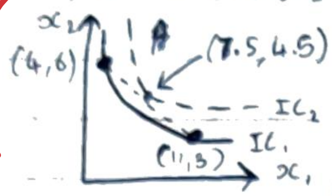
Welfare sheet

Outstanding

3. A_1 : a: (11, 3) b: (4,6) A_2 : a: (5, 3) b (10, 6)
u (x, y) for both a and b

a) Well-behaved preferences are continuous, convex, and monotonic increasing.
This means the ICs are continuous, convex, and downward-sloping.
The consumers are indifferent between the bundles (11, 3) and (4, 6).

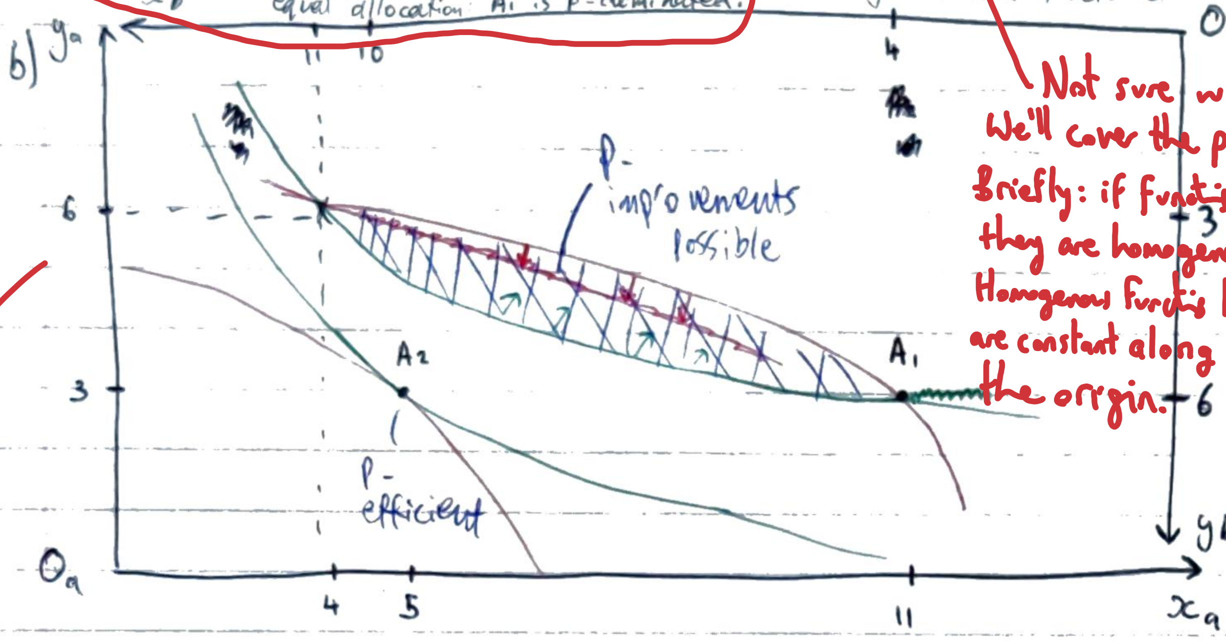


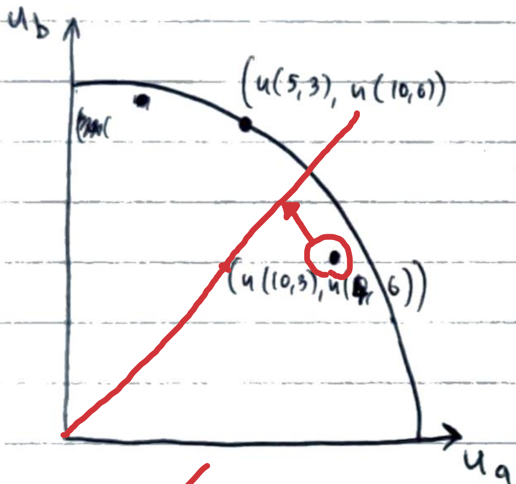
(7.5, 4.5) is feasible and on a higher IC, provided prefs are strictly convex. So both are better off with that equal allocation. A_1 is P-dominated.

