Partial derivatives

4 - Consumption

6. a) $\boxed{\frac{\partial U}{\partial B} = MU_B = 2BL^3, \quad \frac{\partial U}{\partial L} = 3L^2B^2,}$ $MRS_{LB} := -\frac{MU_L}{MU_B} = \cancel{-\frac{3}{2}L^2B^{-1}}$

Well-behaved as Cobb-Douglas ? → MRS_{LB} is ΔB to make up for loss of 1 L.

Doesn't actually say what well-behaved means.

b) Yes, she'll spend it all as we can see there is no satiation point: in this utility function, more of either L or B is always better for utility since $MU_B, MU_L > 0$ for all $\partial B, \partial L > 0$. So since we assume B and L are perfectly divisible, she will consume a combination on the budget constraint to max. U.

c) $1000 = 20B + 10L$, $B = 50 - \frac{1}{2}L$ and we equate gradients, i.e. $MRS = -\frac{p_1}{p_2}$ so that budget constraint tangent on indifference curve.

must be in terms of P_L & P_B ...

$-\frac{3}{2}L^2B^{-1} = -\frac{1}{2}$; $L = 100 - 2B$

Error carried over.

$\frac{3}{2}(100-2B)^2B^{-1} = \frac{1}{2}$, $3(10000 - 400B + 4B^2) = B$

$12B^2 - 1201B + 30000 = 0$ $B = \frac{625}{12}$ or 48

✗ as $L > 0$

So $B = 48$, $L = 4$.

d) Budget constraint: $m = Bp_B + Lp_L$. Same util. func. so $MRS_{LB} = -\frac{3}{2}L^2B^{-1}$

$B = \frac{m}{p_B} - \frac{p_L}{p_B}L$ so $\frac{3}{2}L^2B^{-1} = \frac{p_L}{p_B}$ and $L = \frac{m}{p_L} - \frac{p_B}{p_L}B$

$\frac{3}{2}\left(\frac{m}{p_L} - \frac{p_B}{p_L}B\right)^2B^{-1} = \frac{p_L}{p_B} = \frac{3}{2}\left(\frac{m^2}{p_L^2} - 2\frac{mp_B}{p_L^2}B + \frac{p_B^2}{p_L^2}B^2\right)B^{-1}$

$\frac{2}{3}B\frac{p_L^3}{p_B} = m^2 - 2mp_BB + p_B^2B^2$

$(p_B)^2B^2 + \left(\frac{2}{3}\frac{p_L^3}{p_B} - 2mp_B\right)B + m^2 = 0$ [this is a dead end, too many algebra slips await...]

Start by stating the optimizat. problem. Then use Lagrange.

[proper answer next page]

Let $V = f(U) = 3\ln L + 2\ln B$. Then $\mathcal{L}(B, L, \lambda) = 3\ln L + 2\ln B - \lambda(Bp_B + Lp_L - m)$

$0 = \frac{\partial \mathcal{L}}{\partial B} \Rightarrow 2B^{-1} = \lambda p_B$ (1); $\frac{\partial \mathcal{L}}{\partial L} \Rightarrow 3L^{-1} = \lambda p_L$ (2); $\frac{\partial \mathcal{L}}{\partial \lambda} \Rightarrow Bp_B + Lp_L = m$ (3)

(1) ÷ (2): $\frac{p_B}{p_L} = \frac{2}{3}\frac{L}{B}$

and (3): $L = \frac{m}{p_L} - \frac{p_B}{p_L}B$, so $\frac{2/B}{3/\left(\frac{m}{p_L} - \frac{p_B}{p_L}B\right)} = \frac{p_B}{p_L}$

i.e. $p_B = \frac{2}{3} \times \frac{B}{m - Bp_B}$; $B = \frac{2}{5}\frac{m}{p_B}$ ✓

d) Transform the utility function so that $u' = \log(U)$; $V = \frac{u'}{5}$

i.e. $V = \frac{2}{5}\ln B + \frac{3}{5}\ln L$. This is equivalent to U, as the monotonic transf.

Then $\frac{\partial V}{\partial B} = \frac{2}{5}B^{-1}$; $\frac{\partial V}{\partial L} = \frac{3}{5}L^{-1}$ and $\frac{MV_L}{MV_B}$ $\boxed{MRS_{LB} = -\frac{3}{2}\times\frac{B}{L}}$ $= -\frac{P_L}{P_B}$

ok, but why do you equate them?

With budget constraint, $m = p_B B + p_L L \Rightarrow L = \frac{m}{p_L} - \frac{p_B}{p_L}B$

should be the same as in part (a)

$\frac{3}{2}\times B = \frac{p_L}{p_B}\left(\frac{m}{p_L} - \frac{p_B}{p_L}B\right)$ $\therefore$ $\frac{3}{2}Bp_B = m - Bp_B$

i.e. $B = \frac{2}{5}\times\frac{m}{p_B}$, so we find that in the general case demand for books is proportional to the maximum number of books purchasable with the budget ($\frac{m}{p_B}$), with constant of proportionality taken directly from the coefficient of books in the logged C-D utility function. Demand for B is independent of P_L.

so could have been a way to spot your error.

e) $x_B = \frac{2}{5}\times\frac{m}{p_B}$; $x_L = \frac{3}{5}\frac{m}{p_L}$ $YED_x := \frac{\partial x}{\partial m}\times\frac{m}{x}$

$\frac{\partial x_B}{\partial m} = \frac{2}{5}\times\frac{1}{p_B}$ $\frac{\partial x_L}{\partial m} = \frac{3}{5}\times\frac{1}{p_L}$ $m = 100$, $x_B = 48$, $x_L = 4$

Correct.

