Rohan - Merton - Week 6. Natural deduction

6.1 i) $$\dfrac{\dfrac{[P]}{R \lor P}\ \lor\text{-Intro}}{P \to R \lor P}\ \to\text{-Intro.}\ \square$$

ii) $$\dfrac{\dfrac{R \land Q}{Q} \qquad \dfrac{R \land Q}{R}}{Q \land R}\ \square$$

iii) $$\dfrac{\dfrac{\dfrac{P \to Q \qquad [P]}{Q} \qquad [\neg Q]}{\neg P}}{\neg Q \to \neg P}\ \square$$

iv) $$\dfrac{\dfrac{[P] \qquad \dfrac{[P] \qquad [P \to \neg P]}{\neg P}}{\neg P}}{(P \to \neg P) \to \neg P}\ \square$$

v) $$\dfrac{\dfrac{P \leftrightarrow Q \qquad [P]}{Q} \qquad \neg Q}{\neg P}\ \square$$

vi) $$\dfrac{\dfrac{\dfrac{\dfrac{[P]^2\ [Q]^1}{P \land Q} \qquad P \land Q \to R}{R}}{Q \to R}\ \to I_1}{P \to (Q \to R)}\ \to I_2\ \square$$

vii) $$\dfrac{\dfrac{\dfrac{[\neg P]^2 \qquad [P]^1}{Q}}{P \to Q}\ \to I_1 \qquad \neg(P \to Q)}{P}\ \neg E_2\ \square$$

6.2

want to copy whole circled left subproof above here (★)

$$\dfrac{\dfrac{\dfrac{[R]^2 \quad \boxed{\neg R}}{Q}\ (\star) \qquad \dfrac{\dfrac{\dfrac{P_1 \quad P_1 \to P}{P} \qquad P \to (Q \lor R) \land \neg(Q \land R)}{(Q \lor R) \land \neg(Q \land R)}}{Q \lor R} \qquad [Q]^2}{Q}\ 2 \qquad \neg R}{Q \land \neg R}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{P_2 \to Q_1 \qquad \dfrac{R \to P_2 \quad [R]^1}{P_2}}{Q_1} \qquad Q_1 \to Q}{Q} \qquad [R]^1}{Q \land R} \qquad \dfrac{\dfrac{\dfrac{P_1 \quad P_1 \to P}{P} \qquad P \to (Q \lor R) \land \neg(Q \land R)}{(Q \lor R) \land \neg(Q \land R)}}{\neg(Q \land R)}}{\neg R}\ 1$$

[Here it feels like I'm proving the same thing multiple times, because with, e.g. $\land$-elim, you can only derive one of the subsentences at once. So if you reached $\phi \land \psi$ somehow, you have to reach it "twice" to use both ϕ and ψ subsequently. Is that right? Other forms of natural deduction seem not to have this inefficiency]

6.3 i)

$$\forall x(Px \to P_1 x)$$
$$Pa \to P_1 a \qquad [Pa]$$
$$P_1 a \qquad \neg P_1 a$$
$$\neg Pa \quad \Box$$

ii)

$$\forall x(Px \to Qx)$$
$$Pa \to Qa \qquad Pa$$
$$Qa$$
$$\exists y\, Qy \quad \Box$$

iii)

$$[\neg Qa]^1$$
$$\exists x \neg Qx \qquad [\neg \exists x \neg Qx]^2$$
1
$$Qa$$
$$\forall x Qx \qquad \neg \forall x Qx$$
2
$$\exists x \neg Qx \quad \Box$$

iv)

$$[\forall y\, Pya]^1$$
$$Pba \qquad [\neg Pba]^2$$
1
$$\exists x \neg Pxa \qquad \neg \forall y\, Pya$$
2
$$\neg \forall y\, Pya$$
$$\exists z \neg \forall y\, Pyz \quad \Box$$

v)

$$[\forall z \forall x_1\, Pabzx_1]^1$$
$$\forall x_1\, Pabcx_1$$
$$Pabca_1$$
$$\exists y\, Payca_1$$
$$\forall x_1 \exists y\, Paycx_1$$
$$\exists x \forall x_1 \exists y\, Pxycx_1$$
$$\forall z \exists x \forall x_1 \exists y\, Pxyzx_1 \qquad [\exists y \forall z \forall x_1\, Payzx_1]^2$$
1
$$\exists x \exists y \forall z \forall x_1\, Pxyzx_1 \qquad \forall z \exists x \forall x_1 \exists y\, Pxyzx_1$$
2
$$\forall z \exists x \forall x_1 \exists y\, Pxyzx_1 \quad \Box$$

6.4

$$\forall x((Px \land Qx) \to Rx) \vdash \forall x(Px \to Qx) \to \forall x(Px \to Rx).$$

P': ... is a philosopher Q': ... has studied logic R': ... knows Gödel

$$[\forall x(Px \to Qx)]^2$$

$$\dfrac{{}^1[Pa] \qquad \dfrac{{}^1[Pa] \qquad Pa \to Qa}{Qa}}{Pa \land Qa} \qquad \dfrac{\forall x((Px \land Qx) \to Rx)}{(Pa \land Qa) \to Ra}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{Pa \land Qa \qquad (Pa \land Qa) \to Ra}{Ra}\,1}{Pa \to Ra}}{\forall x(Px \to Rx)}\,2}{\forall x(Px \to Qx) \to \forall x(Px \to Rx)} \; \Box$$

6.5

$$\dfrac{[\forall y(Ray \leftrightarrow \neg Ryy)]^2}{Raa \leftrightarrow \neg Raa} \qquad [Raa]^1$$

$$\dfrac{\neg Raa}{} \; 1 \qquad [Raa]^1 \qquad \neg Raa$$

$$\dfrac{\forall y(Ray \leftrightarrow Ryy)}{Raa \leftrightarrow \neg Raa} \qquad \dfrac{}{Raa} \qquad \neg Raa \qquad Q$$

[copy here also]

$$\dfrac{\exists x \forall y(Rxy \leftrightarrow Ryy) \qquad Q}{Q}\,2 \quad \Box$$

again, feels ⊛ inefficient to have to rederive already-proved sentences.