General equilibrium

1.

1. $u^a(x_1^a, x_2^a) = 2\ln x_1^a + 3\ln x_2^a$; $u^b(x_1^b, x_2^b) = 2\ln x_1^b + \ln x_2^b$

$w_1^a = 20$, $w_2^a = 0$, $w_1^b = 0$, $w_2^b = 12$ where w_n^i is person i's endowment of good n.

a) Each consumer maximises their utility s.t. budget constraint. These are Cobb-Douglas preferences, so at the optimal bundle the consumer will spend fixed proportions of their income on each good. Specifically,

OK for our tutes. But in general,
- Set up full optimizatn prob.
- Argue it can be simplified
- Write the simpler problem.
- Solve.

$$x_1^a = \frac{2}{5} \cdot \frac{m_a}{p_1}, \quad x_2^a = \frac{3}{5} \cdot \frac{m_a}{p_2}, \quad x_1^b = \frac{2}{3} \cdot \frac{m_b}{p_1}, \quad x_2^b = \frac{1}{3} \cdot \frac{m_b}{p_2}$$

where p_n is the price of good n; m_i is the budget of person i.

But since m_i is simply the cash value of i's endowment, and as equilibrium allocations are homogeneous of degree zero in prices, we can suppose $p_2 = 1$ and thus $p_1 = p$, meaning that

$$m_a = 20p, \quad m_b = 12 \Rightarrow x_1^a = 8, \quad x_2^a = 12p, \quad x_1^b = \frac{8}{p}, \quad x_2^b = 4.$$

b) Aggregate excess demand for a good n := $\underbrace{\sum_{i=1}^{k}(x_n^i - w_n^i)}_{z^n}$ over all k people.

So
$$z^1 = (8 - 20) + (\tfrac{8}{p} - 0) = \tfrac{8}{p} - 12$$
$$z^2 = (12p - 0) + (4 - 12) = 12p - 8$$

c) Walras's law states that at any price ["vector"?] level, the sum of the values of excess demand is zero. $\sum_{n=1}^{N} p_n \cdot z^n = 0$.

This holds: $(\frac{8}{p} - 12) \cdot p + (12p - 8) = (8 - 12p) + (12p - 8) = 0$.

It is saying p_1 itself is meaningless. Only the rates at which one good is traded for another matter.
Eg. if you set $P_{apples} = 1$. Then the price of every other good is just telling you its unit value in apples.

Is this basically just "yeah, we don't need to check ⊛ the price is consistent?" by WL — or more than that?

a) At the Walrasian equilibrium, the excess demand for each good is zero.

This is saying that total value is preserved. What you have is a closed system.

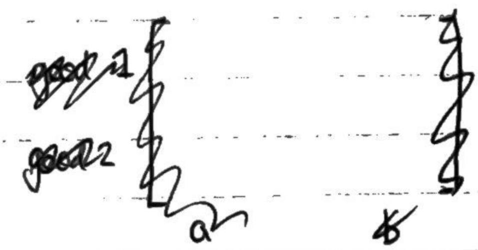
If $z^1 = \frac{8}{p} - 12 = 0$, then $p = \frac{2}{3}$.

Since the values of excess demands must sum to zero at this price level (like all others), we do not need to solve separately by WL for good 2. To verify though, if $z^2 = 12p - 8 = 0$, again $p = \frac{2}{3}$.

So, $x_1^a = 8$, $x_2^a = 8$, $x_1^b = 12$, $x_2^b = 4$

~~and after trade of endowments the allocations are~~ [this is what we mean by allocations, right?]

also, with eg. 3 goods do you need to work with p_1/p_2, p_2/p_3, etc? is this when it's actually useful?



3. $u_a(x_a, y_a) = x_a + \ln y_a$; $u_b(x_b, y_b) = x_b + \ln y_b$

a) Let $p_y / p_x =: p$, then normalising $p_x = 1$, $p_y = p$.

Consumer a is solving the problem

$$\max_{x_a, y_a} u_a(x_a, y_a) = x_a + \ln y_a \quad \text{s.t.} \quad x_a + p y_a \le m_a, \quad x_a \ge 0, \quad y_a \ge 0$$

where m_a is the cash value of their endowment, i.e. $m_a = 4$.

need to include +veity constraints? ⊛

Good

is this sort of argument ⊛ ok this year?

