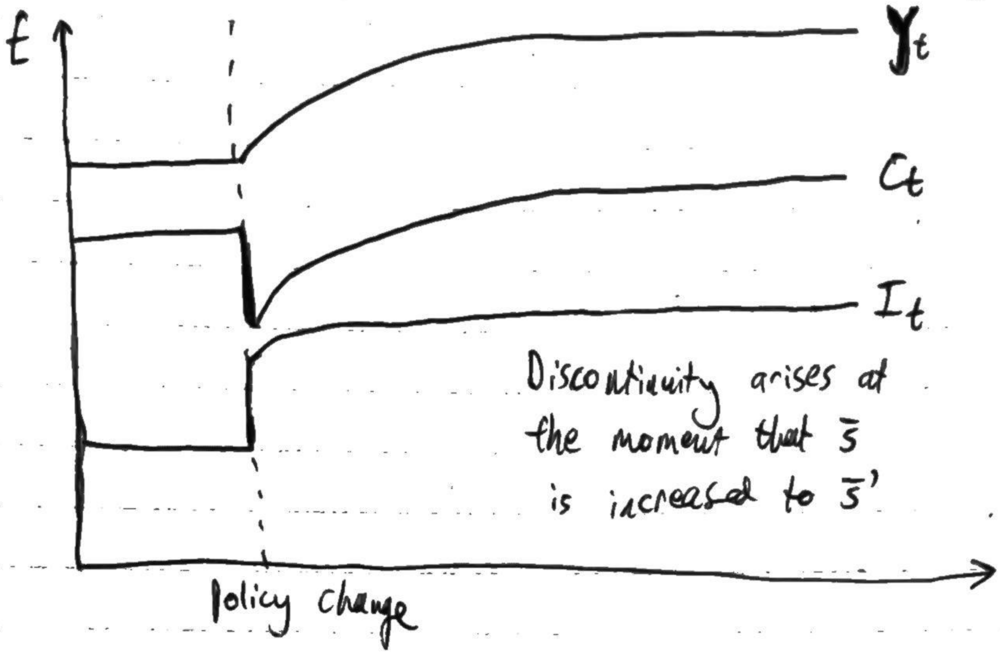
v. • Practically every country seems to be investing too little; China and Singapore invest a tiny amount too much ($>33.3\%$)

• Increasing $\bar{s}$ (to $\bar{s}_{gold}$) by a policy intervention will lead to greater steady-state c^* but transition dynamics mean that c_t decreases in the short run: it takes time for the gradual accumulation in capital due to $\bar{s}\, y_t^* > \bar{d}\, k_t$ to increase output enough to offset the negative effect on c_t from saving more (i.e. spending less)

vi.



Reasons that perhaps increasing \bar s wouldn't even lead to an increase in consumption (absent politics):

- might have different production functions so different \alpha and ideal savings rate
- might not be possible to increase \bar s if people living on subsistence
- might not be in steady state so they'll see growth without even having to increase \bar s

Unlikely to gain political support for reasons noted above. The benefits of increasing $\bar{s}$ accrue to future agents, and the costs are felt now. Since future people can't vote (nor young people), unless there are ~~good~~ strong enough intergenerational linkages and preferences over future outcomes from current voters (and a long-term enough vision to compute these), the present costs would lead the proposal to be unpopular.

2. i. In period 2, ℓ_2 is optimally 0 since production of ideas is worthless (given no subsequent periods) and comes at the cost of output in period 2, which is valued.

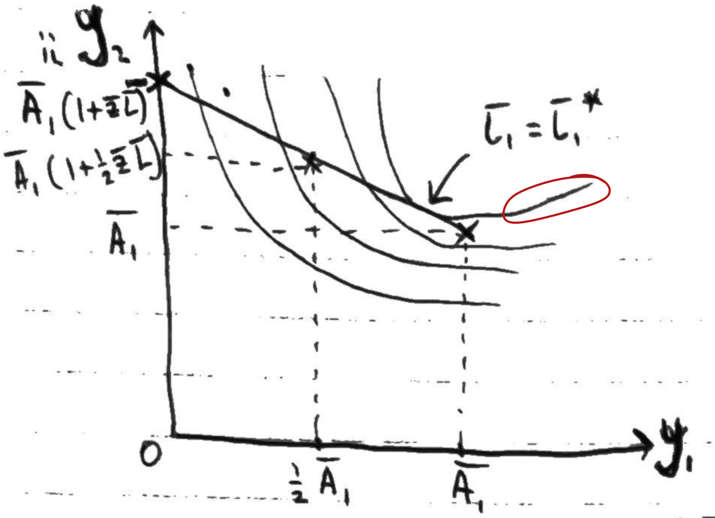
ii.

~~[illegible]~~ A

~~[illegible]~~

(Assume in all cases $\bar{l}_2 = 0$, since this is optimal)

ii.



