Solve Weighing Pool Ball Puzzle

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1 Weighing Pool Ball Puzzle

The problem is described at `https://www.mathsisfun.com/pool_balls.html` with a solution. But the given solution is undetermined. Subsequent measures are depends on the result of the previous one. Ferenc Rákóczi mentioned, he already had a determined solution for this problem, but he forgot it.

1.1 The problem

You have 12 balls identical in size and appearance but 1 is an odd weight (could be either light or heavy).

You have a set of balance scales which will give 3 possible readings:

- Left = Right
- Left > Right
- Left < Right (ie Left and Right have equal weight, Left is Heavier, or Left is Lighter).

You have *only 3 chances* to weigh the balls in any combination using the scales. Find which ball is the odd one and if it's heavier or lighter than the rest.

2 Find one solution

2.1 Generate possible measures

At first I calculated the number of possible measures. We should put equal number of balls onto both arms to get a valid result, so pick even number of balls for measure, then pick half of the balls for Left arm and put remaining balls into Right arm. We can halve the numbers, because putting same set of balls to Right arm is same as to Left arm:

$$\sum_{i=1}^{6} \frac{\binom{12}{2i}\binom{2i}{i}}{2} = 36829$$

All three measures can be one of the 36829 possible ones, so the number of all possible solutions could be:

$$36829^3 = 49953943750789$$

Using information above we can generate possible measures one-by-one:

```
ids = (1..12).to_a                              # array [1, 2, .., 12] for balls
(1..6).each do |level|                          # how much balls to put on an arm
  ids.combination(level*2).each do |subset| # pick a combination for both arms
    reverse = []
    subset.combination(level).each do |left| # pick half of them for left arm
      next if reverse.include?(left)          # go to next if reversed already checked
      right = subset - left
      reverse << right
```

2.2 A subsection

More text.