Rohan Merton week 7. Formalisation in predicate.

7.1 i) ∃-elim done incorrectly on second line of both branches. ('a' appears in undischarged assumptions)

$$\dfrac{\dfrac{\dfrac{\dfrac{[Pa \land Qa]^1}{Pa}}{\exists x Px} \qquad \dfrac{\dfrac{[Pa \land Qa]^1}{Qa}}{\exists x Qx}}{\exists x Px \land \exists x Qx} \qquad \exists x (Px \land Qx)}{\exists x Px \land \exists x Qx}\,1 \quad \square$$

ii) ∃-elim done incorrectly on second line (the constant 'a' can't appear in the initial assumption and the formula you conclude after ∃-elim)

Counter example: $D_A = \{0, 1\}$
$|R^2|_A = \{\langle 0,0\rangle, \langle 1,1\rangle\}$

~~Then note that~~

$|\exists y \forall x Rxy|_A = T$ iff $|\forall x Rxy|_A^\alpha$ under some assignment α over A

Case 1: $|y|_A^\alpha = 0$. But $\langle 1,0\rangle \notin |R^2|_A$ so $|Rxy|_A^\alpha = F$ so $|\forall x Rxy|_A^\alpha = F$

Case 2: $|y|_A^\alpha = 1$. $\langle 0,1\rangle \notin |R^2|_A$ so $|Rxy|_A^\alpha = F$ so $|\forall Rxy|_A^\alpha = F$

So there is no assignment α s.t. $|\forall x Rxy|_A^\alpha = T$ so $|\exists y \forall x Rxy|_A = F$

$\forall x \exists y Rxy \not\models \exists y \forall x Rxy$, since $|\forall x \exists y Rxy|_A = T$ (by inspection: $\langle 0,0\rangle$ and $\langle 1,1\rangle$)

iii) There's an assumption of Pa which has no valid reason to be discharged before ∀-I in line 3, so the constant 'a' still appears undischarged in the proof of $Pa \Rightarrow Qa$.

Counterexample: $D_A = \{0, 1\}$ $|P|_A = \{0, 1\}$ $|Q|_A = \{1\}$

$|\exists y (Py \Rightarrow Qy)|_A = T$ since in the assignment α where $|y|_A^\alpha = 1$, $|Qy|_A^\alpha = T$ as $1 \in |Q|_A$ so $|Py \Rightarrow Qy|_A^\alpha$.

But $|\forall x (Px \Rightarrow Qx)|_A = F$ since in the assignment where $|x|_A^\alpha = 0$, $|Qx|_A = F$ and $|Px|_A = T$ so $|Px \Rightarrow Qx|_A = F$.

So $\exists y (Py \Rightarrow Qy) \not\models \forall x (Px \Rightarrow Qx)$.

7.2 i) $\neg\forall x(Px \to Qx)$ P: ... is a book author Q: ... is famous

or equally $\exists x(Px \land \neg Qx)$

ii) $\exists x(Px \land Qx)$ P: ... is a book Q: ... is famous

iii) ~~$\forall x((Px \land Qx) \leftrightarrow Rx))$~~

$\forall x(Px \to (Qx \leftrightarrow Rx))$ as above, ~~nn~~ R: ... is well-written

~~Is this equiv to $\forall x(Px \land Qx$~~

iv) $\neg P_1 a$ P_1: ... believes that not every book author is famous a: Tom

(extensional ambiguity here otherwise?)

7.3 i) $\forall x(Px \to Qax)$ or $\exists x(Px \land Qax)$ P^1: ... is a logician

Q^2: ... despises ... a: Ben

ii) $Pab \land Pac$ P^2: ... slanders ... a: Harry b: Ron

c: Harry's parents OR Ron's parents

iii) $\exists x \exists y(Px \land Qy \land Rxy)$ or $\forall x \forall y(Px \land Qy \to Rxy)$ P: ... is a student

Q: ... is a tutor R^2: ... is better than ...

iv) $\forall x(Px \land Qx \to Rx)$ or $\forall x(Px \to Rx \land Qx)$

R: ... s rich Q: ... is German

P: ... buys a house in Munich

v) $\forall x(Px \to Rax)$ P: ... is a fast car

or $\exists x(Px \land Rax)$

R: ... likes ... a = James

vi) $\forall x \exists y(Py \land Qxy)$ P: ... is a mistake

Q: ... made ...

or $\exists y(Py \land \forall x\, Qxy)$

7.4 i) (There exists a set such that for all sets, they are an element of that set iff they're not an element of themself.) assume that is domain of discourse

i.e. There is a set containing all sets which don't contain themselves.

ii) There's a set which all sets have iff they don't have themselves.

iii) From 6.5, ~~we~~ we established that $\exists x \forall y(Ryx \leftrightarrow \neg Ryy) \vdash P$.

So any sentence P can be proved from it, so any set Γ containing that as a premiss can be used to prove any sentence P. So there is no P s.t. $\Gamma \nvdash P$, so any such set Γ must be syntactically inconsistent.

iv) Some such sets, like 'the set of all x such that x does not contain itself', may entail a contradiction (and thus not exist?)

7.5 $\forall x \exists y Pxy \vdash \exists y \forall x Pxy$ P: ... has ... as its cause.

- This isn't valid in predicate logic – see 7.1 ii) for a counterexample.
- ~~If we take 'a cause of everything' to mean 'some thing from which a forwards-only causal chain can be drawn to every other thing' then this argument might be logically valid if there is a finite number of objects (otherwise object n+1 could be the cause of n, etc.). But here what we define terms as matters a lot: what does it mean for x to 'cause' y?~~
- In English the argument seems not to be logically valid either: if there is a cause of everything, then that must itself also have a cause, since everything does (as a premiss) and so it can't be <u>the</u> cause of everything.

- Also note that in the reflexive case, it's perfectly possible for everything to have a cause (itself) and there to be no single cause of everything.