cogsci 131 - assignment5

March 5, 2020

[3]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import random
```

1 CogSci 131 - Assignment 5

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1.1 Question 1

The first way I could convert these to distances would be to subtract the similarity from 1 and do that for every cell in the row. The second way to convert these into distances would be to take the inverse of the similarity, i.e. 1/similarity, to find the distance. The third way to convert would be to take the logarithm of 1-similarity.

There is an inverse relationship between similarity and the distance: the more similar something is (closer to 1), the less distance there is between then. All of these methods preserve this negative relationship, but taking the inverse and having an arbitrary reference point may not be the most accurate nor optimal way to do so. The inverse may create some hyperbolic and polynomial curves that won't be best for regression fitting, and the logarithm may lead to distorted graphs and scaling issues.

For this assignment, I will use "1 minus similarity" definition, which is the most efficient and consistent way to find the distances because they all retain the same scale. Furthermore, a linear transformation like this will prevent any strange polynomial errors or approximations.

[13]:
```
#converting similarities to distances

similarities = np.loadtxt(open("Assignment5-similarities.csv", "rb"),␣
 ↪delimiter=",", skiprows=1)

testdistances = [1-x for x in similarities]
distances = [1-x for x in similarities]
testdistances
```

[13]:
```
[array([0.        , 0.81481481, 0.51851852, 0.85185185, 0.25925926,
        0.92592593, 0.22222222, 0.22222222, 0.11111111, 0.88888889,
```

```
        0.85185185, 0.59259259, 0.14814815, 0.07407407, 0.2962963 ,
        0.96296296, 1.        , 0.25925926, 0.92592593, 0.51851852,
        0.96296296]),
 array([0.81481481, 0.        , 0.66666667, 0.81481481, 0.03703704,
        1.        , 0.        , 0.07407407, 0.40740741, 0.37037037,
        1.        , 0.59259259, 0.77777778, 0.62962963, 0.18518519,
        1.        , 0.2962963 , 0.22222222, 0.88888889, 0.92592593,
        1.        ]),
 array([0.51851852, 0.66666667, 0.        , 0.40740741, 0.11111111,
        0.85185185, 0.22222222, 0.25925926, 0.37037037, 0.81481481,
        0.88888889, 0.92592593, 0.33333333, 0.11111111, 0.59259259,
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        0.81481481]),
 array([0.85185185, 0.81481481, 0.40740741, 0.        , 0.11111111,
        0.88888889, 0.18518519, 0.48148148, 0.55555556, 0.85185185,
        0.55555556, 0.59259259, 0.85185185, 0.55555556, 0.85185185,
        0.81481481, 0.14814815, 0.37037037, 0.92592593, 0.81481481,
        0.7037037 ]),
 array([0.25925926, 0.03703704, 0.11111111, 0.11111111, 0.        ,
        1.        , 0.48148148, 0.92592593, 0.18518519, 0.51851852,
        0.96296296, 0.96296296, 0.88888889, 0.25925926, 0.66666667,
        0.96296296, 0.74074074, 0.33333333, 0.92592593, 0.77777778,
        0.88888889]),
 array([0.92592593, 1.        , 0.85185185, 0.88888889, 1.        ,
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        0.18518519, 0.62962963, 0.81481481, 0.96296296, 0.88888889,
        0.37037037, 0.66666667, 0.77777778, 0.11111111, 0.74074074,
        0.88888889]),
 array([0.22222222, 0.        , 0.22222222, 0.18518519, 0.48148148,
        0.88888889, 0.        , 0.88888889, 0.74074074, 0.96296296,
        0.88888889, 0.55555556, 0.88888889, 0.14814815, 0.85185185,
        1.        , 0.92592593, 0.33333333, 1.        , 0.88888889,
        0.81481481]),
 array([0.22222222, 0.07407407, 0.25925926, 0.48148148, 0.92592593,
        0.92592593, 0.88888889, 0.        , 0.37037037, 0.88888889,
        0.96296296, 0.77777778, 0.74074074, 0.81481481, 0.07407407,
        0.85185185, 0.48148148, 0.25925926, 1.        , 0.18518519,
        0.92592593]),
 array([0.11111111, 0.40740741, 0.37037037, 0.55555556, 0.18518519,
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        1.        , 1.        , 0.59259259, 0.51851852, 0.66666667,
        0.66666667, 0.96296296, 0.77777778, 0.96296296, 0.55555556,
        0.92592593]),
 array([0.88888889, 0.37037037, 0.81481481, 0.85185185, 0.51851852,
        0.92592593, 0.96296296, 0.88888889, 0.03703704, 0.        ,
        0.66666667, 0.81481481, 0.59259259, 0.37037037, 0.48148148,
        0.55555556, 1.        , 0.33333333, 0.96296296, 0.7037037 ,
```

