac{\omega \textcolor{teal}{ct}}{c}-k\,z\big]\,f^{(0,1)}[x,y]-\text{Cos}\big[\frac{\omega \textcolor{teal}{ct}}{c}-k\,z\big]\,f^{(1,0)}[x,y]}{k} & 0 \end{pmatrix}$$

(2)

The Landau-Lifshitz pseudotensor

$$\frac{1}{16\,\mathcal{G}\,\pi}\,c^4\left(-\frac{1}{4}\left(\nabla^j h_n^n\right)\left(\nabla^k h_\rho^\rho\right)+\frac{1}{2}\left(\nabla^j h_n^n\right)\left(\nabla^k h_\rho^q\right)+\frac{1}{8}\,\eta^{ik}\left(\nabla_l h_n^n\right)\left(\nabla^l h_\rho^\rho\right)-\frac{1}{4}\,\eta^{ik}\left(\nabla_l h_n^q\right)\left(\nabla^l h_\rho^n\right)-\left(\nabla_l h^{il}\right)\left(\nabla_m h^{km}\right)+\left(\nabla_l h^{ik}\right)\left(\nabla_m h^{lm}\right)+\left(\nabla_n h^{il}\right)\left(\nabla^n h^k{}_l\right)-\left(\nabla^k h_\rho^n\right)\left(\nabla_\rho h^{in}\right)-\left(\nabla^j h_\rho^n\right)\left(\nabla_\rho h^{kn}\right)+\frac{1}{2}\,\eta^{ik}\left(\nabla_n h^\rho{}_l\right)\left(\nabla_\rho h^{ln}\right)\right)$$

(3)

$$\left(\frac{c^4\left(\frac{4\,\omega^4\,f[x,y]^2}{c^2}+2\,\omega^2\left(f^{(0,1)}[x,y]^2+f^{(1,0)}[x,y]^2\right)+c^2\left(4\,f^{(1,1)}[x,y]^2+f^{(0,2)}[x,y]-f^{(2,0)}[x,y]\right)^2\right)-4\,\omega^2\,f[x,y]\left(f^{(0,2)}[x,y]+f^{(2,0)}[x,y]\right)}{64\,\mathcal{G}\,\omega^2\,\pi}-\frac{c^5\left(2\,f^{(1,0)}[x,y]\,f^{(1,1)}[x,y]+f^{(0,1)}[x,y]\left(\frac{4\,\omega^2\,f[x,y]}{c^2}+f^{(0,2)}[x,y]-f^{(2,0)}[x,y]\right)\right)}{32\,\mathcal{G}\,\omega\,\pi}-\frac{c^5\left(2\,f^{(0,1)}[x,y]\,f^{(1,1)}[x,y]+f^{(1,0)}[x,y]\left(\frac{4\,\omega^2\,f[x,y]}{c^2}-f^{(0,2)}[x,y]+f^{(2,0)}[x,y]\right)\right)}{32\,\mathcal{G}\,\omega\,\pi}-\frac{c^2\,f[x,y]\left(\omega^2\,f[x,y]-c^2\left(f^{(0,2)}[x,y]+f^{(2,0)}[x,y]\right)\right)}{16\,\mathcal{G}\,\pi}-\frac{c^4\left(6\,\omega^2\,f^{(0,1)}[x,y]^2+2\,\omega^2\,f^{(1,0)}[x,y]^2+c^2\left(4\,f^{(1,1)}[x,y]^2+\left(f^{(0,2)}[x,y]-f^{(2,0)}[x,y]\right)^2\right)+2\,\omega^2\,f[x,y]\left(f^{(0,2)}[x,y]-f^{(2,0)}[x,y]\right)\right)}{64\,\mathcal{G}\,\omega^2\,\pi}-\frac{c^4\left(f^{(0,1)}[x,y]\,f^{(1,0)}[x,y]+f[x,y]\,f^{(1,1)}[x,y]\right)}{16\,\mathcal{G}\,\pi}-\frac{c^3\,\omega\,f[x,y]\,f^{(1,0)}[x,y]}{8\,\mathcal{G}\,\pi}-\frac{c^4\left(\frac{4\,\omega^4\,f[x,y]^2}{c^2}-2\,\omega^2\left(f^{(0,1)}[x,y]^2+f^{(1,0)}[x,y]^2\right)-4\,\omega^2\,f[x,y]\left(f^{(0,2)}[x,y]+f^{(2,0)}[x,y]\right)+c^2\left(f^{(0,2)}[x,y]^2-4\,f^{(1,1)}[x,y]^2+6\,f^{(0,2)}[x,y]\,f^{(2,0)}[x,y]+f^{(2,0)}[x,y]^2\right)\right)}{64\,\mathcal{G}\,\omega^2\,\pi}\right)$$

(4)

The Einstein pseudotensor

$$\frac{c^4\left(-\frac{1}{2}\,\eta_{mn}\left(\nabla_a h_{rs}\right)\left(\nabla^a h^{rs}\right)+\left(\nabla_m h_{rs}\right)\left(\nabla_n h^{rs}\right)\right)}{32\,\mathcal{G}\,\pi}$$

