

Homework 2

Exercises

1-5. Problems from *Mathematical Reasoning: Writing and Proof* [1]:

- Section 2.3, #2b, 5
- Section 5.3, #2, 3
- Section 5.5, #1a-d

Remember, we use the notation $A \subseteq B$ to indicate that A is a subset of B (that A contains some, but not necessarily all, of the objects that B contains); we use the notation $x \in A$ to indicate that x is an element of A . We also write $A \not\subseteq B$ and $x \notin A$ to indicate the negation of these conditions.

6. Let $A = \{2, 4, 6, 8, 10\}$ and $B = \{1, 3, 6, 7, 8\}$. Indicate whether the following are true or false:

- (a) $11 \in B$
- (b) $A \subseteq \mathbb{Z}^+$
- (c) $A \in \mathbb{Z}^+$
- (d) $4 \notin B$

7. Indicate whether the following are true or false:

- (a) $\emptyset = \{\emptyset\}$
- (b) $\emptyset \subseteq \{\emptyset\}$
- (c) $\emptyset \in \{\emptyset\}$

8. Suppose that $A \subseteq B$. Show that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

9. Let A and B be sets. Show that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

10. If $X = \{a, c\}$ and $Y = \{a, b, d\}$, write down the elements of $X \times Y$, $Y \times X$, and $X \times X$.

11. If A and B are subsets of some universe U , we will define $\overline{A \times B}$ as $(U \times U) \setminus (A \times B)$. Prove that

$$\overline{A \times B} = (\overline{A} \times B) \cup (\overline{A} \times \overline{B}) \cup (A \times \overline{B}).$$

12. Let $k \in \mathbb{Z}^+$ and consider the families $A_k := [1, 1 + \frac{1}{k}]$ and $B_k := (1, 1 + \frac{1}{k})$, where we use parenthesis to denote open intervals of real numbers and brackets to denote closed intervals of real numbers. Compute the following:

$$\bigcap_{k=1}^{\infty} A_k, \quad \bigcup_{k=1}^{\infty} A_k, \quad \bigcap_{k=1}^{\infty} B_k, \quad \text{and} \quad \bigcup_{k=1}^{\infty} B_k.$$

(No proofs needed)

13. Given the theorems you know regarding distance in Euclidean geometry, give an informal proof of the following claim:

Let S be a square whose sides have length 1. Given any five points p_1, p_2, \dots, p_5 that lie inside S , show that the distance between at least two of these points is less than or equal to $\frac{1}{\sqrt{2}}$.

Hint: Pretend the points are pigeons, then try coming up with some reasonable holes to stuff them into.

References

- [1] Ted Sundstrom. *Mathematical Reasoning: Writing and Proof*. Grand Valley State University Libraries, 3rd edition, 2020.