```
       0.96296296]),
array([0.85185185, 1.        , 0.88888889, 0.55555556, 0.96296296,
       0.18518519, 0.88888889, 0.96296296, 1.        , 0.66666667,
       0.        , 0.51851852, 0.96296296, 0.85185185, 0.85185185,
       0.07407407, 0.96296296, 0.88888889, 0.03703704, 0.66666667,
       0.22222222]),
array([0.59259259, 0.59259259, 0.92592593, 0.59259259, 0.96296296,
       0.62962963, 0.55555556, 0.77777778, 1.        , 0.81481481,
       0.51851852, 0.        , 0.92592593, 0.7037037 , 0.48148148,
       0.59259259, 0.74074074, 0.85185185, 0.44444444, 0.88888889,
       0.37037037]),
array([0.14814815, 0.77777778, 0.33333333, 0.85185185, 0.88888889,
       0.81481481, 0.88888889, 0.74074074, 0.59259259, 0.59259259,
       0.96296296, 0.92592593, 0.        , 0.96296296, 0.88888889,
       0.96296296, 0.92592593, 1.        , 0.92592593, 0.22222222,
       0.7037037 ]),
array([0.07407407, 0.62962963, 0.11111111, 0.55555556, 0.25925926,
       0.96296296, 0.14814815, 0.81481481, 0.51851852, 0.37037037,
       0.85185185, 0.7037037 , 0.96296296, 0.        , 0.14814815,
       0.96296296, 0.92592593, 0.44444444, 0.92592593, 0.92592593,
       0.66666667]),
array([0.2962963 , 0.18518519, 0.59259259, 0.85185185, 0.66666667,
       0.88888889, 0.85185185, 0.07407407, 0.66666667, 0.48148148,
       0.85185185, 0.48148148, 0.88888889, 0.14814815, 0.        ,
       0.96296296, 0.55555556, 0.55555556, 0.92592593, 0.48148148,
       0.81481481]),
array([0.96296296, 1.        , 0.96296296, 0.81481481, 0.96296296,
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       0.07407407, 0.59259259, 0.96296296, 0.96296296, 0.96296296,
       0.        , 0.88888889, 0.92592593, 0.11111111, 0.96296296,
       0.74074074]),
array([1.        , 0.2962963 , 0.85185185, 0.14814815, 0.74074074,
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       0.96296296, 0.74074074, 0.92592593, 0.92592593, 0.55555556,
       0.88888889, 0.        , 0.18518519, 0.92592593, 0.59259259,
       0.81481481]),
array([0.25925926, 0.22222222, 0.85185185, 0.37037037, 0.33333333,
       0.77777778, 0.33333333, 0.25925926, 0.77777778, 0.33333333,
       0.88888889, 0.85185185, 1.        , 0.44444444, 0.55555556,
       0.92592593, 0.18518519, 0.        , 0.88888889, 0.88888889,
       0.92592593]),
array([0.92592593, 0.88888889, 0.96296296, 0.92592593, 0.92592593,
       0.11111111, 1.        , 1.        , 0.96296296, 0.96296296,
       0.03703704, 0.44444444, 0.92592593, 0.92592593, 0.92592593,
       0.11111111, 0.92592593, 0.88888889, 0.        , 0.96296296,
       0.66666667]),
array([0.51851852, 0.92592593, 0.77777778, 0.81481481, 0.77777778,
```

```
        0.74074074, 0.88888889, 0.18518519, 0.55555556, 0.7037037 ,
        0.66666667, 0.88888889, 0.22222222, 0.92592593, 0.48148148,
        0.96296296, 0.59259259, 0.88888889, 0.96296296, 0.        ,
        0.37037037]),
 array([0.96296296, 1.        , 0.81481481, 0.7037037 , 0.88888889,
        0.88888889, 0.81481481, 0.92592593, 0.92592593, 0.96296296,
        0.22222222, 0.37037037, 0.7037037 , 0.66666667, 0.81481481,
        0.74074074, 0.81481481, 0.92592593, 0.66666667, 0.37037037,
        0.        ])]
```

1.2 Question 2

[14]:
```
#euclidean distance function
def euclid_distance(x, y):
    # x = [i, j], y=[u, v]
    return ((x[1]-x[0])**2 + (y[1] - y[0])**2)**0.5

#computing stress
#equation: sum of (psi - dist(pi, pj))^2
def stress(positions):
    summation = []
    #iterating over each row and column
    for i in np.arange(len(positions)):
        #j is 2d
        for j in np.arange(1):
            #ignoring diagonals
            if i != j:
                summation.append((distances[i][j] -␣
 ↪euclid_distance(positions[i], positions[j]))**2)
            else:
                pass
    return np.cumsum(summation)[-1]

#values: 2 dimensions, 21 sports
testpoints = np.random.uniform(0,1,size=(21,2))
#test
stress(testpoints)
```

[14]: 2.813157081812429

1.3 Question 3

[15]:
```
##'helper' functions to make the final computation easier
#function to add deltas
def delta_helper(positions, i, j, delt):
    #turn matrix into an array
```

```
    array = np.array(positions)
    #add the change in delta
    array[i, j] = array[i,j] + delt
    return array

#gradient for one individual point at position i and j
def gradient(positions, i, j, delta):
    plus_delta = stress(delta_helper(positions, i, j, delta))
    #multiplying delta by -1 will subtract delta off
    minus_delta = stress(delta_helper(positions, i, j, -1*delta))
    gradient_result = (plus_delta - minus_delta)/(2*delta)
    return gradient_result
#test
gradient(testpoints, 5, 0, 0.01)
```

[15]: 0.02488808451643898

[16]:
```
#gradient of all of the points
def gradient_all(positions):
    #blank array
    gradient_array = np.empty([21,2])
    for i in np.arange(len(positions)):
        # replace empty array dimension poisition with the **calculated
 ↪gradient**. iterating over rows
        #gradient in x
        grad_x = gradient(positions, i, 0, 0.01)
        #gradient in y
        grad_y = gradient(positions, i, 1, 0.01)
        gradient_array[i] = [grad_x, grad_y]
    return gradient_array
#test
gradient_all(testpoints)
```

[16]:
```
array([[ 2.81205456, -2.81205456],
       [-0.11851094,  0.11851094],
       [-0.08607281,  0.08607281],
       [ 0.45744646, -0.45744646],
       [ 0.01347761, -0.01347761],
       [ 0.02488808, -0.02488808],
       [-0.65381248,  0.65381248],
       [ 0.52468218, -0.52468218],
       [ 0.46387511, -0.46387511],
       [-0.50312891,  0.50312891],
       [ 0.44078332, -0.44078332],
       [ 0.15886737, -0.15886737],
       [-1.02129437,  1.02129437],
       [-0.03524393,  0.03524393],
```

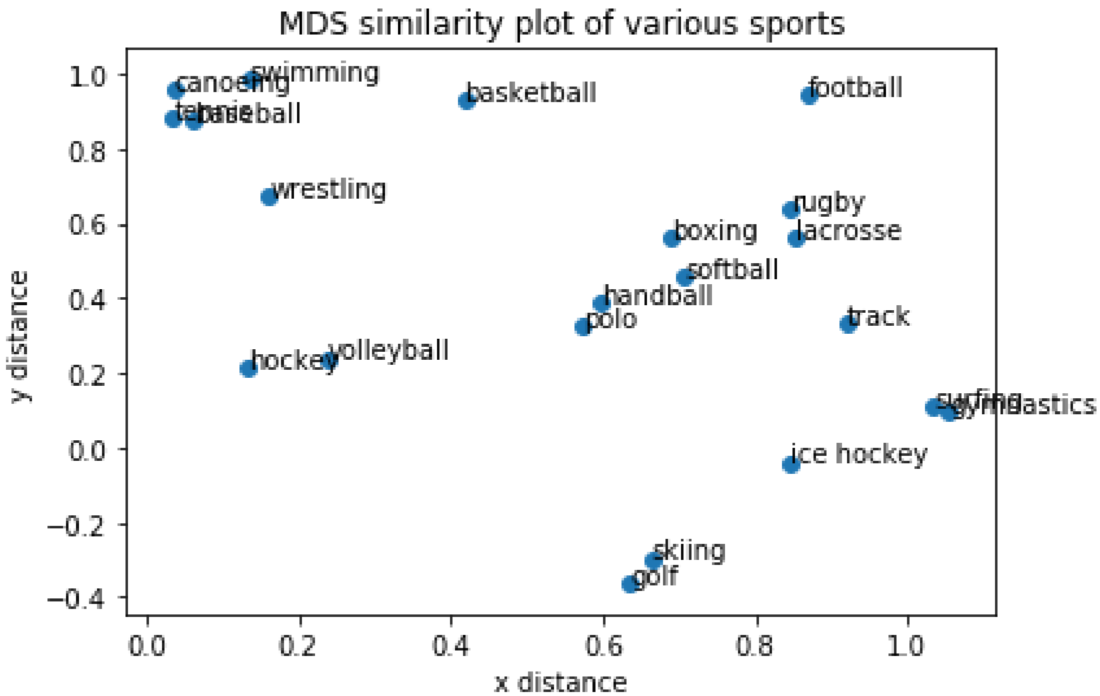
```
       [ 0.70785458, -0.70785458],
       [-0.03593102,  0.03593102],
       [-0.08902916,  0.08902916],
       [ 0.01515623, -0.01515623],
       [ 0.17492891, -0.17492891],
       [-0.07495241,  0.07495241],
       [ 0.60552865, -0.60552865]])
```

1.4 Question 4

[18]:

