cogsci 131 - assignment7

March 20, 2020

[2]:
```python
import os
import numpy as np
import matplotlib.pyplot as plt
```

1 Cogsci 131 - Assignment 7

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1.1 Question 1

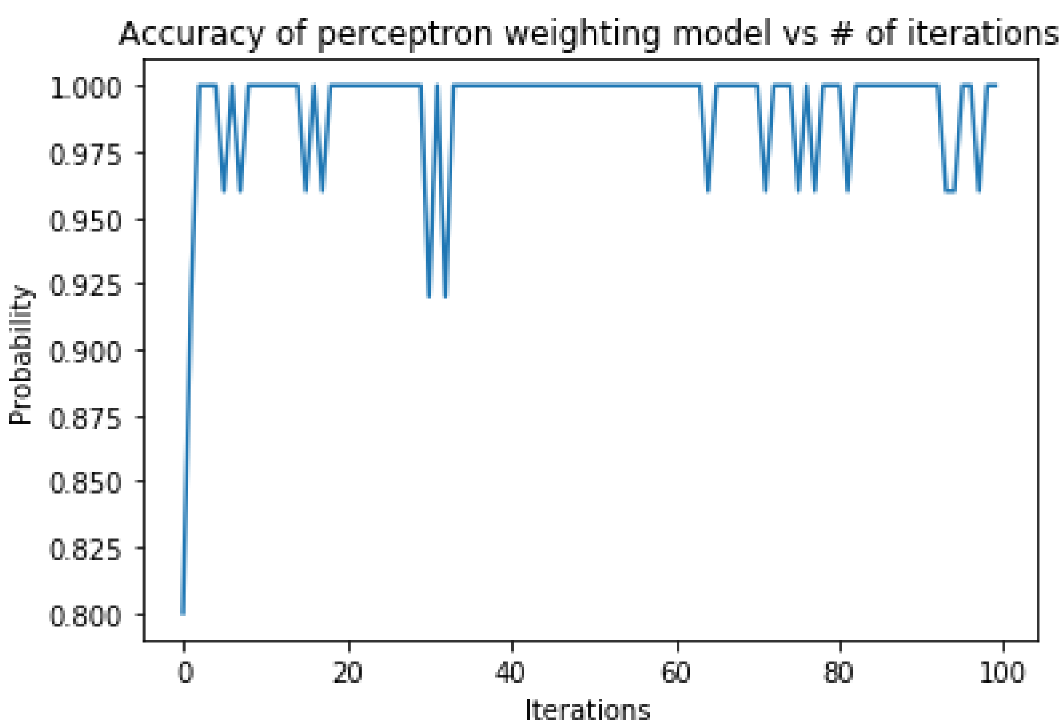
[7]:
```python
import os
import numpy

# Functions that might be useful (please read the documentation)
# x.flatten() (take a N-dimensional numpy array and make it one-dimensional)
# numpy.random.choice -- choose from the list of images
# numpy.dot -- compute the dot product
# numpy.random.normal -- set up random initial weights

DIM = (28,28) #these are the dimensions of the image

def load_image_files(n, path="images/"):
    # helper file to help load the images
    # returns a list of numpy vectors
    images = []
    for f in os.listdir(os.path.join(path,str(n))): # read files in the path
        p = os.path.join(path,str(n),f)
        if os.path.isfile(p):
            i = numpy.loadtxt(p)
            assert i.shape == DIM # just check the dimensions here
            # i is loaded as a matrix, but we are going to flatten it into a␣
↪single vector
            images.append(i.flatten())
    return images

# Load up these image files
A = load_image_files(0)
```

```
B = load_image_files(1)

N = len(A[0]) # the total size
assert N == DIM[0]*DIM[1] # just check our sizes to be sure

# set up some random initial weights
weights = numpy.random.normal(0,1,size=N)
```