⊕ If perf subs isn't A_1 P-efficient? Or we ~~can~~ somehow show they're not p-subs?)
[confused by argument on p 533 of MWG about Edgeworth box diagonal and Pareto set (for CRS / perf (subs)]

Not sure what this is. We'll cover the point in class. Briefly: if functs are CRS, they are homogeneous of degree 1. Homogeneous functs have MRS, that are constant along rays from the origin.

b)





The UPF is downward-sloping because, given the fixed amount of resources in the economy, at a Pareto-efficient point — by definition — an increase in a's utility must lead to a decrease in b's utility. And the UPF is simply the set of P-efficient ~~points~~ allocations

utilities associated with

c) i. ~~Yes~~ Although A_2 is P-efficient and A_1 is not, this does not mean that A_2 P-dominates A_1. Indeed, by inspection it doesn't, since a is strictly worse off, as they have less of x and prefs are monotonic (with no more y). So, they aren't comparable using the Pareto criterion.

ii. A_1 is better under a maximin SWF. Person a in A_2 is worse-off than the worst-off person in A_1 (which is either person, as they're equally well-off), ~~for~~ for the reasons above. So we can say that A_2 is worse according to maximin.

Great

iii. Insufficient information as we need cardinal information to calculate whether the losses to a are outweighed by gains for b in moving from $A_1 \to A_2$ under a utilitarian SWF.

d). Advantages

- Feasible with only ordinal info, whereas utilit. needs cardinal (Rawlsian also feasible with ordinal, though)
- Helps pick out allocative improvements that nobody would object to - seems intuitive and uncontroversial

Disadvantages

- It's a very weak criterion. Many points are Pareto-efficient, so how does the policymaker choose ~~bet~~ between them?
- Says nothing about inequality, or how to make tradeoffs between individuals - even making someone fractionally worse-off to benefit everyone else hugely. A SWF is required to answer these harder Qs.

The PPF is the set of Pareto efficient production output bundles. The max amount of y produced (for a given x) is $(48-x)^{1/2}$. So $\partial y/\partial x = -\frac{1}{2}(48-x)^{-0.5}$. ~~optimum~~

4. $u(x,y) = \ln x + \ln y \qquad \omega = (48, 0). \qquad Y = X^{1/2}$

a) MRS is the slope of the indifference curves: $= -\frac{MU_x}{MU_y}$

$$= -\frac{\partial u/\partial x}{\partial u/\partial y} = -\frac{1/x}{1/y} = -y/x$$

Good

MRT is the slope of the PPF ~~isoquants~~

b) ~~The consumer tries to maximise utility subject to a budget constraint, ie $\max_{x,y} \ln x + \ln y$ s.t.~~

An allocation is efficient iff there is no other allocation where every individual is no worse off and at least one individual is strictly better off. In this economy, it needs to be the case that no other allocation makes the consumer strictly better off. Since there's only one person, the social welfare function is just their utility. So, we want to maximise their SWF subject to the PPF, which occurs when the

MRS (here, slope of SWF) = slope of PPF (the MRT).

$$-y/x = -\frac{1}{2}(48-x)^{-0.5}$$

$$2y = x(48-x)^{-0.5}$$

Plugging in $(32, 4)$,

$$2\times4 = 8 = 32(48-32)^{-0.5} = \frac{32}{4}$$

So it is efficient, by the tangency condition.

c) A competitive equilibrium is a price vector and allocation s.t.
- each agent is behaving optimally given the prices
- all markets clear

The consumer solves the problem

$$\max_{x,y} \ln x + \ln y \quad \text{s.t.} \quad p_x x + p_y y \leq m, \ x \geq 0, \ y \geq 0$$

where m is their income.

