I'm assuming this is the consumption of x of a single type b, but not entirely clear in Q.

ok

Externalities sheet

1. $u_a(x_a, y_a, x_b) = \ln x_a + 2\ln y_a - \ln x_b$ $\quad$ $w_a: (6,0)$

$u_b(x_b, y_b) = \ln x_b + 2\ln y_b$ $\quad$ $w_b \; (0,0)$

a) The Walrasian equilibrium is a price vector and allocation s.t.
- given the price vector, each agent is acting optimally
- all markets clear.

✓

Type a consumers have no control over the quantity of good x demanded by type ~~a~~ b? consumers, so their util-max problem is

$$\max_{x_a, y_a} \ln x_a + 2\ln y_a - \ln x_b \quad \text{s.t. } x_a \geq 0,\ y_a \geq 0,$$
$$p_x x_a + p_y y_a \leq m$$

where m is the value of their endowment i.e. $m = 6p_x$. ✓

- As preferences are monotonic increasing, the BC binds. ✓
- Since $MU^a_x, MU^a_y \to \infty$ as $x_a, x_b \to 0$, they consume strictly +ve quantities of both goods.
- As allocations in Walrasian equilibrium are HD0 in prices, we ~~can~~ are ~~only~~ interested in relative prices $p := p_x/p_y$ only, and can normalise $p_y = 1$. ✓

So, we can think of a type a consumer solving the problem

$$\max_{x_a, y_a} \ln x_a + 2\ln y_a - \ln x_b \quad \text{s.t. } p x_a + y = 6p,$$
$$x_a > 0,\ y_a > 0$$

which, ~~as~~ since the preferences are Cobb-Douglas, yields the Marshallian demands

$$x_a = \frac{1}{3}\cdot\frac{6p}{p} = 2 \qquad y_a = \frac{2}{3}\cdot\frac{6p}{1} = 4p$$ ✓

Similarly for a type b ~~consumers~~ consumer, we obtain

$m = 6$

$$x_b = \frac{1}{3}\cdot\frac{6}{p} = \frac{2}{p} \qquad y_b = \frac{2}{3}\cdot\frac{6}{1} = 4$$ ✓

Excellent

For the market for x to clear, excess demand must equal zero, so

$$2n + \frac{2}{p}n - 6n = 0 \implies p = 1/2$$ where n is # type a = # type b consumers $\neq 0$ ✓

By WL, the market for y also clears at this price level.

✓

So the equilibrium is $p_x/p_y = 1/2$ with each type a consumer having $(2,2)$ and type b having $(4,4)$.

✓

b). No, the presence of externalities means that the market equilibrium will not be Pareto efficient, since prices don't capture the effect of type b consumers on type a ones.

Consider the social welfare function ~~$W(u_a, u_b)$~~

$$W(u_a^1, u_a^2, \dots, u_b^1, u_b^2, \dots) = u_a^1 + u_a^2 + \dots + u_b^1 + u_b^2 + \dots$$

(*)

$$= n(\ln x_a + 2\ln y_a + 2\ln y_b)$$

this argument feels a bit suspect, not quite sure why it does (or doesn't) work either way... was the best I could think of though. (*)

Note that since the full amount of each endowment must be allocated, we know that $x_a + x_b = 6$, so implicitly x_b is a function of x_a. We can therefore rewrite u_a as $u_a(x_a, y_a) = \ln x_a + 2\ln y_a - \ln(6 - x_a)$

$$MRS^a := -\frac{MU_x^a}{MU_y^a} = -\frac{1/x_a + \frac{1}{6-x_a}}{2/y_a}$$

$$MRS^b := -\frac{MU_x^b}{MU_y^b} = -\frac{1/x_b}{2/y_b} = -\frac{y_b}{2x_b}$$

(+) part d) If p_x/p_y ↑, then it's harder for b to afford x as their endowment is all in y, and at equilibrium they consume less, as desired.

see in class

A different argument: explicitly find an allocation that Pareto dominates the C.E.

Actually, I'm not sure about this. p_x/p_y tells us the wealth of a and b, but do we expect a to be relatively wealthier in this new scenario? I guess we want b to buy less of it, and so if x means b affordable...

At a Pareto-efficient outcome, $MRS^a = MRS^b$, or we are at a corner solution. We are not at a corner solution since $x_a, x_b, y_a, y_b > 0$. $MRS^a = -\frac{\frac{1}{2} + \frac{1}{4}}{1} = -\frac{3}{4}$

$MRS^b = -\frac{4}{8} = -\frac{1}{2} \neq -\frac{3}{4}$. So, we are not at a P-efficient equilibrium. A transfer of some x from b's to a's and y from a's to b's could make both better off.

