80.9

Excellent work!

MERTON COLLEGE

COLLECTION PAPER

Please complete in BLOCK CAPITALS

Full name: Rohan Selva-Radov	Honour School: PPE
Term of collection: HT 25	Title of collection: Micro
Tutor marking the collection: Bassel Tarbush	

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A. 1.

$$u_a(x_a, y_a) = 2\ln x_a + 2\ln y_a$$
$$u_b(x_b, y_b) = \ln x_b + \ln y_b.$$

$$\omega_a = (18, 6) \quad ; \quad \omega_b = (6, 18)$$

a) A competitive equilibrium is a price vector and allocation such that:
- all markets clear, and
- given the price vector, each agent is optimising their utility

i.e. where the excess demand in each market is zero.

We can solve for it as follows. First, find the demand functions of each consumer.

A solves the problem $\max_{x_a, y_a} 2\ln x_a + 2\ln y_a$

s.t. the budget constraint $p x_a + y_a \leq \underbrace{18p + 6}$

and $x_a \geq 0$, $y_a \geq 0$. the value of her endowment $(18, 6)$

- Since her preferences are monotonic increasing, the BC binds with equality.
- Since as the quantity of a good $j \to 0$, the $MU_j \to \infty$ we know she consumes strictly +ve quantities of each.

So the problem simplifies to

$$\max_{x_a, y_a} u_a(x_a, y_a) \text{ s.t. } p x_a + y_a = 18p + 6$$
$$x_a > 0, \quad y_a > 0$$

~~For which the FOCs are $\frac{\partial u_a}{\partial x_a} = 0$~~

As this is Cobb-Douglas, we know the demands will be

$$x_a = \frac{1}{2} \cdot \frac{18p + 6}{p} \quad ; \quad y_a = \frac{1}{2} \cdot (18p + 6)$$

as there are equal weights on the goods, so she spends half her income on each.

Similarly for B: $x_b = \frac{1}{2} \cdot \frac{6p + 18}{p}$; $y_b = \frac{1}{2}(6p + 18)$

So the ~~excess~~ total demands of each good are

for x: $\frac{1}{2}\left(\frac{24p + 24}{p}\right) = 12 + \frac{12}{p}$; for y: $\frac{1}{2}(24p + 24) = 12p + 12$

and thus excess demands $z^j := \sum_i \omega_{ij} - \sum_i x_{ij}$

for good x: $12 - \frac{12}{p}$ for y: $12 - 12p$.

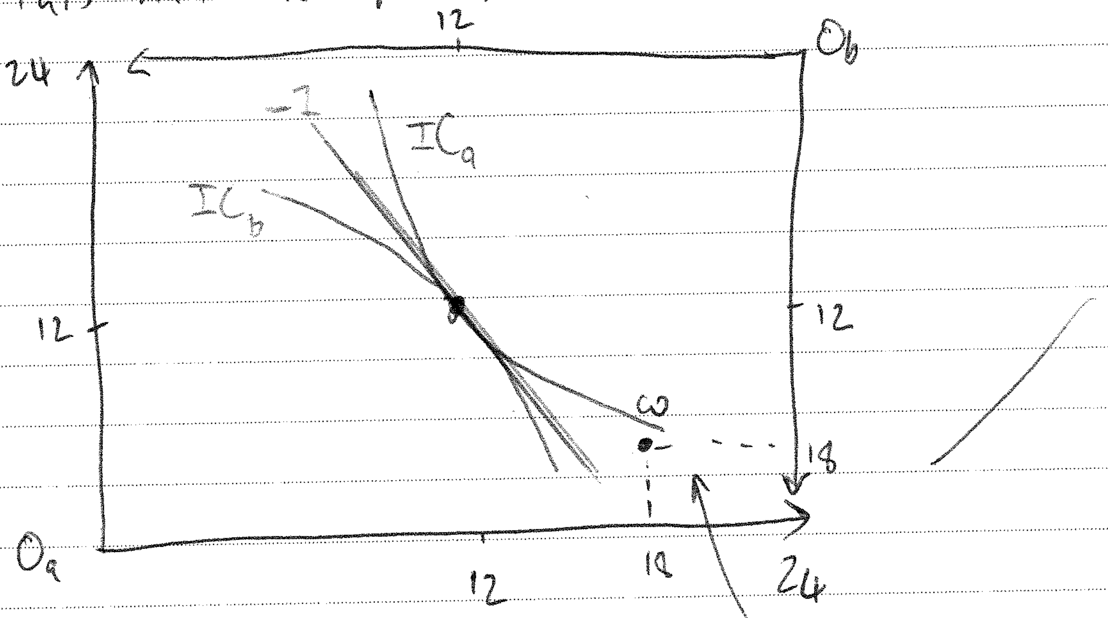
As these must each be zero at comp. eq*,

let $z^x = 0 \Rightarrow 12 - 12/p = 0 \Rightarrow p = 1$ as required.

