

⋮ **ValidateSymbol:** Symbol ValidateSymbol appears in multiple contexts {xAct`xCore`, Global}; definitions in context xAct`xCore` may shadow or be shadowed by other definitions.

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Package xAct`xPerm` version 1.2.3, {2015, 8, 23}  
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Connecting to external linux executable...  
Connection established.

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Package xAct`xTensor` version 1.2.0, {2021, 10, 17}  
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Package xAct`xPlain` version 0.0.0-developer, {2024, 5, 8}  
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Package xAct`xPert` version 1.0.6, {2018, 2, 28}  
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and Guillermo A. Mena Marugan, under the General Public License.  
\*\* Variable \$PrePrint assigned value ScreenDollarIndices  
\*\* Variable \$CovDFormat changed from Prefix to Postfix  
\*\* Option AllowUpperDerivatives of ContractMetric changed from False to True  
\*\* Option MetricOn of MakeRule changed from None to All  
\*\* Option ContractMetrics of MakeRule changed from False to True

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Package xAct`Invar` version 2.0.5, {2013, 7, 1}

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\*\* DefConstantSymbol: Defining constant symbol sigma.

\*\* DefConstantSymbol: Defining constant symbol dim.

\*\* Option CurvatureRelations of DefCovD changed from True to False

\*\* Variable \$CommuteCovDsOnScalars changed from True to False

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Package xAct`xCoba` version 0.8.6, {2021, 2, 28}

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Package xAct`SymManipulator` version 0.9.5, {2021, 9, 14}

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Package xAct`xTras` version 1.4.2, {2014, 10, 30}

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\*\* Variable \$CovDFormat changed from Postfix to Prefix

\*\* Option CurvatureRelations of DefCovD changed from False to True

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# The Gertsenshtein

# effect in gauge theories of gravity

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# Components of the fields, background, and perturbations

The background metric tensor.

$$g_{ab} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (1)$$

The metric perturbation in the transverse-traceless gauge.

$$h_{ab} \rightarrow \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+[t, z] & h_\times[t, z] & 0 \\ 0 & h_\times[t, z] & -h_+[t, z] & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2)$$

The background Faraday tensor.

$$F_{ab} \rightarrow \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & B_x \\ 0 & 0 & -B_x & 0 \end{pmatrix} \quad (3)$$

The perturbative Faraday tensor.

$$\mathcal{F}_{ab} \rightarrow \begin{pmatrix} 0 & \epsilon_x[t, x, y, z] & \epsilon_y[t, x, y, z] & \epsilon_z[t, x, y, z] \\ -\epsilon_x[t, x, y, z] & 0 & 0 & -b[t, z] \\ -\epsilon_y[t, x, y, z] & 0 & 0 & 0 \\ -\epsilon_z[t, x, y, z] & b[t, z] & 0 & 0 \end{pmatrix} \quad (4)$$

The perturbative torsion vectors.

$$Q_a \rightarrow \begin{pmatrix} q_t[t, z] \\ 0 \\ q_y[t, z] \\ 0 \end{pmatrix} \quad (5)$$

$$\mathcal{U}_a \rightarrow \begin{pmatrix} u_t[t, z] \\ 0 \\ u_y[t, z] \\ 0 \end{pmatrix} \quad (6)$$

The background torsion vector.

$$Q_a \rightarrow \begin{pmatrix} Q_0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (7)$$

The unit-timelike vector.

$$u_a \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (8)$$

## Making contact with Mike's conventions

Since my work from the pandemic era is based on Mike's conventions, and Selma's work is based on the xAct conventions for Riemann-Cartan geometry, we need to make contact between these in order to correctly understand the constant-torsion background.

The first step is to define Mike's Riemann tensor. Obviously, xAct won't know that this is the Riemann tensor.

$$\mathcal{R}^{ab}_{cd} \quad (9)$$

Now we define the Ricci tensor in Mike's conventions.

$$\mathcal{R}_{ab} \rightarrow \mathcal{R}^c_{a \ bc} \quad (10)$$

Now we define the Ricci scalar in Mike's conventions.

$$\mathcal{R} \rightarrow \mathcal{R}^a_a \quad (11)$$

Now we define the torsion tensor in Mike's conventions.

$$\mathcal{T}^a_{bc} \quad (12)$$

We can probably identify the torsion tensor in Mike's conventions with the torsion tensor in xAct

without worrying too much about numerical factors (we should check this eventually).

$$\mathcal{T}^a_{bc} \rightarrow T[D]^a_{bc} \quad (13)$$

Now we define the contraction of the torsion tensor in Mike's conventions.

$$\mathcal{T}_a \rightarrow -\mathcal{T}^b_{ab} \quad (14)$$

Now we define the contorsion tensor in Mike's conventions, as defined below Eq. (47) on page 12 of arXiv:1510.06699.

$$\mathcal{K}_{abc} \rightarrow -\frac{1}{2} \mathcal{T}_{abc} + \frac{1}{2} \mathcal{T}_{bac} + \frac{1}{2} \mathcal{T}_{cab} \quad (15)$$

Now we use Eq. (110a) on page 20 of arXiv:1510.06699 to define the (flat) post-Riemannian expansion of Mike's curvature tensor.

$$\mathcal{R}^{ab}_{cd} \rightarrow \mathcal{K}^{ae}_d \mathcal{K}^b_{ec} - \mathcal{K}^{ae}_c \mathcal{K}^b_{ed} + \nabla_c \mathcal{K}^{ab}_d - \nabla_d \mathcal{K}^{ab}_c \quad (16)$$

By expanding the contorsion using Eqs. (16), (15), and (13) we obtain the following.

$$\mathcal{R}^{ab}_{cd} \quad (17)$$

$$\begin{aligned} & \frac{1}{4} T[D]^a_d{}^e T[D]^b_{ce} - \frac{1}{4} T[D]^a_c{}^e T[D]^b_{de} - \frac{1}{4} T[D]^b_d{}^e T[D]^a_{ce} + \frac{1}{4} T[D]^a_d{}^e T[D]^b_{ce} + \frac{1}{4} T[D]^b_c{}^e T[D]^a_{de} + \\ & \frac{1}{4} T[D]^b_c{}^e T[D]^a_{de} - \frac{1}{4} T[D]^a_c{}^e T[D]^b_{de} - \frac{1}{4} T[D]^a_c{}^e T[D]^b_{de} - \frac{1}{4} T[D]^b_c{}^e T[D]^a_{de} - \frac{1}{4} T[D]^b_d{}^e T[D]^a_{ce} - \\ & \frac{1}{4} T[D]^b_d{}^e T[D]^a_{ce} + \frac{1}{4} T[D]^b_c{}^e T[D]^a_{de} + \frac{1}{4} T[D]^b_c{}^e T[D]^a_{de} + \frac{1}{4} T[D]^a_d{}^e T[D]^b_{ce} + \\ & \frac{1}{4} T[D]^a_d{}^e T[D]^b_{ce} - \frac{1}{4} T[D]^a_c{}^e T[D]^b_{de} - \frac{1}{4} T[D]^a_c{}^e T[D]^b_{de} - \frac{1}{4} T[D]^b_c{}^e T[D]^a_{de} + \frac{1}{4} T[D]^b_c{}^e T[D]^a_{de} - \\ & \frac{1}{2} (\nabla_c T[D]^{ab}_d) + \frac{1}{2} (\nabla_c T[D]^{ba}_d) + \frac{1}{2} (\nabla_c T[D]^{ab}_d) + \frac{1}{2} (\nabla_d T[D]^{ab}_c) - \frac{1}{2} (\nabla_d T[D]^{ba}_c) - \frac{1}{2} (\nabla_d T[D]^{ab}_c) \end{aligned} \quad (18)$$

Now we try the same expansion using the xAct conventions (note the different order of indices, which is the point of this whole exercise).

$$R[D]^{ab}_{cd} \quad (19)$$

$$\begin{aligned} & \frac{1}{4} T[D]^a_d{}^e T[D]^b_{ce} - \frac{1}{4} T[D]^a_{ce} T[D]^b_d{}^e - \frac{1}{4} T[D]^b_d{}^e T[D]^a_{ce} + \frac{1}{4} T[D]^a_d{}^e T[D]^b_{ce} + \frac{1}{4} T[D]^b_c{}^e T[D]^a_{de} + \\ & \frac{1}{4} T[D]^b_c{}^e T[D]^a_{de} - \frac{1}{4} T[D]^a_c{}^e T[D]^b_{de} - \frac{1}{4} T[D]^a_c{}^e T[D]^b_{de} + \frac{1}{4} T[D]^b_d{}^e T[D]^a_{ce} - \\ & \frac{1}{4} T[D]^a_d{}^e T[D]^b_{ce} + \frac{1}{4} T[D]^b_c{}^e T[D]^a_{de} - \frac{1}{4} T[D]^a_c{}^e T[D]^b_{de} - \frac{1}{4} T[D]^b_c{}^e T[D]^a_{de} - \end{aligned} \quad (20)$$

$$\begin{aligned} & \frac{1}{4} T[D]_c^b T[D]_d^e + \frac{1}{4} T[D]_{ec}^b T[D]_d^e + \frac{1}{4} T[D]_{ce}^a T[D]_d^e + \frac{1}{4} T[D]_c^a T[D]_d^e - \frac{1}{4} T[D]_{ec}^a T[D]_d^e + \\ & \frac{1}{2} \left( \nabla_c T[D]_d^a \right) - \frac{1}{2} \left( \nabla_c T[D]_d^b \right) + \frac{1}{2} \left( \nabla_c T[D]_d^{ab} \right) - \frac{1}{2} \left( \nabla_d T[D]_c^a \right) + \frac{1}{2} \left( \nabla_d T[D]_c^b \right) - \frac{1}{2} \left( \nabla_d T[D]_c^{ab} \right) \end{aligned}$$

Now we subtract the two expressions in Eqs. (18), and (20) to see if they are equal.

$$0 \quad (21)$$

Good: we are now able to claim the following index matchings when going from Mike's conventions to those of xAct.

$$\mathcal{R}^{ab}_{cd} \rightarrow R[D]^{ab}_{cd} \quad (22)$$

As a final definition, we define the Weyl field strength tensor in Mike's conventions.

$$\mathcal{H}_{ab} \rightarrow -\left( \nabla_a T[D]_{bc}^c \right) + \nabla_b T[D]_{ac}^c \quad (23)$$

**Key observation:** We think of Eqs. (13), (22), and (23) as being on the same footing.

## Defining the Lagrangian

First we define the fully covariant Lagrangian as given in Eqs. (2) and (3) on page 3 of arXiv:1510.06699.

$$\begin{aligned} & \frac{1}{36} \sqrt{-\tilde{g}} \left( 36 \theta F^{ab} \mathcal{H}_{ab} - \xi \mathcal{H}_{ab} \mathcal{H}^{ab} + 12 \chi \mathcal{H}_{ab} \mathcal{R}^{ab} + 36 \alpha_2 \mathcal{R}_{ab} \mathcal{R}^{ab} - \right. \\ & 9 F_{ab} \left( F^{ab} - 4 \zeta \mathcal{R}^{ab} \right) + 36 \alpha_3 \mathcal{R}^{ab} \mathcal{R}_{ba} + 36 \alpha_0 \mathcal{R} + 36 \alpha_1 \mathcal{R}^2 + 36 \alpha_4 \mathcal{R}_{abcd} \mathcal{R}^{abcd} + \\ & \left. 36 \alpha_5 \mathcal{R}_{abcd} \mathcal{R}^{abcd} + 36 \alpha_6 \mathcal{R}^{abcd} \mathcal{R}_{cdab} + 36 \beta_3 \mathcal{T}_a \mathcal{T}^a + 36 \beta_1 \mathcal{T}_{abc} \mathcal{T}^{abc} + 36 \beta_2 \mathcal{T}^{abc} \mathcal{T}_{bac} \right) \end{aligned} \quad (24)$$

**Key observation:** We use Eqs. (13), (22), and (23) to move Eq. (24) into Selma's xAct conventions.

$$\begin{aligned} & \frac{1}{36} \sqrt{-\tilde{g}} \left( -9 F_{ab} \left( F^{ab} + 4 \zeta R[D]^{ab} \right) + \right. \\ & 2 \left( 18 \alpha_2 R[D]_{ab} R[D]^{ab} + 18 \alpha_0 R[D] + 18 \alpha_1 R[D]^2 + 18 \alpha_4 R[D]_{abcd} R[D]^{abcd} + 18 \alpha_5 R[D]_{abcd} R[D]^{abcd} + \right. \\ & 18 \alpha_6 R[D]^{abcd} R[D]_{cdab} + 18 \beta_1 T[D]_{abc} T[D]^{abc} + 18 \beta_2 T[D]^{abc} T[D]_{bac} - 18 \beta_3 T[D]_a^b T[D]_{bc}^c + \\ & 36 \theta F^{ab} \left( \nabla_b T[D]_{ac}^c \right) + 6 R[D]^{ab} \left( 3 \alpha_3 R[D]_{ba} + \chi \left( \nabla_a T[D]_{bc}^c \right) - \chi \left( \nabla_b T[D]_{ac}^c \right) \right) + \\ & \left. \left. \xi g^{ab} \left( \nabla_a T[D]_c^d \right) \left( \nabla_b T[D]_{de}^e \right) + \xi \left( \nabla_b T[D]_{da}^a \right) \left( \nabla^d T[D]_{bc}^c \right) \right) \right) \end{aligned} \quad (25)$$

# Special theories of interest

Now we define the Karananas couplings, and give them in terms of Mike's couplings. These equations are taken from Eq. (B.23b) on page 143 of my thesis.

$\alpha_0 == 2 \lambda$	$\beta_1 == \frac{\lambda}{4} + \frac{t_1}{3} + \frac{t_2}{12}$	$\beta_2 == \frac{\lambda}{2} + \frac{t_1}{3} - \frac{t_2}{6}$	$\beta_3 == -\lambda - \frac{t_1}{3} + \frac{2t_2}{3}$	$\alpha_1 == r_6$
$\alpha_2 == r_4 + r_5$	$\alpha_3 == r_4 - r_5$	$\alpha_4 == \frac{r_1}{3} + \frac{r_2}{6}$	$\alpha_5 == \frac{2r_1}{3} - \frac{2r_2}{3}$	$\alpha_6 == \frac{r_1}{3} + \frac{r_2}{6} - r_3$

(26)

Now we introduce the cosmological couplings, and give them in terms of Mike's couplings. These equations are taken from Eq. (B.24a) on page 144 of my thesis.

$\sigma_1 == \frac{3\alpha_1}{2} + \frac{\alpha_2}{4} + \frac{\alpha_3}{4} + \frac{\alpha_5}{4} - \frac{\alpha_6}{2}$	$\sigma_2 == \frac{3\alpha_1}{2} + \frac{\alpha_2}{2} + \frac{\alpha_3}{2} + \frac{3\alpha_4}{2} - \frac{\alpha_5}{4} + \frac{\alpha_6}{2}$	$\sigma_3 == \frac{3\alpha_1}{2} + \frac{\alpha_2}{2} + \frac{\alpha_3}{2} + \frac{\alpha_4}{2} + \frac{\alpha_5}{4} + \frac{\alpha_6}{2}$	$u_1 == -2\beta_1 + 2\beta_2$	$u_2 == 2\beta_1 + \beta_2 + 3\beta_3$
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(27)

Now we define Case 2 from arXiv:1910.14197 in terms of Karananas' couplings. These equations are taken from Table 1 on page 5/6 of arXiv:1910.14197.

$r_1 == 0$	$\frac{r_3}{2} - r_4 == 0$	$t_1 == 0$	$\lambda == 0$	$r_6 == 0$
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(28)

Now we use Eqs. (26), and (28) define Case 2 from arXiv:1910.14197 in terms of Mike's couplings.

$4\alpha_4 + \alpha_5 == 0$	$\alpha_2 + \alpha_3 + \alpha_6 == \alpha_4$	$\alpha_0 == 4\beta_1 + 2\beta_2$	$\alpha_0 == 0$	$\alpha_1 == 0$
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(29)

Now we use Eq. (29) define Case 2 from arXiv:1910.14197 as a set of rules.

$\alpha_0 \rightarrow 0$	$\alpha_1 \rightarrow 0$	$\alpha_5 \rightarrow -4\alpha_4$	$\alpha_6 \rightarrow -\alpha_2 - \alpha_3 + \alpha_4$	$\beta_2 \rightarrow -2\beta_1$
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(30)

Now we use Eqs. (27), and (29) to consider the effect of Case 2 from arXiv:1910.14197 on the cosmological couplings.

$3(\alpha_2 + \alpha_3 - 2\alpha_4) == 4\sigma_1$	$3\alpha_4 == \sigma_2$	$\sigma_3 == 0$	$u_1 == -6\beta_1$	$3\beta_3 == u_2$
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(31)

We see that only one of the "special" cosmological conditions actually follows from Case 2 from arXiv:1910.14197, i.e. the vanishing of the third sigma parameter. Next, we will explore the combined implications of Case 2 from arXiv:1910.14197 with the preferred cosmological interpretation.

Here are the preferred cosmological conditions, which are collected from comments in Chapters 2 and 3 of my thesis.



$\sigma_1 == \sigma_2$	$\sigma_3 == 0$	$u_1 == \frac{\Lambda \sigma_1}{\mathcal{M}_{\text{Pl}}^2}$	$u_2 == -\frac{4 \mathcal{M}_{\text{Pl}}^2}{3}$
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(32)

Here are the combined implications of Case 2 from arXiv:1910.14197 with the preferred cosmological interpretation expressed as a set of rules. We obtain these by combining Eqs. (32), and (29).

$\alpha_0 \rightarrow 0$	$\alpha_1 \rightarrow 0$	$\alpha_4 \rightarrow \frac{\alpha_2}{6} + \frac{\alpha_3}{6}$	$\alpha_5 \rightarrow -\frac{2\alpha_2}{3} - \frac{2\alpha_3}{3}$	$\alpha_6 \rightarrow -\frac{5\alpha_2}{6} - \frac{5\alpha_3}{6}$
$\beta_1 \rightarrow -\frac{\alpha_2 \Lambda}{12 \mathcal{M}_{\text{Pl}}^2} - \frac{\alpha_3 \Lambda}{12 \mathcal{M}_{\text{Pl}}^2}$	$\beta_2 \rightarrow \frac{\alpha_2 \Lambda}{6 \mathcal{M}_{\text{Pl}}^2} + \frac{\alpha_3 \Lambda}{6 \mathcal{M}_{\text{Pl}}^2}$	$\beta_3 \rightarrow -\frac{4 \mathcal{M}_{\text{Pl}}^2}{9}$		

(33)

**Key observation:** For Anthony. You will presumably wish to apply Eq. (33) to your setup, or perhaps the less restrictive Eq. (30).

## Analysis of GR

**Key observation:** Analysis of GR without any constant-torsion background.

Here is the list of rules.

$\alpha_0 \rightarrow -\frac{\mathcal{M}_{\text{Pl}}^2}{2}$	$\alpha_1 \rightarrow 0$	$\alpha_2 \rightarrow 0$	$\alpha_3 \rightarrow 0$	$\alpha_4 \rightarrow 0$
$\alpha_5 \rightarrow 0$	$\alpha_6 \rightarrow 0$	$\beta_1 \rightarrow 0$	$\beta_2 \rightarrow 0$	$\beta_3 \rightarrow 0$
$Q_0 \rightarrow 0$	$\zeta \rightarrow 0$	$\theta \rightarrow 0$	$\chi \rightarrow 0$	$\xi \rightarrow 0$

(34)

Here is the non-linear Lagrangian.

$$\frac{1}{36} \sqrt{-\tilde{g}} \left( -9 F_{ab} F^{ab} - 18 \mathcal{M}_{\text{Pl}}^2 R[D] \right) \quad (35)$$

## Here are the zeroth-order equations.

The Cartan components.

(36)

The Einstein components.

$B_x == 0$

(37)

The Maxwell components.

(38)

## Here are the first-order equations.

The Cartan components.

$\mathcal{M}_{\text{Pl}} u_y[t, z] == 0$	(39)
$\mathcal{M}_{\text{Pl}} q_y[t, z] == 0$	
$\mathcal{M}_{\text{Pl}} u_t[t, z] == 0$	
$\mathcal{M}_{\text{Pl}} q_t[t, z] == 0$	

The Einstein components.

$\mathcal{M}_{\text{Pl}} u_t^{(1,0)}[t, z] == 0$	(40)
$2 B_x \epsilon_z[t, x, y, z] + 3 \mathcal{M}_{\text{Pl}}^2 u_y^{(1,0)}[t, z] == 0$	
$2 B_x \epsilon_y[t, x, y, z] == 3 \mathcal{M}_{\text{Pl}}^2 u_t^{(0,1)}[t, z]$	
$\mathcal{M}_{\text{Pl}} h_+^{(0,2)}[t, z] == \mathcal{M}_{\text{Pl}} h_+^{(2,0)}[t, z]$	
$2 B_x b[t, z] + \mathcal{M}_{\text{Pl}}^2 h_x^{(0,2)}[t, z] == \mathcal{M}_{\text{Pl}}^2 h_x^{(2,0)}[t, z]$	
$\mathcal{M}_{\text{Pl}} u_y^{(0,1)}[t, z] == 0$	

The Maxwell components.

$B_x h_+^{(0,2)}[t, z] + \epsilon_z^{(1,0,1,0)}[t, x, y, z] == \epsilon_y^{(1,0,0,1)}[t, x, y, z]$	(41)
$B_x h_x^{(0,2)}[t, z] + \epsilon_x^{(1,0,0,1)}[t, x, y, z] == b^{(0,2)}[t, z] + \epsilon_z^{(1,1,0,0)}[t, x, y, z]$	
$\epsilon_x^{(1,0,1,0)}[t, x, y, z] == \epsilon_y^{(1,1,0,0)}[t, x, y, z]$	

## Here is the reduced set of first-order equations.

After simplification, we have 2 equations.

$B_x b[t, z] + \frac{1}{2} \mathcal{M}_{\text{Pl}}^2 h_x^{(0,2)}[t, z] - \frac{1}{2} \mathcal{M}_{\text{Pl}}^2 h_x^{(2,0)}[t, z]$	(42)
$-2 B_x h_x^{(0,2)}[t, z] + 2 b^{(0,2)}[t, z] - 2 b^{(2,0)}[t, z]$	

## Analysis of GR with extra couplings

**Key observation:** Analysis of GR without any constant-torsion background but with extra couplings of torsion to the Maxwell field.

Here is the list of rules.

$\alpha_0 \rightarrow -\frac{\mathcal{M}_{\text{Pl}}^2}{2}$	$\alpha_1 \rightarrow 0$	$\alpha_2 \rightarrow 0$	$\alpha_3 \rightarrow 0$	$\alpha_4 \rightarrow 0$
$\alpha_5 \rightarrow 0$	$\alpha_6 \rightarrow 0$	$\beta_1 \rightarrow 0$	$\beta_2 \rightarrow 0$	$\beta_3 \rightarrow 0$
$Q_0 \rightarrow 0$				

(43)

Here is the non-linear Lagrangian.

$$\begin{aligned}
& \frac{1}{36} \sqrt{-\tilde{g}} \left( -9 F_{ab} \left( F^{ab} + 4 \zeta R[D]^{ab} \right) + \right. \\
& 2 \left( -9 \mathcal{M}_{\text{Pl}}^2 R[D] + 36 \theta F^{ab} \left( \nabla_b T[D]_{ac}^c \right) + 6 R[D]^{ab} \left( \chi \left( \nabla_a T[D]_{bc}^c \right) - \chi \left( \nabla_b T[D]_{ac}^c \right) \right) + \right. \\
& \left. \left. \xi g^{ab} \left( \nabla_a T[D]_{c \ d}^c \right) \left( \nabla_b T[D]_{de}^e \right) + \xi \left( \nabla_b T[D]_{da}^a \right) \left( \nabla^d T[D]_{c \ b}^{cb} \right) \right) \right)
\end{aligned}$$
(44)

## Here are the zeroth-order equations.

The Cartan components.

(45)

The Einstein components.

$B_x == 0$

(46)

The Maxwell components.

(47)

## Here are the first-order equations.

The Cartan components.

$B_x \zeta q_t[t, z] == (2 \theta + \zeta) (B_x h_x^{(0,1)}[t, z] - b^{(0,1)}[t, z]) + 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$6 \mathcal{M}_{\text{Pl}}^2 u_y[t, z] + 6 B_x (2 \theta + \zeta) h_+^{(0,1)}[t, z] + (5 \chi - \xi) u_y^{(0,2)}[t, z] + (-2 \chi + \xi) u_y^{(2,0)}[t, z] == 12 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z]$
$3 B_x \zeta u_y[t, z] + (-2 \chi + \xi) u_t^{(1,1)}[t, z] + 12 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] == 0$
$B_x \zeta q_y[t, z] == 0$
$\mathcal{M}_{\text{Pl}}^2 q_y[t, z] + B_x \zeta h_x^{(1,0)}[t, z] == 2 \zeta b^{(1,0)}[t, z]$
$B_x \zeta (2 u_t[t, z] + h_+^{(1,0)}[t, z]) + \chi u_y^{(1,1)}[t, z] == 0$

$6 \mathcal{M}_{\text{Pl}}^2 u_t[t, z] + (5 \chi - \xi) u_t^{(0,2)}[t, z] ==$ $6 (2 \theta + \zeta) (\epsilon_z^{(0,0,0,1)}[t, x, y, z] + \epsilon_y^{(0,0,1,0)}[t, x, y, z]) + 12 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$
$B_x \zeta q_y[t, z] + 2 \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z] == 0$
$\mathcal{M}_{\text{Pl}}^2 q_y[t, z] == B_x \zeta h_x^{(1,0)}[t, z] + 2 \zeta \epsilon_z^{(0,1,0,0)}[t, x, y, z]$
$6 \mathcal{M}_{\text{Pl}}^2 u_y[t, z] + 3 B_x (4 \theta + \zeta) h_+^{(0,1)}[t, z] + (5 \chi - \xi) (u_y^{(0,2)}[t, z] - u_y^{(2,0)}[t, z]) ==$ $6 (2 \theta + \zeta) \epsilon_y^{(1,0,0,0)}[t, x, y, z]$
$3 B_x \zeta u_y[t, z] + (-5 \chi + \xi) u_t^{(1,1)}[t, z] + 6 (2 \theta + \zeta) \epsilon_z^{(1,0,0,0)}[t, x, y, z] == 0$
$\mathcal{M}_{\text{Pl}} q_t[t, z] == 0$
$\zeta (B_x q_y[t, z] + \epsilon_x^{(0,0,1,0)}[t, x, y, z]) == 0$
$6 \mathcal{M}_{\text{Pl}}^2 u_t[t, z] + (5 \chi - \xi) u_t^{(0,2)}[t, z] ==$ $6 ((2 \theta + \zeta) \epsilon_z^{(0,0,0,1)}[t, x, y, z] + 2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + (2 \theta + \zeta) \epsilon_x^{(0,1,0,0)}[t, x, y, z])$
$B_x \zeta (u_t[t, z] + h_+^{(1,0)}[t, z]) == 2 \zeta \epsilon_z^{(0,0,1,0)}[t, x, y, z]$
$B_x \zeta q_t[t, z] + B_x (4 \theta + \zeta) h_x^{(0,1)}[t, z] == 2 (2 \theta + \zeta) (b^{(0,1)}[t, z] - \epsilon_x^{(1,0,0,0)}[t, x, y, z])$
$6 B_x \zeta u_y[t, z] + (5 \chi - \xi) u_t^{(1,1)}[t, z] == 6 (2 \theta + \zeta) \epsilon_z^{(1,0,0,0)}[t, x, y, z]$
$\mathcal{M}_{\text{Pl}}^2 q_y[t, z] + 2 \zeta \epsilon_x^{(0,0,0,1)}[t, x, y, z] == 0$
$B_x \zeta u_t[t, z] + \chi u_y^{(1,1)}[t, z] + 2 \zeta \epsilon_y^{(0,0,0,1)}[t, x, y, z] == 0$
$6 \mathcal{M}_{\text{Pl}}^2 u_t[t, z] + (2 \chi - \xi) u_t^{(0,2)}[t, z] ==$ $12 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + 6 (2 \theta + \zeta) (\epsilon_y^{(0,0,1,0)}[t, x, y, z] + \epsilon_x^{(0,1,0,0)}[t, x, y, z])$
$B_x \zeta q_t[t, z] + B_x (4 \theta + \zeta) h_x^{(0,1)}[t, z] + 2 (2 \theta + \zeta) \epsilon_x^{(1,0,0,0)}[t, x, y, z] == 4 \theta b^{(0,1)}[t, z]$
$6 \mathcal{M}_{\text{Pl}}^2 u_y[t, z] + 3 B_x (4 \theta + \zeta) h_+^{(0,1)}[t, z] + (2 \chi - \xi) u_y^{(0,2)}[t, z] + (-5 \chi + \xi) u_y^{(2,0)}[t, z] ==$ $6 (2 \theta + \zeta) \epsilon_y^{(1,0,0,0)}[t, x, y, z]$

The Einstein components.

$B_x \zeta u_y^{(0,1)}[t, z] == 6 \mathcal{M}_{\text{Pl}}^2 u_t^{(1,0)}[t, z]$
$2 B_x (\epsilon_z[t, x, y, z] + (3 \theta + \zeta) u_t^{(0,1)}[t, z]) + 3 \mathcal{M}_{\text{Pl}}^2 u_y^{(1,0)}[t, z] == 0$
$2 B_x \epsilon_y[t, x, y, z] == 3 \mathcal{M}_{\text{Pl}}^2 u_t^{(0,1)}[t, z] + 2 B_x (3 \theta + \zeta) u_y^{(1,0)}[t, z]$
$B_x \zeta u_y^{(0,1)}[t, z] + \mathcal{M}_{\text{Pl}}^2 h_+^{(2,0)}[t, z] == \mathcal{M}_{\text{Pl}}^2 h_+^{(0,2)}[t, z]$
$2 B_x b[t, z] + \mathcal{M}_{\text{Pl}}^2 h_x^{(0,2)}[t, z] == \mathcal{M}_{\text{Pl}}^2 h_x^{(2,0)}[t, z]$
$B_x (12 \theta + 5 \zeta) u_y^{(0,1)}[t, z] + \mathcal{M}_{\text{Pl}}^2 (h_+^{(0,2)}[t, z] - h_+^{(2,0)}[t, z]) == 0$
$\mathcal{M}_{\text{Pl}} u_y^{(0,1)}[t, z] == 0$
$B_x (12 \theta + 5 \zeta) u_y^{(0,1)}[t, z] == 0$

(49)

The Maxwell components.

$B_x h_+^{(0,2)}[t, z] + \epsilon_z^{(1,0,1,0)}[t, x, y, z] = (3\theta + \zeta)(u_y^{(0,3)}[t, z] - u_y^{(2,1)}[t, z]) + \epsilon_y^{(1,0,0,1)}[t, x, y, z]$	(50)
$B_x h_x^{(0,2)}[t, z] + \epsilon_x^{(1,0,0,1)}[t, x, y, z] = b^{(0,2)}[t, z] + \epsilon_z^{(1,1,0,0)}[t, x, y, z]$	
$\epsilon_x^{(1,0,1,0)}[t, x, y, z] = \epsilon_y^{(1,1,0,0)}[t, x, y, z]$	

## Here is the reduced set of first-order equations.

After simplification, we have 36 equations.