To be on the highest IC possible, they want to be at a point of tangency between an IC and their budget line (assuming an interior solution; if one of the demands is negative then a corner solution is best). This implies their MRS $= -p_x/p_y$, so

$-y_a = -1/p_y \Rightarrow y_a = 1/p$; $x_a = \frac{3}{p_x} = 3$

Good

but we can finesse it.
See in class —

For consumer b, with endowment value $4p$, their demand for y is the same, and the remainder is spent on x (as they have the same tastes) (demand ⊥ income here)

$$y_b = 1/p_y \; ; \qquad x_b = \frac{4p - 1}{p_x} = 4p - 1$$

The excess demands are $z^x = 2 - 4p$

$z^y = 4 - 2/p_y$

By WL, we need only to solve in one market, and at eq., $z^n = 0 \; \forall n$, so $p = 1/2$

The allocations for a are (3, 2) and b (1, 2).

b) Since they have identical tastes, we can simply consider this a new problem a) where we rename a to b and vice versa. So, the price ratio is still 1/2, and allocations are
a : (1, 2) b : (3, 2).

c) The optimal demands for y are unchanged, provided we have an interior solution. Assuming this, we'd get

$y_a = 1/p \; ; \quad x_a = p - 1 \; ; \quad y_b = 1/p \; ; \quad x_b = 4 + 3p - 1 = 3p + 3$

