Foundations of an Alternative Approach to Reification in RDF

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Abstract

This document defines extensions of the RDF data model and of the SPARQL query language that capture an alternative approach to represent statement-level metadata. While this alternative approach is backwards compatible with RDF reification as defined by the RDF standard, the approach aims to address usability and data management shortcomings of RDF reification. One of the great advantages of the proposed approach is that it clarifies a means to (i) understand sparse matrices, the property graph model, hypergraphs, and other data structures with an emphasis on link attributes, (ii) map such data onto RDF, and (iii) query such data using SPARQL. Further, the proposal greatly expands both the freedom that database designers enjoy when creating physical indexing schemes and query plans for graph data annotated with link attributes and the interoperability of those database solutions.

1 Introduction

The RDF standard introduces the notion of reification as an approach to provide a set of RDF triples that describe some other RDF triple [HPS14]. This form of statement-level metadata about a reified triple has to include four additional RDF triples to refer to the reified triple.

Example 1. *Consider the following two RDF triples—given in Turtle syntax [PC14]—that indicate the age of somebody named Bob* [1]

```
:bob foaf:name "Bob" ; foaf:age 23 .
```

To capture metadata about a given RDF triple as per RDF reification, we have to introduce an IRI or a blank node and use this IRI or blank node as the subject of four RDF triples that reify the given triple by applying the RDF reification vocabulary [HPS14]. Then, the IRI or blank node can be used to provide metadata about the reified triple. For instance, by using a blank node, say _:s, *we may reify the second of the two example triples as follows:*

```
_:s rdf:type rdf:Statement ;
    rdf:subject :bob ;
    rdf:predicate foaf:age ;
    rdf:object 23 .
```

---

[1] Prefix declarations are omitted in all examples in this document. The prefixes used are the usual prefixes as can be found via the http://prefix.cc service.

Now, we can use the blank node to provide metadata about the triple:

```
_:s dct:creator <http://example.com/crawlers#c1> ;
    dct:source <http://example.net/homepage-listing.html> .
```

RDF reification as demonstrated in the example has two major shortcomings. First, adding four reification triples for every reified triple is inefficient for exchanging as well as for managing RDF data that includes statement-level metadata. Second, writing queries to access statement-level metadata is cumbersome because any metadata-related (sub)expression in a query has to be accompanied by another subexpression to match the corresponding four reification triples.

Example 2. *Consider the data (including the metadata) from Example 1. To query this data we may use SPARQL, the standard query language for RDF [HSP13]. For instance, if we are interested in the age of Bob, including the source of this information, we may write a SPARQL query such as the following.*

```
SELECT ?age ?src WHERE {
    ?bob foaf:name "Bob" ;
         foaf:age ?age .
    ?r rdf:type rdf:Statement ;
       rdf:subject ?b ;
       rdf:predicate foaf:age ;
       rdf:object ?age ;
       dct:source ?src .
}
```

Note that the given query contains four triple patterns to identify the reified triple whose metadata we want to see. If we were also interested in potential metadata about the corresponding `foaf:name` *triple, we would have to add another four, reification-related triple patterns.*

During a Dagstuhl seminar on "Semantic Data Management" [ACA+12] several participants of the seminar—including Bryan Thompson (Systap, LLC), Orri Erling and Yrjänä Rankka (Open-Link Software), and Olaf Hartig (then Humboldt Universität zu Berlin)—discussed an alternative approach to reification that addresses the aforementioned shortcomings.

This document provides a formal foundation for this approach. The document is structured as follows: Section 2 outlines the approach informally. Thereafter, Section 3 introduces an extension of the RDF data model that makes metadata statements a first class citizen. Finally, Sections 4 and 5 extends the query language SPARQL to enable users to benefit from the extended data model.

2   An Alternative Approach to Reification in RDF

The alternative approach to reification is based on the idea of using a triple directly in the subject position or object position of (other) triples that represent metadata about the embedded triple.

Example 3. *Assume a possible extension of the Turtle syntax that implements the idea of embedding RDF triples into other RDF triples by enclosing any embedded triple in '<<' and '>>' (Section 3.3 shall introduce such an extension). Then, the data from Example 1 (including the metadata) could be represented as follows.*

```
:bob foaf:name "Bob" .
<<:bob foaf:age 23>> dct:creator <http://example.com/crawlers#c1> ;
                     dct:source <http://example.net/homepage-listing.html> .
```

Embedding triples into (metadata) triples as demonstrated in the example achieves a more compact representation of statement-level metadata than standard RDF reification. Such a compact representation may improve comprehensibility for users who have to inspect RDF documents directly (e.g., in a text editor). Such a representation may also reduce the size of RDF documents that include statement-level metadata and, thus, may be advantageous for data exchange. Furthermore, embedded triples, conceived of as a form of self-referencing identifiers, corresponds naturally to the concept of triple identifiers that some RDF data management systems such as Systap's Bigdata [TPC14] use internally to avoid the overhead of keeping a physical representation of four reification triples per reified triple.

Given that triples are embedded into other triples, the idea of such an embedding can be carried over to SPARQL queries.

