Week 1 logic

1a) i. ($\Rightarrow$) Assume $V_I(\sim\phi \to \psi) = 1$.

Then $V_I(\sim\phi) = 0$ or $V_I(\psi) = 1$, by def.

Case 1	Case 2
$V_I(\sim\phi) = 0$	$V_I(\psi) = 1$
$\therefore V_I(\phi) = 1$	Then $V_I(\phi) = 1$ or $V_I(\psi) = 1$
Then $V_I(\phi) = 1$ or $V_I(\psi) = 1$	

No need to explicitly appeal to the semantic definitions; can just use them, as done here. ✓

Either way, if $V_I(\sim\phi \to \psi) = 1$ then $V_I(\phi) = 1$ or $V_I(\psi) = 1$

($\Leftarrow$) Assume $V(\phi) = 1$ or $V(\psi) = 1$

Case 1	Case 2
$V(\phi) = 1$	$V(\psi) = 1$
$\therefore V(\sim\phi) = 0$	
$\therefore V(\sim\phi \to \psi) = 1$	$\therefore V(\sim\phi \to \psi) = 1$

Either way, if $V(\phi) = 1$ or $V(\psi) = 1$, $V(\sim\phi \to \psi) = 1$. $\square$

ii. $V(\sim(\phi \to \sim\psi)) = 1$

$\Leftrightarrow$ $V(\phi \to \sim\psi) = 0$

$\Leftrightarrow$ $V(\phi) = 1$ and $V(\sim\psi) = 0$ ✓ As done here,

$\Leftrightarrow$ $V(\phi) = 1$ and $V(\psi) = 1$ $\square$ be careful to not use object-level symbols ('$\rightarrow$' and '$\leftrightarrow$') in the metalanguage: use '$\Rightarrow$' and '$\Leftrightarrow$' instead.

iii. $V(\sim((\phi \to \psi) \to \sim(\psi \to \phi))) = 1$

$\Leftrightarrow$ $V((\phi \to \psi) \to \sim(\psi \to \phi)) = 0$

$\Leftrightarrow$ $V(\phi \to \psi) = 1$ and $V(\sim(\psi \to \phi)) = 0$

$\Leftrightarrow$ $V(\phi \to \psi) = 1$ and $V(\psi \to \phi) = 1$

($\Rightarrow$) $[V(\phi) = 0$ or $V(\psi) = 1]$ and $[V(\psi) = 0$ or $V(\phi) = 1]$ by def.

$\therefore V(\phi) = V(\psi)$, as ~~otherwise the above is~~ not satisfied.

($\Leftarrow$) Let $V(\phi) = V(\psi)$.

Case 1

$v(\phi) = v(\psi) = 1$

$\therefore v(\phi \to \psi) = 1$

and $v(\psi \to \phi) = 1$

Case 2

$v(\phi) = v(\psi) = 0$

$\therefore v(\phi \to \psi) = 1$

and $v(\psi \to \phi) = 1$

So either way $v(\psi \to \phi) = 1$ and $v(\phi \to \psi) = 1$, so you can continue back along the proof. □

✓

b) i. ~~This follows from the truth tables~~

i. This is simply given in the definition of PL-valuations, and is presented the same in Halbach.

ii. Similarly, given by definition and the same as Halbach.

iii-v. The main difference here is that Halbach presents $\wedge$, $\vee$, and $\leftrightarrow$ as primitive connectives, whereas Sider defines them in terms of $\to$ and $\sim$, and we can show that those definitions are equivalent to the direct truth conditions given here and in Halbach.

good

iii. $v(\phi \vee \psi) = 1$ is an abbreviation of $v(\sim\phi \to \psi) = 1$

$\Leftrightarrow$ ~~which~~ means ~~that~~ so $v(\sim\phi) = 0$ or $v(\psi) = 1$, ~~true~~.

