Neoclassical consumption; unemployment; arbitrage

Jones 16.5 a) ① $\frac{c_f}{c_t} = \beta(1+R)$ as $u'(c_t) = \beta(1+R)\, u'(c_f)$, with $u(c) = \ln(c)$

② $c_t + \frac{c_f}{1+R} = \bar{x}$ where $\bar{x} := y_t + f_t + \frac{y_f}{1+R}$ (budget constraint)

From ①, $\frac{c_f}{1+R} = \beta c_t$, and by ② $c_t(1+\beta) = \bar{x}$

So $c_t = \frac{\bar{x}}{1+\beta}$, $c_f = (1+R)\frac{\beta \bar{x}}{\beta(1+\beta)}$ ✓

b) When $\beta = 1$, $c_t = \frac{1}{2}\bar{x}$ and $c_f = (1+R)\cdot\frac{1}{2}\cdot\bar{x}$ as required ✓

c) For $\beta < 1$, the denominator $1+\beta$ in $c_t = \frac{\bar{x}}{1+\beta}$ is smaller, so c_t rises. Intuitively, this is because the agent values current consumption relatively more than future, so naturally they consume more today. ✓

Unemployment i. In the steady state, $\Delta U^* = 0$ i.e. $\bar{s}E^* = \bar{f}U^*$ where $E^* = \bar{L} - U^*$

so $U^*(\bar{f} + \bar{s}) = \bar{s}\bar{L}$, $U^* = \frac{\bar{s}\bar{L}}{\bar{f}+\bar{s}}$

and $u^* := \frac{U^*}{\bar{L}} = \frac{\bar{s}}{\bar{f}+\bar{s}}$ ✓

ii. $f(b) = 0.5 - 0.05b$ where $b \in [0, 10]$

$s(c) = 0.1 - 0.02c$ where $c \in [0, 5]$

(Surely then if $f(e) = e$, then $e \in [0, 0.5]$ not $[0, 1]$)

I think so too, given the available values for b – but I can't think it matters hugely

If benefits are higher for the unemployed, people will be less motivated to look for work since the marginal benefits are smaller, so people find new jobs more slowly. Making it costlier for firms to sack people means they do it less, so s↓ (In reality, making it costlier to sack people also makes firms less willing to hire, so f↓)

well expressed

clearly this does depend on their previous wage in employment

iii. Using i., $u^* = \frac{0.1 - 0.02c}{0.5 - 0.05b + 0.1 - 0.02c}$

$= \frac{1 - 0.2c}{6 - 0.5b - 0.2c}$ ✓

So u^* depends ~~negatively~~ positively on b, as one would expect (since ↑b means denominator ↓; higher benefits makes work relatively less attractive so more people unemployed). Where $b \neq 10$ (in this case unemployment = 100% as $f = 0$; numerator and denominator the same so c irrelevant), u^* depends negatively on c, again as expected. (since the larger fixed term in denominator means when c↑, the numerator proportionally decreases more and u^*↓)

(You could also partially differentiate wrt b then c to be more "precise")

Absolutely. We have to be really careful in macro to think broadly about the consequences / incentive changes that result from policy.

This might not be particularly realistic: if the government increases c then we'd expect firms to be more hesitant about hiring (put differently, s and f are modelled as independent which they probably are not).

iv. To $\min_{b,c} \; u^* = \dfrac{1-0.2c}{6-0.5b-0.2c}$ we have FOCs:

$$\frac{\partial u^*}{\partial b} = 0 \qquad\qquad \frac{\partial u^*}{\partial c} = 0$$

$$\frac{0.5(1-0.2c)}{(6-0.5b-0.2c)^2} = 0 \qquad\qquad \frac{-0.2(6-0.5b-0.2c)+0.2(1-0.2c)}{(6-0.5b-0.2c)^2} = 0$$

$$\frac{0.5-0.1c}{(6-0.5b-0.2c)^2} = 0 \qquad\qquad \frac{0.1b-1}{(6-0.5b-0.2c)^2} = 0$$

this condition isn't met when $b=0$, $c=5$?

(*) We know by inspection that if $0.1b-1=0$ i.e. $b=10$, u^* is maximised. And intuitively the costs of firing should be as high as possible, which is achieved when $0.5-0.1c=0$ i.e. $c=5$.

Explain this a bit more: what would such policies actually look like in reality? Are they desirable?