And by Walras's law, we only need to solve for $k-1$ markets. Since allocations are HOD in prices

An allocation is P-efficient iff it is not possible to increase one agent's utility without harming some other's. That is, there does not exist another allocation that every agent weakly prefers and some ~~st~~ strictly prefers.

By the first FWT, a competitive equilibrium in the exchange economy is P-efficient. Substituting in $p=1$ to our demands, we find the allocation is indeed (12, 12; 12, 12). So this must be P-efficient.



(imagine the budget line goes through ω)

b. In this economy, both agents have identical preferences. Note that u_a is simply a strictly positive transformation of u_b, ~~a~~ created by doubling the utility associated with any bundle of goods owned by that consumer. ~~This means they have~~

Also, note that Cobb-Douglas preferences are homothetic = the slope is equal all along a given ~~each~~ ~~any~~ ray from the origin.

The relative ~~eq*~~ price, or the slope of the budget constraint, would not change due to vertical movements in endowment point ω, because the identical preferences mean that ~~they bel~~ the ICs are all tangent along a single ray

c. They solve the problem

$$\max_{x_a, y_a} \; 2\ln x_a + \ln x_b + 2\ln y_a + \ln y_b$$

s.t.

2a. $U_a(q, x_a) = 2q - q^2 + x_a$

$U_b(q, x_b) = q - \frac{1}{2}q^2 + x_b$

a) Alice solves $\max_{q, x_a} 2q - q^2 + x_a$ s.t. $q \cdot c + x_a \leq m$, $q \geq 0$, $x_a \geq 0$

And for q-linear utility the FOC will be

$2 - 2q = c$ as we require $MRS = \frac{c}{1}$ where $MRS := \frac{MU_q}{MU_{x_a}}$

So ~~opt~~ she buys for herself $q = 1 - \frac{c}{2}$ if B buys nothing, given that $x_a > 0$ and we have an interior sol'n.

Similarly for Bob, if A buys nothing he would purchase q s.t.

$1 - q = c \Rightarrow q = 1 - c$

if A bought nothing.

A strategy profile is Nash if no agent has a strict unilateral incentive to deviate, i.e. each plays a best response to the other's played strategy.

~~The~~ ~~so~~ It's Nash to have Alice buy $1 - \frac{c}{2}$ and Bob buy zero as:

- As shown, if B buys zero A buys $1 - \frac{c}{2}$, so A plays a best response.
- For q-linear goods, agents optimally do not demand any more of it once they have attained their satiation quantity, which for Bob is $1 - c < 1 - \frac{c}{2}$. So it's a BR for Bob to buy zero of the good, given ...

A is buying $1 - \frac{c}{2}$.

b) The Samuelson rule says ~~that~~ $\sum_i MU_i(x) = MC(x)$ at the optimal quantity to be supplied x of a public good.

Here, $MC(q) = c$ for all q, and
$\sum_i MU(q) = (2-2q) + (1-q) = 3-3q$

So optimally, we have $3-3q = c \Rightarrow q = 1 - \frac{c}{3}$

which is $> 1 - \frac{c}{2}$, i.e. there is, as expected, undersupply of the public good in (a).

c) Lindahl's scheme will, if agents report truthfully, by construction lead to the optimal quantity being supplied according to the the Samuelson rule, and ensure the government has a balanced budget, since
$\sum_i MU_i(q^*) = c$ is specified.

So, this seems an attractive way to reach the socially optimal outcome.

But,
- Agents have no incentive to report truthfully as it's not a dominant strategy

For full marks, state prices + allocations, and give example of beneficial lying.

- It is informationally very hard for agents to know their whole marginal utility function

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3a)

	L	R
T	2, $\underline{1}$	$\underline{1}$, x
B	$\underline{3}$, 1	0, $\underline{2}$

0.5

$0 < x < 1$

We can find the pure NE by underlining best-response payoffs, but there is none.

