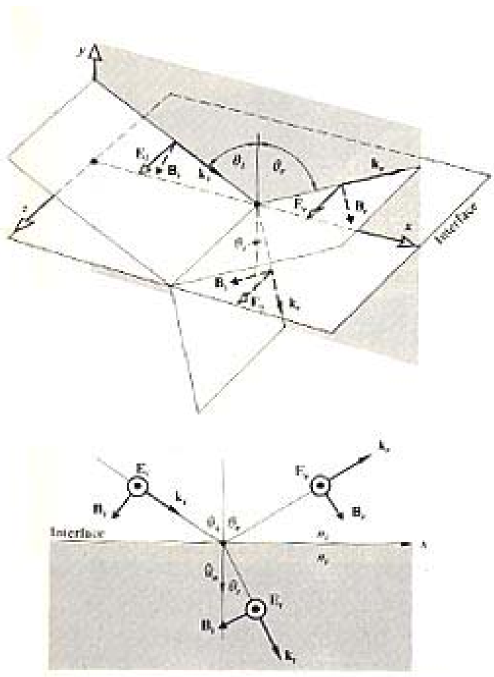
The laws of reflection and refraction

The full treatment of reflection and refraction is dervived from the Maxwell's equations of electrodynamics. It is known as Fresnel equations. First of all, we suppose that a plane monochromatic wave is incident on the planar surface separating two isotropic media with nearly equal permittivity ($\mu_1=\mu_2=\mu_0$). Whatever the polarization of the wave we shall resolve its electric **E** and magnetic **B** fields into components parallel and perpendiculat to the plane of incidence and treat these constituents seperately.

<u>Case 1. **E** perpendicular to the plane of incidence (s component)</u>



An incoming wave whose E-field is normal to the plane of incidence.

The most common forms of the *amplitude* reflection coefficient r_s and the amplitude transmission coefficient t_s are simply:

$$r_s = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_s = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

Case 2. **E** parallel to the plane of incidence (p component)

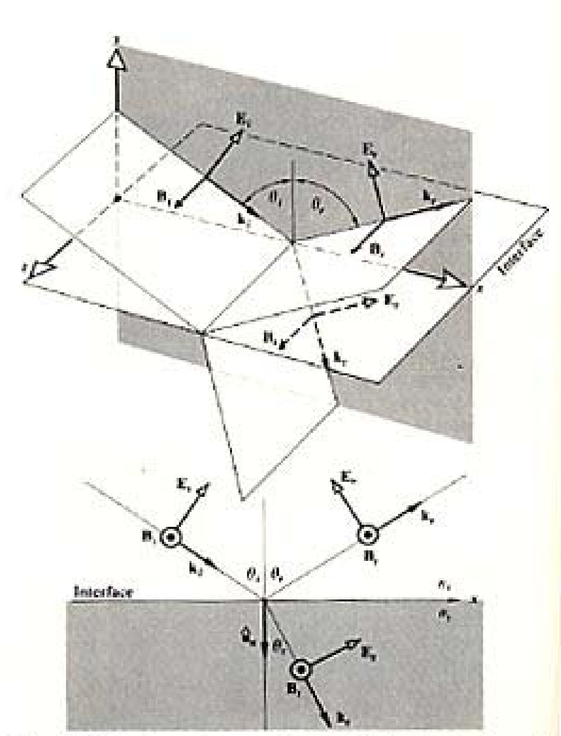


Fig. 4.19 An incoming wave whose E-field is in the plane of incidence.

The most common forms of the *amplitude* reflection coefficient r_p and the amplitude transmission coefficient t_p are simply:

$$r_p = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i}$$

$$t_p = \frac{2n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i}$$

One further notational simplification can be made by using Snell's law, where upon the Fresnel equations for dielectric media become

$$r_s = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)} \qquad t_s = \frac{2\sin\theta_i \cos\theta_t}{\sin(\theta_i + \theta_t)}$$

$$r_p = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)} \qquad t_p = \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