By monotonicity of prefences, the BC binds. Since $MU_x (MU_y) \to \infty$ as $x(y) \to 0$, the consumer buys strictly +ve amounts of each.

Beautiful

So, we know that there is an interior solution for these Cobb-Douglas prefs. where $MRS = -p_x/p_y$ i.e. $-y/x = -p_x/p_y$. And $y = 4$, $x = 32$ if markets clear, so $P_x / P_y = 1/8$.

The firm's profits $\pi(y) = P_y \cdot y - C(y)$, where the cost function can be derived from the production function following solving the cost-min problem.

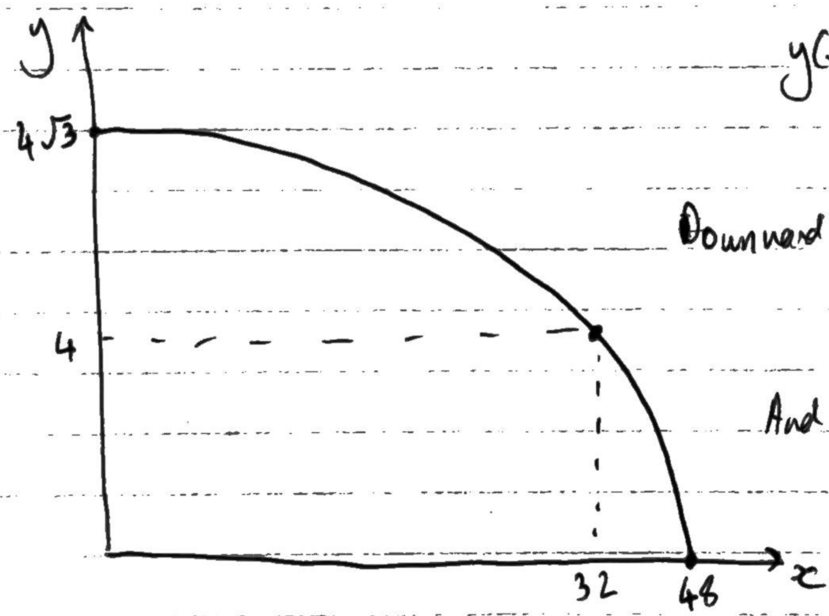
The cost of producing 4 units of y is equal to the value of the inputs required, which is $y^2 = 16$ of x. ~~The valu~~

OK

So $\pi(4) = 8 \cdot 4 - 16 = 16$, by normalising $P_x = 1$. ~~by [illegible]~~ as allocations are HDO in prices.

The consumer spends $32 \times 1 + 8 \times 4 = 64$ on consumption. They have an endowment worth 48×1, and earn profits of 16, so the BC binds as expected: $48 + 16 = 64$

d)



$y(x) = (48-x)^{1/2}$ as above.

Downward-sloping as $\frac{\partial y}{\partial x} = -\frac{1}{2}(48-x)^{-0.5} < 0$ for $0 \leq x \leq 48$

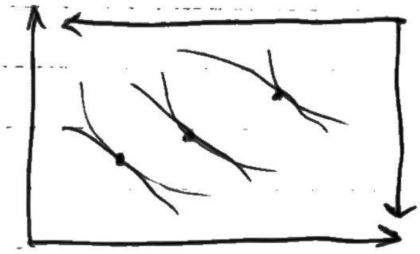
And concave as

$$\frac{\partial^2 y}{\partial x^2} = -\frac{1}{4}(48-x)^{-1.5} < 0$$

for $0 \leq x < 48$.