By the Nash theorem, there will be at least one mixed equilibrium. (A NE is a strategy profile s.t. no player has a strict unilateral incentive to deviate)

At this mixed eq^{m}, each agent must be indifferent between the pure strategies they're mixing between or else they would simply play their preferred pure strategy given the other's strategy.

Suppose R plays T with probability p, and B otherwise.

Then for C, the expected payoff from L is 1 (payoff doesn't vary with R's choice) and expected payoff from R is

$$px + (1-p)\cdot 2 = 2 + (x-2)p$$

To be indifferent b'ween L and R, we need

$$2 + (x-2)p = 1$$

so $\underline{p = \frac{1}{2-x}}$

Similarly let C play L w.p. q and R otherwise.

Then for R, the expected payoff from T is $2q + (1-q)$ and from B is $3q$.

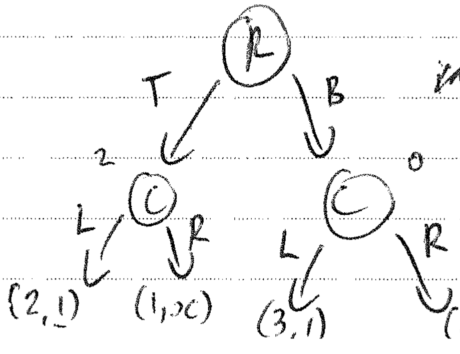
For indifference, $2q + (1-q) = 3q$

so $\underline{q = \frac{1}{2}}$

And R has an expected payoff of ~~1.5~~ in 1.5, L has an expected payoff of 1.

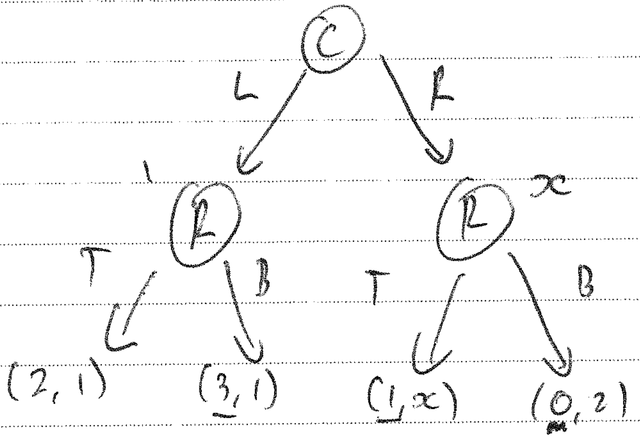
With $x \in (0,1)$, eqm payoffs do not change – all that happens is that as x grows, R is more ~~less~~ likely to play Top in order ~~that~~ to keep C indifferent between Left and Right.

b) Suppose R goes first:



By backwards induction, ~~insPE RR would choo the subgame perfect eqm is~~ R would choose T and C replies L, for (2, 1).

So ~~R would prefer to move first.~~



For Col if they went first they'd choose ~~L~~ L and Row replies with B, for ~~payoffs~~ (3, 1).

So, both players would weakly prefer either of them to move first, compared to going simultaneously. does either of them strictly benefit? how?

c)

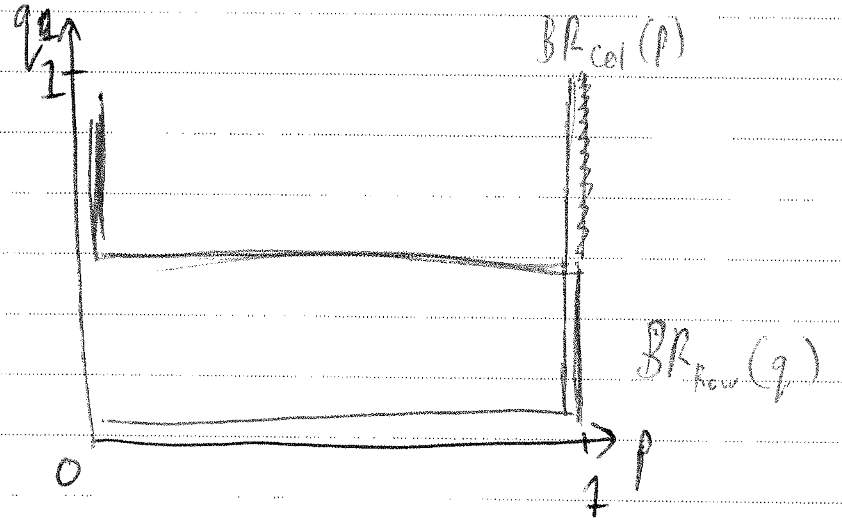
	L	R
T		
B		

d)

	L	R
T	2, 1	1, 1
B	3, 1	0, 2

(T, R) is a pure NE.

