Homework 0

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1. (2.1 Question 2)
(a) False. We are given that $P \to Q$. We know the logical equivalency $P \to Q \equiv \neg P \vee Q$. We are given $\neg Q$, so we must have $\neg P$, thus we cannot have P.
(b) False. We are given $\neg Q$, so we cannot have $P \wedge Q$.
(c) False. We are given $P \to Q$, and we know that $P \to Q \equiv \neg P \vee Q$. We are given $\neg Q$, thus we must have $\neg P$. Thus, we have $\neg Q \wedge \neg P$. $\neg Q \wedge \neg P \equiv \neg(P \vee Q)$, so $P \vee Q$ is false.

2. (2.1 Question 3)
(a) True. We know $\neg P \to Q \equiv P \vee Q$. We are given that $P \to Q$ is false, which is logically equivalent to $\neg(\neg P \vee Q) \equiv P \wedge \neg Q$. From this, we know P, thus $P \vee Q$ is satisfied, and $\neg P \to Q$ is true.
(b) True. We know that $Q \to P \equiv \neg Q \vee P$. Similarly to (a), we have $\neg(P \to Q) \equiv P \wedge \neg Q$, so we have P. Thus, we have $\neg Q \vee P$, so we have. $Q \to P$.
(c) True. We are given $\neg(P \to Q) \equiv P \wedge \neg Q$. We have P, so we have $P \vee Q$.

3. (2.1 Question 7)
We construct the following truth table:

p	q	r	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
1	1	1	1	1
1	0	1	1	1
1	1	0	1	1
1	0	0	0	0
0	1	1	0	0
0	0	1	0	0
0	1	0	0	0
0	0	0	0	0

We find that $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ based on their truth table.

4. (2.2 Question 1)
(a) Converse: If $a^2 = 25$, then $a = 5$. Contrapositive: If $a^2 \neq 25$, then $a \neq 5$.
(b) Converse: If Laura is playing golf, then it is not raining. Contrapositive: If Laura is not playing golf, then it is raining.
(c) Converse: If $a^4 \neq b^4$, then $a \neq b$. Contrapositive: If $a^4 = b^4$, then $a = b$.
(d) Converse: If $3a$ is odd, then a is odd. Contrapositive: If $3a$ is not odd (even), then a is not odd (even).

5. (2.2 Question 3(e) and 3(f))
(e) I will not wash the car and I will not mow the lawn.
(f) I will graduate from college and I will not get a job and I will not go to graduate school.

6. (2.2 Question 4)
(a) We construct the following truth table:

p	q	$p \Leftrightarrow q$	$(p \to q) \wedge (q \to p)$
1	1	1	1
1	0	0	0
0	1	0	0
0	0	1	1

Thus, $p \Leftrightarrow q \equiv [(p \rightarrow q) \wedge (q \rightarrow p)]$.

(c) We construct the following truth table:

p	q	$p \Leftrightarrow q$	$\neg p \Leftrightarrow \neg q$
1	1	1	1
1	0	0	0
0	1	0	0
0	0	1	1

Thus, $p \Leftrightarrow q \equiv \neg p \Leftrightarrow \neg q$.

7. (2.2 Question 7)
(a) We are given $(P \wedge Q) \rightarrow R$, which is logically equivalent to $\neg(P \wedge Q) \vee R \equiv \neg P \vee \neg Q \vee R$. By idempotency, $\neg P \vee \neg Q \vee R \equiv \neg P \vee \neg Q \vee R \vee R$. By the associativity of conjunctions and disjunctions, this is equivalent to $(\neg P \vee R) \vee (\neg Q \vee R)$, which is in turn equivalent to $(P \rightarrow R) \vee (Q \rightarrow R)$.
(b) We are given $P \rightarrow (Q \wedge R)$. This is logically equivalent to $\neg P \vee (Q \wedge R)$. By idempotency, $\neg P \vee (Q \wedge R) \equiv \neg P \vee (Q \wedge R) \vee \neg P$. By the associativity of conjunctions and disjunctions, $\neg P \vee (Q \wedge R) \vee \neg P \equiv (\neg P \vee Q) \wedge (\neg P \vee R)$, which is equivalent to $(P \rightarrow Q) \wedge (P \rightarrow R)$.

8. (2.2 Question 10(a), (b), and (c))
(a) The statement is the converse, and is neither a negation or an equivalent.
(b) The statement is the inverse, and is neither a negation or an equivalent.
(c) The statement is the contrapositive, and is therefore logically equivalent.

9. Question 9
If $P \downarrow Q$ is true exactly when P and Q are both false, then an equivalent expression is $\neg P \wedge \neg Q$.

10. Question 10
A logical equivalent to $\neg P$ solely in terms of the neither operator is $P \downarrow P$. By both the definition of the neither operator and idempotency, $P \downarrow P \equiv \neg P \wedge \neg P \equiv \neg P$.
A logical equivalent for $P \vee Q$ solely in terms of the neither operator is $(P \downarrow Q) \downarrow (P \downarrow Q)$. By the definition of the neither operator, as well as the definition of idempotency, $(P \downarrow Q) \downarrow (P \downarrow Q) \equiv \neg(\neg P \wedge \neg Q) \wedge \neg(\neg P \wedge \neg Q)$. Negating, we get $(P \vee Q) \wedge (P \vee Q) \equiv P \vee Q$.
A similar approach can be taken for $P \wedge Q$. $P \wedge Q \equiv (P \downarrow P) \downarrow (Q \downarrow Q)$. By definition of the neither operator, as well as the fact that $\neg P \equiv P \downarrow P$, we can show that $(P \downarrow P) \downarrow (Q \downarrow Q) \equiv \neg(\neg P) \wedge \neg(\neg Q)$. Negating, we get $P \wedge Q$.