Tutorial 1 - models.

4. $U = \ln(C) - 2L^2$

i. $C = Y = AL^\alpha$ so $U = \ln(AL^\alpha) - 2L^2$

To maximise U, the first-order condition is that $\frac{dU}{dL} = 0$

i.e. $\frac{\alpha A L^{\alpha-1}}{AL^\alpha} = 4L$

$\alpha L^{-1} = 4L$, $L = \sqrt{\frac{\alpha}{4}}$ at the optimal level.

L^* is independent of the productivity parameter A, so productivity growth would have no impact on labour supply. (The yeomen would work the same hours but produce more output and thus have greater utility).

ii. The worker's ~~maximi~~ optimisation problem is

~~$\max_L U = \ln(wL + \pi) - 2L^2$ for which the first-order condition is $\frac{1}{L} - 4L$~~

$\max_L U = \ln(C) - 2L^2$ s.t. $C \leq wL + \pi$

for which the first-order condition is $\frac{dU}{dL} = 0$

[should've just left in terms of C] $\therefore \frac{w}{wL+\pi} = 4L$, ~~$w = 4wL^2 + 4\pi L$~~ i.e. $\frac{w}{C} = 4L$

so $L = \frac{w}{4C}$ at the optimum level.

iii. To $\max_N \pi = AN^\alpha - wN$ we have FOC $\frac{d\pi}{dN} = 0$

$\therefore \alpha A N^{\alpha-1} = w$

$N = \left(\frac{w}{\alpha A}\right)^{\frac{1}{\alpha-1}}$ is the labour demand curve.

If $MP_L := \frac{\partial Y}{\partial N}$ where Y is output, assumed to be production function $Y = AN^\alpha$, then we arrive again at $MP_L = w$, i.e. the firm pays a real wage equal to marginal product of labour (which makes sense - they buy units of labour until marginal cost of an additional unit is equal to marginal revenue [prices normalised to 1]) where Y is output per firm (?)

iv. $w = MP_L = \alpha A N^{\alpha-1}$, $L = \frac{w}{4C}$, $C = nY = n A N^{\alpha}$

So $L = \frac{\alpha A N^{\alpha-1}}{4 n A N^{\alpha}} = \frac{\alpha}{4} \cdot \frac{1}{nN}$ and $N = \left(\frac{w}{\alpha A}\right)^{\frac{1}{\alpha-1}}$

seems to yield $L = \frac{w}{4} \cdot \frac{1}{nA} \cdot \left(\frac{w}{\alpha A}\right)^{-(\alpha/\alpha-1)}$

[...but w is a model parameter (as perfectly competitive firms) so shouldn't have substituted that out]

$\therefore L = \frac{\alpha}{4} \cdot \frac{1}{n} \cdot \left(\frac{w}{\alpha A}\right)^{-\frac{1}{\alpha-1}}$ with $\alpha < 1$ as diminishing returns (?) (and $\alpha > 0$)

(*) Unlike in part i., this now depends ~~negatively~~ ~~positively~~ on productivity A. Increases in that will lead to ~~greater~~ more ~~fewer~~ [fewer [? I think] equilibrium hours worked.

v. The wage rate w is unchanged and equal to the MP_L, which is given by firms' production functions and not dependent on n. However, each worker's earnings wL will fall, since $L = nN$ and n has fallen (with N constant).

Output per head is still nY where Y is output per firm, given by their production function $Y = AN^{\alpha}$. Since A, α, w are unchanged, so too is N and hence Y. So output per head nY is smaller. And by the circular flow model, we know that this is equal to consumption per head, which must thus also be smaller.

vi. $\pi = AN^{\alpha} - wN$ where $N = \frac{L}{n}$ and $w = \alpha A N^{\alpha-1}$

so $\pi = A\left(\frac{L}{n}\right)^{\alpha} - w\frac{L}{n}$

$= A\left(\frac{L}{n}\right)^{\alpha} - \alpha A\left(\frac{L}{n}\right)^{\alpha-1}\left(\frac{L}{n}\right) = A(1-\alpha)\left(\frac{L}{n}\right)^{\alpha}$

[In long-run competitive equilibrium, free entry and exit drive profits π to 0?] ~~So, we expect to see~~ ~~$A\left(\frac{L}{n}\right)$~~

(*)

Assuming A and α are exogenous, if n falls, we might see L also fall ~~and~~ ~~n remain constant, or L remain constant and n fall~~ such that their ratio is unchanged, and profits stay at the same level?