For mixed NE, we can plot BR curves in (p, q) space, and find the intersections. Again, let Row play T w.p. p and Col play L w.p. q.



For Row, T is better iff

$$2q + 1 - q > 3q \Rightarrow q < \frac{1}{2}$$

For Col, L is better iff

$$1 > p + 2(1-p) \Rightarrow p > 1$$

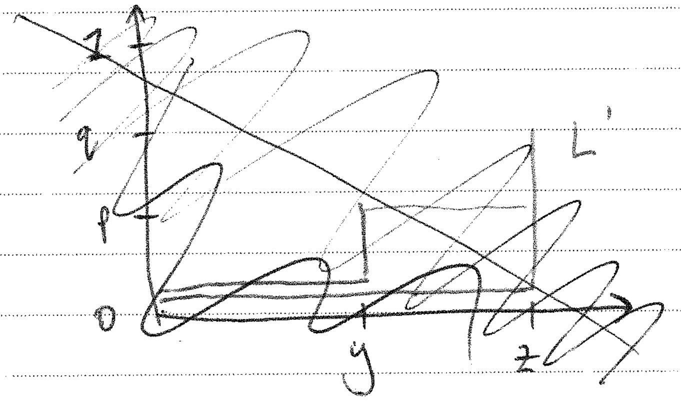
So we have an infinite number of mixing eq* where Row plays T w.p. $p = 1$ and Col plays L w.p. $q \leq \frac{1}{2}$.

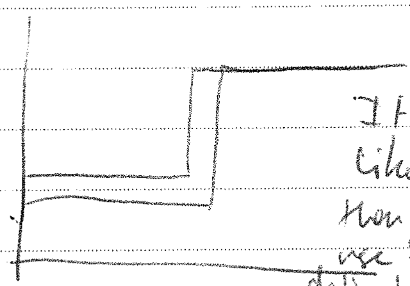
4(a) $L = [1-p, p; 0, y]$ and $L' = [1-q, q; 0, z]$

where $z > y > 0$.

a) Lottery L_a FOSDs L_b iff the probability of having at least w ~~wealth~~ payoff is weakly greater under L_a than L_b for all w, and strictly for some.

If $q \geq p$ then ~~the probabili~~ yes, L' must FOSD L. This is because the CDF of L' is everywhere lower than L: the probability of getting 0 payoff is smaller, and the probability of at least y is greater (since $q \geq p$), and P(at least z) is greater (as $z > y$).





It'll look like this, then you can use the CDF definition of FOSD

b) ~~Even~~ If L_a FOSDs L_b then every EU-maximiser prefers L_a to L_b. So if Ava prefers L to L' then it cannot be FOSD-ed, so it must be that $p > q$, following (a).

For L', ~~then~~ $EU := \sum_i p_i \cdot u(x_i)$

$$= (1-q)\cdot 0 + q \cdot 1 = q$$

and for L, $EU = (1-p)\cdot 0 + p \cdot u(y) = p \cdot u(y)$

so, ~~as~~ as to prefer L over L' then $EU(L) > EU(L')$ we require $p \cdot u(y) > q$ i.e. $u(y) > \frac{q}{p}$ where

$q, p > 0$ and $q < p$, so all we can say is that $0 < u(y) < 1$. This is expected – since her preferences are monotonic, it cannot be that $u(y) > u(z)$, but otherwise $u(y)$ could e.g. be very low, and still she prefers L given a suitably low relative probability of q to p.

c) EU preferences are invariant under strictly increasing affine transformations.

Note that $\mathbb{E}[a + bX] = a + b \cdot \mathbb{E}[X]$ where X is a random variable, by linearity of $\mathbb{E}$.