If $\bar{l}_1 = 1$, then $Y_1 = 0 = y_1$

and $A_2 = \bar{A}_1(1 + \bar{z}\bar{L})$

so $y_2 := \frac{Y_2}{\bar{L}} = \bar{A}_1(1+\bar{z}\bar{L})$

If $\bar{l}_1 = 0$ then $Y_1 = Y_2 = \bar{A}_1\bar{L}$

so $y_1 = y_2 = \bar{A}_1$

If $\bar{l}_1 = \frac{1}{2}$, $y_1 = \frac{1}{2}\bar{A}_1$ and $y_2 = \bar{A}_1(1+\frac{1}{2}\bar{z}\bar{L})$

More generally, $Y_2 = (\bar{A}_1 + \bar{z}\bar{A}_1\bar{l}_1\bar{L})\bar{L} \Rightarrow y_2 = \bar{A}_1(1+\bar{z}\bar{l}_1\bar{L})$

and $Y_1 = \bar{A}_1(1-\bar{l}_1)\bar{L} \Rightarrow y_1 = \bar{A}_1(1-\bar{l}_1)$

Lovely

iii. From above, the optimisation is

$$\max_{\bar{l}_1} U = \bar{A}_1^2(1-\bar{l}_1)(1+\bar{z}\bar{l}_1\bar{L})$$

for which the FOC is $\frac{\partial U}{\partial \bar{l}_1} = 0$,

i.e. $\cancel{\bar{A}_1^2}(1+\bar{z}\bar{l}_1\bar{L}) = \cancel{\bar{A}_1^2}(1-\bar{l}_1)\bar{z}\bar{L}$ by product rule

$\therefore 2\bar{z}\bar{l}_1\bar{L} = \bar{z}\bar{L} - 1$

so $\bar{l}_1 = \frac{\bar{z}\bar{L}-1}{2\bar{z}\bar{L}}$ ~~at~~ the optimum, $\bar{l}_1^*$

or $\bar{l}_1^* = \frac{1}{2}(1-\frac{1}{\bar{z}\bar{L}})$

To show the indifference curves, consider some level of utility $\bar{U}$

Then $\bar{U} = y_1 y_2$ i.e. $y_2 = \bar{U}\cdot(y_1)^{-1}$

and $\frac{dy_2}{dy_1} = \underbrace{-\bar{U}\cdot(y_1)^{-2}}_{<0 \text{ for all } y_1}$, $\frac{d^2y_2}{dy_1^2} = \underbrace{2\bar{U}\cdot(y_1)^{-3}}_{>0 \text{ for all } y_2}$

So they are convex and downward-sloping, and one is ~~[illegible]~~ tangent to the linear constraint trading off y_1 with y_2 at exactly the point where $\bar{l}_1 = \bar{l}_1^*$

Really nicely done showing convexity

Specifically, where $y_1 = \bar{A}_1\left(\frac{\bar{z}\bar{L}+1}{2\bar{z}\bar{L}}\right)$, $y_2 = \bar{A}_1\left(\frac{\bar{z}\bar{L}+1}{2}\right)$

$= \frac{1}{2}\bar{A}_1\left(1+\frac{1}{\bar{z}\bar{L}}\right)$ $= \frac{1}{2}\bar{A}_1(\bar{z}\bar{L}+1)$

iv. So, $\bar{l}_1^*$ increases with $\bar{z}$: more productive idea workers means more of them is optimal.

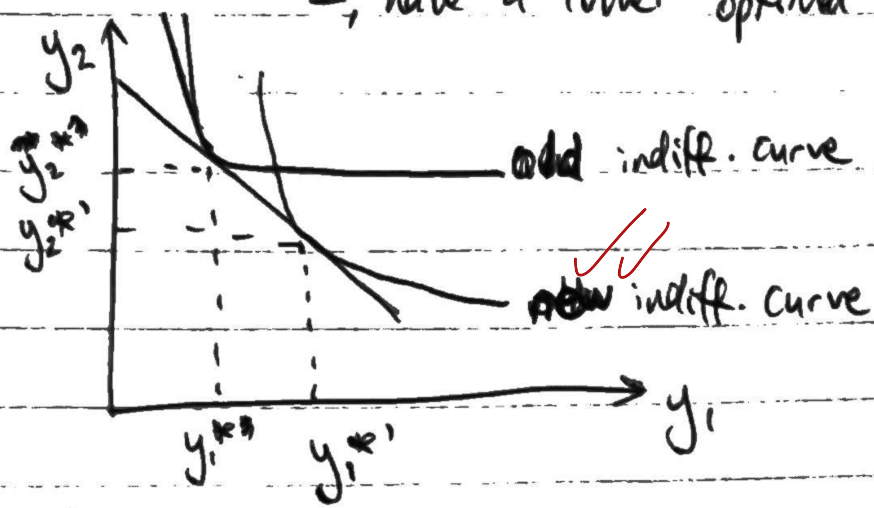
$$U = y_1^{\phi} y_2$$

v. For Cobb-Douglas preferences like ~~these~~, $MRS_{y_1, y_2} = -\frac{\phi y_2}{y_1}$

Increasing the value of the exponent ϕ means the indifference curves become ~~steeper at all values of y_1~~ more squashed towards the origin. steeper?

This is because for the gain of a given ~~levels of~~ amount of additional output in period y_1, consumers are willing to sacrifice a relatively larger amount of output in period 2, as compared to when ϕ is smaller. ✓✓ Great intuitive link

So, ~~naturally~~ you would expect agents to produce more in period 1, have a lower optimal $\bar{l}_1$, and produce less in period 2,



Graphically, to remain tangent to the fixed linear constraint, the more-squashed new indifference curves must shift rightwards. 4.

Shift essay Q3