Question 1:

(a) $[18] = \{\, 4n + 2 \mid n \in \mathbb{Z} \,\}$.

(b) $[31] = \{\, 4n + 2 \mid n \in \mathbb{Z} \,\}$.

(c) There are 4 equivalence classes.

Question 2: *Proof.* Need to prove that the relation is reflective, symmetric, and transitive. □

Question 3: *Proof.* Suppose $a, b, c, d \in \mathbb{Z}$, and $a \equiv_n c$ and $b \equiv_n d$. Then,

$$\begin{gathered} a - c = nk, k \in \mathbb{Z} \\ b - d = nl, l \in \mathbb{Z}. \end{gathered}$$

Adding the equations, we get

$$a + b - c - d = n(k + l).$$

Simplifying, we get

$$(a + b) - (c + d) = n(k + l).$$

Thus, by definition $a + b \equiv_n c + d$. □

Question 4:

(a) *Proof.* The theorem given in the proposition is equivalent to claiming that 'If ab is even, then a is even or b is even'. We have proven this theorem to be true, so it is the case that if $ab \equiv_2 0$, then $a \equiv_2 0$ or $b \equiv_2 0$. □