$\Leftrightarrow$ $v(\phi) = 1$ or $v(\psi) = 1$ as shown in (a)

✓

iv. Proof given in Sider, $v(\phi \wedge \psi) = 1$ abbreviates $v(\sim(\phi \to \sim\psi)) = 1$

$\Leftrightarrow$ $v(\phi \to \sim\psi) = 0$

$\Leftrightarrow$ $v(\phi) = 1$ and $v(\sim\psi) = 0$

$\Leftrightarrow$ $v(\phi) = 1$ and $v(\psi) = 1$ as shown in (a)

v. $V(\phi \leftrightarrow \psi) = 1$ abbreviates $V((\phi \to \psi) \wedge (\psi \to \phi))$, which abbreviates $v(\sim((\phi \to \psi) \to \sim(\psi \to \phi))) = 1$.

As shown in (a), this holds iff $v(\phi) = v(\psi)$.

2 a) Suppose, for #, that $V_I(\phi \to (\psi \to \phi)) = 0$, where I is any arbitrary PL-interpretation.

$\therefore V_I(\phi) = 1$ and $V_I(\psi \to \phi) = 0$

✓ $\therefore V_I(\phi) = 1$ and $V_I(\psi) = 1$ and $V_I(\phi) = 0$ #

(*) will a proof of validity necessarily be a reductio?

It's usually most convenient to do it that way, but there are always different approaches: any reductio proof can be recast as a non-reductio one with sufficient ingenuity (and vice versa).

b) Let I be any PL-interpretation. Suppose, for #, that

$V_I(\phi \to (\psi \to \chi)) \to ((\phi \to \psi) \to (\phi \to \chi))) = 0$

$\therefore V_I(\phi \to (\psi \to \chi)) = 1$ and $V_I((\phi \to \psi) \to (\phi \to \chi)) = 0$

$\therefore V_I(\phi) = 0$ or $V_I(\psi \to \chi) = 1$ and $V_I(\phi \to \psi) = 1$ and $V_I(\phi \to \chi) = 0$

$\therefore V_I(\phi) = 0$ or $V_I(\psi) = 0$ or $V_I(\chi) = 1$ and $V_I(\phi) = 0$ or $V_I(\psi) = 1$ and $V_I(\phi) = 1$ and $V_I(\chi) = 0$

Be a bit clearer here - I think this is a place where a few linking words would help.

$\therefore$ " and $V_I(\psi) = 1$ and $V(\phi) = 1$ and $V(\chi) = 0$

✓ #

c) ~~Suppose~~ Let I be any PL-interp. Suppose, for #, that

$V_I((\sim\psi \to \sim\phi) \to ((\sim\psi \to \phi) \to \psi)) = 0$

$\therefore V_I(\sim\psi \to \sim\phi) = 1$ and $V_I((\sim\psi \to \phi) \to \psi) = 0$

$\therefore V_I(\sim\psi) = 0$ or $V_I(\sim\phi) = 1$ and $V_I(\sim\psi \to \phi) = 1$ and $V_I(\psi) = 0$

$\therefore V_I(\psi) = 1$ or $V_I(\phi) = 0$ (*) and $V_I(\sim\psi) = 0$ or $V_I(\phi) = 1$ (#) and $V_I(\psi) = 0$ (+)

✓ $\therefore V_I(\psi) = 0$ and $V_I(\phi) = 0$, from (*) with (+)

but this is # with (#)

d) Let I be some PL-interp where $V_I(\phi) = 1$ and $V_I(\phi \to \psi) = 1$.

~~Suppose, for #, $V_I(\psi) = 0$.~~

Then $V_I(\phi) = 0$ or $V_I(\psi) = 1$, by def $\to$

But $V_I(\phi) = 1 \neq 0$, so $V_I(\psi) = 1$ ∎

~~Sol~~ Let $D_K = \{1\}$ be the set of designated values in Kleene; similarly $D_Ł = \{1\}$ and $D_{LP} = \{1, \#\}$ for Lukasiewicz and Logic of paradox respectively.

a) i. Assume $KV(P) \in D_K$ and $KV(P \to Q) \in D_K$. Then

$KV(P) = 1$ and $KV(P) = 0$ or $KV(Q) = 1$

Put $KV(P) = 1 \neq 0$ so $KV(Q) = 1 \in D_K$, so the
i. the claim is true.

ii.