$2 B_x \theta h_x^{(0,1)}[t, z] + B_x \zeta h_x^{(0,1)}[t, z] - 2 \theta b^{(0,1)}[t, z] - \zeta b^{(0,1)}[t, z] + 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$-\mathcal{M}_{\text{Pl}}^2 u_y[t, z] - 2 B_x \theta h_+^{(0,1)}[t, z] - B_x \zeta h_+^{(0,1)}[t, z] +$ $\frac{1}{3} \chi u_y^{(2,0)}[t, z] - \frac{1}{6} \xi u_y^{(2,0)}[t, z] + 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z]$
$\frac{1}{2} B_x \zeta u_y[t, z] + 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z]$
$-2 B_x \theta h_x^{(0,1)}[t, z] - B_x \zeta h_x^{(0,1)}[t, z] + 2 \theta b^{(0,1)}[t, z] + \zeta b^{(0,1)}[t, z] - 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$\frac{1}{2} B_x \zeta h_x^{(1,0)}[t, z] - \zeta b^{(1,0)}[t, z]$
$\mathcal{M}_{\text{Pl}}^2 u_y[t, z] + 2 B_x \theta h_+^{(0,1)}[t, z] + B_x \zeta h_+^{(0,1)}[t, z] -$ $\frac{1}{3} \chi u_y^{(2,0)}[t, z] + \frac{1}{6} \xi u_y^{(2,0)}[t, z] - 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z]$
$-B_x \zeta u_t[t, z] - \frac{1}{2} B_x \zeta h_+^{(1,0)}[t, z]$
$-\frac{1}{2} B_x \zeta u_y[t, z] - 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z]$
$-\frac{1}{2} B_x \zeta h_x^{(1,0)}[t, z] + \zeta b^{(1,0)}[t, z]$
$B_x \zeta u_t[t, z] + \frac{1}{2} B_x \zeta h_+^{(1,0)}[t, z]$
$-\mathcal{M}_{\text{Pl}}^2 u_t[t, z] - \frac{5}{6} \chi u_t^{(0,2)}[t, z] + \frac{1}{6} \xi u_t^{(0,2)}[t, z] +$ $2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + \zeta \epsilon_z^{(0,0,0,1)}[t, x, y, z] +$ $2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + \zeta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$
$-\frac{1}{2} B_x \zeta h_x^{(1,0)}[t, z] - \zeta \epsilon_z^{(0,1,0,0)}[t, x, y, z]$
$\mathcal{M}_{\text{Pl}}^2 u_t[t, z] + \frac{5}{6} \chi u_t^{(0,2)}[t, z] - \frac{1}{6} \xi u_t^{(0,2)}[t, z] - 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] - \zeta \epsilon_z^{(0,0,0,1)}[t, x, y, z] -$ $2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] - \zeta \epsilon_y^{(0,0,1,0)}[t, x, y, z] - 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$
$-\mathcal{M}_{\text{Pl}}^2 u_y[t, z] - 2 B_x \theta h_+^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_+^{(0,1)}[t, z] + \frac{5}{6} \chi u_y^{(2,0)}[t, z] -$ $\frac{1}{6} \xi u_y^{(2,0)}[t, z] + 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_y^{(1,0,0,0)}[t, x, y, z]$
$\frac{1}{2} B_x \zeta u_y[t, z] + 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_z^{(1,0,0,0)}[t, x, y, z]$
$\mathcal{M}_{\text{Pl}}^2 u_y[t, z] + 2 B_x \theta h_+^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_+^{(0,1)}[t, z] - \frac{5}{6} \chi u_y^{(2,0)}[t, z] +$ $\frac{1}{6} \xi u_y^{(2,0)}[t, z] - 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_y^{(1,0,0,0)}[t, x, y, z]$

$\frac{1}{2} B_x \zeta h_x^{(1,0)}[t, z] + \zeta \epsilon_z^{(0,1,0,0)}[t, x, y, z]$
$-\frac{1}{2} B_x \zeta u_y[t, z] - 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_z^{(1,0,0,0)}[t, x, y, z]$
$-\mathcal{M}_{\text{Pl}}^2 u_t[t, z] - \frac{5}{6} \chi u_t^{(0,2)}[t, z] + \frac{1}{6} \xi u_t^{(0,2)}[t, z] +$ $2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + \zeta \epsilon_z^{(0,0,0,1)}[t, x, y, z] +$ $2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z] + \zeta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$
$\frac{1}{2} B_x \zeta u_t[t, z] + \frac{1}{2} B_x \zeta h_+^{(1,0)}[t, z] - \zeta \epsilon_z^{(0,0,1,0)}[t, x, y, z]$
$-2 B_x \theta h_x^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_x^{(0,1)}[t, z] + 2 \theta b^{(0,1)}[t, z] +$ $\zeta b^{(0,1)}[t, z] - 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$\mathcal{M}_{\text{Pl}}^2 u_t[t, z] + \frac{5}{6} \chi u_t^{(0,2)}[t, z] - \frac{1}{6} \xi u_t^{(0,2)}[t, z] - 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] - \zeta \epsilon_z^{(0,0,0,1)}[t, x, y, z] -$ $2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] - 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z] - \zeta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$
$2 B_x \theta h_x^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_x^{(0,1)}[t, z] - 2 \theta b^{(0,1)}[t, z] -$ $\zeta b^{(0,1)}[t, z] + 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$-B_x \zeta u_y[t, z] + 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_z^{(1,0,0,0)}[t, x, y, z]$
$-\frac{1}{2} B_x \zeta u_t[t, z] - \frac{1}{2} B_x \zeta h_+^{(1,0)}[t, z] + \zeta \epsilon_z^{(0,0,1,0)}[t, x, y, z]$
$B_x \zeta u_y[t, z] - 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_z^{(1,0,0,0)}[t, x, y, z]$
$-\frac{1}{2} B_x \zeta u_t[t, z] - \zeta \epsilon_y^{(0,0,0,1)}[t, x, y, z]$
$-\mathcal{M}_{\text{Pl}}^2 u_t[t, z] - \frac{1}{3} \chi u_t^{(0,2)}[t, z] + \frac{1}{6} \xi u_t^{(0,2)}[t, z] +$ $2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + 2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] +$ $\zeta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z] + \zeta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$
$-2 B_x \theta h_x^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_x^{(0,1)}[t, z] +$ $2 \theta b^{(0,1)}[t, z] - 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$\frac{1}{2} B_x \zeta u_t[t, z] + \zeta \epsilon_y^{(0,0,0,1)}[t, x, y, z]$
$\mathcal{M}_{\text{Pl}}^2 u_t[t, z] + \frac{1}{3} \chi u_t^{(0,2)}[t, z] - \frac{1}{6} \xi u_t^{(0,2)}[t, z] -$ $2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] - 2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] -$ $\zeta \epsilon_y^{(0,0,1,0)}[t, x, y, z] - 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z] - \zeta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$
$2 B_x \theta h_x^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_x^{(0,1)}[t, z] -$ $2 \theta b^{(0,1)}[t, z] + 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$-B_x \epsilon_z[t, x, y, z] - 3 B_x \theta u_t^{(0,1)}[t, z] - B_x \zeta u_t^{(0,1)}[t, z] - \frac{3}{2} \mathcal{M}_{\text{Pl}}^2 u_y^{(1,0)}[t, z]$
$B_x \epsilon_y[t, x, y, z] - \frac{3}{2} \mathcal{M}_{\text{Pl}}^2 u_t^{(0,1)}[t, z] - 3 B_x \theta u_y^{(1,0)}[t, z] - B_x \zeta u_y^{(1,0)}[t, z]$
$B_x b[t, z] + \frac{1}{2} \mathcal{M}_{\text{Pl}}^2 h_x^{(0,2)}[t, z] - \frac{1}{2} \mathcal{M}_{\text{Pl}}^2 h_x^{(2,0)}[t, z]$
$-2 B_x h_x^{(0,2)}[t, z] + 2 b^{(0,2)}[t, z] - 2 b^{(2,0)}[t, z]$

# Analysis of Case 2

**Key observation:** Analysis of Case 2 as defined by Yun-Cherng.

Here is the list of rules.

$$\boxed{\alpha_0 \rightarrow 0 \mid \alpha_1 \rightarrow 0 \mid \alpha_5 \rightarrow -4 \alpha_4 \mid \alpha_6 \rightarrow -\alpha_2 - \alpha_3 + \alpha_4 \mid \beta_2 \rightarrow -2 \beta_1} \quad (52)$$

Here is the non-linear Lagrangian.

$$\begin{aligned} & \frac{1}{36} \sqrt{-\tilde{g}} \\ & \left( -9 F_{ab} \left( F^{ab} + 4 \zeta R[D]^{ab} \right) + 2 \left( 18 \alpha_2 R[D]_{ab} R[D]^{ab} + 18 \alpha_4 R[D]_{abcd} R[D]^{abcd} - 72 \alpha_4 R[D]_{acbd} R[D]^{abcd} + \right. \right. \\ & \quad 18 (-\alpha_2 - \alpha_3 + \alpha_4) R[D]^{abcd} R[D]_{cdab} + 18 \beta_1 T[D]_{abc} T[D]^{abc} - 36 \beta_1 T[D]^{abc} T[D]_{bac} - 18 \beta_3 \\ & \quad T[D]_{a \ b}^a \ T[D]_{bc}^c + 36 \theta F^{ab} \left( \nabla_b T[D]_{ac}^c \right) + 6 R[D]^{ab} \left( 3 \alpha_3 R[D]_{ba} + \chi \left( \nabla_a T[D]_{bc}^c \right) - \chi \left( \nabla_b T[D]_{ac}^c \right) \right) + \\ & \quad \left. \left. \xi g^{ab} \left( \nabla_a T[D]_{c \ d}^c \right) \left( \nabla_b T[D]_{de}^e \right) + \xi \left( \nabla_b T[D]_{da}^a \right) \left( \nabla^d T[D]_{c \ b}^b \right) \right) \right) \end{aligned} \quad (53)$$

## Here are the zeroth-order equations.

The Cartan components.

$$\boxed{B_x Q_0 \zeta == 0} \quad (54)$$

$$\boxed{\beta_1 Q_0 == 0}$$

The Einstein components.

$$\boxed{\beta_1 Q_0 == 0} \quad (55)$$

$$\boxed{B_x == 0}$$

The Maxwell components.

$$\quad (56)$$

## Here are the first-order equations.

The Cartan components.

$$\boxed{2 B_x \zeta q_t[t, z] + 2 (5 \alpha_2 + 3 \alpha_3) q_y^{(1,1)}[t, z] == 2 (2 \theta + \zeta) (B_x h_x^{(0,1)}[t, z] - b^{(0,1)}[t, z]) + (10 \alpha_2 + 6 \alpha_3 + \chi) Q_0 u_y^{(0,1)}[t, z] + 4 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z]}$$

$3(-\beta_3 + (\alpha_2 + \alpha_3) Q_0^2) u_y[t, z] +$ $\frac{1}{6} ((-12 \alpha_2 - 5 \chi + \xi) u_y^{(0,2)}[t, z] - 6(5 \alpha_2 + 3 \alpha_3) Q_0 q_y^{(1,0)}[t, z] + (2 \chi - \xi) u_y^{(2,0)}[t, z]) +$ $2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] == Q_0 \zeta b[t, z] + B_x (2 \theta + \zeta) h_+^{(0,1)}[t, z]$
$3 B_x \zeta u_y[t, z] + (-2 \chi + \xi) u_t^{(1,1)}[t, z] + 12 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] == 6(\alpha_2 + 3 \alpha_3) Q_0 q_t^{(0,1)}[t, z]$
$B_x \zeta q_y[t, z] + 2 Q_0 \zeta \epsilon_z[t, x, y, z] + 2(5 \alpha_2 + 3 \alpha_3 - 6 \alpha_4) q_t^{(1,1)}[t, z] ==$ $(10 \alpha_2 + 6 \alpha_3 + 12 \alpha_4 + \chi) Q_0 u_t^{(0,1)}[t, z]$
$24 \beta_1 q_y[t, z] + 2 Q_0 \zeta \epsilon_y[t, x, y, z] + B_x \zeta h_x^{(1,0)}[t, z] + (10 \alpha_2 + 6 \alpha_3 + \chi) Q_0 u_y^{(1,0)}[t, z] ==$ $2 \zeta b^{(1,0)}[t, z] + 2(5 \alpha_2 + 3 \alpha_3) q_y^{(2,0)}[t, z]$
$\xi u_y^{(0,2)}[t, z] + 2 \chi u_y^{(2,0)}[t, z] + 12 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] ==$ $6 Q_0 \zeta b[t, z] + 18(\beta_3 - (\alpha_2 + \alpha_3) Q_0^2) u_y[t, z] + 6 B_x (2 \theta + \zeta) h_+^{(0,1)}[t, z] +$ $(12 \alpha_2 + 5 \chi) u_y^{(0,2)}[t, z] + 6(5 \alpha_2 + 3 \alpha_3) Q_0 q_y^{(1,0)}[t, z] + \xi u_y^{(2,0)}[t, z]$
$2 B_x \zeta u_t[t, z] + 2 Q_0 \zeta \epsilon_x[t, x, y, z] +$ $4 \alpha_2 Q_0 q_y^{(0,1)}[t, z] + B_x \zeta h_+^{(1,0)}[t, z] + (4 \alpha_2 + \chi) u_y^{(1,1)}[t, z] == 0$
$18(\beta_3 - 4 \alpha_4 Q_0^2) u_t[t, z] + (12 \alpha_2 + 5 \chi - \xi) u_t^{(0,2)}[t, z] ==$ $9(\alpha_2 + \alpha_3) Q_0^2 h_+^{(1,0)}[t, z] + 6(-6(\alpha_2 + \alpha_3 - 2 \alpha_4) Q_0 q_t^{(1,0)}[t, z] +$ $3(\alpha_2 + \alpha_3)(2 Q_0 h_x^{(1,1)}[t, z] - h_+^{(1,2)}[t, z] + h_+^{(3,0)}[t, z]) +$ $(2 \theta + \zeta)(\epsilon_z^{(0,0,0,1)}[t, x, y, z] + \epsilon_y^{(0,0,1,0)}[t, x, y, z]) + 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$
$\frac{1}{2} (Q_0 \zeta \epsilon_z[t, x, y, z] +$ $3(4 \alpha_4 Q_0 u_t^{(0,1)}[t, z] + 4 \alpha_4 q_t^{(1,1)}[t, z] + (\alpha_2 + \alpha_3)(Q_0^2 h_x^{(1,0)}[t, z] + 2 h_x^{(3,0)}[t, z])) ==$ $\frac{1}{2} B_x \zeta q_y[t, z] + \frac{1}{4} (16 \alpha_2 + 12 \alpha_3 + \chi) Q_0 u_t^{(0,1)}[t, z] +$ $3(\alpha_2 + \alpha_3)(2 Q_0 h_+^{(1,1)}[t, z] + h_x^{(1,2)}[t, z]) + \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z]$
$48 \beta_1 q_y[t, z] ==$ $2 Q_0 \zeta \epsilon_y[t, x, y, z] + 2 B_x \zeta h_x^{(1,0)}[t, z] + (4 \alpha_2 + \chi) Q_0 u_y^{(1,0)}[t, z] + 4 \zeta \epsilon_z^{(0,1,0,0)}[t, x, y, z]$
$3 Q_0 \zeta b[t, z] + 9(-2 \beta_3 + (\alpha_2 + \alpha_3) Q_0^2) u_y[t, z] + \xi u_y^{(0,2)}[t, z] +$ $(12 \alpha_2 + 5 \chi) u_y^{(2,0)}[t, z] + 6(2 \theta + \zeta) \epsilon_y^{(1,0,0,0)}[t, x, y, z] == 3 B_x (4 \theta + \zeta) h_+^{(0,1)}[t, z] +$ $(12 \alpha_2 + 5 \chi) u_y^{(0,2)}[t, z] + 3(\alpha_2 + 3 \alpha_3) Q_0 q_y^{(1,0)}[t, z] + \xi u_y^{(2,0)}[t, z]$
$3 B_x \zeta u_y[t, z] + \xi u_t^{(1,1)}[t, z] + 9(\alpha_2 + \alpha_3)(4 Q_0^2 h_+^{(0,1)}[t, z] + 5 Q_0 h_x^{(0,2)}[t, z] + 2 h_+^{(2,1)}[t, z]) +$ $6(2 \theta + \zeta) \epsilon_z^{(1,0,0,0)}[t, x, y, z] == 3(5 \alpha_2 + 3 \alpha_3) Q_0 q_t^{(0,1)}[t, z] +$ $18(\alpha_2 + \alpha_3) h_+^{(0,3)}[t, z] + (12 \alpha_2 + 5 \chi) u_t^{(1,1)}[t, z] + 9(\alpha_2 + \alpha_3) Q_0 h_x^{(2,0)}[t, z]$
$\frac{1}{2} B_x \zeta q_y[t, z] + \frac{1}{4} (16 \alpha_2 + 12 \alpha_3 + \chi) Q_0 u_t^{(0,1)}[t, z] +$ $3(\alpha_2 + \alpha_3)(2 Q_0 h_+^{(1,1)}[t, z] + h_x^{(1,2)}[t, z]) + \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z] == \frac{1}{2} (Q_0 \zeta \epsilon_z[t, x, y, z] +$ $3(4 \alpha_4 Q_0 u_t^{(0,1)}[t, z] + 4 \alpha_4 q_t^{(1,1)}[t, z] + (\alpha_2 + \alpha_3)(Q_0^2 h_x^{(1,0)}[t, z] + 2 h_x^{(3,0)}[t, z]))$
$8 \beta_1 q_t[t, z] + (\alpha_2 + \alpha_3)(5 Q_0 h_+^{(0,2)}[t, z] + 2(h_x^{(0,3)}[t, z] + Q_0 u_t^{(1,0)}[t, z])) ==$ $4(\alpha_2 + \alpha_3) Q_0^2 h_x^{(0,1)}[t, z] + 4 \alpha_4 Q_0 u_t^{(1,0)}[t, z] +$ $4 \alpha_4 q_t^{(2,0)}[t, z] + (\alpha_2 + \alpha_3)(Q_0 h_+^{(2,0)}[t, z] + 2 h_x^{(2,1)}[t, z])$



$\frac{1}{4} \left( (16 \alpha_2 + 12 \alpha_3 + \chi) Q_0 u_t^{(0,1)}[t, z] + 6 (\alpha_2 + \alpha_3) (Q_0^2 h_x^{(1,0)}[t, z] + 2 h_x^{(3,0)}[t, z]) \right) =$ $B_x \zeta q_y[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_z[t, x, y, z] +$ $6 (\alpha_4 Q_0 u_t^{(0,1)}[t, z] + (\alpha_2 + \alpha_3) Q_0 h_+^{(1,1)}[t, z] + \alpha_4 q_t^{(1,1)}[t, z]) +$ $3 (\alpha_2 + \alpha_3) h_x^{(1,2)}[t, z] + \zeta \epsilon_x^{(0,0,1,0)}[t, x, y, z]$
$18 (\beta_3 - 4 \alpha_4 Q_0^2) u_t[t, z] + (12 \alpha_2 + 5 \chi - \xi) u_t^{(0,2)}[t, z] +$ $9 ((\alpha_2 + \alpha_3) Q_0^2 h_+^{(1,0)}[t, z] + 4 (\alpha_2 + \alpha_3 - 2 \alpha_4) Q_0 q_t^{(1,0)}[t, z] +$ $2 (\alpha_2 + \alpha_3) (2 Q_0 h_x^{(1,1)}[t, z] - h_+^{(1,2)}[t, z] + h_+^{(3,0)}[t, z])) =$ $6 ((2 \theta + \zeta) \epsilon_z^{(0,0,0,1)}[t, x, y, z] + 2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + (2 \theta + \zeta) \epsilon_x^{(0,1,0,0)}[t, x, y, z])$
$\zeta (Q_0 \epsilon_x[t, x, y, z] + B_x (u_t[t, z] + h_+^{(1,0)}[t, z])) =$ $(\alpha_2 + 3 \alpha_3) Q_0 q_y^{(0,1)}[t, z] + 2 \zeta \epsilon_z^{(0,0,1,0)}[t, x, y, z]$
$B_x \zeta q_y[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_z[t, x, y, z] + \frac{1}{4} Q_0$ $(-((16 \alpha_2 + 12 \alpha_3 - 24 \alpha_4 + \chi) u_t^{(0,1)}[t, z]) - 6 (\alpha_2 + \alpha_3) (Q_0 h_x^{(1,0)}[t, z] - 4 h_+^{(1,1)}[t, z])) +$ $6 \alpha_4 q_t^{(1,1)}[t, z] + 3 (\alpha_2 + \alpha_3) h_x^{(1,2)}[t, z] + \zeta \epsilon_x^{(0,0,1,0)}[t, x, y, z] = 3 (\alpha_2 + \alpha_3) h_x^{(3,0)}[t, z]$
$2 B_x (\zeta q_t[t, z] + (4 \theta + \zeta) h_x^{(0,1)}[t, z]) =$ $(16 \alpha_2 + 12 \alpha_3 + \chi) Q_0 u_y^{(0,1)}[t, z] + 4 (2 \theta + \zeta) (b^{(0,1)}[t, z] - \epsilon_x^{(1,0,0,0)}[t, x, y, z])$
$8 \beta_1 q_t[t, z] + (\alpha_2 + \alpha_3) (Q_0 (4 Q_0 h_x^{(0,1)}[t, z] + 2 u_t^{(1,0)}[t, z] + h_+^{(2,0)}[t, z]) + 2 h_x^{(2,1)}[t, z]) =$ $5 (\alpha_2 + \alpha_3) Q_0 h_+^{(0,2)}[t, z] + 2 (\alpha_2 + \alpha_3) h_x^{(0,3)}[t, z] + 4 \alpha_4 (Q_0 u_t^{(1,0)}[t, z] + q_t^{(2,0)}[t, z])$
$(16 \alpha_2 + 12 \alpha_3 + \chi) Q_0 u_y^{(0,1)}[t, z] + 4 (2 \theta + \zeta) (b^{(0,1)}[t, z] - \epsilon_x^{(1,0,0,0)}[t, x, y, z]) =$ $2 B_x (\zeta q_t[t, z] + (4 \theta + \zeta) h_x^{(0,1)}[t, z])$
$6 B_x \zeta u_y[t, z] + 36 (\alpha_2 + \alpha_3) Q_0^2 h_+^{(0,1)}[t, z] +$ $3 (5 \alpha_2 + 3 \alpha_3) Q_0 q_t^{(0,1)}[t, z] + 45 \alpha_2 Q_0 h_x^{(0,2)}[t, z] + 45 \alpha_3 Q_0 h_x^{(0,2)}[t, z] +$ $12 \alpha_2 u_t^{(1,1)}[t, z] + 5 \chi u_t^{(1,1)}[t, z] + 18 (\alpha_2 + \alpha_3) h_+^{(2,1)}[t, z] =$ $\xi u_t^{(1,1)}[t, z] + 9 (\alpha_2 + \alpha_3) (2 h_+^{(0,3)}[t, z] + Q_0 h_x^{(2,0)}[t, z]) + 6 (2 \theta + \zeta) \epsilon_z^{(1,0,0,0)}[t, x, y, z]$
$48 \beta_1 q_y[t, z] + 4 (5 \alpha_2 + 3 \alpha_3) q_y^{(0,2)}[t, z] + 4 \zeta \epsilon_x^{(0,0,0,1)}[t, x, y, z] =$ $2 Q_0 \zeta \epsilon_y[t, x, y, z] + (4 \alpha_2 + \chi) Q_0 u_y^{(1,0)}[t, z]$
$B_x \zeta u_t[t, z] + Q_0 \zeta \epsilon_x[t, x, y, z] + (5 \alpha_2 + 3 \alpha_3) Q_0 q_y^{(0,1)}[t, z] +$ $(4 \alpha_2 + \chi) u_y^{(1,1)}[t, z] + 2 \zeta \epsilon_y^{(0,0,0,1)}[t, x, y, z] = 0$
$18 (\beta_3 - 4 \alpha_4 Q_0^2) u_t[t, z] + (2 \chi - \xi) u_t^{(0,2)}[t, z] + 36 (\alpha_2 + \alpha_3 - 2 \alpha_4) Q_0 q_t^{(1,0)}[t, z] =$ $12 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + 6 (2 \theta + \zeta) (\epsilon_y^{(0,0,1,0)}[t, x, y, z] + \epsilon_x^{(0,1,0,0)}[t, x, y, z])$
$12 \beta_1 q_t[t, z] + (5 \alpha_2 + 3 \alpha_3) q_t^{(0,2)}[t, z] + 3 (\alpha_2 + \alpha_3 - 2 \alpha_4) Q_0 u_t^{(1,0)}[t, z] = 6 \alpha_4 q_t^{(2,0)}[t, z]$
$2 B_x \zeta q_t[t, z] + 2 B_x (4 \theta + \zeta) h_x^{(0,1)}[t, z] + 4 (5 \alpha_2 + 3 \alpha_3) q_y^{(1,1)}[t, z] +$ $4 (2 \theta + \zeta) \epsilon_x^{(1,0,0,0)}[t, x, y, z] = 8 \theta b^{(0,1)}[t, z] + (4 \alpha_2 + \chi) Q_0 u_y^{(0,1)}[t, z]$
$3 Q_0 \zeta b[t, z] + 9 (-2 \beta_3 + (\alpha_2 + \alpha_3) Q_0^2) u_y[t, z] + (-2 \chi + \xi) u_y^{(0,2)}[t, z] +$ $(12 \alpha_2 + 5 \chi - \xi) u_y^{(2,0)}[t, z] + 6 (2 \theta + \zeta) \epsilon_y^{(1,0,0,0)}[t, x, y, z] =$ $3 B_x (4 \theta + \zeta) h_+^{(0,1)}[t, z] + 3 (\alpha_2 + 3 \alpha_3) Q_0 q_y^{(1,0)}[t, z]$

The Einstein components.

$18 (\alpha_2 + 3 \alpha_3) Q_0^3 q_t[t, z] + B_x \zeta u_y^{(0,1)}[t, z] +$ $(\alpha_2 + \alpha_3) (9 Q_0^2 u_t^{(1,0)}[t, z] + 2 u_t^{(1,2)}[t, z] + 6 u_t^{(3,0)}[t, z]) = 0$
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$Q_0 (2 B_x \zeta u_t[t, z] + 2 Q_0 \zeta \epsilon_x[t, x, y, z] + 4 \alpha_2 Q_0 q_y^{(0,1)}[t, z] + (4 \alpha_2 + \chi) u_y^{(1,1)}[t, z] + \zeta (B_x h_+^{(1,0)}[t, z] + 2 \epsilon_y^{(0,0,0,1)}[t, x, y, z] - 2 \epsilon_z^{(0,0,1,0)}[t, x, y, z])) = 0$
$Q_0^2 \zeta \epsilon_y[t, x, y, z] + \frac{1}{2} B_x Q_0 \zeta h_x^{(1,0)}[t, z] + 6 \alpha_2 Q_0^2 u_y^{(1,0)}[t, z] + 4 \alpha_3 Q_0^2 u_y^{(1,0)}[t, z] + \frac{1}{2} \chi Q_0^2 u_y^{(1,0)}[t, z] + \alpha_2 Q_0 q_y^{(2,0)}[t, z] + \alpha_3 Q_0 q_y^{(2,0)}[t, z] + \alpha_2 u_y^{(3,0)}[t, z] + \alpha_3 u_y^{(3,0)}[t, z] + Q_0 \zeta \epsilon_z^{(0,1,0,0)}[t, x, y, z] = 12 \beta_1 Q_0 q_y[t, z] + B_x \epsilon_z[t, x, y, z] + B_x (3 \theta + \zeta) u_t^{(0,1)}[t, z] + 2 \alpha_2 Q_0 q_y^{(0,2)}[t, z] + (\alpha_2 + \alpha_3) u_y^{(1,2)}[t, z] + Q_0 \zeta \epsilon_x^{(0,0,0,1)}[t, x, y, z]$
$B_x \epsilon_y[t, x, y, z] + Q_0^2 \zeta \epsilon_z[t, x, y, z] + (\alpha_2 + \alpha_3) (4 Q_0 q_t^{(1,1)}[t, z] + 5 u_t^{(2,1)}[t, z]) + Q_0 \zeta \epsilon_x^{(0,0,1,0)}[t, x, y, z] = B_x Q_0 \zeta q_y[t, z] + \frac{1}{2} (4 \alpha_2 + \chi) Q_0^2 u_t^{(0,1)}[t, z] + (\alpha_2 + \alpha_3) u_t^{(0,3)}[t, z] + B_x (3 \theta + \zeta) u_y^{(1,0)}[t, z] + 6 \alpha_4 Q_0 q_t^{(1,1)}[t, z] + Q_0 \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z]$
$\frac{3}{2} (\alpha_2 + \alpha_3) (Q_0 (12 Q_0 h_+^{(0,2)}[t, z] + 8 h_x^{(0,3)}[t, z] + Q_0 u_t^{(1,0)}[t, z] + 4 q_t^{(2,0)}[t, z]) + 2 (2 h_+^{(2,2)}[t, z] + u_t^{(3,0)}[t, z])) = 3 (8 \beta_1 Q_0 + 3 (\alpha_2 + 3 \alpha_3) Q_0^3) q_t[t, z] + 12 (\alpha_2 + \alpha_3) Q_0^3 h_x^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta u_y^{(0,1)}[t, z] + 4 \alpha_2 Q_0 q_t^{(0,2)}[t, z] + 3 \alpha_2 h_+^{(0,4)}[t, z] + 3 \alpha_3 h_+^{(0,4)}[t, z] + 3 \alpha_2 u_t^{(1,2)}[t, z] + 3 \alpha_3 u_t^{(1,2)}[t, z] + 3 \alpha_2 Q_0^2 h_+^{(2,0)}[t, z] + 3 \alpha_3 Q_0^2 h_+^{(2,0)}[t, z] + 12 \alpha_4 Q_0 q_t^{(2,0)}[t, z] + 12 \alpha_2 Q_0 h_x^{(2,1)}[t, z] + 12 \alpha_3 Q_0 h_x^{(2,1)}[t, z] + 3 (\alpha_2 + \alpha_3) h_+^{(4,0)}[t, z]$
$2 B_x b[t, z] + 3 B_x Q_0 \zeta u_y[t, z] = 6 (\alpha_2 + \alpha_3) (h_x^{(0,4)}[t, z] + Q_0 (Q_0 (-4 Q_0 h_+^{(0,1)}[t, z] - 6 h_x^{(0,2)}[t, z] + h_x^{(2,0)}[t, z]) + 4 (h_+^{(0,3)}[t, z] - h_+^{(2,1)}[t, z])) - 2 h_x^{(2,2)}[t, z] + h_x^{(4,0)}[t, z])$
$(4 \alpha_2 + \chi) Q_0 u_y^{(0,2)}[t, z] = 0$
$\frac{3}{2} (\alpha_2 + \alpha_3) (Q_0 (Q_0 (8 Q_0 h_x^{(0,1)}[t, z] + u_t^{(1,0)}[t, z] + 2 h_+^{(2,0)}[t, z]) + 4 (q_t^{(2,0)}[t, z] + 2 h_x^{(2,1)}[t, z])) + 2 (h_+^{(0,4)}[t, z] + u_t^{(3,0)}[t, z] + h_+^{(4,0)}[t, z])) = 3 (8 \beta_1 Q_0 + 3 (\alpha_2 + 3 \alpha_3) Q_0^3) q_t[t, z] + \frac{1}{2} B_x (12 \theta + 5 \zeta) u_y^{(0,1)}[t, z] + 18 (\alpha_2 + \alpha_3) Q_0^2 h_+^{(0,2)}[t, z] + 4 \alpha_2 Q_0 q_t^{(0,2)}[t, z] + 12 \alpha_2 Q_0 h_x^{(0,3)}[t, z] + 12 \alpha_3 Q_0 h_x^{(0,3)}[t, z] + 3 \alpha_2 u_t^{(1,2)}[t, z] + 3 \alpha_3 u_t^{(1,2)}[t, z] + 12 \alpha_4 Q_0 q_t^{(2,0)}[t, z] + 6 (\alpha_2 + \alpha_3) h_+^{(2,2)}[t, z]$
$(\alpha_2 + \alpha_3) Q_0^2 u_y^{(0,1)}[t, z] + \alpha_3 Q_0 q_y^{(1,1)}[t, z] + (\alpha_2 + \alpha_3) u_y^{(2,1)}[t, z] = Q_0 \zeta b^{(0,1)}[t, z] + (\alpha_2 + \alpha_3) u_y^{(0,3)}[t, z] + \alpha_2 Q_0 q_y^{(1,1)}[t, z]$
$B_x \left( -6 \theta - \frac{5 \zeta}{2} \right) u_y^{(0,1)}[t, z] + \frac{1}{2} (\alpha_2 + \alpha_3) (4 Q_0 q_t^{(0,2)}[t, z] + 3 Q_0^2 u_t^{(1,0)}[t, z] + 2 u_t^{(1,2)}[t, z] + 12 Q_0 q_t^{(2,0)}[t, z] + 6 u_t^{(3,0)}[t, z]) = 3 (8 \beta_1 Q_0 + 3 (\alpha_2 + 3 \alpha_3) Q_0^3) q_t[t, z] + 12 \alpha_4 Q_0 q_t^{(2,0)}[t, z]$

The Maxwell components.

$B_x h_+^{(0,2)}[t, z] + \epsilon_z^{(1,0,1,0)}[t, x, y, z] = (3\theta + \zeta)(u_y^{(0,3)}[t, z] - u_y^{(2,1)}[t, z]) + \epsilon_y^{(1,0,0,1)}[t, x, y, z]$	(59)
$B_x h_x^{(0,2)}[t, z] + \epsilon_x^{(1,0,0,1)}[t, x, y, z] = b^{(0,2)}[t, z] + \epsilon_z^{(1,1,0,0)}[t, x, y, z]$	
$\epsilon_x^{(1,0,1,0)}[t, x, y, z] = \epsilon_y^{(1,1,0,0)}[t, x, y, z]$	

## Here is the reduced set of first-order equations.

After simplification, we have 60 equations.

