Week 8

1a) The truth clause for $\Box\!\!\rightarrow$ is defined as

(I) $V_M(\phi \Box\!\!\rightarrow \psi, w) = 1$ iff $V_M(\psi, u) = 1$ for every ϕ-world u maximally close to w,

This is not exactly what the question is asking for. You need to define a function!

or equivalently

(II) $V_M(\phi \Box\!\!\rightarrow \psi, w) = 1$ iff $V_M(\psi, u) = 1$ in the closest ϕ-world u to w, or $V_M(\phi, u) = 0$ for every $u \in W$

(since on SC, limit implies there is a unique closest ϕ-world if there are any ϕ-worlds).

But note that we can easily reformulate the first disjunct in (I) to pick out u from our selection f, where $f(\phi, w)$ returns the closest ϕ-world to w and is undefined otherwise.

b) Suppose for # that $\nvDash_{SC}$ the wff. Then there exists some assignment g in some world w in some model M s.t.

(i) $V_{M,g}(\phi \Box\!\!\rightarrow (\psi \vee \chi), w) = 1$ and (ii) $V_{M,g}(\phi \Box\!\!\rightarrow \psi \vee \phi \Box\!\!\rightarrow \chi, w) = 0$

From (ii), $V(\phi \Box\!\!\rightarrow \psi, w) = 0$ and $V(\phi \Box\!\!\rightarrow \chi, w) = 0$.

So there must be at least one ϕ-world, ~~and~~ but no ϕ-worlds in which ψ holds or χ holds, by semantics of $\Box\!\!\rightarrow$.

But (i) requires either that there are no ϕ-worlds (which is not the case) or that in the maximally-close ϕ-world, $\psi \vee \chi$ holds. Since there are no ϕ-worlds in which ~~ψ~~ ψ holds or χ holds, $\psi \vee \chi$ certainly doesn't hold in the closest ϕ-world. So this is #.

[Alternatively you could do a ~~constructive~~ proof by cases on the existence of ϕ-worlds.]

c) In LC, there need not be a single closest ϕ-world. So consider the following counterexample model:

I recommend drawing them out in questions like these!

~~There is~~ Take some w, g such that there is a countably infinite number of ϕ-worlds $u_1, u_2, \ldots$ ~~increasing~~ in closeness to ~~from~~ w.
Let $V_g(\psi, u_n) = 1$ iff n is odd
and $V_g(\chi, u_n) = 1$ iff n is even

Also, the problem should usually be solvable without infinitely many models!

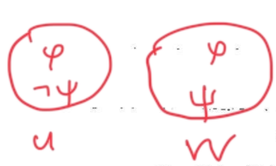
Then although there exists a world u s.t. for all $u' \lesssim_w u$, $LV(\phi \rightarrow \cancel{\psi\chi}, u') = 1$ ~~and~~ (e.g., u_ϕ),

there is no ~~a~~ analagous u for ~~both~~ $\phi \rightarrow \psi_n$ or $\phi \rightarrow \chi$.
So the antecedent holds at w but the consequent is false.

2a) i. Suppose $V(\phi, w) = 0$, $V(\phi, u) = 1$, ~~and~~ $V(\psi, w) = 1$, $V(\psi, u) = 0$.
(with $W = \{u, w\}$).

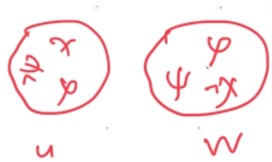
Then $V(\phi \Box\!\!\rightarrow \psi, w) = 0$ even though $V(\psi, w) = 1$, since ψ is false at the closest ϕ-world, u.

ii. Consider $W = \{u, w\}$, $V(\phi, u) = V(\phi, w) = 1$,
$V(\psi, w) = 1$, $V(\psi, u) = 0$.



