1 Virtual Machine Abstract Syntax

$$
\begin{array}{llll}
S & ::= & d; S & \textit{statements} \\
  & | & e & \\
\\
d & ::= & \texttt{val}\ f = e & \textit{declarations} \\
  & | & \texttt{def}\ m(\overline{x : \tau}) : \tau = S & \\
  & | & \boxed{\texttt{type}\ t \Rightarrow z : \overline{\sigma}} & \\
  & | & \texttt{subtype}\ \tau\ \texttt{extends}\ \beta & \\
  & | & \boxed{\texttt{type}\ t = \tau} & \\
\\
p & ::= & x & \textit{paths} \\
  & | & p.f & \\
\\
e & ::= & p & \textit{expressions} \\
  & | & \texttt{new}\ x : \tau\ \{\overline{d}\} & \\
  & | & e.m(\overline{e}) & \\
  & | & \mathscr{L} & \\
\\
B & ::= & \leq & \textit{type bound} \\
  & | & \geq & \\
  & | & = & \\
\\
\mathscr{L} & ::= & n & \textit{literals} \\
  & | & \textit{unit} &
\end{array}
$$

$$
\begin{array}{llll}
\beta & ::= & \texttt{Unit} & \textit{base type} \\
  & | & \boxed{\bot} & \\
  & | & L & \\
  & | & \boxed{p.t} & \\
\\
\tau & ::= & \beta\{\overline{\sigma}\} & \textit{type} \\
\\
\sigma & ::= & \texttt{val}\ f : \tau & \textit{decl type} \\
  & | & \texttt{def}\ m(\overline{x : \tau}) : \tau & \\
  & | & \boxed{\texttt{type}\ t \Rightarrow z : \overline{\sigma}} & \\
  & | & \texttt{subtype}\ \tau\ \texttt{extends}\ \beta & \\
  & | & \boxed{\texttt{type}\ t\ B\ \tau} &
\end{array}
$$

Notation: overbar means a list of elements, as in Java

2 Standard prelude

```
type Int
    def +(i:Int):Int
    def −(i:Int):Int
    def ∗(i:Int):Int
    def /(i:Int):Int
```

3 Virtual Machine Typing Rules

$\boxed{\Gamma \vdash e : \tau}$

$$\frac{\Gamma \vdash d : \sigma \quad \Gamma, \sigma \vdash S : \tau}{\Gamma \vdash d; S : \tau} \text{ (T-Stmt)}$$

$$\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \text{ (T-Var)}$$

$$\boxed{\frac{\mathit{unfold}_\Gamma(\tau) = z : \overline{\sigma} \quad \Gamma \mid x : \tau \vdash \overline{d} : \overline{\sigma}}{\Gamma \vdash \texttt{new } x : \tau \ \{\overline{d}\} : \tau} \text{ (T-New)}}$$

$$\boxed{\frac{\Gamma \vdash e_1 : \tau \quad \mathit{unfold}_\Gamma(\tau) = z : \overline{\sigma} \quad \texttt{def } m(\overline{x_a : \tau_a}) : \tau_r \in \overline{\sigma} \quad \Gamma \vdash \overline{e_2} : \overline{\tau'} \quad \Gamma \vdash \overline{\tau'} <: [e_1/z]\overline{\tau_a}}{\Gamma \vdash e_1.m(\overline{e_2}) : [e_1, \overline{e_2}/z, \overline{x_a}]\tau_r} \text{ (T-Invk)}}$$

$$\boxed{\frac{\Gamma \vdash e : \tau \quad \mathit{unfold}_\Gamma(\tau) = z : \overline{\sigma} \quad \texttt{val } f : \tau_v \in \overline{\sigma}}{\Gamma \vdash e.f : [e/z]\tau_v} \text{ (T-Field)}}$$

$$\frac{}{\Gamma \vdash n : \mathit{Int}} \text{ (T-Int)}$$

$$\frac{}{\Gamma \vdash \mathit{unit} : \texttt{Unit}} \text{ (T-Unit)}$$

Technically Γ is a list of σ, but we often write $x : \tau$ for $\texttt{val } x : \tau$.