$-B_x \zeta q_t[t, z] + 2B_x \theta h_x^{(0,1)}[t, z] + B_x \zeta h_x^{(0,1)}[t, z] - 2\theta b^{(0,1)}[t, z] - \zeta b^{(0,1)}[t, z] + 5\alpha_2 Q_0 u_y^{(0,1)}[t, z] + 3\alpha_3 Q_0 u_y^{(0,1)}[t, z] + \frac{1}{2} \chi Q_0 u_y^{(0,1)}[t, z] - 5\alpha_2 q_y^{(1,1)}[t, z] - 3\alpha_3 q_y^{(1,1)}[t, z] + 2\theta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$-Q_0 \zeta b[t, z] - 3\beta_3 u_y[t, z] + 3\alpha_2 Q_0^2 u_y[t, z] + 3\alpha_3 Q_0^2 u_y[t, z] - 2B_x \theta h_+^{(0,1)}[t, z] - B_x \zeta h_+^{(0,1)}[t, z] - 2\alpha_2 u_y^{(0,2)}[t, z] - \frac{5}{6} \chi u_y^{(0,2)}[t, z] + \frac{1}{6} \xi u_y^{(0,2)}[t, z] - 5\alpha_2 Q_0 q_y^{(1,0)}[t, z] - 3\alpha_3 Q_0 q_y^{(1,0)}[t, z] + \frac{1}{3} \chi u_y^{(2,0)}[t, z] - \frac{1}{6} \xi u_y^{(2,0)}[t, z] + 2\theta \epsilon_y^{(1,0,0,0)}[t, x, y, z]$
$\frac{1}{2} B_x \zeta u_y[t, z] - \alpha_2 Q_0 q_t^{(0,1)}[t, z] - 3\alpha_3 Q_0 q_t^{(0,1)}[t, z] - \frac{1}{3} \chi u_t^{(1,1)}[t, z] + \frac{1}{6} \xi u_t^{(1,1)}[t, z] + 2\theta \epsilon_z^{(1,0,0,0)}[t, x, y, z]$
$B_x \zeta q_t[t, z] - 2B_x \theta h_x^{(0,1)}[t, z] - B_x \zeta h_x^{(0,1)}[t, z] + 2\theta b^{(0,1)}[t, z] + \zeta b^{(0,1)}[t, z] - 5\alpha_2 Q_0 u_y^{(0,1)}[t, z] - 3\alpha_3 Q_0 u_y^{(0,1)}[t, z] - \frac{1}{2} \chi Q_0 u_y^{(0,1)}[t, z] + 5\alpha_2 q_y^{(1,1)}[t, z] + 3\alpha_3 q_y^{(1,1)}[t, z] - 2\theta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$-\frac{1}{2} B_x \zeta q_y[t, z] - Q_0 \zeta \epsilon_z[t, x, y, z] + 5\alpha_2 Q_0 u_t^{(0,1)}[t, z] + 3\alpha_3 Q_0 u_t^{(0,1)}[t, z] + 6\alpha_4 Q_0 u_t^{(0,1)}[t, z] + \frac{1}{2} \chi Q_0 u_t^{(0,1)}[t, z] - 5\alpha_2 q_t^{(1,1)}[t, z] - 3\alpha_3 q_t^{(1,1)}[t, z] + 6\alpha_4 q_t^{(1,1)}[t, z]$
$12\beta_1 q_y[t, z] + Q_0 \zeta \epsilon_y[t, x, y, z] + \frac{1}{2} B_x \zeta h_x^{(1,0)}[t, z] - \zeta b^{(1,0)}[t, z] + 5\alpha_2 Q_0 u_y^{(1,0)}[t, z] + 3\alpha_3 Q_0 u_y^{(1,0)}[t, z] + \frac{1}{2} \chi Q_0 u_y^{(1,0)}[t, z] - 5\alpha_2 q_y^{(2,0)}[t, z] - 3\alpha_3 q_y^{(2,0)}[t, z]$
$Q_0 \zeta b[t, z] + 3\beta_3 u_y[t, z] - 3\alpha_2 Q_0^2 u_y[t, z] - 3\alpha_3 Q_0^2 u_y[t, z] + 2B_x \theta h_+^{(0,1)}[t, z] + B_x \zeta h_+^{(0,1)}[t, z] + 2\alpha_2 u_y^{(0,2)}[t, z] + \frac{5}{6} \chi u_y^{(0,2)}[t, z] - \frac{1}{6} \xi u_y^{(0,2)}[t, z] + 5\alpha_2 Q_0 q_y^{(1,0)}[t, z] + 3\alpha_3 Q_0 q_y^{(1,0)}[t, z] - \frac{1}{3} \chi u_y^{(2,0)}[t, z] + \frac{1}{6} \xi u_y^{(2,0)}[t, z] - 2\theta \epsilon_y^{(1,0,0,0)}[t, x, y, z]$
$\frac{1}{2} B_x \zeta q_y[t, z] + Q_0 \zeta \epsilon_z[t, x, y, z] - 5\alpha_2 Q_0 u_t^{(0,1)}[t, z] - 3\alpha_3 Q_0 u_t^{(0,1)}[t, z] - 6\alpha_4 Q_0 u_t^{(0,1)}[t, z] - \frac{1}{2} \chi Q_0 u_t^{(0,1)}[t, z] + 5\alpha_2 q_t^{(1,1)}[t, z] + 3\alpha_3 q_t^{(1,1)}[t, z] - 6\alpha_4 q_t^{(1,1)}[t, z]$
$-B_x \zeta u_t[t, z] - Q_0 \zeta \epsilon_x[t, x, y, z] - 2\alpha_2 Q_0 q_y^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_+^{(1,0)}[t, z] - 2\alpha_2 u_y^{(1,1)}[t, z] - \frac{1}{2} \chi u_y^{(1,1)}[t, z]$

$-\frac{1}{2} B_x \zeta u_y[t, z] + \alpha_2 Q_0 q_t^{(0,1)}[t, z] + 3 \alpha_3 Q_0 q_t^{(0,1)}[t, z] +$ $\frac{1}{3} \chi u_t^{(1,1)}[t, z] - \frac{1}{6} \xi u_t^{(1,1)}[t, z] - 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z]$
$-12 \beta_1 q_y[t, z] - Q_0 \zeta \epsilon_y[t, x, y, z] - \frac{1}{2} B_x \zeta h_x^{(1,0)}[t, z] + \zeta b^{(1,0)}[t, z] - 5 \alpha_2 Q_0 u_y^{(1,0)}[t, z] -$ $3 \alpha_3 Q_0 u_y^{(1,0)}[t, z] - \frac{1}{2} \chi Q_0 u_y^{(1,0)}[t, z] + 5 \alpha_2 q_y^{(2,0)}[t, z] + 3 \alpha_3 q_y^{(2,0)}[t, z]$
$B_x \zeta u_t[t, z] + Q_0 \zeta \epsilon_x[t, x, y, z] + 2 \alpha_2 Q_0 q_y^{(0,1)}[t, z] +$ $\frac{1}{2} B_x \zeta h_+^{(1,0)}[t, z] + 2 \alpha_2 u_y^{(1,1)}[t, z] + \frac{1}{2} \chi u_y^{(1,1)}[t, z]$
$-3 \beta_3 u_t[t, z] + 12 \alpha_4 Q_0^2 u_t[t, z] - 2 \alpha_2 u_t^{(0,2)}[t, z] - \frac{5}{6} \chi u_t^{(0,2)}[t, z] +$ $\frac{1}{6} \xi u_t^{(0,2)}[t, z] + \frac{3}{2} \alpha_2 Q_0^2 h_+^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0^2 h_+^{(1,0)}[t, z] - 6 \alpha_2 Q_0 q_t^{(1,0)}[t, z] -$ $6 \alpha_3 Q_0 q_t^{(1,0)}[t, z] + 12 \alpha_4 Q_0 q_t^{(1,0)}[t, z] + 6 \alpha_2 Q_0 h_x^{(1,1)}[t, z] +$ $6 \alpha_3 Q_0 h_x^{(1,1)}[t, z] - 3 \alpha_2 h_+^{(1,2)}[t, z] - 3 \alpha_3 h_+^{(1,2)}[t, z] + 3 \alpha_2 h_+^{(3,0)}[t, z] +$ $3 \alpha_3 h_+^{(3,0)}[t, z] + 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + \zeta \epsilon_z^{(0,0,0,1)}[t, x, y, z] +$ $2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + \zeta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$
$-\frac{1}{2} B_x \zeta q_y[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_z[t, x, y, z] - 4 \alpha_2 Q_0 u_t^{(0,1)}[t, z] - 3 \alpha_3 Q_0 u_t^{(0,1)}[t, z] +$ $6 \alpha_4 Q_0 u_t^{(0,1)}[t, z] - \frac{1}{4} \chi Q_0 u_t^{(0,1)}[t, z] + \frac{3}{2} \alpha_2 Q_0^2 h_x^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0^2 h_x^{(1,0)}[t, z] -$ $6 \alpha_2 Q_0 h_+^{(1,1)}[t, z] - 6 \alpha_3 Q_0 h_+^{(1,1)}[t, z] + 6 \alpha_4 q_t^{(1,1)}[t, z] - 3 \alpha_2 h_x^{(1,2)}[t, z] -$ $3 \alpha_3 h_x^{(1,2)}[t, z] + 3 \alpha_2 h_x^{(3,0)}[t, z] + 3 \alpha_3 h_x^{(3,0)}[t, z] - \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z]$
$12 \beta_1 q_y[t, z] - \frac{1}{2} Q_0 \zeta \epsilon_y[t, x, y, z] - \frac{1}{2} B_x \zeta h_x^{(1,0)}[t, z] -$ $\alpha_2 Q_0 u_y^{(1,0)}[t, z] - \frac{1}{4} \chi Q_0 u_y^{(1,0)}[t, z] - \zeta \epsilon_z^{(0,1,0,0)}[t, x, y, z]$
$3 \beta_3 u_t[t, z] - 12 \alpha_4 Q_0^2 u_t[t, z] + 2 \alpha_2 u_t^{(0,2)}[t, z] + \frac{5}{6} \chi u_t^{(0,2)}[t, z] -$ $\frac{1}{6} \xi u_t^{(0,2)}[t, z] - \frac{3}{2} \alpha_2 Q_0^2 h_+^{(1,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0^2 h_+^{(1,0)}[t, z] + 6 \alpha_2 Q_0 q_t^{(1,0)}[t, z] +$ $6 \alpha_3 Q_0 q_t^{(1,0)}[t, z] - 12 \alpha_4 Q_0 q_t^{(1,0)}[t, z] - 6 \alpha_2 Q_0 h_x^{(1,1)}[t, z] -$ $6 \alpha_3 Q_0 h_x^{(1,1)}[t, z] + 3 \alpha_2 h_+^{(1,2)}[t, z] + 3 \alpha_3 h_+^{(1,2)}[t, z] - 3 \alpha_2 h_+^{(3,0)}[t, z] -$ $3 \alpha_3 h_+^{(3,0)}[t, z] - 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] - \zeta \epsilon_z^{(0,0,0,1)}[t, x, y, z] -$ $2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] - \zeta \epsilon_y^{(0,0,1,0)}[t, x, y, z] - 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$
$\frac{1}{2} Q_0 \zeta b[t, z] - 3 \beta_3 u_y[t, z] + \frac{3}{2} \alpha_2 Q_0^2 u_y[t, z] + \frac{3}{2} \alpha_3 Q_0^2 u_y[t, z] -$ $2 B_x \theta h_+^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_+^{(0,1)}[t, z] - 2 \alpha_2 u_y^{(0,2)}[t, z] - \frac{5}{6} \chi u_y^{(0,2)}[t, z] +$ $\frac{1}{6} \xi u_y^{(0,2)}[t, z] - \frac{1}{2} \alpha_2 Q_0 q_y^{(1,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0 q_y^{(1,0)}[t, z] + 2 \alpha_2 u_y^{(2,0)}[t, z] +$ $\frac{5}{6} \chi u_y^{(2,0)}[t, z] - \frac{1}{6} \xi u_y^{(2,0)}[t, z] + 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_y^{(1,0,0,0)}[t, x, y, z]$
$\frac{1}{2} B_x \zeta u_y[t, z] + 6 \alpha_2 Q_0^2 h_+^{(0,1)}[t, z] + 6 \alpha_3 Q_0^2 h_+^{(0,1)}[t, z] -$ $\frac{5}{2} \alpha_2 Q_0 q_t^{(0,1)}[t, z] - \frac{3}{2} \alpha_3 Q_0 q_t^{(0,1)}[t, z] + \frac{15}{2} \alpha_2 Q_0 h_x^{(0,2)}[t, z] +$ $\frac{15}{2} \alpha_3 Q_0 h_x^{(0,2)}[t, z] - 3 \alpha_2 h_+^{(0,3)}[t, z] - 3 \alpha_3 h_+^{(0,3)}[t, z] - 2 \alpha_2 u_t^{(1,1)}[t, z] -$ $\frac{5}{6} \chi u_t^{(1,1)}[t, z] + \frac{1}{6} \xi u_t^{(1,1)}[t, z] - \frac{3}{2} \alpha_2 Q_0 h_x^{(2,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0 h_x^{(2,0)}[t, z] +$ $3 \alpha_2 h_+^{(2,1)}[t, z] + 3 \alpha_3 h_+^{(2,1)}[t, z] + 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_z^{(1,0,0,0)}[t, x, y, z]$

$\begin{aligned} & \frac{1}{2} B_x \zeta q_y[t, z] - \frac{1}{2} Q_0 \zeta \epsilon_z[t, x, y, z] + 4 \alpha_2 Q_0 u_t^{(0,1)}[t, z] + 3 \alpha_3 Q_0 u_t^{(0,1)}[t, z] - \\ & 6 \alpha_4 Q_0 u_t^{(0,1)}[t, z] + \frac{1}{4} \chi Q_0 u_t^{(0,1)}[t, z] - \frac{3}{2} \alpha_2 Q_0^2 h_x^{(1,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0^2 h_x^{(1,0)}[t, z] + \\ & 6 \alpha_2 Q_0 h_+^{(1,1)}[t, z] + 6 \alpha_3 Q_0 h_+^{(1,1)}[t, z] - 6 \alpha_4 q_t^{(1,1)}[t, z] + 3 \alpha_2 h_x^{(1,2)}[t, z] + \\ & 3 \alpha_3 h_x^{(1,2)}[t, z] - 3 \alpha_2 h_x^{(3,0)}[t, z] - 3 \alpha_3 h_x^{(3,0)}[t, z] + \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z] \end{aligned}$
$\begin{aligned} & -\frac{1}{2} Q_0 \zeta b[t, z] + 3 \beta_3 u_y[t, z] - \frac{3}{2} \alpha_2 Q_0^2 u_y[t, z] - \frac{3}{2} \alpha_3 Q_0^2 u_y[t, z] + \\ & 2 B_x \theta h_+^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_+^{(0,1)}[t, z] + 2 \alpha_2 u_y^{(0,2)}[t, z] + \frac{5}{6} \chi u_y^{(0,2)}[t, z] - \\ & \frac{1}{6} \xi u_y^{(0,2)}[t, z] + \frac{1}{2} \alpha_2 Q_0 q_y^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0 q_y^{(1,0)}[t, z] - 2 \alpha_2 u_y^{(2,0)}[t, z] - \\ & \frac{5}{6} \chi u_y^{(2,0)}[t, z] + \frac{1}{6} \xi u_y^{(2,0)}[t, z] - 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_y^{(1,0,0,0)}[t, x, y, z] \end{aligned}$
$\begin{aligned} & -12 \beta_1 q_t[t, z] + 6 \alpha_2 Q_0^2 h_x^{(0,1)}[t, z] + 6 \alpha_3 Q_0^2 h_x^{(0,1)}[t, z] - \\ & \frac{15}{2} \alpha_2 Q_0 h_+^{(0,2)}[t, z] - \frac{15}{2} \alpha_3 Q_0 h_+^{(0,2)}[t, z] - 3 \alpha_2 h_x^{(0,3)}[t, z] - 3 \alpha_3 h_x^{(0,3)}[t, z] - \\ & 3 \alpha_2 Q_0 u_t^{(1,0)}[t, z] - 3 \alpha_3 Q_0 u_t^{(1,0)}[t, z] + 6 \alpha_4 Q_0 u_t^{(1,0)}[t, z] + \frac{3}{2} \alpha_2 Q_0 h_+^{(2,0)}[t, z] + \\ & \frac{3}{2} \alpha_3 Q_0 h_+^{(2,0)}[t, z] + 6 \alpha_4 q_t^{(2,0)}[t, z] + 3 \alpha_2 h_x^{(2,1)}[t, z] + 3 \alpha_3 h_x^{(2,1)}[t, z] \end{aligned}$
$\begin{aligned} & -12 \beta_1 q_y[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_y[t, x, y, z] + \frac{1}{2} B_x \zeta h_x^{(1,0)}[t, z] + \\ & \alpha_2 Q_0 u_y^{(1,0)}[t, z] + \frac{1}{4} \chi Q_0 u_y^{(1,0)}[t, z] + \zeta \epsilon_z^{(0,1,0,0)}[t, x, y, z] \end{aligned}$
$\begin{aligned} & -\frac{1}{2} B_x \zeta u_y[t, z] - 6 \alpha_2 Q_0^2 h_+^{(0,1)}[t, z] - 6 \alpha_3 Q_0^2 h_+^{(0,1)}[t, z] + \\ & \frac{5}{2} \alpha_2 Q_0 q_t^{(0,1)}[t, z] + \frac{3}{2} \alpha_3 Q_0 q_t^{(0,1)}[t, z] - \frac{15}{2} \alpha_2 Q_0 h_x^{(0,2)}[t, z] - \\ & \frac{15}{2} \alpha_3 Q_0 h_x^{(0,2)}[t, z] + 3 \alpha_2 h_+^{(0,3)}[t, z] + 3 \alpha_3 h_+^{(0,3)}[t, z] + 2 \alpha_2 u_t^{(1,1)}[t, z] + \\ & \frac{5}{6} \chi u_t^{(1,1)}[t, z] - \frac{1}{6} \xi u_t^{(1,1)}[t, z] + \frac{3}{2} \alpha_2 Q_0 h_x^{(2,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0 h_x^{(2,0)}[t, z] - \\ & 3 \alpha_2 h_+^{(2,1)}[t, z] - 3 \alpha_3 h_+^{(2,1)}[t, z] - 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_z^{(1,0,0,0)}[t, x, y, z] \end{aligned}$
$\begin{aligned} & 12 \beta_1 q_t[t, z] - 6 \alpha_2 Q_0^2 h_x^{(0,1)}[t, z] - 6 \alpha_3 Q_0^2 h_x^{(0,1)}[t, z] + \\ & \frac{15}{2} \alpha_2 Q_0 h_+^{(0,2)}[t, z] + \frac{15}{2} \alpha_3 Q_0 h_+^{(0,2)}[t, z] + 3 \alpha_2 h_x^{(0,3)}[t, z] + 3 \alpha_3 h_x^{(0,3)}[t, z] + \\ & 3 \alpha_2 Q_0 u_t^{(1,0)}[t, z] + 3 \alpha_3 Q_0 u_t^{(1,0)}[t, z] - 6 \alpha_4 Q_0 u_t^{(1,0)}[t, z] - \frac{3}{2} \alpha_2 Q_0 h_+^{(2,0)}[t, z] - \\ & \frac{3}{2} \alpha_3 Q_0 h_+^{(2,0)}[t, z] - 6 \alpha_4 q_t^{(2,0)}[t, z] - 3 \alpha_2 h_x^{(2,1)}[t, z] - 3 \alpha_3 h_x^{(2,1)}[t, z] \end{aligned}$
$\begin{aligned} & -B_x \zeta q_y[t, z] - \frac{1}{2} Q_0 \zeta \epsilon_z[t, x, y, z] + 4 \alpha_2 Q_0 u_t^{(0,1)}[t, z] + 3 \alpha_3 Q_0 u_t^{(0,1)}[t, z] - \\ & 6 \alpha_4 Q_0 u_t^{(0,1)}[t, z] + \frac{1}{4} \chi Q_0 u_t^{(0,1)}[t, z] + \frac{3}{2} \alpha_2 Q_0^2 h_x^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0^2 h_x^{(1,0)}[t, z] - \\ & 6 \alpha_2 Q_0 h_+^{(1,1)}[t, z] - 6 \alpha_3 Q_0 h_+^{(1,1)}[t, z] - 6 \alpha_4 q_t^{(1,1)}[t, z] - 3 \alpha_2 h_x^{(1,2)}[t, z] - \\ & 3 \alpha_3 h_x^{(1,2)}[t, z] + 3 \alpha_2 h_x^{(3,0)}[t, z] + 3 \alpha_3 h_x^{(3,0)}[t, z] - \zeta \epsilon_x^{(0,0,1,0)}[t, x, y, z] \end{aligned}$
$\begin{aligned} & -3 \beta_3 u_t[t, z] + 12 \alpha_4 Q_0^2 u_t[t, z] - 2 \alpha_2 u_t^{(0,2)}[t, z] - \frac{5}{6} \chi u_t^{(0,2)}[t, z] + \\ & \frac{1}{6} \xi u_t^{(0,2)}[t, z] - \frac{3}{2} \alpha_2 Q_0^2 h_+^{(1,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0^2 h_+^{(1,0)}[t, z] - 6 \alpha_2 Q_0 q_t^{(1,0)}[t, z] - \\ & 6 \alpha_3 Q_0 q_t^{(1,0)}[t, z] + 12 \alpha_4 Q_0 q_t^{(1,0)}[t, z] - 6 \alpha_2 Q_0 h_x^{(1,1)}[t, z] - \\ & 6 \alpha_3 Q_0 h_x^{(1,1)}[t, z] + 3 \alpha_2 h_+^{(1,2)}[t, z] + 3 \alpha_3 h_+^{(1,2)}[t, z] - 3 \alpha_2 h_+^{(3,0)}[t, z] - \\ & 3 \alpha_3 h_+^{(3,0)}[t, z] + 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + \zeta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + \\ & 2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z] + \zeta \epsilon_x^{(0,1,0,0)}[t, x, y, z] \end{aligned}$

$\frac{1}{2} B_x \zeta u_t[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_x[t, x, y, z] - \frac{1}{2} \alpha_2 Q_0 q_y^{(0,1)}[t, z] -$ $\frac{3}{2} \alpha_3 Q_0 q_y^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_+^{(1,0)}[t, z] - \zeta \epsilon_z^{(0,0,1,0)}[t, x, y, z]$
$B_x \zeta q_y[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_z[t, x, y, z] - 4 \alpha_2 Q_0 u_t^{(0,1)}[t, z] - 3 \alpha_3 Q_0 u_t^{(0,1)}[t, z] +$ $6 \alpha_4 Q_0 u_t^{(0,1)}[t, z] - \frac{1}{4} \chi Q_0 u_t^{(0,1)}[t, z] - \frac{3}{2} \alpha_2 Q_0^2 h_x^{(1,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0^2 h_x^{(1,0)}[t, z] +$ $6 \alpha_2 Q_0 h_+^{(1,1)}[t, z] + 6 \alpha_3 Q_0 h_+^{(1,1)}[t, z] + 6 \alpha_4 q_t^{(1,1)}[t, z] + 3 \alpha_2 h_x^{(1,2)}[t, z] +$ $3 \alpha_3 h_x^{(1,2)}[t, z] - 3 \alpha_2 h_x^{(3,0)}[t, z] - 3 \alpha_3 h_x^{(3,0)}[t, z] + \zeta \epsilon_x^{(0,0,1,0)}[t, x, y, z]$
$-\frac{1}{2} B_x \zeta q_t[t, z] - 2 B_x \theta h_x^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_x^{(0,1)}[t, z] +$ $2 \theta b^{(0,1)}[t, z] + \zeta b^{(0,1)}[t, z] + 4 \alpha_2 Q_0 u_y^{(0,1)}[t, z] + 3 \alpha_3 Q_0 u_y^{(0,1)}[t, z] +$ $\frac{1}{4} \chi Q_0 u_y^{(0,1)}[t, z] - 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$12 \beta_1 q_t[t, z] + 6 \alpha_2 Q_0^2 h_x^{(0,1)}[t, z] + 6 \alpha_3 Q_0^2 h_x^{(0,1)}[t, z] -$ $\frac{15}{2} \alpha_2 Q_0 h_+^{(0,2)}[t, z] - \frac{15}{2} \alpha_3 Q_0 h_+^{(0,2)}[t, z] - 3 \alpha_2 h_x^{(0,3)}[t, z] - 3 \alpha_3 h_x^{(0,3)}[t, z] +$ $3 \alpha_2 Q_0 u_t^{(1,0)}[t, z] + 3 \alpha_3 Q_0 u_t^{(1,0)}[t, z] - 6 \alpha_4 Q_0 u_t^{(1,0)}[t, z] + \frac{3}{2} \alpha_2 Q_0 h_+^{(2,0)}[t, z] +$ $\frac{3}{2} \alpha_3 Q_0 h_+^{(2,0)}[t, z] - 6 \alpha_4 q_t^{(2,0)}[t, z] + 3 \alpha_2 h_x^{(2,1)}[t, z] + 3 \alpha_3 h_x^{(2,1)}[t, z]$
$3 \beta_3 u_t[t, z] - 12 \alpha_4 Q_0^2 u_t[t, z] + 2 \alpha_2 u_t^{(0,2)}[t, z] + \frac{5}{6} \chi u_t^{(0,2)}[t, z] -$ $\frac{1}{6} \xi u_t^{(0,2)}[t, z] + \frac{3}{2} \alpha_2 Q_0^2 h_+^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0^2 h_+^{(1,0)}[t, z] + 6 \alpha_2 Q_0 q_t^{(1,0)}[t, z] +$ $6 \alpha_3 Q_0 q_t^{(1,0)}[t, z] - 12 \alpha_4 Q_0 q_t^{(1,0)}[t, z] + 6 \alpha_2 Q_0 h_x^{(1,1)}[t, z] +$ $6 \alpha_3 Q_0 h_x^{(1,1)}[t, z] - 3 \alpha_2 h_+^{(1,2)}[t, z] - 3 \alpha_3 h_+^{(1,2)}[t, z] + 3 \alpha_2 h_+^{(3,0)}[t, z] +$ $3 \alpha_3 h_+^{(3,0)}[t, z] - 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] - \zeta \epsilon_z^{(0,0,0,1)}[t, x, y, z] -$ $2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] - 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z] - \zeta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$
$\frac{1}{2} B_x \zeta q_t[t, z] + 2 B_x \theta h_x^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_x^{(0,1)}[t, z] -$ $2 \theta b^{(0,1)}[t, z] - \zeta b^{(0,1)}[t, z] - 4 \alpha_2 Q_0 u_y^{(0,1)}[t, z] - 3 \alpha_3 Q_0 u_y^{(0,1)}[t, z] -$ $\frac{1}{4} \chi Q_0 u_y^{(0,1)}[t, z] + 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$-B_x \zeta u_y[t, z] - 6 \alpha_2 Q_0^2 h_+^{(0,1)}[t, z] - 6 \alpha_3 Q_0^2 h_+^{(0,1)}[t, z] -$ $\frac{5}{2} \alpha_2 Q_0 q_t^{(0,1)}[t, z] - \frac{3}{2} \alpha_3 Q_0 q_t^{(0,1)}[t, z] - \frac{15}{2} \alpha_2 Q_0 h_x^{(0,2)}[t, z] -$ $\frac{15}{2} \alpha_3 Q_0 h_x^{(0,2)}[t, z] + 3 \alpha_2 h_+^{(0,3)}[t, z] + 3 \alpha_3 h_+^{(0,3)}[t, z] - 2 \alpha_2 u_t^{(1,1)}[t, z] -$ $\frac{5}{6} \chi u_t^{(1,1)}[t, z] + \frac{1}{6} \xi u_t^{(1,1)}[t, z] + \frac{3}{2} \alpha_2 Q_0 h_x^{(2,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0 h_x^{(2,0)}[t, z] -$ $3 \alpha_2 h_+^{(2,1)}[t, z] - 3 \alpha_3 h_+^{(2,1)}[t, z] + 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_z^{(1,0,0,0)}[t, x, y, z]$
$-\frac{1}{2} B_x \zeta u_t[t, z] - \frac{1}{2} Q_0 \zeta \epsilon_x[t, x, y, z] + \frac{1}{2} \alpha_2 Q_0 q_y^{(0,1)}[t, z] +$ $\frac{3}{2} \alpha_3 Q_0 q_y^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_+^{(1,0)}[t, z] + \zeta \epsilon_z^{(0,0,1,0)}[t, x, y, z]$
$-12 \beta_1 q_t[t, z] - 6 \alpha_2 Q_0^2 h_x^{(0,1)}[t, z] - 6 \alpha_3 Q_0^2 h_x^{(0,1)}[t, z] +$ $\frac{15}{2} \alpha_2 Q_0 h_+^{(0,2)}[t, z] + \frac{15}{2} \alpha_3 Q_0 h_+^{(0,2)}[t, z] + 3 \alpha_2 h_x^{(0,3)}[t, z] + 3 \alpha_3 h_x^{(0,3)}[t, z] -$ $3 \alpha_2 Q_0 u_t^{(1,0)}[t, z] - 3 \alpha_3 Q_0 u_t^{(1,0)}[t, z] + 6 \alpha_4 Q_0 u_t^{(1,0)}[t, z] - \frac{3}{2} \alpha_2 Q_0 h_+^{(2,0)}[t, z] -$ $\frac{3}{2} \alpha_3 Q_0 h_+^{(2,0)}[t, z] + 6 \alpha_4 q_t^{(2,0)}[t, z] - 3 \alpha_2 h_x^{(2,1)}[t, z] - 3 \alpha_3 h_x^{(2,1)}[t, z]$

$B_x \zeta u_y[t, z] + 6 \alpha_2 Q_0^2 h_+^{(0,1)}[t, z] + 6 \alpha_3 Q_0^2 h_+^{(0,1)}[t, z] +$ $\frac{5}{2} \alpha_2 Q_0 q_t^{(0,1)}[t, z] + \frac{3}{2} \alpha_3 Q_0 q_t^{(0,1)}[t, z] + \frac{15}{2} \alpha_2 Q_0 h_x^{(0,2)}[t, z] +$ $\frac{15}{2} \alpha_3 Q_0 h_x^{(0,2)}[t, z] - 3 \alpha_2 h_+^{(0,3)}[t, z] - 3 \alpha_3 h_+^{(0,3)}[t, z] + 2 \alpha_2 u_t^{(1,1)}[t, z] +$ $\frac{5}{6} \chi u_t^{(1,1)}[t, z] - \frac{1}{6} \xi u_t^{(1,1)}[t, z] - \frac{3}{2} \alpha_2 Q_0 h_x^{(2,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0 h_x^{(2,0)}[t, z] +$ $3 \alpha_2 h_+^{(2,1)}[t, z] + 3 \alpha_3 h_+^{(2,1)}[t, z] - 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_z^{(1,0,0,0)}[t, x, y, z]$
$-12 \beta_1 q_y[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_y[t, x, y, z] - 5 \alpha_2 q_y^{(0,2)}[t, z] -$ $3 \alpha_3 q_y^{(0,2)}[t, z] + \alpha_2 Q_0 u_y^{(1,0)}[t, z] + \frac{1}{4} \chi Q_0 u_y^{(1,0)}[t, z] - \zeta \epsilon_x^{(0,0,0,1)}[t, x, y, z]$
$-\frac{1}{2} B_x \zeta u_t[t, z] - \frac{1}{2} Q_0 \zeta \epsilon_x[t, x, y, z] - \frac{5}{2} \alpha_2 Q_0 q_y^{(0,1)}[t, z] -$ $\frac{3}{2} \alpha_3 Q_0 q_y^{(0,1)}[t, z] - 2 \alpha_2 u_y^{(1,1)}[t, z] - \frac{1}{2} \chi u_y^{(1,1)}[t, z] - \zeta \epsilon_y^{(0,0,0,1)}[t, x, y, z]$
$-3 \beta_3 u_t[t, z] + 12 \alpha_4 Q_0^2 u_t[t, z] - \frac{1}{3} \chi u_t^{(0,2)}[t, z] + \frac{1}{6} \xi u_t^{(0,2)}[t, z] - 6 \alpha_2 Q_0 q_t^{(1,0)}[t, z] -$ $6 \alpha_3 Q_0 q_t^{(1,0)}[t, z] + 12 \alpha_4 Q_0 q_t^{(1,0)}[t, z] + 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + 2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] +$ $\zeta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z] + \zeta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$
$12 \beta_1 q_y[t, z] - \frac{1}{2} Q_0 \zeta \epsilon_y[t, x, y, z] + 5 \alpha_2 q_y^{(0,2)}[t, z] +$ $3 \alpha_3 q_y^{(0,2)}[t, z] - \alpha_2 Q_0 u_y^{(1,0)}[t, z] - \frac{1}{4} \chi Q_0 u_y^{(1,0)}[t, z] + \zeta \epsilon_x^{(0,0,0,1)}[t, x, y, z]$
$-12 \beta_1 q_t[t, z] - 5 \alpha_2 q_t^{(0,2)}[t, z] - 3 \alpha_3 q_t^{(0,2)}[t, z] -$ $3 \alpha_2 Q_0 u_t^{(1,0)}[t, z] - 3 \alpha_3 Q_0 u_t^{(1,0)}[t, z] + 6 \alpha_4 Q_0 u_t^{(1,0)}[t, z] + 6 \alpha_4 q_t^{(2,0)}[t, z]$
$-\frac{1}{2} B_x \zeta q_t[t, z] - 2 B_x \theta h_x^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_x^{(0,1)}[t, z] +$ $2 \theta b^{(0,1)}[t, z] + \alpha_2 Q_0 u_y^{(0,1)}[t, z] + \frac{1}{4} \chi Q_0 u_y^{(0,1)}[t, z] - 5 \alpha_2 q_y^{(1,1)}[t, z] -$ $3 \alpha_3 q_y^{(1,1)}[t, z] - 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$\frac{1}{2} B_x \zeta u_t[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_x[t, x, y, z] + \frac{5}{2} \alpha_2 Q_0 q_y^{(0,1)}[t, z] +$ $\frac{3}{2} \alpha_3 Q_0 q_y^{(0,1)}[t, z] + 2 \alpha_2 u_y^{(1,1)}[t, z] + \frac{1}{2} \chi u_y^{(1,1)}[t, z] + \zeta \epsilon_y^{(0,0,0,1)}[t, x, y, z]$
$12 \beta_1 q_t[t, z] + 5 \alpha_2 q_t^{(0,2)}[t, z] + 3 \alpha_3 q_t^{(0,2)}[t, z] +$ $3 \alpha_2 Q_0 u_t^{(1,0)}[t, z] + 3 \alpha_3 Q_0 u_t^{(1,0)}[t, z] - 6 \alpha_4 Q_0 u_t^{(1,0)}[t, z] - 6 \alpha_4 q_t^{(2,0)}[t, z]$
$-\frac{1}{2} Q_0 \zeta b[t, z] + 3 \beta_3 u_y[t, z] - \frac{3}{2} \alpha_2 Q_0^2 u_y[t, z] - \frac{3}{2} \alpha_3 Q_0^2 u_y[t, z] +$ $2 B_x \theta h_+^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_+^{(0,1)}[t, z] + \frac{1}{3} \chi u_y^{(0,2)}[t, z] - \frac{1}{6} \xi u_y^{(0,2)}[t, z] +$ $\frac{1}{2} \alpha_2 Q_0 q_y^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0 q_y^{(1,0)}[t, z] - 2 \alpha_2 u_y^{(2,0)}[t, z] - \frac{5}{6} \chi u_y^{(2,0)}[t, z] +$ $\frac{1}{6} \xi u_y^{(2,0)}[t, z] - 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_y^{(1,0,0,0)}[t, x, y, z]$
$3 \beta_3 u_t[t, z] - 12 \alpha_4 Q_0^2 u_t[t, z] + \frac{1}{3} \chi u_t^{(0,2)}[t, z] - \frac{1}{6} \xi u_t^{(0,2)}[t, z] + 6 \alpha_2 Q_0 q_t^{(1,0)}[t, z] +$ $6 \alpha_3 Q_0 q_t^{(1,0)}[t, z] - 12 \alpha_4 Q_0 q_t^{(1,0)}[t, z] - 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] - 2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] -$ $\zeta \epsilon_y^{(0,0,1,0)}[t, x, y, z] - 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z] - \zeta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$
$\frac{1}{2} B_x \zeta q_t[t, z] + 2 B_x \theta h_x^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_x^{(0,1)}[t, z] -$ $2 \theta b^{(0,1)}[t, z] - \alpha_2 Q_0 u_y^{(0,1)}[t, z] - \frac{1}{4} \chi Q_0 u_y^{(0,1)}[t, z] + 5 \alpha_2 q_y^{(1,1)}[t, z] +$ $3 \alpha_3 q_y^{(1,1)}[t, z] + 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$



