Rohan - Merton - Week 5 - Predicate Semantics

5.1 i) $1 \notin \{2\}$ so $|P^1a|_S = F$ as $|a|_S \notin |P^1|_S$

ii) $\langle 1,3\rangle \in |R^2|_S$ so $|R^2ab|_S = T$ as $\langle |a|_S, |b|_S\rangle \in |R^2|_S$

iii) $\langle 3,1\rangle \notin |R^2|_S$ so $|R^2ba|_S = F$ as $\langle |b|_S, |a|_S\rangle \notin |R^2|_S$

iv) $|R^2ab|_S = T$ and $|R^2ba|_S = F$ (above). So as $|R^2ab|_S \neq |R^2ba|_S$, $|R^2ab \leftrightarrow R^2ba|_S = F$

v) $|P^1a|_S = F$ (above) so $|\neg P^1a|_S = T$; $\langle 1,1\rangle \notin |R^2|_S$ so $\langle |a|_S, |a|_S\rangle \notin |R^2|_S$
so $|R^2aa|_S = F$ so $|\neg R^2aa|_S = T$ so $|\neg P^1a \wedge \neg R^2aa|_S = T$
so $|R^2bb \vee (\neg P^1a \wedge \neg R^2aa)|_S = T$

vi) Let α be a variable assignment over A. Then change $|x|_S^\alpha$ to 2, call this assignment β, differing from α in x only. Then $\langle 1,2\rangle \in |R^2|_S^\beta$
so $\langle |a|_S^\beta, |x|_S^\beta\rangle \in |R^2|_S^\beta$ so $|R^2ax|_S^\beta = T$ and so $|\exists x\, Rax|_S = T$

vii) Let α be a var. assign. Then change $|x|_S^\alpha$ to 2, call this β.
~~then~~ $|R^2ax|_S^\beta = T$ (above). $\langle 2,3\rangle \in |R^2|_S$ so $\langle |x|_S^\beta, |b|_S\rangle \in |R^2|_S$
so $|Rxb|_S^\beta = T$ so $|(Rax \wedge Rxb)|_S^\beta = T$ and β differs from α by x only so $|\exists x(Rax \wedge Rxb)|_S = T$

(never mind) viii) Done by Völker :)

ix) Let α be a var. assign. Then change $|x|_S^\alpha$ to 3, call this β.
There is no ordered pair in $|R^2|_S$ with 3 as the first element,
i.e. $\langle 3,1\rangle \notin |R^2|_S$ and $\langle 3,2\rangle \notin |R^2|_S$ and $\langle 3,3\rangle \notin |R^2|_S$.
So $\langle |x|_S^\beta, |y|_S^\beta\rangle \notin R$ so $|R^2xy|_S^\beta = F$. β differs from α only by x so $|R^2xy|_S^\beta \neq T$ for some β differing from α only in x
so $|\forall x \exists y\, R^2xy|_S = F$

is $\exists y(Rxy \wedge Ryx)$ not the same as $\exists y Rxy \wedge \exists y Ryx$?

x) Let α be a var. assign. ~~change $|x|_S^\alpha$ to 2, call this β.~~
Case 1: $|x|_S^\alpha = 2$. Then $|x|_S^\alpha \in |P^1|_S$ so $|Px|_S^\alpha = T$
$|y|_S^\alpha = 1$ or 2 or 3.

More efficient way to do it?

~~If $|y|_S^\alpha = 1$ then $\langle |x|_S^\alpha, |y|_S^\alpha\rangle \notin |R^2|_S$ as $\langle 2,1\rangle \notin |R^2|_S$; $|R^2xy|_S = F$~~
~~If $|y|_S^\alpha = 2$ then $\langle |x|_S^\alpha, |y|_S^\alpha\rangle \notin |R^2|_S$ as $\langle 2,2\rangle \notin |R^2|_S$; $|R^2xy|_S^\alpha = F$~~
~~If $|y|_S^\alpha = 3$ then $\langle |y|_S^\alpha, |x|_S^\alpha\rangle \notin |R^2|_S$ as $\langle 3,2\rangle \notin |R^2|_S$; $|R^2yx|_S^\alpha = F$~~
~~so in all cases,~~