P	Q	P → Q	(P → Q) → Q	P → ((P → Q) → Q)
1	1			1 1 1 1 1 1 1
1	#			1 # 1 # # # #
1	0			1 1 1 0 0 1 0
#	1			# 1 # 1 1 1 1
#	#			# # # # # # #
#	0			# # # # 0 # 0
0	1			0 1 0 1 1 1 1
0	#			0 # 0 1 # # #
0	0			0 1 0 1 0 0 0

No, not true - doesn't always take a designated value.
[hmm, something about "no tautologies in Kleene?"] good

iii.

P	~P	P ∧ ~P
1	0	0
#	#	#
0	1	0

Vacuously, there are no K-interpretations where the premisses both take designated values.

So, the claim is true because ~~it is never the case~~ ~~that~~ there does not exist any I s.t. $KV_I(P) \in D_K$, $KV_I(\sim P) \in D_K$, and $KV_I(Q) \notin D_K$.

iv. Not true.

P	Q	(P ∧ ~P)	→ Q
1		1 0 0 1	1
#		# # # #	
0		0 0 1 0	1

designated

⇐ ~~true~~ only if $KV_I(Q) = 1$, so not 0-valid ~~under all the interpretations~~

5b). i. Assume $LV(P) \in D_L$ and $LV(P \to Q) \in D_L$

Then $LV(P) = 1$ and $LV(P) = 0$ or $LV(Q) = 1$ or $LV(P) = LV(Q) = \#$

But $LV(P) = 1 \neq 0 \neq \#$ so $LV(Q) = 1$ ✓

$\therefore$ the claim is true.

ii.

P	Q	P	$\to$	$((P$	$\to$	$Q)$	$\to$	$Q)$
1	1	1	1	1	1	1	1	1
1	#	1	1	1	#	#	1	#
1	0	1	1	1	0	0	1	0
#	1	#	1	#	1	1	1	1
#	#	#	1	#	1	#	#	#
#	0	#	1	#	#	0	#	0
0	1	0	1	0	1	1	1	1
0	#	0	1	0	1	#	#	#
0	0	0	1	0	1	0	0	0

✓

$\therefore$ Claim true as valid under all L-interpretations.

iii. True as in (a). ✓

iv.

P	$(P$	$\wedge$	$\sim$	$P)$	$\to$	Q
1	1	0	0	1	1	
#	#	#	#	#		
0	0	0	1	0	1	

$\in$ ~~True~~ Designated only if $LV_I(Q) \neq 0$ so formula is not D-valid. ✓

$\therefore$ claim not true.

c) i Assume ~~LP~~ $PV_I(P) \in D_{LP}$ and $PV_I(P \to Q) \in D_{LP}$

~~Then $PV(P) = 1$ or #~~ ~~and $PV(P) = 0$ or $PV(Q) = 1$~~ ~~or $PV(P)$~~

Suppose that $PV_I(Q) \notin D_{LP}$ ✓

$\therefore PV_I(Q) = 0$

We could have I s.t. $PV_I(P) = \#$, and then $PV_I(P \to Q) = \# \in D_{LP}$

So this claim is not true, as Q is not a D-consequence of P, $P \to Q$.

ii.

P	Q	P →	((P →	Q) →	Q)
1	1	1	1	1	1
1	#	1	1	#	#
1	0	1	1	0	0
#	1	#	#	1	1
#	#	#	#	#	#
#	0	#	#	0	0
0	1	0	0	1	1
0	#	0	0	#	#
0	0	0	0	0	0

Truth-table identical to in (a), but now the formula always takes a designated value. So, the claim is true under LP.

✓

iii. Not true, as ~~if~~ under ~~the~~ an interpretation I s.t. $PV_I(P) = \#$, the premises take a designated value but the conclusion Q may not do so.

✓

iv. True. Same truth table as in (a), but now the formula always takes a designated value since if $PV(\phi) = 0$ or $\#$ then $PV(\phi \to \psi) = 1$ or $\#$.

✓

7.a) (I). A zero-year-old ~~seems~~ young; a centenerian ~~seems~~ old

(II). If someone ~~is~~ a year older than you is ~~old~~ young, ~~then~~ ~~seems like~~ surely you ~~should be~~ are young too

(III). Similarly, if you're ~~young~~ old then ~~it seems~~ surely ~~the~~ someone ~~just~~ one year older ~~should also be~~ still ~~young~~ is also old.