$\begin{aligned} & \frac{1}{2} Q_0 \zeta b[t, z] - 3 \beta_3 u_y[t, z] + \frac{3}{2} \alpha_2 Q_0^2 u_y[t, z] + \frac{3}{2} \alpha_3 Q_0^2 u_y[t, z] - \\ & 2 B_x \theta h_+^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_+^{(0,1)}[t, z] - \frac{1}{3} \chi u_y^{(0,2)}[t, z] + \frac{1}{6} \xi u_y^{(0,2)}[t, z] - \\ & \frac{1}{2} \alpha_2 Q_0 q_y^{(1,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0 q_y^{(1,0)}[t, z] + 2 \alpha_2 u_y^{(2,0)}[t, z] + \frac{5}{6} \chi u_y^{(2,0)}[t, z] - \\ & \frac{1}{6} \xi u_y^{(2,0)}[t, z] + 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_y^{(1,0,0,0)}[t, x, y, z] \end{aligned}$
$\begin{aligned} & 9 \alpha_2 Q_0^3 q_t[t, z] + 27 \alpha_3 Q_0^3 q_t[t, z] + \frac{1}{2} B_x \zeta u_y^{(0,1)}[t, z] + \frac{9}{2} \alpha_2 Q_0^2 u_t^{(1,0)}[t, z] + \\ & \frac{9}{2} \alpha_3 Q_0^2 u_t^{(1,0)}[t, z] + \alpha_2 u_t^{(1,2)}[t, z] + \alpha_3 u_t^{(1,2)}[t, z] + 3 \alpha_2 u_t^{(3,0)}[t, z] + 3 \alpha_3 u_t^{(3,0)}[t, z] \end{aligned}$
$\begin{aligned} & B_x Q_0 \zeta u_t[t, z] + Q_0^2 \zeta \epsilon_x[t, x, y, z] + 2 \alpha_2 Q_0^2 q_y^{(0,1)}[t, z] + \frac{1}{2} B_x Q_0 \zeta h_+^{(1,0)}[t, z] + \\ & 2 \alpha_2 Q_0 u_y^{(1,1)}[t, z] + \frac{1}{2} \chi Q_0 u_y^{(1,1)}[t, z] + Q_0 \zeta \epsilon_y^{(0,0,0,1)}[t, x, y, z] - Q_0 \zeta \epsilon_z^{(0,0,1,0)}[t, x, y, z] \end{aligned}$
$\begin{aligned} & -12 \beta_1 Q_0 q_y[t, z] + Q_0^2 \zeta \epsilon_y[t, x, y, z] - B_x \epsilon_z[t, x, y, z] - 3 B_x \theta u_t^{(0,1)}[t, z] - B_x \zeta u_t^{(0,1)}[t, z] - \\ & 2 \alpha_2 Q_0 q_y^{(0,2)}[t, z] + \frac{1}{2} B_x Q_0 \zeta h_x^{(1,0)}[t, z] + 6 \alpha_2 Q_0^2 u_y^{(1,0)}[t, z] + 4 \alpha_3 Q_0^2 u_y^{(1,0)}[t, z] + \\ & \frac{1}{2} \chi Q_0^2 u_y^{(1,0)}[t, z] - \alpha_2 u_y^{(1,2)}[t, z] - \alpha_3 u_y^{(1,2)}[t, z] + \alpha_2 Q_0 q_y^{(2,0)}[t, z] + \alpha_3 Q_0 q_y^{(2,0)}[t, z] + \\ & \alpha_2 u_y^{(3,0)}[t, z] + \alpha_3 u_y^{(3,0)}[t, z] - Q_0 \zeta \epsilon_x^{(0,0,0,1)}[t, x, y, z] + Q_0 \zeta \epsilon_z^{(0,1,0,0)}[t, x, y, z] \end{aligned}$
$\begin{aligned} & -B_x Q_0 \zeta q_y[t, z] + B_x \epsilon_y[t, x, y, z] + Q_0^2 \zeta \epsilon_z[t, x, y, z] - 2 \alpha_2 Q_0^2 u_t^{(0,1)}[t, z] - \\ & \frac{1}{2} \chi Q_0^2 u_t^{(0,1)}[t, z] - \alpha_2 u_t^{(0,3)}[t, z] - \alpha_3 u_t^{(0,3)}[t, z] - 3 B_x \theta u_y^{(1,0)}[t, z] - \\ & B_x \zeta u_y^{(1,0)}[t, z] + 4 \alpha_2 Q_0 q_t^{(1,1)}[t, z] + 4 \alpha_3 Q_0 q_t^{(1,1)}[t, z] - 6 \alpha_4 Q_0 q_t^{(1,1)}[t, z] + \\ & 5 \alpha_2 u_t^{(2,1)}[t, z] + 5 \alpha_3 u_t^{(2,1)}[t, z] + Q_0 \zeta \epsilon_x^{(0,0,1,0)}[t, x, y, z] - Q_0 \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z] \end{aligned}$
$\begin{aligned} & -24 \beta_1 Q_0 q_t[t, z] - 9 \alpha_2 Q_0^3 q_t[t, z] - 27 \alpha_3 Q_0^3 q_t[t, z] - 12 \alpha_2 Q_0^3 h_x^{(0,1)}[t, z] - \\ & 12 \alpha_3 Q_0^3 h_x^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta u_y^{(0,1)}[t, z] + 18 \alpha_2 Q_0^2 h_+^{(0,2)}[t, z] + \\ & 18 \alpha_3 Q_0^2 h_+^{(0,2)}[t, z] - 4 \alpha_2 Q_0 q_t^{(0,2)}[t, z] + 12 \alpha_2 Q_0 h_x^{(0,3)}[t, z] + \\ & 12 \alpha_3 Q_0 h_x^{(0,3)}[t, z] - 3 \alpha_2 h_+^{(0,4)}[t, z] - 3 \alpha_3 h_+^{(0,4)}[t, z] + \frac{3}{2} \alpha_2 Q_0^2 u_t^{(1,0)}[t, z] + \\ & \frac{3}{2} \alpha_3 Q_0^2 u_t^{(1,0)}[t, z] - 3 \alpha_2 u_t^{(1,2)}[t, z] - 3 \alpha_3 u_t^{(1,2)}[t, z] - 3 \alpha_2 Q_0^2 h_+^{(2,0)}[t, z] - \\ & 3 \alpha_3 Q_0^2 h_+^{(2,0)}[t, z] + 6 \alpha_2 Q_0 q_t^{(2,0)}[t, z] + 6 \alpha_3 Q_0 q_t^{(2,0)}[t, z] - 12 \alpha_4 Q_0 q_t^{(2,0)}[t, z] - \\ & 12 \alpha_2 Q_0 h_x^{(2,1)}[t, z] - 12 \alpha_3 Q_0 h_x^{(2,1)}[t, z] + 6 \alpha_2 h_+^{(2,2)}[t, z] + 6 \alpha_3 h_+^{(2,2)}[t, z] + \\ & 3 \alpha_2 u_t^{(3,0)}[t, z] + 3 \alpha_3 u_t^{(3,0)}[t, z] - 3 \alpha_2 h_+^{(4,0)}[t, z] - 3 \alpha_3 h_+^{(4,0)}[t, z] \end{aligned}$
$\begin{aligned} & B_x b[t, z] + \frac{3}{2} B_x Q_0 \zeta u_y[t, z] + 12 \alpha_2 Q_0^3 h_+^{(0,1)}[t, z] + 12 \alpha_3 Q_0^3 h_+^{(0,1)}[t, z] + \\ & 18 \alpha_2 Q_0^2 h_x^{(0,2)}[t, z] + 18 \alpha_3 Q_0^2 h_x^{(0,2)}[t, z] - 12 \alpha_2 Q_0 h_+^{(0,3)}[t, z] - \\ & 12 \alpha_3 Q_0 h_+^{(0,3)}[t, z] - 3 \alpha_2 h_x^{(0,4)}[t, z] - 3 \alpha_3 h_x^{(0,4)}[t, z] - 3 \alpha_2 Q_0^2 h_x^{(2,0)}[t, z] - \\ & 3 \alpha_3 Q_0^2 h_x^{(2,0)}[t, z] + 12 \alpha_2 Q_0 h_+^{(2,1)}[t, z] + 12 \alpha_3 Q_0 h_+^{(2,1)}[t, z] + \\ & 6 \alpha_2 h_x^{(2,2)}[t, z] + 6 \alpha_3 h_x^{(2,2)}[t, z] - 3 \alpha_2 h_x^{(4,0)}[t, z] - 3 \alpha_3 h_x^{(4,0)}[t, z] \end{aligned}$
$2 \alpha_2 Q_0 u_y^{(0,2)}[t, z] + \frac{1}{2} \chi Q_0 u_y^{(0,2)}[t, z]$



$$\begin{aligned}
& -24 \beta_1 Q_0 q_t[t, z] - 9 \alpha_2 Q_0^3 q_t[t, z] - 27 \alpha_3 Q_0^3 q_t[t, z] + 12 \alpha_2 Q_0^3 h_x^{(0,1)}[t, z] + \\
& 12 \alpha_3 Q_0^3 h_x^{(0,1)}[t, z] - 6 B_x \theta u_y^{(0,1)}[t, z] - \frac{5}{2} B_x \zeta u_y^{(0,1)}[t, z] - 18 \alpha_2 Q_0^2 h_+^{(0,2)}[t, z] - \\
& 18 \alpha_3 Q_0^2 h_+^{(0,2)}[t, z] - 4 \alpha_2 Q_0 q_t^{(0,2)}[t, z] - 12 \alpha_2 Q_0 h_x^{(0,3)}[t, z] - \\
& 12 \alpha_3 Q_0 h_x^{(0,3)}[t, z] + 3 \alpha_2 h_+^{(0,4)}[t, z] + 3 \alpha_3 h_+^{(0,4)}[t, z] + \frac{3}{2} \alpha_2 Q_0^2 u_t^{(1,0)}[t, z] + \\
& \frac{3}{2} \alpha_3 Q_0^2 u_t^{(1,0)}[t, z] - 3 \alpha_2 u_t^{(1,2)}[t, z] - 3 \alpha_3 u_t^{(1,2)}[t, z] + 3 \alpha_2 Q_0^2 h_+^{(2,0)}[t, z] + \\
& 3 \alpha_3 Q_0^2 h_+^{(2,0)}[t, z] + 6 \alpha_2 Q_0 q_t^{(2,0)}[t, z] + 6 \alpha_3 Q_0 q_t^{(2,0)}[t, z] - 12 \alpha_4 Q_0 q_t^{(2,0)}[t, z] + \\
& 12 \alpha_2 Q_0 h_x^{(2,1)}[t, z] + 12 \alpha_3 Q_0 h_x^{(2,1)}[t, z] - 6 \alpha_2 h_+^{(2,2)}[t, z] - 6 \alpha_3 h_+^{(2,2)}[t, z] + \\
& 3 \alpha_2 u_t^{(3,0)}[t, z] + 3 \alpha_3 u_t^{(3,0)}[t, z] + 3 \alpha_2 h_+^{(4,0)}[t, z] + 3 \alpha_3 h_+^{(4,0)}[t, z] \\
& - Q_0 \zeta b^{(0,1)}[t, z] + \alpha_2 Q_0^2 u_y^{(0,1)}[t, z] + \alpha_3 Q_0^2 u_y^{(0,1)}[t, z] - \alpha_2 u_y^{(0,3)}[t, z] - \\
& \alpha_3 u_y^{(0,3)}[t, z] - \alpha_2 Q_0 q_y^{(1,1)}[t, z] + \alpha_3 Q_0 q_y^{(1,1)}[t, z] + \alpha_2 u_y^{(2,1)}[t, z] + \alpha_3 u_y^{(2,1)}[t, z] \\
& - 24 \beta_1 Q_0 q_t[t, z] - 9 \alpha_2 Q_0^3 q_t[t, z] - 27 \alpha_3 Q_0^3 q_t[t, z] - 6 B_x \theta u_y^{(0,1)}[t, z] - \\
& \frac{5}{2} B_x \zeta u_y^{(0,1)}[t, z] + 2 \alpha_2 Q_0 q_t^{(0,2)}[t, z] + 2 \alpha_3 Q_0 q_t^{(0,2)}[t, z] + \frac{3}{2} \alpha_2 Q_0^2 u_t^{(1,0)}[t, z] + \\
& \frac{3}{2} \alpha_3 Q_0^2 u_t^{(1,0)}[t, z] + \alpha_2 u_t^{(1,2)}[t, z] + \alpha_3 u_t^{(1,2)}[t, z] + 6 \alpha_2 Q_0 q_t^{(2,0)}[t, z] + \\
& 6 \alpha_3 Q_0 q_t^{(2,0)}[t, z] - 12 \alpha_4 Q_0 q_t^{(2,0)}[t, z] + 3 \alpha_2 u_t^{(3,0)}[t, z] + 3 \alpha_3 u_t^{(3,0)}[t, z] \\
& - 2 B_x h_+^{(0,2)}[t, z] + 6 \theta u_y^{(0,3)}[t, z] + 2 \zeta u_y^{(0,3)}[t, z] - 6 \theta u_y^{(2,1)}[t, z] - 2 \zeta u_y^{(2,1)}[t, z] \\
& - 2 B_x h_x^{(0,2)}[t, z] + 2 b^{(0,2)}[t, z] - 2 b^{(2,0)}[t, z]
\end{aligned}$$

# Analysis of CTEG

**Key observation:** Analysis of constant-torsion emergent gravity.

Here is the list of rules.

$\alpha_0 \rightarrow 0$	$\alpha_1 \rightarrow 0$	$\alpha_4 \rightarrow \frac{\alpha_2}{6} + \frac{\alpha_3}{6}$	$\alpha_5 \rightarrow -\frac{2\alpha_2}{3} - \frac{2\alpha_3}{3}$	$\alpha_6 \rightarrow -\frac{5\alpha_2}{6} - \frac{5\alpha_3}{6}$
$\beta_1 \rightarrow -\frac{\alpha_2 \Lambda}{12 \mathcal{M}_{\text{Pl}}^2} - \frac{\alpha_3 \Lambda}{12 \mathcal{M}_{\text{Pl}}^2}$	$\beta_2 \rightarrow \frac{\alpha_2 \Lambda}{6 \mathcal{M}_{\text{Pl}}^2} + \frac{\alpha_3 \Lambda}{6 \mathcal{M}_{\text{Pl}}^2}$	$\beta_3 \rightarrow -\frac{4 \mathcal{M}_{\text{Pl}}^2}{9}$		

(61)

Here is the non-linear Lagrangian.

$$\begin{aligned}
& \frac{1}{36} \sqrt{-\tilde{g}} \left( -9 F_{ab} \left( F^{ab} + 4 \zeta R[D]^{ab} \right) + \right. \\
& 2 \left( 18 \alpha_2 R[D]_{ab} R[D]^{ab} + 18 \left( \frac{\alpha_2}{6} + \frac{\alpha_3}{6} \right) R[D]_{abcd} R[D]^{abcd} + 18 \left( -\frac{2\alpha_2}{3} - \frac{2\alpha_3}{3} \right) R[D]_{acbd} R[D]^{abcd} + \right. \\
& \left. \left. 18 \left( -\frac{5\alpha_2}{6} - \frac{5\alpha_3}{6} \right) R[D]^{abcd} R[D]_{cdab} + 18 \left( -\frac{\alpha_2 \Lambda}{12 \mathcal{M}_{\text{Pl}}^2} - \frac{\alpha_3 \Lambda}{12 \mathcal{M}_{\text{Pl}}^2} \right) T[D]_{abc} T[D]^{abc} + \right. \right.
\end{aligned}$$

(62)

$$\begin{aligned}
& 18 \left( \frac{\alpha_2 \wedge}{6 \mathcal{M}_{\text{Pl}}^2} + \frac{\alpha_3 \wedge}{6 \mathcal{M}_{\text{Pl}}^2} \right) T[D]^{abc} T[D]_{bac} + 8 \mathcal{M}_{\text{Pl}}^2 T[D]_a^b T[D]_{bc}^c + \\
& 36 \theta F^{ab} \left( \nabla_b T[D]_{ac}^c \right) + 6 R[D]^{ab} \left( 3 \alpha_3 R[D]_{ba} + \chi \left( \nabla_a T[D]_{bc}^c \right) - \chi \left( \nabla_b T[D]_{ac}^c \right) \right) + \\
& \xi g^{ab} \left( \nabla_a T[D]_c^d \right) \left( \nabla_b T[D]_{de}^e \right) + \xi \left( \nabla_b T[D]_{da}^a \right) \left( \nabla^d T[D]_{c}^{cb} \right) \Bigg)
\end{aligned}$$

## Here are the zeroth-order equations.

The Cartan components.

$B_x Q_0 \zeta == 0$	(63)
$\frac{(\alpha_2 + \alpha_3) \wedge Q_0}{\mathcal{M}_{\text{Pl}}} == 0$	

The Einstein components.

$\frac{(\alpha_2 + \alpha_3) \wedge Q_0}{\mathcal{M}_{\text{Pl}}} == 0$	(64)
$B_x == 0$	

The Maxwell components.

	(65)
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## Here are the first-order equations.

The Cartan components.

$2 B_x \zeta q_t[t, z] + 2 (5 \alpha_2 + 3 \alpha_3) q_y^{(1,1)}[t, z] == 2 (2 \theta + \zeta) (B_x h_x^{(0,1)}[t, z] - b^{(0,1)}[t, z]) +$ $(10 \alpha_2 + 6 \alpha_3 + \chi) Q_0 u_y^{(0,1)}[t, z] + 4 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$2 (4 \mathcal{M}_{\text{Pl}}^2 + 9 (\alpha_2 + \alpha_3) Q_0^2) u_y[t, z] + \xi u_y^{(0,2)}[t, z] + 2 \chi u_y^{(2,0)}[t, z] +$ $12 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] == 6 Q_0 \zeta b[t, z] + 6 B_x (2 \theta + \zeta) h_+^{(0,1)}[t, z] +$ $(12 \alpha_2 + 5 \chi) u_y^{(0,2)}[t, z] + 6 (5 \alpha_2 + 3 \alpha_3) Q_0 q_y^{(1,0)}[t, z] + \xi u_y^{(2,0)}[t, z]$
$3 B_x \zeta u_y[t, z] + (-2 \chi + \xi) u_t^{(1,1)}[t, z] + 12 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] == 6 (\alpha_2 + 3 \alpha_3) Q_0 q_t^{(0,1)}[t, z]$
$B_x \zeta q_y[t, z] + 2 Q_0 \zeta \epsilon_z[t, x, y, z] + 4 (2 \alpha_2 + \alpha_3) q_t^{(1,1)}[t, z] == (12 \alpha_2 + 8 \alpha_3 + \chi) Q_0 u_t^{(0,1)}[t, z]$
$\frac{1}{2} (2 Q_0 \zeta \epsilon_y[t, x, y, z] + B_x \zeta h_x^{(1,0)}[t, z] + (10 \alpha_2 + 6 \alpha_3 + \chi) Q_0 u_y^{(1,0)}[t, z]) ==$ $\frac{(\alpha_2 + \alpha_3) \wedge q_y[t, z]}{\mathcal{M}_{\text{Pl}}^2} + \zeta b^{(1,0)}[t, z] + (5 \alpha_2 + 3 \alpha_3) q_y^{(2,0)}[t, z]$
$2 B_x \zeta u_t[t, z] + 2 Q_0 \zeta \epsilon_x[t, x, y, z] +$ $4 \alpha_2 Q_0 q_y^{(0,1)}[t, z] + B_x \zeta h_+^{(1,0)}[t, z] + (4 \alpha_2 + \chi) u_y^{(1,1)}[t, z] == 0$

$\frac{(\alpha_2 + \alpha_3) \wedge q_y[t, z]}{\mathcal{M}_{\text{Pl}}^2} + \zeta b^{(1,0)}[t, z] + (5 \alpha_2 + 3 \alpha_3) q_y^{(2,0)}[t, z] ==$ $\frac{1}{2} (2 Q_0 \zeta \epsilon_y[t, x, y, z] + B_x \zeta h_x^{(1,0)}[t, z] + (10 \alpha_2 + 6 \alpha_3 + \chi) Q_0 u_y^{(1,0)}[t, z])$
$4 (2 \mathcal{M}_{\text{Pl}}^2 + 3 (\alpha_2 + \alpha_3) Q_0^2) u_t[t, z] + (-12 \alpha_2 - 5 \chi + \xi) u_t^{(0,2)}[t, z] + 3 (\alpha_2 + \alpha_3)$ $(Q_0 (3 Q_0 h_+^{(1,0)}[t, z] - 8 q_t^{(1,0)}[t, z] + 12 h_x^{(1,1)}[t, z]) - 6 h_+^{(1,2)}[t, z] + 6 h_+^{(3,0)}[t, z]) +$ $6 (2 \theta + \zeta) (\epsilon_z^{(0,0,0,1)}[t, x, y, z] + \epsilon_y^{(0,0,1,0)}[t, x, y, z]) + 12 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z] == 0$
$\frac{1}{2} (Q_0 \zeta \epsilon_z[t, x, y, z] + (\alpha_2 + \alpha_3) (3 Q_0^2 h_x^{(1,0)}[t, z] + 2 q_t^{(1,1)}[t, z] + 6 h_x^{(3,0)}[t, z])) ==$ $\frac{1}{2} B_x \zeta q_y[t, z] + \frac{1}{4} (12 \alpha_2 + 8 \alpha_3 + \chi) Q_0 u_t^{(0,1)}[t, z] +$ $3 (\alpha_2 + \alpha_3) (2 Q_0 h_+^{(1,1)}[t, z] + h_x^{(1,2)}[t, z]) + \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z]$
$\frac{4 (\alpha_2 + \alpha_3) \wedge q_y[t, z]}{\mathcal{M}_{\text{Pl}}} + \mathcal{M}_{\text{Pl}} (2 Q_0 \zeta \epsilon_y[t, x, y, z] + 2 B_x \zeta h_x^{(1,0)}[t, z] +$ $(4 \alpha_2 + \chi) Q_0 u_y^{(1,0)}[t, z] + 4 \zeta \epsilon_z^{(0,1,0,0)}[t, x, y, z]) == 0$
$3 Q_0 \zeta b[t, z] + (8 \mathcal{M}_{\text{Pl}}^2 + 9 (\alpha_2 + \alpha_3) Q_0^2) u_y[t, z] + \xi u_y^{(0,2)}[t, z] +$ $(12 \alpha_2 + 5 \chi) u_y^{(2,0)}[t, z] + 6 (2 \theta + \zeta) \epsilon_y^{(1,0,0,0)}[t, x, y, z] == 3 B_x (4 \theta + \zeta) h_+^{(0,1)}[t, z] +$ $(12 \alpha_2 + 5 \chi) u_y^{(0,2)}[t, z] + 3 (\alpha_2 + 3 \alpha_3) Q_0 q_y^{(1,0)}[t, z] + \xi u_y^{(2,0)}[t, z]$
$3 B_x \zeta u_y[t, z] + \xi u_t^{(1,1)}[t, z] + 9 (\alpha_2 + \alpha_3) (4 Q_0^2 h_+^{(0,1)}[t, z] + 5 Q_0 h_x^{(0,2)}[t, z] + 2 h_+^{(2,1)}[t, z]) +$ $6 (2 \theta + \zeta) \epsilon_z^{(1,0,0,0)}[t, x, y, z] == 3 (5 \alpha_2 + 3 \alpha_3) Q_0 q_t^{(0,1)}[t, z] +$ $18 (\alpha_2 + \alpha_3) h_+^{(0,3)}[t, z] + (12 \alpha_2 + 5 \chi) u_t^{(1,1)}[t, z] + 9 (\alpha_2 + \alpha_3) Q_0 h_x^{(2,0)}[t, z]$
$\frac{1}{2} B_x \zeta q_y[t, z] + \frac{1}{4} (12 \alpha_2 + 8 \alpha_3 + \chi) Q_0 u_t^{(0,1)}[t, z] +$ $3 (\alpha_2 + \alpha_3) (2 Q_0 h_+^{(1,1)}[t, z] + h_x^{(1,2)}[t, z]) + \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z] ==$ $\frac{1}{2} (Q_0 \zeta \epsilon_z[t, x, y, z] + (\alpha_2 + \alpha_3) (3 Q_0^2 h_x^{(1,0)}[t, z] + 2 q_t^{(1,1)}[t, z] + 6 h_x^{(3,0)}[t, z]))$
$\frac{(\alpha_2 + \alpha_3) (2 \wedge q_t[t, z] + \mathcal{M}_{\text{Pl}}^2 (-6 h_x^{(0,3)}[t, z] + Q_0 (12 Q_0 h_x^{(0,1)}[t, z] - 15 h_+^{(0,2)}[t, z] - 4 u_t^{(1,0)}[t, z] + 3 h_+^{(2,0)}[t, z]) + 2 q_t^{(2,0)}[t, z] + 6 h_x^{(2,1)}[t, z]))}{\mathcal{M}_{\text{Pl}}}$ $== 0$
$\frac{1}{4} ((12 \alpha_2 + 8 \alpha_3 + \chi) Q_0 u_t^{(0,1)}[t, z] + 6 (\alpha_2 + \alpha_3) (Q_0^2 h_x^{(1,0)}[t, z] + 2 h_x^{(3,0)}[t, z])) ==$ $B_x \zeta q_y[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_z[t, x, y, z] +$ $(\alpha_2 + \alpha_3) (6 Q_0 h_+^{(1,1)}[t, z] + q_t^{(1,1)}[t, z] + 3 h_x^{(1,2)}[t, z]) + \zeta \epsilon_x^{(0,0,1,0)}[t, x, y, z]$
$4 (2 \mathcal{M}_{\text{Pl}}^2 + 3 (\alpha_2 + \alpha_3) Q_0^2) u_t[t, z] +$ $(-12 \alpha_2 - 5 \chi + \xi) u_t^{(0,2)}[t, z] + 6 (2 \theta + \zeta) \epsilon_z^{(0,0,0,1)}[t, x, y, z] +$ $12 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + 6 (2 \theta + \zeta) \epsilon_x^{(0,1,0,0)}[t, x, y, z] == 3 (\alpha_2 + \alpha_3)$ $(Q_0 (3 Q_0 h_+^{(1,0)}[t, z] + 8 q_t^{(1,0)}[t, z] + 12 h_x^{(1,1)}[t, z]) - 6 h_+^{(1,2)}[t, z] + 6 h_+^{(3,0)}[t, z])$
$\zeta (Q_0 \epsilon_x[t, x, y, z] + B_x (u_t[t, z] + h_+^{(1,0)}[t, z])) ==$ $(\alpha_2 + 3 \alpha_3) Q_0 q_y^{(0,1)}[t, z] + 2 \zeta \epsilon_z^{(0,0,1,0)}[t, x, y, z]$
$B_x \zeta q_y[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_z[t, x, y, z] +$ $(\alpha_2 + \alpha_3) (6 Q_0 h_+^{(1,1)}[t, z] + q_t^{(1,1)}[t, z] + 3 h_x^{(1,2)}[t, z]) + \zeta \epsilon_x^{(0,0,1,0)}[t, x, y, z] ==$ $\frac{1}{4} ((12 \alpha_2 + 8 \alpha_3 + \chi) Q_0 u_t^{(0,1)}[t, z] + 6 (\alpha_2 + \alpha_3) (Q_0^2 h_x^{(1,0)}[t, z] + 2 h_x^{(3,0)}[t, z]))$

$2 B_x (\zeta q_t[t, z] + (4 \theta + \zeta) h_x^{(0,1)}[t, z]) =$ $(16 \alpha_2 + 12 \alpha_3 + \chi) Q_0 u_y^{(0,1)}[t, z] + 4 (2 \theta + \zeta) (b^{(0,1)}[t, z] - \epsilon_x^{(1,0,0,0)}[t, x, y, z])$
$\frac{1}{\mathcal{M}_{\text{Pl}}} (\alpha_2 + \alpha_3)$ $(-2 \wedge q_t[t, z] + \mathcal{M}_{\text{Pl}}^2 (Q_0 (12 Q_0 h_x^{(0,1)}[t, z] - 15 h_+^{(0,2)}[t, z] + 4 u_t^{(1,0)}[t, z] + 3 h_+^{(2,0)}[t, z]) -$ $2 (3 h_x^{(0,3)}[t, z] + q_t^{(2,0)}[t, z] - 3 h_x^{(2,1)}[t, z])) = 0$
$(16 \alpha_2 + 12 \alpha_3 + \chi) Q_0 u_y^{(0,1)}[t, z] + 4 (2 \theta + \zeta) (b^{(0,1)}[t, z] - \epsilon_x^{(1,0,0,0)}[t, x, y, z]) =$ $2 B_x (\zeta q_t[t, z] + (4 \theta + \zeta) h_x^{(0,1)}[t, z])$
$6 B_x \zeta u_y[t, z] + 36 (\alpha_2 + \alpha_3) Q_0^2 h_+^{(0,1)}[t, z] +$ $3 (5 \alpha_2 + 3 \alpha_3) Q_0 q_t^{(0,1)}[t, z] + 45 \alpha_2 Q_0 h_x^{(0,2)}[t, z] + 45 \alpha_3 Q_0 h_x^{(0,2)}[t, z] +$ $12 \alpha_2 u_t^{(1,1)}[t, z] + 5 \chi u_t^{(1,1)}[t, z] + 18 (\alpha_2 + \alpha_3) h_+^{(2,1)}[t, z] =$ $\xi u_t^{(1,1)}[t, z] + 9 (\alpha_2 + \alpha_3) (2 h_+^{(0,3)}[t, z] + Q_0 h_x^{(2,0)}[t, z]) + 6 (2 \theta + \zeta) \epsilon_z^{(1,0,0,0)}[t, x, y, z]$
$\frac{(\alpha_2 + \alpha_3) \wedge q_y[t, z]}{\mathcal{M}_{\text{Pl}}^2} + \frac{1}{4} Q_0 (2 \zeta \epsilon_y[t, x, y, z] + (4 \alpha_2 + \chi) u_y^{(1,0)}[t, z]) =$ $(5 \alpha_2 + 3 \alpha_3) q_y^{(0,2)}[t, z] + \zeta \epsilon_x^{(0,0,0,1)}[t, x, y, z]$
$B_x \zeta u_t[t, z] + Q_0 \zeta \epsilon_x[t, x, y, z] + (5 \alpha_2 + 3 \alpha_3) Q_0 q_y^{(0,1)}[t, z] +$ $(4 \alpha_2 + \chi) u_y^{(1,1)}[t, z] + 2 \zeta \epsilon_y^{(0,0,0,1)}[t, x, y, z] = 0$
$4 (2 \mathcal{M}_{\text{Pl}}^2 + 3 (\alpha_2 + \alpha_3) Q_0^2) u_t[t, z] + (-2 \chi + \xi) u_t^{(0,2)}[t, z] +$ $6 (-4 (\alpha_2 + \alpha_3) Q_0 q_t^{(1,0)}[t, z] + 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] +$ $(2 \theta + \zeta) (\epsilon_y^{(0,0,1,0)}[t, x, y, z] + \epsilon_x^{(0,1,0,0)}[t, x, y, z])) = 0$
$(5 \alpha_2 + 3 \alpha_3) q_y^{(0,2)}[t, z] + \zeta \epsilon_x^{(0,0,0,1)}[t, x, y, z] =$ $\frac{(\alpha_2 + \alpha_3) \wedge q_y[t, z]}{\mathcal{M}_{\text{Pl}}^2} + \frac{1}{4} Q_0 (2 \zeta \epsilon_y[t, x, y, z] + (4 \alpha_2 + \chi) u_y^{(1,0)}[t, z])$
$\frac{(\alpha_2 + \alpha_3) \wedge q_t[t, z]}{\mathcal{M}_{\text{Pl}}} + (\alpha_2 + \alpha_3) \mathcal{M}_{\text{Pl}} (-2 Q_0 u_t^{(1,0)}[t, z] + q_t^{(2,0)}[t, z]) = (5 \alpha_2 + 3 \alpha_3) \mathcal{M}_{\text{Pl}} q_t^{(0,2)}[t, z]$
$2 B_x \zeta q_t[t, z] + 2 B_x (4 \theta + \zeta) h_x^{(0,1)}[t, z] + 4 (5 \alpha_2 + 3 \alpha_3) q_y^{(1,1)}[t, z] +$ $4 (2 \theta + \zeta) \epsilon_x^{(1,0,0,0)}[t, x, y, z] = 8 \theta b^{(0,1)}[t, z] + (4 \alpha_2 + \chi) Q_0 u_y^{(0,1)}[t, z]$
$3 Q_0 \zeta b[t, z] + (8 \mathcal{M}_{\text{Pl}}^2 + 9 (\alpha_2 + \alpha_3) Q_0^2) u_y[t, z] + (-2 \chi + \xi) u_y^{(0,2)}[t, z] +$ $(12 \alpha_2 + 5 \chi - \xi) u_y^{(2,0)}[t, z] + 6 (2 \theta + \zeta) \epsilon_y^{(1,0,0,0)}[t, x, y, z] =$ $3 B_x (4 \theta + \zeta) h_+^{(0,1)}[t, z] + 3 (\alpha_2 + 3 \alpha_3) Q_0 q_y^{(1,0)}[t, z]$

The Einstein components.