[9]:
```
#using section template
# y == output, x = randomly selected, o = output of perceptron
average_accuracy = []
weights = numpy.random.normal(0,1,size=N)
##add 2 matrices together to randomly select
zero_and_one = A + B
j = 0

while j < 100:
    correct = 0
    selected_array = []
    #select the positions of the files to pick
    actual_digits = []
    ##these will also be the indexes for which one we pick
    selection = np.random.choice(range(len(zero_and_one)), 25)
    ##combine images into one arrray
    for i in selection:
        selected_array.append(zero_and_one[i])
    ##what is the true value?
    ## we know that the left half of the array equals digit 0, and right half␣
↪of array equals digit 1
    for i in selection:
        if i < len(A):
            actual_digit = 0
            actual_digits.append(actual_digit)
        else:
            actual_digit = 1
            actual_digits.append(actual_digit)
#few iterations and see the trend of accuracy
    for i in range(25):
        x = selected_array[i]
        y = actual_digits[i]
        #input; xi*wi for all
        sum_of_inputs_into_weights = np.dot(weights, x)
        if sum_of_inputs_into_weights >= 0:
            o = 1
        else:
            o = 0
        if o == 1 and y == 0: # where o is the output of the perceptron
```

```
            weights -= x
        elif o == 0 and y == 1:
            weights += x
        #both are 0 or 1, it is correct
        else:
            correct += 1
    accuracy = correct/25
    #for each chunk of 25
    average_accuracy.append(accuracy)
    j += 1
#plot formatting
plt.plot(average_accuracy)
plt.title("Accuracy of perceptron weighting model vs # of iterations")
plt.xlabel("Iterations")
plt.ylabel("Probability")
```

[9]: Text(0, 0.5, 'Probability')

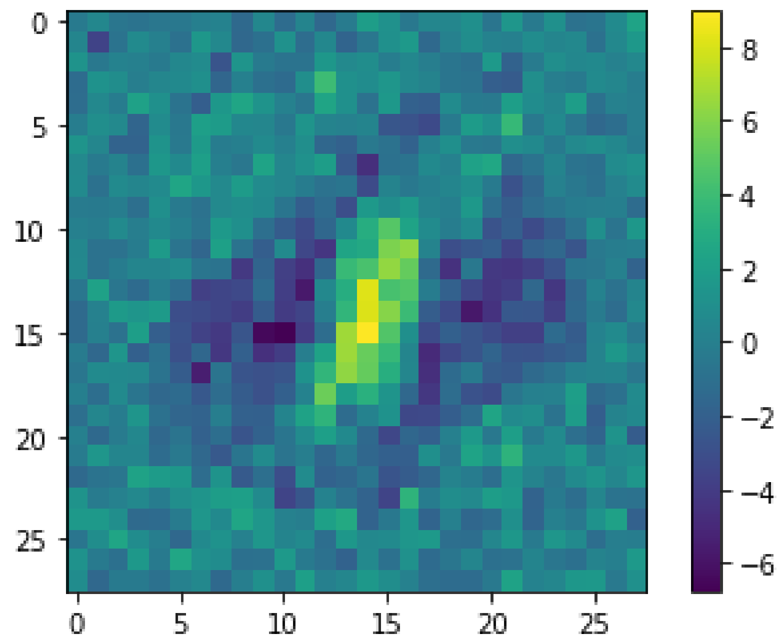


1.2 Question 2

My code does converge to 100%: we see that there are a few dips below 100%, but the majority of the trials do go to a percentage close to 100%, especially at the end. This means that 0 and 1 can be linearly separable in this particular feature space, and that this is a linear binary classifier.

1.3 Question 3

[237]:
```
reshape = np.reshape(weights, (28, 28))
plt.imshow(reshape)
plt.colorbar();
```



This graph shows the weights for each square on the graph. The large negative values (darker blue squares) are the ones that are associated to the 0 digit, and the large positive values (yellow squares) are the ones that are associated with the 1 digit. Alternatively, the squares that correspond to the value 0 mean that the weight isn't associated much with either 0 or 1. We can see that out of all of the weights, the ones in the center are more closely correlated with the 1 digit, and the squares surrounding the yellow line are more negative and thus correspond to the 0 digit. This matrix kind of looks like the digits that we were training on in the beginning: The dark blue is in a circle, which is in the same shape as a 0, and the middle line is a 1, which looks like a 1.