5a) $u^a(x_1^a, x_2^a) = 2\ln x_1^a + 3\ln x_2^a \qquad u^b(x_1^b, x_2^b) = 2\ln x_1^b + \ln x_2^b$

a). $MRS^a := -\frac{MU_1^a}{MU_2^a} = -\frac{2/x_1^a}{3/x_2^a} = -\frac{2}{3}\frac{x_2^a}{x_1^a}$



$MRS^b := -\frac{MU_1^b}{MU_2^b} = -\frac{2/x_1^b}{1/x_2^b} = -2\frac{x_2^b}{x_1^b}$

Yes, good

We have a Pareto-efficient allocation whenever $MRS^a = MRS^b$, and ~~by~~ for feasibility we require

$x_1^a + x_1^b = 20 \quad ; \quad x_2^a + x_2^b = 12$

So $x_1^a = 20 - x_1^b, \quad x_2^a = 12 - x_2^b$

~~$2\frac{x_2^b}{x_1^b} = \frac{2}{3}\frac{20 - x_1^b}{12 - x_2^b}$~~

~~$20x_2^b - 3x_1^b x_2^b = \frac{2}{3}(12 - x_2^b)x_1^b$~~

~~$x_2^b =$~~

~~The set of Pareto-efficient allocations~~

~~$2\frac{x_2^b}{x_1^b} = \frac{2}{3}\frac{x_2^a}{x_1^a} \Rightarrow 3x_1^a x_2^b = x_2^a x_1^b$~~

$2\frac{x_2^b}{x_1^b} = \frac{2}{3}\frac{x_2^a}{x_1^a} \Rightarrow 3x_1^a x_2^b = x_2^a x_1^b$

$\therefore \; 3(20 - x_1^b)x_2^b = (12 - x_2^b)x_1^b$

$60x_2^b - 3x_1^b x_2^b = 12x_1^b - x_1^b x_2^b$

$x_2^b(60 - 2x_1^b) = 12x_1^b$

So $x_1^b \in [0, 20]$, $x_2^b = \frac{12x_1^b}{60 - 2x_1^b} = \frac{6x_1^b}{30 - x_1^b}$,

$x_1^a = 20 - x_1^b$

$x_2^a = 12 - \frac{6x_1^b}{30 - x_1^b}$

is the set of Pareto-efficient allocations

Not entirely sure I follow. It may be equivalent... I get $x_2^a = \frac{18x_1^a}{10 + x_1^a}$.

b). At the equilibrium prices and allocation,

- each consumer is behaving optimally given the prices
- all markets clear.

Consumer a is solving the problem

$\max_{x_1^a, x_2^a} 2\ln x_1^a + 3\ln x_2^a \quad$ s.t. $x_1^a \geq 0, \; x_2^a \geq 0,$

$p_1 x_1^a + p_2 x_2^a \leq 20p_1$

Since preferences are monotonic, the BC binds. And as MU_1^a, $MU_2^a \to \infty$ as $x_1^a, x_2^a \to 0$, they consume strictly +ve quantities of each good.

So, we can solve $\max_{x_1^a, x_2^a} 2\ln x_1^a + 3\ln x_2^a$ s.t. $p_1 x_1^a + p_2 x_2^a = 20p_1$

which has an interior solution where the $MRS = -p_1/p_2$, as this satisfies the tangency condition between their ICs and BC.

As preferences are Cobb-Douglas, they spend fixed proportions of their income on each good. So Marshallian demands are $x_1^a = \frac{2}{5} \cdot \frac{m^a}{p_1}$; $x_2^a = \frac{3}{5} \cdot \frac{m^a}{p_2}$ where $m^a = 20p_1$

Similarly, $x_1^b = \frac{2}{3} \frac{m^b}{p_1}$, $x_2^b = \frac{1}{3} \frac{m^b}{p_2}$ where $m^b = 12p_2$.

Since allocations are HD0 in prices, let's define $p := p_1/p_2$ and normalise $p_2 = 1$.

Then $x_1^a = 8$, $x_2^a = 12p$, $x_1^b = \frac{8}{p}$, $x_2^b = 4$

For markets to clear, excess demand in each market must be zero.

