Games

2a)

	L	C	R
T	3, 1	2, 2	0, 0
B	2, 2	0, 0	3, 1

(I assume the payoffs are structured

	column
row	

)

i. For the Column player, R is strictly dominated by L whether Row plays T or B, the payoff from L is greater, so Column would never play R. ✓

ii. Now, for the Row player, B is strictly dominated by T. ✓

iii. For Column, C strictly dominates L

So we can predict the outcome will be the strategy profile (T, C) with payoffs (2, 2). T & C survive ISD.

b)

	L	C	R
T	5, 7	3, 2	1, 0
B	2, 1	3, 3	4, 6

- No strategy for either player strictly dominates another, so we cannot even get started with IESDS.

Dominate using mixed strategies.

- If we delete weakly dominated strategies, we ~~can~~ cannot eliminate any either: for T vs B consider

$$\begin{array}{c} T \\ B \end{array}\begin{bmatrix} \underline{5} & 3 & 1 \\ 2 & 3 & \underline{4} \end{bmatrix}$$

and L vs C ; ~~wsL~~ L vs R ; C vs R

$$\begin{bmatrix} \underline{7} & 2 \\ 1 & \underline{3} \end{bmatrix} , \begin{bmatrix} \underline{7} & 0 \\ 1 & \underline{6} \end{bmatrix} ; \begin{bmatrix} \underline{3} & 0 \\ 3 & \underline{6} \end{bmatrix}$$

c) Consider T vs B

$$\begin{array}{c} T \\ B \end{array}\begin{bmatrix} 4 & 0 & \underline{3} \\ 4 & \underline{1} & 1 \end{bmatrix}$$

So neither is strictly dominated

$$\begin{array}{ccc} L \text{ vs } C & L \text{ vs } R & C \text{ vs } R \\ \begin{bmatrix} 6 & 6 \\ \underline{3} & 2 \end{bmatrix} & \begin{bmatrix} \underline{6} & 5 \\ 3 & 3 \end{bmatrix} & \begin{bmatrix} \underline{6} & 5 \\ 2 & \underline{3} \end{bmatrix} \end{array}$$

again, none is strictly dominated.

We can see that C is weakly dominated by L, as is R. Proceeding to delete from this, we'd conclude that the outcome will be (T, L) or (B, L), but

- if we eliminate both C and R first, we can't say which out of (T, L) or (B, L) happens
- if we elimate C, then eliminate Row's weakly dominated strategy B, then elimate R, we'd say the outcome is definitely (T, L)
- if we elimate R, then T (which would now be weakly dominated), then C, we'd say the outcome is definitely (B, L).

Great

So, using IEWDS is not appropriate because it can remove NE and means that our conclusions depend on arbitrary choices about the order in which we eliminate strategies.

d) Yes, true. No player will employ a strategy which is strictly dominated then another since it's by definition always worse for them. So if a unique strategy remains for each every player will certainly play that strategy, and the collection of them will be a NE from which there's no unilateral incentive to deviate.

e) ~~Not necessarily~~ Yes. Though it's not the case that a strategy that's part of a NE necessarily is a strictly best response – since e.g. in a mixed-strategy NE the agent is indifferent between playing the pure strategies in response; an agent may simply have constant payoffs from all strategies and so none is a strictly best response — it's not possible for a strategy that would be elimated by IESDS to be part of a NE. (This is why IESDS is useful: it does not delete any ~~NE~~ NE of the game.) Suppose s_i doesn't survive IESDS. Then ~~there~~ is another strategy s_i' which performs better than it for all actions from opponents. But then s_i cannot be part of a NE since it would not be a best response to the other players' strategies - s_i' is strictly better.

Since for Column, R is a ~~[illegible]~~ dominant strategy, there's no MSNE.

5a)

	L	R
U	0, ~~1~~ 1	<u>2</u>, <u>2</u>
D	<u>3</u>, 0	0, <u>3</u>

I have underlined the best-response payoffs to find the ~~strictly dominant strategies~~ NE.

⇒ There is a pure NE of (U, R).

~~Since there's exactly one pure PSNE, there are no MSNE.~~

~~· Let r be the probability of Column playing R. Then for Row,~~

~~$u_R(U) = 2r$~~

~~$u_R(D) = 3(1-r)$~~

~~So Row would be indifferent about playing U when~~

~~$2r = 3-3r \Rightarrow r = 3/5$~~

(*) [is this right? why?]

↳ OK, it's wrong, but what's the intuition?

...Intuition for what?