Example 4. *By adopting the extended syntax outlined in Example 3, we could represent the query from Example 2 in the following, more compact form.*

```
SELECT ?age ?src WHERE {
   ?bob foaf:name "Bob" .
   <<?bob foaf:age ?age>> dct:source ?src .
}
```

An alternative form is to use BIND clauses as follows.

```
SELECT ?age ?src WHERE {
   ?bob foaf:name "Bob" .
   BIND( <<?bob foaf:age ?age>> AS ?t )
   ?t dct:source ?src .
}
```

The remainder of this document provides a formal definition of the approach outlined in this section. An important characteristic of this formalization is its backward compatibility with standard RDF reification.

3   RDF* – A Metadata Extension of RDF

This section introduces an extension of the RDF data model [CWL$^+$14] that makes metadata statements a first class citizen. Hereafter, the extended data model is referred to as RDF*.

3.1   Concepts

Assume pairwise disjoint sets $\mathcal{I}$ (all IRIs), $\mathcal{B}$ (blank nodes), and $\mathcal{L}$ (literals). As usual, an *RDF triple* is a tuple $\langle s, p, o \rangle \in (\mathcal{I} \cup \mathcal{B}) \times \mathcal{I} \times (\mathcal{I} \cup \mathcal{B} \cup \mathcal{L})$ and an *RDF graph* is a set of RDF triples.

RDF* extends such triples by permitting the embedding of a given triple in the subject or object position of another triple. Triples whose subject or object is an embedded triple represent some

form of metadata. An embedded triple may itself be a metadata triple and, thus, may also contain embedded triples; and so forth. The following definition captures this notion.

Definition 1. Let $\widetilde{\mathcal{T}} = (\widetilde{\mathcal{T}} \cup \mathcal{I} \cup \mathcal{B}) \times \mathcal{I} \times (\widetilde{\mathcal{T}} \cup \mathcal{I} \cup \mathcal{B} \cup \mathcal{L})$, and call a tuple $\langle s, p, o \rangle \in \widetilde{\mathcal{T}}$ *k-nested* if k is a natural number such that either (i) $k = 0$ and $\langle s, p, o \rangle$ is an (ordinary) RDF triple or (ii) $k > 0$ and each (embedded) tuple $\langle s', p', o' \rangle \in (\{s, o\} \cap \widetilde{\mathcal{T}})$ is $(k-1)$-nested. Then, an **RDF* triple** is a tuple $t \in \widetilde{\mathcal{T}}$ for which there exists a natural number k such that t is k-nested. A set of RDF* triples is called an **RDF* graph**.

Note that the notion of *k-nestedness* in Definition 1 rules out infinitely deep nesting of RDF* triples.

Hereafter, the set of *all RDF* triples* is denoted by $\mathcal{T}^\star$, and, for any RDF* triple $t \in \mathcal{T}^\star$, $\mathrm{Elmts}^+(t)$ denotes the set of all RDF terms and all RDF* triples mentioned in t; i.e., if $t = \langle s, p, o \rangle$, then $\mathrm{Elmts}^+(t) = \{s, p, o\} \cup \{x' \in \mathrm{Elmts}^+(x) \mid x \in \{s, o\} \cap \mathcal{T}^\star\}$. An RDF* triple t with $\mathrm{Elmts}^+(t) \cap \mathcal{T}^\star \neq \emptyset$ is called a *metadata triple* (note that any other RDF* triple is an ordinary RDF triple).

Overloading function Elmts^+, for any RDF* graph $G^\star$, $\mathrm{Elmts}^+(G^\star) = \bigcup_{t \in G^\star} \mathrm{Elmts}^+(t)$. Furthermore, $\mathrm{Emb}^+(G^\star)$ denotes the set of all RDF* triples that are (recursively) embedded in RDF* triples of RDF* graph $G^\star$ (i.e., $\mathrm{Emb}^+(G^\star) = \mathrm{Elmts}^+(G^\star) \cap \mathcal{T}^\star$).

Example 5. *The data represented in Example 3 can be parsed into the following RDF* graph.*

$$G^\star_{\mathrm{ex}} = \big\{ \langle \texttt{:bob}, \texttt{foaf:name}, \texttt{"Bob"} \rangle,$$

$$\langle \langle \texttt{:bob}, \texttt{foaf:age}, \texttt{23} \rangle, \texttt{dct:creator}, \texttt{http://example.com/crawlers\#c1} \rangle,$$

$$\langle \langle \texttt{:bob}, \texttt{foaf:age}, \texttt{23} \rangle, \texttt{dct:source}, \texttt{http://example.net/homepage-listing.html} \rangle \big\}$$

Hence, this RDF* graph consists of three RDF* triples, and its set of embedded RDF* triples contains a single triple, that is, $\mathrm{Emb}^+(G^\star_{\mathrm{ex}}) = \big\{ \langle \texttt{:bob}, \texttt{foaf:age}, \texttt{23} \rangle \big\}$.

3.2 RDF* Semantics

To support a model-theoretic interpretation of RDF* graphs in terms of the standard RDF semantics [HPS14] this section defines a transformation from RDF* graphs to ordinary RDF graphs. This transformation may also be used to enable ordinary RDF data management systems (that do not support RDF*) to process data that is represented as an RDF* graph.

