Question 1: (Section 2.3, #2b) $B = \{ -\pi^x \mid x \leq 0 \}$.

Question 2: (Section 2.3, #5)

(a) $S = \{ x \in \mathbb{Z} \mid x \geq 5 \}$.

(b) $S = \{ 2x + 1 \mid x \in \mathbb{Z} \}$.

(c) $S = \{ x \in \mathbb{Q} \mid x > 0 \}$.

(d) $S = \{ x \in \mathbb{R} \mid 1 < x < 7 \}$.

(e) $S = \{ x \in \mathbb{R} \mid x^2 > 0 \}$.

Question 3: (Section 5.3, #2)

Proof. In order to prove equality, we show that $A \cup (B \cap C) \subseteq (A \cap B) \cup (A \cap C)$ and $(A \cap B) \cup (A \cap C) \subseteq A \cup (B \cap C)$.

($\subseteq$) Suppose x is an arbitrary element in $A \cap (B \cup C)$. Then, $x \in A \wedge (x \in B \vee x \in C)$. By the distributive property of conjunctions and disjunctions, $(x \in A \wedge x \in B) \vee (x \in A \vee x \in C)$. Thus, $x \in (A \cap B) \cup (A \cap C)$.

($\supseteq$) The argument is symmetric.

$\square$

Question 4: (Section 5.3, #3)

Proof. In order to prove equality, we show that $\overline{(A \cap B)} \subseteq \overline{A} \cup \overline{B}$ and $\overline{A} \cup \overline{B} \subseteq \overline{(A \cap B)}$.

($\subseteq$) Let x be an arbitrary element of $\overline{(A \cap B)}$. That is, $(x \notin A \vee x \notin B) \rightarrow (x \in \overline{A} \vee x \in \overline{B})$. Thus, $x \in \overline{A} \vee x \in \overline{B}$, so $x \in \overline{A} \cup \overline{B}$.

($\supseteq$) The argument is symmetric.

$\square$

Question 5: (Section 5.5, #1a-d)

(a) $\{ 3, 4 \}$.

(b) $\{ 1, 2, 3, 4, 5, 6 \}$.

(c) $\varnothing$.

(d) $\{ 3, 4, 5, 6, 7, 8, 9, 10 \}$.

Question 6: (a) False. $11 \notin B$.

(b) True. Every element in A is also in $\mathbb{Z}^+$.

(c) False. A is not an integer, and therefore is not a subset of $\mathbb{Z}^+$.

(d) True.

Question 7: (a) False. The empty set has no elements, and the set on the RHS has 1 element.

(b) True. The empty set is a subset of every set.

(c) True. The empty set appears in the set on the RHS, and therefore is an element.

Question 8: *Proof.* Suppose A and B are sets, and $A \subseteq B$. Let X be an arbitrary element of $\mathcal{P}(A)$. Then, $X \subseteq A \subseteq B$, so $X \subseteq B$. Thus, $X \in \mathcal{P}(B)$. So, an aribtrary element of $\mathcal{P}(A)$ is in $\mathcal{P}(B)$, so $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

□

Question 9: *Proof.* In order to show that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$, we show that they are both are a subset of each other.

($\subseteq$) Suppose A and B are sets, and $C \in \mathcal{P}(A \cap B)$. Then, $C \subseteq (A \cap B) \to C \subseteq A$ and $C \subseteq B$. $C \subseteq A \to C \in \mathcal{P}(A)$. Similarly, $C \subseteq B \to C \in \mathcal{P}(B)$. Thus, $C \in \mathcal{P}(A) \cap \mathcal{P}(B)$.

($\supseteq$) The argument is symmetric.

□

Question 10:

$$X \times Y = \{\, (a,a), (a,b), (a,d), (c,a), (c,b), (c,d) \,\}.$$
$$Y \times X = \{\, (a,a), (b,a), (d,a), (a,c), (b,c), (d,c) \,\}.$$
$$X \times X = \{\, (a,a), (a,c), (c,a), (c,c) \,\}.$$

Question 11: In order to show that $\overline{A \times B} = (\overline{A} \times B) \cup (\overline{A} \times \overline{B}) \cup (A \times \overline{B})$, we show they are both subsets of each other.

($\subseteq$) Let (x,y) be an element of $\overline{A \times B}$, so $x \notin A$ or $y \notin B$. There are three cases that satisfy this conjunction:

(Case 1:) $x \notin A$ and $y \notin B$. Thus, $(x,y) \in \overline{A} \times \overline{B}$.

(Case 2:) $x \in A$ and $y \notin B$. Thus, $(x,y) \in A \times \overline{B}$.

(Case 3:) $x \notin A$ and $y \in B$. Thus, $(x,y) \in \overline{A} \times B$.

So, $(x,y) \in (\overline{A} \times B) \cup (\overline{A} \times \overline{B}) \cup (A \times \overline{B})$, thus $\overline{A \times B} = (\overline{A} \times B) \cup (\overline{A} \times \overline{B}) \cup (A \times \overline{B})$.

($\supseteq$) Let (x,y) be an element of $(\overline{A} \times B) \cup (\overline{A} \times \overline{B})$. In all cases, it is never the case that $x \notin A \vee y \notin B$, so in all cases we have $(x,y) \notin A \times B$. Thus, $(x,y) \in \overline{A \times B}$, so $(\overline{A} \times B) \cup (\overline{A} \times \overline{B}) \cup (A \times \overline{B}) \subseteq \overline{A \times B}$.

Question 12:

$$\bigcap_{k=1}^{\infty} A_k = \{1\}$$

$$\bigcup_{k=1}^{\infty} A_k = [1, 2]$$
$$\bigcap_{k=1}^{\infty} B_k = \varnothing$$
$$\bigcup_{k=1}^{\infty} A_k = (1, 2)$$

Question 13: *Proof.* Let S be square with vertices $ABCD$ and side length 1, and let there be 5 points, $p_1, p_2, \ldots, p_5$. Construct 4 circles of radius $\frac{\sqrt{2}}{2}$, centered about A, B, C, D respectively. We place one point from $p1, \ldots, p_4$ into a valid configuration: that is, where each point $p_1, \ldots, p_4$ is on the boundary or inside of only one circle. We notice that p_5 must placed on or inside one or more circles, that of which already have a point inside them. Thus, the distance between p_5 and at least one other point must be less than or equal to $\frac{1}{\sqrt{2}}$.

□