$\boxed{\Gamma \mid x : \tau \vdash d : \sigma}$

$$\frac{\Gamma, x{:}\tau \vdash \overline{\tau} \ \mathit{wf} \quad \Gamma, \ x{:}\tau, \ \overline{y{:}\tau} \vdash e : \tau_2' \quad \Gamma, x{:}\tau \vdash \tau_2' <: \tau_2}{\Gamma \mid x : \tau \vdash \texttt{def } m(\overline{y : \tau}) : \tau_2 = e \ : \ \texttt{def } m(\overline{y : \tau}) : \tau_2} \text{ (T-Def)}$$

$$\frac{\Gamma \vdash e : \tau}{\Gamma \mid x : \tau \vdash \texttt{val } f = e \ : \ \texttt{val } f : \tau} \text{ (T-Val)}$$

$$\boxed{\frac{\Gamma, z : t, x : \tau \vdash \overline{\sigma} \ \mathit{wf}}{\Gamma \mid x : \tau \vdash \texttt{type } t \Rightarrow z : \overline{\sigma} \ : \ \texttt{type } t \Rightarrow z : \overline{\sigma}} \text{ (T-Type)}}$$

$$\frac{\Gamma, x : \tau \vdash \texttt{subtype } \tau \texttt{ extends } \beta \ \mathit{wf}}{\Gamma \mid x : \tau \vdash \texttt{subtype } \tau \texttt{ extends } \beta : \texttt{subtype } \tau \texttt{ extends } \beta} \text{ (T-Subtype)}$$

$$\boxed{\frac{\Gamma \vdash \tau \ \mathit{wf}}{\Gamma \vdash \texttt{type } t = \tau : \texttt{type } t = \tau} \text{ (T-Type)}}$$

$\boxed{\Gamma \vdash \tau_1 <: \tau_2}$

$$\frac{\Gamma \vdash \overline{\sigma_1} <: \overline{\sigma_2}}{\Gamma \vdash \beta\{\overline{\sigma_1}\} <: \beta\{\overline{\sigma_2}\}}$$

$$\frac{\Gamma \vdash \tau_1 <: \tau_2 \quad \Gamma \vdash \tau_2 <: \tau_3}{\Gamma \vdash \tau_1 <: \tau_3}$$

$$\frac{}{\Gamma \vdash \tau <: \texttt{Unit}}$$

$$\boxed{\frac{}{\Gamma \vdash \bot <: \tau}}$$

$$\boxed{\frac{\Gamma \vdash \beta_1\{\overline{\sigma_1}\} <: \beta_2 \quad \mathit{unfold}_\Gamma(\beta_1\{\overline{\sigma_1}\}) = z : \overline{\sigma_3} \quad \Gamma, z : \beta_1\{\overline{\sigma_1}\} \vdash \overline{\sigma_3} <: \overline{\sigma_2}}{\Gamma \vdash \beta_1\{\overline{\sigma_1}\} <: \beta_2\{\overline{\sigma_2}\}}}$$

$$\boxed{\frac{\Gamma \vdash p : \tau_p \quad \mathit{unfold}_\Gamma(\tau_p) = z : \overline{\sigma_2} \quad \texttt{type}\ t \leqq \beta_t\{\overline{\sigma_t}\} \in \overline{\sigma_2} \quad \Gamma \vdash [p/z](\beta_t\{\overline{\sigma_t}\} \leftarrow \{\overline{\sigma_1}\}) <: \tau_2}{\Gamma \vdash p.t\{\overline{\sigma_1}\} <: \tau_2}}$$

$$\boxed{\frac{\Gamma \vdash p : \tau_p \quad \mathit{unfold}_\Gamma(\tau_p) = z : \overline{\sigma_2} \quad \texttt{type}\ t \geqq \beta_t\{\overline{\sigma_t}\} \in \overline{\sigma_2} \quad \Gamma \vdash \tau_1 <: [p/z](\beta_t\{\overline{\sigma_t}\} \leftarrow \{\overline{\sigma_1}\})}{\Gamma \vdash \tau_1 <: p.t\{\overline{\sigma_1}\}}}$$