$|\exists y\, Rxy|_S^\alpha = T$ as, for β when we change only $|y|_S^\alpha$ to 3,
$|R^2xy|_S^\beta = T$ since $\langle |x|_S^\beta, |y|_S^\beta\rangle \in |R^2|_S$ since $\langle 2,3\rangle \in |R^2|_S$
And similarly if we take γ changing only $|y|_S^\alpha$ to 1 then $\langle |y|_S^\gamma, |x|_S^\gamma\rangle \in |R^2|_S$
so $|R^2xy|_S^\gamma = T$ and hence $|\exists y\, Ryx|_S^\alpha = T$

So $|\exists y Ryx \wedge \exists y Rxy|^{\alpha}_{s} = T$, so $|Px \to (\exists y Ryx \wedge \exists y Rxy)|^{\alpha}_{s} = T$

Case 2: $|x|^{\alpha}_{s} \neq 2$ (i.e. 1 or 3). Then $|Px|^{\alpha}_{s} = F$ since 1 and 3 are not in $|P^1|_s$

So $|Px \to (\sim)|^{\alpha}_{s} = T$

So for all variable assignments differing only in x, $|Px \to (\sim)|^{\alpha}_{s} = T$

so $|\forall x(Px \to (\exists y Ryx \wedge \exists y Rxy))|_s = T$

xi) Consider var. assign. α where $|x|^{\alpha}_{s} = 2$.

Then $|Px|^{\alpha}_{s} = T$ as above.

Now change the value $|y|^{\alpha}_{s}$ to make a var. assign. β

$|y|^{\beta}_{s} = 1$ or 2 or 3.

If $|y|^{\beta}_{s} = 1$ then $|Rxy|^{\beta}_{s} = F$ as $\langle 2,1\rangle \notin |R^2|_s$

If $|y|^{\beta}_{s} = 2$ then $|Rxy|^{\beta}_{s} = F$ as $\langle 2,2\rangle \notin |R^2|_s$

$\Big\}$ so $\langle |x|^{\beta}_{s}, |y|^{\beta}_{s}\rangle \notin |R^2|_s$

If $|y|^{\beta}_{s} = 3$ then $|Ryx|^{\beta}_{s} = F$ as $\langle 3,2\rangle \notin |R^2|_s$ so $\langle |y|^{\beta}_{s}, |x|^{\beta}_{s}\rangle \notin |R^2|_s$

So there ~~exists~~ is no assignment β differing from α only in y where

$|Ryx \wedge Rxy|^{\beta}_{s} = T$, so $|\exists y(Ryx \wedge Rxy)|^{\alpha}_{s} = F$

so there is an assignment α with $|Px \to \exists y(\sim)|^{\alpha}_{s} = F$ so

$|\forall x(\sim)|_s = F$

5.2 i) Let $D_s = \{1\}$, $|a|_s = 1$, $|P^1|_s = \{1\}$, $|Q^1|_s = \emptyset$

ii) Let $D_s = \{1, 2\}$, $|P^1|_s = \{1\}$, $|R^2|_s = \{\langle 1,2\rangle\}$. then note e.g. $|x|^{\alpha}_{s} = 1$, $|y|^{\alpha}_{s} = 2$

iii) Let $D_s = \{1\}$, $|R^2|_s = \{\langle 1,1\rangle\}$, $|a|^{\alpha}_{s} = 2$.

5.3. $\Gamma \models \phi \leftrightarrow$ there exists no L_2-structure A where $|\gamma|_A = T$ for all $\gamma \in \Gamma$ and $|\phi|_A = F$

$\leftrightarrow$ the set $\Gamma \cup \{\neg\phi\}$ is semant. incons.

5.4 i) $\exists x \exists y \exists z(Px \wedge \neg Py \wedge Qy \wedge \neg Qz \wedge \neg Pz)$ i.e. $|x|_A \neq |y|_A$; $|x|_A \neq |z|_A$, etc.

ii) $\exists x \exists y Rxy$ where e.g. $|R^2|_A = \{\langle d, e\rangle : d$ is less than $e\}$

iii) $\exists x \exists y \exists z(Rxy \wedge Rxz \wedge Ryz)$ where e.g. $|R^2|_A = \ldots$ is less than...

iv) $\forall x \exists y Rxy$ where e.g. $|R^2|_A = \ldots$ is less than ...