~~· We can similarly let d be the probability Row plays D. Then for Column~~

~~$u_c(L) = 1-d$~~

~~$u_c(R) = 2(1-d) + 3d$~~

~~So Column would be indifferent about playing R when~~

~~$1-d = 2-2d+3d \Rightarrow d = -1/2$~~

(*) How would I have known

~~But $0 \le d \le 1$,~~

~~So we don't have a MSNE.~~ ~~where Column plays R w.p. 3/5 and Row plays D w.p. 2/5.~~

b)

	L	R
U	0, $\underline{3}$	$\underline{2}$, 2
D	$\underline{3}$, 0	0, $\underline{3}$

again, underlining best-response payoffs

There is no pure strategy NE.

To find the MSNE, let Row play U_p w.p. p. Then

$u_{column}(L) = 3p$

$u_{column}(R) = 2p + 3(1-p)$

So Column is indifferent between L and R when

$3p = 2p + 3 - 3p \Rightarrow p = 3/4$

Let Column play Left w.p. q, then

$u_{Row}(U) = 2(1-q)$

$u_{Row}(D) = 3q$

and they're indifferent between U and Down when

$2(1-q) = 3q \Rightarrow q = 2/5$

[how are you meant to / actually do you justify this?]

So at the MSNE Row plays (U_p, Down) w.p. $(3/4, 1/4)$ and Column plays (Left, Right) w.p. $(2/5, 3/5)$.

Justify what aspect of it? In an MNE, no player has a strict unilateral incentive to deviate.

c)

	L	R
U	0, $\underline{3}$	$\underline{2}$, $\underline{2}$
D	$\underline{3}$, 0	0, $\underline{3}$

(U, R) is a PSNE. But as neither player has a dominant strategy there's also a MSNE.

Let Row play U w.p. p. $u_c(L) = 2p$; $u_c(R) = 2p + 3(1-p)$

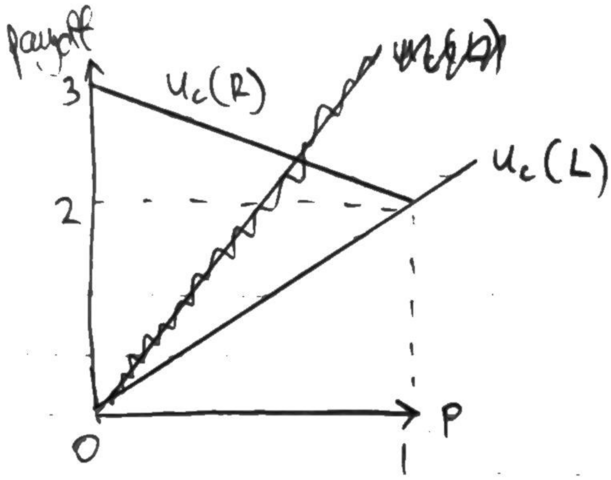
intuition? ⊛

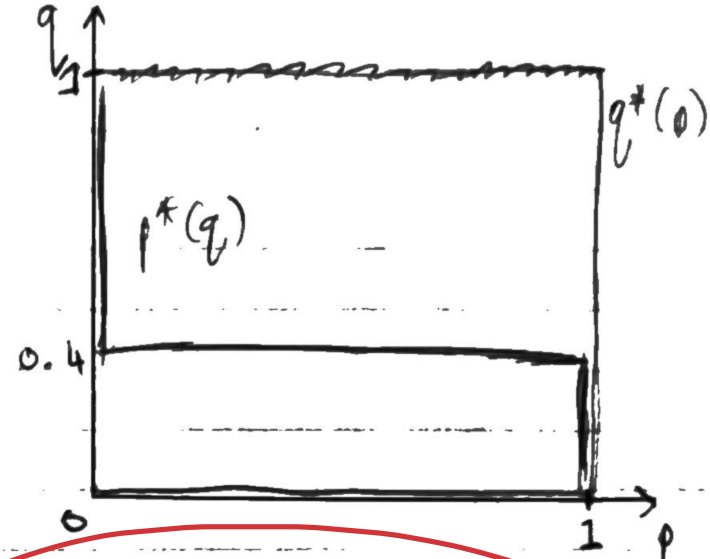
So for column to be indifferent, we require $p = 1$, i.e. Row always ~~plays~~ U.