The measured reflectance R and transmittance T however are the ratio of flux (not amplitude), such that

$$R = r^2$$

$$T = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i} \right) t^2$$

$$R + T = 1$$

Hence

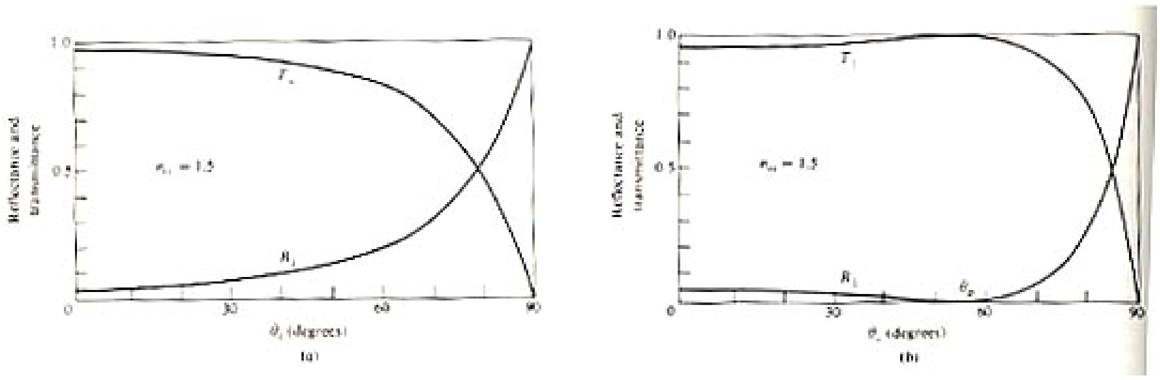
$$R_s = r_s^{\ 2}$$

$$R_p = r_p^{\ 2}$$

$$T_s = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i} \right)^2 t_s^{\ 2}$$

$$T_p = \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i} \right)^2 t_s^{\ 2}$$

The reflectance and transmittance versus incident angle is shown below for refractive index of 1.5. Note the existence of Brewster's angle in one case



Note that the law is applicable at the point of incidence on the surface. Absorption, if any, is after penetration into the media and therefore does not enter into the Fresnel equations.

Brewster's Angle and Polarization

I. Introduction

Light is a wave phenomenon with its electric field (or its magnetic field) vibrating in time and in space. If the electric field of a light beam is vibrating in a fixed plane in space, such a light beam is referred to as **plane-polarized** light or **linearly polarized** light. This fixed plane is named as the **plane of vibration.** Light from most lasers is plane-polarized. On the other hand, light form the Sun is said to be un-polarized. However, sunlight can be decomposed into two plane-polarized light beams with their planes of vibration perpendicular to each other.

When un-polarized light is incident on the surface of a dielectric (such as a glass), it can be decomposed into two components, denoted as "*s*" and "*p*" component. The plane of vibration of the "*s*" or "*p*" component is perpendicular or parallel to the plane of incidence, respectively. Their intensities depend on the angle of incidence. At a specific angle of incidence, the intensity of "*p*" component of reflected light is zero; and the reflected light becomes plane-polarized with only the "*s*" component. This phenomenon was discovered by Sir David Brewster and, thus, the specific angle is called **Brewster's angle** or **polarization angle.** From his experiments, Brewster confirmed that the reflected ray and the refracted ray are 90 degrees apart when the incident angle is set at Brewster's angle.

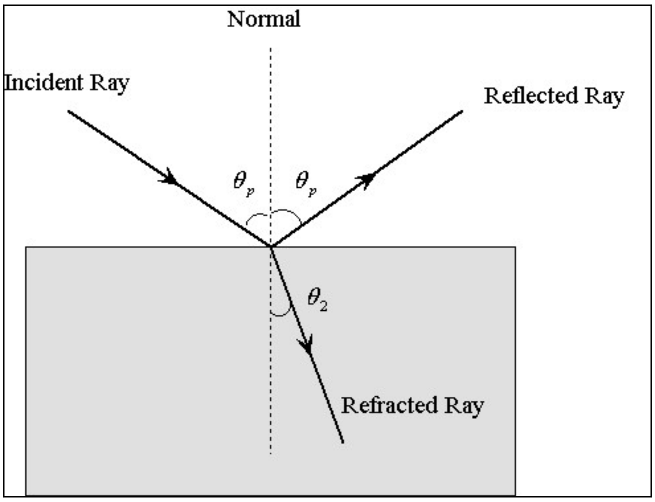


Figure 1. Reflection and Refraction of Un-polarized Light

Shell's law states that:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{1}$$

where n_1 and n_2 are the refractive index of the air and the dielectric, respectively; θ_1 and θ_2 are the angle of incidence and refraction, respectively. Because of $\theta_p + \theta_2 = 90^\circ, \theta_2 = 90^\circ - \theta_p$, and $\sin\theta_2 = \sin(90^\circ - \theta_p) = \cos\theta_p$, we have the following equation at Brewster's angle.

$$n_1 \sin\theta_p = n_2 \cos\theta_p \,. \tag{2}$$

Finally, we arrive at

$$\tan\theta_p = \frac{n_2}{n_1} \tag{3}$$

Equation (3) clearly shows that Brewster's angle depends on the refractive index of the dielectric.