✓✓ Great.

c) Yes, Coasean bargaining here will be Pareto-efficient (assuming zero transaction costs) because the specific property rights in endowments mean that a and b can (and will) make mutually-beneficial transfers between each other until no more are possible, i.e. P-efficient.

d) As noted, a will have more of x and less of y, and vice versa.

(+) (They must have strictly less y otherwise b couldn't be better off.) If they just engage in bargaining, prices don't matter but this outcome would affect a price level where x is relatively more expensive than before, as a's endowment

2.

	$h=0$	$h=1$
Firm	x	$x+b$
Consumer	y	$y-c$

a) No, it does not follow. Suppose the true values of benefit and cost are $b_t = 3$, $c_t = 1$. Then it is Pareto optimal to allow the externality, and the firm could make a transfer to the consumer $1 < t < 3$ that would make everybody better-off compared to the externality being banned.

Yes But the firm would have no obligation to make this transfer, and so wouldn't, if the externality were ~~just permitted~~ simply allowed. So the consumer has no incentive to truthfully state their cost, and may exaggerate e.g. reporting $c_r = 1000$, to avoid the externality (since they don't benefit from the firm being allowed to produce externality). Likewise the firm would exaggerate its benefit, so both state as high values as possible and there's no reason that the gov't's decision would be Pareto-optimal, since this mechanism is not incentive-compatible.

b) Now the payoffs are

	$h=0$	$h=1$
Firm	x	$x+b_t-c_r$
Consumer	y	$y+b_r-c_t$

And $h=1$ iff $b_r > c_r$, where x_t = true value, x_r = reported.
The government spends $b_r - c_r > 0$ more than when $h=0$, ~~in~~ which
~~Suppose $b_t > c_t$. Then it is Pareto optimal for h to equal 1.~~
~~If the firm states $b_r > b_t$ it makes no difference, assuming $c_t = c_r$~~
~~If the consumer states $c_r > c_t$~~
they must finance from elsewhere in the budget.

[no need to assume this, maybe?] ~~Let's assume first that $c_t = c_r$.~~ Then, for the firm,

- if they state $b_r > b_t$, they risk $x + b_t - c_r < x$ with $h=1$, as it could be that $b_r > c_r > b_t$
- if they state $b_r < b_t$ then the externality might be banned even if $x + b_t - c_r > x$, since potentially $b_t > c_r > b_r$.

- So, they have no incentive to report $b_r \neq b_t$.

For the consumer,
- if they report $c_r > c_t$, the externality may be banned when it'd have benefitted them, if $c_r > b_r > c_t$,
- if they report $c_r < c_t$, it may be permitted and make them worse off, if $c_r < b_r < c_t$,
- So, again no incentive to report $c_r \neq c_t$.

[what's a (*) concise way to say "and these are the only cases where outcomes differ from lying vs telling truth"?]

Both have a weakly dominant strategy to tell the truth, so the Pareto-efficient outcome will be arrived at.

(couldn't you get another Nash equilibrium where e.g. both exaggerate their benefits / costs a lot? Would this still be Pareto-efficient?)

see notes (or in class).

6. $\pi(h; \eta) = w_f + \eta h - 2h^2$; $\phi(h) = w_c - h^2$

a) i. The firm solves the problem

$\max_h \pi = w_f + 24h - 2h^2$ for which the FOC is

$$\frac{\partial \pi}{\partial h} = 0 = 24 - 4h \Rightarrow h = 6$$

✓

ii. The socially optimal level of pollution can be found by solving the social planner's problem

$\max_h \underbrace{\pi(h) + \phi(h)}_{W} = w_f + w_c + 24h - 3h^2$ for which the FOC is

$$\frac{\partial W}{\partial h} = 0 = 24 - 6h \Rightarrow h = 4$$ is the optimal quota.