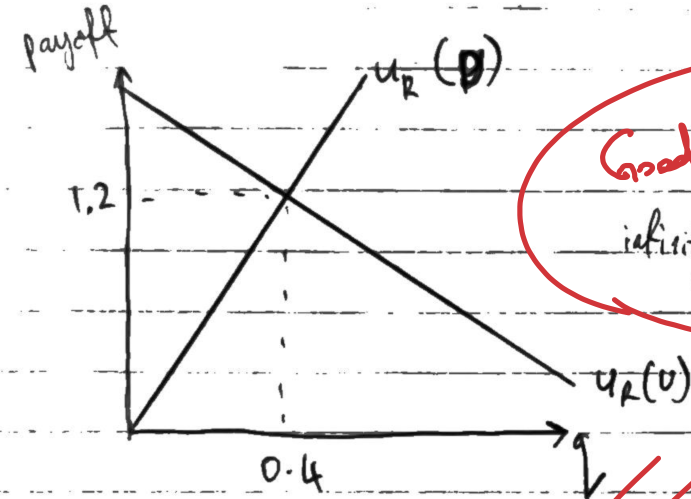
Let Column play L w.p. q. $u_R(U) = \cancel{2(1-q)}$; $u_R(D) = 3q$

So for Row to be indifferent, we require $q = 2/5$.

Therefore there is a MSNE ~~in~~ where ~~where~~ ~~that~~ Row plays U_p and Column plays (L, R) w.p. $(2/5, 3/5)$.







Good. In fact, there are infinitely many mixed-strategy NE characterised as:

Row plays (U, D) w.p. (1, 0)
Column plays (L, R) w.p. (q, 1-q)
where $q \in [0, 2/5]$

We'll look at this in class -

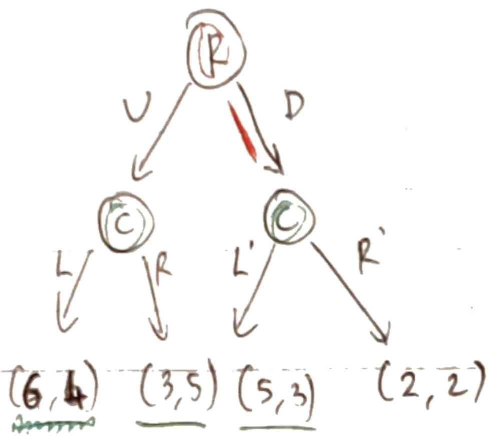
[Do you have to draw out the graphs to know whether there are infinitely many MSNE? e.g. in part b) ~~how~~ can you read off the payoff matrix that there will be only one, etc? In general, would be keen to understand 'shortcuts' for knowing the form of solution + why these work].

6. a)

	L	R
U	6, 4	3, 5
D	5, 3	2, 2

The pure strategy NE is (U, R)

b)



We can find SPE by backwards induction, as a SPE is a strategy profile that's a NE in every subgame.

- For Col it's optimal to play R after U and L after D
- So for Row it's optimal to play D.

(D, [L if D, R if U])

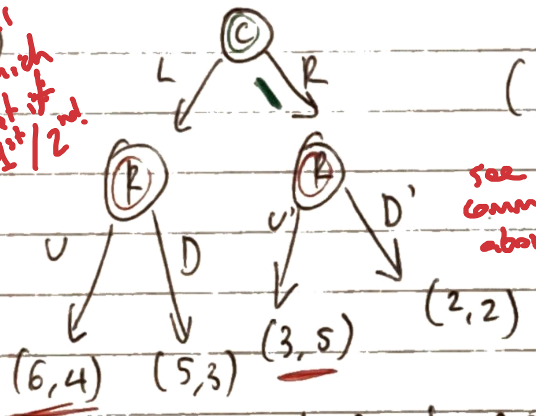
Can we say in general whether 1st or 2nd player does better/worse in sequential game vs simultaneous?

No. It will depend on the payoff structures of the players. There are classes of games for which we do know that it is better to move 1st/2nd.

The unique SPE is ~~(D,R)~~ ∧ this is not a NE in the simultaneous version of the game, because in that case Row would be better off from playing U, so they'd have an incentive to deviate. Here, because Col can punish Row for going Up by responding with L, this outcome is stable.

Can't properly compare: they are different games with different strategy spaces.

[I don't think I have a good intuitive grasp of why things have worked out differently going from simul. to seq.] In the sequential form of the game, yes the SPE is a NE. (see below).



The SPE is ([U if L, U if R], R), (which is the NE in the simultaneous game, and again ~~it is a strictly~~ a NE ~~dominant strategy~~ in the seq. game.

see comment above.

b)

	LL	LR	RL	RR
U	6, 4	6, 4	3, 5	3, 5
D	5, 3	2, 2	5, 3	2, 2

Nice

SPE

not credible: if R goes D C should go L

c)

	L	R
UU	6, 4	3, 5
UD	6, 4	2, 2
DU	5, 3	3, 5
DD	5, 3	2, 2

SPE

not credible: if C goes R, R should go U

10 a)

	C	D
C	5, 5	1, 8
D	8, 1	4, 4

(D, D) is the unique NE; defecting is strictly dominant for each player.

b) The only NE in the finitely repeated game is for each player to defect on every turn. By backwards induction: ~~was the~~
- in the N^{th} period, both will defect. ~~as~~ this is equivalent to a nonrepeated simultaneous game as there are no future rounds to be concerned with
- in the $(N-1)^{th}$ period, both know the next time they'll defect, which behaviour in this period won't change. So they maximise their payoffs from that round alone, which means D
- this is true rolling back one period all the way to the start

So the only NE is to defect every time.