Then since ϕ, ψ are true at w, $V(\phi \Box\!\!\rightarrow \psi, w) = 1$.
But at u, the closest $\sim\psi$-world, ϕ holds, so
$V(\sim\psi \Box\!\!\rightarrow \sim\phi, w) = 0$.

iii. $W = \{u, w\}$, $V(\phi, w) = V(\psi, w) = 1$
$V(\chi, w) = 0$, ~~[illegible]~~



$V(\chi, u) = V(\phi, u) = 1$, $V(\psi, u) = 0$.

~~[illegible]~~ w is the closest ϕ-world to itself, and ψ holds, but ψ is false at u, the closest $\phi \wedge \chi$-world

iv. Consider $W = \{u, v, w\}$ ~~where~~ and $v \leq_w u$.

Let $V(\phi \wedge \psi, u) = 1 = V(\chi, u)$

$V(\phi, w) = 1$, $V(\psi, w) = 0$

$V(\psi, v) = 1$, $V(\chi, v) = 0$

At w, the closest $\phi \wedge \psi$-world is u, where χ holds. The closest ϕ world is w, so we evaluate $\psi \Box\!\!\rightarrow \chi$ there to see if the consequent holds.

The closest ψ-world to w is v, but χ is false there, so the inference is invalid.

b) In each case, SC's invalidation of the inference makes ~~it~~ $\Box\!\!\rightarrow$ a better symbolisation of the English counterfactual conditional.

i. 'It rained yesterday' doesn't imply 'If it were clear yesterday, it would've rained yesterday'

I suppose. But not. language is tricky: doesn't "were clear" mean "¬(rained)"?

ii. ~~'If it'd rained heavily, it would've rained' doesn't imply~~ ~~'If it~~ 'If it'd rained, it wouldn't have rained heavily' doesn't imply 'If it'd rained heavily, it wouldn't have rained'

iii. 'If it'd rained, you'd be wet' doesn't imply 'If it'd rained and you were indoors, you'd be wet'.

(however, note that we could object to this as a counterexample, by saying that the first sentence involves an implicit assumption that you weren't ~~outdoors~~ indoors) ~~and is so false otherwise~~

Quite true! All the implicit assumptions in natural language expressions.

iv. 'If it had rained and you went for a walk, you'd have ~~got~~ wet'

I think (iv) is not like the others, and $\varphi \wedge \psi \,\Box\!\!\rightarrow \chi \vDash \varphi \,\Box\!\!\rightarrow (\psi \,\Box\!\!\rightarrow \chi)$ actually is valid in English!

doesn't imply 'If it'd rained, then if you'd gone for a walk you'd have got wet', because in the consequent it's more natural to expect you'd have taken an umbrella. (but again, same objection as iii)

3a) (I) $P \,\Box\!\!\rightarrow (Q \vee R)$ P: flip coin Q: heads R: tails

(II) $\neg$ ~~□~~ $(P \,\Box\!\!\rightarrow Q)$? ← Seems strange to not have a □ to symbolise 'definitely', but I think we need to leave it out for validity.

$\vDash_{LC} P \,\Box\!\!\rightarrow R$

Hmm!

Take some arbitrary world $w \in W$ in any model under any assignment. From (I), either there's no P-world u (in which case the conclusion follows trivially) or in the closest P-world to w, $Q \vee R$ holds.
From (II), in ~~that~~ same closest P-world, u, ~~it's not true~~ Q doesn't hold.
So R must hold in u.

b) Much like 1(c), since LC doesn't require a unique, identifiable closest world, we can have a sequence of ever-closer-to-w P-worlds, u_n where in odd-indexed ones Q holds and ~~in~~ the rest R holds. Then (I) holds but both $\neg(P \,\Box\!\!\rightarrow Q)$ and $\neg(P \,\Box\!\!\rightarrow R)$ are true.

Try to come up with a 3-world model that does the trick!

c) No, the conclusion doesn't intuitively follow. It seems genuinely open whether, had you flipped the coin, H or T would've come up. We're uncertain. (It does seem like this would be a good place to use the ◇ operator, however.)

Still, this gives us a reason to prefer LC over SC.