Tute sheet 7 - market failure

1a) $u_1 = w_0 + kA \log(1 + e_1 + e_2) - e_1$

b) To maximise u_1 with e_1 as the choice variable, the first-order optimality condition is when we set $\frac{\partial u_1}{\partial e_1} = 0$

~~$\frac{\partial u_1}{\partial e_1} = \frac{kA}{e_1} \log(1 + e_1 + e_2)$~~

$$\frac{\partial u_1}{\partial e_1} = \frac{kA}{1 + e_1 + e_2} - 1 \Rightarrow e_1 = kA - \hat{e}_2 - 1$$

or $\max[0, kA - \hat{e}_2 - 1]$ so non-negative

c) We symmetrically obtain $e_2 = kA - \hat{e}_1 - 1$.
At equilibrium, $\hat{e}_1 = \hat{e}_2$, so by substituting $\hat{e}_1 = \hat{e}_2 = \frac{kA - 1}{2}$

d) The NE effort leads each agent to have utility $w_0 + kA \log(kA) - \frac{kA-1}{2}$.
If we merge their utility functions to obtain

$$U = 2(w_0 + kA \log(1 + e_1 + e_2)) - e_1 - e_2$$

we can maximise aggregate utility using FOCs as above; $\max_{e_1, e_2} U$

so set $\frac{\partial U}{\partial e_1} = 0$, $\frac{\partial U}{\partial e_2} = 0$

$$\Rightarrow 2 \frac{kA}{1 + e_1 + e_2} = 1 \qquad \Rightarrow 2 \frac{kA}{1 + e_1 + e_2} = 1$$

$\tilde{e}_1 = 2kA - \tilde{e}_2 - 1$ and at the symmetric solution $\tilde{e}_1 = \tilde{e}_2$

so $\tilde{e}_1 = \tilde{e}_2 = kA - \frac{1}{2}$

which leads to each agent having utility $w_0 + kA \log(2kA) - (kA - \frac{1}{2})$

Is this an improvement? Let us check if

$$\cancel{w_0} + kA \log(2kA) - (kA - \cancel{\tfrac{1}{2}}) > \cancel{w_0} + kA \log(kA) - \left(\frac{kA}{2} - \cancel{\tfrac{1}{2}}\right)$$

$$\log(2kA) - 1 > \log(kA) - \frac{1}{2}$$

$$\log\left(\frac{2kA}{kA}\right) > \frac{1}{2}$$

In words (for the maths may be wrong): they can get higher utility by both credibly committing to putting in more effort, and doing that.

$\log 2 = 0.69 > \frac{1}{2}$ so yes utility is higher for both agents; we have made a Pareto improvement.

2a) $\Pi_1 = P_f \frac{f(b)}{b} - \frac{P_f}{4}$

b) When the returns from sending out a boat are 0, $\Pi = 0$ so

$P_f \frac{f(\hat{b})}{\hat{b}} = \frac{P_f}{4}$, $\Rightarrow \hat{b} = 4 f(\hat{b})$. This means there will be 4x as many boats as there are fish caught, which seems inefficient.

✓

c) The firm seeks to ~~max Π, for which the FOC is~~

$\max_b \Pi_2 = P_f f(b) - b \frac{P_f}{4}$ which is achieved with FOC $\frac{\partial \Pi}{\partial b} = 0$

✓

$\Rightarrow P_f f'(b^*) = \frac{P_f}{4}$, $f'(b^*) = \frac{1}{4}$

They choose the value of b^* such that the marginal number of fish it catches is $\frac{1}{4}$, as this means the marginal cost of the last boat will be equal to its marginal benefit i.e. the market price × number of fish caught

does this phrase make sense in this context?

d) In the sense that [it is a Pareto-efficient outcome?] there is no alternative number of boats to be sent out which would lead to more profit for the firm.

Yes

e) by substitution

✓ $\hat{b} = 4 \times 8 \hat{b}^{\frac{1}{4}} \therefore \hat{b}^{\frac{3}{4}} = 32, \therefore \hat{b} = 102$

✓ $f'(b) = 2b^{-\frac{3}{4}}$, $2b^{*-\frac{3}{4}} = \frac{1}{4}, \therefore b^* = 16$

by substitution

private costs = private benefit?

f) The tax t should increase the cost of sending a marginal boat so that it = benefit at the optimal level, that is, when $b = b^* = 16$

~~So We know from c) that $f'(b^*) = \frac{1}{4}$ = MB at the optimum.~~

~~So currently the marginal cost is $\frac{P_f}{4}$, we want $\frac{P_f}{4} + t = \frac{1}{4}$~~

This implies that $\Pi_3(b) = 0$ for $b = 16$, where

$\Pi_3 = P_f \frac{f(b)}{b} - \frac{P_f}{4} - t$

(*) why doesn't solving for $\frac{P_f}{4} + t = f'(b^*) = \frac{1}{4}$ work here?

✓ So $t = P_f \left(\frac{8 \times 16^{0.25}}{16} - \frac{1}{4} \right) = 0.75 P_f$

3a) If she is acting to maximise utility then $u_s = 10 \times 100^{0.5} = 100$

b) Let us merge the utility functions to obtain $U(D, m_s, m_f)$

[hold on, we're doing interpersonal utility comparisons and tradeoffs, yet these are ordinal quantities, not cardinal ones (unlike profits. But doing the merging of utility func's in Q1. seems a bit suspect too now.). Wouldn't different social preferences and weights on utilities lead to different results?]

Samuelson Rule will give you the correct outcome ...

$$\max_D U = 10D^{\frac{1}{2}} + m_s + 10(100-D)^{\frac{1}{2}} + m_f$$

and when utility is quasi-linear, maxing the sum will give you a condition that is the same as the Samuelson Rule

FOC is when we set $\frac{\partial U}{\partial D} = 0 \Rightarrow \cancel{5}D^{-0.5} = \cancel{5}(100-D)^{-0.5}$

$$D = 100 - D, \quad D = 50$$

c) ~~The~~ $u_f(D=100) - u_f(D=50) = 10(\sqrt{0} - \sqrt{50}) = -50\sqrt{2}$

So the Fellow would be willing to pay a maximum of £$(50\sqrt{2}) \approx$ £71 to reduce the music to the Pareto-optimal level, as she would be indifferent between paying that amount and reducing the noise, or not paying + not reducing it.

Let's check if $u_s(50, £50) \geq u_s(100, 0)$, i.e. if the student would accept an offer of £50 to turn down the music

$$u_s(50, 50) = 50\sqrt{2} + 50 \approx 121 > u_s(100, 0) = 100$$

So yes, £50 would be sufficient.