$18 (\alpha_2 + 3 \alpha_3) Q_0^3 q_t[t, z] + B_x \zeta u_y^{(0,1)}[t, z] +$ $(\alpha_2 + \alpha_3) (9 Q_0^2 u_t^{(1,0)}[t, z] + 2 u_t^{(1,2)}[t, z] + 6 u_t^{(3,0)}[t, z]) = 0$
$Q_0 (2 B_x \zeta u_t[t, z] + 2 Q_0 \zeta \epsilon_x[t, x, y, z] + 4 \alpha_2 Q_0 q_y^{(0,1)}[t, z] + (4 \alpha_2 + \chi) u_y^{(1,1)}[t, z] +$ $\zeta (B_x h_+^{(1,0)}[t, z] + 2 \epsilon_y^{(0,0,0,1)}[t, x, y, z] - 2 \epsilon_z^{(0,0,1,0)}[t, x, y, z])) = 0$

$\frac{(\alpha_2 + \alpha_3) \wedge Q_0 q_y[t, z]}{\mathcal{M}_{\text{Pl}}^2} + Q_0^2 \zeta \epsilon_y[t, x, y, z] + \frac{1}{2} B_x Q_0 \zeta h_x^{(1,0)}[t, z] + 6 \alpha_2 Q_0^2 u_y^{(1,0)}[t, z] +$ $4 \alpha_3 Q_0^2 u_y^{(1,0)}[t, z] + \frac{1}{2} \chi Q_0^2 u_y^{(1,0)}[t, z] + \alpha_2 Q_0 q_y^{(2,0)}[t, z] +$ $\alpha_3 Q_0 q_y^{(2,0)}[t, z] + \alpha_2 u_y^{(3,0)}[t, z] + \alpha_3 u_y^{(3,0)}[t, z] + Q_0 \zeta \epsilon_z^{(0,1,0,0)}[t, x, y, z] =$ $B_x \epsilon_z[t, x, y, z] + B_x (3 \theta + \zeta) u_t^{(0,1)}[t, z] + 2 \alpha_2 Q_0 q_y^{(0,2)}[t, z] +$ $(\alpha_2 + \alpha_3) u_y^{(1,2)}[t, z] + Q_0 \zeta \epsilon_x^{(0,0,0,1)}[t, x, y, z]$
$B_x \epsilon_y[t, x, y, z] + Q_0^2 \zeta \epsilon_z[t, x, y, z] + (\alpha_2 + \alpha_3) (3 Q_0 q_t^{(1,1)}[t, z] + 5 u_t^{(2,1)}[t, z]) +$ $Q_0 \zeta \epsilon_x^{(0,0,1,0)}[t, x, y, z] = B_x Q_0 \zeta q_y[t, z] + \frac{1}{2} (4 \alpha_2 + \chi) Q_0^2 u_t^{(0,1)}[t, z] +$ $(\alpha_2 + \alpha_3) u_t^{(0,3)}[t, z] + B_x (3 \theta + \zeta) u_y^{(1,0)}[t, z] + Q_0 \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z]$
$Q_0 \left( \frac{2(\alpha_2 + \alpha_3) \wedge}{\mathcal{M}_{\text{Pl}}^2} - 9(\alpha_2 + 3 \alpha_3) Q_0^2 \right) q_t[t, z] +$ $\frac{1}{2} (\alpha_2 + \alpha_3) (Q_0 (36 Q_0 h_+^{(0,2)}[t, z] + 24 h_x^{(0,3)}[t, z] + 3 Q_0 u_t^{(1,0)}[t, z] + 8 q_t^{(2,0)}[t, z]) +$ $6 (2 h_+^{(2,2)}[t, z] + u_t^{(3,0)}[t, z])) =$ $12 (\alpha_2 + \alpha_3) Q_0^3 h_x^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta u_y^{(0,1)}[t, z] + 4 \alpha_2 Q_0 q_t^{(0,2)}[t, z] +$ $3 (\alpha_2 + \alpha_3) (h_+^{(0,4)}[t, z] + u_t^{(1,2)}[t, z] + Q_0^2 h_+^{(2,0)}[t, z] + 4 Q_0 h_x^{(2,1)}[t, z] + h_+^{(4,0)}[t, z])$
$2 B_x b[t, z] + 3 B_x Q_0 \zeta u_y[t, z] =$ $6 (\alpha_2 + \alpha_3) (h_x^{(0,4)}[t, z] + Q_0 (Q_0 (-4 Q_0 h_+^{(0,1)}[t, z] - 6 h_x^{(0,2)}[t, z] + h_x^{(2,0)}[t, z]) +$ $4 (h_+^{(0,3)}[t, z] - h_+^{(2,1)}[t, z])) - 2 h_x^{(2,2)}[t, z] + h_x^{(4,0)}[t, z])$
$(4 \alpha_2 + \chi) Q_0 u_y^{(0,2)}[t, z] = 0$
$Q_0 \left( \frac{2(\alpha_2 + \alpha_3) \wedge}{\mathcal{M}_{\text{Pl}}^2} - 9(\alpha_2 + 3 \alpha_3) Q_0^2 \right) q_t[t, z] + \frac{1}{2} (\alpha_2 + \alpha_3)$ $(Q_0 (3 Q_0 (8 Q_0 h_x^{(0,1)}[t, z] + u_t^{(1,0)}[t, z] + 2 h_+^{(2,0)}[t, z]) + 8 (q_t^{(2,0)}[t, z] + 3 h_x^{(2,1)}[t, z])) +$ $6 (h_+^{(0,4)}[t, z] + u_t^{(3,0)}[t, z] + h_+^{(4,0)}[t, z])) =$ $\frac{1}{2} B_x (12 \theta + 5 \zeta) u_y^{(0,1)}[t, z] + 18 (\alpha_2 + \alpha_3) Q_0^2 h_+^{(0,2)}[t, z] + 4 \alpha_2 Q_0 q_t^{(0,2)}[t, z] +$ $3 (\alpha_2 + \alpha_3) (4 Q_0 h_x^{(0,3)}[t, z] + u_t^{(1,2)}[t, z] + 2 h_+^{(2,2)}[t, z])$
$(\alpha_2 + \alpha_3) Q_0^2 u_y^{(0,1)}[t, z] + \alpha_3 Q_0 q_y^{(1,1)}[t, z] + (\alpha_2 + \alpha_3) u_y^{(2,1)}[t, z] =$ $Q_0 \zeta b^{(0,1)}[t, z] + (\alpha_2 + \alpha_3) u_y^{(0,3)}[t, z] + \alpha_2 Q_0 q_y^{(1,1)}[t, z]$
$\frac{4(\alpha_2 + \alpha_3) \wedge Q_0 q_t[t, z]}{\mathcal{M}_{\text{Pl}}} + (\alpha_2 + \alpha_3) \mathcal{M}_{\text{Pl}}$ $(4 Q_0 q_t^{(0,2)}[t, z] + 3 Q_0^2 u_t^{(1,0)}[t, z] + 2 u_t^{(1,2)}[t, z] + 8 Q_0 q_t^{(2,0)}[t, z] + 6 u_t^{(3,0)}[t, z]) =$ $18 (\alpha_2 + 3 \alpha_3) \mathcal{M}_{\text{Pl}} Q_0^3 q_t[t, z] + B_x \mathcal{M}_{\text{Pl}} (12 \theta + 5 \zeta) u_y^{(0,1)}[t, z]$

The Maxwell components.

$B_x h_+^{(0,2)}[t, z] + \epsilon_z^{(1,0,1,0)}[t, x, y, z] = (3 \theta + \zeta) (u_y^{(0,3)}[t, z] - u_y^{(2,1)}[t, z]) + \epsilon_y^{(1,0,0,1)}[t, x, y, z]$
$B_x h_x^{(0,2)}[t, z] + \epsilon_x^{(1,0,0,1)}[t, x, y, z] = b^{(0,2)}[t, z] + \epsilon_z^{(1,1,0,0)}[t, x, y, z]$
$\epsilon_x^{(1,0,1,0)}[t, x, y, z] = \epsilon_y^{(1,1,0,0)}[t, x, y, z]$

(68)

## Here is the reduced set of first-order equations.

After simplification, we have 60 equations.

$-B_x \zeta q_t[t, z] + 2 B_x \theta h_x^{(0,1)}[t, z] + B_x \zeta h_x^{(0,1)}[t, z] -$ $2 \theta b^{(0,1)}[t, z] - \zeta b^{(0,1)}[t, z] + 5 \alpha_2 Q_0 u_y^{(0,1)}[t, z] + 3 \alpha_3 Q_0 u_y^{(0,1)}[t, z] +$ $\frac{1}{2} \chi Q_0 u_y^{(0,1)}[t, z] - 5 \alpha_2 q_y^{(1,1)}[t, z] - 3 \alpha_3 q_y^{(1,1)}[t, z] + 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$-Q_0 \zeta b[t, z] + \frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_y[t, z] + 3 \alpha_2 Q_0^2 u_y[t, z] + 3 \alpha_3 Q_0^2 u_y[t, z] - 2 B_x \theta h_+^{(0,1)}[t, z] -$ $B_x \zeta h_+^{(0,1)}[t, z] - 2 \alpha_2 u_y^{(0,2)}[t, z] - \frac{5}{6} \chi u_y^{(0,2)}[t, z] + \frac{1}{6} \xi u_y^{(0,2)}[t, z] - 5 \alpha_2 Q_0 q_y^{(1,0)}[t, z] -$ $3 \alpha_3 Q_0 q_y^{(1,0)}[t, z] + \frac{1}{3} \chi u_y^{(2,0)}[t, z] - \frac{1}{6} \xi u_y^{(2,0)}[t, z] + 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z]$
$\frac{1}{2} B_x \zeta u_y[t, z] - \alpha_2 Q_0 q_t^{(0,1)}[t, z] - 3 \alpha_3 Q_0 q_t^{(0,1)}[t, z] -$ $\frac{1}{3} \chi u_t^{(1,1)}[t, z] + \frac{1}{6} \xi u_t^{(1,1)}[t, z] + 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z]$
$B_x \zeta q_t[t, z] - 2 B_x \theta h_x^{(0,1)}[t, z] - B_x \zeta h_x^{(0,1)}[t, z] + 2 \theta b^{(0,1)}[t, z] +$ $\zeta b^{(0,1)}[t, z] - 5 \alpha_2 Q_0 u_y^{(0,1)}[t, z] - 3 \alpha_3 Q_0 u_y^{(0,1)}[t, z] - \frac{1}{2} \chi Q_0 u_y^{(0,1)}[t, z] +$ $5 \alpha_2 q_y^{(1,1)}[t, z] + 3 \alpha_3 q_y^{(1,1)}[t, z] - 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$-\frac{1}{2} B_x \zeta q_y[t, z] - Q_0 \zeta \epsilon_z[t, x, y, z] + 6 \alpha_2 Q_0 u_t^{(0,1)}[t, z] +$ $4 \alpha_3 Q_0 u_t^{(0,1)}[t, z] + \frac{1}{2} \chi Q_0 u_t^{(0,1)}[t, z] - 4 \alpha_2 q_t^{(1,1)}[t, z] - 2 \alpha_3 q_t^{(1,1)}[t, z]$
$-\frac{\alpha_2 \wedge q_y[t, z]}{\mathcal{M}_{\text{Pl}}^2} - \frac{\alpha_3 \wedge q_y[t, z]}{\mathcal{M}_{\text{Pl}}^2} + Q_0 \zeta \epsilon_y[t, x, y, z] + \frac{1}{2} B_x \zeta h_x^{(1,0)}[t, z] -$ $\zeta b^{(1,0)}[t, z] + 5 \alpha_2 Q_0 u_y^{(1,0)}[t, z] + 3 \alpha_3 Q_0 u_y^{(1,0)}[t, z] +$ $\frac{1}{2} \chi Q_0 u_y^{(1,0)}[t, z] - 5 \alpha_2 q_y^{(2,0)}[t, z] - 3 \alpha_3 q_y^{(2,0)}[t, z]$
$Q_0 \zeta b[t, z] - \frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_y[t, z] - 3 \alpha_2 Q_0^2 u_y[t, z] - 3 \alpha_3 Q_0^2 u_y[t, z] + 2 B_x \theta h_+^{(0,1)}[t, z] +$ $B_x \zeta h_+^{(0,1)}[t, z] + 2 \alpha_2 u_y^{(0,2)}[t, z] + \frac{5}{6} \chi u_y^{(0,2)}[t, z] - \frac{1}{6} \xi u_y^{(0,2)}[t, z] + 5 \alpha_2 Q_0 q_y^{(1,0)}[t, z] +$ $3 \alpha_3 Q_0 q_y^{(1,0)}[t, z] - \frac{1}{3} \chi u_y^{(2,0)}[t, z] + \frac{1}{6} \xi u_y^{(2,0)}[t, z] - 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z]$
$\frac{1}{2} B_x \zeta q_y[t, z] + Q_0 \zeta \epsilon_z[t, x, y, z] - 6 \alpha_2 Q_0 u_t^{(0,1)}[t, z] -$ $4 \alpha_3 Q_0 u_t^{(0,1)}[t, z] - \frac{1}{2} \chi Q_0 u_t^{(0,1)}[t, z] + 4 \alpha_2 q_t^{(1,1)}[t, z] + 2 \alpha_3 q_t^{(1,1)}[t, z]$
$-B_x \zeta u_t[t, z] - Q_0 \zeta \epsilon_x[t, x, y, z] - 2 \alpha_2 Q_0 q_y^{(0,1)}[t, z] -$ $\frac{1}{2} B_x \zeta h_+^{(1,0)}[t, z] - 2 \alpha_2 u_y^{(1,1)}[t, z] - \frac{1}{2} \chi u_y^{(1,1)}[t, z]$
$-\frac{1}{2} B_x \zeta u_y[t, z] + \alpha_2 Q_0 q_t^{(0,1)}[t, z] + 3 \alpha_3 Q_0 q_t^{(0,1)}[t, z] +$ $\frac{1}{3} \chi u_t^{(1,1)}[t, z] - \frac{1}{6} \xi u_t^{(1,1)}[t, z] - 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z]$

$\frac{\alpha_2 \wedge q_y[t, z]}{\mathcal{M}_{\text{Pl}}^2} + \frac{\alpha_3 \wedge q_y[t, z]}{\mathcal{M}_{\text{Pl}}^2} - Q_0 \zeta \epsilon_y[t, x, y, z] - \frac{1}{2} B_x \zeta h_x^{(1,0)}[t, z] + \zeta b^{(1,0)}[t, z] - 5 \alpha_2 Q_0 u_y^{(1,0)}[t, z] - 3 \alpha_3 Q_0 u_y^{(1,0)}[t, z] - \frac{1}{2} \chi Q_0 u_y^{(1,0)}[t, z] + 5 \alpha_2 q_y^{(2,0)}[t, z] + 3 \alpha_3 q_y^{(2,0)}[t, z]$
$B_x \zeta u_t[t, z] + Q_0 \zeta \epsilon_x[t, x, y, z] + 2 \alpha_2 Q_0 q_y^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_+^{(1,0)}[t, z] + 2 \alpha_2 u_y^{(1,1)}[t, z] + \frac{1}{2} \chi u_y^{(1,1)}[t, z]$
$\frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_t[t, z] + 2 \alpha_2 Q_0^2 u_t[t, z] + 2 \alpha_3 Q_0^2 u_t[t, z] - 2 \alpha_2 u_t^{(0,2)}[t, z] - \frac{5}{6} \chi u_t^{(0,2)}[t, z] + \frac{1}{6} \xi u_t^{(0,2)}[t, z] + \frac{3}{2} \alpha_2 Q_0^2 h_+^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0^2 h_+^{(1,0)}[t, z] - 4 \alpha_2 Q_0 q_t^{(1,0)}[t, z] - 4 \alpha_3 Q_0 q_t^{(1,0)}[t, z] + 6 \alpha_2 Q_0 h_x^{(1,1)}[t, z] + 6 \alpha_3 Q_0 h_x^{(1,1)}[t, z] - 3 \alpha_2 h_+^{(1,2)}[t, z] - 3 \alpha_3 h_+^{(1,2)}[t, z] + 3 \alpha_2 h_+^{(3,0)}[t, z] + 3 \alpha_3 h_+^{(3,0)}[t, z] + 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + \zeta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + 2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + \zeta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$
$-\frac{1}{2} B_x \zeta q_y[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_z[t, x, y, z] - 3 \alpha_2 Q_0 u_t^{(0,1)}[t, z] - 2 \alpha_3 Q_0 u_t^{(0,1)}[t, z] - \frac{1}{4} \chi Q_0 u_t^{(0,1)}[t, z] + \frac{3}{2} \alpha_2 Q_0^2 h_x^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0^2 h_x^{(1,0)}[t, z] - 6 \alpha_2 Q_0 h_+^{(1,1)}[t, z] - 6 \alpha_3 Q_0 h_+^{(1,1)}[t, z] + \alpha_2 q_t^{(1,1)}[t, z] + \alpha_3 q_t^{(1,1)}[t, z] - 3 \alpha_2 h_x^{(1,2)}[t, z] - 3 \alpha_3 h_x^{(1,2)}[t, z] + 3 \alpha_2 h_x^{(3,0)}[t, z] + 3 \alpha_3 h_x^{(3,0)}[t, z] - \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z]$
$-\frac{\alpha_2 \wedge q_y[t, z]}{\mathcal{M}_{\text{Pl}}^2} - \frac{\alpha_3 \wedge q_y[t, z]}{\mathcal{M}_{\text{Pl}}^2} - \frac{1}{2} Q_0 \zeta \epsilon_y[t, x, y, z] - \frac{1}{2} B_x \zeta h_x^{(1,0)}[t, z] - \alpha_2 Q_0 u_y^{(1,0)}[t, z] - \frac{1}{4} \chi Q_0 u_y^{(1,0)}[t, z] - \zeta \epsilon_z^{(0,1,0,0)}[t, x, y, z]$
$-\frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_t[t, z] - 2 \alpha_2 Q_0^2 u_t[t, z] - 2 \alpha_3 Q_0^2 u_t[t, z] + 2 \alpha_2 u_t^{(0,2)}[t, z] + \frac{5}{6} \chi u_t^{(0,2)}[t, z] - \frac{1}{6} \xi u_t^{(0,2)}[t, z] - \frac{3}{2} \alpha_2 Q_0^2 h_+^{(1,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0^2 h_+^{(1,0)}[t, z] + 4 \alpha_2 Q_0 q_t^{(1,0)}[t, z] + 4 \alpha_3 Q_0 q_t^{(1,0)}[t, z] - 6 \alpha_2 Q_0 h_x^{(1,1)}[t, z] - 6 \alpha_3 Q_0 h_x^{(1,1)}[t, z] + 3 \alpha_2 h_+^{(1,2)}[t, z] + 3 \alpha_3 h_+^{(1,2)}[t, z] - 3 \alpha_2 h_+^{(3,0)}[t, z] - 3 \alpha_3 h_+^{(3,0)}[t, z] - 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] - \zeta \epsilon_z^{(0,0,0,1)}[t, x, y, z] - 2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] - \zeta \epsilon_y^{(0,0,1,0)}[t, x, y, z] - 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$
$\frac{1}{2} Q_0 \zeta b[t, z] + \frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_y[t, z] + \frac{3}{2} \alpha_2 Q_0^2 u_y[t, z] + \frac{3}{2} \alpha_3 Q_0^2 u_y[t, z] - 2 B_x \theta h_+^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_+^{(0,1)}[t, z] - 2 \alpha_2 u_y^{(0,2)}[t, z] - \frac{5}{6} \chi u_y^{(0,2)}[t, z] + \frac{1}{6} \xi u_y^{(0,2)}[t, z] - \frac{1}{2} \alpha_2 Q_0 q_y^{(1,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0 q_y^{(1,0)}[t, z] + 2 \alpha_2 u_y^{(2,0)}[t, z] + \frac{5}{6} \chi u_y^{(2,0)}[t, z] - \frac{1}{6} \xi u_y^{(2,0)}[t, z] + 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_y^{(1,0,0,0)}[t, x, y, z]$
$\frac{1}{2} B_x \zeta u_y[t, z] + 6 \alpha_2 Q_0^2 h_+^{(0,1)}[t, z] + 6 \alpha_3 Q_0^2 h_+^{(0,1)}[t, z] - \frac{5}{2} \alpha_2 Q_0 q_t^{(0,1)}[t, z] - \frac{3}{2} \alpha_3 Q_0 q_t^{(0,1)}[t, z] + \frac{15}{2} \alpha_2 Q_0 h_x^{(0,2)}[t, z] + \frac{15}{2} \alpha_3 Q_0 h_x^{(0,2)}[t, z] - 3 \alpha_2 h_+^{(0,3)}[t, z] - 3 \alpha_3 h_+^{(0,3)}[t, z] - 2 \alpha_2 u_t^{(1,1)}[t, z] - \frac{5}{6} \chi u_t^{(1,1)}[t, z] + \frac{1}{6} \xi u_t^{(1,1)}[t, z] - \frac{3}{2} \alpha_2 Q_0 h_x^{(2,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0 h_x^{(2,0)}[t, z] + 3 \alpha_2 h_+^{(2,1)}[t, z] + 3 \alpha_3 h_+^{(2,1)}[t, z] + 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_z^{(1,0,0,0)}[t, x, y, z]$

$$\begin{aligned}
& \frac{1}{2} B_x \zeta q_y[t, z] - \frac{1}{2} Q_0 \zeta \epsilon_z[t, x, y, z] + 3 \alpha_2 Q_0 u_t^{(0,1)}[t, z] + 2 \alpha_3 Q_0 u_t^{(0,1)}[t, z] + \\
& \frac{1}{4} \chi Q_0 u_t^{(0,1)}[t, z] - \frac{3}{2} \alpha_2 Q_0^2 h_x^{(1,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0^2 h_x^{(1,0)}[t, z] + 6 \alpha_2 Q_0 h_+^{(1,1)}[t, z] + \\
& 6 \alpha_3 Q_0 h_+^{(1,1)}[t, z] - \alpha_2 q_t^{(1,1)}[t, z] - \alpha_3 q_t^{(1,1)}[t, z] + 3 \alpha_2 h_x^{(1,2)}[t, z] + \\
& 3 \alpha_3 h_x^{(1,2)}[t, z] - 3 \alpha_2 h_x^{(3,0)}[t, z] - 3 \alpha_3 h_x^{(3,0)}[t, z] + \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z] \\
& - \frac{1}{2} Q_0 \zeta b[t, z] - \frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_y[t, z] - \frac{3}{2} \alpha_2 Q_0^2 u_y[t, z] - \frac{3}{2} \alpha_3 Q_0^2 u_y[t, z] + \\
& 2 B_x \theta h_+^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_+^{(0,1)}[t, z] + 2 \alpha_2 u_y^{(0,2)}[t, z] + \frac{5}{6} \chi u_y^{(0,2)}[t, z] - \\
& \frac{1}{6} \xi u_y^{(0,2)}[t, z] + \frac{1}{2} \alpha_2 Q_0 q_y^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0 q_y^{(1,0)}[t, z] - 2 \alpha_2 u_y^{(2,0)}[t, z] - \\
& \frac{5}{6} \chi u_y^{(2,0)}[t, z] + \frac{1}{6} \xi u_y^{(2,0)}[t, z] - 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_y^{(1,0,0,0)}[t, x, y, z] \\
& \frac{\alpha_2 \wedge q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} + \frac{\alpha_3 \wedge q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} + 6 \alpha_2 Q_0^2 h_x^{(0,1)}[t, z] + 6 \alpha_3 Q_0^2 h_x^{(0,1)}[t, z] - \\
& \frac{15}{2} \alpha_2 Q_0 h_+^{(0,2)}[t, z] - \frac{15}{2} \alpha_3 Q_0 h_+^{(0,2)}[t, z] - 3 \alpha_2 h_x^{(0,3)}[t, z] - 3 \alpha_3 h_x^{(0,3)}[t, z] - \\
& 2 \alpha_2 Q_0 u_t^{(1,0)}[t, z] - 2 \alpha_3 Q_0 u_t^{(1,0)}[t, z] + \frac{3}{2} \alpha_2 Q_0 h_+^{(2,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0 h_+^{(2,0)}[t, z] + \\
& \alpha_2 q_t^{(2,0)}[t, z] + \alpha_3 q_t^{(2,0)}[t, z] + 3 \alpha_2 h_x^{(2,1)}[t, z] + 3 \alpha_3 h_x^{(2,1)}[t, z] \\
& \frac{\alpha_2 \wedge q_y[t, z]}{\mathcal{M}_{\text{Pl}}^2} + \frac{\alpha_3 \wedge q_y[t, z]}{\mathcal{M}_{\text{Pl}}^2} + \frac{1}{2} Q_0 \zeta \epsilon_y[t, x, y, z] + \frac{1}{2} B_x \zeta h_x^{(1,0)}[t, z] + \\
& \alpha_2 Q_0 u_y^{(1,0)}[t, z] + \frac{1}{4} \chi Q_0 u_y^{(1,0)}[t, z] + \zeta \epsilon_z^{(0,1,0,0)}[t, x, y, z] \\
& - \frac{1}{2} B_x \zeta u_y[t, z] - 6 \alpha_2 Q_0^2 h_+^{(0,1)}[t, z] - 6 \alpha_3 Q_0^2 h_+^{(0,1)}[t, z] + \\
& \frac{5}{2} \alpha_2 Q_0 q_t^{(0,1)}[t, z] + \frac{3}{2} \alpha_3 Q_0 q_t^{(0,1)}[t, z] - \frac{15}{2} \alpha_2 Q_0 h_x^{(0,2)}[t, z] - \\
& \frac{15}{2} \alpha_3 Q_0 h_x^{(0,2)}[t, z] + 3 \alpha_2 h_+^{(0,3)}[t, z] + 3 \alpha_3 h_+^{(0,3)}[t, z] + 2 \alpha_2 u_t^{(1,1)}[t, z] + \\
& \frac{5}{6} \chi u_t^{(1,1)}[t, z] - \frac{1}{6} \xi u_t^{(1,1)}[t, z] + \frac{3}{2} \alpha_2 Q_0 h_x^{(2,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0 h_x^{(2,0)}[t, z] - \\
& 3 \alpha_2 h_+^{(2,1)}[t, z] - 3 \alpha_3 h_+^{(2,1)}[t, z] - 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_z^{(1,0,0,0)}[t, x, y, z] \\
& - \frac{\alpha_2 \wedge q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} - \frac{\alpha_3 \wedge q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} - 6 \alpha_2 Q_0^2 h_x^{(0,1)}[t, z] - 6 \alpha_3 Q_0^2 h_x^{(0,1)}[t, z] + \\
& \frac{15}{2} \alpha_2 Q_0 h_+^{(0,2)}[t, z] + \frac{15}{2} \alpha_3 Q_0 h_+^{(0,2)}[t, z] + 3 \alpha_2 h_x^{(0,3)}[t, z] + 3 \alpha_3 h_x^{(0,3)}[t, z] + \\
& 2 \alpha_2 Q_0 u_t^{(1,0)}[t, z] + 2 \alpha_3 Q_0 u_t^{(1,0)}[t, z] - \frac{3}{2} \alpha_2 Q_0 h_+^{(2,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0 h_+^{(2,0)}[t, z] - \\
& \alpha_2 q_t^{(2,0)}[t, z] - \alpha_3 q_t^{(2,0)}[t, z] - 3 \alpha_2 h_x^{(2,1)}[t, z] - 3 \alpha_3 h_x^{(2,1)}[t, z] \\
& - B_x \zeta q_y[t, z] - \frac{1}{2} Q_0 \zeta \epsilon_z[t, x, y, z] + 3 \alpha_2 Q_0 u_t^{(0,1)}[t, z] + 2 \alpha_3 Q_0 u_t^{(0,1)}[t, z] + \\
& \frac{1}{4} \chi Q_0 u_t^{(0,1)}[t, z] + \frac{3}{2} \alpha_2 Q_0^2 h_x^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0^2 h_x^{(1,0)}[t, z] - 6 \alpha_2 Q_0 h_+^{(1,1)}[t, z] - \\
& 6 \alpha_3 Q_0 h_+^{(1,1)}[t, z] - \alpha_2 q_t^{(1,1)}[t, z] - \alpha_3 q_t^{(1,1)}[t, z] - 3 \alpha_2 h_x^{(1,2)}[t, z] - \\
& 3 \alpha_3 h_x^{(1,2)}[t, z] + 3 \alpha_2 h_x^{(3,0)}[t, z] + 3 \alpha_3 h_x^{(3,0)}[t, z] - \zeta \epsilon_x^{(0,0,1,0)}[t, x, y, z] \\
& \frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_t[t, z] + 2 \alpha_2 Q_0^2 u_t[t, z] + 2 \alpha_3 Q_0^2 u_t[t, z] - 2 \alpha_2 u_t^{(0,2)}[t, z] - \\
& \frac{5}{6} \chi u_t^{(0,2)}[t, z] + \frac{1}{6} \xi u_t^{(0,2)}[t, z] - \frac{3}{2} \alpha_2 Q_0^2 h_+^{(1,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0^2 h_+^{(1,0)}[t, z] - \\
& 4 \alpha_2 Q_0 q_t^{(1,0)}[t, z] - 4 \alpha_3 Q_0 q_t^{(1,0)}[t, z] - 6 \alpha_2 Q_0 h_x^{(1,1)}[t, z] - \\
& 6 \alpha_3 Q_0 h_x^{(1,1)}[t, z] + 3 \alpha_2 h_+^{(1,2)}[t, z] + 3 \alpha_3 h_+^{(1,2)}[t, z] - 3 \alpha_2 h_+^{(3,0)}[t, z] - \\
& 3 \alpha_3 h_+^{(3,0)}[t, z] + 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + \zeta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + \\
& 2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z] + \zeta \epsilon_x^{(0,1,0,0)}[t, x, y, z]
\end{aligned}$$