(5)

$$\left(\frac{2\,c^2\,\omega^4\,f[x,y]^2+3\,c^4\,\omega^4\left(f^{(0,1)}[x,y]^2+f^{(1,0)}[x,y]^2\right)+c^6\left(f^{(0,2)}[x,y]^2+2\,f^{(1,1)}[x,y]^2+f^{(2,0)}[x,y]^2\right)}{32\,\mathcal{G}\,\omega^2\,\pi}-\frac{c^5\left(f^{(1,0)}[x,y]\,f^{(1,1)}[x,y]-f^{(0,1)}[x,y]\,f^{(2,0)}[x,y]\right)}{16\,\mathcal{G}\,\omega\,\pi}-\frac{c^5\left(f^{(0,2)}[x,y]\,f^{(1,0)}[x,y]-f^{(0,1)}[x,y]\,f^{(1,1)}[x,y]\right)}{16\,\mathcal{G}\,\omega\,\pi}-\frac{c^2\left(\omega^2\,f[x,y]^2+c^2\left(f^{(0,1)}[x,y]^2+f^{(1,0)}[x,y]^2\right)\right)}{16\,\mathcal{G}\,\pi}-\frac{c^5\left(f^{(1,0)}[x,y]\,f^{(1,1)}[x,y]-f^{(0,1)}[x,y]\,f^{(2,0)}[x,y]\right)}{16\,\mathcal{G}\,\omega\,\pi}-\frac{c^4\,\omega^2\left(-f^{(0,1)}[x,y]^2+f^{(1,0)}[x,y]^2\right)+c^6\left(-f^{(0,2)}[x,y]^2+f^{(2,0)}[x,y]^2\right)}{32\,\mathcal{G}\,\omega^2\,\pi}-\frac{c^4\,\omega^2\,f^{(0,1)}[x,y]\,f^{(1,0)}[x,y]+c^6\,f^{(1,1)}[x,y]\left(f^{(0,2)}[x,y]+f^{(2,0)}[x,y]\right)}{16\,\mathcal{G}\,\omega^2\,\pi}-\frac{c^5\left(-f^{(1,0)}[x,y]\,f^{(1,1)}[x,y]+f^{(0,1)}[x,y]\,f^{(2,0)}[x,y]\right)}{16\,\mathcal{G}\,\omega\,\pi}-\frac{c^5\left(f^{(0,2)}[x,y]\,f^{(1,0)}[x,y]-f^{(0,1)}[x,y]\,f^{(1,1)}[x,y]\right)}{16\,\mathcal{G}\,\omega\,\pi}-\frac{c^4\,\omega^2\,f^{(0,1)}[x,y]\,f^{(1,0)}[x,y]+c^6\,f^{(1,1)}[x,y]\left(f^{(0,2)}[x,y]+f^{(2,0)}[x,y]\right)}{16\,\mathcal{G}\,\omega^2\,\pi}-\frac{c^4\,\omega^2\left(f^{(0,1)}[x,y]^2-f^{(1,0)}[x,y]^2\right)+c^6\left(f^{(0,2)}[x,y]^2-f^{(2,0)}[x,y]^2\right)}{32\,\mathcal{G}\,\omega^2\,\pi}-\frac{c^5\left(-f^{(0,2)}[x,y]\,f^{(1,0)}[x,y]+f^{(0,1)}[x,y]\,f^{(1,1)}[x,y]\right)}{16\,\mathcal{G}\,\omega\,\pi}-\frac{c^2\left(\omega^2\,f[x,y]^2+c^2\left(f^{(0,1)}[x,y]^2+f^{(1,0)}[x,y]^2\right)\right)}{16\,\mathcal{G}\,\pi}-\frac{c^5\left(-f^{(1,0)}[x,y]\,f^{(1,1)}[x,y]+f^{(0,1)}[x,y]\,f^{(2,0)}[x,y]\right)}{16\,\mathcal{G}\,\omega\,\pi}-\frac{c^5\left(-f^{(0,2)}[x,y]\,f^{(1,0)}[x,y]+f^{(0,1)}[x,y]\,f^{(1,1)}[x,y]\right)}{16\,\mathcal{G}\,\omega\,\pi}-\frac{2\,c^2\,\omega^4\,f[x,y]^2+c^4\,\omega^2\left(f^{(0,1)}[x,y]^2+f^{(1,0)}[x,y]^2\right)-c^6\left(f^{(0,2)}[x,y]^2+2\,f^{(1,1)}[x,y]^2+f^{(2,0)}[x,y]^2\right)}{32\,\mathcal{G}\,\omega^2\,\pi}\right)$$

(6)