```
#mds function
def min_mds():
    #generate actual points and distance matrices inside function
    points = np.random.uniform(0,1,size=(21,2))
    distances = [1-x for x in similarities]
    #labels for sports
    sport = ["football", "baseball", "basketball", "tennis", "softball",␣
 ↪"canoeing",
             "handball", "rugby", "hockey", "ice hockey", "swimming", "track",␣
 ↪"boxing",
             "volleyball", "lacrosse", "skiing", "golf", "polo", "surfing",␣
 ↪"wrestling", "gymnastics"]
    #following gradient downhill to minimize stress as per lecture
    #scaling it down
    for i in np.arange(100):
        for point in points:
            points -= gradient_all(points)*0.01
    #appending values
    x_vals=[]
    y_vals=[]
    for position in points:
        x_vals.append(position[0])
        y_vals.append(position[1])
    #plot format
    plt.figure()
    plt.title('MDS similarity plot of various sports')
    plt.xlabel('x distance')
    plt.ylabel('y distance')
    plt.scatter(x_vals, y_vals)
    #adding labels
    for i, sport in enumerate(sport):
        plt.annotate(sport, (x_vals[i], y_vals[i]))

min_mds()
```



These results somewhat are in alignment with my intution: sports that are more similar to each other tend to be near the same area, while radically different sports are on opposite ends of the graph. While I may not personally agree that some of these sports are more similar than others and where they are placed in relation to each other, it gives a very rough estimate of where things are placed in the MDS space.

1.5 Question 5

[ ]:
```
#pairwise distances of mds--same thing as q4 but finding the distances

def dist(x, y):
    return (y**2 - x**2)**0.5

def mds_q5():
    #generate actual points and distance matrices inside function
    points = np.random.uniform(0,1,size=(21,2))
    distances = [1-x for x in similarities]
    #labels for sports
    sport = ["football", "baseball", "basketball", "tennis", "softball",␣
 ↪"canoeing",
          "handball", "rugby", "hockey", "ice hockey", "swimming", "track",␣
 ↪"boxing",
          "volleyball", "lacrosse", "skiing", "golf", "polo", "surfing",␣
 ↪"wrestling", "gymnastics"]
```