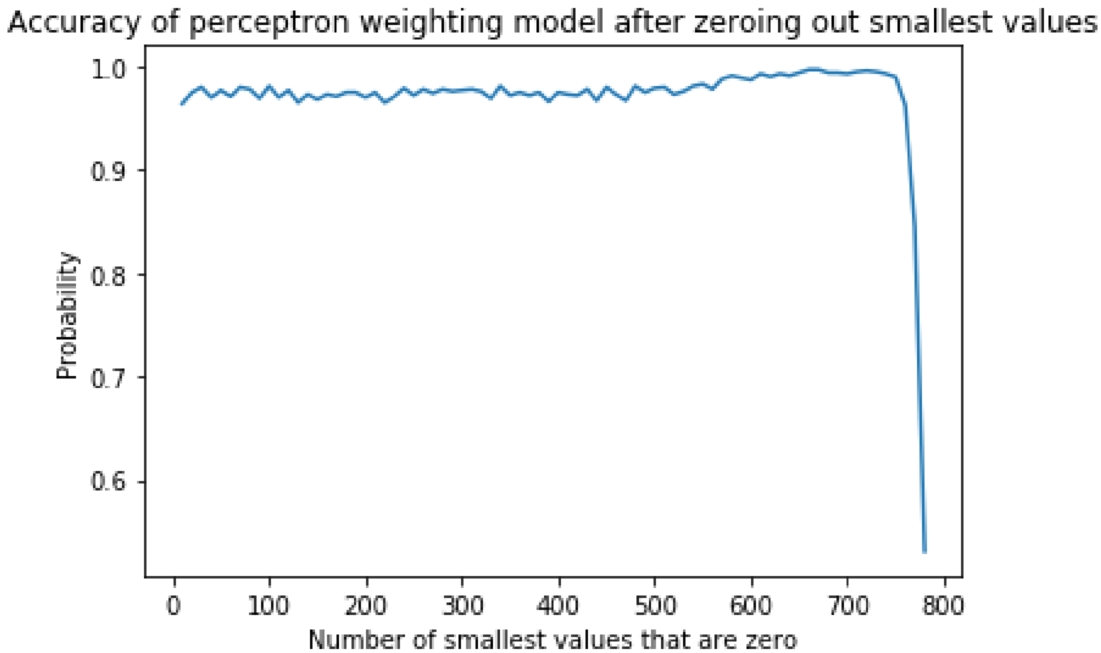
1.4 Question 4

If I set the weight elements of the vector to be actually zero, then I would still expect to see a pretty high level of accuracy. In the graph below we see that this is true; however, at the largest values, we see a dramatic decrease in accuracy.

[24]:
```
weight_to_zero = range(10, 790, 10)
accuracy_4 = []
for i in weight_to_zero:
    #sets them equal to zero
    new_weights = np.sort(abs(weights))
    for j in range(i):
```

```
        new_weights[j] = 0
    #same as question 2
    correct = 0
    selected_array = []
    #select the positions of the files to pick
    actual_digits = []
    #indexes
    selection = np.random.choice(range(len(zero_and_one)), 1000)
    ##to get all the images into one array
    for k in selection:
        selected_array.append(zero_and_one[k])
    ## what is the true value? derived from the combined array
    ## we know that the left half of the array equals digit 0, and right half␣
 ↪of array equals digit 1
    for l in selection:
        if l < len(A):
            actual_digit = 0
            actual_digits.append(actual_digit)
        else:
            actual_digit = 1
            actual_digits.append(actual_digit)
#few iterations and see the trend of accuracy
##counting accuracies
    for m in range(1000):
        x = selected_array[m]
        y = actual_digits[m]
        #input; xi*wi for all
        sum_of_inputs_into_weights = np.dot(new_weights, x)
        if sum_of_inputs_into_weights >= 0:
            o = 1
        else:
            o = 0
        if (o == 1 and y == 1):
            correct += 1
        elif (o==0 and y == 0):
            correct += 1
    accuracy = correct/1000
    accuracy_4.append(accuracy)
#plot formatting
plt.plot(weight_to_zero, accuracy_4)
plt.title("Accuracy of perceptron weighting model after zeroing out smallest␣
 ↪values")
plt.xlabel("Number of smallest values that are zero")
plt.ylabel("Probability")
```