The objectives of this experiment are to determine both the Brewster's angle and the refractive index of acrylic.

II. Experimental Set-up

1. Set up the laser diode, the polarizer assembly, the collimating slits, the D lens, the square analyzing polarizer, and the light sensor on the bench, as shown in Figure 2.

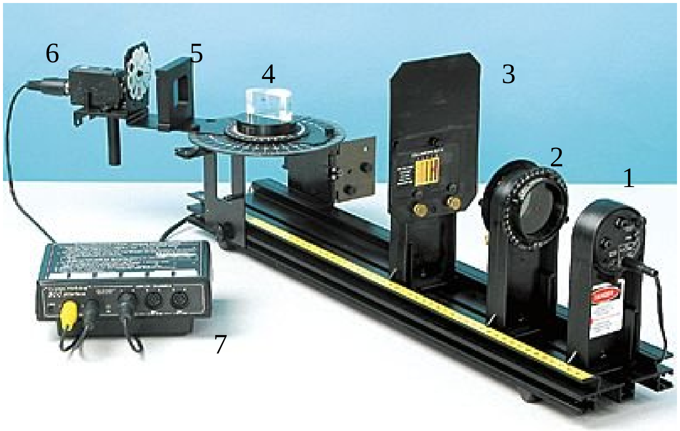


Figure 2. Experiment set-up

1: Laser Diode. 2: Polarizer Assembly (1 lens holder and 2 polarizers). 3: Collimating slits. `

4: D lens. 5: Square analyzing polarizer. 6: Light Sensor. 7: *SicenceWorkshop*® Interface.

2. In the polarizer assembly, there are two round polarizers. Rotate the second polarizer (counting from the laser diode) to 45 degrees, as indicated on the lens holder and lock it in place by tightening the brass screw. By varying the angle of the first polarizer, the intensity of the transmitted laser light is adjustable. The laser light, transmitted through this assembly, is plane-polarized with 45 degree to the plane of incidence when the laser light is incident on the D lens. Therefore, the incident light can be decomposed to "*s*" and "*p*" components that are perpendicular and parallel to the plane of incidence, respectively. Effectively, such an incident laser beam is treated as un-polarized light.

3. The square analyzing polarizer has its transmission axis marked. Rotate this transmission axis to an angle at which it is parallel to the plane of incidence or the plane of vibration for the "*p*" beam. Hence, the light sensor can only measure the intensity of "*p*" beam of reflected light.

4. Mount the metal lens holder on the Brewster's angle base disk, as shown in Figure 2. The metal lens holder has two zero marks. For measurements of reflected light, use the mark that is on the side with the higher step. The D lens is mounted with its magnet side sitting on the metal lens holder. Note that the D lens is made of material called acrylic.

5. To align laser beam with the Light Sensor, remove the square analyzing polarizer, the collimating slits, and the D lens. Set the Arm at 180 degrees. Turn on the Laser Diode and use the vertical adjust on the laser to get the laser beam at the center of the Light Sensor slit. The Light Sensor slit should be set on #4. Mount the collimating slits on the track and adjust its slit position so that the laser beam passes through the #4 slit. Mount the D lens back on the metal lens holder with its flat surface towards the laser light. If the laser spot is not still centered on the slit of the Light Sensor, slide the D lens to center it. Make sure the lens is mounted firmly against the step of the metal lens holder.

6. For software setup, connect the Rotary Motion Sensor into Channel 1 and 2 of the *SicenceWorkshop®* Interface. Connect the Light Sensor into Channel A. In *DataStudio*, create the following two equations:

$$Angle = [180\text{-}abs(x)/12.5]/2,$$

where x is the Angular Position (from Channel 1 and 2); and

$$Light\ Intensity = smooth(8, I),$$

where I is the Light Intensity (from Channel A).

From the *Data* list, drag the *Angle*, *Light Intensity* to the *digits* displays to monitor both the angle and the light intensity.