The affine tensor of Butcher

$$\frac{c^4\left(-\frac{1}{8}\,\eta_{pq}\left(\left(\nabla_c h_{ab}\right)\left(\nabla^c h^{ab}\right)-\frac{1}{2}\left(\nabla_c h^a{}_a\right)\left(\nabla^c h^b{}_b\right)\right)+\frac{1}{4}\left(\nabla_\rho h_{ab}\right)\left(\nabla_q h^{ab}\right)-\frac{1}{8}\left(\nabla_\rho h^a{}_a\right)\left(\nabla_q h^b{}_b\right)\right)}{8\,\mathcal{G}\,\pi}$$

(7)

$$\left(\frac{2\,c^2\,\omega^4\,f[x,y]^2+3\,c^4\,\omega^2\left(f^{(0,1)}[x,y]^2+f^{(1,0)}[x,y]^2\right)+c^6\left(f^{(0,2)}[x,y]^2+2\,f^{(1,1)}[x,y]^2+f^{(2,0)}[x,y]^2\right)}{32\,\mathcal{G}\,\omega^2\,\pi}-\frac{c^5\left(f^{(1,0)}[x,y]\,f^{(1,1)}[x,y]-f^{(0,1)}[x,y]\,f^{(2,0)}[x,y]\right)}{16\,\mathcal{G}\,\omega\,\pi}-\frac{c^5\left(f^{(0,2)}[x,y]\,f^{(1,0)}[x,y]-f^{(0,1)}[x,y]\,f^{(1,1)}[x,y]\right)}{16\,\mathcal{G}\,\omega\,\pi}-\frac{c^2\left(\omega^2\,f[x,y]^2+c^2\left(f^{(0,1)}[x,y]^2+f^{(1,0)}[x,y]^2\right)\right)}{16\,\mathcal{G}\,\pi}-\frac{c^5\left(f^{(1,0)}[x,y]\,f^{(1,1)}[x,y]-f^{(0,1)}[x,y]\,f^{(2,0)}[x,y]\right)}{16\,\mathcal{G}\,\omega\,\pi}-\frac{c^4\,\omega^2\left(-f^{(0,1)}[x,y]^2+f^{(1,0)}[x,y]^2\right)+c^6\left(-f^{(0,2)}[x,y]^2+f^{(2,0)}[x,y]^2\right)}{32\,\mathcal{G}\,\omega^2\,\pi}-\frac{c^4\,\omega^2\,f^{(0,1)}[x,y]\,f^{(1,0)}[x,y]+c^6\,f^{(1,1)}[x,y]\left(f^{(0,2)}[x,y]+f^{(2,0)}[x,y]\right)}{16\,\mathcal{G}\,\omega^2\,\pi}-\frac{c^5\left(-f^{(1,0)}[x,y]\,f^{(1,1)}[x,y]+f^{(0,1)}[x,y]\,f^{(2,0)}[x,y]\right)}{16\,\mathcal{G}\,\omega\,\pi}-\frac{c^5\left(f^{(0,2)}[x,y]\,f^{(1,0)}[x,y]-f^{(0,1)}[x,y]\,f^{(1,1)}[x,y]\right)}{16\,\mathcal{G}\,\omega\,\pi}-\frac{c^4\,\omega^2\,f^{(0,1)}[x,y]\,f^{(1,0)}[x,y]+c^6\,f^{(1,1)}[x,y]\left(f^{(0,2)}[x,y]+f^{(2,0)}[x,y]\right)}{16\,\mathcal{G}\,\omega^2\,\pi}-\frac{c^4\,\omega^2\left(f^{(0,1)}[x,y]^2-f^{(1,0)}[x,y]^2\right)+c^6\left(f^{(0,2)}[x,y]^2-f^{(2,0)}[x,y]^2\right)}{32\,\mathcal{G}\,\omega^2\,\pi}-\frac{c^5\left(-f^{(0,2)}[x,y]\,f^{(1,0)}[x,y]+f^{(0,1)}[x,y]\,f^{(1,1)}[x,y]\right)}{16\,\mathcal{G}\,\omega\,\pi}-\frac{c^2\left(\omega^2\,f[x,y]^2+c^2\left(f^{(0,1)}[x,y]^2+f^{(1,0)}[x,y]^2\right)\right)}{16\,\mathcal{G}\,\pi}-\frac{c^5\left(-f^{(1,0)}[x,y]\,f^{(1,1)}[x,y]+f^{(0,1)}[x,y]\,f^{(2,0)}[x,y]\right)}{16\,\mathcal{G}\,\omega\,\pi}-\frac{c^5\left(-f^{(0,2)}[x,y]\,f^{(1,0)}[x,y]+f^{(0,1)}[x,y]\,f^{(1,1)}[x,y]\right)}{16\,\mathcal{G}\,\omega\,\pi}-\frac{2\,c^2\,\omega^4\,f[x,y]^2+c^4\,\omega^2\left(f^{(0,1)}[x,y]^2+f^{(1,0)}[x,y]^2\right)-c^6\left(f^{(0,2)}[x,y]^2+2\,f^{(1,1)}[x,y]^2+f^{(2,0)}[x,y]^2\right)}{32\,\mathcal{G}\,\omega^2\,\pi}\right)$$

(8)

Key observation: This is the end of the script.