Suppose $L_a := [\underline{f}_a ; \underline{x}_a] \succsim L_b := [\underline{f}_b ; \underline{x}_b]$

Then $EU(L_a) \geq EU(L_b)$, as they're an EU-maximiser, i.e. $\sum \{ f_a \cdot u(\underline{x}_a) \} \geq \sum \{ f_b \cdot u(\underline{x}_b) \}$

But, as above, this is $\iff$

$$\sum \{ f_a \cdot v(\underline{x}_a) \} \geq \sum \{ f_b \cdot v(\underline{x}_b) \}$$

Since you can simply factor out α and β from the sum. And thus the risk preferences are unchanged by this transformation.

So, we can pin down $u(0)$ and $u(2)$ wherever convenient by scaling and translating Ava's utility function without affecting the underlying risk prefs being represented.

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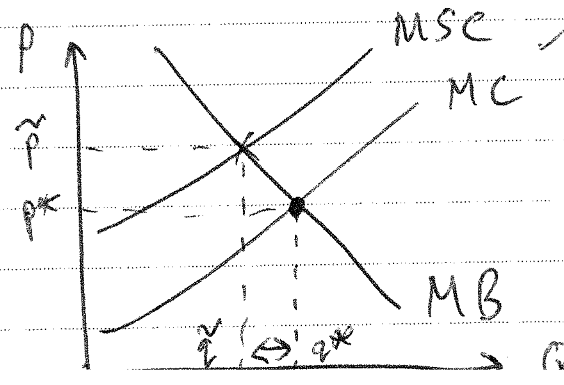
6. Greenhouse gas emissions (hereafter GHGs) are a negative externality, where the social cost is greater than the private cost to the individual producing them. Without remedy, this leads to market failure where an inefficient quantity (excess) will be produced. Government intervention can address this, and as I explain, although each of the measures proposed would help to an extent, a cap-and trade scheme is most likely appropriate, in conjunction with a carbon border adjustment mechanism (CB



The market outcome will be (q^*, p^*), but $(\tilde{q}, \tilde{p})$ would be the efficient outcome, with the gap $\Leftrightarrow$ excess emissions.

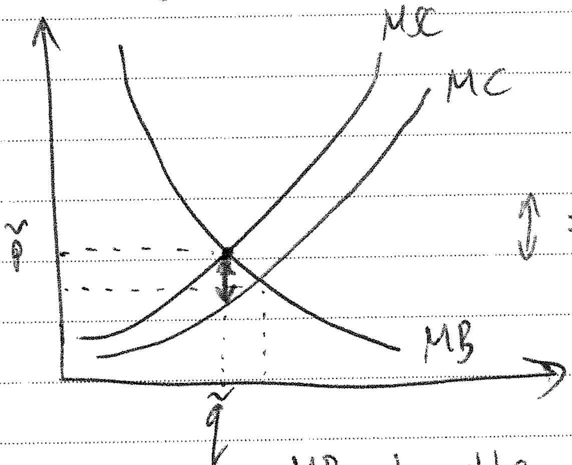
(i) The Coase theorem states that, given perfect information and zero transaction costs, any assignment of enforceable property rights over a public resource combined with bargaining between agents will lead to a Pareto-efficient allocation. This theorem is the idea motivating the suggestion. In practice, it is extremely unlikely such an approach could be implemented, let alone lead to efficiency.

Second. ~~First~~, GHGs are a <u>global</u> issue. So, there property rights would need to be respected by every government, requiring a substantial degree of international cooperation. Monitoring for violations would be difficult, as they could occur anywhere on earth.

First, it's unclear what the rights would be, and to whom they would be assigned. This isn't insurmountable – maybe the rights could be to a certain quantity of CO_2e/year and handed out to some group of corporations. (And other approaches must also deal with what exactly to regulate, e.g. carbon, or methane, etc.). But there would be large distributional concerns from simply endowing one or a few agents with the right to pollute, as although trades would occur, the initial owners would capture all the surplus. Having a government own these rights may be preferred, but functionally this is similar to (ii) or (iii).

Third, there are not zero transaction costs. Cosean bargaining requires time + effort from agents to e.g. get lawyers ~~to~~ review contracts, match up with rights owners, etc. So this would create further inefficiencies.

(ii) Pigouvian taxes ~~increase~~ would be levied by the gov't on polluters, and set at a per-unit quantity such that at the optimal pollution level, ~~the~~ marginal ~~tax~~ private cost = marginal social cost.



This internalises the externality and ensures that producers are incentivised to pollute only if the MB to them is greater than MSC.

One benefit here is that revenue is raised by the gov't which can be spent as it sees fit, perhaps with social welfare goals in mind. However, taxation is politically challenging and may be especially so in the ~~GHG~~ context since benefits accrue not only to future citizens but also people in other countries.