$\boxed{\Gamma \vdash \beta_1\{\overline{\sigma}\} <: \beta_2}$ and $\boxed{\Gamma \vdash \overline{\sigma_1} <: \overline{\sigma_2}}$ and $\boxed{\Gamma \vdash \sigma_1 <: \sigma_2}$

$$\frac{}{\Gamma \vdash \beta\{\overline{\sigma}\} <: \beta}$$

$$\frac{\texttt{subtype}\ L\{\overline{\sigma_1'}\}\ \texttt{extends}\ \beta' \in \Gamma \quad \Gamma \vdash \overline{\sigma_1} <: \overline{\sigma_1'} \quad \Gamma \vdash \beta'\{\overline{\sigma_1}\} <: \beta}{\Gamma \vdash L\{\overline{\sigma_1}\} <: \beta}$$

$$\boxed{\frac{\Gamma \vdash p : \tau \quad \mathit{unfold}_\Gamma(\tau) = z : \overline{\sigma} \quad \texttt{subtype}\ p.t\{\overline{\sigma_1'}\}\ \texttt{extends}\ \beta' \in [p/z]\overline{\sigma} \quad \overline{\sigma_1} <: \overline{\sigma_1'} \quad \Gamma \vdash \beta'\{\overline{\sigma_1}\} <: \beta}{\Gamma \vdash p.t\{\overline{\sigma_1}\} <: \beta}}$$

$$\frac{\sigma_1' \subset \sigma_1 \quad \overline{\Gamma \vdash \sigma_1' <: \sigma_2}}{\Gamma \vdash \overline{\sigma_1} <: \overline{\sigma_2}}$$

$$\frac{\Gamma \vdash \tau <: \tau'}{\Gamma \vdash \texttt{val}\ f : \tau <: \texttt{val}\ f : \tau'}$$

$$\frac{\Gamma \vdash \tau_2 <: \tau_2' \quad \Gamma \vdash \overline{\tau_1'} <: \overline{\tau_1}}{\Gamma \vdash \texttt{def}\ m(\overline{x : \tau_1}) : \tau_2 <: \texttt{def}\ m(\overline{x : \tau_1'}) : \tau_2'}$$

$$\boxed{\frac{\overline{\sigma} = [z/z']\overline{\sigma'}}{\Gamma \vdash \texttt{type}\ t \Rightarrow z : \overline{\sigma} <: t \Rightarrow z' : \overline{\sigma'}}}$$

$$\boxed{\frac{}{\Gamma \vdash \texttt{type}\ t = \tau <: \texttt{type}\ t = \tau}}$$

$$\boxed{\frac{\Gamma \vdash \tau_1 <: \tau_2}{\Gamma \vdash \texttt{type}\ t \overset{\leq}{\equiv} \tau_1 <: \texttt{type}\ t \leq \tau_2}}$$

$$\boxed{\frac{\Gamma \vdash \tau_2 <: \tau_1}{\Gamma \vdash \texttt{type}\ t \overset{\geq}{\equiv} \tau_1 <: \texttt{type}\ t \geq \tau_2}}$$

$\boxed{\mathit{unfold}_\Gamma(\tau) = z : \overline{\sigma}}$

$$\frac{}{\mathit{unfold}_\Gamma(\texttt{Unit}) = z : \bullet}$$

$$\frac{\texttt{type}\ L : z : \overline{\sigma} \in \Gamma}{\mathit{unfold}_\Gamma(L) = z : \overline{\sigma}}$$

$$\frac{\Gamma \vdash p : \tau \quad \mathit{unfold}_\Gamma(\tau) = z' : \overline{\sigma'} \quad \texttt{type}\ t \Rightarrow z : \overline{\sigma} \in \overline{\sigma'}}{\mathit{unfold}_\Gamma(p.t) = z : [p/z']\overline{\sigma}}$$

