# Some Title

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#### Abstract

Doing cool stuff to find cooler stuff

#### 1 Introduction

The very important physical effect has applications to something in the world.

# 2 Background

Gravitational theories and the Gertsenshtein effect combined in groundbreaking ways. We use a (1,-1,-1,-1) metric.

The paper used the Einstein-Maxwell Lagrangian

$$\mathcal{L} = \sqrt{g}(R + \kappa F_{\mu\nu}F^{\mu\nu}),\tag{1}$$

where g is the determinant of the metric, R is the Ricci Scalar, and F is the electromagnetic field tensor.

Linearizing the action and varying it gives the Maxwell field equations and the linearized gravitational wave equation. Using a transverse traceless Lorentz gauge for h, a small metric perturbation, with  $\hbar_{\mu\nu} = \hbar_{\mu\nu}(t,z)$ , they evaluated the components of the field equations (making a lot of assumptions about the EM wave and background), giving two mixing equations:

$$\ddot{\mathscr{E}}_{y} - \mathscr{E}_{y}^{"} = B_0 \mathring{\mathscr{R}}_{12}^{"} \tag{2}$$

$$\ddot{\mathcal{R}}_{12} - \mathcal{R}_{12}^{"} = 4B_0 \mathcal{E}_y \tag{3}$$

Here, we have a background magnetic field  $\mathbf{B_0} = \mathbf{B}_{0x}$ , and a magnetic perturbation  $\mathbf{b} = \mathbf{b}_y(t, z)$ . We have no electric field in the background, only in the wave.

We recreated the calculation with torsion, and got back the same equations.

# 3 Our stuff

We see what we get for the mixing equations with more complicated couplings in the lagrangian, with a torsion theory. We have a perturbation on the torsion given by

$$\mathcal{T}^{\mu}_{\nu\sigma} = \epsilon^{\mu}_{\nu\sigma\lambda} \mathcal{Q}^{\lambda} + \delta^{\mu}_{[\nu} \mathcal{U}_{\sigma]} \tag{4}$$

with

$$Q_{\mu} = [q_0(t, z), 0, q_2(t, z), 0] \tag{5}$$

and

$$\mathcal{U}_{\mu} = [u_0(t, z), 0, u_2(t, z), 0]. \tag{6}$$

When we add inn a background torsion T, we use U=0 and

$$Q_{\mu} = [q_0, 0, 0, 0] \tag{7}$$

Lagrangian 1 -  $R_{\mu\nu}F^{\mu\nu}$ 

$$\mathcal{L} = \sqrt{g}(R + \zeta R_{\mu\nu}F^{\mu\nu} + \kappa F \mu\nu F^{\mu\nu}) \tag{8}$$

With no background torsion:

Maxwell and Einstein gives us nothing new on the face of it. The torsion field equations give us a bunch of mixings (notably with only first-order derivatives):

$$\frac{1}{2}B_0 \mathcal{R}'_+ - \partial_t \mathcal{E}_y = 0 \tag{9}$$

$$\frac{1}{2}B_0u_0 + \frac{1}{2}B_0\dot{R}_+ - \partial_y\mathcal{E}_z = 0 \tag{10}$$

$$\frac{1}{2}B_0\dot{R}_{\times} - \dot{\mathcal{E}} = 0 \tag{11}$$

We also have these three equations, which have some interesting impact on the original mixing equations we get from Maxwell and Einstein:

$$\frac{1}{2}B_0 \hat{n}_{\times}' - \hat{\mathscr{E}}' + \partial_t \mathscr{E}_x = 0 \tag{12}$$

$$\frac{1}{2}B_0 \mathscr{R}_{\times}' - \mathscr{E}' = 0 \tag{13}$$

$$\frac{1}{2}B_0 \hat{n}_{\times}' + \partial_t \mathcal{E}_x = 0 \tag{14}$$

meaning that each of the three terms individually are zero, removing the z-derivatives of both  $\mathcal{R}_{\times}$  and  $\mathcal{E}$  in our original mixing equations, leaving

$$\ddot{\mathscr{E}}_{\nu} = 0 \tag{15}$$

$$\ddot{\mathcal{R}}_{12} = 4B_0 \mathcal{E}_y \tag{16}$$

This lagrangian does not allow for a finite background torsion.

Lagrangian 2 -  $R_{\mu\nu}R^{\mu\nu}$ 

$$\mathcal{L} = \sqrt{g}(R + \zeta R_{\mu\nu}R^{\mu\nu} + \kappa F \mu\nu F^{\mu\nu}) = 0 \tag{17}$$

With no background torsion:

The torsion field equations have no EM-components, and only serve to help us simplify the other equations. The maxwell field equations are unchanged.

For the Einstein equation we get some higher-order derivatives, but no torsion contribution:

$$-4\kappa B_0 \mathcal{E} - \mathcal{H}_{\times}^{"} + \ddot{\mathcal{H}}_{\times} + \zeta \mathcal{H}^{""} + 2\zeta \ddot{\mathcal{H}}_{\times}^{"} + \zeta \ddot{\mathcal{H}}_{\times} = 0$$

$$\tag{18}$$

With background torsion:

Computer still running. Only expecting change in Einstein one.

We also did a run with the indices on the second Ricci tensor swapped, but achieved the same mixing equations (different torsion field equations, but that had no effect on the mixing equations).

Lagrangian 3 -  $R_{\mu\nu\sigma\lambda}R^{\mu\nu\sigma\lambda}$ 

$$\mathcal{L} = \sqrt{g}(R + \zeta R_{\mu\nu\sigma\lambda}R^{\mu\nu\sigma\lambda} + \kappa F \mu\nu F^{\mu\nu}) \tag{19}$$

With no background torsion:

Maxwell still gives the same equations as expected.

Torsion still only gives some substitutions

Einstein is not done evaluating.

With background torsion: Not started

Lagrangian 4 -  $\epsilon_{\mu\nu\sigma\lambda}R^{\mu\nu}F^{\sigma\lambda}$ 

$$\mathcal{L} = \sqrt{g}(R + \zeta \epsilon_{\mu\nu\sigma\lambda} R^{\mu\nu} F^{\sigma\lambda} + \kappa F \mu\nu F^{\mu\nu}) \tag{20}$$

With no background torsion:

No change in Maxwell, Einstein gave same mixing equation. Torsion field eqs gave some equations though:

$$2\mathscr{E}$$
 (21)

FIX NOTATION - DERIVATIVES AND by VS b AND PROBABLY MORE

With background torsion: Not started. Is it worth trying?

### 4 Conclusions

The world is forever changed

# References

[1] A source, available at https://wevbarker.com/.