Some Title

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July 26, 2024

Abstract

Doing cool stuff to find cooler stuff

1 Introduction

The very important physical effect has applications to something in the world.

Formatting stuff to consider:

1. Do we want to use ∂-notation or $\dot{A}'$-notation?
2. Do we want to be reminded what variables are fcns off (eg $\mathscr{b}(t,z)$) or is that too messy?
3. Do we want to use h_{12} or $h_\times$?
4. I am mixing between [t,x,y,z] and [0,1,2,3] which is maybe confusing? (ex. $\mathscr{E}_x$ vs $\mathscr{u}_1$)

2 Background

Gravitational theories and the Gertsenshtein effect combined in groundbreaking ways. We use a (1,-1,-1,-1) metric.

The paper used the Einstein-Maxwell Lagrangian

$$\mathscr{L} = \sqrt{g}(R + \kappa F_{\mu\nu}F^{\mu\nu}), \tag{1}$$

where g is the determinant of the metric, R is the Ricci Scalar, and F is the electromagnetic field tensor.

Linearizing the action and varying it gives the Maxwell field equations and the linearized gravitational wave eqaution. Using a transverse traceless Lorentz gauge for h, a small metric perturbation, with $\mathscr{h}_{\mu\nu} = \mathscr{h}_{\mu\nu}(t,z)$, they evaluated the components of the field equations (making assumptions about the EM wave and background), giving two mixing equations:

$$\ddot{\mathscr{b}}_y - \mathscr{b}_y'' = B_0 \mathscr{h}_{12}'' \tag{2}$$

$$\ddot{\mathscr{h}}_{12} - \mathscr{h}_{12}'' = 4B_0 \mathscr{b}_y \tag{3}$$

Here, we have a background magnetic field $\mathbf{B_0} = \mathbf{B}_{0x}$, and a magnetic perturbation $\mathbf{b} = \mathbf{b}_y(t,z)$. We have no electric field in the background, only in the wave.

We recreated the calculation with torsion, and got back the same equations.

3 Our stuff

We see what we get for the mixing equations with more complicated couplings in the lagrangian, with a torsion theory. We have a perturbation on the torsion given by

$$\mathscr{T}^{\mu}_{\nu\sigma} = \epsilon^{\mu}_{\nu\sigma\lambda}\mathscr{Q}^{\lambda} + \delta^{\mu}_{[\nu}\mathscr{U}_{\sigma]} \tag{4}$$

with

$$\mathscr{Q}_{\mu} = [\mathscr{q}_0(t,z), 0, \mathscr{q}_2(t,z), 0] \tag{5}$$

and

$$\mathscr{U}_{\mu} = [\mathscr{u}_0(t,z), 0, \mathscr{u}_2(t,z), 0]. \tag{6}$$

When we add inn a background torsion T, given by

$$T^{\mu}_{\nu\sigma} = \epsilon^{\mu}_{\nu\sigma\lambda}Q^{\lambda} \tag{7}$$

with

$$Q_{\mu} = [q_0, 0, 0, 0] \tag{8}$$

Lagrangian 1 - $R_{\mu\nu}F^{\mu\nu}$

$$\mathscr{L} = \sqrt{g}(R + \zeta R_{\mu\nu}F^{\mu\nu} + \kappa F\mu\nu F^{\mu\nu}) \tag{9}$$

With no background torsion:

Maxwell and Einstein gives us nothing new on the face of it. The torsion field equations give us a bunch of mixings (notably with only first-order derivatives):

$$\frac{1}{2}B_0\mathscr{h}'_{11} - \partial_t\mathscr{E}_y = 0 \tag{10}$$

$$\frac{1}{2}B_0\mathscr{u}_0 + \frac{1}{2}B_0\dot{\mathscr{h}}_{11} - \partial_y\mathscr{E}_z = 0 \tag{11}$$

$$\frac{1}{2}B_0\dot{\mathscr{h}}_{12} - \dot{\mathscr{b}}_y = 0 \tag{12}$$

We also have these three equations, which have some interesting impact on the original mixing equations we get from Maxwell and Einstein:

$$\frac{1}{2}B_0\mathscr{h}'_{12} - \mathscr{b}'_y + \partial_t\mathscr{E}_x = 0 \tag{13}$$

$$\frac{1}{2}B_0\mathscr{h}'_{12} - \mathscr{b}'_y = 0 \tag{14}$$

$$\frac{1}{2}B_0\mathscr{h}'_{12} + \partial_t\mathscr{E}_x = 0 \tag{15}$$

meaning that each of the three terms individually are zero, removing the z-derivatives of both $\mathscr{h}_\times$ and $\mathscr{b}$ in our original mixing equations, leaving

$$\ddot{\mathscr{b}}_y = 0 \tag{16}$$

$$\ddot{\mathscr{h}}_{12} = 4B_0\mathscr{b}_y \tag{17}$$

This lagrangian does not allow for a finite background torsion.

