Industrial organisation

2. $C_k(q_k) = c_k q_k$; $p(q) = a - q$ where $q = q_1 + q_2$

a). Firm 1 solves the profit-maximisation problem

✓ $$\max_{q_1} \pi_1(q_1) = (p(q) - c_1 \cancel{q_1})q_1$$

$$= (a - q_1 - q_2 - c_1 \cancel{q_1}) q_1$$

For which the FOC is $\frac{\partial \pi_1}{\partial q_1} = 0$

$$\therefore (-1 - c_1)q_1 + (a - q_1 - q_2 - c_1 \cancel{q_1}) = 0$$

$$a - 2q_1 - q_2 - c_1 = 0 \quad (1)$$

✓ with SOC $\frac{\partial^2 \pi_1}{\partial q_1^2} = -2 < 0$ so a maximum ~~at opt~~

Rearranging (1) for q_1, we find

✓ $$BR_1(q_2) = \cancel{\frac{a - q_2}{2(1+c_1)}} \quad \frac{a - q_2 - c_1}{2}$$

✓ and ~~by symmetry~~ similarly, $BR_2(q_1) = \frac{a - q_1 - c_2}{2}$ as required.

At the NE with quantities q_1^*, q_2^*, each firm plays the best response to its opponent's equilibrium strategy.

are all these equalities true / necessary?

So $q_1(q_2^*) = q_1^*$; $q_2(q_1^*) = q_2^*$; $q_1 = q_1^*$, $q_2 = q_2^*$

You can just say - looking for the intersection of the BR curves and go straight to ⇒

i.e. $q_1^* = \frac{a - q_2^* - c_1}{2}$ and $q_2^* = \frac{a - q_1^* - c_2}{2}$

$$\therefore q_2^* = \frac{a - \left(\frac{a - q_2^* - c_1}{2}\right) - c_2}{2}$$

$$\frac{3}{2} q_2^* = \frac{1}{2} a + \frac{1}{2} c_1 - c_2$$

$$q_2^* = \frac{1}{3}(a + c_1 - 2c_2)$$

and $q_1^c = \frac{1}{3}(a + c_2 - 2c_1)$ by symmetry

✓

b) From a), $\pi_1^c(q_1^c(c_1), q_2^c(c_1)) = (a - q_1^c(c_1) - q_2^c(c_1) - c_1)\, q_1^c$

$$\frac{d\pi_1^c}{dc_1} = \underbrace{\frac{\partial \pi_1}{\partial c_1}}_{\text{(i) direct effect}} + \underbrace{\frac{\partial \pi_1}{\partial q_1}\cdot\frac{dq_1}{dc_1}}_{\text{(ii) change in own quantity}} + \underbrace{\frac{\partial \pi_1}{\partial q_2}\cdot\frac{dq_2}{dc_1}}_{\text{(iii) change in other quantity}}$$ by total differentiation

✓

- At equilibrium, firm 1 is optimising its choice of q_1. But as ~~expected~~ the FOC for this we require $\frac{\partial \pi_1}{\partial q_1} = 0$, so the effect of (ii) is 0. ✓

- (i) is just $-q_1^c$ ✓

  (iii) is $-q_1^c \cdot \frac{dq_2}{dc_1}$ and from (a), $\frac{dq_2}{dc_1} = \frac{1}{3}$

  so $\frac{d\pi_1^c}{dc_1} = -\frac{4}{3} q_1^c$ ✓

- By the same argument as above, (ii) will be 0 in the effects of a change in c_2 on firm 1's profit. The direct effect (i) is also 0 since c_2 does not enter directly into π_1. So the only effect is from (iii):

$$\frac{d\pi_1^c}{dc_2} = \frac{\partial \pi_1}{\partial q_2}\cdot\frac{dq_2}{dc_2}$$

$$= -q_1^c \cdot -\frac{2}{3} = \frac{2}{3} q_1^c$$ ✓

- The size of the change in profit is twice as large when the firm's own cost changes. This is because an ↑ in costs not only leads the rival to produce more in response to firm 1 producing less, but also has a direct effect on firm 1's margins. When the other firm's ~~price~~ costs change, there is only the strategic effect.