The transformation is based on the following three functions. First, the transformation uses a function that associates every embedded RDF* triple $t \in \mathrm{Emb}^+(G^\star)$ in an RDF* graph $G^\star$ with a fresh and unique blank node. Hence, this function is called a *bnode assignment function*.

Definition 2. A **bnode assignment function** id for an RDF* graph $G^\star$ is a bijective function $id : \mathrm{Emb}^+(G^\star) \to B$ such that $B \subseteq \mathcal{B}$ is a set of blank nodes that has the following two properties: (i) $|B| = \big|\mathrm{Emb}^+(G^\star)\big|$ and (ii) $B \cap \mathrm{Elmts}^+(G^\star) = \emptyset$.

Second, the transformation uses a *reification function* that associates every embedded RDF* triple in an RDF* graph with a corresponding set of four reification triples.

Definition 3. Let $G^\star$ be an RDF* graph and let id be a bnode assignment function for $G^\star$. The id-specific **reification function** for $G^\star$ is a function $\text{reif}^{id} : \text{Emb}^+(G^\star) \to 2^{\mathcal{T}^\star}$ that, for every (embedded) RDF* triple $t \in \text{Emb}^+(G^\star)$, is defined as follows:

$$\text{reif}^{id}(t) = \{\langle id^*(t), \texttt{rdf:type}, \texttt{rdf:Statement}\rangle, \langle id^*(t), \texttt{rdf:subject}, id^*(s)\rangle,$$
$$\langle id^*(t), \texttt{rdf:predicate}, id^*(p)\rangle, \langle id^*(t), \texttt{rdf:object}, id^*(o)\rangle\},$$

where $id^*(t) = id(t)$ for all $t \in \text{Emb}^+(G^\star)$ and $id^*(x) = x$ for all $x \notin \text{Emb}^+(G^\star)$.

The third function for the transformation unfolds (potentially nested) RDF* triples recursively.

Definition 4. Let $G^\star$ be an RDF* graph and let id be a bnode assignment function for $G^\star$. The id-specific **unfold function** for $G^\star$ is a function $\text{rdf}^{id} : \left(G^\star \cup \text{Emb}^+(G^\star)\right) \to 2^{\mathcal{T}}$ that, for every RDF* triple $t \in \left(G^\star \cup \text{Emb}^+(G^\star)\right)$ with $t = \langle s, p, o\rangle$, is defined as follows:

$$\text{rdf}^{id}(t) = \begin{cases} \{\langle id(s), p, o\rangle\} \cup \text{reif}^{id}(s) \cup \text{rdf}^{id}(s) & \text{if } s \in \mathcal{T}^\star \text{ and } o \notin \mathcal{T}^\star, \\ \{\langle s, p, id(o)\rangle\} \cup \text{reif}^{id}(o) \cup \text{rdf}^{id}(o) & \text{if } s \notin \mathcal{T}^\star \text{ and } o \in \mathcal{T}^\star, \\ \{\langle id(s), p, id(o)\rangle\} \cup \text{reif}^{id}(s) \cup \text{rdf}^{id}(s) & \text{if } s \in \mathcal{T}^\star \text{ and } o \in \mathcal{T}^\star, \\ \qquad \cup \text{reif}^{id}(o) \cup \text{rdf}^{id}(o) & \\ \{\langle s, p, o\rangle\} & \text{else.} \end{cases}$$

Given these three functions, the transformation itself is defined as follows.

Definition 5. Let $G^\star$ be an RDF* graph and let id be a bnode assignment function for $G^\star$. The id-specific **unfolded RDF graph** of $G^\star$, denoted by $\text{rdf}^{id}(G^\star)$, is an RDF graph that is defined as follows:

$$\text{rdf}^{id}(G^\star) = \bigcup_{t \in G^\star} \text{rdf}^{id}(t).$$

Remark 1. Due to the definition of the unfold function, the transformation as given in Definition 5 entails any RDF triple $t \in \text{Emb}^+(G^\star)$ that is embedded in some metadata triple in an RDF* graph $G^\star$. Hence, the given transformation captures the use case of RDF reification in which RDF graphs that contain a reification of an RDF triple t also contain t itself.

Example 6. *An unfolded RDF graph of the RDF* graph in Example 5 is given as follows:*

$$\text{rdf}^{id_{\text{ex}}}(G^\star_{\text{ex}}) = \{\langle\texttt{:bob}, \texttt{foaf:name}, \texttt{"Bob"}\rangle,$$
$$\langle\texttt{:bob}, \texttt{foaf:age}, 23\rangle,$$
$$\langle b, \texttt{rdf:type}, \texttt{rdf:Statement}\rangle,$$
$$\langle b, \texttt{rdf:subject}, \texttt{:bob}\rangle,$$
$$\langle b, \texttt{rdf:predicate}, \texttt{foaf:age}\rangle,$$
$$\langle b, \texttt{rdf:object}, 23\rangle,$$
$$\langle b, \texttt{dct:creator}, \texttt{http://example.com/crawlers\#c1}\rangle,$$
$$\langle b, \texttt{dct:source}, \texttt{http://example.net/homepage-listing.html}\rangle\}$$

Note that this example uses a bnode assignment function id_{ex} that associates the (embedded) RDF* triple $\langle$:bob, foaf:age, 23$\rangle$ with blank node $b \in \mathcal{B}$.