Lagrangian 2 - $\epsilon_{\mu\nu\sigma\lambda}R^{\mu\nu}F^{\sigma\lambda}$

$$\mathscr{L} = \sqrt{g}(R + \zeta\epsilon_{\mu\nu\sigma\lambda}R^{\mu\nu}F^{\sigma\lambda} + \kappa F\mu\nu F^{\mu\nu}) \tag{18}$$

<u>With no background torsion:</u>

No change in Maxwell, Einstein gave same mixing equation. Torsion field eqs gave some equations though:

$$2\mathscr{b}_y^{'} - B_0\mathscr{h}_{12}^{'} + B_0\mathscr{q}_0 = 0 \tag{19}$$

$$-B_0\dot{\mathscr{h}}_{11} + 2\partial_z\mathscr{E}_y = 0 \tag{20}$$

$$-2\dot{\mathscr{b}}_y - B_0\dot{\mathscr{h}}_{12} + 2\partial_z\mathscr{E}_x = 0 \tag{21}$$

$$B_0\dot{\mathscr{h}}_{12} - 2\partial_z\mathscr{E}_x + 2\partial_x\mathscr{E}_z = 0 \tag{22}$$

Here, the two last equations give us

$$-2\dot{\mathscr{b}}_y - B_0\dot{\mathscr{h}}_{12} = 0 \tag{23}$$

with

$$2\partial_z\mathscr{E}_x = 0 \tag{24}$$

since

$$-\partial_z\mathscr{E}_x + \partial_x\mathscr{E}_z = \dot{\mathscr{b}}_y \tag{25}$$

meaning that in this case,

$$\partial_x\mathscr{E}_z = \dot{\mathscr{b}}_y \tag{26}$$

which might be worth noting for further calculations, but has no immediate effect.

<u>With background torsion:</u> Not started.

Lagrangian 3 - $R_{\mu\nu}R^{\mu\nu}$

$$\mathscr{L} = \sqrt{g}(R + \zeta R_{\mu\nu}R^{\mu\nu} + \kappa F\mu\nu F^{\mu\nu}) = 0 \tag{27}$$

<u>With no background torsion:</u>

The torsion field equations have no EM-components, and only serve to help us simplify the other equations. The maxwell field equations are unchanged.

For the Einstein equation we get some higher-order derivatives, but no torsion contribution:

$$-4\kappa B_0\mathscr{b}_y - \mathscr{h}_{12}^{''} + \ddot{\mathscr{h}}_{12} + \zeta\mathscr{h}_{12}^{''''} + 2\zeta\ddot{\mathscr{h}}_{12}^{''} + \zeta\ddot{\ddot{\mathscr{h}}}_{12} = 0 \tag{28}$$

<u>With background torsion:</u> Not finsihed running.

We also did a run with the indices on the second Ricci tensor swapped, but achieved the same mixing equations (different torsion field equations, but that had no effect on the mixing equations).

Lagrangian 4 - $R_{\mu\nu\sigma\lambda}R^{\mu\nu\sigma\lambda}$

$$\mathscr{L} = \sqrt{g}(R + \zeta R_{\mu\nu\sigma\lambda}R^{\mu\nu\sigma\lambda} + \kappa F\mu\nu F^{\mu\nu}) \tag{29}$$

With no background torsion:

Maxwell still gives the same equations as expected.

Torsion still only gives some substitutions

Einstein is not done evaluating.

With background torsion: Not started

Lagrangian 5 - $R_{\mu\nu\sigma\lambda}R^{\mu\sigma\nu\lambda}$ Not finished running

Lagrangian 6 - $R_{\mu\nu\sigma\lambda}R^{\sigma\lambda\mu\nu}$ Not finished running

4 Conclusions

The world is forever changed

References

[1] *A source*, available at `https://wevbarker.com/`.