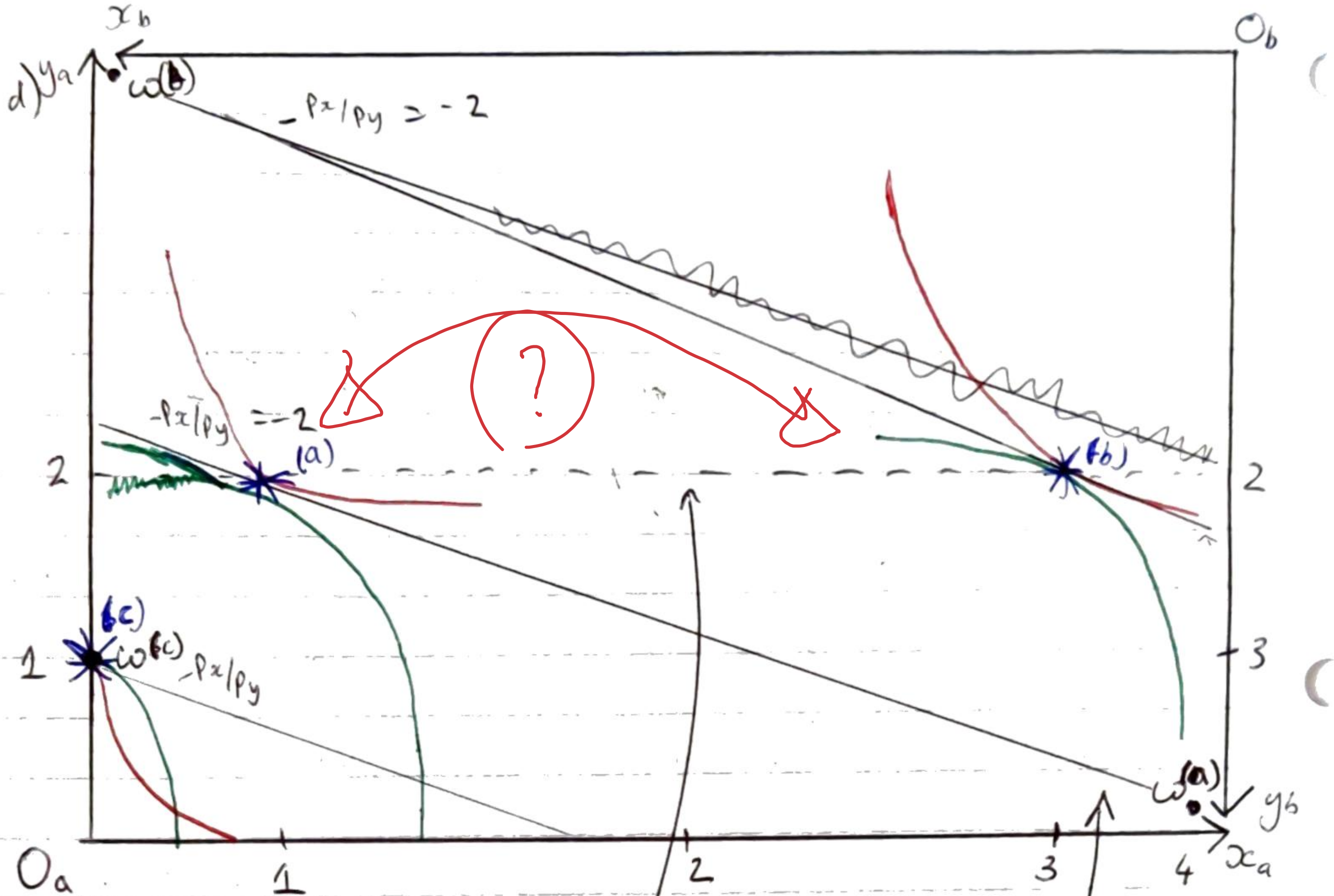
with excess demands $z^y = 4 - 2/p$

$z^x = 2 - 4p$

implying $p = 1/2$. But this would mean that a consumes $-1/2$ of good x, which is not possible. We have a corner solution.

I would leave prices out of this. Make the argument in terms of MRSs only.

Thus, the optimal consumption for a is to demand only good y, as they're not wealthy enough to satisfy their demand for it. There are thus no gains from trade, as a does not benefit (indeed, is strictly worse off) from exchanging any of their endowment of y for some x with b, as the MU 1\$ is greater for y.



~~(imagine the budget lines are a bit steeper so that *2 lies on it.)~~

because MRS depends only on y, the ICs are tangent everywhere (and only) along the line $y = 2$. Any endowment where the budget constraint can pass through here will have a Walrasian equilibrium with gains from trade, except for if they have equal endowments, since they're then indifferent to trade given identical tastes.

is it a "W. eq" in part c)'s corner soln? at what price?

~~When the endowment is in the top left or bottom right quadrant, no price level induces the consumer with < 2~~

when the endowment gives a consumer < 2 of y and 0 x, (i.e. on bottom half of y-axis), no price level can lead to trade, as the consumer would not give up a marginal unit of y for any amount of x. If they have < 2 y and ε x, they'd trade their ε x for y until they have 2 of it, potentially with a corner solution [what does this mean for e* price level? can y be arbitrarily expensive and they still trade x for it?]

5a) $u(t,l) = \ln t + \ln(1-l)$; $f(L,F) = L^{1/2} F^{1/2}$

a) The farmer spends $t \cdot p_t$ on consuming turnips, where $p_t = 1$. Their income comes from profits, rent, and wages.

(*) The firm's profits ~~[illegible]~~ are equal to 0, as the industry is competitive. [?!]

Not quite. They are zero because the Firm is a price taker with CRS production.

So, their income will be the wages they earn plus the rent they charge, which is simply $wl + r$. Thus $t \leq wl + r$.

b) The farmer is solving $\max_{t,l} \ln t + \ln(1-l)$ s.t. $t \leq wl + r$

Good

By monotonicity of preferences, this constraint is met with equality.

need to argue (*) SOCs on q-concavity?

Not necessary this year. We can talk about this in class. But yes here you have a concave objective with linear constraints.

Let $\mathcal{L}(t,l,\lambda) = \ln t + \ln(1-l) + \lambda(wl + r - t)$

to maximise $\mathcal{L}$, the FOCs are

$$\frac{\partial \mathcal{L}}{\partial t} = 0 \Rightarrow \frac{1}{t} - \lambda = 0 \Rightarrow \lambda = \frac{1}{t} \quad (I)$$

$$\frac{\partial \mathcal{L}}{\partial l} = 0 \Rightarrow -\frac{1}{1-l} + \lambda w = 0 \Rightarrow \lambda = \frac{1}{w(1-l)} \quad (II)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow t = wl + r. \quad (III)$$

by I and II, $w(1-l) = t$ and by III $= wl + r$.

so $l = \frac{w-r}{2w}$, $t = \frac{w+r}{2}$ and l is +ve iff $w > r$.

c) The cost function $C(F,L) = rF + wL$.

d) $\min_{F,L} rF + wL$ s.t. $f(L,F) = L^{1/2} F^{1/2} = \bar{y}$

[Minimising $f(x)$ is ~~equal~~ $\equiv$ to maximising $-f(x)$.]

