

12) ~~Ans.~~

3b) Perhaps we can use the Samuelson condition here? ^{set} $\sum MRS_{12} = \cancel{MRS} - \frac{P_1}{P_2}$
 i.e. social benefit = social cost for a marginal unit of the good, at the optimal output level.

$$\frac{\partial u_S}{\partial D} = 5D^{-\frac{1}{2}}, \quad \frac{\partial u_S}{\partial M_S} = 1, \quad MRS_{Dm}^S = 5D^{-\frac{1}{2}}$$

$$\frac{\partial u_F}{\partial D} = -5(100-D)^{-\frac{1}{2}}, \quad \frac{\partial u_F}{\partial M_F} = 1, \quad MRS_{Dm}^F = -5(100-D)^{-\frac{1}{2}}$$

and $\cancel{MRS} - \frac{P_1}{P_2} = -\frac{0}{1} = 0$ so at the optimum,

$$5D^{-\frac{1}{2}} - 5(100-D)^{-\frac{1}{2}} = 0$$

Ha! Yes exactly but this yields exactly what we had from simply summing the individual utilities (not MRSs) in a social welfare function.
 Is that necessarily so for an obvious reason I'm missing entirely?

It works nicely in the quasilinear case.
 We'd need to be more careful if we had different utility functions.