(if there's extra High, give it to type X after Y)

Information

2(a)

Q	X	Y
H	90	110
M	80	85
L	70	60

[This question seems too vague on the number of bikes and consumers to give a more precise answer?]

Yeah.

a) The Social planner would allocate High to type Y first. Then if some Y remained they'd be given Medium, before giving the Mediums followed by Lows to type X, and finally Low to any remaining Y.

✓✓ [(If we assume demand is never satiated, then Y has all H + M, and X has all L)]

b). Given perfect information, zero transaction costs, and all agents having no pricing power, the market equilibrium will produce the exact same distribution of resources as the Pareto optimal allocation chosen by the social planner in (a), by FWT 1. ✓✓

Intuitively, the $WTA_X(High) < WTP_Y(High)$ and $WTA_X(Med) < WTP_Y(Med)$, so there are gains from trade to be had in X selling High and Medium to Y at some price $90 < p_H < 110$ and $80 < p_M < 85$. There are no gains to be had from transfer of low from X to Y. ✓

c). For type X, the expected valuation of a bike is 80; for type Y it is 85. So type Ys will purchase all the bikes at some price $80 < p < 85$. ✓

d). Suppose Y is considering a contract to buy a bike at a price $p_1 \geq 90$. Then Xs would be happy to sell any bike at this price, and so for Y the expected value of the bike purchased would be $(110 + 85 + 60)/3 = 85$. But this is less than they paid for it, so they wouldn't contract at this price. ✓

Maybe, instead, they are looking to buy at a price $80 \leq p_2 < 90$. Then only Ms and Ls will be sold by the Xs, since $WTA_X(H) = 90$. But for Y the expected value of the bike purchased would then be $(80+70)/2 = 75 < 80$, so again they wouldn't contract. ✓

Finally, type Y_s wouldn't contract at any price $p_3 < 80$, because they'd know that they'd only get L_s, which they value less than X_s, so no mutually-beneficial trades of L_s are possible at all.

Great

So no bikes would be exchanged and the market collapses.

e) As informational conditions worsen, the outcome becomes less efficient. (b) is fully efficient, (c) is not fully efficient as Y_s end up with some L_s but nonetheless is closer to the Pareto frontier than the initial allocation, and in (d) the asymmetric information means that no gains from trade are realised at all.

5.a) H-types want to signal to firms their type, because a firm's WTP for a H-type is greater than that for a L-type (since ↑ productivity means ↑ MP_L and thus more valuable). If H-types can credibly signal their type then firms will pay them more, and the workers could capture more of the value they create.

For this signal to be credible, it must be from something that is costlier for L-types to produce in such a way that they have no incentive to "fake" being a H-type. (One could also model it more simply as a signal that L-types are <u>unable</u> to produce, no matter the effort they put in, e.g. some high score on an intelligence test you can't prepare for, etc.)

b) The firms will pay the expected marginal revenue product of a worker as the wage, as any less than that means they would be unable to recruit workers given competition in the labour market. This is equal to $\frac{3}{4} \times 80 + \frac{1}{4} \times 100 = 85$.

+ All workers work at 85 since there are no outside options.

c) Suppose some worker has an education. The most a firm would pay them is 100, ~~in the~~ as that is the H-type's productivity. If this worker were really an L-type, they would end up with utility $(100 - 22) = 78 < 80$, i.e. they are worse-off than what the firm is willing to pay them knowing their status as an L-type. So no L-types would obtain an education. For the H-type, they end up with utility $(100 - 12) = 88$ ~~> 85~~, so they're better-off than in the pooling eq* where the types are indistinguishable.

See in class

No! If H doesn't signal they get the "no-signal" wage of 80, not 85.

Thus, we get a separating eq* where only H-types get an education, firms are able to perfectly distinguish between workers, and $w_L = 80$, $w_H = 100$.

d) • H-types would have no incentive to obtain the education because even if the firm paid them 100 based on the signal, $100 - 16 = 84 < \cancel{85}$ so they're worse-off than the pooling eq* (doesn't satisfy individual rationality)

• Further, firms wouldn't pay 100 based on the signal because it's no longer incentive-compatible: the ~~smaller gap in~~ reduced costs ~~between~~ for L ~~and H~~ types means that ~~the expense~~ they would actually be willing to get an education were the wage to be 100, since $100 - 18 = 82 > 80$. But then the firm would offer expected ~~...~~ productivity salaries of 85 to all educated workers, and as above, H-types are worse-off compared to not having had an education.

is this argument correct on its own, or only in combination with the second bullet point?

Same comment as above.

So nobody gets an education as the small cost gap between H and L means it can't serve as a credible signal.

Assume everyone else is playing their part of the profile and consider only unilateral deviations.

e) In the no-education world (a), total surplus is simply wages, at 85 per worker (as no other opportunities ⇒ reservation price $= 0$). In the signalling world (b), H-types are better-off but L-types worse off, and total surplus is $\frac{1}{4} \times (100 - 12) + \frac{3}{4} \times 80 = 82$

per worker. So, on the assumption that education is purely serving a signalling role (i.e. it does not increase human capital / productivity and so WTP(educated H) = WTP(non-educated H)), then yes, the costliness of this signal is socially wasteful.