3.3 Turtle* − An RDF* Extension of Turtle

Example 3 outlines a possible extension of the Turtle syntax to write an RDF* graph. This section defines this extension, called Turtle*.

Turtle* extends the Turtle grammar (as given in [PC14, Section 6.5]) with the following three additional productions.

> tripleX ::= '<<' subjectX predicate objectX '>>'
> subjectX ::= iri | BlankNode | tripleX
> objectX ::= iri | BlankNode | literal | tripleX

Any string that matches production tripleX is to be mapped to an RDF* triple $\langle s, p, o \rangle$ such that (i) s is the RDF term or the (embedded) RDF* triple that can be obtained by parsing the substring that matches subjectX, (ii) p is the RDF term obtained by parsing the substring that matches predicate, and (iii) o is the RDF term or the (embedded) RDF* triple obtained by parsing the substring that matches objectX.

In addition to adding these three productions to the grammar, Turtle* extends the productions labeled [10] and [12] in the standard Turtle grammar as follows (the extension to the productions are given in bold font).

> subject ::= iri | BlankNode | collection **| tripleX**
> object ::= iri | BlankNode | collection | blankNodePropertyList | literal **| tripleX**

A Turtle* parser is a Turtle parser that is extended to take into account the productions defined in this section. Hence, such a parser constructs a set of RDF* triples (i.e., an RDF* graph) that can be processed by an RDF*-aware system.

Note that ordinary RDF data management systems (that do not support RDF*) may easily be enabled to read a Turtle* document and process the data; they only need to use a Turtle* parser equipped with a transformation component that transforms the RDF* graph given in the document to an unfolded RDF graph as defined in Section 3.2.

4 SPARQL* − A Metadata Extension of SPARQL

This section introduces SPARQL*, which is an RDF*-aware extension of the RDF query language SPARQL; i.e., SPARQL* can be used to query RDF* graphs. To fully benefit from the extended data model, SPARQL* adds new features that enable users to directly access metadata triples in queries. In particular, SPARQL* introduces the possibility to bind RDF* triples to query variables; such a variable may then be used in a triple pattern in order to ask for matching metadata triples. Furthermore, as a shortcut, (recursively nested) triple patterns may be embedded directly in triple patterns (as demonstrated in Example 4).

In the following, Section 4.1 introduces basic terminology and concepts. Section 4.2 defines SPARQL* based on Pérez et al.'s algebraic syntax of SPARQL [PAG09]. Thereafter, Section 5 provides the corresponding extension of the W3C specification of SPARQL [HSP13].

4.1 Basic Terminology and Concepts

The basic concepts for defining SPARQL queries and their semantics are triple patterns and solution mappings. A *triple pattern* is a tuple $tp \in (\mathcal{V} \cup \mathcal{I} \cup \mathcal{B} \cup \mathcal{L}) \times (\mathcal{V} \cup \mathcal{I}) \times (\mathcal{V} \cup \mathcal{I} \cup \mathcal{B} \cup \mathcal{L})$ where $\mathcal{V}$ is a set of query variables that is disjoint from $\mathcal{I}$, $\mathcal{B}$, and $\mathcal{L}$, respectively. A *solution mapping* is a partial mapping $\mu : \mathcal{V} \to (\mathcal{I} \cup \mathcal{B} \cup \mathcal{L})$. SPARQL* extends these concepts by introducing a notion of *triple* patterns* and *solution* mappings*.

Definition 6. Let $\widetilde{\mathcal{TP}} = (\mathcal{V} \cup \widetilde{\mathcal{TP}} \cup \mathcal{I} \cup \mathcal{B} \cup \mathcal{L}) \times (\mathcal{V} \cup \mathcal{I}) \times (\mathcal{V} \cup \widetilde{\mathcal{TP}} \cup \mathcal{I} \cup \mathcal{B} \cup \mathcal{L})$, and call a tuple $\langle s, p, o \rangle \in \widetilde{\mathcal{TP}}$ *k-nested* if k is a natural number such that either (i) $k = 0$ and $\langle s, p, o \rangle$ is an (ordinary) triple pattern or (ii) $k > 0$ and each (embedded) tuple $\langle s', p', o' \rangle \in (\{s, o\} \cap \widetilde{\mathcal{TP}})$ is $(k\!-\!1)$-nested and free of blank nodes (i.e., $s' \notin \mathcal{B}$ and $o' \notin \mathcal{B}$). Then, a **triple* pattern** is a tuple $tp \in \widetilde{\mathcal{TP}}$ for which there exists a natural number k such that tp is k-nested. $\mathcal{TP}^\star$ denotes the set of all triple* patterns; i.e. $\mathcal{TP}^\star \subset \widetilde{\mathcal{TP}}$.

Definition 7. A **solution* mapping** is a partial mapping $\eta : \mathcal{V} \to (\mathcal{T}^\star \cup \mathcal{I} \cup \mathcal{B} \cup \mathcal{L})$.

Note that, in contrast to standard solution mappings that bind variables only to an IRI, a blank node, or a literal, a solution* mapping may bind a variable also to an RDF* triple.

The following three definitions adapt the standard notions of *compatibility* of solution mappings, *merging* of solution mappings, and *application* of solution mappings to solution* mappings.