Good

So, it fits with our ~~intuitions~~ common sense about the statements P_n.

b) By ~~hypothesis~~ (I), $I(P_0) = 1$ and $I(P_{100}) = 0$.

From (II), if any value $I(P_n)$ is 1 then all previous values must also be 1. And from (III), if any value $I(P_n)$ is 0 all subsequent must be 0. So we must have only 1s until the final 1, then some weakly +ve number of #s, then only 0s from the first 0. This satisfies the statement that the values must be weakly monotonic decreasing

A bit informal but ok

c) i. α does not hold; you can do modus ponens all along the chain to reach the conclusion.
β holds, since to obtain a faithful interpretation I we require $I(P_0) \neq I(P_{100})$, but if all premisses are true then (since the conclusion is entailed by the premisses), ~~[illegible]~~ $I(P_{100})$ would be $1 = I(P_0)$

ii. α, again, does not hold.
β does not hold. We could have a faithful interpretation where $I(P_1) = \dots = I(P_{99}) = \#$, ~~and~~ such that ~~not~~ premiss is false but all the material conditionals are # (i.e. not designated) and thus we can maintain $I(P_{100}) = 0$ even despite the falsity of α.

iii. α holds. As noted in ii, we could let $I(P_0) = 1$ and s.t. $I(P_n \to P_{n+1}) = \#$ for all n, which would make every premiss designated under LP. But this would not semantically entail the conclusion because you can't do modus ponens with conditionals whose truth-value is #.

[is this right? how do you explain it properly?]

β does not hold, because we can have a faithful interpretation as described above where no premiss is false (and indeed ~~[illegible]~~ under LP, every premiss would be designated; it is just that the conclusion doesn't follow from the premisses that allows us to have a faithful interp.)

d) This seems to undermine the principle of modus ponens, by claiming that we cannot follow a chain of implications to reach a true conclusion from a true ~~[illegible]~~ antecedent.

Good

e) This just doesn't seem to fit with our intuitions about age. ~~[illegible]~~ It seems like it ought to be the case that a faithful interpretation exists without any premiss being false, as they all seem reasonable claims. However, I think it's actually not so bad to uphold β – I don't fully trust our intuitions here, and ~~think~~ there is also something a little suspicious-seeming about all the premisses, though needing to ~~drawing~~ a sharp boundary between "old" and "young" is undesirable.

~~f)~~

Δ here is a 'definitely' operator [f) [Didn't understand what was being asked here]

g) In this case of vagueness, at least, Kleene's logic seems to have an advantage over both PL and LP. Classical logic forces us to impose a binary value onto the question of whether someone is young when it's a vague, continuous property; the logic of paradox forces us to give up on modus ponens, which is philosophically unsatisfying in this context.

Induction on complexity of formulae: define a mapping $cp(\phi)$ from the set of all formulae into ~~the~~ $\mathbb{N}$ such that $cp(\phi)$ is the number of connectives in ϕ.

3. ~~The~~ Inductive hypothesis: For all ϕ with $cp(\phi) < n$, if ϕ is s.t. no sentence letter occurs more than once, then there exists I_1, I_2 s.t. $V_{I_1}(\phi) = 1$, ~~or~~ $V_{I_2}(\phi) = 0$.

Base case: $cp(\phi) = 0$ so ϕ is atomic. Then let I_1 be s.t. $V_{I_1}(\phi) = 1$ and I_2 be s.t. $V_{I_2}(\phi) = 0$.

Inductive case: $cp(\phi) > 0$.

either ϕ is $\neg\psi$, and proceed from there

or ϕ is $\psi_1 \to \psi_2$, and proceed from there

to show that given the inductive hypothesis, ~~φ~~ ϕ ~~no~~ satisfies the relevant property.

So for all n: for all $m < n$, if every ϕ s.t. $cp(\phi) = m$ ~~then~~ and ϕ satisfies the property then every ϕ' s.t. $cp(\phi') = n$ also satisfies the property.