$\frac{1}{2} B_x \zeta u_t[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_x[t, x, y, z] - \frac{1}{2} \alpha_2 Q_0 q_y^{(0,1)}[t, z] -$ $\frac{3}{2} \alpha_3 Q_0 q_y^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_+^{(1,0)}[t, z] - \zeta \epsilon_z^{(0,0,1,0)}[t, x, y, z]$
$B_x \zeta q_y[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_z[t, x, y, z] - 3 \alpha_2 Q_0 u_t^{(0,1)}[t, z] - 2 \alpha_3 Q_0 u_t^{(0,1)}[t, z] -$ $\frac{1}{4} \chi Q_0 u_t^{(0,1)}[t, z] - \frac{3}{2} \alpha_2 Q_0^2 h_x^{(1,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0^2 h_x^{(1,0)}[t, z] + 6 \alpha_2 Q_0 h_+^{(1,1)}[t, z] +$ $6 \alpha_3 Q_0 h_+^{(1,1)}[t, z] + \alpha_2 q_t^{(1,1)}[t, z] + \alpha_3 q_t^{(1,1)}[t, z] + 3 \alpha_2 h_x^{(1,2)}[t, z] +$ $3 \alpha_3 h_x^{(1,2)}[t, z] - 3 \alpha_2 h_x^{(3,0)}[t, z] - 3 \alpha_3 h_x^{(3,0)}[t, z] + \zeta \epsilon_x^{(0,0,1,0)}[t, x, y, z]$
$-\frac{1}{2} B_x \zeta q_t[t, z] - 2 B_x \theta h_x^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_x^{(0,1)}[t, z] +$ $2 \theta b^{(0,1)}[t, z] + \zeta b^{(0,1)}[t, z] + 4 \alpha_2 Q_0 u_y^{(0,1)}[t, z] + 3 \alpha_3 Q_0 u_y^{(0,1)}[t, z] +$ $\frac{1}{4} \chi Q_0 u_y^{(0,1)}[t, z] - 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$-\frac{\alpha_2 \wedge q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} - \frac{\alpha_3 \wedge q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} + 6 \alpha_2 Q_0^2 h_x^{(0,1)}[t, z] + 6 \alpha_3 Q_0^2 h_x^{(0,1)}[t, z] -$ $\frac{15}{2} \alpha_2 Q_0 h_+^{(0,2)}[t, z] - \frac{15}{2} \alpha_3 Q_0 h_+^{(0,2)}[t, z] - 3 \alpha_2 h_x^{(0,3)}[t, z] - 3 \alpha_3 h_x^{(0,3)}[t, z] +$ $2 \alpha_2 Q_0 u_t^{(1,0)}[t, z] + 2 \alpha_3 Q_0 u_t^{(1,0)}[t, z] + \frac{3}{2} \alpha_2 Q_0 h_+^{(2,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0 h_+^{(2,0)}[t, z] -$ $\alpha_2 q_t^{(2,0)}[t, z] - \alpha_3 q_t^{(2,0)}[t, z] + 3 \alpha_2 h_x^{(2,1)}[t, z] + 3 \alpha_3 h_x^{(2,1)}[t, z]$
$-\frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_t[t, z] - 2 \alpha_2 Q_0^2 u_t[t, z] - 2 \alpha_3 Q_0^2 u_t[t, z] + 2 \alpha_2 u_t^{(0,2)}[t, z] +$ $\frac{5}{6} \chi u_t^{(0,2)}[t, z] - \frac{1}{6} \xi u_t^{(0,2)}[t, z] + \frac{3}{2} \alpha_2 Q_0^2 h_+^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0^2 h_+^{(1,0)}[t, z] +$ $4 \alpha_2 Q_0 q_t^{(1,0)}[t, z] + 4 \alpha_3 Q_0 q_t^{(1,0)}[t, z] + 6 \alpha_2 Q_0 h_x^{(1,1)}[t, z] +$ $6 \alpha_3 Q_0 h_x^{(1,1)}[t, z] - 3 \alpha_2 h_+^{(1,2)}[t, z] - 3 \alpha_3 h_+^{(1,2)}[t, z] + 3 \alpha_2 h_+^{(3,0)}[t, z] +$ $3 \alpha_3 h_+^{(3,0)}[t, z] - 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] - \zeta \epsilon_z^{(0,0,0,1)}[t, x, y, z] -$ $2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] - 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z] - \zeta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$
$\frac{1}{2} B_x \zeta q_t[t, z] + 2 B_x \theta h_x^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_x^{(0,1)}[t, z] -$ $2 \theta b^{(0,1)}[t, z] - \zeta b^{(0,1)}[t, z] - 4 \alpha_2 Q_0 u_y^{(0,1)}[t, z] - 3 \alpha_3 Q_0 u_y^{(0,1)}[t, z] -$ $\frac{1}{4} \chi Q_0 u_y^{(0,1)}[t, z] + 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$-B_x \zeta u_y[t, z] - 6 \alpha_2 Q_0^2 h_+^{(0,1)}[t, z] - 6 \alpha_3 Q_0^2 h_+^{(0,1)}[t, z] -$ $\frac{5}{2} \alpha_2 Q_0 q_t^{(0,1)}[t, z] - \frac{3}{2} \alpha_3 Q_0 q_t^{(0,1)}[t, z] - \frac{15}{2} \alpha_2 Q_0 h_x^{(0,2)}[t, z] -$ $\frac{15}{2} \alpha_3 Q_0 h_x^{(0,2)}[t, z] + 3 \alpha_2 h_+^{(0,3)}[t, z] + 3 \alpha_3 h_+^{(0,3)}[t, z] - 2 \alpha_2 u_t^{(1,1)}[t, z] -$ $\frac{5}{6} \chi u_t^{(1,1)}[t, z] + \frac{1}{6} \xi u_t^{(1,1)}[t, z] + \frac{3}{2} \alpha_2 Q_0 h_x^{(2,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0 h_x^{(2,0)}[t, z] -$ $3 \alpha_2 h_+^{(2,1)}[t, z] - 3 \alpha_3 h_+^{(2,1)}[t, z] + 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_z^{(1,0,0,0)}[t, x, y, z]$
$-\frac{1}{2} B_x \zeta u_t[t, z] - \frac{1}{2} Q_0 \zeta \epsilon_x[t, x, y, z] + \frac{1}{2} \alpha_2 Q_0 q_y^{(0,1)}[t, z] +$ $\frac{3}{2} \alpha_3 Q_0 q_y^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_+^{(1,0)}[t, z] + \zeta \epsilon_z^{(0,0,1,0)}[t, x, y, z]$
$\frac{\alpha_2 \wedge q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} + \frac{\alpha_3 \wedge q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} - 6 \alpha_2 Q_0^2 h_x^{(0,1)}[t, z] - 6 \alpha_3 Q_0^2 h_x^{(0,1)}[t, z] +$ $\frac{15}{2} \alpha_2 Q_0 h_+^{(0,2)}[t, z] + \frac{15}{2} \alpha_3 Q_0 h_+^{(0,2)}[t, z] + 3 \alpha_2 h_x^{(0,3)}[t, z] + 3 \alpha_3 h_x^{(0,3)}[t, z] -$ $2 \alpha_2 Q_0 u_t^{(1,0)}[t, z] - 2 \alpha_3 Q_0 u_t^{(1,0)}[t, z] - \frac{3}{2} \alpha_2 Q_0 h_+^{(2,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0 h_+^{(2,0)}[t, z] +$ $\alpha_2 q_t^{(2,0)}[t, z] + \alpha_3 q_t^{(2,0)}[t, z] - 3 \alpha_2 h_x^{(2,1)}[t, z] - 3 \alpha_3 h_x^{(2,1)}[t, z]$

$$\begin{aligned}
& B_x \zeta u_y[t, z] + 6 \alpha_2 Q_0^2 h_+^{(0,1)}[t, z] + 6 \alpha_3 Q_0^2 h_+^{(0,1)}[t, z] + \\
& \frac{5}{2} \alpha_2 Q_0 q_t^{(0,1)}[t, z] + \frac{3}{2} \alpha_3 Q_0 q_t^{(0,1)}[t, z] + \frac{15}{2} \alpha_2 Q_0 h_x^{(0,2)}[t, z] + \\
& \frac{15}{2} \alpha_3 Q_0 h_x^{(0,2)}[t, z] - 3 \alpha_2 h_+^{(0,3)}[t, z] - 3 \alpha_3 h_+^{(0,3)}[t, z] + 2 \alpha_2 u_t^{(1,1)}[t, z] + \\
& \frac{5}{6} \chi u_t^{(1,1)}[t, z] - \frac{1}{6} \xi u_t^{(1,1)}[t, z] - \frac{3}{2} \alpha_2 Q_0 h_x^{(2,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0 h_x^{(2,0)}[t, z] + \\
& 3 \alpha_2 h_+^{(2,1)}[t, z] + 3 \alpha_3 h_+^{(2,1)}[t, z] - 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_z^{(1,0,0,0)}[t, x, y, z] \\
& \frac{\alpha_2 \wedge q_y[t, z]}{\mathcal{M}_{\text{Pl}}^2} + \frac{\alpha_3 \wedge q_y[t, z]}{\mathcal{M}_{\text{Pl}}^2} + \frac{1}{2} Q_0 \zeta \epsilon_y[t, x, y, z] - 5 \alpha_2 q_y^{(0,2)}[t, z] - \\
& 3 \alpha_3 q_y^{(0,2)}[t, z] + \alpha_2 Q_0 u_y^{(1,0)}[t, z] + \frac{1}{4} \chi Q_0 u_y^{(1,0)}[t, z] - \zeta \epsilon_x^{(0,0,0,1)}[t, x, y, z] \\
& -\frac{1}{2} B_x \zeta u_t[t, z] - \frac{1}{2} Q_0 \zeta \epsilon_x[t, x, y, z] - \frac{5}{2} \alpha_2 Q_0 q_y^{(0,1)}[t, z] - \\
& \frac{3}{2} \alpha_3 Q_0 q_y^{(0,1)}[t, z] - 2 \alpha_2 u_y^{(1,1)}[t, z] - \frac{1}{2} \chi u_y^{(1,1)}[t, z] - \zeta \epsilon_y^{(0,0,0,1)}[t, x, y, z] \\
& \frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_t[t, z] + 2 \alpha_2 Q_0^2 u_t[t, z] + 2 \alpha_3 Q_0^2 u_t[t, z] - \frac{1}{3} \chi u_t^{(0,2)}[t, z] + \frac{1}{6} \xi u_t^{(0,2)}[t, z] - \\
& 4 \alpha_2 Q_0 q_t^{(1,0)}[t, z] - 4 \alpha_3 Q_0 q_t^{(1,0)}[t, z] + 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + 2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + \\
& \zeta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z] + \zeta \epsilon_x^{(0,1,0,0)}[t, x, y, z] \\
& -\frac{\alpha_2 \wedge q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} - \frac{\alpha_3 \wedge q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} - \frac{1}{2} Q_0 \zeta \epsilon_y[t, x, y, z] + 5 \alpha_2 q_y^{(0,2)}[t, z] + \\
& 3 \alpha_3 q_y^{(0,2)}[t, z] - \alpha_2 Q_0 u_y^{(1,0)}[t, z] - \frac{1}{4} \chi Q_0 u_y^{(1,0)}[t, z] + \zeta \epsilon_x^{(0,0,0,1)}[t, x, y, z] \\
& \frac{\alpha_2 \wedge q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} + \frac{\alpha_3 \wedge q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} - 5 \alpha_2 q_t^{(0,2)}[t, z] - 3 \alpha_3 q_t^{(0,2)}[t, z] - \\
& 2 \alpha_2 Q_0 u_t^{(1,0)}[t, z] - 2 \alpha_3 Q_0 u_t^{(1,0)}[t, z] + \alpha_2 q_t^{(2,0)}[t, z] + \alpha_3 q_t^{(2,0)}[t, z] \\
& -\frac{1}{2} B_x \zeta q_t[t, z] - 2 B_x \theta h_x^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_x^{(0,1)}[t, z] + \\
& 2 \theta b^{(0,1)}[t, z] + \alpha_2 Q_0 u_y^{(0,1)}[t, z] + \frac{1}{4} \chi Q_0 u_y^{(0,1)}[t, z] - 5 \alpha_2 q_y^{(1,1)}[t, z] - \\
& 3 \alpha_3 q_y^{(1,1)}[t, z] - 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_x^{(1,0,0,0)}[t, x, y, z] \\
& \frac{1}{2} B_x \zeta u_t[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_x[t, x, y, z] + \frac{5}{2} \alpha_2 Q_0 q_y^{(0,1)}[t, z] + \\
& \frac{3}{2} \alpha_3 Q_0 q_y^{(0,1)}[t, z] + 2 \alpha_2 u_y^{(1,1)}[t, z] + \frac{1}{2} \chi u_y^{(1,1)}[t, z] + \zeta \epsilon_y^{(0,0,0,1)}[t, x, y, z] \\
& -\frac{\alpha_2 \wedge q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} - \frac{\alpha_3 \wedge q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} + 5 \alpha_2 q_t^{(0,2)}[t, z] + 3 \alpha_3 q_t^{(0,2)}[t, z] + \\
& 2 \alpha_2 Q_0 u_t^{(1,0)}[t, z] + 2 \alpha_3 Q_0 u_t^{(1,0)}[t, z] - \alpha_2 q_t^{(2,0)}[t, z] - \alpha_3 q_t^{(2,0)}[t, z] \\
& -\frac{1}{2} Q_0 \zeta b[t, z] - \frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_y[t, z] - \frac{3}{2} \alpha_2 Q_0^2 u_y[t, z] - \frac{3}{2} \alpha_3 Q_0^2 u_y[t, z] + \\
& 2 B_x \theta h_+^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_+^{(0,1)}[t, z] + \frac{1}{3} \chi u_y^{(0,2)}[t, z] - \frac{1}{6} \xi u_y^{(0,2)}[t, z] + \\
& \frac{1}{2} \alpha_2 Q_0 q_y^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0 q_y^{(1,0)}[t, z] - 2 \alpha_2 u_y^{(2,0)}[t, z] - \frac{5}{6} \chi u_y^{(2,0)}[t, z] + \\
& \frac{1}{6} \xi u_y^{(2,0)}[t, z] - 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_y^{(1,0,0,0)}[t, x, y, z] \\
& -\frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_t[t, z] - 2 \alpha_2 Q_0^2 u_t[t, z] - 2 \alpha_3 Q_0^2 u_t[t, z] + \frac{1}{3} \chi u_t^{(0,2)}[t, z] - \frac{1}{6} \xi u_t^{(0,2)}[t, z] + \\
& 4 \alpha_2 Q_0 q_t^{(1,0)}[t, z] + 4 \alpha_3 Q_0 q_t^{(1,0)}[t, z] - 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] - 2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] - \\
& \zeta \epsilon_y^{(0,0,1,0)}[t, x, y, z] - 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z] - \zeta \epsilon_x^{(0,1,0,0)}[t, x, y, z]
\end{aligned}$$

$\frac{1}{2} B_x \zeta q_t[t, z] + 2 B_x \theta h_x^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_x^{(0,1)}[t, z] -$ $2 \theta b^{(0,1)}[t, z] - \alpha_2 Q_0 u_y^{(0,1)}[t, z] - \frac{1}{4} \chi Q_0 u_y^{(0,1)}[t, z] + 5 \alpha_2 q_y^{(1,1)}[t, z] +$ $3 \alpha_3 q_y^{(1,1)}[t, z] + 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$\frac{1}{2} Q_0 \zeta b[t, z] + \frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_y[t, z] + \frac{3}{2} \alpha_2 Q_0^2 u_y[t, z] + \frac{3}{2} \alpha_3 Q_0^2 u_y[t, z] -$ $2 B_x \theta h_+^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_+^{(0,1)}[t, z] - \frac{1}{3} \chi u_y^{(0,2)}[t, z] + \frac{1}{6} \xi u_y^{(0,2)}[t, z] -$ $\frac{1}{2} \alpha_2 Q_0 q_y^{(1,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0 q_y^{(1,0)}[t, z] + 2 \alpha_2 u_y^{(2,0)}[t, z] + \frac{5}{6} \chi u_y^{(2,0)}[t, z] -$ $\frac{1}{6} \xi u_y^{(2,0)}[t, z] + 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_y^{(1,0,0,0)}[t, x, y, z]$
$9 \alpha_2 Q_0^3 q_t[t, z] + 27 \alpha_3 Q_0^3 q_t[t, z] + \frac{1}{2} B_x \zeta u_y^{(0,1)}[t, z] + \frac{9}{2} \alpha_2 Q_0^2 u_t^{(1,0)}[t, z] +$ $\frac{9}{2} \alpha_3 Q_0^2 u_t^{(1,0)}[t, z] + \alpha_2 u_t^{(1,2)}[t, z] + \alpha_3 u_t^{(1,2)}[t, z] + 3 \alpha_2 u_t^{(3,0)}[t, z] + 3 \alpha_3 u_t^{(3,0)}[t, z]$
$B_x Q_0 \zeta u_t[t, z] + Q_0^2 \zeta \epsilon_x[t, x, y, z] + 2 \alpha_2 Q_0^2 q_y^{(0,1)}[t, z] + \frac{1}{2} B_x Q_0 \zeta h_+^{(1,0)}[t, z] +$ $2 \alpha_2 Q_0 u_y^{(1,1)}[t, z] + \frac{1}{2} \chi Q_0 u_y^{(1,1)}[t, z] + Q_0 \zeta \epsilon_y^{(0,0,0,1)}[t, x, y, z] - Q_0 \zeta \epsilon_z^{(0,0,1,0)}[t, x, y, z]$
$\frac{\alpha_2 \wedge Q_0 q_y[t, z]}{\mathcal{M}_{\text{Pl}}^2} + \frac{\alpha_3 \wedge Q_0 q_y[t, z]}{\mathcal{M}_{\text{Pl}}^2} + Q_0^2 \zeta \epsilon_y[t, x, y, z] - B_x \epsilon_z[t, x, y, z] -$ $3 B_x \theta u_t^{(0,1)}[t, z] - B_x \zeta u_t^{(0,1)}[t, z] - 2 \alpha_2 Q_0 q_y^{(0,2)}[t, z] + \frac{1}{2} B_x Q_0 \zeta h_x^{(1,0)}[t, z] +$ $6 \alpha_2 Q_0^2 u_y^{(1,0)}[t, z] + 4 \alpha_3 Q_0^2 u_y^{(1,0)}[t, z] + \frac{1}{2} \chi Q_0^2 u_y^{(1,0)}[t, z] - \alpha_2 u_y^{(1,2)}[t, z] -$ $\alpha_3 u_y^{(1,2)}[t, z] + \alpha_2 Q_0 q_y^{(2,0)}[t, z] + \alpha_3 Q_0 q_y^{(2,0)}[t, z] + \alpha_2 u_y^{(3,0)}[t, z] +$ $\alpha_3 u_y^{(3,0)}[t, z] - Q_0 \zeta \epsilon_x^{(0,0,0,1)}[t, x, y, z] + Q_0 \zeta \epsilon_z^{(0,1,0,0)}[t, x, y, z]$
$-B_x Q_0 \zeta q_y[t, z] + B_x \epsilon_y[t, x, y, z] + Q_0^2 \zeta \epsilon_z[t, x, y, z] - 2 \alpha_2 Q_0^2 u_t^{(0,1)}[t, z] -$ $\frac{1}{2} \chi Q_0^2 u_t^{(0,1)}[t, z] - \alpha_2 u_t^{(0,3)}[t, z] - \alpha_3 u_t^{(0,3)}[t, z] - 3 B_x \theta u_y^{(1,0)}[t, z] -$ $B_x \zeta u_y^{(1,0)}[t, z] + 3 \alpha_2 Q_0 q_t^{(1,1)}[t, z] + 3 \alpha_3 Q_0 q_t^{(1,1)}[t, z] + 5 \alpha_2 u_t^{(2,1)}[t, z] +$ $5 \alpha_3 u_t^{(2,1)}[t, z] + Q_0 \zeta \epsilon_x^{(0,0,1,0)}[t, x, y, z] - Q_0 \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z]$
$\frac{2 \alpha_2 \wedge Q_0 q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} + \frac{2 \alpha_3 \wedge Q_0 q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} - 9 \alpha_2 Q_0^3 q_t[t, z] - 27 \alpha_3 Q_0^3 q_t[t, z] - 12 \alpha_2 Q_0^3 h_x^{(0,1)}[t, z] -$ $12 \alpha_3 Q_0^3 h_x^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta u_y^{(0,1)}[t, z] + 18 \alpha_2 Q_0^2 h_+^{(0,2)}[t, z] + 18 \alpha_3 Q_0^2 h_+^{(0,2)}[t, z] -$ $4 \alpha_2 Q_0 q_t^{(0,2)}[t, z] + 12 \alpha_2 Q_0 h_x^{(0,3)}[t, z] + 12 \alpha_3 Q_0 h_x^{(0,3)}[t, z] - 3 \alpha_2 h_+^{(0,4)}[t, z] -$ $3 \alpha_3 h_+^{(0,4)}[t, z] + \frac{3}{2} \alpha_2 Q_0^2 u_t^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0^2 u_t^{(1,0)}[t, z] - 3 \alpha_2 u_t^{(1,2)}[t, z] -$ $3 \alpha_3 u_t^{(1,2)}[t, z] - 3 \alpha_2 Q_0^2 h_+^{(2,0)}[t, z] - 3 \alpha_3 Q_0^2 h_+^{(2,0)}[t, z] + 4 \alpha_2 Q_0 q_t^{(2,0)}[t, z] +$ $4 \alpha_3 Q_0 q_t^{(2,0)}[t, z] - 12 \alpha_2 Q_0 h_x^{(2,1)}[t, z] - 12 \alpha_3 Q_0 h_x^{(2,1)}[t, z] + 6 \alpha_2 h_+^{(2,2)}[t, z] +$ $6 \alpha_3 h_+^{(2,2)}[t, z] + 3 \alpha_2 u_t^{(3,0)}[t, z] + 3 \alpha_3 u_t^{(3,0)}[t, z] - 3 \alpha_2 h_+^{(4,0)}[t, z] - 3 \alpha_3 h_+^{(4,0)}[t, z]$
$B_x b[t, z] + \frac{3}{2} B_x Q_0 \zeta u_y[t, z] + 12 \alpha_2 Q_0^3 h_+^{(0,1)}[t, z] + 12 \alpha_3 Q_0^3 h_+^{(0,1)}[t, z] +$ $18 \alpha_2 Q_0^2 h_x^{(0,2)}[t, z] + 18 \alpha_3 Q_0^2 h_x^{(0,2)}[t, z] - 12 \alpha_2 Q_0 h_+^{(0,3)}[t, z] -$ $12 \alpha_3 Q_0 h_+^{(0,3)}[t, z] - 3 \alpha_2 h_x^{(0,4)}[t, z] - 3 \alpha_3 h_x^{(0,4)}[t, z] - 3 \alpha_2 Q_0^2 h_x^{(2,0)}[t, z] -$ $3 \alpha_3 Q_0^2 h_x^{(2,0)}[t, z] + 12 \alpha_2 Q_0 h_+^{(2,1)}[t, z] + 12 \alpha_3 Q_0 h_+^{(2,1)}[t, z] +$ $6 \alpha_2 h_x^{(2,2)}[t, z] + 6 \alpha_3 h_x^{(2,2)}[t, z] - 3 \alpha_2 h_x^{(4,0)}[t, z] - 3 \alpha_3 h_x^{(4,0)}[t, z]$
$2 \alpha_2 Q_0 u_y^{(0,2)}[t, z] + \frac{1}{2} \chi Q_0 u_y^{(0,2)}[t, z]$

$$\begin{aligned}
& \frac{2 \alpha_2 \wedge Q_0 q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} + \frac{2 \alpha_3 \wedge Q_0 q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} - 9 \alpha_2 Q_0^3 q_t[t, z] - 27 \alpha_3 Q_0^3 q_t[t, z] + \\
& 12 \alpha_2 Q_0^3 h_x^{(0,1)}[t, z] + 12 \alpha_3 Q_0^3 h_x^{(0,1)}[t, z] - 6 B_x \theta u_y^{(0,1)}[t, z] - \frac{5}{2} B_x \zeta u_y^{(0,1)}[t, z] - \\
& 18 \alpha_2 Q_0^2 h_+^{(0,2)}[t, z] - 18 \alpha_3 Q_0^2 h_+^{(0,2)}[t, z] - 4 \alpha_2 Q_0 q_t^{(0,2)}[t, z] - \\
& 12 \alpha_2 Q_0 h_x^{(0,3)}[t, z] - 12 \alpha_3 Q_0 h_x^{(0,3)}[t, z] + 3 \alpha_2 h_+^{(0,4)}[t, z] + 3 \alpha_3 h_+^{(0,4)}[t, z] + \\
& \frac{3}{2} \alpha_2 Q_0^2 u_t^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0^2 u_t^{(1,0)}[t, z] - 3 \alpha_2 u_t^{(1,2)}[t, z] - 3 \alpha_3 u_t^{(1,2)}[t, z] + \\
& 3 \alpha_2 Q_0^2 h_+^{(2,0)}[t, z] + 3 \alpha_3 Q_0^2 h_+^{(2,0)}[t, z] + 4 \alpha_2 Q_0 q_t^{(2,0)}[t, z] + 4 \alpha_3 Q_0 q_t^{(2,0)}[t, z] + \\
& 12 \alpha_2 Q_0 h_x^{(2,1)}[t, z] + 12 \alpha_3 Q_0 h_x^{(2,1)}[t, z] - 6 \alpha_2 h_+^{(2,2)}[t, z] - 6 \alpha_3 h_+^{(2,2)}[t, z] + \\
& 3 \alpha_2 u_t^{(3,0)}[t, z] + 3 \alpha_3 u_t^{(3,0)}[t, z] + 3 \alpha_2 h_+^{(4,0)}[t, z] + 3 \alpha_3 h_+^{(4,0)}[t, z] \\
& - Q_0 \zeta b^{(0,1)}[t, z] + \alpha_2 Q_0^2 u_y^{(0,1)}[t, z] + \alpha_3 Q_0^2 u_y^{(0,1)}[t, z] - \alpha_2 u_y^{(0,3)}[t, z] - \\
& \alpha_3 u_y^{(0,3)}[t, z] - \alpha_2 Q_0 q_y^{(1,1)}[t, z] + \alpha_3 Q_0 q_y^{(1,1)}[t, z] + \alpha_2 u_y^{(2,1)}[t, z] + \alpha_3 u_y^{(2,1)}[t, z] \\
& \frac{2 \alpha_2 \wedge Q_0 q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} + \frac{2 \alpha_3 \wedge Q_0 q_t[t, z]}{\mathcal{M}_{\text{Pl}}^2} - 9 \alpha_2 Q_0^3 q_t[t, z] - 27 \alpha_3 Q_0^3 q_t[t, z] - \\
& 6 B_x \theta u_y^{(0,1)}[t, z] - \frac{5}{2} B_x \zeta u_y^{(0,1)}[t, z] + 2 \alpha_2 Q_0 q_t^{(0,2)}[t, z] + 2 \alpha_3 Q_0 q_t^{(0,2)}[t, z] + \\
& \frac{3}{2} \alpha_2 Q_0^2 u_t^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0^2 u_t^{(1,0)}[t, z] + \alpha_2 u_t^{(1,2)}[t, z] + \alpha_3 u_t^{(1,2)}[t, z] + \\
& 4 \alpha_2 Q_0 q_t^{(2,0)}[t, z] + 4 \alpha_3 Q_0 q_t^{(2,0)}[t, z] + 3 \alpha_2 u_t^{(3,0)}[t, z] + 3 \alpha_3 u_t^{(3,0)}[t, z] \\
& - 2 B_x h_+^{(0,2)}[t, z] + 6 \theta u_y^{(0,3)}[t, z] + 2 \zeta u_y^{(0,3)}[t, z] - 6 \theta u_y^{(2,1)}[t, z] - 2 \zeta u_y^{(2,1)}[t, z] \\
& - 2 B_x h_x^{(0,2)}[t, z] + 2 b^{(0,2)}[t, z] - 2 b^{(2,0)}[t, z]
\end{aligned}$$

## Analysis of flat CTEG

**Key observation:** Analysis of constant-torsion emergent gravity without the cosmological constant.

Here is the list of rules.

$\alpha_0 \rightarrow 0$	$\alpha_1 \rightarrow 0$	$\alpha_4 \rightarrow \frac{\alpha_2}{6} + \frac{\alpha_3}{6}$	$\alpha_5 \rightarrow -\frac{2 \alpha_2}{3} - \frac{2 \alpha_3}{3}$	$\alpha_6 \rightarrow -\frac{5 \alpha_2}{6} - \frac{5 \alpha_3}{6}$
$\beta_1 \rightarrow 0$	$\beta_2 \rightarrow 0$	$\beta_3 \rightarrow -\frac{4 \mathcal{M}_{\text{Pl}}^2}{9}$	$\Lambda \rightarrow 0$	

(70)

Here is the non-linear Lagrangian.

$$\begin{aligned}
& \frac{1}{36} \sqrt{-\tilde{g}} \\
& \left( -9 F_{ab} \left( F^{ab} + 4 \zeta R[D]^{ab} \right) + 2 \left( 18 \alpha_2 R[D]_{ab} R[D]^{ab} + 18 \left( \frac{\alpha_2}{6} + \frac{\alpha_3}{6} \right) R[D]_{abcd} R[D]^{abcd} + 18 \left( -\frac{2 \alpha_2}{3} - \frac{2 \alpha_3}{3} \right) \right. \right. \\
& \quad \left. \left. R[D]_{acbd} R[D]^{abcd} + 18 \left( -\frac{5 \alpha_2}{6} - \frac{5 \alpha_3}{6} \right) R[D]^{abcd} R[D]_{cdab} + 8 \mathcal{M}_{\text{Pl}}^2 T[D]_a^a{}^b{}_b T[D]_c{}^c{}_{bc} + \right. \right.
\end{aligned}$$
(71)

$$36 \theta F^{ab} \left( \nabla_b T[D]^c_{ac} \right) + 6 R[D]^{ab} \left( 3 \alpha_3 R[D]_{ba} + \chi \left( \nabla_a T[D]^c_{bc} \right) - \chi \left( \nabla_b T[D]^c_{ac} \right) \right) + \\ \xi g^{ab} \left( \nabla_a T[D]^c_d \right) \left( \nabla_b T[D]^e_{de} \right) + \xi \left( \nabla_b T[D]^a_{da} \right) \left( \nabla^d T[D]^c_{cb} \right) \Bigg)$$

## Here are the zeroth-order equations.

The Cartan components.

$$B_x Q_0 \zeta == 0$$

(72)

The Einstein components.

$$B_x == 0$$

(73)

The Maxwell components.

(74)

## Here are the first-order equations.

The Cartan components.

$$\begin{aligned} & 2 B_x \zeta q_t[t, z] + 2 (5 \alpha_2 + 3 \alpha_3) q_y^{(1,1)}[t, z] == 2 (2 \theta + \zeta) (B_x h_x^{(0,1)}[t, z] - b^{(0,1)}[t, z]) + \\ & (10 \alpha_2 + 6 \alpha_3 + \chi) Q_0 u_y^{(0,1)}[t, z] + 4 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] \\ & 2 (4 \mathcal{M}_{\text{Pl}}^2 + 9 (\alpha_2 + \alpha_3) Q_0^2) u_y[t, z] + \xi u_y^{(0,2)}[t, z] + 2 \chi u_y^{(2,0)}[t, z] + \\ & 12 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] == 6 Q_0 \zeta b[t, z] + 6 B_x (2 \theta + \zeta) h_+^{(0,1)}[t, z] + \\ & (12 \alpha_2 + 5 \chi) u_y^{(0,2)}[t, z] + 6 (5 \alpha_2 + 3 \alpha_3) Q_0 q_y^{(1,0)}[t, z] + \xi u_y^{(2,0)}[t, z] \\ & 3 B_x \zeta u_y[t, z] + (-2 \chi + \xi) u_t^{(1,1)}[t, z] + 12 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] == 6 (\alpha_2 + 3 \alpha_3) Q_0 q_t^{(0,1)}[t, z] \\ & B_x \zeta q_y[t, z] + 2 Q_0 \zeta \epsilon_z[t, x, y, z] + 4 (2 \alpha_2 + \alpha_3) q_t^{(1,1)}[t, z] == (12 \alpha_2 + 8 \alpha_3 + \chi) Q_0 u_t^{(0,1)}[t, z] \\ & 2 Q_0 \zeta \epsilon_y[t, x, y, z] + B_x \zeta h_x^{(1,0)}[t, z] + (10 \alpha_2 + 6 \alpha_3 + \chi) Q_0 u_y^{(1,0)}[t, z] == \\ & 2 \zeta b^{(1,0)}[t, z] + 2 (5 \alpha_2 + 3 \alpha_3) q_y^{(2,0)}[t, z] \\ & 2 B_x \zeta u_t[t, z] + 2 Q_0 \zeta \epsilon_x[t, x, y, z] + \\ & 4 \alpha_2 Q_0 q_y^{(0,1)}[t, z] + B_x \zeta h_+^{(1,0)}[t, z] + (4 \alpha_2 + \chi) u_y^{(1,1)}[t, z] == 0 \\ & 4 (2 \mathcal{M}_{\text{Pl}}^2 + 3 (\alpha_2 + \alpha_3) Q_0^2) u_t[t, z] + (-12 \alpha_2 - 5 \chi + \xi) u_t^{(0,2)}[t, z] + 3 (\alpha_2 + \alpha_3) \\ & (Q_0 (3 Q_0 h_+^{(1,0)}[t, z] - 8 q_t^{(1,0)}[t, z] + 12 h_x^{(1,1)}[t, z]) - 6 h_+^{(1,2)}[t, z] + 6 h_+^{(3,0)}[t, z]) + \\ & 6 (2 \theta + \zeta) (\epsilon_z^{(0,0,0,1)}[t, x, y, z] + \epsilon_y^{(0,0,1,0)}[t, x, y, z]) + 12 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z] == 0 \\ & \frac{1}{2} (Q_0 \zeta \epsilon_z[t, x, y, z] + (\alpha_2 + \alpha_3) (3 Q_0^2 h_x^{(1,0)}[t, z] + 2 q_t^{(1,1)}[t, z] + 6 h_x^{(3,0)}[t, z])) == \\ & \frac{1}{2} B_x \zeta q_y[t, z] + \frac{1}{4} (12 \alpha_2 + 8 \alpha_3 + \chi) Q_0 u_t^{(0,1)}[t, z] + \\ & 3 (\alpha_2 + \alpha_3) (2 Q_0 h_+^{(1,1)}[t, z] + h_x^{(1,2)}[t, z]) + \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z] \end{aligned}$$