Definition 8. Two solution* mappings η and η' are **compatible**, denoted by $\eta \sim \eta'$, if, for every variable $?v \in (\mathrm{dom}(\eta) \cap \mathrm{dom}(\eta'))$, $\eta(?v) = \eta'(?v)$.

Definition 9. Let η and η' be two solution* mappings that are compatible. The **merge** of η and η', denoted by $\eta \cup \eta'$, is a solution* mapping η'' that has the following three properties:

1. $\mathrm{dom}(\eta'') = \mathrm{dom}(\eta) \cup \mathrm{dom}(\eta')$,

2. $\eta''(?v) = \eta(?v)$ for all $?v \in \mathrm{dom}(\eta)$, and

3. $\eta''(?v) = \eta'(?v)$ for all $?v \in \mathrm{dom}(\eta') \setminus \mathrm{dom}(\eta)$.

Definition 10. The **application** of a solution* mapping η to a triple* pattern tp, denoted by $\eta[tp]$, is the triple* pattern that can be obtained by replacing all variables in tp according to η (unbound variables must not be replaced).

4.2 (Algebraic) Syntax and Semantics of SPARQL*

This section defines the semantics of the core fragment of SPARQL*, which is represented based on an algebraic syntax that extends the algebraic SPARQL syntax introduced by Pérez et al. [PAG09].

Definition 11. A **SPARQL* expression** is defined recursively as follows:

1. Any finite set of triple* patterns is a SPARQL* expression, which is called a *BGP**.

2. If tp is a triple* pattern and $?v$ is a variable, then $(tp \text{ AS } ?v)$ is a SPARQL* expression.

3. If P_1 and P_2 are SPARQL* expressions and R is a filter condition[2], then $(P_1 \text{ AND } P_2)$, $(P_1 \text{ UNION } P_2)$, $(P_1 \text{ OPT } P_2)$, and $(P_1 \text{ FILTER } R)$ are SPARQL* expressions.

Example 7. *The first query pattern of Example 4 can be represented as a BGP**

$$P_{\mathsf{ex}} = \big\{ \langle ?bob, \texttt{foaf:name}, \texttt{"Bob"} \rangle, \langle \langle ?bob, \texttt{foaf:age}, ?age \rangle, \texttt{dct:source}, ?src \rangle \big\},$$

which consists of two triple* patterns. The second query pattern of Example 4 can be represented as a semantically equivalent SPARQL* expression P_{ex2} that has the following form:

$$\Big(\big(\langle ?bob, \texttt{foaf:age}, ?age \rangle \text{ AS } ?t \big) \text{ AND } \big\{ \langle ?bob, \texttt{foaf:name}, \texttt{"Bob"} \rangle, \langle ?t, \texttt{dct:source}, ?src \rangle \big\} \Big).$$

The basis for defining the semantics of SPARQL* is an algebra over multisets of solution* mappings that resembles the standard SPARQL algebra (which is defined over multisets of ordinary solution mappings). Formally, a multiset solution* mappings is a pair $M = \langle \Omega, card \rangle$ where Ω is the underlying set (of solution* mappings) and $card$ is the corresponding *cardinality function*; i.e., $card : \Omega \to \{1, 2, \dots\}$. Then, the SPARQL*-specific algebra operators are defined as follows.

Definition 12. Let $M_1 = \langle \Omega_1, card_1 \rangle$ and $M_2 = \langle \Omega_2, card_2 \rangle$ be multisets of solution* mappings.

- The **join** of M_1 and M_2, denoted by $M_1 \bowtie M_2$, is a multiset of solution* mappings $\langle \Omega, card \rangle$ such that
$$\Omega = \big\{ \eta_1 \cup \eta_2 \mid \eta_1 \in \Omega_1 \text{ and } \eta_2 \in \Omega_2 \text{ and } \eta_1 \sim \eta_2 \big\}$$
and, for every $\eta \in \Omega$,
$$card(\eta) = \sum_{(\eta_1, \eta_2) \in \Omega^\eta} card_1(\eta_1) \cdot card_2(\eta_2),$$
where $\Omega^\eta = \big\{ (\eta_1, \eta_2) \in \Omega_1 \times \Omega_2 \mid \eta_1 \cup \eta_2 = \eta \big\}$.

- The (multiset) **union** of M_1 and M_2, denoted by $M_1 \uplus M_2$, is a multiset of solution* mappings $\langle \Omega, card \rangle$ such that $\Omega = \Omega_1 \cup \Omega_2$ and, for every $\eta \in \Omega$,
$$card(\eta) = \begin{cases} card_1(\eta) + card_2(\eta) & \text{if } \eta \in (\Omega_1 \cup \Omega_2), \\ card_1(\eta) & \text{if } \eta \in (\Omega_1 \setminus \Omega_2), \\ card_2(\eta) & \text{else.} \end{cases}$$

- The (multiset) **difference** of M_1 and M_2, denoted by $M_1 \setminus\!\!\setminus M_2$, is a multiset of solution* mappings $\langle \Omega, card \rangle$ such that $\Omega = \big\{ \eta \in \Omega_1 \mid \eta \not\sim \eta' \text{ for all } \eta' \in \Omega_2 \big\}$ and $card(\eta) = card_1(\eta)$ for all $\eta \in \Omega$.