$\frac{\partial x}{\partial m}\cdot\frac{m}{x} = \frac{2}{5}\frac{1}{P_B}\cdot\frac{m}{\left(\frac{2}{5}\frac{m}{P_B}\right)} = 1$

~~so $YED_B = \frac{2}{5}\times\frac{1}{20}\times\frac{1000}{48} = \frac{5}{12} < 1$ so books are a necessity.~~

~~$YED_L = \frac{3}{5}\times\frac{1}{10}\times\frac{1000}{4} = 15 > 1$ so luxuries are a luxury~~

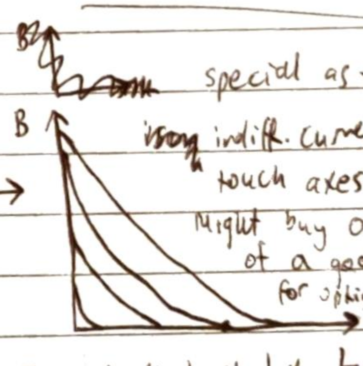
f) $MU_B = \frac{1}{\sqrt{B}}$; $MU_L = 1$, $MRS_{LB} = -\sqrt{B}$

budget: $B = 50 - \frac{1}{2}L$

So equating $MRS = -\frac{P_1}{P_2}$, $B = \frac{1}{4}$

(ignore) and so $L = 99.5 \Rightarrow V = 100.5$

$B = \left(\frac{U}{2}\right)^2 - 2U + L^2 \rightarrow$



special as the indiff. curves touch axes. Might buy 0 of a good for optimum.

But check the corners: at $B = 0$, $L = 100$, $U = 100$, at $L = 0$, $B = 50$, $V = 14.1$. No better.

So the optimum for the consumer is to buy 0.25 books and 99.5 luxuries.

In general case, $x_B = \frac{m}{p_B} - \frac{p_L}{p_B}x_L$ with budget, so equating tangents, $MRS = -\frac{p_L}{p_B}$

must deal with corners.

so $x_B = \left(\frac{p_L}{p_B}\right)^2$, substituting we get $x_L = \frac{m}{p_L} - \frac{p_L}{p_B}$ $\rightarrow$

If $\left(\frac{p_L}{p_B}\right)^2 > \frac{m}{p_L}$ then implied $x_L < 0$ which is impossible, so the demand is:

$x_B = \left(\frac{p_L}{p_B}\right)^2$ and $x_L = \frac{m}{p_L} - \left(\frac{p_L}{p_B}\right)^2$ if $\left(\frac{p_L}{p_B}\right)^2 < \frac{m}{p_L}$

and otherwise, $x_B = \frac{m}{p_B}$, $x_L = 0$

am I right to be doing elasticities at that particular point? they do change along curve, so couldn't evaluate in general?

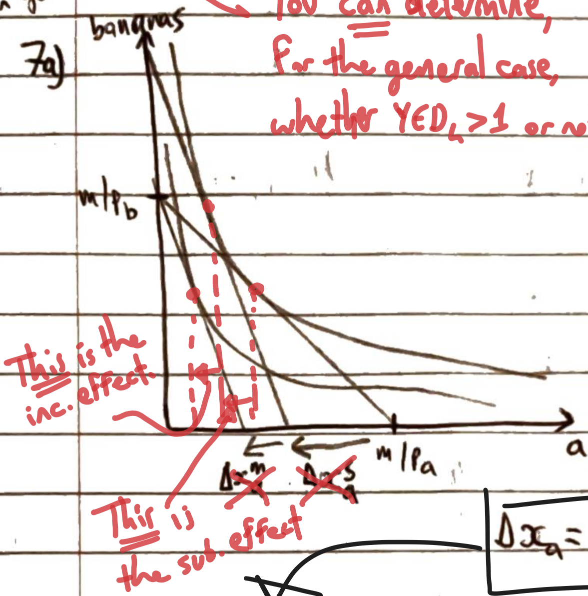
$\frac{\partial x_L}{\partial m} = \frac{1}{p_L}$ and $YED_L := \frac{\partial x_L}{\partial m} \times \frac{m}{x_L}$

So at the optimum earlier where $x_L = 99.5$,

$YED_L = \frac{1}{p_L} \times \frac{m}{x_L} = \frac{1}{10} \times \frac{1000}{99.5} = 1.005 > 1$ so (very weakly) luxury

You can determine, for the general case, whether $YED_L > 1$ or not.