6. $\pi_L = 60 < \pi_H$; $P(\pi_H \mid e=0) = 2/5$; $P(\pi_H \mid e=1) = 3/5$

$u(w,e) = \sqrt{w} - e$, $u_0 = 8$

a). • The principal solves the cost-minimisation problem

$$\min_{w_e} w_e \quad \text{s.t.} \quad \underbrace{u(w_e, e) \geq u_0 = 8}_{\text{IR constraint from agent}}$$

fix

The constraint will be met with equality, as otherwise the principal could have paid less and the agent would've still contracted. So for e=0, we require w_0 st.

$$\sqrt{w_0} - 0 = 8 \Rightarrow \underline{w_0 = 64}$$

and for e=1, similarly $\sqrt{w_1} - 1 = 8 \Rightarrow \underline{w_1 = 81}$

• The principal is an expected utility maximiser. Since they're risk-neutral, $v(m) = m$, i.e. their Bernoulli utility function is linear in money. They're solving the problem i.e.

$$\max_{w} \mathbb{E}[\pi(w)].$$

hmm, how do you do this notation to show the distribution of π is a function of w?

Condition on e. See in class.

Expected net profits from the contract $\{0, w_0\}$ are $\left(\frac{2}{5}\cdot\pi_H + \frac{3}{5}\cdot\pi_L\right) - w_0 := \mathbb{E}[\pi(w_0)]$

and similarly $\{1, w_1\}$ has expected net profits $\left(\frac{3}{5}\cdot\pi_H + \frac{2}{5}\pi_L\right) - w_1$ $:= \mathbb{E}[\pi(w_1)]$

• If $\pi_H = 195$ then $\mathbb{E}[\pi(w_0)] = 50 < \mathbb{E}[\pi(w_1)] = 60$ so $\{1, w_1\}$ is optimal

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• When $\pi_H = 165$, $\mathbb{E}[\pi(w_0)] = 36 < \mathbb{E}[\pi(w_1)] = 42$. So again $\{1, w_1\}$ optimal

• when $\pi_H = 135$, $\mathbb{E}[\pi(w_0)] = 26 > \mathbb{E}[\pi(w_1)] = 24$

so $\{0, w_0\}$ is optimal

b) ~~If the principal wants low effort they~~

- Imagine you are the agent and are paid a fixed salary independent of outcomes. Then, since your utility is strictly decreasing in effort, certainly you will choose $e=0$, if you agree to contract.
- If the principal wants the agent to contract, they must provide in expectation at least the reservation utility of 8, i.e. pay in expectation the wage w_0 from (a). Paying any more than this would be suboptimal because the agent does not need any more wages to agree to contract. Furthermore, paying based on outcomes would lead to them paying more in expectation than just a fixed wage, as the risk-averse agent will demand a risk premium for contracts where wages are not fixed, but the risk-neutral agent cares only about expected costs.

- IR: the agent must be willing to contract, i.e. in expectation be at least as well-off as their reservation utility
  $P_L \cdot \sqrt{w(\pi_L)} + P_H \sqrt{w(\pi_H)} - e^* \geq 8$ where $P_L + P_H = 1$, $P_L = 3/5$ if $e^*=0$ and $2/5$ otherwise and e^* is the agent's optimal effort given the contract
- IC: the agent must be incentivised to choose $e=1$
  $$3/5 \cdot \sqrt{w(\pi_L)} + 2/5 \cdot \sqrt{w(\pi_H)} \leq 2/5 \cdot \sqrt{w(\pi_L)} + 3/5 \cdot \sqrt{w(\pi_H)} - 1$$

Using the IC constraint, since we're trying to induce $e=1$, we can simplify IR:
$$2/5 \cdot \sqrt{w(\pi_L)} + 3/5 \sqrt{w(\pi_H)} - 1 \geq 8$$

~~And again, the principal wants to maximise expected profits~~
The ~~const~~ risk-neutral firm is trying to minimise expected costs, given $e=1$.
We know, therefore, that

- IR will be met with equality – otherwise the principal would simply reduce both $w(\pi_L)$ and $w(\pi_H)$ while still providing higher EU to agent than their reservation utility
- IC will be met with equality – otherwise the principal could transfer less risk to the agent by making $w(\pi_H)$ and $w(\pi_L)$ closer together, and thereby reduce expected costs (as ↓ risk premium paid to agent).

$$\sqrt{w(\pi_H)} - \sqrt{w(\pi_L)} = 5$$

So, IC: $\frac{3}{5}\sqrt{w(\pi_L)} + \frac{2}{5}\sqrt{w(\pi_H)} = \frac{2}{5}\sqrt{w(\pi_L)} + \frac{3}{5}\sqrt{w(\pi_H)} - 1$