Let

$$\mathcal{L}(F, L, \lambda) = rF + wL + \lambda(\bar{y} - F^{1/2}L^{1/2})$$

for which the FOCs are

again, argue ⊛ SOCs and q-convex (?) ?

$$\frac{\partial \mathcal{L}}{\partial F} = 0 \Rightarrow r = \frac{1}{2}\lambda F^{-1/2} L^{1/2} \quad (I)$$

$$\frac{\partial \mathcal{L}}{\partial L} = 0 \Rightarrow w = \frac{1}{2}\lambda F^{1/2} L^{-1/2} \quad (II)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow F^{1/2} L^{1/2} = \bar{y} \quad (III)$$

by (I), $\frac{\lambda}{2} = wL^{1/2}F^{-1/2}$; by (II) $\frac{\lambda}{2} = rF^{1/2}L^{-1/2}$

$$\text{so } wL^{1/2}F^{-1/2} = rF^{1/2}L^{-1/2} \Rightarrow wL = rF$$

As expected, since this is Cobb-Douglas with equal weights, they spend the same amount on fields and labour.

by (III), $F = y^2/L$; $= \frac{y^2 \cdot w}{rF}$ so $F = y \cdot \sqrt{\frac{w}{r}}$

and by symmetry, $L = y \cdot \sqrt{\frac{r}{w}}$

λ is marginal cost, i.e. the amount by which the optimal value of $C(F,L)$ rises when you increase the production constraint by a marginal unit.

⊛ This is because (a) $\frac{\partial \mathcal{L}}{\partial \lambda} = 0$ (FOC) meaning $\mathcal{L}^*(F,L,\lambda) = C^*(F,L)$ and (b) $\frac{\partial \mathcal{L}^*}{\partial \bar{y}} = \lambda^*$, ~~and by the envelope theorem~~ which using the envelope theorem means that

$$\frac{d\mathcal{L}^*}{d\bar{y}} = \frac{\partial \mathcal{L}^*}{\partial \bar{y}} = \lambda^*.$$

e) ~~At a W eq. all excess demands are zero.~~

~~Given the turnip market~~ is competitive, the firm profits

$\cancel{\pi = t - wL - rF = 0.}$

Using the solution to the cost-min problem,

$$\max_t \pi(t) = t - w\cdot t\sqrt{\frac{r}{w}} - r\cdot t\sqrt{\frac{w}{r}}$$

$$= t(1-2\sqrt{rw})$$

for which the FOC is $\frac{\partial \pi}{\partial t} = 0 \Rightarrow \sqrt{rw} = \frac{1}{2}$

so $rw = \frac{1}{4}$ as $r>0,\ w>0$

which, as expected in a competitive industry, means that the profits are 0.

is this right? seems like a bit too soon to assert this. (*)

Since the factor markets are competitive, the rental rate for fields = MRP_F, and there is no surplus supply. Hence, $F=1$.

Not sure I follow your point...

This means $t\cdot\sqrt{\frac{w}{r}} = 1 \Rightarrow t = \sqrt{\frac{r}{w}}$, and $L = \sqrt{\frac{r}{w}}\cdot\sqrt{\frac{r}{w}} = \frac{r}{w}$

In a W eq*, all markets clear and excess demands are zero.

So $z^F = 0 \Rightarrow 1-F = 0 \Rightarrow F=1$ as above.

$z^L = 0 \Rightarrow$ ~~and so turnip production F(1,1)=1.~~

$z^L = \frac{w-r}{2w} - \frac{r}{w}$

$\Rightarrow w = 3r$

so $r = \sqrt{\frac{1}{12}} = \frac{\sqrt{3}}{6}$

$w = \frac{\sqrt{3}}{2}$

$L = \frac{1}{3}$

$z^t = 0 \Rightarrow$ by WL, we only need to solve k-1 markets as the sum of values of excess demands is zero, but here the supply and demand of turnips is $\frac{\sqrt{3}}{3} = \frac{1}{3}^{1/2}\cdot 1 = \frac{\sqrt{3}/6 + \sqrt{3}/2}{2}$

$\Rightarrow F(\frac{1}{3}, 1) = \frac{\sqrt{3}}{3}$

Nice

Doing national income accounting, Σ expenditure = Σ value of output = Σ income and profits $= \sqrt{3}/3$.