So $8 + \frac{8}{p} = 20 \Rightarrow p = 2/3$

and by WL, we only need to solve in $k-1$ markets.

Allocation is a: $(8, 8)$ b: $(12, 4)$

From part a) above, $x_1^b \in [0, 20]$ and $x_1^a = 20 - x_1^b = 8$

and $x_2^b = 4 = \frac{6 \times 12}{30-12}$ and $x_2^a = 12 - \frac{6 \times 12}{30-12}$

So it's a P-efficient allocation.

By the 1st FWT we could've concluded this directly since ~~under~~ if (p^*, x^*) is a competitive equilibrium in an exchange economy, it must be Pareto efficient.

c) Again, $x_1^b \in [0, 20]$, $x_1^a = 20 - x_1^b$, $x_2^b = 6 = \frac{6 \times 15}{30-15}$, $x_2^a = 12 - x_2^b$

So it's Pareto efficient.

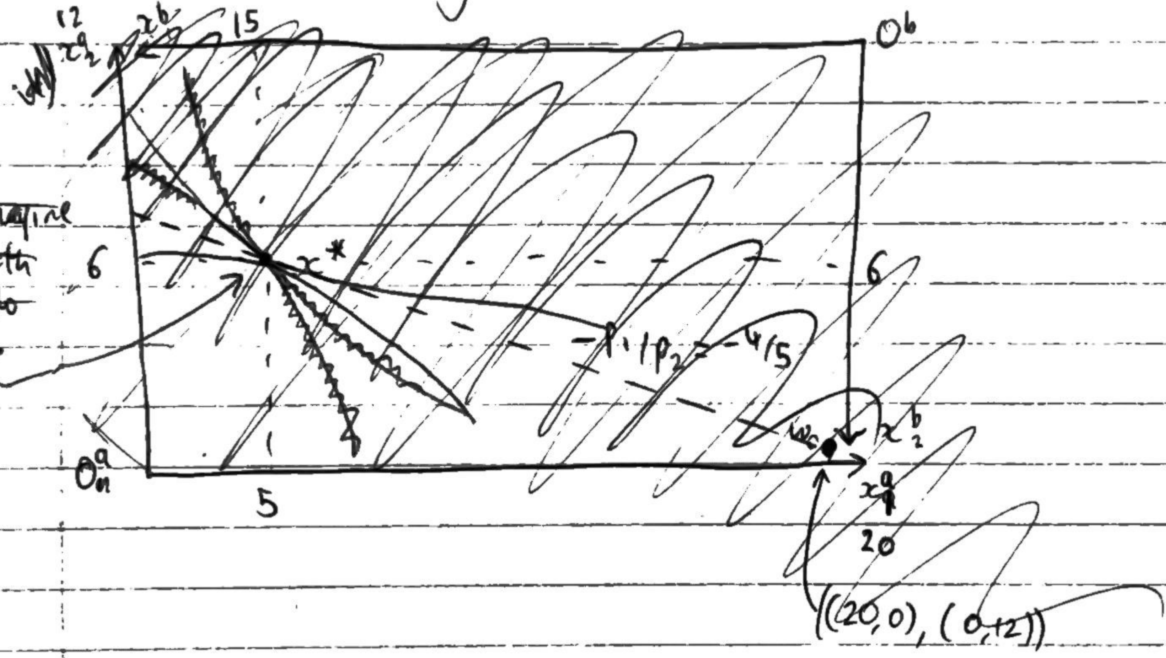
Yes, by 2nd FWT any Pareto efficient allocation can be achieved as competitive equilibrium given suitable initial endowments post-redistribution.

