Tute sheet 3

college 2 a) ii) $AC = \frac{2}{3}y^2 + 1 + 36y^{-1}$, $\frac{dAC}{dy} = \frac{4}{3}y - 36y^{-2} = 0$

$\therefore \frac{4}{3}y^3 = 36$, $y = \sqrt[3]{27} = 3$ $\boxed{\text{to min } AC}$ $\quad \left.\frac{d^2AC}{dy^2}\right|_3 = \frac{4}{3} + 72y^{-3} = 4 > 0$ so min.

$VC = \frac{2}{3}y^3 + y$

$AVC = \frac{2}{3}y^2 + 1$, $\frac{dAVC}{dy} = \frac{4}{3}y = 0$ so output 0 to min AVC, at cost = 1.

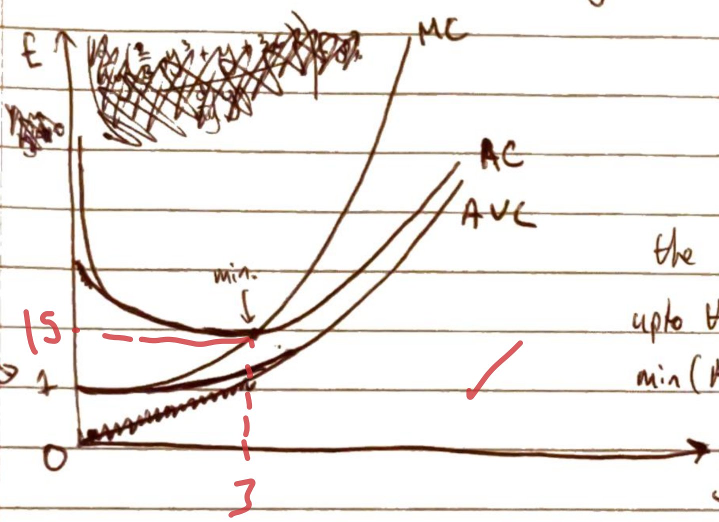
$\left.\frac{d^2AVC}{dy^2}\right|_0 = \frac{4}{3} > 0$ so min. which seems odd, maybe not undefined AVC with l'Hôpital's rule or something?

$MC = \frac{dC}{dy} = 2y^2 + 1$, at $y = 3$ $MC = 19$

at $y = 0$ $MC = 1$

iii)

MC and AVC have same y-intercept



iv) In general, where the market price $p >$ min(AVC), then the supply curve is simply the MC curve, since the firm maximises profit by producing upto the unit where $p = MR = MC$. Since min(AVC) = 1, at all prices $p > 1$ the firm can recover its variable costs by producing some output, so supply = MC curve, with discontinuity to produce 0 output when $p < 1$

b) i) $AC = c + Fy^{-1}$, $\frac{dAC}{dy} = -Fy^{-2}$ and as $\frac{1}{y^2} > 0$ for all $y \neq 0$,

$-Fy^{-2} < 0$ for all output $y \neq 0$ since $F > 0$

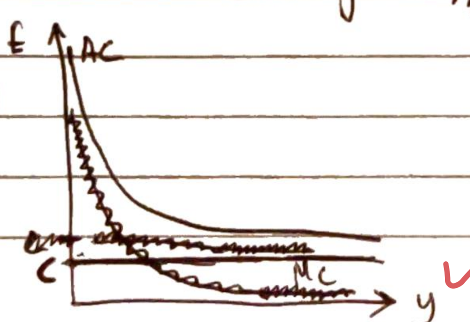
ii) $MC_y = \left.\frac{dC}{dy}\right|_y = c$

so $AC_y - MC_y = (c + Fy^{-1}) - (c) = Fy^{-1}$

For all $y > 0$, $y^{-1} > 0$, and since $F > 0$,

$Fy^{-1} > 0$ i.e. $AC_y - MC_y > 0$, $\therefore AC_y > MC_y$

iii)



as $y \to \infty$, $AC \to c$

c) $AC = \frac{C(y)}{y}$, $\cancel{\frac{dAC}{dy}} = $ ~~—~~

let $MC = \frac{d}{dy}(C(y)) = C'(y)$

Then with quotient rule, $\frac{d}{dy}(AC) = \frac{C'(y)\,y - C(y)}{y^2} = \frac{MC\,y - C(y)}{y^2}$

[is there a better way to deal with $y=0$ cases in denominator, generally?]