$\Rightarrow$ So the policymaker would want to set $b=0$ and $c=5$ so that s is minimised and f maximised, in order to minimise u^*

- But the $\dfrac{\partial u^*}{\partial c}=0$ FOC seems not to be met? Maybe due to the fact that we have constraints on b and c's values. yes.

v. If $c=2.5$ then ~~[illegible]~~. If we want $u^*=0.1$,

~~[illegible]~~ since $u^* = \dfrac{1-0.2c}{6-0.2c-0.5b}$

$$u^* = 0.1 = \frac{0.5}{5.5-0.5b} = \frac{0.1}{1.1-0.1b}$$

$$1.1-0.1b = 1 \Rightarrow \underline{b=1}$$

Generations i. Today: $c_t = wL - s$ where s are their savings today

Future: $c_f = (1+R)s$

So $\dfrac{c_f}{1+R} = s$ and substituting into first equation,

Intertemporal: $c_t + \dfrac{c_f}{1+R} = wL$

ii. $\max_{L,c_t,c_f} U = \left[\log(c_t) + \log(c_f) + \log(1-L)\right]$ s.t. $c_t + \frac{c_f}{1+R} = wL$

Clear set-up. Good. :)

Which is the same as $\max_{L,c_t}\left[\log(c_t) + \log\left((1+R)(wL-c_t)\right) + \log(1-L)\right]$

For which the FOCs are

$$\frac{\partial U}{\partial L} = 0 \qquad\qquad \frac{\partial U}{\partial c_t} = 0$$

$$\frac{\cancel{(1+R)}\,w}{\cancel{(1+R)}(wL-c_t)} - \frac{1}{1-L} = 0 \quad (\text{II}) \qquad \frac{1}{c_t} - \frac{\cancel{(1+R)}\cdot 1}{\cancel{(1+R)}(wL-c_t)} = 0 \quad (\text{I})$$

From (I) $c_t = wL - c_t$ and we know that $c_f = (1+R)(wL - c_t)$

(A) $c_t = \frac{1}{2}wL$ so $c_f = (1+R)(wL - \frac{1}{2}wL)$
$= \frac{1}{2}(1+R)wL$

From (II) $wL - c_t = w - wL$

(B) $L = \frac{1}{2}\cdot\frac{c_t + w}{w}$

We have two equations with two unknowns; substitute (A) into (B)

$$L = \frac{1}{2}\cdot\frac{\frac{1}{2}wL + w}{w} = \frac{1}{2}\cdot\left(\frac{1}{2}L + 1\right)$$

so $\frac{3}{4}L = \frac{1}{2}$; $L = \frac{2}{3}$, $c_t = \frac{1}{3}w$, $c_f = \frac{1}{3}(1+R)w$

Lovely work. This you can also solve using Lagrangian multipliers (see any of the maths workbooks)

- Log utility means that the income and substitution effects of a wage increase cancel out and optimum labour proportion L is independent of w. The agent ~~spend~~ gains the same utility per unit of consumption now as in future, so they consume equal amounts (in present value) in both periods.

Excellent (could show this on a backwards bending labour supply curve)

iii. We ~~will~~ have $c_t = wL - s - T$ where T = lump-sum tax and $c_f = (1+R)s + p$ where p is the pension.
So $c_t + \frac{c_f}{1+R} = wL + \frac{p}{1+R} - T$ i.e. lifetime consumption = lifetime wealth.

If the young and old populations are equal in size and the government sets the pension such that $p = (1+R)T$ then this would have no effect. But probably they directly move money from young to old, rather than "saving tax take in meantime".

We can actually show that there is a change & it's especially noticeable with the impact on L (which actually increases)

so the young will have less income over their lifetimes and consume less. If we ignore this possible effect, though, the permanent income hypothesis and consumption smoothing should mean the young's consumption is unchanged, provided they can borrow. For the currently-old, the unexpected windfall from the policy change would increase consumption.

iv. Now the intertemporal constraint is $c_t + \frac{c_f}{1+R} = (1-t)wL + \frac{p}{1+R}$.

Letting $R=0$, $c_t + c_f = (1-t)wL + p$.

(what we find by crunching through the maths here is that L falls in this case. We'll try to walk it through in tutorial if we have time)

Stock pricing i. $R \times p_t = d_t + (p_{t+1} - p_t)$ so $(1+R)p_t = d_t + p_{t+1}$

$$p_t = \frac{d_t + p_{t+1}}{1+R}$$

ii. $p_{t+1} = \frac{d_{t+1} + p_{t+2}}{1+R}$ so iii. $p_t = \frac{d_t + \frac{d_{t+1} + p_{t+2}}{1+R}}{1+R}$

Express more simply $p_t = \frac{d_t}{(1+R)} + \frac{d_{t+1}}{(1+R)^2} + \frac{p_{t+2}}{(1+R)^2}$

iv. $p_t = \sum_{t=1}^{\infty} d_t \cdot (1+R)^{-t}$ So the price of a risk-free stock is equal to the present value of all future dividends.