Great

Can we go over why in sequential game you can substitute at this stage, but in simultaneous not? (*)

← can't read this but answer is yes.

c) Firm 2's best response to firm 1's move is, from a), $q_2 = \frac{a - q_1 - c_2}{2}$

Firm 1 must choose q_1 to maximise profits

$$\pi_1(q_1, q_2) = (a - q_1 - q_2 - c_1)\, q_1$$

$$= \left(a - q_1 - \frac{a - q_1 - c_2}{2} - c_1\right) q_1$$ Since they move first

$$\left(\tfrac{1}{2}a - \tfrac{1}{2}q_1 + \tfrac{1}{2}c_2 - c_1\right) q_1$$

with FOC $\frac{\partial \pi_1}{\partial q_1} = 0 \Rightarrow \left(a - q_1 - \frac{a - q_1 - c_2}{2} - c_1\right) - \frac{1}{2} q_1 = 0$

$$2a - 2q_1 - a + q_1 + c_2 - 2c_1 - q_1 = 0$$

$q^s_1 = \frac{1}{2}(a + c_2 - 2c_1)$ ✓ whereas $q^c_1 = \frac{1}{3}(a + c_2 - 2c_1)$

so $q^s_1 = \frac{3}{2} q^c_1$

$q^s_1 > q^c_1$

as both are true ✓

and $q^s_2 = \frac{1}{2}\left(a - \frac{1}{2}(a + c_2 - 2c_1) - c_2\right)$ ✓

$$= \tfrac{1}{4}a - \tfrac{3}{4}c_2 + \tfrac{1}{2}c_1$$

Firm 1's eq* q^s_1 is larger than q^c_1 because their choice has a strategic effect on firm 2. When choosing simultaneously under Cournot, firm 2 has no strict unilateral incentive to produce more than the Cournot-Nash eq* quantity, by definition. But moving to the sequential game, they are incentivised to produce more than q^c_1 since the best-response function of firm 2 is unchanged and quantity choices are strategic substitutes. So, firm 1 producing more leads firm 2 to produce less in order to maximise profits, and this means firm 1 gains market share and has higher profits. ✓ ✓

does total q generally ↑ when sequential or stay constant? (*) if q ↑, then we'd see p↓ so couldn't that hurt firm 1 overall from producing more than Cournot?

Good question

d) If firm 1 moves first the total output is

$$q^s_1 + q^s_2 = \tfrac{3}{4}a - \tfrac{1}{2}c_1 - \tfrac{1}{4}c_2 =: q_T$$

if firm 2 moved first, output would be

$$\tfrac{1}{2}(a + c_1 - 2c_2) + \left(\tfrac{1}{4}a - \tfrac{5}{4}c_1 + \tfrac{1}{2}c_2\right)$$

$$= \tfrac{3}{4}a - \tfrac{1}{4}c_1 - \tfrac{1}{2}c_2 =: q_T'$$

~~Supposing firm 1 is the high-cost firm,~~

For some fixed values of c_1 and c_2, ~~where~~ ~~...~~ Supposing $c_1 > c_2$, then $q_T < q_T'$ as $q_T' - q_T = \frac{1}{4}c_1 - \frac{1}{4}c_2$ which is > 0 when $c_1 > c_2$.

So consumers prefer it when the ~~high~~ low-cost firm moves ~~second~~ first as this leads to a greater quantity being supplied and hence a lower price with larger consumer surplus. ✓✓

[why is this?] more of the market supply is produced by the lower cost firm... which gets passed on to consumers -

4. $C_1(q_1) = 2q_1$; $C_2(q_2) = F + 4q_2$; $p(q) = 40 - q$

a) Profits for firm 2 are $\pi_2(q_1, q_2; F) = p(q_1, q_2)\cdot q_2 - C_2(q_2)$

$$= (40 - q_1 - q_2)\cdot q_2 - (F + 4q_2)$$

where in this case $F = 0$, so

$$= (36 - q_1 - q_2)\cdot q_2$$

The FOC is $\frac{\partial \pi_2}{\partial q_2} = 0 \Rightarrow -q_2 + (36 - q_1 - q_2) = 0$