Generally? No. But we also won't worry about such cases much.

$\frac{d}{dy}(AC) < 0$ (i.e. AC decreasing) iff $MC \times y - C(y) < 0$ since $y^2 > 0$ for all $y \neq 0$.

i.e. $MC < \frac{C(y)}{y} = AC$

Conversely, $\frac{d}{dy}(AC) > 0$ iff $MC \times y - C(y) > 0$

i.e. $MC > \frac{C(y)}{y} = AC$

Intuitively, if you're adding extra (marginal) units whose production cost is greater than average to your output, clearly that will increase the average. The converse is also true: the new average is nudged somewhat towards the value (i.e. production cost) of the new item added to the group.

College 3 a) Constant, as by Cobb-Douglas function $a + b = \frac{1}{2} + \frac{1}{2} = 1$.

Prove it from first principles.

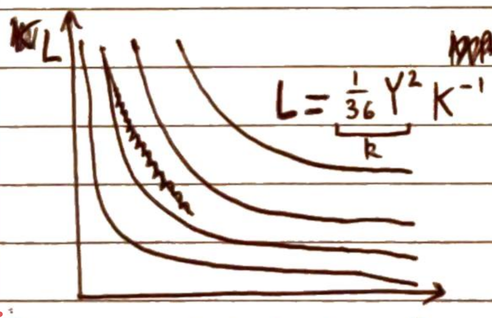
i.e. it is homogeneous of degree 1

let $f(K, L) = 6K^{\frac{1}{2}}L^{\frac{1}{2}}$,

$f(\lambda K, \lambda L) = 6(\lambda K)^{\frac{1}{2}}(\lambda L)^{\frac{1}{2}} = 6\lambda^{\frac{1}{2}}K^{\frac{1}{2}}\lambda^{\frac{1}{2}}L^{\frac{1}{2}} = 6\lambda K^{\frac{1}{2}}L^{\frac{1}{2}} = \lambda f(K, L)$

Ah

b)

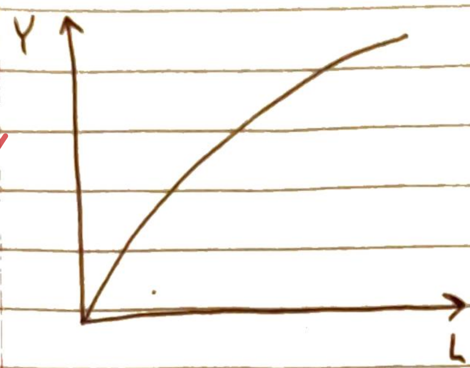


As K increases the |MRTS| i.e. slope of isoquant decreases. MRTS gives you how many units of L you give up to keep output constant after an increase of K by 1 unit. So this becomes smaller because additional units of K become less valuable the more of K you have, i.e. MP_K exhibits diminishing returns (and so too with MP_L by reverse argument).

[how to actually "work these out"?]

via MRTS.

Calculate MRTS explicitly for this problem. It is $-\frac{L}{K}$, which shows that the isoquants are downward sloping and convex.

c) 

$Y = 12\sqrt{L}$

$MP_L = \frac{dY}{dL} = 6L^{-\frac{1}{2}}$

Diminishing MP_L; $\frac{d^2Y}{dL^2} = -3L^{-\frac{3}{2}}$ and since $L^{-3/2} > 0$ for all $L > 0$, $-3L^{-3/2} < 0$ for all $L > 0$, i.e. MP_L is a decreasing function.

d) The cost of a marginal unit of labour is w. The revenue brought in by a marginal unit of labour is the output it produces × price of that output at market, i.e. $MP_L \times p$. The firm maximises profits when ~~these~~ it selects a labour input amount such that these are equal, otherwise they are producing units with $MC > MR$ (too much labour) or failing to produce units where $MR > MC$ (too little), in either case reducing profits.