Uniqueness comes from the fact that the NE of the stage game is unique

[I think the above argument shows only that the unique SPE is to defect every time, rather than the stronger claim that there's only one NE. But I couldn't think of any others. What does a more thorough answer look like?]

c) An alternative ? strategy which may be a better response would be to cooperate for some number of turns then switch to defecting for the rest of time. (You wouldn't always cooperate after your defection as you know the opponent will always be defecting and always defecting is the best response to that.)

Not sure what you're doing here... Are you analysing the same strategy profile that was given in the question?

Let n be the ~~number of~~ turn on which you first defect.

Then your payoff from the initial cooperation is

$$\frac{5 \times (\delta^{n-1} - 1)}{\delta - 1},$$

this answer was right, he just misread it

your payoff from the first defection is $8 \times \delta^{n-1}$, and your payoff from the rest of time is $\frac{4 \times \delta^n}{1-\delta}$.

So, your total payoff will be

$$u(n;\delta) \quad \frac{5\times(\delta^{n-1}-1)}{\delta-1} + 8\times\delta^{n-1} + \frac{4\times\delta^{n}}{1-\delta}$$

and we are interested in the values of δ for which $u(n;\delta)$ is nondecreasing in n, i.e. it is ~~always~~ better to contine cooperating indefinitely in response to the opponent's trigger strategy.

[in the slides they just show that for some certain δ, you're better off continuing to cooperate forever compared to playing D on the first turn + always after. But I don't understand why they can be sure that this particular strategy (as opposed to, e.g. [C, C, C, D, D, ...]) is the relevant one to compare to.]

$$\frac{\partial u}{\partial n} = \frac{\delta^{n-1}\cdot \ln(\delta)\cdot(4\delta-3)}{\delta-1}$$

For $0<\delta<1$ and $n\in\mathbb{N}^+$,
$\delta^{n-1}>0$; $\ln(\delta)<0$; $\delta-1<0$

So $u(n;\delta)$ is increasing in n iff $4\delta-3>0$
$\iff \delta>3/4$

i.e. the trigger strategies are a NE when $\delta>0.75$

Not quite right, this would only be true if the strategy were to always defect after *anybody* (i.e. not just your opponent) defects

d) A strategy profile is SPE if it's a NE in every subgame. We can distinguish between subgames where:

- Both have always cooperated in the past — in this case, the situation is like starting from the first turn and so a NE as shown above.
- Someone defected in the past — also NE in these, because we know the opponent will always defect in future, to which the best response is to defect also, as the strategy does.

ok.

e) minmax punishment: the lowest payoff a player i can be forced to receive by others, assuming that i is trying to maximise their payoff. It's $\min_{s_{-i}} [\max_{s_i} u(s_i, s_{-i})]$.

I think the min and max are the wrong way around.

It's a "punishment" in this context because the threat of being forced into this lower payoff from defecting is what sustains cooperation in the repeated game.

individually rational: a strategy s_i is ~~thus~~ if it yields a payoff at least as high as the minmax value. If the payoff is lower, then the agent can do better by simply playing their minmax strategy on every turn, as it'd be guaranteed to be better for them

feasible payoff pair: a pair of payoffs, which can be achieved (on average per turn over the long run) as a weighted average of payoffs from pure strategy profiles

["convex combination" any different?] No.

It's not possible to get an average payoff that's better for someone than their best pure strategy payoff, or e.g. in this case for player 1 to get 8 while player 2 also gets 8 (or indeed even 5).

f) The Folk Theorem says that with sufficiently patient agents (i.e. when δ is near enough to 1), any average payoff profile which is individually rational and is feasible can be achieved as a SPE in an infinitely-repeated game. In this context, the presence of the credible threat to punish a defecting ~~player~~ opponent by you yourself defecting forever is enough to sustain mutually-beneficial cooperation. This is because, with large enough δ, the ~~potential~~ loss from punishment is bigger than the gain from defecting a single time, so it is not rational to defect at all.

Sure ... but will impose particular conditions on the preference structure –

→ Intertemporal tradeoffs.

[Intuition for why moving to infinite horizon helps with cooperation? I saw on Wikipedia there is a folk theorem for finite horizon games too.]