- The **left outer join** of M_1 and M_2, denoted by $M_1 \rimmeljoin M_2$, is a multiset of solution* mappings that is defined by:
$$M_1 \rimmeljoin M_2 = \big(M_1 \bowtie M_2 \big) \uplus \big(M_1 \setminus\!\!\setminus M_2 \big).$$

- Given a filter condition R, the R-specific **selection** of M_1, denoted by $\sigma_R(M_1)$, is a multiset of solution* mappings $\langle \Omega, card \rangle$ such that $\Omega = \{\eta \in \Omega_1 \mid \eta$ satisfies $R\}$ and $card(\eta) = card_1(\eta)$ for all $\eta \in \Omega$, where a solution* mapping η *satisfies* filter condition R if any of the following holds:

  1. R is bound($?v$) where $?v \in \mathcal{V}$ and $?v \in \mathrm{dom}(\eta)$;
  2. R is $?v = c$ where $?v \in \mathcal{V}$ and $c \in (\mathcal{I} \cup \mathcal{L})$, $?v \in \mathrm{dom}(\eta)$, and $\eta(?v) = c$;
  3. R is $?x = ?y$ where $?x, ?y \in \mathcal{V}$, $?x \in \mathrm{dom}(\eta)$, $?y \in \mathrm{dom}(\eta)$, and $\eta(?x) = \eta(?y)$;
  4. R is $(\neg R')$ where R' is a filter condition and η does not satisfy R';
  5. R is $(R_1 \vee R_2)$ where R_1 and R_2 are filter conditions and η satisfies R_1 or R_2; or
  6. R is $(R_1 \wedge R_2)$ where R_1 and R_2 are filter conditions and η satisfies both R_1 and R_2.

Given these algebra operators, the semantics of any SPARQL* expression is defined by the following evaluation function.

Definition 13. Let P be a SPARQL* expression and let $G^\star$ be an RDF* graph. The **evaluation** of P over $G^\star$, denoted by $[\![P]\!]_{G^\star}$, is a multiset of solution* mappings $\langle \Omega, card \rangle$ that is defined recursively as follows:

1. If P is a BGP*, then

$$\Omega = \{\eta \mid \eta[\rho[P]] \subseteq (\mathrm{Emb}^+(G^\star) \cup G^\star) \text{ for some } P\text{-bnodes mapping } \rho\}$$

   and, for every $\eta \in \Omega$,

$$card(\eta) = \left| \{\rho \mid \rho \text{ is a } P\text{-bnodes mapping such that } \eta[\rho[P]] \subseteq (\mathrm{Emb}^+(G^\star) \cup G^\star)\} \right|,$$

   where a *P-bnodes mapping* is a mapping $\rho : \mathrm{bn}(P) \to (\mathcal{T}^\star \cup \mathcal{I} \cup \mathcal{B} \cup \mathcal{L})$ and

$$\eta[\rho[P]] = \{\eta[tp] \mid tp \text{ is a triple}^\star \text{ pattern obtained by replacing all} \\ \text{blank nodes in some triple}^\star \text{ pattern } tp' \in P \text{ according to } \rho\}.$$

2. If P is $(tp$ AS $?v)$, then

$$\Omega = \{\eta \mid \exists \eta' \in \Omega' : \eta' \sim \eta \text{ and } \mathrm{dom}(\eta) = (\mathrm{dom}(\eta') \cup \{?v\}) \text{ and } \eta(?v) = \eta'[tp]\}$$

   and, for every $\eta \in \Omega$,

$$card(\eta) = \sum_{\eta' \in \Omega'_\eta} card'(\eta'),$$

   where $\langle \Omega', card' \rangle = [\![\{tp\}]\!]_{G^\star}$ and $\Omega'_\eta = \{\eta' \in \Omega' \mid \eta' \sim \eta\}$ for all $\eta \in \Omega$.

3. If P is $(P_1$ AND $P_2)$, then $\langle \Omega, card \rangle = [\![P_1]\!]_{G^\star} \bowtie [\![P_2]\!]_{G^\star}$.

4. If P is $(P_1$ UNION $P_2)$, then $\langle \Omega, card \rangle = [\![P_1]\!]_{G^\star} \uplus [\![P_2]\!]_{G^\star}$.

5. If P is $(P_1$ OPT $P_2)$, then $\langle \Omega, card \rangle = [\![P_1]\!]_{G^\star} \leftouterjoin [\![P_2]\!]_{G^\star}$.

6. If P is $(P'$ FILTER $R)$, then $\langle \Omega, card \rangle = \sigma_R([\![P']\!]_{G^\star})$.

Example 8. *The evaluation of SPARQL* expression P_{ex} (cf. Example 7) over RDF* graph $G_{\mathsf{ex}}^\star$ (cf. Example 5) consists of a single solution* mapping η_1, which has the following properties:*

1. $\mathrm{dom}(\eta_1) = \{?bob, ?age, ?src\}$,

2. $\eta_1(?bob) = $ `:bob`,

3. $\eta_1(?age) = $ `23`, *and*

4. $\eta_1(?src) = $ `http://example.net/homepage-listing.html`.