$2 Q_0 \zeta \epsilon_y[t, x, y, z] + 2 B_x \zeta h_x^{(1,0)}[t, z] + (4 \alpha_2 + \chi) Q_0 u_y^{(1,0)}[t, z] + 4 \zeta \epsilon_z^{(0,1,0,0)}[t, x, y, z] == 0$
$3 Q_0 \zeta b[t, z] + (8 \mathcal{M}_{Pl}^2 + 9 (\alpha_2 + \alpha_3) Q_0^2) u_y[t, z] + \xi u_y^{(0,2)}[t, z] +$ $(12 \alpha_2 + 5 \chi) u_y^{(2,0)}[t, z] + 6 (2 \theta + \zeta) \epsilon_y^{(1,0,0,0)}[t, x, y, z] == 3 B_x (4 \theta + \zeta) h_+^{(0,1)}[t, z] +$ $(12 \alpha_2 + 5 \chi) u_y^{(0,2)}[t, z] + 3 (\alpha_2 + 3 \alpha_3) Q_0 q_y^{(1,0)}[t, z] + \xi u_y^{(2,0)}[t, z]$
$3 B_x \zeta u_y[t, z] + \xi u_t^{(1,1)}[t, z] + 9 (\alpha_2 + \alpha_3) (4 Q_0^2 h_+^{(0,1)}[t, z] + 5 Q_0 h_x^{(0,2)}[t, z] + 2 h_+^{(2,1)}[t, z]) +$ $6 (2 \theta + \zeta) \epsilon_z^{(1,0,0,0)}[t, x, y, z] == 3 (5 \alpha_2 + 3 \alpha_3) Q_0 q_t^{(0,1)}[t, z] +$ $18 (\alpha_2 + \alpha_3) h_+^{(0,3)}[t, z] + (12 \alpha_2 + 5 \chi) u_t^{(1,1)}[t, z] + 9 (\alpha_2 + \alpha_3) Q_0 h_x^{(2,0)}[t, z]$
$\frac{1}{2} B_x \zeta q_y[t, z] + \frac{1}{4} (12 \alpha_2 + 8 \alpha_3 + \chi) Q_0 u_t^{(0,1)}[t, z] +$ $3 (\alpha_2 + \alpha_3) (2 Q_0 h_+^{(1,1)}[t, z] + h_x^{(1,2)}[t, z]) + \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z] ==$ $\frac{1}{2} (Q_0 \zeta \epsilon_z[t, x, y, z] + (\alpha_2 + \alpha_3) (3 Q_0^2 h_x^{(1,0)}[t, z] + 2 q_t^{(1,1)}[t, z] + 6 h_x^{(3,0)}[t, z]))$
$(\alpha_2 + \alpha_3) (-6 h_x^{(0,3)}[t, z] + Q_0 (12 Q_0 h_x^{(0,1)}[t, z] - 15 h_+^{(0,2)}[t, z] - 4 u_t^{(1,0)}[t, z] + 3 h_+^{(2,0)}[t, z]) +$ $2 q_t^{(2,0)}[t, z] + 6 h_x^{(2,1)}[t, z]) == 0$
$\frac{1}{4} ((12 \alpha_2 + 8 \alpha_3 + \chi) Q_0 u_t^{(0,1)}[t, z] + 6 (\alpha_2 + \alpha_3) (Q_0^2 h_x^{(1,0)}[t, z] + 2 h_x^{(3,0)}[t, z])) ==$ $B_x \zeta q_y[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_z[t, x, y, z] +$ $(\alpha_2 + \alpha_3) (6 Q_0 h_+^{(1,1)}[t, z] + q_t^{(1,1)}[t, z] + 3 h_x^{(1,2)}[t, z]) + \zeta \epsilon_x^{(0,0,1,0)}[t, x, y, z]$
$4 (2 \mathcal{M}_{Pl}^2 + 3 (\alpha_2 + \alpha_3) Q_0^2) u_t[t, z] +$ $(-12 \alpha_2 - 5 \chi + \xi) u_t^{(0,2)}[t, z] + 6 (2 \theta + \zeta) \epsilon_z^{(0,0,0,1)}[t, x, y, z] +$ $12 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + 6 (2 \theta + \zeta) \epsilon_x^{(0,1,0,0)}[t, x, y, z] == 3 (\alpha_2 + \alpha_3)$ $(Q_0 (3 Q_0 h_+^{(1,0)}[t, z] + 8 q_t^{(1,0)}[t, z] + 12 h_x^{(1,1)}[t, z]) - 6 h_+^{(1,2)}[t, z] + 6 h_+^{(3,0)}[t, z])$
$\zeta (Q_0 \epsilon_x[t, x, y, z] + B_x (u_t[t, z] + h_+^{(1,0)}[t, z])) ==$ $(\alpha_2 + 3 \alpha_3) Q_0 q_y^{(0,1)}[t, z] + 2 \zeta \epsilon_z^{(0,0,1,0)}[t, x, y, z]$
$B_x \zeta q_y[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_z[t, x, y, z] +$ $(\alpha_2 + \alpha_3) (6 Q_0 h_+^{(1,1)}[t, z] + q_t^{(1,1)}[t, z] + 3 h_x^{(1,2)}[t, z]) + \zeta \epsilon_x^{(0,0,1,0)}[t, x, y, z] ==$ $\frac{1}{4} ((12 \alpha_2 + 8 \alpha_3 + \chi) Q_0 u_t^{(0,1)}[t, z] + 6 (\alpha_2 + \alpha_3) (Q_0^2 h_x^{(1,0)}[t, z] + 2 h_x^{(3,0)}[t, z]))$
$2 B_x (\zeta q_t[t, z] + (4 \theta + \zeta) h_x^{(0,1)}[t, z]) ==$ $(16 \alpha_2 + 12 \alpha_3 + \chi) Q_0 u_y^{(0,1)}[t, z] + 4 (2 \theta + \zeta) (b^{(0,1)}[t, z] - \epsilon_x^{(1,0,0,0)}[t, x, y, z])$
$(\alpha_2 + \alpha_3) (Q_0 (12 Q_0 h_x^{(0,1)}[t, z] - 15 h_+^{(0,2)}[t, z] + 4 u_t^{(1,0)}[t, z] + 3 h_+^{(2,0)}[t, z]) -$ $2 (3 h_x^{(0,3)}[t, z] + q_t^{(2,0)}[t, z] - 3 h_x^{(2,1)}[t, z])) == 0$
$(16 \alpha_2 + 12 \alpha_3 + \chi) Q_0 u_y^{(0,1)}[t, z] + 4 (2 \theta + \zeta) (b^{(0,1)}[t, z] - \epsilon_x^{(1,0,0,0)}[t, x, y, z]) ==$ $2 B_x (\zeta q_t[t, z] + (4 \theta + \zeta) h_x^{(0,1)}[t, z])$
$6 B_x \zeta u_y[t, z] + 36 (\alpha_2 + \alpha_3) Q_0^2 h_+^{(0,1)}[t, z] +$ $3 (5 \alpha_2 + 3 \alpha_3) Q_0 q_t^{(0,1)}[t, z] + 45 \alpha_2 Q_0 h_x^{(0,2)}[t, z] + 45 \alpha_3 Q_0 h_x^{(0,2)}[t, z] +$ $12 \alpha_2 u_t^{(1,1)}[t, z] + 5 \chi u_t^{(1,1)}[t, z] + 18 (\alpha_2 + \alpha_3) h_+^{(2,1)}[t, z] ==$ $\xi u_t^{(1,1)}[t, z] + 9 (\alpha_2 + \alpha_3) (2 h_+^{(0,3)}[t, z] + Q_0 h_x^{(2,0)}[t, z]) + 6 (2 \theta + \zeta) \epsilon_z^{(1,0,0,0)}[t, x, y, z]$
$2 Q_0 \zeta \epsilon_y[t, x, y, z] + (4 \alpha_2 + \chi) Q_0 u_y^{(1,0)}[t, z] ==$ $4 (5 \alpha_2 + 3 \alpha_3) q_y^{(0,2)}[t, z] + 4 \zeta \epsilon_x^{(0,0,0,1)}[t, x, y, z]$

$B_x \zeta u_t[t, z] + Q_0 \zeta \epsilon_x[t, x, y, z] + (5 \alpha_2 + 3 \alpha_3) Q_0 q_y^{(0,1)}[t, z] + (4 \alpha_2 + \chi) u_y^{(1,1)}[t, z] + 2 \zeta \epsilon_y^{(0,0,0,1)}[t, x, y, z] == 0$
$4 (2 \mathcal{M}_{\text{Pl}}^2 + 3 (\alpha_2 + \alpha_3) Q_0^2) u_t[t, z] + (-2 \chi + \xi) u_t^{(0,2)}[t, z] + 6 (-4 (\alpha_2 + \alpha_3) Q_0 q_t^{(1,0)}[t, z] + 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + (2 \theta + \zeta) (\epsilon_y^{(0,0,1,0)}[t, x, y, z] + \epsilon_x^{(0,1,0,0)}[t, x, y, z])) == 0$
$(5 \alpha_2 + 3 \alpha_3) q_t^{(0,2)}[t, z] + (\alpha_2 + \alpha_3) (2 Q_0 u_t^{(1,0)}[t, z] - q_t^{(2,0)}[t, z]) == 0$
$2 B_x \zeta q_t[t, z] + 2 B_x (4 \theta + \zeta) h_x^{(0,1)}[t, z] + 4 (5 \alpha_2 + 3 \alpha_3) q_y^{(1,1)}[t, z] + 4 (2 \theta + \zeta) \epsilon_x^{(1,0,0,0)}[t, x, y, z] == 8 \theta b^{(0,1)}[t, z] + (4 \alpha_2 + \chi) Q_0 u_y^{(0,1)}[t, z]$
$3 Q_0 \zeta b[t, z] + (8 \mathcal{M}_{\text{Pl}}^2 + 9 (\alpha_2 + \alpha_3) Q_0^2) u_y[t, z] + (-2 \chi + \xi) u_y^{(0,2)}[t, z] + (12 \alpha_2 + 5 \chi - \xi) u_y^{(2,0)}[t, z] + 6 (2 \theta + \zeta) \epsilon_y^{(1,0,0,0)}[t, x, y, z] == 3 B_x (4 \theta + \zeta) h_+^{(0,1)}[t, z] + 3 (\alpha_2 + 3 \alpha_3) Q_0 q_y^{(1,0)}[t, z]$

The Einstein components.

$18 (\alpha_2 + 3 \alpha_3) Q_0^3 q_t[t, z] + B_x \zeta u_y^{(0,1)}[t, z] + (\alpha_2 + \alpha_3) (9 Q_0^2 u_t^{(1,0)}[t, z] + 2 u_t^{(1,2)}[t, z] + 6 u_t^{(3,0)}[t, z]) == 0$
$Q_0 (2 B_x \zeta u_t[t, z] + 2 Q_0 \zeta \epsilon_x[t, x, y, z] + 4 \alpha_2 Q_0 q_y^{(0,1)}[t, z] + (4 \alpha_2 + \chi) u_y^{(1,1)}[t, z] + \zeta (B_x h_+^{(1,0)}[t, z] + 2 \epsilon_y^{(0,0,0,1)}[t, x, y, z] - 2 \epsilon_z^{(0,0,1,0)}[t, x, y, z])) == 0$
$Q_0^2 \zeta \epsilon_y[t, x, y, z] + \frac{1}{2} B_x Q_0 \zeta h_x^{(1,0)}[t, z] + 6 \alpha_2 Q_0^2 u_y^{(1,0)}[t, z] + 4 \alpha_3 Q_0^2 u_y^{(1,0)}[t, z] + \frac{1}{2} \chi Q_0^2 u_y^{(1,0)}[t, z] + \alpha_2 Q_0 q_y^{(2,0)}[t, z] + \alpha_3 Q_0 q_y^{(2,0)}[t, z] + \alpha_2 u_y^{(3,0)}[t, z] + \alpha_3 u_y^{(3,0)}[t, z] + Q_0 \zeta \epsilon_z^{(0,1,0,0)}[t, x, y, z] == B_x \epsilon_z[t, x, y, z] + B_x (3 \theta + \zeta) u_t^{(0,1)}[t, z] + 2 \alpha_2 Q_0 q_y^{(0,2)}[t, z] + (\alpha_2 + \alpha_3) u_y^{(1,2)}[t, z] + Q_0 \zeta \epsilon_x^{(0,0,0,1)}[t, x, y, z]$
$B_x \epsilon_y[t, x, y, z] + Q_0^2 \zeta \epsilon_z[t, x, y, z] + (\alpha_2 + \alpha_3) (3 Q_0 q_t^{(1,1)}[t, z] + 5 u_t^{(2,1)}[t, z]) + Q_0 \zeta \epsilon_x^{(0,0,1,0)}[t, x, y, z] == B_x Q_0 \zeta q_y[t, z] + \frac{1}{2} (4 \alpha_2 + \chi) Q_0^2 u_t^{(0,1)}[t, z] + (\alpha_2 + \alpha_3) u_t^{(0,3)}[t, z] + B_x (3 \theta + \zeta) u_y^{(1,0)}[t, z] + Q_0 \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z]$
$\frac{1}{2} (\alpha_2 + \alpha_3) (Q_0 (36 Q_0 h_+^{(0,2)}[t, z] + 24 h_x^{(0,3)}[t, z] + 3 Q_0 u_t^{(1,0)}[t, z] + 8 q_t^{(2,0)}[t, z]) + 6 (2 h_+^{(2,2)}[t, z] + u_t^{(3,0)}[t, z])) == 9 (\alpha_2 + 3 \alpha_3) Q_0^3 q_t[t, z] + 12 (\alpha_2 + \alpha_3) Q_0^3 h_x^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta u_y^{(0,1)}[t, z] + 4 \alpha_2 Q_0 q_t^{(0,2)}[t, z] + 3 \alpha_2 h_+^{(0,4)}[t, z] + 3 \alpha_3 h_+^{(0,4)}[t, z] + 3 \alpha_2 u_t^{(1,2)}[t, z] + 3 \alpha_3 u_t^{(1,2)}[t, z] + 3 \alpha_2 Q_0^2 h_+^{(2,0)}[t, z] + 3 \alpha_3 Q_0^2 h_+^{(2,0)}[t, z] + 12 \alpha_2 Q_0 h_x^{(2,1)}[t, z] + 12 \alpha_3 Q_0 h_x^{(2,1)}[t, z] + 3 (\alpha_2 + \alpha_3) h_+^{(4,0)}[t, z]$
$2 B_x b[t, z] + 3 B_x Q_0 \zeta u_y[t, z] == 6 (\alpha_2 + \alpha_3) (h_x^{(0,4)}[t, z] + Q_0 (Q_0 (-4 Q_0 h_+^{(0,1)}[t, z] - 6 h_x^{(0,2)}[t, z] + h_x^{(2,0)}[t, z]) + 4 (h_+^{(0,3)}[t, z] - h_+^{(2,1)}[t, z])) - 2 h_x^{(2,2)}[t, z] + h_x^{(4,0)}[t, z])$
$(4 \alpha_2 + \chi) Q_0 u_y^{(0,2)}[t, z] == 0$

(76)

$$\begin{aligned}
& \frac{1}{2} (\alpha_2 + \alpha_3) \\
& (Q_0 (3 Q_0 (8 Q_0 h_x^{(0,1)}[t, z] + u_t^{(1,0)}[t, z] + 2 h_+^{(2,0)}[t, z]) + 8 (q_t^{(2,0)}[t, z] + 3 h_x^{(2,1)}[t, z])) + \\
& 6 (h_+^{(0,4)}[t, z] + u_t^{(3,0)}[t, z] + h_+^{(4,0)}[t, z])) = \\
& 9 (\alpha_2 + 3 \alpha_3) Q_0^3 q_t[t, z] + \frac{1}{2} B_x (12 \theta + 5 \zeta) u_y^{(0,1)}[t, z] + \\
& 2 Q_0 (9 (\alpha_2 + \alpha_3) Q_0 h_+^{(0,2)}[t, z] + 2 \alpha_2 q_t^{(0,2)}[t, z] + 6 (\alpha_2 + \alpha_3) h_x^{(0,3)}[t, z]) + \\
& 3 (\alpha_2 + \alpha_3) u_t^{(1,2)}[t, z] + 6 (\alpha_2 + \alpha_3) h_+^{(2,2)}[t, z] \\
& (\alpha_2 + \alpha_3) Q_0^2 u_y^{(0,1)}[t, z] + \alpha_3 Q_0 q_y^{(1,1)}[t, z] + (\alpha_2 + \alpha_3) u_y^{(2,1)}[t, z] = \\
& Q_0 \zeta b^{(0,1)}[t, z] + (\alpha_2 + \alpha_3) u_y^{(0,3)}[t, z] + \alpha_2 Q_0 q_y^{(1,1)}[t, z] \\
& 18 (\alpha_2 + 3 \alpha_3) Q_0^3 q_t[t, z] + B_x (12 \theta + 5 \zeta) u_y^{(0,1)}[t, z] = \\
& (\alpha_2 + \alpha_3) (4 Q_0 q_t^{(0,2)}[t, z] + 3 Q_0^2 u_t^{(1,0)}[t, z] + 2 u_t^{(1,2)}[t, z] + 8 Q_0 q_t^{(2,0)}[t, z] + 6 u_t^{(3,0)}[t, z])
\end{aligned}$$

The Maxwell components.

$$\begin{aligned}
& B_x h_+^{(0,2)}[t, z] + \epsilon_z^{(1,0,1,0)}[t, x, y, z] = (3 \theta + \zeta) (u_y^{(0,3)}[t, z] - u_y^{(2,1)}[t, z]) + \epsilon_y^{(1,0,0,1)}[t, x, y, z] \\
& B_x h_x^{(0,2)}[t, z] + \epsilon_x^{(1,0,0,1)}[t, x, y, z] = b^{(0,2)}[t, z] + \epsilon_z^{(1,1,0,0)}[t, x, y, z] \\
& \epsilon_x^{(1,0,1,0)}[t, x, y, z] = \epsilon_y^{(1,1,0,0)}[t, x, y, z]
\end{aligned}$$

(77)

## Here is the reduced set of first-order equations.

After simplification, we have 60 equations.

$$\begin{aligned}
& -B_x \zeta q_t[t, z] + 2 B_x \theta h_x^{(0,1)}[t, z] + B_x \zeta h_x^{(0,1)}[t, z] - \\
& 2 \theta b^{(0,1)}[t, z] - \zeta b^{(0,1)}[t, z] + 5 \alpha_2 Q_0 u_y^{(0,1)}[t, z] + 3 \alpha_3 Q_0 u_y^{(0,1)}[t, z] + \\
& \frac{1}{2} \chi Q_0 u_y^{(0,1)}[t, z] - 5 \alpha_2 q_y^{(1,1)}[t, z] - 3 \alpha_3 q_y^{(1,1)}[t, z] + 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] \\
& -Q_0 \zeta b[t, z] + \frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_y[t, z] + 3 \alpha_2 Q_0^2 u_y[t, z] + 3 \alpha_3 Q_0^2 u_y[t, z] - 2 B_x \theta h_+^{(0,1)}[t, z] - \\
& B_x \zeta h_+^{(0,1)}[t, z] - 2 \alpha_2 u_y^{(0,2)}[t, z] - \frac{5}{6} \chi u_y^{(0,2)}[t, z] + \frac{1}{6} \xi u_y^{(0,2)}[t, z] - 5 \alpha_2 Q_0 q_y^{(1,0)}[t, z] - \\
& 3 \alpha_3 Q_0 q_y^{(1,0)}[t, z] + \frac{1}{3} \chi u_y^{(2,0)}[t, z] - \frac{1}{6} \xi u_y^{(2,0)}[t, z] + 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] \\
& \frac{1}{2} B_x \zeta u_y[t, z] - \alpha_2 Q_0 q_t^{(0,1)}[t, z] - 3 \alpha_3 Q_0 q_t^{(0,1)}[t, z] - \\
& \frac{1}{3} \chi u_t^{(1,1)}[t, z] + \frac{1}{6} \xi u_t^{(1,1)}[t, z] + 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] \\
& B_x \zeta q_t[t, z] - 2 B_x \theta h_x^{(0,1)}[t, z] - B_x \zeta h_x^{(0,1)}[t, z] + 2 \theta b^{(0,1)}[t, z] + \\
& \zeta b^{(0,1)}[t, z] - 5 \alpha_2 Q_0 u_y^{(0,1)}[t, z] - 3 \alpha_3 Q_0 u_y^{(0,1)}[t, z] - \frac{1}{2} \chi Q_0 u_y^{(0,1)}[t, z] + \\
& 5 \alpha_2 q_y^{(1,1)}[t, z] + 3 \alpha_3 q_y^{(1,1)}[t, z] - 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] \\
& -\frac{1}{2} B_x \zeta q_y[t, z] - Q_0 \zeta \epsilon_z[t, x, y, z] + 6 \alpha_2 Q_0 u_t^{(0,1)}[t, z] + \\
& 4 \alpha_3 Q_0 u_t^{(0,1)}[t, z] + \frac{1}{2} \chi Q_0 u_t^{(0,1)}[t, z] - 4 \alpha_2 q_t^{(1,1)}[t, z] - 2 \alpha_3 q_t^{(1,1)}[t, z]
\end{aligned}$$



$Q_0 \zeta \epsilon_y[t, x, y, z] + \frac{1}{2} B_x \zeta h_x^{(1,0)}[t, z] - \zeta b^{(1,0)}[t, z] + 5 \alpha_2 Q_0 u_y^{(1,0)}[t, z] +$ $3 \alpha_3 Q_0 u_y^{(1,0)}[t, z] + \frac{1}{2} \chi Q_0 u_y^{(1,0)}[t, z] - 5 \alpha_2 q_y^{(2,0)}[t, z] - 3 \alpha_3 q_y^{(2,0)}[t, z]$
$Q_0 \zeta b[t, z] - \frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_y[t, z] - 3 \alpha_2 Q_0^2 u_y[t, z] - 3 \alpha_3 Q_0^2 u_y[t, z] + 2 B_x \theta h_+^{(0,1)}[t, z] +$ $B_x \zeta h_+^{(0,1)}[t, z] + 2 \alpha_2 u_y^{(0,2)}[t, z] + \frac{5}{6} \chi u_y^{(0,2)}[t, z] - \frac{1}{6} \xi u_y^{(0,2)}[t, z] + 5 \alpha_2 Q_0 q_y^{(1,0)}[t, z] +$ $3 \alpha_3 Q_0 q_y^{(1,0)}[t, z] - \frac{1}{3} \chi u_y^{(2,0)}[t, z] + \frac{1}{6} \xi u_y^{(2,0)}[t, z] - 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z]$
$\frac{1}{2} B_x \zeta q_y[t, z] + Q_0 \zeta \epsilon_z[t, x, y, z] - 6 \alpha_2 Q_0 u_t^{(0,1)}[t, z] -$ $4 \alpha_3 Q_0 u_t^{(0,1)}[t, z] - \frac{1}{2} \chi Q_0 u_t^{(0,1)}[t, z] + 4 \alpha_2 q_t^{(1,1)}[t, z] + 2 \alpha_3 q_t^{(1,1)}[t, z]$
$-B_x \zeta u_t[t, z] - Q_0 \zeta \epsilon_x[t, x, y, z] - 2 \alpha_2 Q_0 q_y^{(0,1)}[t, z] -$ $\frac{1}{2} B_x \zeta h_+^{(1,0)}[t, z] - 2 \alpha_2 u_y^{(1,1)}[t, z] - \frac{1}{2} \chi u_y^{(1,1)}[t, z]$
$-\frac{1}{2} B_x \zeta u_y[t, z] + \alpha_2 Q_0 q_t^{(0,1)}[t, z] + 3 \alpha_3 Q_0 q_t^{(0,1)}[t, z] +$ $\frac{1}{3} \chi u_t^{(1,1)}[t, z] - \frac{1}{6} \xi u_t^{(1,1)}[t, z] - 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z]$
$-Q_0 \zeta \epsilon_y[t, x, y, z] - \frac{1}{2} B_x \zeta h_x^{(1,0)}[t, z] + \zeta b^{(1,0)}[t, z] - 5 \alpha_2 Q_0 u_y^{(1,0)}[t, z] -$ $3 \alpha_3 Q_0 u_y^{(1,0)}[t, z] - \frac{1}{2} \chi Q_0 u_y^{(1,0)}[t, z] + 5 \alpha_2 q_y^{(2,0)}[t, z] + 3 \alpha_3 q_y^{(2,0)}[t, z]$
$B_x \zeta u_t[t, z] + Q_0 \zeta \epsilon_x[t, x, y, z] + 2 \alpha_2 Q_0 q_y^{(0,1)}[t, z] +$ $\frac{1}{2} B_x \zeta h_+^{(1,0)}[t, z] + 2 \alpha_2 u_y^{(1,1)}[t, z] + \frac{1}{2} \chi u_y^{(1,1)}[t, z]$
$\frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_t[t, z] + 2 \alpha_2 Q_0^2 u_t[t, z] + 2 \alpha_3 Q_0^2 u_t[t, z] - 2 \alpha_2 u_t^{(0,2)}[t, z] -$ $\frac{5}{6} \chi u_t^{(0,2)}[t, z] + \frac{1}{6} \xi u_t^{(0,2)}[t, z] + \frac{3}{2} \alpha_2 Q_0^2 h_+^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0^2 h_+^{(1,0)}[t, z] -$ $4 \alpha_2 Q_0 q_t^{(1,0)}[t, z] - 4 \alpha_3 Q_0 q_t^{(1,0)}[t, z] + 6 \alpha_2 Q_0 h_x^{(1,1)}[t, z] +$ $6 \alpha_3 Q_0 h_x^{(1,1)}[t, z] - 3 \alpha_2 h_+^{(1,2)}[t, z] - 3 \alpha_3 h_+^{(1,2)}[t, z] + 3 \alpha_2 h_+^{(3,0)}[t, z] +$ $3 \alpha_3 h_+^{(3,0)}[t, z] + 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + \zeta \epsilon_z^{(0,0,0,1)}[t, x, y, z] +$ $2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + \zeta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$
$-\frac{1}{2} B_x \zeta q_y[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_z[t, x, y, z] - 3 \alpha_2 Q_0 u_t^{(0,1)}[t, z] - 2 \alpha_3 Q_0 u_t^{(0,1)}[t, z] -$ $\frac{1}{4} \chi Q_0 u_t^{(0,1)}[t, z] + \frac{3}{2} \alpha_2 Q_0^2 h_x^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0^2 h_x^{(1,0)}[t, z] - 6 \alpha_2 Q_0 h_+^{(1,1)}[t, z] -$ $6 \alpha_3 Q_0 h_+^{(1,1)}[t, z] + \alpha_2 q_t^{(1,1)}[t, z] + \alpha_3 q_t^{(1,1)}[t, z] - 3 \alpha_2 h_x^{(1,2)}[t, z] -$ $3 \alpha_3 h_x^{(1,2)}[t, z] + 3 \alpha_2 h_x^{(3,0)}[t, z] + 3 \alpha_3 h_x^{(3,0)}[t, z] - \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z]$
$-\frac{1}{2} Q_0 \zeta \epsilon_y[t, x, y, z] - \frac{1}{2} B_x \zeta h_x^{(1,0)}[t, z] -$ $\alpha_2 Q_0 u_y^{(1,0)}[t, z] - \frac{1}{4} \chi Q_0 u_y^{(1,0)}[t, z] - \zeta \epsilon_z^{(0,1,0,0)}[t, x, y, z]$
$-\frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_t[t, z] - 2 \alpha_2 Q_0^2 u_t[t, z] - 2 \alpha_3 Q_0^2 u_t[t, z] + 2 \alpha_2 u_t^{(0,2)}[t, z] +$ $\frac{5}{6} \chi u_t^{(0,2)}[t, z] - \frac{1}{6} \xi u_t^{(0,2)}[t, z] - \frac{3}{2} \alpha_2 Q_0^2 h_+^{(1,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0^2 h_+^{(1,0)}[t, z] +$ $4 \alpha_2 Q_0 q_t^{(1,0)}[t, z] + 4 \alpha_3 Q_0 q_t^{(1,0)}[t, z] - 6 \alpha_2 Q_0 h_x^{(1,1)}[t, z] -$ $6 \alpha_3 Q_0 h_x^{(1,1)}[t, z] + 3 \alpha_2 h_+^{(1,2)}[t, z] + 3 \alpha_3 h_+^{(1,2)}[t, z] - 3 \alpha_2 h_+^{(3,0)}[t, z] -$ $3 \alpha_3 h_+^{(3,0)}[t, z] - 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] - \zeta \epsilon_z^{(0,0,0,1)}[t, x, y, z] -$ $2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] - \zeta \epsilon_y^{(0,0,1,0)}[t, x, y, z] - 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$

$\begin{aligned} & \frac{1}{2} Q_0 \zeta b[t, z] + \frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_y[t, z] + \frac{3}{2} \alpha_2 Q_0^2 u_y[t, z] + \frac{3}{2} \alpha_3 Q_0^2 u_y[t, z] - \\ & 2 B_x \theta h_+^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_+^{(0,1)}[t, z] - 2 \alpha_2 u_y^{(0,2)}[t, z] - \frac{5}{6} \chi u_y^{(0,2)}[t, z] + \\ & \frac{1}{6} \xi u_y^{(0,2)}[t, z] - \frac{1}{2} \alpha_2 Q_0 q_y^{(1,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0 q_y^{(1,0)}[t, z] + 2 \alpha_2 u_y^{(2,0)}[t, z] + \\ & \frac{5}{6} \chi u_y^{(2,0)}[t, z] - \frac{1}{6} \xi u_y^{(2,0)}[t, z] + 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_y^{(1,0,0,0)}[t, x, y, z] \end{aligned}$
$\begin{aligned} & \frac{1}{2} B_x \zeta u_y[t, z] + 6 \alpha_2 Q_0^2 h_+^{(0,1)}[t, z] + 6 \alpha_3 Q_0^2 h_+^{(0,1)}[t, z] - \\ & \frac{5}{2} \alpha_2 Q_0 q_t^{(0,1)}[t, z] - \frac{3}{2} \alpha_3 Q_0 q_t^{(0,1)}[t, z] + \frac{15}{2} \alpha_2 Q_0 h_x^{(0,2)}[t, z] + \\ & \frac{15}{2} \alpha_3 Q_0 h_x^{(0,2)}[t, z] - 3 \alpha_2 h_+^{(0,3)}[t, z] - 3 \alpha_3 h_+^{(0,3)}[t, z] - 2 \alpha_2 u_t^{(1,1)}[t, z] - \\ & \frac{5}{6} \chi u_t^{(1,1)}[t, z] + \frac{1}{6} \xi u_t^{(1,1)}[t, z] - \frac{3}{2} \alpha_2 Q_0 h_x^{(2,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0 h_x^{(2,0)}[t, z] + \\ & 3 \alpha_2 h_+^{(2,1)}[t, z] + 3 \alpha_3 h_+^{(2,1)}[t, z] + 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_z^{(1,0,0,0)}[t, x, y, z] \end{aligned}$
$\begin{aligned} & \frac{1}{2} B_x \zeta q_y[t, z] - \frac{1}{2} Q_0 \zeta \epsilon_z[t, x, y, z] + 3 \alpha_2 Q_0 u_t^{(0,1)}[t, z] + 2 \alpha_3 Q_0 u_t^{(0,1)}[t, z] + \\ & \frac{1}{4} \chi Q_0 u_t^{(0,1)}[t, z] - \frac{3}{2} \alpha_2 Q_0^2 h_x^{(1,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0^2 h_x^{(1,0)}[t, z] + 6 \alpha_2 Q_0 h_+^{(1,1)}[t, z] + \\ & 6 \alpha_3 Q_0 h_+^{(1,1)}[t, z] - \alpha_2 q_t^{(1,1)}[t, z] - \alpha_3 q_t^{(1,1)}[t, z] + 3 \alpha_2 h_x^{(1,2)}[t, z] + \\ & 3 \alpha_3 h_x^{(1,2)}[t, z] - 3 \alpha_2 h_x^{(3,0)}[t, z] - 3 \alpha_3 h_x^{(3,0)}[t, z] + \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z] \end{aligned}$
$\begin{aligned} & -\frac{1}{2} Q_0 \zeta b[t, z] - \frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_y[t, z] - \frac{3}{2} \alpha_2 Q_0^2 u_y[t, z] - \frac{3}{2} \alpha_3 Q_0^2 u_y[t, z] + \\ & 2 B_x \theta h_+^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_+^{(0,1)}[t, z] + 2 \alpha_2 u_y^{(0,2)}[t, z] + \frac{5}{6} \chi u_y^{(0,2)}[t, z] - \\ & \frac{1}{6} \xi u_y^{(0,2)}[t, z] + \frac{1}{2} \alpha_2 Q_0 q_y^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0 q_y^{(1,0)}[t, z] - 2 \alpha_2 u_y^{(2,0)}[t, z] - \\ & \frac{5}{6} \chi u_y^{(2,0)}[t, z] + \frac{1}{6} \xi u_y^{(2,0)}[t, z] - 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_y^{(1,0,0,0)}[t, x, y, z] \end{aligned}$
$\begin{aligned} & 6 \alpha_2 Q_0^2 h_x^{(0,1)}[t, z] + 6 \alpha_3 Q_0^2 h_x^{(0,1)}[t, z] - \frac{15}{2} \alpha_2 Q_0 h_+^{(0,2)}[t, z] - \\ & \frac{15}{2} \alpha_3 Q_0 h_+^{(0,2)}[t, z] - 3 \alpha_2 h_x^{(0,3)}[t, z] - 3 \alpha_3 h_x^{(0,3)}[t, z] - 2 \alpha_2 Q_0 u_t^{(1,0)}[t, z] - \\ & 2 \alpha_3 Q_0 u_t^{(1,0)}[t, z] + \frac{3}{2} \alpha_2 Q_0 h_+^{(2,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0 h_+^{(2,0)}[t, z] + \\ & \alpha_2 q_t^{(2,0)}[t, z] + \alpha_3 q_t^{(2,0)}[t, z] + 3 \alpha_2 h_x^{(2,1)}[t, z] + 3 \alpha_3 h_x^{(2,1)}[t, z] \end{aligned}$
$\begin{aligned} & \frac{1}{2} Q_0 \zeta \epsilon_y[t, x, y, z] + \frac{1}{2} B_x \zeta h_x^{(1,0)}[t, z] + \\ & \alpha_2 Q_0 u_y^{(1,0)}[t, z] + \frac{1}{4} \chi Q_0 u_y^{(1,0)}[t, z] + \zeta \epsilon_z^{(0,1,0,0)}[t, x, y, z] \end{aligned}$
$\begin{aligned} & -\frac{1}{2} B_x \zeta u_y[t, z] - 6 \alpha_2 Q_0^2 h_+^{(0,1)}[t, z] - 6 \alpha_3 Q_0^2 h_+^{(0,1)}[t, z] + \\ & \frac{5}{2} \alpha_2 Q_0 q_t^{(0,1)}[t, z] + \frac{3}{2} \alpha_3 Q_0 q_t^{(0,1)}[t, z] - \frac{15}{2} \alpha_2 Q_0 h_x^{(0,2)}[t, z] - \\ & \frac{15}{2} \alpha_3 Q_0 h_x^{(0,2)}[t, z] + 3 \alpha_2 h_+^{(0,3)}[t, z] + 3 \alpha_3 h_+^{(0,3)}[t, z] + 2 \alpha_2 u_t^{(1,1)}[t, z] + \\ & \frac{5}{6} \chi u_t^{(1,1)}[t, z] - \frac{1}{6} \xi u_t^{(1,1)}[t, z] + \frac{3}{2} \alpha_2 Q_0 h_x^{(2,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0 h_x^{(2,0)}[t, z] - \\ & 3 \alpha_2 h_+^{(2,1)}[t, z] - 3 \alpha_3 h_+^{(2,1)}[t, z] - 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_z^{(1,0,0,0)}[t, x, y, z] \end{aligned}$
$\begin{aligned} & -6 \alpha_2 Q_0^2 h_x^{(0,1)}[t, z] - 6 \alpha_3 Q_0^2 h_x^{(0,1)}[t, z] + \frac{15}{2} \alpha_2 Q_0 h_+^{(0,2)}[t, z] + \\ & \frac{15}{2} \alpha_3 Q_0 h_+^{(0,2)}[t, z] + 3 \alpha_2 h_x^{(0,3)}[t, z] + 3 \alpha_3 h_x^{(0,3)}[t, z] + 2 \alpha_2 Q_0 u_t^{(1,0)}[t, z] + \\ & 2 \alpha_3 Q_0 u_t^{(1,0)}[t, z] - \frac{3}{2} \alpha_2 Q_0 h_+^{(2,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0 h_+^{(2,0)}[t, z] - \\ & \alpha_2 q_t^{(2,0)}[t, z] - \alpha_3 q_t^{(2,0)}[t, z] - 3 \alpha_2 h_x^{(2,1)}[t, z] - 3 \alpha_3 h_x^{(2,1)}[t, z] \end{aligned}$

