Rohan 2-propositional logic

2.1 i) "Potassium" ii) "Snow is white", "snow is white"
iii) "John", "Jane", "Jeremy" iv) "'George'", "George"
v) "Tom", "Reginald" OR "Tom ... Reginald"

2.2 i) P_1 is, so $P_1 \to P_1$ is, so $(P_1 \to P_1) \to P_1$. Q is, so
$((P_1 \to P_1) \to P_1) \vee Q$ is

ii) P_2 and R are, so $(P_2 \wedge R)$ is. Q_4 is so $((P_2 \wedge R) \to Q_4)$ is
(but note that with extra brackets, $(((P_2 \wedge R)) \to Q_4)$ is not)
(?)

iii) P is, so $\neg P$ is, so $(P \to \neg P)$ is
iv) P is, but $P\neg$ is not, so $P\neg \to P$ is not
v) As iii), yes $(\neg P \to P)$ is
vi) R is so $\neg R$ is so $(R \vee \neg R)$ is so $\neg(R \vee \neg R)$ is
so ... $\neg\neg\neg(R \vee \neg R)$ is. P is so $P \to \neg\neg\neg(R \vee \neg R)$ is
vii) P is so $P \to \neg Q$ is, given Q is and so $\neg Q$ is. So
$(P \to (P \to \neg Q))$ is and so $\neg((P \to (P \to \neg Q))$ is.
R_2, R_7 also sentences, so $P \vee R_7$ is one too as is $\neg(P \vee R_7)$
and thus $(R_2 \leftrightarrow \neg(P \vee R_7))$ is, as is $\neg(R_2 \leftrightarrow \neg(P \vee R_7))$ and
also the double negation. So, yes the whole thing is an L sentence.

2.3 i) $(\neg P \wedge Q)$ ii) $(((P \wedge \neg Q) \wedge R) \leftrightarrow ((\neg P \vee P) \vee R_5))$

iii) $(\neg\neg\neg(P \to Q) \leftrightarrow P)$

nope, precedence is $\neg, \wedge, \vee, \to, \leftrightarrow$. So outer brackets are needed here?

2.4 i) $((\neg P \to \neg Q) \vee Q_2) \wedge P$

do $\wedge$ and $\vee$ have equal precedence, just left to right?

ii) $(\neg P \to \neg Q) \wedge Q_2 \wedge P$
iii) $\neg(P \wedge (P \to \neg Q) \wedge Q_1 \wedge P)$

just an exercise? ?

why do we need this leading column section? seems to be always left blank, in the textbook.

2.5 i) $P \quad Q \;\|\; P \wedge (P \to Q) \to Q$

$\qquad\qquad\quad\;\, T_3 \;\; T_2 \;\; T_5 \; T_4 \; ? \;\; F \;\; F_1$

ii) $P \quad Q \;\|\; \neg Q \wedge (P \to Q) \to \neg P$

$\qquad\qquad\quad\;\, T_4 \, F_5 \; T_3 \; T_7 \; ? \; F_6 \;\; F \; F_1 \, T_2$

iii) $P \;\|\; P \vee \neg P$

$\qquad\quad\; F_1 \;\; F \;\; F_2 \; ?$

iv) $P \;\|\; \neg (P \wedge \neg P)$

$\qquad\quad F \; T_2 \; T_1 \; T_3 \; ?$

(*) v) $P \;\|\; (\neg P \to P) \to P$

$\qquad\quad F_4 \; ? \; T_2 \; F_3 \;\; F \;\; F_1$

ohh - because not - P entails a contradiction, it can't be ?

vi) $P \quad Q \;\|\; (P \to Q) \wedge (\neg P \to Q) \to Q$

$\qquad\qquad\quad F_7 \; T_3 \; F_6 \;\; T_2 \; F_8 \, ? \; T_4 \; F_5 \;\; F \;\; F_1$

vii) $P \quad Q \;\|\; \neg (P \wedge Q) \leftrightarrow (\neg P \vee \neg Q)$

1. $\underline{F_1} \; T_4 \; T_3 \; T_5 \quad F \quad ? \; T_8 \; \underline{T_2} \; F_7 \; T_6$

2. $\underline{T_1} \; ? \; F_3 \; T_7 \quad F \quad F_4 \; T_5 \; \underline{F_2} \; F_3 \; T_6$

ix) $P \;\|\; P \wedge \neg P \to Q$

$\qquad\quad T_3 \; T_2 \; T_4 \; ? \;\; F \;\; F_1$

viii) $P \quad Q \;\|\; \neg (P \vee Q) \leftrightarrow (\neg P \wedge \neg Q)$

$\qquad\qquad\quad \underline{T_1} \; F_5 \; F_3 \; F_4 \quad F \quad ? \; F_8 \; \underline{F_2} \; T_7 \; F_6$

$\qquad\qquad\quad \underline{F_1} \; ? \; T_7 \; F_8 \quad F \quad T_3 \; F_4 \; \underline{T_2} \; T_5 \; F_6$

SEE END for 2.6 (sorry).

ix) ~~Neither~~ I can't tell immediately by inspection

2.6 i) ~~Tautology~~ ~~(contingent?)~~ ~~(*) i) P Q R || ((P → Q) → R) ↔ (P → (Q → R))~~ 1.1

ii) Tautology? ~~2.~~ 1.2

iii) Tautology ~~(v) Tautology~~

~~iv) ((P → Q) → R) ↔~~

2.7. $|\phi \vee \psi|_A = F$ iff $|\phi|_A = F$ and $|\psi|_A = F$; $|\phi \to \psi|_A = F$ iff $|\phi|_A = T$ and $|\psi|_A = F$

$|\phi \leftrightarrow \psi|_A = F$ iff $|\phi|_A = |\neg \psi|_A$ i.e. $|\phi|_A \neq |\psi|_A$ ie.

$[|\phi|_A = F$ and $|\psi|_A = T]$ or $[|\phi|_A = T$ and $|\psi|_A = F]$

(*) 2.8 $\Gamma \models \phi$ iff for all L_1-structures A,

If $|\psi|_A = T$ for all $\psi \in \Gamma$, then $|\phi|_A = T$

So, iff $\Gamma \models \phi$, there does not exist any A such that $|\psi|_A = T$ for all $\psi \in \Gamma$ and $|\phi|_A = F$, i.e. $|\neg\phi|_A = T$

In other words, the set β of sentences $\{\Gamma, \neg\phi\}$ is inconsistent iff $\Gamma \models \phi$, as there cannot be any L_1-structure A whereby $|\psi|_A = T$ for all $\psi \in \beta$.

(*) 2.6 i) Neither

ii)

P	Q	R	$((P \to Q) \to R) \leftrightarrow (P \to (Q \to R))$	
T	T	T	$T_4\ T_3\ T_5\ T_2\ T_6\quad T\quad T_7\ T_1\ T_9\ T_8\ T_{10}$	∴ not contradict.
F	F	F	$F_6\ T_3\ F_8\ F_1\ F_4\quad F\quad F_7\ T_2\ F_9\ T_{10}\ F_5$	∴ not tautology ⇒ Neither

iii)

P	Q	R	$(P \leftrightarrow (Q \leftrightarrow R)) \leftrightarrow ((P \leftrightarrow Q) \leftrightarrow R)$	Seems like taut.
			$T\quad T\ F\quad T\ T$	
			$F_3\ T_1\ F_4\quad F\quad F_5\ F_2$	1.1
			$F_2\quad T_1$	2

iv) Tautology