For the other expression from Example 7 we obtain the same result: $[\![P_{\mathsf{ex2}}]\!]_{G_{\mathsf{ex}}^\star} = \langle \Omega_{\mathsf{ex2}}, card_{\mathsf{ex2}} \rangle$ *where $\Omega_{\mathsf{ex2}} = \{\eta_1\}$ and $card_{\mathsf{ex2}}(\eta_1) = 1$.*

5   Extension of the W3C Specification of SPARQL

After defining SPARQL* based on an algebraic syntax, the remainder of this document defines a corresponding extension of the formalization of SPARQL 1.1 that is given by the W3C specification [HSP13]. This extension assumes that any mention of *"RDF triple"* in the specification is understood as an RDF* triple; similarly, *"RDF graph"*, *"triple pattern"*, *"basic graph pattern"* (or *"basic graph pattern"*, and *"solution mapping"* are understood as RDF* graph, triple* pattern, BGP*, and solution* mapping, respectively. Furthermore, the understanding of a *"property path pattern"* includes the possibility to use a triple* pattern as subject or object of such a pattern.

Section 5.1 introduces the grammar of SPARQL* as an extension of the grammar of SPARQL 1.1. Section 5.2 specifies how to support the extended grammar during the conversion of query strings into algebra expressions; this specification includes the introduction of a new algebra symbol which corresponds to SPARQL* expressions of the form (tp AS $?v$). Section 5.3 defines the evaluation semantics for the resulting algebra expressions.

5.1   Grammar

This section specifies the SPARQL* grammar as an extension of the standard SPARQL 1.1 grammar [HSP13]. Elements of the grammar that are not specified explicitly in this section are defined as given in [HSP13, Section 19.8].

An *embedded triple pattern* is a new syntax element that conforms to the following, new grammar rules:

> EmbTP ::= '<<' VarOrIRIOrLitOrEmbTP Verb VarOrIRIOrLitOrEmbTP '>>'
>
> VarOrIRIOrLitOrEmbTP ::= Var | iri | RDFLiteral | NumericLiteral | BooleanLiteral | EmbTP

As the given grammar rules indicate, an embedded triple pattern may contain other embedded triple patterns. Embedded triple patterns may be used in a query in the following two ways: (i) they are part of a BIND clause (which corresponds to SPARQL* expressions of the form (tp AS $?v$)), or (ii) they are embedded in a triple* pattern or in a property path pattern. Both of these options are specified and discussed in the following.

SPARQL introduces the BIND clause for assigning the value of evaluating a given expression to a variable. To enable the use of embedded triple patterns in BIND clauses (instead of an expression), the following two extensions to the grammar are necessary. First, a new grammar rule is added:

ExpressionOrEmbTP ::= Expression **|** EmbTP

Second, the original grammar rule [60] is redefined as follows:[3]

Bind ::= 'BIND' '(' **ExpressionOrEmbTP** 'AS' Var ')'

An embedded triple pattern may not only be used in a BIND clause but it may also be embedded in a property path pattern or in a triple* pattern. More precisely, in the extended syntax the subject or object of a property path pattern can be an embedded triple pattern (instead of an RDF term or a variable). Similarly, triple* patterns may have an embedded triple pattern in the subject position or in the object position (cf. Definition 6). To this end, the grammar is extended with a new rule:

VarOrTermOrEmbTP ::= Var | GraphTerm | EmbTP

Moreover, the original grammar rules [80] and [81] are redefined:

Object ::= GraphNode **| EmbTP**

TriplesSameSubjectPath ::= **VarOrTermOrEmbTP** PropertyListPathNotEmpty |

TriplesNodePath PropertyListPath

5.2 Translation to the Algebra

Based on the SPARQL grammar the SPARQL specification *"defines the process of converting graph patterns and solution modifiers in a SPARQL query string into a SPARQL algebra expression"* [HSP13, Section 18.2]. This process must be adjusted to consider the extended grammar introduced in the previous section. In the following, any step of the conversion process that requires adjustment is discussed.

5.2.1 Variable Scope

As a basis of the translation, the SPARQL specification introduces a notion of *in-scope variables* [HSP13, Section 18.2.1]. To cover the new syntax elements introduced in Section 5.1 this notion must be extended as follows.

- A variable is in-scope of a BGP* B if the variable occurs in B, which includes an occurrence in any embedded triple pattern in BGP* (independent of the level of nesting).

- A variable is in-scope of a property path pattern if the variable occurs in that pattern, which includes an occurrence in any embedded triple pattern in the pattern (independent of the level of nesting).

- A variable is in-scope of a BIND clause of the form `BIND ( T AS v )` (where `T` is an embedded triple pattern) if the variable is variable `v` or the variable occurs in the embedded triple pattern `T`. As for standard BIND clauses with expressions, variable `v` must *"not [be] in-scope from the preceeding elements in the group graph pattern in which* [the BIND clause] *is used"* [HSP13, Section 18.2.1].

---

[3]The adjusted part in which a redefined grammar rule differs from the original rule in [HSP13, Section 19.8] is given in bold font.