III. Experiment

1. With the D lens removed, calibrate the angle for the Rotary Motion Sensor: rotate the base disk so the laser beam is centered on the Light Sensor slit. The angle of the base disk should be near 180 degrees. However, it is OK if the angle is slightly off.

2. In *DataStudio*, click the *Start* button. Rotate the disk while watching the reading on the *digits* display for the light intensity. Stop at the position that gives the maximum intensity by clicking on the *Stop* button. Then restart you will see the angle on the *digits* display reads 90 degrees.

   Note that do not move the Arm until the program is started to take the actual data run. This ensures the angle you read from the *digits* display is the actual angle.

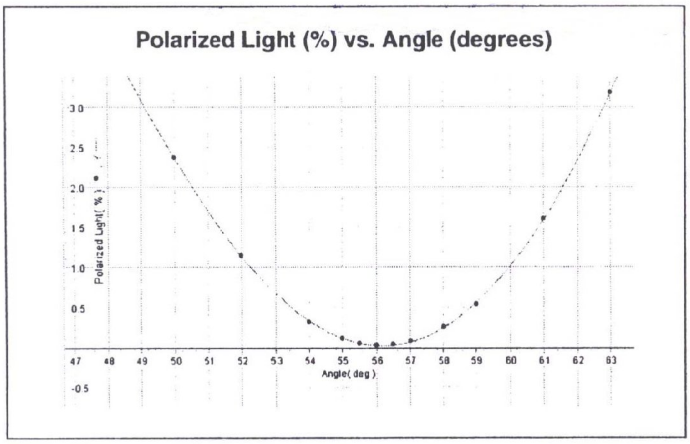
3. Mount the D lens back on the metal lens holder. Turn off the room lights. Click on the *Start* button. Do not click on the *Stop* button until all the procedure steps are completed. Set the angle of the metal lens holder to 85 degrees by monitoring the *digits* display. The square analyzing polarizer should not be in place. Write down the angle of the metal lens holder.

4. Rotate the base disk for maximum reading on the *digits* display for the light intensity. (Please do not rotate the Arm because it may not be very firmly fixed to the disk and thus causing errors.) If the light intensity is too high or too low, rotate the first polarizer (nearest to the laser) to moderate. The Light Sensor should be set at a gain of 1. Write down the maximum reading for the total reflected light intensity. Insert the square analyzing polarizer between the D lens and the Light Sensor silt with its transmission axis horizontal, and then write down the reading for the intensity of “*p*” beam of reflected light.

5. Remove the square analyzing polarizer, rotate the metal lens holder to smaller angles with an interval of 5 degrees, and repeat **Steps 3** and **4** above. When approaching Brewster’s angle, the interval should be set at one degree; and switch the gain of the Light Sensor to 10 or 100 if the reading for the light intensity is too low.

6. Date processing: for each pair of the intensity data, calculate the ratio of the light intensity measured with the polarizer to the light intensity measured without the polarizer. Use Microsoft’s Excel to plot this ratio as function of the incident angle in the range from 30 to 85 degrees. The following plot is an example. Then, determine Brewster’s angle at which the ratio is a minimum. By using any reference book, find out the refractive index of air at room temperature. Substitute both the Brewster’s angle and refractive index of air into Equation (3) for determination of the refractive index of the D lens.

Sample Data:



VI. Questions

1. Discuss in detail how to determine the error in the measured refractive index.
2. What are major sources for the error?
3. From the reference books listed below, it can be derived that the ratio of the intensity of "*p*" beam of reflected light to the intensity of total reflected light is expressed by

$$\frac{Intesnity \quad of \quad "p" \quad beam \quad of \quad reflected \quad light}{Intesnity \quad of \quad total \quad reflected \quad light} = \frac{\cos^2(\theta_1 + \theta_2)}{\cos^2(\theta_1 + \theta_2) + \cos^2(\theta_1 - \theta_2)}$$

   Fit your results with the above expression. Discuss why if there is discrepancy.
4. What is the total reflectance at Brewster's angle?
5. If a blue laser light is used instead of the red laser light, what kind of change (increase or decrease) in the measured Brewster's angle do you predict?

V. References

1. Eugene Hecht, Optics (4^{th} Edition, Addison Wesley, 2002)
2. Jenkins and White, Fundamentals of Optics (4^{th} Edition, McGraw Hill, 1981)