7a)



① Change the ~~slope~~ budget constraint given the new higher value of p_a, holding utility constant. Quantity of apples falls. (Subs.)

② Now consider the change in purchasing power, given the new relative price level and holding that constant. Quantity of apples (and bananas) falls. (Income effect).

$$\boxed{\Delta x_a = \Delta x_a^s + \Delta x_a^m}$$

use more standard notation

b) Δx_a^s is always ≤ 0 since ~~demand~~ indifference curves are ~~convex and~~ downward-sloping. If ~~...~~ ~~negative~~ ~~non~~ YED_L is very negative, i.e. apples are a strongly inferior good, then * it may be that $\Delta x_a^m > |\Delta x_a^s|$, meaning $\Delta x_a > 0$ for an increase in price. In other words, the +ve income effect outweighs the -ve substitution effect. Apples would in this case have +ve own-price elasticity and be a Giffen good.

(after p↑ leads to m↓)

*Since $YED := \frac{\partial x}{\partial m}\frac{m}{x}$ and $\frac{\partial m}{\partial p_a} = -p_a x_a$

c) ~~$MRS_{ab} = \frac{p_a}{p_b}$ since optimising, so $= -1$ initially. And $MRS_{ab} = -\frac{MU_a}{MU_b}$ so $MU_a = MU_b$. Her effective income $m = 3p_a + 1p_b = 4$. If p_a now $= 2$, since therefore $MRS_{ab} = -2$~~

$\frac{\partial x_a}{\partial p_a} = \frac{\partial x}{\partial p_a}\Big|_u - \frac{\partial x_a}{\partial m} x_a$, with $\frac{\partial x_a}{\partial m} < 0$.

c) Her effective income is ~~initially~~ $3p_a + p_b$, initially $= 4$. At the new price, her income is effectively 7. At this new price level, she can afford ~~there~~ $2\times$ apples $+ 3\times$ bananas. With a well-behaved utility func this is weakly better than $2\times$ apples $+ 2\times$ bananas since she gains a banana. [She is a net supplier of ~~bananas~~ apples at initial eq. Does that entail a price increase helping her? Or the converse? Or neither?]

ok yes. Point is, the original bundle is still affordable so can't (*) be worse off.

d) Her effective income is now 2.5. She can afford to buy 2 apples and 1.5 bananas. ~~Knowing~~ Indifference curves are thin and don't cross, so she is on a lower indifference curve than initially ($2\times a + 1.5\times b$ is worse than $2\times a + 2\times b$) and hence worse off. → Depends... see in class.

8. $\left(\frac{\partial x_1}{\partial p_1} := \frac{\partial x_1}{\partial p_1}\Big|_u - \frac{\partial x_1}{\partial m} x_1 \right)$ $\left(\frac{\partial m}{\partial p_1} = -x_1 \right)$

(*) Hicksian vs Marshallian; compensated vs uncompens.

Properly deriving Slutsky

(From $x_1 = \beta \frac{m}{p_1}$, $\frac{\partial x_1}{\partial p_1} = -\beta \frac{m}{(p_1)^2}$) let $U(x_1, x_2) = x_1^{\beta} x_2^{1-\beta}$ as this is a Cobb-Douglas function.

and $\frac{\partial x_1}{\partial m} = \beta \frac{1}{p_1}$

So $\frac{\partial x_1}{\partial p_1}\Big|_u = -\beta \frac{m}{p_1^2} + \beta \frac{1}{p_1} \cdot \left(\beta \frac{m}{p_1} \right)$

Then $x_2 = (1-\beta) \frac{m}{p_2}$.

So $U(x_1, x_2) = \beta^{\beta} (1-\beta)^{1-\beta} \frac{m}{p_1^{\beta} p_2^{1-\beta}}$

??

Say p_1 ~~increases~~ changes by a factor λ, ~~m~~ to λp_1.

The substitution effect is x_1' holding U constant and letting m vary. To hold U at the original level, given only m and p_1 ~~have~~ changed, and $U \propto \frac{m}{p_1^{\beta}}$, m must increase by a factor λ^{β}. The new $x_1' = \beta \frac{\lambda^{\beta} m}{\lambda p_1} = \beta \frac{\lambda^{\beta-1} m}{p_1} = \lambda^{\beta-1} x_1$. So the substitution effect $\Delta^s x_1 = x_1 (1 - \lambda^{\beta-1})$

(Note that for all $\lambda > 1$, $\Delta^s x_1 < 0$)

The total effect Δx_1 is easily found as at the final equilibrium x_1^*, we have the same budget m as initially and hence $x_1^* = \beta \frac{m}{\lambda p_1}$; $\Delta x_1 = \beta \frac{m}{p_1} - \beta \frac{m}{\lambda p_1}$

Since $\Delta x_1 := \Delta^s x_1 + \Delta^m x_1$ where income effect is $\Delta^m x_1$, $\Delta^m x_1 = \Delta x_1 - \Delta^s x_1$

i.e. $\left(\beta \frac{m}{p_1} - \beta \frac{m}{\lambda p_1} \right) - \left(x_1 (1 - \lambda^{\beta-1}) \right)$ is the income effect.