Week 6

1a) Suppose for # that the wff is false at some world w under some assignment g. Then $V_g(\Box \forall \alpha \phi, w) = 1$ (i) and $V_g(\forall \alpha \Box \phi, w) = 0$. (ii)

From (i), $V_g(\forall \alpha \phi, w') = 1$ for every w', and for every $u \in D$, $V_{g^{\alpha}_{u}}(\phi, w') = 1$

But this is # with (ii) which claims that there is some $u' \in D$ s.t. in some possible world w'' $V_{g^{\alpha}_{u'}}(\phi, w'') = 0$.

Valid. ■

b) Suppose for # the wff is false at some world w under some assignment g. Then $V_g(\forall \alpha \Box \phi, w) = 1$ (i) and $V_g(\Box \forall \alpha \phi, w) = 0$. (ii)

From (i), $V_{g^{\alpha}_{u}}(\phi, v) = 1$ for every $u \in D$ and $v \in W$. (+)
From (ii), there exists some w' where there is some $d \in D$ s.t. $V_{g^{\alpha}_{d}}(\phi, w') = 0$. But this is # with (+)

Valid. ■

c) ~~False~~ Invalid. Consider the model:

$$W = \{v, w\}$$
$$D = \{d, e\}$$
$$I(\phi) = \{\langle d, v \rangle, \langle e, w \rangle\}$$

d) Suppose for # the wff is false at some world w under some assignment g. Then $V_g(\exists \alpha \Box \phi, w) = 1$ (i) and (ii) $V_g(\Box \exists \alpha \phi, w) = 0$.

From (ii), there is some world w' where, for all $d \in D$, $V_{g^{\alpha}_{d}}(\phi, w') = 0$.

But (i) implies that there is some $u \in D$ such that in all $v \in W$, $V_{g^{\alpha}_{u}}(\phi, v) = 1$ ※ Valid ■

2a) i. Px iv. $\Box \forall x\, Px$
ii. $Px \wedge \Box Q x$ v. undefined, y occurs freely after $\forall x$.
iii. Qz vi. $(\forall x\, Px \wedge Ryy)$

b) ① Only the αs free in ϕ are to be changed: the definition accounts for this as $(\forall \alpha \phi)(\beta/\alpha) = \forall \alpha \phi$, leaving the bound αs unchanged

② All free occurrences must be changed: this happens thanks to the recursive definition, so every free α term will be substituted out and replaced with β.

③ The resulting occurrences of the instantiated term can't be bound: we prevent variable capture by disallowing substitutions of β for α where α occurs free in a subformula after $\forall \beta$.

3a) 1. $\forall x\, Fx \rightarrow Fy$ PC1
2. $\forall y(\forall x\, Fx \rightarrow Fy)$ 1, UG
3. $\forall y(\forall x\, Fx \rightarrow Fy) \rightarrow (\forall x\, Fx \rightarrow \forall y\, Fy)$ PC2
4. $\forall x\, Fx \rightarrow \forall y\, Fy$ 2, 3, MP

Good

b) 1. $\forall \alpha(\phi \rightarrow \psi) \rightarrow (\phi \rightarrow \psi)$ PC1
2. $\forall \alpha \phi \rightarrow \phi$ PC1
3. $((\phi \rightarrow \psi) \wedge \phi) \rightarrow \psi$ PL
4. $(\forall \alpha(\phi \rightarrow \psi) \wedge \forall \alpha \phi) \rightarrow \psi$ 1, 2, 3, PL
5. $(\forall \alpha(\phi \rightarrow \psi) \wedge \forall \alpha \phi) \rightarrow \forall \alpha \psi$ 4, UG, PC2, MP
6. $\forall \alpha(\phi \rightarrow \psi) \rightarrow (\forall \alpha \phi \rightarrow \forall \alpha \psi)$ 5, PL

c)

Try invoking an instance of the claim proved in (b). To get you started:
1. Forall a (phi —> ~psi) —> (Forall a phi —> Forall a ~ psi)
2. Forall a ~(phi and psi) —> (Forall a phi —> Forall a ~Psi)
3. Forall a phi —> (Forall a ~ (phi and psi —> Forall a ~Psi)

(i) $\alpha = \alpha$

(ii) $\forall\beta({\sim}\beta = \alpha) \rightarrow {\sim}\alpha = \alpha$

(iii) ${\sim}\forall\beta({\sim}\beta = \alpha)$

(iv) $\Box{\sim}\forall\beta({\sim}\beta = \alpha)$

(v) $\forall\alpha\Box{\sim}\forall\beta({\sim}\beta = \alpha)$

(vi) $\Box\forall\alpha\Box{\sim}\forall\beta({\sim}\beta = \alpha)$

(vii) $\Box\forall\alpha\Box\exists\beta(\beta = \alpha)$

d) 1. $\forall\alpha\phi \rightarrow \phi$ PC1
2. $\Box\forall\alpha\phi \rightarrow \Box\phi$ 1, Becker
3. $\forall\alpha(\Box\forall\alpha\phi \rightarrow \Box\phi)$ 2, UG
4. $\Box\forall\alpha\phi \rightarrow \forall\alpha\Box\phi$ 3, PC2, MP

e) 1. $\forall\alpha\Box\phi \rightarrow \Box\phi$ PC1
2. $\Box\phi \rightarrow \phi$ T
3. $\forall\alpha\Box\phi \rightarrow \phi$ 1, 2, PL (Syll.)
4. $\forall\alpha\Box\phi \rightarrow \forall\alpha\phi$ 3, UG, PC2, MP

Some hints in this note

hmm, but how to get $\Box$ here?

[illegible]

f) 1. $\Box\forall\alpha\exists\beta(\beta = \alpha) \rightarrow \forall\alpha\Box\exists\beta(\beta = \alpha)$ from d) with $\phi = \exists\beta(\beta = \alpha)$
2.

For question 4, we are basically proving SOUNDNESS, and this is carried out by induction on the length of proofs. ~~For the base cases of the induction, we want to prove that all SQML axioms are valid (this is quite easy, make the~~ semantic arguments for each axiom) and then prove that the SQML rules preserve validity (the rules are NEC, MP and ~~UG — you want to prove that assuming everything before application of the rule is valid, the result of applying the rule~~ is also valid)