In addition, as Weitzmann discusses, there is uncertainty about the MB curve and thus the optimal $\tilde{q}$. Since in GHGs, the MSC is likely to be relatively steep compared to MB ~~due~~ to the existence of climate tipping points the expected DWL is smaller from setting quantities.

(iii) Under perfect information, cap and ~~trade~~ is economically identical to Pigouvian taxes; it simply uses a quantity instrument rather than a price one. However, uncertainty makes it preferable in this case: the government chooses the optimal ~~polluti~~ GHG level and auctions off (or gives away) permits summing to that $\tilde{q}$. Trades of permits will mean they end up with the firms who value them highest, and so efficiency is attained.

One disadvantage is that pinning down $\tilde{q}$ may lead to volatility in prices e.g. as new technologies emerge to abate GHGs in different industries. Because ~~prices~~ these price changes won't be passed down supply chains immediately, the volatility could lead to inefficiency in permit allocation, and menu costs for firms.

Also, regulating domestic GHGs only would put local firms at a disadvantage compared to competitors abroad. To deal with this, a CBA would be desired to levy a tax/tarrif on imports from countries w/out equivalent regulation. This helps with efficiency (not simply offshoring GHGs) and political palatability (don't put local firms at a disadvantage).

This is clearly a market that isn't reaching efficient outcomes.

(iv) This is the least sensible approach. Direct regulation makes it impossible for market forces to reach an efficient allocation of rights to produce GHGs. Differen firms have their own GHG abatement cost schedule, and so value 1 CO2e permit differently. It therefore is best to allow for trades so that ~~ma~~ firms who value GHGs more will pay cleaner ones for their right to pollute. This achieves efficiency, and also creates the right incentive firms to invest in new technology to cut GHGs, rather than making it so there's no benefit of doing so.

One merit of this approach is distributional – the gov't can choose which sources are allowed to pollute and by how much, with full control over the outcome. But a better means to the same objective is selling permits and using the proceeds. In the case of GHGs, the problem is not localised at all – we only care about aggregate emissions. So there is no benefit to micromanaging where emissions come from, since gov't has less info about abatement costs than firms themselves.

not necessarily true if we think that parts of the economy are inefficient.

for

To conclude, either of the market mechanisms (ii) or (iii) would be suitable, though Cap-and-trade with a CBA is best for the GHG case in particular. ~~The~~ (i) is unlikely to be efficient or equitable in practice, and (iv) forgoes the price/quantity discovery properties of market-based solutions and is thus inefficient even in theory.

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B 10a)

	H: £400	M: £280	L: £100
e_h :	$\frac{1}{2}$,	0 ,	$\frac{1}{2}$
e_l :	$\frac{1}{4}$,	$\frac{1}{4}$,	$\frac{1}{2}$

$u(w, e) = \sqrt{100 + w} - e$, $u_0 = 11$

a) Moral hazard is a ~~market~~ phenomenon where the outcome of a contract depends on the actions of one party, but the other party cannot enforce, monitor, or verify those actions. Since the firm can't observe effort and outcomes depend only stochastically on it, they can't simply tell the agent to put in ~~any~~ high effort, as this isn't incentive-compatible: the agent would ~~[illegible]~~ prefer to just put in low effort, all else equal. So the firm must find a way to incentivise high effort through the contract if desired, as they can't check whether the agent actually did do e_h and pay accordingly.

If e_L is desired, the problem is simple and there's no tradeoff needed between efficient risk-sharing and provision of incentives. The firm should offer a fixed wage to the agent regardless of outcome, at the lowest rate possible they'll accept, i.e. such that $\sqrt{100+w} - e = 11$ where $e = 0$ so $w = 21$.