IR: $\frac{2}{5}\sqrt{w(\pi_L)} + \frac{3}{5}\sqrt{w(\pi_H)} - 1 = 8$

$\rightarrow 2\sqrt{w(\pi_L)} + 3\sqrt{w(\pi_H)} = 45$

$\cancel{\frac{3}{5}\sqrt{w(\pi_L)} + \frac{2}{5}\sqrt{w(\pi_H)} = 8 \quad (I)}$

So $2\sqrt{w(\pi_L)} + 3\left(\sqrt{w(\pi_L)} + 5\right) = 45$

~~and the problem is to~~ $\cancel{\min_{w(\pi_L), w(\pi_H)} \frac{2}{5}w(\pi_L) + \frac{3}{5}w(\pi_H)}$

and thus $\sqrt{w(\pi_L)} = 6 \Rightarrow$ $w(\pi_L) = 36$

$w(\pi_H) = 121$

~~From (I),~~ $\cancel{w(\pi_H) = (40 - 3\sqrt{w(\pi_L)})^2}$

~~So we must~~ $\cancel{\min_{w(\pi_L)} \frac{2}{5}w(\pi_L) + \frac{3}{5}(40 - 3\sqrt{w(\pi_L)})^2}$

~~For which the FOC is~~ $\cancel{\frac{26}{8\sqrt{w(\pi_L)}}}$

Expected costs are $\frac{2}{5}\cdot 36 + \frac{3}{5}\cdot 121 = 87$

- When $\pi_H = 195$ optimally the firm offers contract #2, for expected profits $\frac{2}{5}\cdot 60 + \frac{3}{5}\cdot 195 - 87 = 54$ vs 50 from contract #1 (identical to w_0 in part (a)).
- When $\pi_H = 165$ the firm offers contract #1 with expected profits 38 vs $\frac{2}{5}\cdot 60 + \frac{3}{5}\cdot 165 - 87 = 36$
- When $\pi_H = 135$ the firm offers #1 for 26 vs 18.

Note that in the first two scenarios, the firm is in expectation worse off than w/ full info.

c) $\pi_H = 195$: The optimal approach from the principal is still to induce $e=1$. However, because effort is unobserved and not perfectly correlated with outcomes, results-based compensation transfers risk to the agent, and so the principal must set higher wages in expectation to balance this and satisfy IR for the agent.

$\pi_H = 165$: Although under full information $e=1$ would've been optimal, this is no longer true, because paying the increased costs required to incentivise high effort ~~are~~ not justified by a sufficiently large difference ~~between~~ L and H outcome for the firm.

$\pi_H = 135$: If $e=0$ was optimal in the full info case, certainly it still will be in the moral hazard case, since it costs exactly as much and $e=1$ is more expensive.

13. · Suppose there are two types of consumers: high-risk and low-risk, who have probabilities $1 > p_H > p_L > 0$ of incurring a loss L.
· All individuals have the same endowment ω and identical convex preferences u.
· For a consumer of type i, the possible outcomes are:

	Incurs loss w.p. p_i	No loss w.p. $(1-p_i)$
Insures q	$\omega - L + q - \pi q$	$\omega - \pi q$
No insurance	$\omega - L$	ω

where π is the price per unit of insurance.

· Let's assume there are equal numbers of H and L consumers.
· By competition, insurer profits are zero, so they must be actuarially fair.

what does this mean in this context?

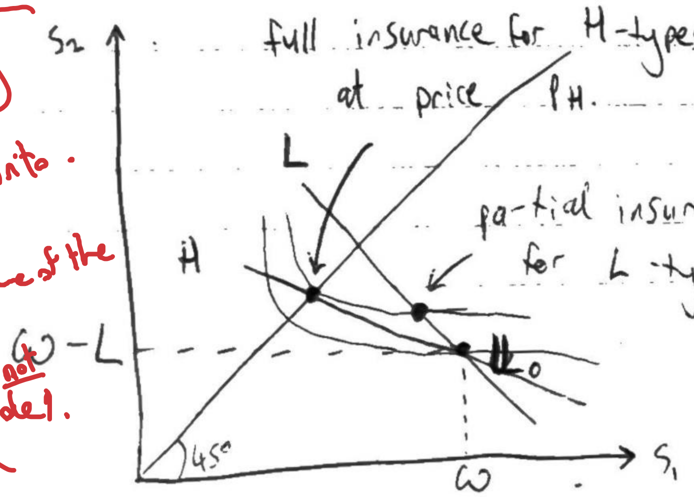
⇒ If firms try to offer one premium to all customers, and allow full insurance, the market will unravel due to adverse selection.
⇒ High-risk consumers will be offered full insurance and buy it all; low-risk will get partial insurance at a level determined by the ICs of Hs, to ensure screening + separating eq.*

do we use these two terms here?

⇒ Firms may try to perform pre-contract screening based on e.g. age, smoking, etc. This will allow them to offer closer to full insurance for low-risk.
⇒ To reduce moral hazard, which is a post-contract issue, firms might require an excess, copayments, offer no-claim benefits, and so on.

This looks like there are two contracts which Hs & Ls self-select into.

...which looks like a possible outcome of the R-S model, but the essay question is not about the R-S model.



This satisfies IR and IC but is not socially optimal (better for all to have full insurance at actuarially fair price, but not feasible).