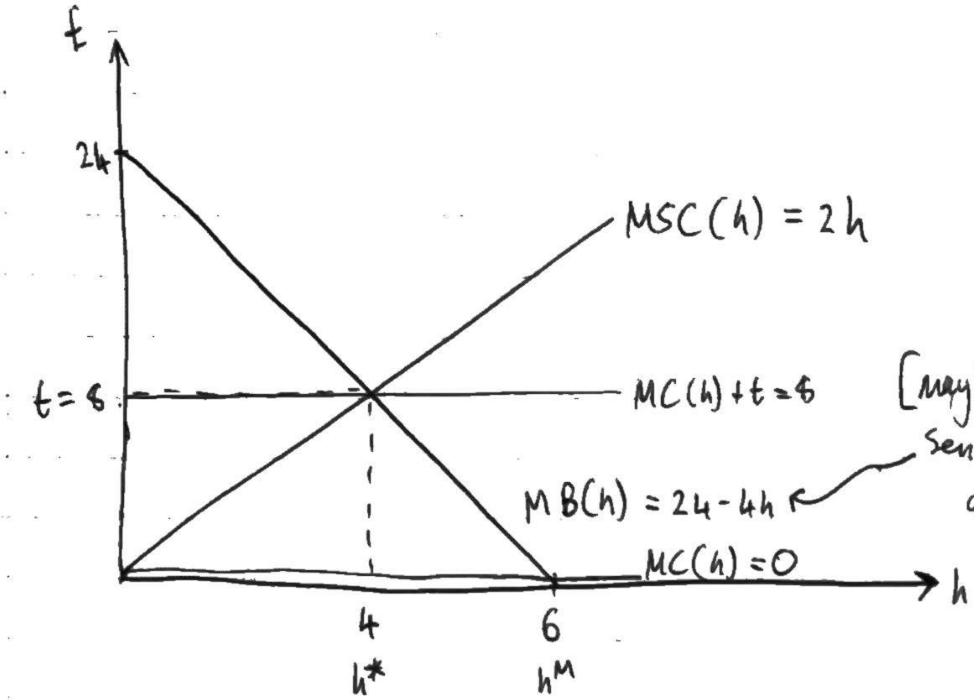
✓

iii. The optimal Pigouvian per unit tax is t such that ~~the marginal external cost~~ $MB(h^*) = MC(h^*) + t$ for the firm, at the optimal pollution level h^*. In other words, $t =$ marginal external cost of pollution at the optimal level.

$MEC(h^*) = 2h\big|_{h=4}$ So we should set $t = 8$.

as $E(h) = h^2$

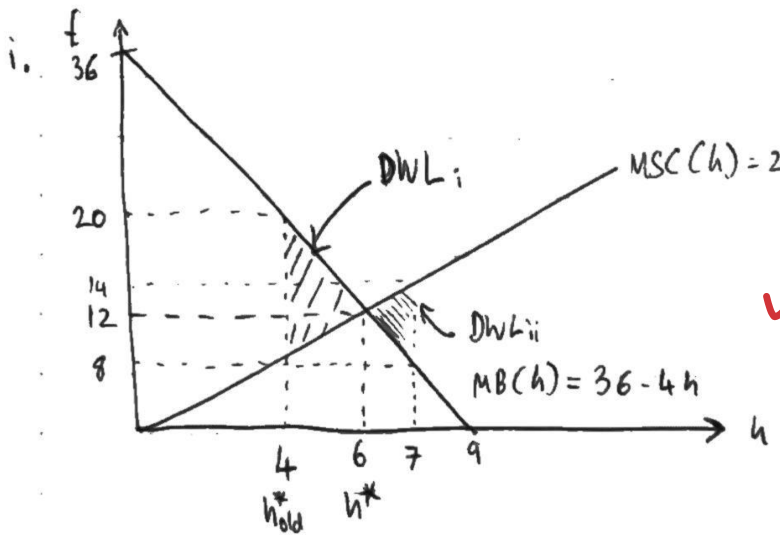
✓



[maybe it makes more sense to say MB = 24 and have MC = 4h?]
MSC = 6h?

b) The efficient level of pollution would now be $h = 6$, from the maximisation $\max_h w_f + w_c + 36h - 3h^2$.

i.



- DWL$_i$ is the difference in social welfare between the efficient outcome and the one with the old quota, where social welfare := CS + PS + gov't revenue.
- From the diagram, we can see it is $\frac{1}{2} \times (6-4) \times (20-8) = 12$

ii. If instead of the quota we set the tax level $t = 8$, we would have $h = 7$ and DWL$_{ii}$ = 3. It is better to set the policy in terms of prices than in terms of quantities when you are uncertain about the true marginal benefits and the MSC curve is relatively shallower than the MB curve, as here.

Good

let's discuss this point in class

[what's the "real world" interpretation of one being shallower than the other?]