$$MP_L = 6L^{-\frac{1}{2}} = 2 \Rightarrow L = 9$$

4a) $F(\lambda K, \lambda L) = (\lambda K)^{\frac{1}{2}} + (\lambda L)^{\frac{1}{2}} = \lambda^{\frac{1}{2}}(K^{\frac{1}{2}} + L^{\frac{1}{2}}) = \lambda^{\frac{1}{2}} F(K,L)$

So ~~diminishing~~ decreasing returns to scale as $\lambda^{\frac{1}{2}} < \lambda$ for $\lambda > 1$

b) $\frac{dK}{dL} = -\frac{\partial F}{\partial L} \div \frac{\partial F}{\partial K} = -\frac{\frac{1}{2}L^{-\frac{1}{2}}}{\frac{1}{2}K^{-\frac{1}{2}}} = -\sqrt{\frac{K}{L}}$

leave in terms of K and L? Yes

c) You reduce K and increase L so long as $\frac{MP_L}{w} > \frac{MP_K}{r}$ because you then can reduce costs by doing so. So $\boxed{\frac{MP_L}{MP_K} = \frac{w}{r} \text{ at the}}$ optimal combination, i.e. $\sqrt{\frac{K}{L}} = \frac{w}{r}$ and $\sqrt{K} + \sqrt{L} = y$,

So $\frac{y - \sqrt{L}}{\sqrt{L}} = \frac{w}{r}$, $y = \sqrt{L}\left(\frac{w}{r} + 1\right)$, $L = \left(\frac{y}{\frac{w}{r}+1}\right)^2 = \left(\frac{yr}{w+r}\right)^2$

$$K = \left(y - \frac{y}{\frac{w}{r}+1}\right)^2 = \left(\frac{yw}{r+w}\right)^2$$

(slope of isoquant = slope of isocost)

relevant here.

Not sure why this is mentioned now.

~~6a~~ d) $\left(y = \sqrt{K} + \sqrt{L},\right)$ cost $= Kr + Lw$

and the ideal values of K and L are known

so min cost $= \left(\frac{yr}{r+w}\right)^2 w + \left(\frac{yw}{r+w}\right)^2 r$

$$= y^2\left(\frac{r^2w + w^2r}{(r+w)^2}\right) = y^2\left(\frac{rw(r+w)}{(r+w)^2}\right)$$

$= y^2\left(\frac{rw}{r+w}\right)$, i.e. min cost $\propto y^2$ → yes. Cost is convex in y. Why? Because F is DRS.

6 a) Supply function is MC curve i.e. $\frac{d}{dy}C(y) = 8y$

So inverse supply function $y(p) = \frac{1}{8}p$

Firm closes when $p <$ min (AVC) but min (AVC) = 0 so no discontinuity.

(*) [ask about when (why) practically we have min (AVC) $\neq 0$ i.e. what algebraically the VC and AVC might be for that to occur] → if production is initially convex and then eventually concave.

Aggregate supply $Y(p) = 3p$

at p^*, $3p^* = 150 - 2p^*$, $p^* = 30$

b) In the long run profits are 0 ← Ok yes, why?

So, ~~[illegible]~~ $\tilde{p}y = 16 + 4y^2$ i.e. revenue = costs

$\tilde{p}y = 16 + 4y^2$ ~~and $150 - 2\tilde{p}$~~

at the LR equilibrium, $\tilde{p} = 75 - \frac{1}{2}Y$, from demand

~~So $\tilde{p} = 25$ and $y = ?$~~

~~$Y = Ny$, so $75 - \frac{1}{2}Ny = 16 + 4y$, $59 = (\frac{1}{2}N + 4)y$~~

Cost function unchanged so $\tilde{p}y = 8y \times y = 16 + 4y^2$;

~~$8y^2 - 4y^2 - 16 = 0$~~ $4y^2 = 16$, $y = 2$ so $\tilde{p} = 8y = 16$

$Y = Ny = 2N = 150 - 2\tilde{p}$, $N = 75 - \tilde{p} = \boxed{59}$ firms