```
    #following gradient downhill to minimize stress as per lecture
    #scaling it down
    for i in np.arange(100):
        for point in points:
            points -= gradient_all(points)*0.01
##calculating pairwise distance
    x_vals=[]
    y_vals=[]
    for position in points:
        x_vals.append(position[0])
        y_vals.append(position[1])
    pairwise_dist = []
    for i in np.arange(len(x_vals)):
        pairwise_dist.append(dist(x_vals[i], y_vals[i]))
    print(pairwise_dist)
    psych_dist = []
    for i in np.arange(len(points)):
        for j in np.arange(1):
            #ignoring diagonals
            if i != j:
                psych_dist.append((distances[i][j]))
#     #plot format
    plt.figure()
    plt.title('Pairwise distances vs psychological distances')
    plt.xlabel('psychlogical distances')
    plt.ylabel('pairwise distances')
    plt.scatter(psych_dist, pairwise_dist)
    #adding labels
    for i, sport in enumerate(sport):
        plt.annotate(sport, (psych_dist[i], pairwise_dist[i]))
    #iterating over each row and column
```

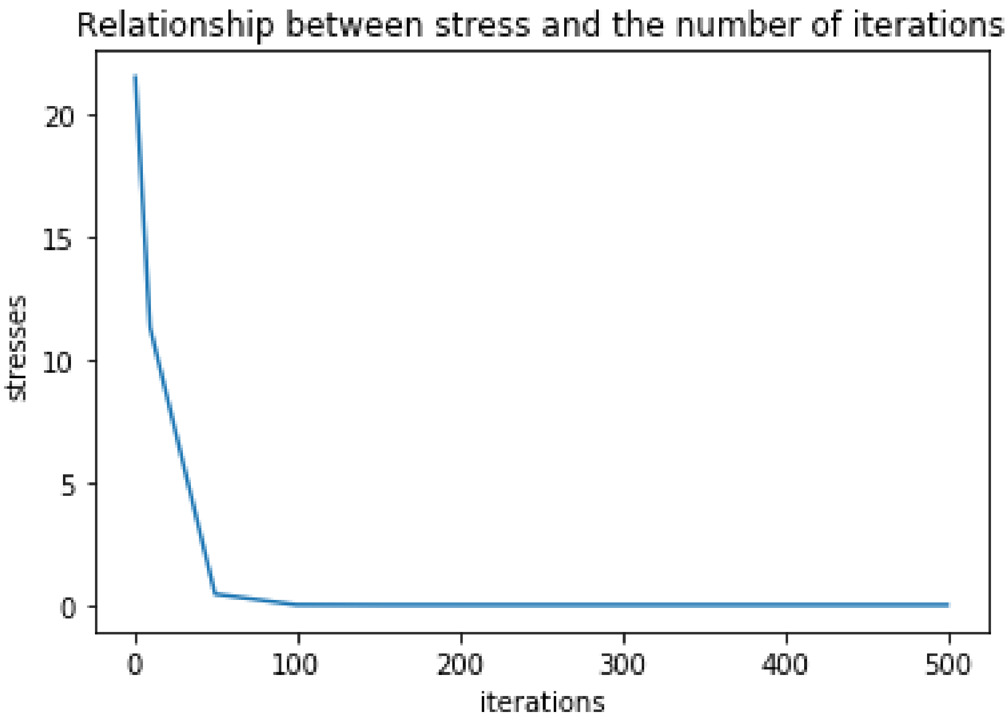
Good plots will show that the difference between the psychological error and the MDS will have a one to one linear relationship.

1.6 Question 6

[19]:
```
def stress_vs_iterations():
    #blank stress, preset number of iterations
    iterations = [1, 10, 50, 100, 150, 200, 250, 300, 350, 400, 450, 500]
    stresses = []
    for i in np.arange(len(iterations)):
        #setting points and steps again, similar to question 4
        points = np.random.normal(0 ,1 ,size=(21,2))
        for steps in np.arange(iterations[i]):
            points -= gradient_all(points)*0.01
        stresses.append(stress(points))
```

```
    print(stresses)
    plt.plot(iterations, stresses)
    plt.xlabel('iterations')
    plt.ylabel('stresses')
    plt.title('Relationship between stress and the number of iterations')
    plt.figure()
stress_vs_iterations()
```

```
[21.493172058270517, 11.323302725254536, 0.44725609331477933,
0.008356861283938288, 0.00011574382872268488, 2.202869698137527e-06,
3.3712649032372206e-08, 7.982383494005966e-10, 1.4367044351008043e-11,
1.7349680544145147e-13, 2.7298272527611056e-15, 2.6035901405332846e-17]
```

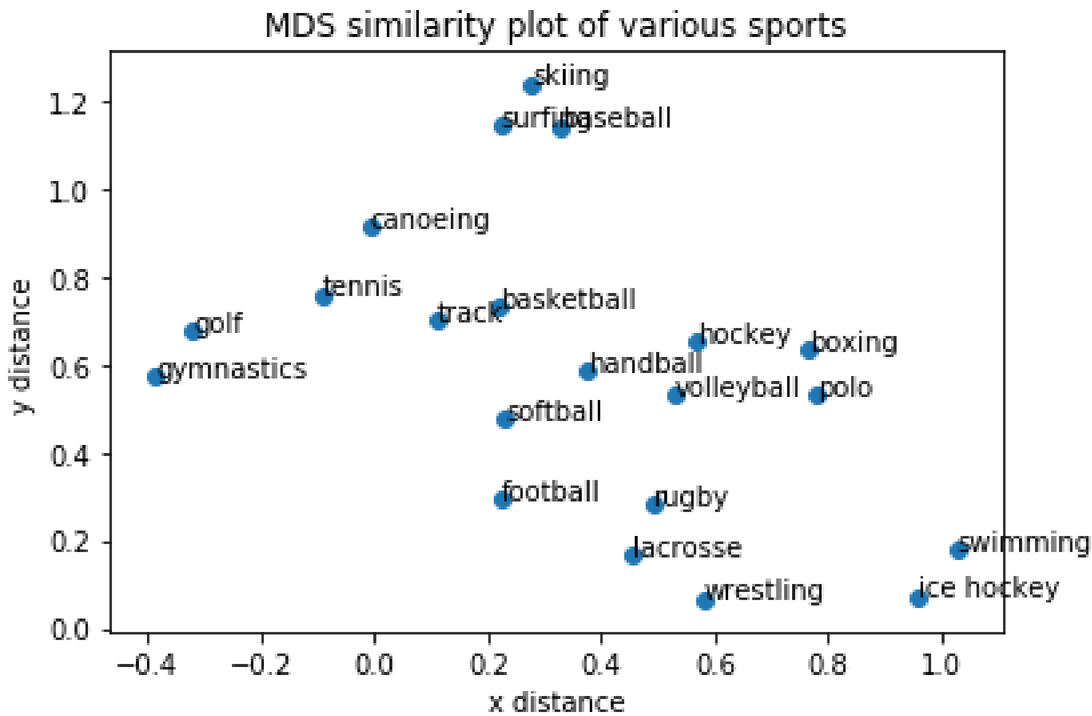


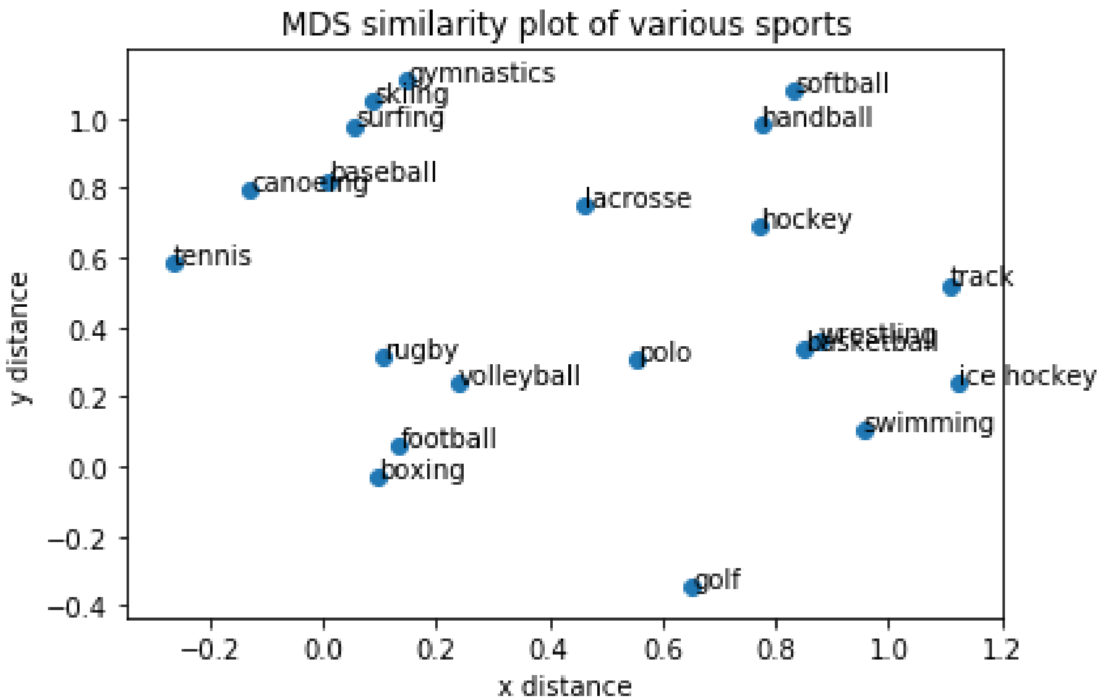