$ \begin{aligned} & -B_x \zeta q_y[t, z] - \frac{1}{2} Q_0 \zeta \epsilon_z[t, x, y, z] + 3 \alpha_2 Q_0 u_t^{(0,1)}[t, z] + 2 \alpha_3 Q_0 u_t^{(0,1)}[t, z] + \\ & \frac{1}{4} \chi Q_0 u_t^{(0,1)}[t, z] + \frac{3}{2} \alpha_2 Q_0^2 h_x^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0^2 h_x^{(1,0)}[t, z] - 6 \alpha_2 Q_0 h_+^{(1,1)}[t, z] - \\ & 6 \alpha_3 Q_0 h_+^{(1,1)}[t, z] - \alpha_2 q_t^{(1,1)}[t, z] - \alpha_3 q_t^{(1,1)}[t, z] - 3 \alpha_2 h_x^{(1,2)}[t, z] - \\ & 3 \alpha_3 h_x^{(1,2)}[t, z] + 3 \alpha_2 h_x^{(3,0)}[t, z] + 3 \alpha_3 h_x^{(3,0)}[t, z] - \zeta \epsilon_x^{(0,0,1,0)}[t, x, y, z] \end{aligned} $
$ \begin{aligned} & \frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_t[t, z] + 2 \alpha_2 Q_0^2 u_t[t, z] + 2 \alpha_3 Q_0^2 u_t[t, z] - 2 \alpha_2 u_t^{(0,2)}[t, z] - \\ & \frac{5}{6} \chi u_t^{(0,2)}[t, z] + \frac{1}{6} \xi u_t^{(0,2)}[t, z] - \frac{3}{2} \alpha_2 Q_0^2 h_+^{(1,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0^2 h_+^{(1,0)}[t, z] - \\ & 4 \alpha_2 Q_0 q_t^{(1,0)}[t, z] - 4 \alpha_3 Q_0 q_t^{(1,0)}[t, z] - 6 \alpha_2 Q_0 h_x^{(1,1)}[t, z] - \\ & 6 \alpha_3 Q_0 h_x^{(1,1)}[t, z] + 3 \alpha_2 h_+^{(1,2)}[t, z] + 3 \alpha_3 h_+^{(1,2)}[t, z] - 3 \alpha_2 h_+^{(3,0)}[t, z] - \\ & 3 \alpha_3 h_+^{(3,0)}[t, z] + 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + \zeta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + \\ & 2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z] + \zeta \epsilon_x^{(0,1,0,0)}[t, x, y, z] \end{aligned} $
$ \begin{aligned} & \frac{1}{2} B_x \zeta u_t[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_x[t, x, y, z] - \frac{1}{2} \alpha_2 Q_0 q_y^{(0,1)}[t, z] - \\ & \frac{3}{2} \alpha_3 Q_0 q_y^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_+^{(1,0)}[t, z] - \zeta \epsilon_z^{(0,0,1,0)}[t, x, y, z] \end{aligned} $
$ \begin{aligned} & B_x \zeta q_y[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_z[t, x, y, z] - 3 \alpha_2 Q_0 u_t^{(0,1)}[t, z] - 2 \alpha_3 Q_0 u_t^{(0,1)}[t, z] - \\ & \frac{1}{4} \chi Q_0 u_t^{(0,1)}[t, z] - \frac{3}{2} \alpha_2 Q_0^2 h_x^{(1,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0^2 h_x^{(1,0)}[t, z] + 6 \alpha_2 Q_0 h_+^{(1,1)}[t, z] + \\ & 6 \alpha_3 Q_0 h_+^{(1,1)}[t, z] + \alpha_2 q_t^{(1,1)}[t, z] + \alpha_3 q_t^{(1,1)}[t, z] + 3 \alpha_2 h_x^{(1,2)}[t, z] + \\ & 3 \alpha_3 h_x^{(1,2)}[t, z] - 3 \alpha_2 h_x^{(3,0)}[t, z] - 3 \alpha_3 h_x^{(3,0)}[t, z] + \zeta \epsilon_x^{(0,0,1,0)}[t, x, y, z] \end{aligned} $
$ \begin{aligned} & -\frac{1}{2} B_x \zeta q_t[t, z] - 2 B_x \theta h_x^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_x^{(0,1)}[t, z] + \\ & 2 \theta b^{(0,1)}[t, z] + \zeta b^{(0,1)}[t, z] + 4 \alpha_2 Q_0 u_y^{(0,1)}[t, z] + 3 \alpha_3 Q_0 u_y^{(0,1)}[t, z] + \\ & \frac{1}{4} \chi Q_0 u_y^{(0,1)}[t, z] - 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_x^{(1,0,0,0)}[t, x, y, z] \end{aligned} $
$ \begin{aligned} & 6 \alpha_2 Q_0^2 h_x^{(0,1)}[t, z] + 6 \alpha_3 Q_0^2 h_x^{(0,1)}[t, z] - \frac{15}{2} \alpha_2 Q_0 h_+^{(0,2)}[t, z] - \\ & \frac{15}{2} \alpha_3 Q_0 h_+^{(0,2)}[t, z] - 3 \alpha_2 h_x^{(0,3)}[t, z] - 3 \alpha_3 h_x^{(0,3)}[t, z] + 2 \alpha_2 Q_0 u_t^{(1,0)}[t, z] + \\ & 2 \alpha_3 Q_0 u_t^{(1,0)}[t, z] + \frac{3}{2} \alpha_2 Q_0 h_+^{(2,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0 h_+^{(2,0)}[t, z] - \\ & \alpha_2 q_t^{(2,0)}[t, z] - \alpha_3 q_t^{(2,0)}[t, z] + 3 \alpha_2 h_x^{(2,1)}[t, z] + 3 \alpha_3 h_x^{(2,1)}[t, z] \end{aligned} $
$ \begin{aligned} & -\frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_t[t, z] - 2 \alpha_2 Q_0^2 u_t[t, z] - 2 \alpha_3 Q_0^2 u_t[t, z] + 2 \alpha_2 u_t^{(0,2)}[t, z] + \\ & \frac{5}{6} \chi u_t^{(0,2)}[t, z] - \frac{1}{6} \xi u_t^{(0,2)}[t, z] + \frac{3}{2} \alpha_2 Q_0^2 h_+^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0^2 h_+^{(1,0)}[t, z] + \\ & 4 \alpha_2 Q_0 q_t^{(1,0)}[t, z] + 4 \alpha_3 Q_0 q_t^{(1,0)}[t, z] + 6 \alpha_2 Q_0 h_x^{(1,1)}[t, z] + \\ & 6 \alpha_3 Q_0 h_x^{(1,1)}[t, z] - 3 \alpha_2 h_+^{(1,2)}[t, z] - 3 \alpha_3 h_+^{(1,2)}[t, z] + 3 \alpha_2 h_+^{(3,0)}[t, z] + \\ & 3 \alpha_3 h_+^{(3,0)}[t, z] - 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] - \zeta \epsilon_z^{(0,0,0,1)}[t, x, y, z] - \\ & 2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] - 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z] - \zeta \epsilon_x^{(0,1,0,0)}[t, x, y, z] \end{aligned} $
$ \begin{aligned} & \frac{1}{2} B_x \zeta q_t[t, z] + 2 B_x \theta h_x^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_x^{(0,1)}[t, z] - \\ & 2 \theta b^{(0,1)}[t, z] - \zeta b^{(0,1)}[t, z] - 4 \alpha_2 Q_0 u_y^{(0,1)}[t, z] - 3 \alpha_3 Q_0 u_y^{(0,1)}[t, z] - \\ & \frac{1}{4} \chi Q_0 u_y^{(0,1)}[t, z] + 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_x^{(1,0,0,0)}[t, x, y, z] \end{aligned} $

$ \begin{aligned} & -B_x \zeta u_y[t, z] - 6 \alpha_2 Q_0^2 h_+^{(0,1)}[t, z] - 6 \alpha_3 Q_0^2 h_+^{(0,1)}[t, z] - \\ & \frac{5}{2} \alpha_2 Q_0 q_t^{(0,1)}[t, z] - \frac{3}{2} \alpha_3 Q_0 q_t^{(0,1)}[t, z] - \frac{15}{2} \alpha_2 Q_0 h_x^{(0,2)}[t, z] - \\ & \frac{15}{2} \alpha_3 Q_0 h_x^{(0,2)}[t, z] + 3 \alpha_2 h_+^{(0,3)}[t, z] + 3 \alpha_3 h_+^{(0,3)}[t, z] - 2 \alpha_2 u_t^{(1,1)}[t, z] - \\ & \frac{5}{6} \chi u_t^{(1,1)}[t, z] + \frac{1}{6} \xi u_t^{(1,1)}[t, z] + \frac{3}{2} \alpha_2 Q_0 h_x^{(2,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0 h_x^{(2,0)}[t, z] - \\ & 3 \alpha_2 h_+^{(2,1)}[t, z] - 3 \alpha_3 h_+^{(2,1)}[t, z] + 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_z^{(1,0,0,0)}[t, x, y, z] \end{aligned} $
$ \begin{aligned} & -\frac{1}{2} B_x \zeta u_t[t, z] - \frac{1}{2} Q_0 \zeta \epsilon_x[t, x, y, z] + \frac{1}{2} \alpha_2 Q_0 q_y^{(0,1)}[t, z] + \\ & \frac{3}{2} \alpha_3 Q_0 q_y^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_+^{(1,0)}[t, z] + \zeta \epsilon_z^{(0,0,1,0)}[t, x, y, z] \end{aligned} $
$ \begin{aligned} & -6 \alpha_2 Q_0^2 h_x^{(0,1)}[t, z] - 6 \alpha_3 Q_0^2 h_x^{(0,1)}[t, z] + \frac{15}{2} \alpha_2 Q_0 h_+^{(0,2)}[t, z] + \\ & \frac{15}{2} \alpha_3 Q_0 h_+^{(0,2)}[t, z] + 3 \alpha_2 h_x^{(0,3)}[t, z] + 3 \alpha_3 h_x^{(0,3)}[t, z] - 2 \alpha_2 Q_0 u_t^{(1,0)}[t, z] - \\ & 2 \alpha_3 Q_0 u_t^{(1,0)}[t, z] - \frac{3}{2} \alpha_2 Q_0 h_+^{(2,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0 h_+^{(2,0)}[t, z] + \\ & \alpha_2 q_t^{(2,0)}[t, z] + \alpha_3 q_t^{(2,0)}[t, z] - 3 \alpha_2 h_x^{(2,1)}[t, z] - 3 \alpha_3 h_x^{(2,1)}[t, z] \end{aligned} $
$ \begin{aligned} & B_x \zeta u_y[t, z] + 6 \alpha_2 Q_0^2 h_+^{(0,1)}[t, z] + 6 \alpha_3 Q_0^2 h_+^{(0,1)}[t, z] + \\ & \frac{5}{2} \alpha_2 Q_0 q_t^{(0,1)}[t, z] + \frac{3}{2} \alpha_3 Q_0 q_t^{(0,1)}[t, z] + \frac{15}{2} \alpha_2 Q_0 h_x^{(0,2)}[t, z] + \\ & \frac{15}{2} \alpha_3 Q_0 h_x^{(0,2)}[t, z] - 3 \alpha_2 h_+^{(0,3)}[t, z] - 3 \alpha_3 h_+^{(0,3)}[t, z] + 2 \alpha_2 u_t^{(1,1)}[t, z] + \\ & \frac{5}{6} \chi u_t^{(1,1)}[t, z] - \frac{1}{6} \xi u_t^{(1,1)}[t, z] - \frac{3}{2} \alpha_2 Q_0 h_x^{(2,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0 h_x^{(2,0)}[t, z] + \\ & 3 \alpha_2 h_+^{(2,1)}[t, z] + 3 \alpha_3 h_+^{(2,1)}[t, z] - 2 \theta \epsilon_z^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_z^{(1,0,0,0)}[t, x, y, z] \end{aligned} $
$ \begin{aligned} & \frac{1}{2} Q_0 \zeta \epsilon_y[t, x, y, z] - 5 \alpha_2 q_y^{(0,2)}[t, z] - 3 \alpha_3 q_y^{(0,2)}[t, z] + \\ & \alpha_2 Q_0 u_y^{(1,0)}[t, z] + \frac{1}{4} \chi Q_0 u_y^{(1,0)}[t, z] - \zeta \epsilon_x^{(0,0,0,1)}[t, x, y, z] \end{aligned} $
$ \begin{aligned} & -\frac{1}{2} B_x \zeta u_t[t, z] - \frac{1}{2} Q_0 \zeta \epsilon_x[t, x, y, z] - \frac{5}{2} \alpha_2 Q_0 q_y^{(0,1)}[t, z] - \\ & \frac{3}{2} \alpha_3 Q_0 q_y^{(0,1)}[t, z] - 2 \alpha_2 u_y^{(1,1)}[t, z] - \frac{1}{2} \chi u_y^{(1,1)}[t, z] - \zeta \epsilon_y^{(0,0,0,1)}[t, x, y, z] \end{aligned} $
$ \begin{aligned} & \frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_t[t, z] + 2 \alpha_2 Q_0^2 u_t[t, z] + 2 \alpha_3 Q_0^2 u_t[t, z] - \frac{1}{3} \chi u_t^{(0,2)}[t, z] + \frac{1}{6} \xi u_t^{(0,2)}[t, z] - \\ & 4 \alpha_2 Q_0 q_t^{(1,0)}[t, z] - 4 \alpha_3 Q_0 q_t^{(1,0)}[t, z] + 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] + 2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + \\ & \zeta \epsilon_y^{(0,0,1,0)}[t, x, y, z] + 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z] + \zeta \epsilon_x^{(0,1,0,0)}[t, x, y, z] \end{aligned} $
$ \begin{aligned} & -\frac{1}{2} Q_0 \zeta \epsilon_y[t, x, y, z] + 5 \alpha_2 q_y^{(0,2)}[t, z] + 3 \alpha_3 q_y^{(0,2)}[t, z] - \\ & \alpha_2 Q_0 u_y^{(1,0)}[t, z] - \frac{1}{4} \chi Q_0 u_y^{(1,0)}[t, z] + \zeta \epsilon_x^{(0,0,0,1)}[t, x, y, z] \end{aligned} $
$ \begin{aligned} & -5 \alpha_2 q_t^{(0,2)}[t, z] - 3 \alpha_3 q_t^{(0,2)}[t, z] - 2 \alpha_2 Q_0 u_t^{(1,0)}[t, z] - \\ & 2 \alpha_3 Q_0 u_t^{(1,0)}[t, z] + \alpha_2 q_t^{(2,0)}[t, z] + \alpha_3 q_t^{(2,0)}[t, z] \end{aligned} $
$ \begin{aligned} & -\frac{1}{2} B_x \zeta q_t[t, z] - 2 B_x \theta h_x^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_x^{(0,1)}[t, z] + \\ & 2 \theta b^{(0,1)}[t, z] + \alpha_2 Q_0 u_y^{(0,1)}[t, z] + \frac{1}{4} \chi Q_0 u_y^{(0,1)}[t, z] - 5 \alpha_2 q_y^{(1,1)}[t, z] - \\ & 3 \alpha_3 q_y^{(1,1)}[t, z] - 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_x^{(1,0,0,0)}[t, x, y, z] \end{aligned} $
$ \begin{aligned} & \frac{1}{2} B_x \zeta u_t[t, z] + \frac{1}{2} Q_0 \zeta \epsilon_x[t, x, y, z] + \frac{5}{2} \alpha_2 Q_0 q_y^{(0,1)}[t, z] + \\ & \frac{3}{2} \alpha_3 Q_0 q_y^{(0,1)}[t, z] + 2 \alpha_2 u_y^{(1,1)}[t, z] + \frac{1}{2} \chi u_y^{(1,1)}[t, z] + \zeta \epsilon_y^{(0,0,0,1)}[t, x, y, z] \end{aligned} $

$5 \alpha_2 q_t^{(0,2)}[t, z] + 3 \alpha_3 q_t^{(0,2)}[t, z] + 2 \alpha_2 Q_0 u_t^{(1,0)}[t, z] +$ $2 \alpha_3 Q_0 u_t^{(1,0)}[t, z] - \alpha_2 q_t^{(2,0)}[t, z] - \alpha_3 q_t^{(2,0)}[t, z]$
$-\frac{1}{2} Q_0 \zeta b[t, z] - \frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_y[t, z] - \frac{3}{2} \alpha_2 Q_0^2 u_y[t, z] - \frac{3}{2} \alpha_3 Q_0^2 u_y[t, z] +$ $2 B_x \theta h_+^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_+^{(0,1)}[t, z] + \frac{1}{3} \chi u_y^{(0,2)}[t, z] - \frac{1}{6} \xi u_y^{(0,2)}[t, z] +$ $\frac{1}{2} \alpha_2 Q_0 q_y^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0 q_y^{(1,0)}[t, z] - 2 \alpha_2 u_y^{(2,0)}[t, z] - \frac{5}{6} \chi u_y^{(2,0)}[t, z] +$ $\frac{1}{6} \xi u_y^{(2,0)}[t, z] - 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] - \zeta \epsilon_y^{(1,0,0,0)}[t, x, y, z]$
$-\frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_t[t, z] - 2 \alpha_2 Q_0^2 u_t[t, z] - 2 \alpha_3 Q_0^2 u_t[t, z] + \frac{1}{3} \chi u_t^{(0,2)}[t, z] - \frac{1}{6} \xi u_t^{(0,2)}[t, z] +$ $4 \alpha_2 Q_0 q_t^{(1,0)}[t, z] + 4 \alpha_3 Q_0 q_t^{(1,0)}[t, z] - 2 \theta \epsilon_z^{(0,0,0,1)}[t, x, y, z] - 2 \theta \epsilon_y^{(0,0,1,0)}[t, x, y, z] -$ $\zeta \epsilon_y^{(0,0,1,0)}[t, x, y, z] - 2 \theta \epsilon_x^{(0,1,0,0)}[t, x, y, z] - \zeta \epsilon_x^{(0,1,0,0)}[t, x, y, z]$
$\frac{1}{2} B_x \zeta q_t[t, z] + 2 B_x \theta h_x^{(0,1)}[t, z] + \frac{1}{2} B_x \zeta h_x^{(0,1)}[t, z] -$ $2 \theta b^{(0,1)}[t, z] - \alpha_2 Q_0 u_y^{(0,1)}[t, z] - \frac{1}{4} \chi Q_0 u_y^{(0,1)}[t, z] + 5 \alpha_2 q_y^{(1,1)}[t, z] +$ $3 \alpha_3 q_y^{(1,1)}[t, z] + 2 \theta \epsilon_x^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_x^{(1,0,0,0)}[t, x, y, z]$
$\frac{1}{2} Q_0 \zeta b[t, z] + \frac{4}{3} \mathcal{M}_{\text{Pl}}^2 u_y[t, z] + \frac{3}{2} \alpha_2 Q_0^2 u_y[t, z] + \frac{3}{2} \alpha_3 Q_0^2 u_y[t, z] -$ $2 B_x \theta h_+^{(0,1)}[t, z] - \frac{1}{2} B_x \zeta h_+^{(0,1)}[t, z] - \frac{1}{3} \chi u_y^{(0,2)}[t, z] + \frac{1}{6} \xi u_y^{(0,2)}[t, z] -$ $\frac{1}{2} \alpha_2 Q_0 q_y^{(1,0)}[t, z] - \frac{3}{2} \alpha_3 Q_0 q_y^{(1,0)}[t, z] + 2 \alpha_2 u_y^{(2,0)}[t, z] + \frac{5}{6} \chi u_y^{(2,0)}[t, z] -$ $\frac{1}{6} \xi u_y^{(2,0)}[t, z] + 2 \theta \epsilon_y^{(1,0,0,0)}[t, x, y, z] + \zeta \epsilon_y^{(1,0,0,0)}[t, x, y, z]$
$9 \alpha_2 Q_0^3 q_t[t, z] + 27 \alpha_3 Q_0^3 q_t[t, z] + \frac{1}{2} B_x \zeta u_y^{(0,1)}[t, z] + \frac{9}{2} \alpha_2 Q_0^2 u_t^{(1,0)}[t, z] +$ $\frac{9}{2} \alpha_3 Q_0^2 u_t^{(1,0)}[t, z] + \alpha_2 u_t^{(1,2)}[t, z] + \alpha_3 u_t^{(1,2)}[t, z] + 3 \alpha_2 u_t^{(3,0)}[t, z] + 3 \alpha_3 u_t^{(3,0)}[t, z]$
$B_x Q_0 \zeta u_t[t, z] + Q_0^2 \zeta \epsilon_x[t, x, y, z] + 2 \alpha_2 Q_0^2 q_y^{(0,1)}[t, z] + \frac{1}{2} B_x Q_0 \zeta h_+^{(1,0)}[t, z] +$ $2 \alpha_2 Q_0 u_y^{(1,1)}[t, z] + \frac{1}{2} \chi Q_0 u_y^{(1,1)}[t, z] + Q_0 \zeta \epsilon_y^{(0,0,0,1)}[t, x, y, z] - Q_0 \zeta \epsilon_z^{(0,0,1,0)}[t, x, y, z]$
$Q_0^2 \zeta \epsilon_y[t, x, y, z] - B_x \epsilon_z[t, x, y, z] - 3 B_x \theta u_t^{(0,1)}[t, z] - B_x \zeta u_t^{(0,1)}[t, z] -$ $2 \alpha_2 Q_0 q_y^{(0,2)}[t, z] + \frac{1}{2} B_x Q_0 \zeta h_x^{(1,0)}[t, z] + 6 \alpha_2 Q_0^2 u_y^{(1,0)}[t, z] + 4 \alpha_3 Q_0^2 u_y^{(1,0)}[t, z] +$ $\frac{1}{2} \chi Q_0^2 u_y^{(1,0)}[t, z] - \alpha_2 u_y^{(1,2)}[t, z] - \alpha_3 u_y^{(1,2)}[t, z] + \alpha_2 Q_0 q_y^{(2,0)}[t, z] + \alpha_3 Q_0 q_y^{(2,0)}[t, z] +$ $\alpha_2 u_y^{(3,0)}[t, z] + \alpha_3 u_y^{(3,0)}[t, z] - Q_0 \zeta \epsilon_x^{(0,0,0,1)}[t, x, y, z] + Q_0 \zeta \epsilon_z^{(0,1,0,0)}[t, x, y, z]$
$-B_x Q_0 \zeta q_y[t, z] + B_x \epsilon_y[t, x, y, z] + Q_0^2 \zeta \epsilon_z[t, x, y, z] - 2 \alpha_2 Q_0^2 u_t^{(0,1)}[t, z] -$ $\frac{1}{2} \chi Q_0^2 u_t^{(0,1)}[t, z] - \alpha_2 u_t^{(0,3)}[t, z] - \alpha_3 u_t^{(0,3)}[t, z] - 3 B_x \theta u_y^{(1,0)}[t, z] -$ $B_x \zeta u_y^{(1,0)}[t, z] + 3 \alpha_2 Q_0 q_t^{(1,1)}[t, z] + 3 \alpha_3 Q_0 q_t^{(1,1)}[t, z] + 5 \alpha_2 u_t^{(2,1)}[t, z] +$ $5 \alpha_3 u_t^{(2,1)}[t, z] + Q_0 \zeta \epsilon_x^{(0,0,1,0)}[t, x, y, z] - Q_0 \zeta \epsilon_y^{(0,1,0,0)}[t, x, y, z]$

$ \begin{aligned} & -9 \alpha_2 Q_0^3 q_t[t, z] - 27 \alpha_3 Q_0^3 q_t[t, z] - 12 \alpha_2 Q_0^3 h_x^{(0,1)}[t, z] - 12 \alpha_3 Q_0^3 h_x^{(0,1)}[t, z] - \\ & \frac{1}{2} B_x \zeta u_y^{(0,1)}[t, z] + 18 \alpha_2 Q_0^2 h_+^{(0,2)}[t, z] + 18 \alpha_3 Q_0^2 h_+^{(0,2)}[t, z] - 4 \alpha_2 Q_0 q_t^{(0,2)}[t, z] + \\ & 12 \alpha_2 Q_0 h_x^{(0,3)}[t, z] + 12 \alpha_3 Q_0 h_x^{(0,3)}[t, z] - 3 \alpha_2 h_+^{(0,4)}[t, z] - 3 \alpha_3 h_+^{(0,4)}[t, z] + \\ & \frac{3}{2} \alpha_2 Q_0^2 u_t^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0^2 u_t^{(1,0)}[t, z] - 3 \alpha_2 u_t^{(1,2)}[t, z] - 3 \alpha_3 u_t^{(1,2)}[t, z] - \\ & 3 \alpha_2 Q_0^2 h_+^{(2,0)}[t, z] - 3 \alpha_3 Q_0^2 h_+^{(2,0)}[t, z] + 4 \alpha_2 Q_0 q_t^{(2,0)}[t, z] + 4 \alpha_3 Q_0 q_t^{(2,0)}[t, z] - \\ & 12 \alpha_2 Q_0 h_x^{(2,1)}[t, z] - 12 \alpha_3 Q_0 h_x^{(2,1)}[t, z] + 6 \alpha_2 h_+^{(2,2)}[t, z] + 6 \alpha_3 h_+^{(2,2)}[t, z] + \\ & 3 \alpha_2 u_t^{(3,0)}[t, z] + 3 \alpha_3 u_t^{(3,0)}[t, z] - 3 \alpha_2 h_+^{(4,0)}[t, z] - 3 \alpha_3 h_+^{(4,0)}[t, z] \end{aligned} $
$ \begin{aligned} & B_x b[t, z] + \frac{3}{2} B_x Q_0 \zeta u_y[t, z] + 12 \alpha_2 Q_0^3 h_+^{(0,1)}[t, z] + 12 \alpha_3 Q_0^3 h_+^{(0,1)}[t, z] + \\ & 18 \alpha_2 Q_0^2 h_x^{(0,2)}[t, z] + 18 \alpha_3 Q_0^2 h_x^{(0,2)}[t, z] - 12 \alpha_2 Q_0 h_+^{(0,3)}[t, z] - \\ & 12 \alpha_3 Q_0 h_+^{(0,3)}[t, z] - 3 \alpha_2 h_+^{(0,4)}[t, z] - 3 \alpha_3 h_+^{(0,4)}[t, z] - 3 \alpha_2 Q_0^2 h_x^{(2,0)}[t, z] - \\ & 3 \alpha_3 Q_0^2 h_x^{(2,0)}[t, z] + 12 \alpha_2 Q_0 h_+^{(2,1)}[t, z] + 12 \alpha_3 Q_0 h_+^{(2,1)}[t, z] + \\ & 6 \alpha_2 h_x^{(2,2)}[t, z] + 6 \alpha_3 h_x^{(2,2)}[t, z] - 3 \alpha_2 h_+^{(4,0)}[t, z] - 3 \alpha_3 h_+^{(4,0)}[t, z] \end{aligned} $
$2 \alpha_2 Q_0 u_y^{(0,2)}[t, z] + \frac{1}{2} \chi Q_0 u_y^{(0,2)}[t, z]$
$ \begin{aligned} & -9 \alpha_2 Q_0^3 q_t[t, z] - 27 \alpha_3 Q_0^3 q_t[t, z] + 12 \alpha_2 Q_0^3 h_x^{(0,1)}[t, z] + 12 \alpha_3 Q_0^3 h_x^{(0,1)}[t, z] - \\ & 6 B_x \theta u_y^{(0,1)}[t, z] - \frac{5}{2} B_x \zeta u_y^{(0,1)}[t, z] - 18 \alpha_2 Q_0^2 h_+^{(0,2)}[t, z] - 18 \alpha_3 Q_0^2 h_+^{(0,2)}[t, z] - \\ & 4 \alpha_2 Q_0 q_t^{(0,2)}[t, z] - 12 \alpha_2 Q_0 h_x^{(0,3)}[t, z] - 12 \alpha_3 Q_0 h_x^{(0,3)}[t, z] + 3 \alpha_2 h_+^{(0,4)}[t, z] + \\ & 3 \alpha_3 h_+^{(0,4)}[t, z] + \frac{3}{2} \alpha_2 Q_0^2 u_t^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0^2 u_t^{(1,0)}[t, z] - 3 \alpha_2 u_t^{(1,2)}[t, z] - \\ & 3 \alpha_3 u_t^{(1,2)}[t, z] + 3 \alpha_2 Q_0^2 h_+^{(2,0)}[t, z] + 3 \alpha_3 Q_0^2 h_+^{(2,0)}[t, z] + 4 \alpha_2 Q_0 q_t^{(2,0)}[t, z] + \\ & 4 \alpha_3 Q_0 q_t^{(2,0)}[t, z] + 12 \alpha_2 Q_0 h_x^{(2,1)}[t, z] + 12 \alpha_3 Q_0 h_x^{(2,1)}[t, z] - 6 \alpha_2 h_+^{(2,2)}[t, z] - \\ & 6 \alpha_3 h_+^{(2,2)}[t, z] + 3 \alpha_2 u_t^{(3,0)}[t, z] + 3 \alpha_3 u_t^{(3,0)}[t, z] + 3 \alpha_2 h_+^{(4,0)}[t, z] + 3 \alpha_3 h_+^{(4,0)}[t, z] \end{aligned} $
$ \begin{aligned} & -Q_0 \zeta b^{(0,1)}[t, z] + \alpha_2 Q_0^2 u_y^{(0,1)}[t, z] + \alpha_3 Q_0^2 u_y^{(0,1)}[t, z] - \alpha_2 u_y^{(0,3)}[t, z] - \\ & \alpha_3 u_y^{(0,3)}[t, z] - \alpha_2 Q_0 q_y^{(1,1)}[t, z] + \alpha_3 Q_0 q_y^{(1,1)}[t, z] + \alpha_2 u_y^{(2,1)}[t, z] + \alpha_3 u_y^{(2,1)}[t, z] \end{aligned} $
$ \begin{aligned} & -9 \alpha_2 Q_0^3 q_t[t, z] - 27 \alpha_3 Q_0^3 q_t[t, z] - 6 B_x \theta u_y^{(0,1)}[t, z] - \\ & \frac{5}{2} B_x \zeta u_y^{(0,1)}[t, z] + 2 \alpha_2 Q_0 q_t^{(0,2)}[t, z] + 2 \alpha_3 Q_0 q_t^{(0,2)}[t, z] + \\ & \frac{3}{2} \alpha_2 Q_0^2 u_t^{(1,0)}[t, z] + \frac{3}{2} \alpha_3 Q_0^2 u_t^{(1,0)}[t, z] + \alpha_2 u_t^{(1,2)}[t, z] + \alpha_3 u_t^{(1,2)}[t, z] + \\ & 4 \alpha_2 Q_0 q_t^{(2,0)}[t, z] + 4 \alpha_3 Q_0 q_t^{(2,0)}[t, z] + 3 \alpha_2 u_t^{(3,0)}[t, z] + 3 \alpha_3 u_t^{(3,0)}[t, z] \end{aligned} $
$-2 B_x h_+^{(0,2)}[t, z] + 6 \theta u_y^{(0,3)}[t, z] + 2 \zeta u_y^{(0,3)}[t, z] - 6 \theta u_y^{(2,1)}[t, z] - 2 \zeta u_y^{(2,1)}[t, z]$
$-2 B_x h_x^{(0,2)}[t, z] + 2 b^{(0,2)}[t, z] - 2 b^{(2,0)}[t, z]$

**Key observation:** This is the end of the script.