This makes sense intuitively: stock is valuable because of the dividends it provides. So any potential capital gains from the stock are ultimately determined by the dividends it pays. Holding the stock has an opportunity cost in each period of not investing in the bank account to appreciate at a rate R, so the dividends are discounted by this factor (1+R) each period.

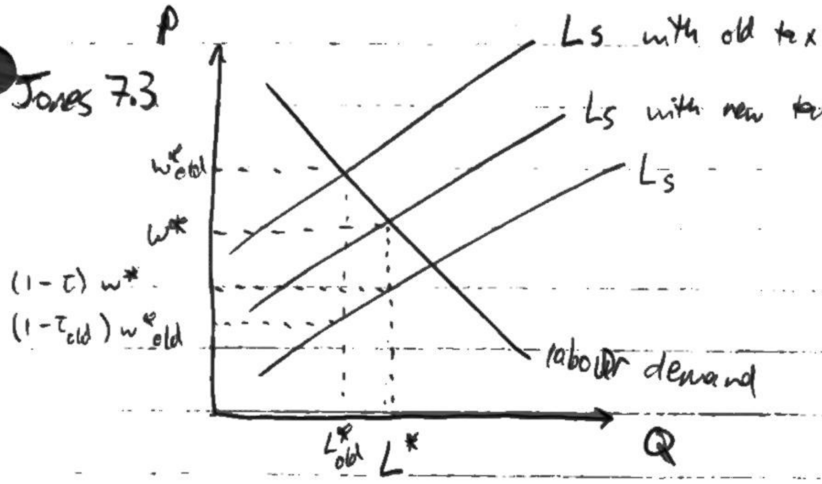
Very nice answer

v. It suggests that the arbitrage equation's RHS is missing some other terms which make stock valuable. We could potentially rationalise the valuation with the existing model if we claimed that investors all believed that Alphabet would in future begin paying substantial dividends, but it seems unlikely that this is the full explanation. Ability to exert control over the company via voting rights, or the chance that Alphabet later buys back the stock at a premium may also play a part.

Yes

Also, irrational behaviour, speculation etc.

Jones 7.3



We can see that wages will fall, (though take-home pay rises), and total labour supplied increases, because the labor supply curve shifts rightward.

The employment rate thus increases.

The effect on unemployment rate is indeterminate. Whilst firms may hire workers who were previously jobseekers, more people may also enter the labour force, so both the numerator & denominator ↑.

7.11 a) For: provides a good safety net allowing workers' living standard to remain
Against: creates a strong incentive not to search for any work for 6 months

b) For: retains incentive for people to search for work and increase their income

Against: may be prone to manipulation by getting fired many times; does not help people whose skills don't match economic needs and are unemployed / reskilling for >10 weeks.

8.4 $M_tV_t = P_tY_t$ where P_t is the key endogenous variable

a) P doubles; twice as much money chasing same amount of goods/output

b) P increases by 10% c) P falls by 2%. d) no change, i.e. $\times \frac{1.03}{1.03} = 1$

8.8 $R_t = i_t - \pi_t$ a) 9%. b) People borrow money from the bank, invest it in machines and make a profit on the difference between nominal rate charged and (return + inflation); bank loses money and must ↑ i_t.

c) People put all their savings into the bank, it has no way to achieve a nominal return of 12% so loses money and must ↓ i_t.

8.11 a) Yes, if nominal rates are zero or near-zero and inflation is positive, real rates can be negative, since $R_t = i_t - \pi_t$. This may be desired by a CB in the short term to stimulate the economy.

b) There's a weak zero lower bound on i_t in normal economies, since people can simply hoard cash rather than "pay" to deposit their savings. It's only a weak bound, i.e. i_t could be slightly -ve, as people do prefer to store their savings in a bank. ✓ (i.e. other reasons for saving)

8.14 Governments printing money to finance deficits leads to persistently high inflation, but in the short-term there can be ~~many~~ other causes of inflation, e.g. supply shocks - but their effects do not persist. ✓

16.4. $\frac{c_f}{c_t} = \beta(1+R)$

a) c_f / c_t = ~~[illegible]~~ 1.05 so growth rate = 5% i.e. = R_t (makes sense: they do no future discounting)

b) $0.95 \times 1.05 - 1 = -0.25\%$ ~~ie.~~

c) $1.02 = 0.95 \times (1+R)$ so $R = 7.4\%$ ✓

16.6 a) Lower taxes today either means a cut in spending, or borrowing. If gov't borrows, then in future citizens will have to pay higher taxes to repay the loan with interest. So, expecting this, they save the money from the tax cut until then, and consumption is unaffected. ✓

b) The credit-constrained individuals would ✓ spend the tax cut income, as they have a high MPC. This is because they would ideally be borrowing to consume more now but are unable to, so the windfall enables them to spend more. So consumption ↑ ✓ Good