Expected profits will be

$$\frac{1}{4} \cdot 400 + \frac{1}{4} \cdot 280 + \frac{1}{2} \cdot 100 - 21$$

$$= £199.$$

b) We require individual rationality and incentive compatibility. (For ease, I will assume in the IR constraint that IC is satisfied, i.e. e_h is preformed)

IR: the agent must prefer the contract to reservation.

$$\frac{1}{2} \cdot \left(\sqrt{100+w_H} - 1\right) + \frac{1}{2} \cdot \left(\sqrt{100+w_L} - 1\right) \geq 11$$

(since $P(M \mid e=1) = 0$, we can ignore)

IC: the agent must prefer to put in high effort.

$$\frac{1}{2}\left(\sqrt{100+w_H} - 1\right) + \frac{1}{2}\left(\sqrt{100+w_L} - 1\right) \geq \frac{1}{4}\left(\sqrt{100+w_H}\right) + \frac{1}{4}\left(\sqrt{100+w_M}\right) + \frac{1}{2}\left(\sqrt{100+w_L}\right)$$

c) If the agent puts in high effort, certainly the medium outcome will not arise. So, setting $w_M = -100$ has no bearing on IR, and merely makes e_L less attractive, i.e. helps with IC. This is an attractive way for the firm to satisfy IC, since they don't have to offer to pay out large wages in the H outcome to make e_h more appealing to the agent. They're instead able to costlessly incentivise e_h.

Since setting $w_M = -100$ is enough to make e_L very unappealing, there is no need to transfer ~~additional~~ risk to the agent and make $w_H > w_L$ as a way to further incentivise e_h. The agent is risk-averse, so transferring risk leads to more costly wages in expectation to satisfy IR. So, setting $w_L = w_H$ will be the cheapest expected contract and thus maximises firm profits, provided $w_M = -100$ is enough to satisfy IC.

IR will bind, otherwise the firm could've offered lower wages in both H and L cases.

$$\text{so} \quad \sqrt{100+w_L} + \sqrt{100+w_H} \geq 24$$

and as by ~~hypothesis~~ assumption $w_L = w_H$, this implies $100 + w_L \geq 144$;

$$w_L = w_H = 44.$$

so the contract should be $(44, -100, 44)$.
And the RHS of IC will be

$$\frac{1}{4}(12) + \frac{1}{4}\cdot(0) + \frac{1}{2}\cdot 12 = 9 < 12 = LHS \text{ so}$$

IC is satisfied.

Firm expected profits will be

$$\frac{1}{2}\cdot £400 + \frac{1}{2}\cdot £100 - 44 = £206 \text{ so}$$

e_h is better.

d) · Both IR and IC will bind, as argued above

∘ If IR doesn't bind, both w_H and w_L can be ↓

∘ If IC doesn't bind, the firm can transfer less risk to make the exp. payoffs closer, so less risky, and have ↓ expected wage costs.

· Certainly $w_M = 0$, since they want to make e_L as unattractive as possible, and so w_M as low as permissible, following the argument in (c), ~~the~~ ~~constraint~~ but now bound by the laws in

so IR: $\sqrt{100+w_H} + \sqrt{100+w_L} = 24$

IC: $\frac{1}{2}\sqrt{100+w_H} + \frac{1}{2}\sqrt{100+w_L} - \cancel{1} = \frac{1}{4}\sqrt{100+w_H} + \frac{1}{4}\cdot 10 + \frac{1}{2}\cdot\sqrt{100+w_L}$

$\therefore \sqrt{100+w_H} + \sqrt{100+w_L} = \frac{1}{2}\sqrt{100+w_H} + \sqrt{100+w_L} + \cancel{7}$

$\sqrt{100+w_H} = 14$; $w_H = 96$

and $\sqrt{100+w_L} = 10$ so $w_L = 0$

(hmm, this seems a bit surprisingly low for w_L and high for w_H)

e) From (d), exp. profits will be

$\frac{1}{2}\cdot 400 + \frac{1}{2}\cdot 100 - \frac{1}{2}\cdot 96 = £202$

so as expected the firm is worse-off vs. (c), but they prefer e_H to e_L even with the law. The agency cost is equal to the difference in firm profits in the full information case vs moral hazard and exactly equal to the risk premium of the wages lottery they offer to the agent. If the firm could monitor effort and require e_H for payment, then they'd be able to pay wages of only 44 vs 48 (expected) so agency cost is £4 - the difference in π between (c) and (d).

f) Negative wages allow for a more efficient allocation of risk, i.e. none for the agent. This increases social welfare. Note that in both cases IR binds, so the agent is actually no better off under the law, all that happens is that the firm has lower profits. The firm could offer the agent £1 to waive their right and keep the remaining £3. Then all are strictly better off.

(In practice, it's unlikely that $P(M \mid e=2)=0$ so the agent may not be so willing to accept the threat of -ve wages. Also, enforcement of -ve wages may be hard - perhaps the agent has no money to pay, etc.)