Rohan - Merton - 8: Identity

8.1 $\forall x \forall y \forall z (((Px \land Py) \land Pz) \rightarrow (((x=y) \lor (y=z)) \lor (x=z)))$

8.2 i) $D_A = \{1, 2\}$ $|Q|_A = \{\langle 1,2\rangle, \langle 2,1\rangle\}$ $|a|^\alpha_A = 1$, $|b|^\alpha_A = 2$

ii) $D_A = \{1\}$ $|P|_A = \emptyset$

(*) 8.3 The first formalisation would suffice if we wanted to conclude, e.g., that $\exists x Px$. Since our statement is an indefinite designator, there isn't a particularly good reason why we should proceed the formalisation with identity, so we might as well use the simpler one.

8.4 i)

$$\dfrac{[a=a]}{\exists y\, y=y}\ \square$$

ii)

$$\dfrac{\exists x \neg Px \qquad \dfrac{\dfrac{\dfrac{\dfrac{[Pa]^2 \qquad \dfrac{[\neg Pb]^3 \qquad [a=b]^2}{\neg Pa}}{\neg a=b}\,1}{\exists y \neg a=y}}{\exists x \exists y \neg x=y} \qquad \exists x Px}{\exists x \exists y \neg x=y}\,2}{\exists x \exists y \neg x=y}\,3 \qquad \exists x \exists y \neg x=y\ \square$$

[What is the sentence capturing, intuitively?]

8.5 i)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\forall y(Py \rightarrow a=y) \land Pa]_2}{\forall y (Py \rightarrow a=y)}}{Pb \rightarrow a=b} \quad [Pb]^1}{a=b} \qquad \dfrac{\dfrac{[\forall y(Py \rightarrow a=y) \land Pa]^2}{Pa} \quad [a=b]^1}{Pb}}{Pb \leftrightarrow a=b}\,1}{\forall y (Py \leftrightarrow a=y)}}{\exists x \forall y (Py \leftrightarrow x=y)} \qquad \exists x(\forall y (Py \rightarrow a=y) \land Px)}{\exists x \forall y (Py \leftrightarrow x=y)}\,2\ \square$$

ii. $[\forall y\,(Py \leftrightarrow a=y)]^2$

$Pb \leftrightarrow a=b \qquad [Pb]^1$

$a=b$ $\quad 1$

$Pb \rightarrow a=b$

$\forall y\,(Py \rightarrow a=y)$

$[\forall y\,(Py \leftrightarrow a=y)]^2$

$Pa \leftrightarrow a=a \qquad [a=a]$

Pa

$\forall y\,(Py \rightarrow a=y) \wedge Pa$

$\exists x(\forall y\,(Py \rightarrow x=y) \wedge Px) \qquad \exists x \forall y\,(Py \leftrightarrow x=y)$ $\quad 2$

$\exists x(\forall y(Py \rightarrow x=y) \wedge Px)$ $\square$

8.6 i) $\exists x\, \exists y\, (Q_1 x \wedge Q_1 y \wedge \neg x=y) \wedge \forall x \forall y \forall z\, (Q_1 x \wedge Q_1 y \wedge Q_1 z \rightarrow x=y \vee x=z \vee y=z)$

ii) $\exists x\,(Px \wedge Qx \wedge \forall y\,(Q_1 y \rightarrow Rxy) \wedge \forall z\,(Pz \wedge Qz \rightarrow z=x))$

iii) $\exists x\,(Q_1 x \wedge \forall y\,(Qy \rightarrow Rxy) \wedge Px \wedge \forall z((Q_1 z \wedge \forall y(Qy \rightarrow Rzy)) \rightarrow z=x))$

iv) $\forall x\, \forall y\, \forall z\, (Qx \wedge Qy \wedge Qz \rightarrow x=y \vee x=z \vee y=z)$