[24]: Text(0, 0.5, 'Probability')



Here, we see that the accuracy still hovers around 1 no matter how many values closest to zero that I set to zero. However, we see a dramatic decrease once we zero out the first ~750 weights. That means only a small proportion of weights and squares are the ones critical towards determining whether or not the number is zero or one.

1.5 Question 5

[21]:
```
##setting up each digit to be loaded
C = load_image_files(2)
D = load_image_files(3)
E = load_image_files(4)
F = load_image_files(5)
G = load_image_files(6)
H = load_image_files(7)
I = load_image_files(8)
J = load_image_files(9)
```

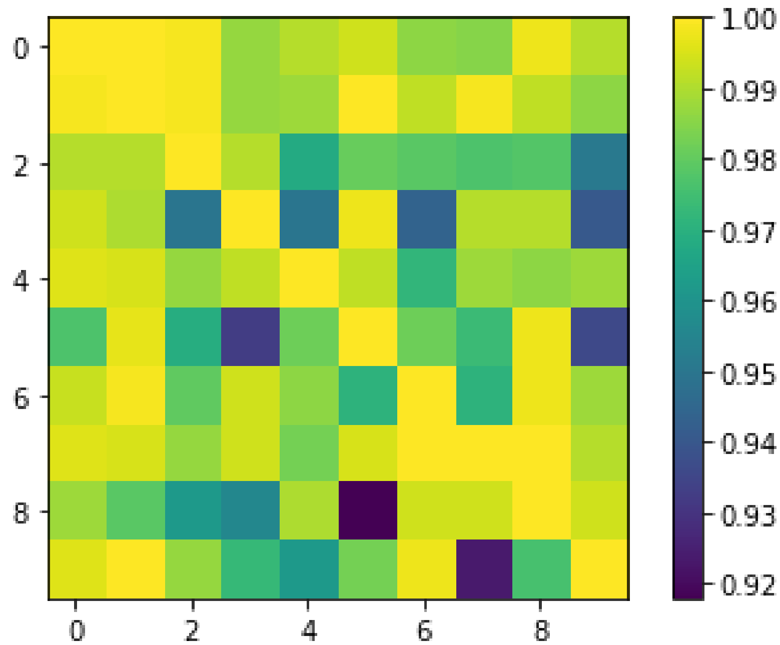
[33]:
```
#dictionary mapping letter to digit
d = {0:A,1:B,2:C,3:D,4:E,5:F,6:G,7:H,8:I,9:J}
accuracy_5 = []
for i in range(10):
    #array to fill in times where i==j
    arr = []
    for j in range(10):
        #if it's the same number, then accuracy = 1
```

```python
        if i == j:
            arr.append(1)
        else:
            #same algorithm as in part 2
            average_accuracy = []
            weights = np.random.normal(0,1,size = N)
            #get a long array of both pairs of digits
            combined = d[i] + d[j]
            for k in range(50):
                correct = 0
                selected_array = []
                #select the positions of the files to pick
                actual_digits = []
                #where we get our indexes
                selection = np.random.choice(range(len(combined)), 1000)
                ##to get all the images into one array
                for k in selection:
                    selected_array.append(combined[k])
                ## what is the true value? derived from the combined array
                ## we know that the left half of the array equals digit i, and right half of array equals digit j
                if i < j:
                    for l in selection:
                        if l < len(d[i]):
                            actual_digit = i
                            actual_digits.append(actual_digit)
                        else:
                            actual_digit = j
                            actual_digits.append(actual_digit)
                # if j>i then reverse logic applies
                elif i > j:
                    for l in selection:
                        if l < len(d[i]):
                            actual_digit = j
                            actual_digits.append(actual_digit)
                        else:
                            actual_digit = i
                            actual_digits.append(actual_digit)
                #weight training
                for m in range(1000):
                    x = selected_array[m]
                    y = actual_digits[m]
                    sum_of_inputs_into_weights = np.dot(weights, x)
                    #input; xi*wi for all
                    if sum_of_inputs_into_weights >= 0:
                        o = 1
                    else:
```

```
                    o = 0
                if o == 1 and y == 0: # where o is the output of the␣
↪perceptron
                    weights -= x
                elif o == 0 and y == 1:
                    weights += x
                #both are 0 or 1, it is correct
                else:
                    correct += 1
            accuracy = correct/1000
            #for each chunk of 1000
            average_accuracy.append(accuracy)
        #append the final result
        arr.append(average_accuracy[-1])
    #if i == j just append p=1
    accuracy_5.append(arr)
```