$$\frac{\Gamma \vdash p : \tau \quad \mathit{unfold}_\Gamma(\tau) = z' : \overline{\sigma'} \quad \texttt{type}\ t \underset{=}{\leq} \tau' \in \overline{\sigma'} \quad \mathit{unfold}_\Gamma([p/z']\tau') = z : \overline{\sigma}}{\mathit{unfold}_\Gamma(p.t) = z : \overline{\sigma}}$$

$$\frac{\Gamma \vdash p : \tau \quad \mathit{unfold}_\Gamma(\tau) = z' : \overline{\sigma'} \quad \texttt{type}\ t \geq \tau' \in \overline{\sigma'}}{\mathit{unfold}_\Gamma(p.t) = z : \bullet}$$

$$\frac{\mathit{unfold}_\Gamma(\beta) = z : \overline{\sigma}}{\mathit{unfold}_\Gamma(\beta\{\overline{\sigma'}\}) = z : \overline{\sigma} \leftarrow \overline{\sigma'}}$$

Note: $\overline{\sigma} \leftarrow \overline{\sigma'}$ means that we append the two lists, except that when the same symbol is defined in both $\overline{\sigma}$ and $\overline{\sigma'}$, we include only the (overriding) definition in $\overline{\sigma'}$.

Now, finally, type and declaration type well-formedness rules:

$\boxed{\Gamma \vdash \tau\ \mathit{wf}}$

$$\frac{}{\Gamma \vdash \texttt{Unit}\ \ \mathit{wf}}$$

$$\frac{}{\Gamma \vdash \bot\ \mathit{wf}}$$

$$\frac{\mathit{unfold}_\Gamma(\beta) = z : \overline{\sigma}}{\Gamma \vdash \beta\ \mathit{wf}}$$

$$\frac{\Gamma \vdash \beta\ \mathit{wf} \quad \mathit{unfold}_\Gamma(\beta\{\overline{\sigma}\}) = z : \overline{\sigma_1} \quad \mathit{unfold}_\Gamma(\beta) = z : \overline{\sigma_2} \quad \Gamma, z : \beta\{\overline{\sigma}\} \vdash \overline{\sigma_1} <: \overline{\sigma_2}}{\Gamma \vdash \beta\{\overline{\sigma}\}\ \mathit{wf}}$$

$$\boxed{\Gamma \vdash \sigma\ wf}$$

$$\frac{\Gamma \vdash \overline{y : \tau_1}\ wf \quad \Gamma, \overline{y : \tau_1} \vdash \tau_2\ wf}{\Gamma \vdash \texttt{def}\ m(\overline{y : \tau_1}) : \tau_2\ wf}$$

$$\frac{\Gamma \vdash \tau\ wf}{\Gamma \vdash \texttt{val}\ f : \tau\ wf}$$

$$\boxed{\frac{\Gamma, z : t, \texttt{type}\ t \Rightarrow z : \overline{\sigma} \vdash \overline{\sigma}\ wf}{\Gamma \vdash \texttt{type}\ t \Rightarrow z : \overline{\sigma}\ wf}}$$

$$\boxed{\frac{unfold_{\Gamma}(\tau) = z : \overline{\sigma} \quad unfold_{\Gamma}(\beta) = z' : \overline{\sigma'} \quad \Gamma, z : \tau, \texttt{subtype}\ \tau\ \texttt{extends}\ \beta \vdash \overline{\sigma} <: [z/z']\overline{\sigma'}}{\Gamma \vdash \texttt{subtype}\ \tau\ \texttt{extends}\ \beta\ wf}}$$

4 Proposed Additional Edges in Type Graph

For each type member declaration `type` $t\ B\ \beta\{\overline{\sigma}\}$ in the definition of type L, add the following edge:

$$\boxed{\frac{\overline{\beta} = \beta_1, \beta_2, ..., \beta_n \quad \Gamma \vdash L\{\overline{\sigma'}\} <: \beta_i}{L \xrightarrow{\overline{\beta}} L :: t}}$$