with SOC $\frac{\partial^2 \pi_2}{\partial q_2^2} = -2 < 0$

so as required the best response is $q_2^*(q_1) = \frac{1}{2}(36 - q_1)$

Profits for firm 1 are $\pi_1(q_1, q_2) = (40 - q_1 - q_2 - 2)\cdot q_1$

but it is a sequential game with 1 moving first

$\therefore \pi_1(q_1) = (38 - q_1 - \frac{1}{2}(36 - q_1))\cdot q_1$

$$= (20 - \tfrac{1}{2}q_1)\cdot q_1$$

for which the FOC is $\frac{\partial \pi_1}{\partial q_1} = 0 \Rightarrow 20 - q_1 = 0 \Rightarrow \underline{\underline{q_1 = 20}}$

So profits for firm 1 are 200 and for firm 2 – which produces 8 – are 64.

b) Since only the fixed cost has changed, the FOC for firm 2 to maximise profits is unchanged, and so neither has the best response q_2^* assuming it enters. We're interested in the values of q_1 for which $\pi_2(q_1, q_2^*; F=4) \le 0$, i.e. that satisfy

$$(40 - q_1 - q_2^*(q_1) - 4)\cdot q_2^*(q_1) - F \le 0$$

$$\Leftrightarrow (36 - q_1 - \tfrac{1}{2}(36 - q_1))\cdot \tfrac{1}{2}(36 - q_1) - 4 \le 0$$

$$\Leftrightarrow 320 - 18q_1 + \tfrac{1}{4}q_1^2 \le 0$$

$$\Leftrightarrow 32 \le q_1 \le 40$$

intuition for the upper limit of 40 on q_1 to deter firm 2? (oh, it's just p becoming -ve, we can ignore it?) – Yes

Since firm 1's profits are quadratic in q_1, we know they will maximise profits with a quantity closest to the optimum. For the case where firm 2 doesn't enter, $\pi_1(q_1) = 38q_1 - q_1^2$ which implies an optimum $q_1^* = 19$. So their greatest profits under the constraints above can't read this ...please bring up in class!

[wait, why is firm 1's duopoly output greater than its monopoly output? I thought it was just ~~total~~ duopoly output is greater]

to deter firm 2 are $\pi_1(32) = 192$. But this is worse for firm 1 than the profits of 200 in (a), which it can still now guarantee by producing 20 as before. In response to this, firm 2 will enter the market and produce q as before, with slightly lower profits of 60 due to $F > 0$.

c) ~~Again~~ Like above, to deter entry we require

$$(40 - q_1 - q_2^*(q_1) - 4) \cdot q_2^*(q_1) - F \leq 0 \quad \text{with } F = 36$$

which implies

$$288 - 18q_1 + \frac{1}{4}q_1^2 \leq 0$$

$$\Leftrightarrow \quad 24 \leq q_1 \leq 48$$

with profits maximised at $q_1 = 24$, of 336. This is greater than the 200 possible by allowing firm 2 to enter, so firm 1 sets $q_1 = 24$ and firm 2 doesn't enter.

d) ~~Simi~~ Similarly, we want the values of q_1 s.t. firm 2 won't enter, that is, s.t. $224 - 18q_1 + \frac{1}{4}q_1^2 \leq 0$

$$\Leftrightarrow \quad 16 \leq q_1 \leq 56$$

Firm 1 is now free to choose their optimal quantity $q_1^* = 19$ whilst still deterring firm 2, and so is again a monopolist but with even greater profits of 361.

Good

e) There are two considerations for firm 1. First, they wish to produce a sufficient quantity to drive down the price to a level that deters the less efficient firm 2 from entering the market as a competitor. Second, they want to select an output level which is not so high as to harm their own profits as a firm with market power. In part (b) firm 1 would need to drive down the ~~cost~~ price too much for it to be profitable for them to deter firm 2 from entering. With sufficiently high F, deterrence becomes an economical strategy for firm 1. In (d), F is so high that the constraint on q_1 to be ≥ 16 for deterrence no longer binds the firm, and it's optimal for them to produce more - specifically, the monopolist's output of 19. This yields the greatest profits possible, even if firm 2 didn't exist at all. In a sense, strategic considerations are no longer important with very high fixed costs.

Great