```
<Figure size 432x288 with 0 Axes>
```

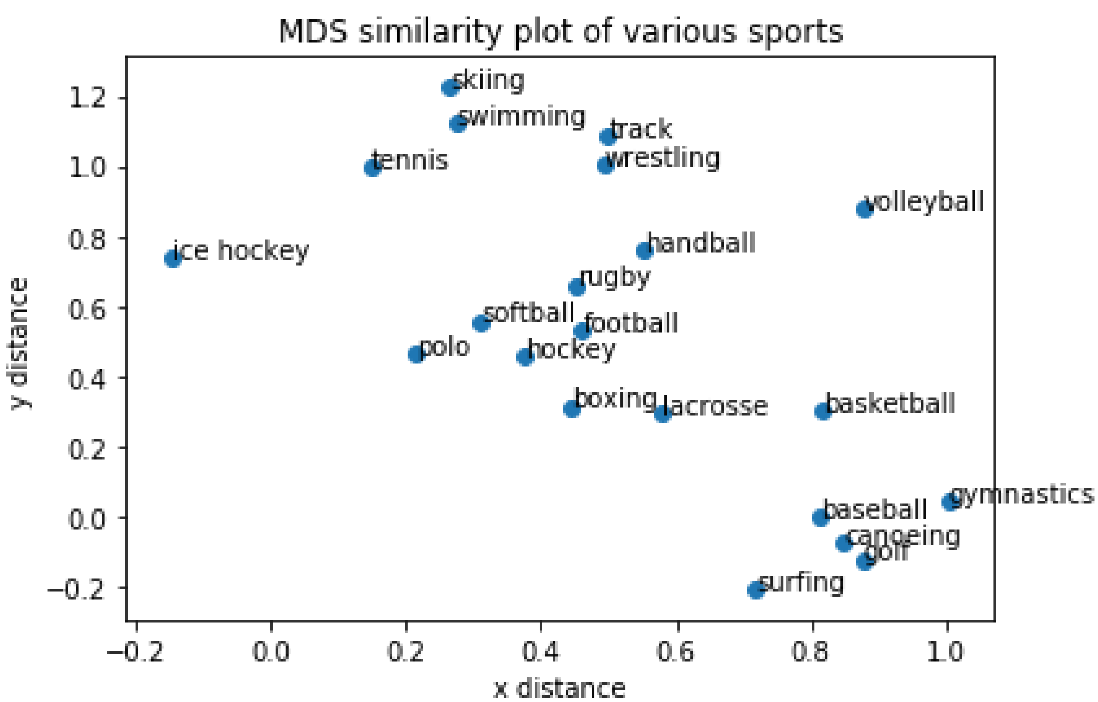
This graph shows what the stress is at various number of iterations. We find the minimum stress on this graph and match it to the number of iterations. We see that between 50 and 100 iterations, the stress drops dramatically from 40+ to 1. This means that the minimum number of iterations we need to perform is around 50 if we would like to minimize stresses while still running an efficient code

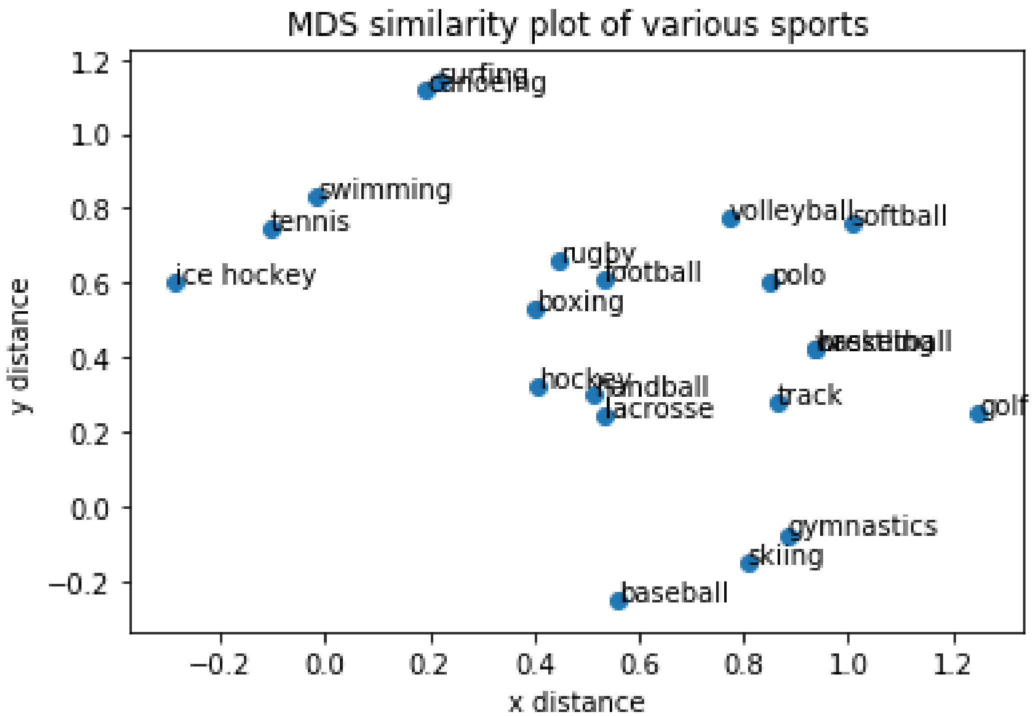
1.7 Question 7

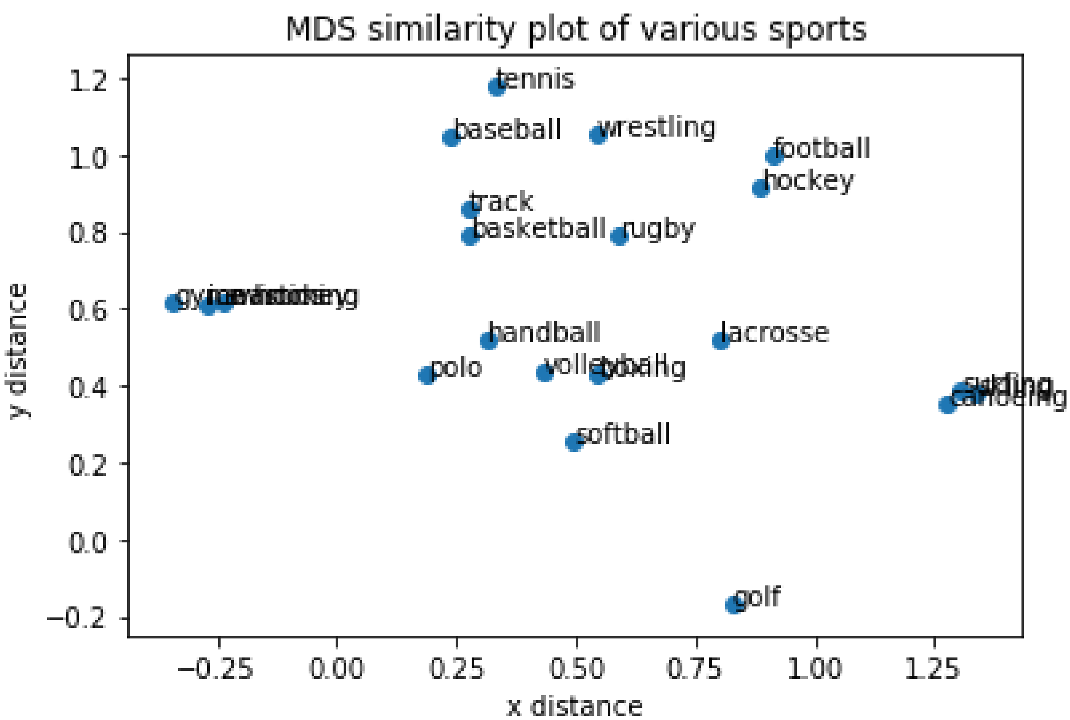
[22]:
```
for i in range(9):
    min_mds()
```

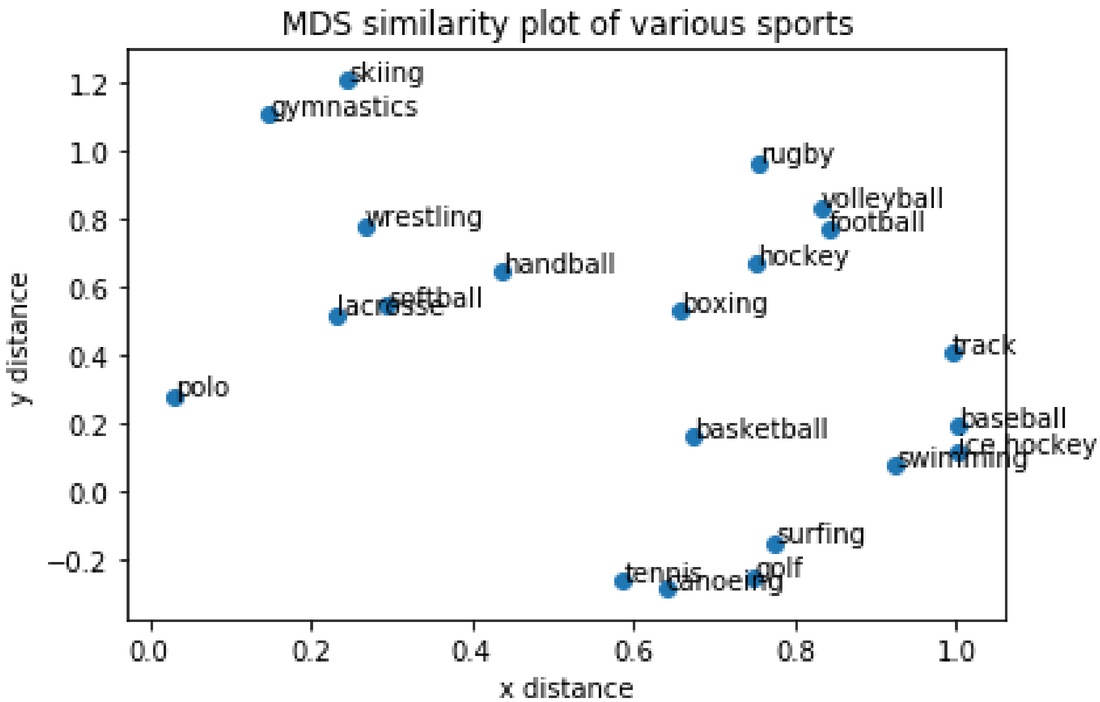


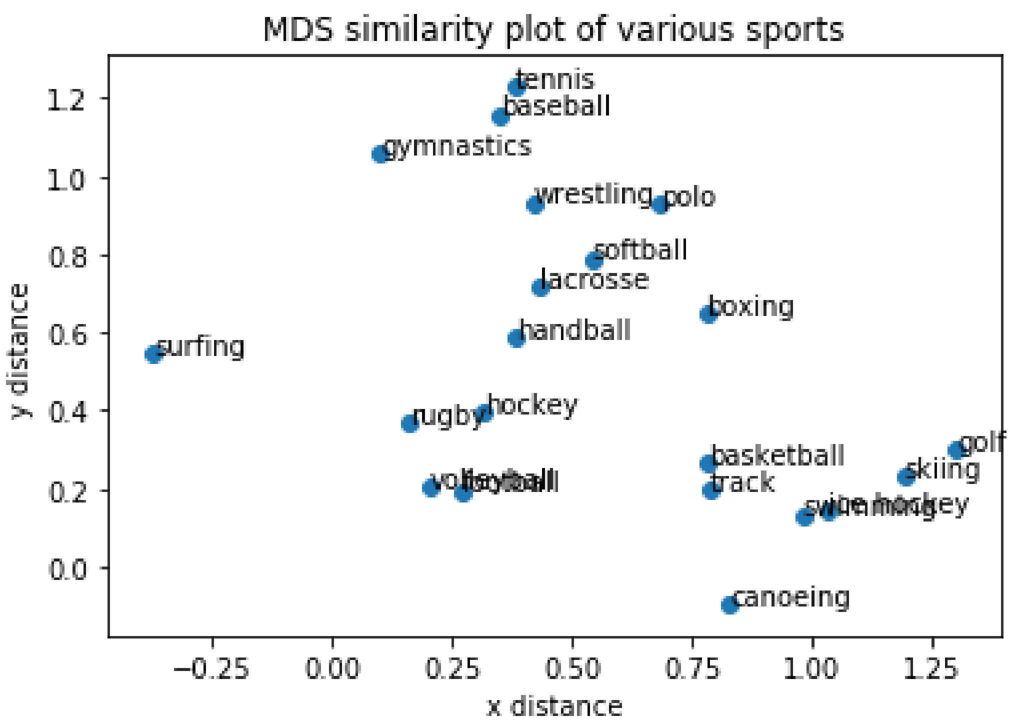

MDS similarity plot of various sports
gymnastics
skiing
surfing
softball
handball
canoeing
baseball
lacrosse
hockey
tennis
track
wrestling
basketball
rugby
polo
volleyball
ice hockey
swimming
football
boxing
golf
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