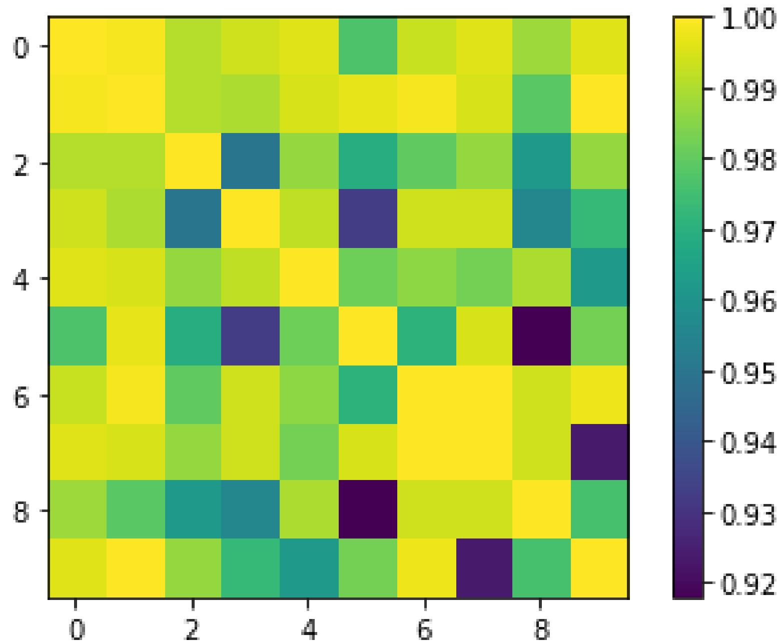
[46]:
```
plot_5 = np.reshape(accuracy_5, (10, 10))
```

[49]:
```
#no correction for symmetry
plt.imshow(plot_5)
plt.colorbar();
```



[54]:
```
#correction for symmetry
```

```
#note these probabilities are all pretty similar anyways, but for aesthetic␣
↪purposes I've
#added it here
plt5 = np.array(accuracy_5)
plt5 = np.reshape(plt5, (10, 10))
#goes through every row and column in order to mirror the image
for i in range(0, 10):
    for j in range(i + 1, 10):
        plt5[i][j] = plt5[j][i]
plt.imshow(plt5)
plt.colorbar();
```



This plot does match my intuitions as to which pairs are easiest and the hardest. Besides comparing a number to itself, we see that the easiest to distinguish are numbers that have different shapes, such as 6 and 9, or 5 and 1. These pairings have much higher rates of accuracy. Conversely, the squares with the lowest accuracy, such as 5 and 8, or 7 and 9, are much harder to recognize because they share a lot of the same features and look somewhat alike.