5.2.2 Expand Syntax Forms

The translation process starts with expanding *"abbreviations for IRIs and triple patterns"* [HSP13, Section 18.2.2.1]. This step must be extended in two ways:

1. Abbreviations for triple patterns with embedded triple patterns must be expanded as if each embedded triple pattern was a variable (or an RDF term). For instance, the following syntax expression

   ```
   <<?c a rdfs:Class>> dct:source ?src ;
                       prov:wasDerivedFrom <<?c a owl:Class>> .
   ```

   must be expanded to

   ```
   <<?c a rdfs:Class>> dct:source ?src .
   <<?c a rdfs:Class>> prov:wasDerivedFrom <<?c a owl:Class>> .
   ```

2. Abbreviations for IRIs in all embedded triple patterns must be expanded. For instance, the embedded triple pattern

   ```
   <<?c a rdfs:Class>>
   ```

   must be expanded to

   ```
   <<?c <http://www.w3.org/1999/02/22-rdf-syntax-ns#type> <http://www.w3.org/2000/01/rdf-schema#Class>>>
   ```

5.2.3 Translate Property Path Patterns

The translation of property path patterns (cf. [HSP13, Section 18.2.2.4]) has to be adjusted because the extended grammar allows for property path patterns whose subject or object is an embedded triple pattern (cf. Section 5.1).

The translation as specified in the W3C specification distinguishes four cases. The first three of these cases do not require adjustment because they are taken care of either by recursion or by the adjusted translation of basic graph patterns (as defined in Section 5.2.4 below). However, the fourth case must be adjusted as follows.

Let `X P Y` be a string that corresponds to the fourth case in [HSP13, Section 18.2.2.4]. Given the grammar introduced in Section 5.1, X and Y may be an RDF term, a variable, or an embedded triple pattern, respectively (and P is a property path expression). The string `X P Y` is translated to the algebra expression Path(X', P, Y') where X' and Y' are the result of calling a function named Lift for X and Y, respectively. For some input string Z (such as X or Y) that can be an RDF term, a variable, or an embedded triple pattern, the function Lift is defined as follows:

```
If Z is an embedded triple pattern <<S,P,O>>
    Return triple* pattern ( Lift(S), P, Lift(O) )
Else
    Return Z
End
```

5.2.4 Translate Basic Graph Patterns

After translating property path patterns, the translation process collects *"any adjacent triple patterns [...] to form a basic graph pattern"* [HSP13, Section 18.2.2.5]. This step has to be adjusted because triple patterns in the extended syntax may have an embedded triple pattern in their subject position or in their object position (or in both). To ensure that every result of this step is a BGP*, before adding a triple pattern to its corresponding collection, its subject and object must be replaced by the result of calling function `Lift` (cf. Section 5.2.3) for the subject and the object, respectively.

5.2.5 Translate BIND Clauses with an Embedded Triple Pattern

The extended grammar in Section 5.1 allows for BIND clauses with an embedded triple pattern. The translation of such a BIND clause to a SPARQL algebra expression requires a new algebra symbol:

- $\mathsf{TR}(\ \text{triple}^\star\ \text{pattern, variable}\ )$

Note that this symbol corresponds to SPARQL* expressions of the form $(tp\ \mathsf{AS}\ ?v)$ (cf. Definition 11).

Then, any string of the form `BIND ( T AS v )` with `T` being an embedded triple pattern (i.e., not a standard BIND expression) is translated to the algebra expression $\mathsf{TR}(\ \text{T'},\ \text{v}\ )$ where `T'` is the result of calling the aforementioned function `Lift` for `T`.

Notice, the translation of BIND clauses with an embedded triple pattern as defined in this section is used during the translation of group graph patterns that is specified in [HSP13, Section 18.2.2.6]. The case of BIND clauses with an embedded triple pattern is covered in this translation of group graph patterns by the last, "catch all other" `IF` statement (i.e., the `IF` statement with the condition `E is any other form`) and *not* by the `IF` statement for BIND clauses with an expression.

5.3 Evaluation Semantics

The SPARQL specification defines a function *"eval(D(G), algebra expression) as the evaluation of an algebra expression with respect to a dataset D having active graph G"* [HSP13, Section 18.6]. Recall that the active graph G in the context of SPARQL* is an RDF* graph, and so is any other graph in dataset D. The definition of function eval is recursive; the two base cases of this definition for SPARQL* are given as follows:

- For any BGP* B, $\mathrm{eval}\big(D(G), B\big) = [\![B]\!]_G$ (where $[\![B]\!]_G$ is the evaluation of B over RDF* graph G as per Definition 13).

- For any algebra expression E of the form $\mathsf{TR}(\ tp, ?v\ )$ where tp is a triple* pattern and $?v$ is a variable (as introduced in Section 5.2.5), $\mathrm{eval}\big(D(G), E\big) = [\![(tp\ \mathsf{AS}\ ?v)]\!]_G$ (where $[\![(tp\ \mathsf{AS}\ ?v)]\!]_G$ is the evaluation of SPARQL* expression $(tp\ \mathsf{AS}\ ?v)$ over RDF* graph G as per Definition 13).

For any other algebra expression, the SPARQL specification defines algebra operators. These definitions can be extended naturally to operate over multisets of solution* mappings (instead of ordinary solution mappings). Given this extension, the recursive steps of the definition of function eval for SPARQL* are the same as in the SPARQL specification.

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