y distance
x distance

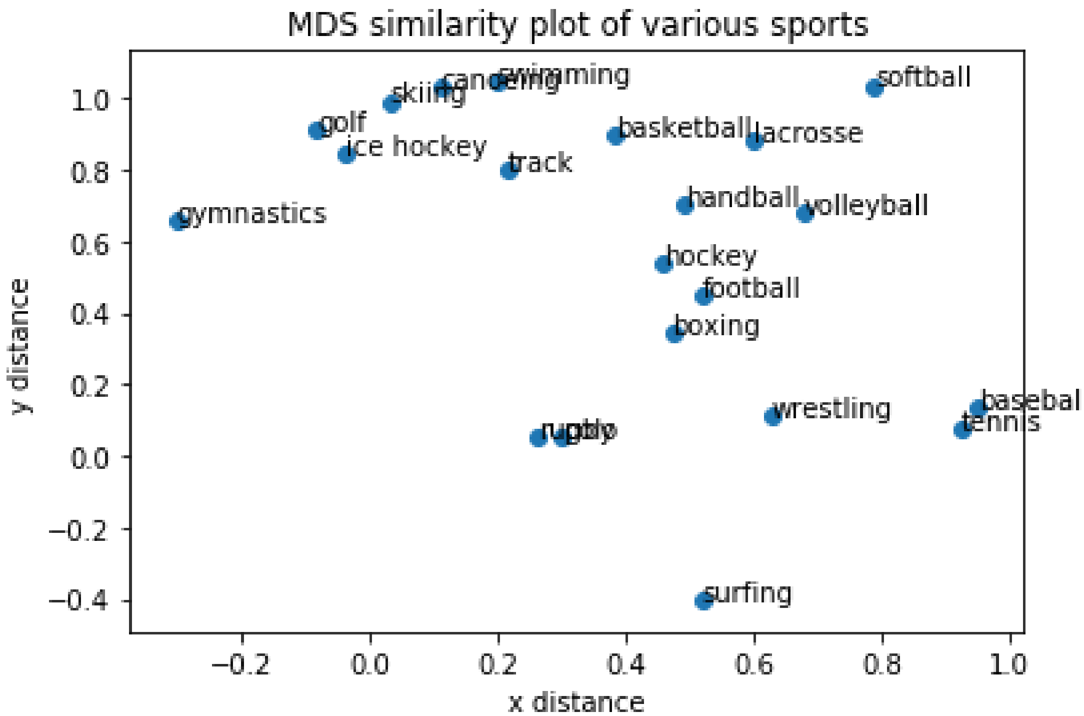

MDS similarity plot of various sports
skiing
swimming
track
tennis
wrestling
volleyball
ice hockey
handball
rugby
softball
football
polo
hockey
boxing
lacrosse
basketball
gymnastics
baseball
canoeing
golf
surfing
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
y distance
x distance

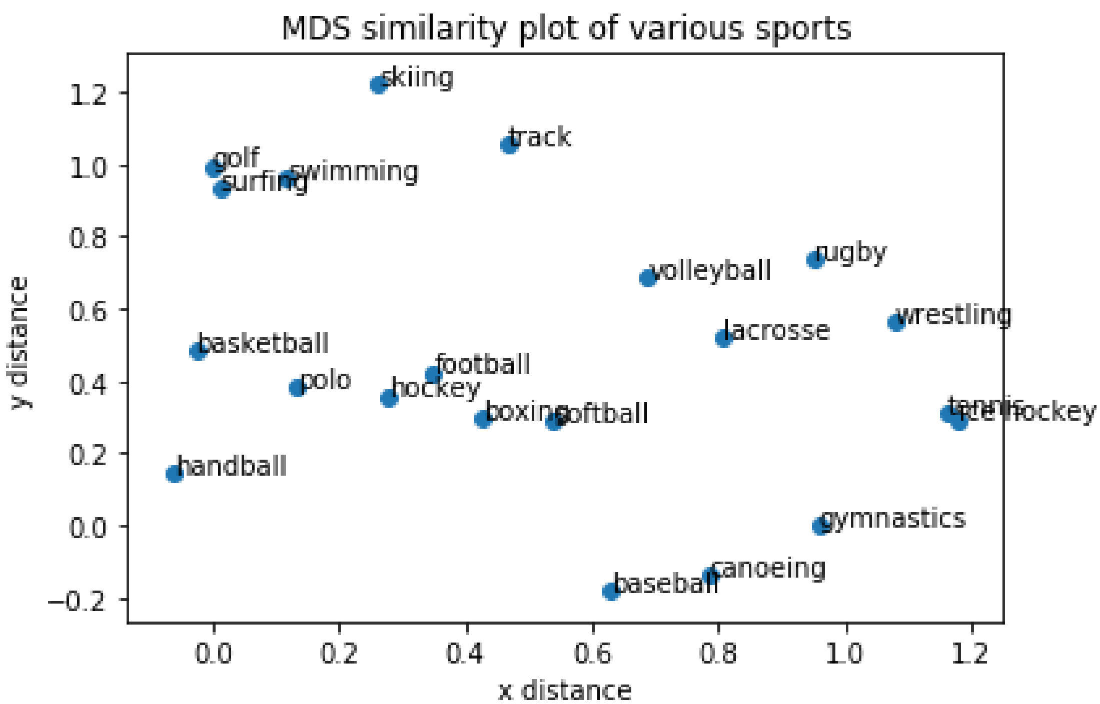

MDS similarity plot of various sports
y distance
x distance
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
swimming
tennis
ice hockey
rugby
football
boxing
volleyball
softball
polo
hockey
handball
lacrosse
track
golf
gymnastics
skiing
baseball


MDS similarity plot of various sports
y distance
x distance
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.25
0.00
0.25
0.50
0.75
1.00
1.25
tennis
baseball
wrestling
football
hockey
track
basketball
rugby
handball
lacrosse
polo
softball
golf

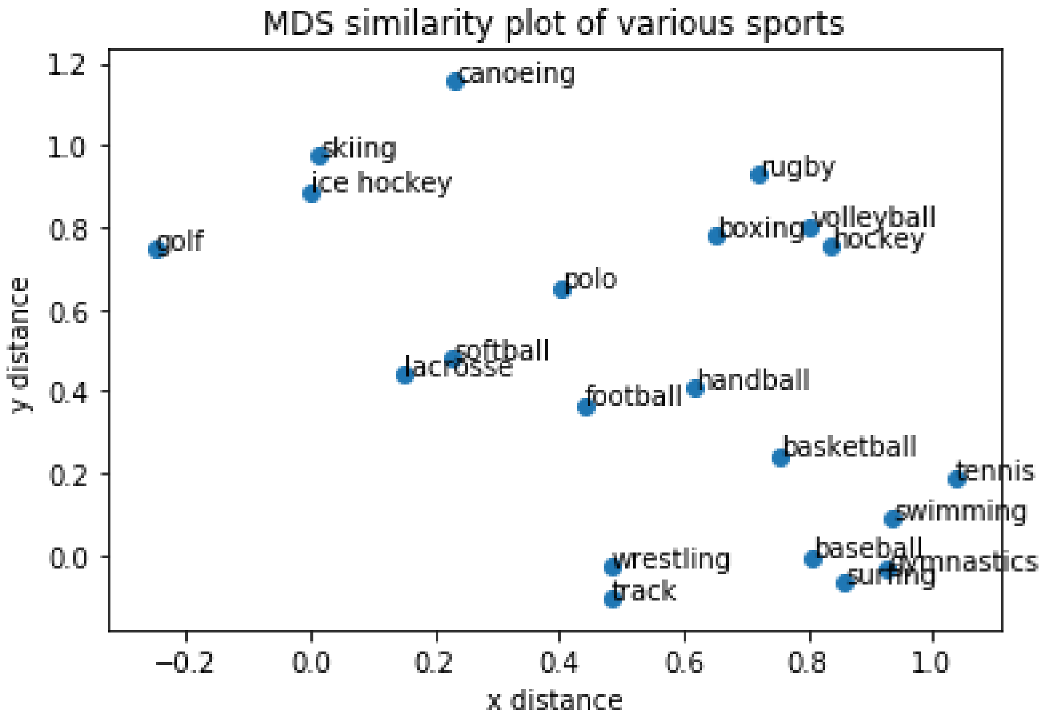