Consider agent a. To be acting optimally given the price level p_1/p_2 and their observed demands,

$$MRS^a = -p_1/p_2 \Rightarrow -\frac{2}{3} \cdot \frac{6}{5} = -p_1/p_2$$

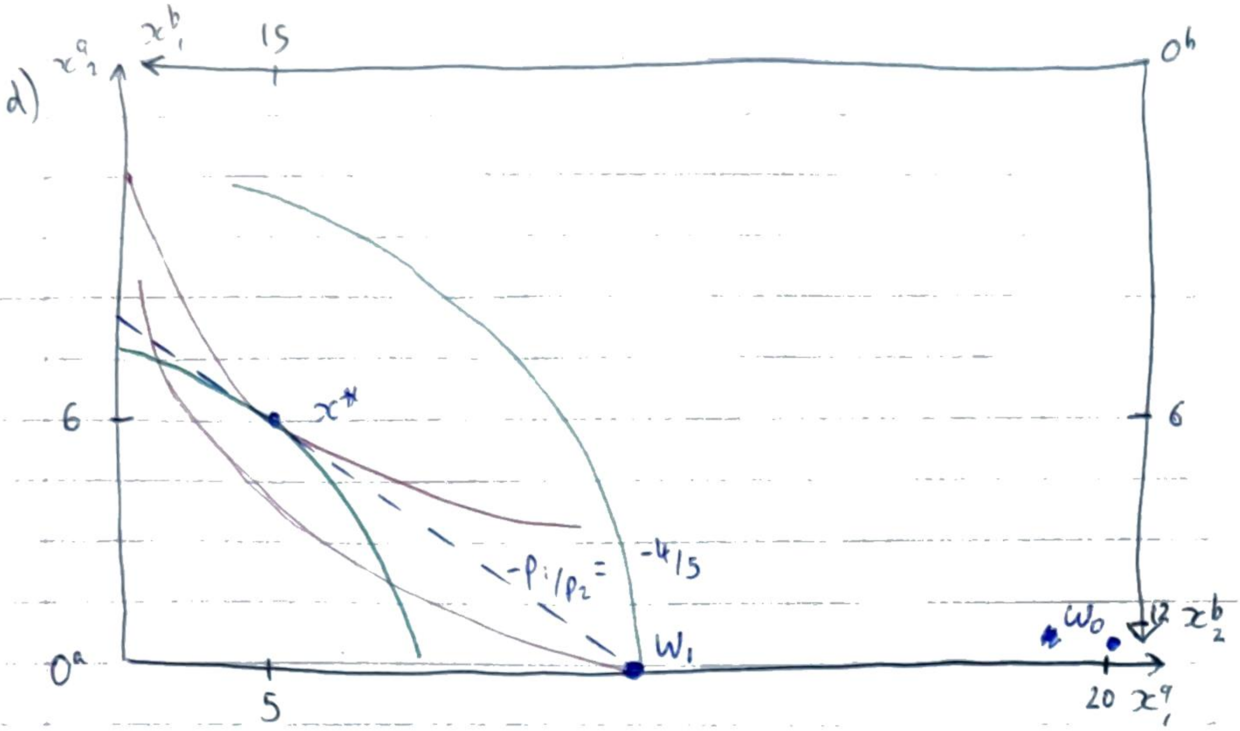
So $p_1/p_2 = \frac{4}{5}$ at equilibrium.

(we can verify if desired that $MRS^b = -2 \cdot \frac{6}{15} = -\frac{4}{5} = -p_1/p_2$)



~~We need the initial endowment to be on an IC which is for both individuals worse than x^*'s.~~

d)



Great

[Redistribution to anywhere along the line with slope $-4/5$ through the point x^* will support a competitive equilibrium at the desired allocation, because, using that price mechanism, x^* is optimal for both consumers. Both are better off at x^* than w_1.

(*) [but there are plenty of other points of tangency between IC's in that region between IC's. are we just taking the price level as something exogeneous that forces them to go to x^*?]

Yes, but please bring this up in class.

The blue line has equation $y = -4/5 x + 10$
so the x-intercept is at $x = 8$, i.e.
she should transfer 12 units of x_1 from a to b.

Great.