MDS similarity plot of various sports
skiing
gymnastics
rugby
volleyball
football
wrestling
handball
hockey
lacrosse
softball
boxing
track
polo
basketball
baseball
ice hockey
swimming
surfing
tennis
canoeing
golf
y distance
x distance
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0.0
0.2
0.4
0.6
0.8
1.0


MDS similarity plot of various sports
tennis
baseball
gymnastics
wrestling
polo
softball
lacrosse
boxing
surfing
handball
rugby
hockey
basketball
golf
skiing
volleyball
football
track
swimming
ice hockey
canoeing
y distance
x distance
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.25
0.00
0.25
0.50
0.75
1.00
1.25


MDS similarity plot of various sports
y distance
x distance
softball
swimming
canoeing
skiing
golf
ice hockey
basketball
lacrosse
track
handball
volleyball
gymnastics
hockey
football
boxing
baseball
wrestling
tennis
rugby
polo
surfing
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0


MDS similarity plot of various sports
y distance
x distance
skiing
track
golf
swimming
surfing
rugby
volleyball
wrestling
lacrosse
basketball
football
polo
hockey
boxing
softball
tennis
ice hockey
handball
gymnastics
canoeing
baseball
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2

[24]:
```
##10th graph
min_mds()
```



None of these graphs are the same because the points variable is random and is set within each function, meaning that it will be called when the function is executed, and it will present a random set of starting points each time. As a result, none of these graphs are the same, because the function to minimize stress relies on these random points in the equation.

1.8 Question 8

If I were to choose the best one to use, I would first run different versions of the function and code them wtih more iterations and a smaller step size to know that I was following the gradient more closely. I could also juse the min() function to find the one that minimizes all stress within the MDS.