FEDERAL STATE AUTONOMOUS EDUCATIONAL INSTITUTION
OF HIGHER EDUCATION
ITMO UNIVERSITY

REPORT

on learning practice No.3

«Sampling of multivariate random variables»

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code

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[1]:
```python
# imports
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.preprocessing import KBinsDiscretizer
from sklearn.metrics import accuracy_score
from pgmpy.estimators import HillClimbSearch
from pgmpy.estimators import K2Score, BicScore
from pgmpy.models import BayesianNetwork
from pgmpy.inference import VariableElimination
import networkx as nx
from pgmpy.base import DAG
from pgmpy.models import BayesianModel
from pgmpy.sampling import BayesianModelSampling
from sklearn.metrics import classification_report

import warnings
warnings.filterwarnings("ignore")
```

[2]:
```python
source_data_path = "./../tcs_stock.csv"
row_df = pd.read_csv(source_data_path)
```

0.1 Step 1. Choose variables for sampling from your dataset (overall – about 10 variables, 3-4 –target variables, the rest - predictors).

[3]:
```python
row_df.head()
```

[3]:
```
         Date Symbol Series  Prev Close    Open     High      Low     Last  \
0  2015-01-01    TCS     EQ     2558.25  2567.0  2567.00  2541.00  2550.00
1  2015-01-02    TCS     EQ     2545.55  2551.0  2590.95  2550.60  2588.40
2  2015-01-05    TCS     EQ     2579.45  2581.0  2599.90  2524.65  2538.10
3  2015-01-06    TCS     EQ     2540.25  2529.1  2529.10  2440.00  2450.05
4  2015-01-07    TCS     EQ     2446.60  2470.0  2479.15  2407.45  2426.90

     Close     VWAP   Volume      Turnover  Trades  Deliverable Volume  \
0  2545.55  2548.51   183415  4.674345e+13    8002               52870
1  2579.45  2568.19   462870  1.188740e+14   27585              309350
```

```
2  2540.25  2563.94   877121  2.248886e+14   43234              456728
3  2446.60  2466.90  1211892  2.989615e+14   84503              714306
4  2417.70  2433.96  1318166  3.208362e+14  101741              886368

   %Deliverble
0       0.2883
1       0.6683
2       0.5207
3       0.5894
4       0.6724
```

Target: High, Low, Trades #### Predictors: Prev Close, Open, Close, VWAP, Volume, Deliverable Volume

[4]:
```
df = row_df[['High', 'Low', 'Trades', 'Prev Close', 'Open', 'Close', 'VWAP',␣
 ↪'Volume', 'Deliverable Volume', 'Turnover']]
df.head()
```

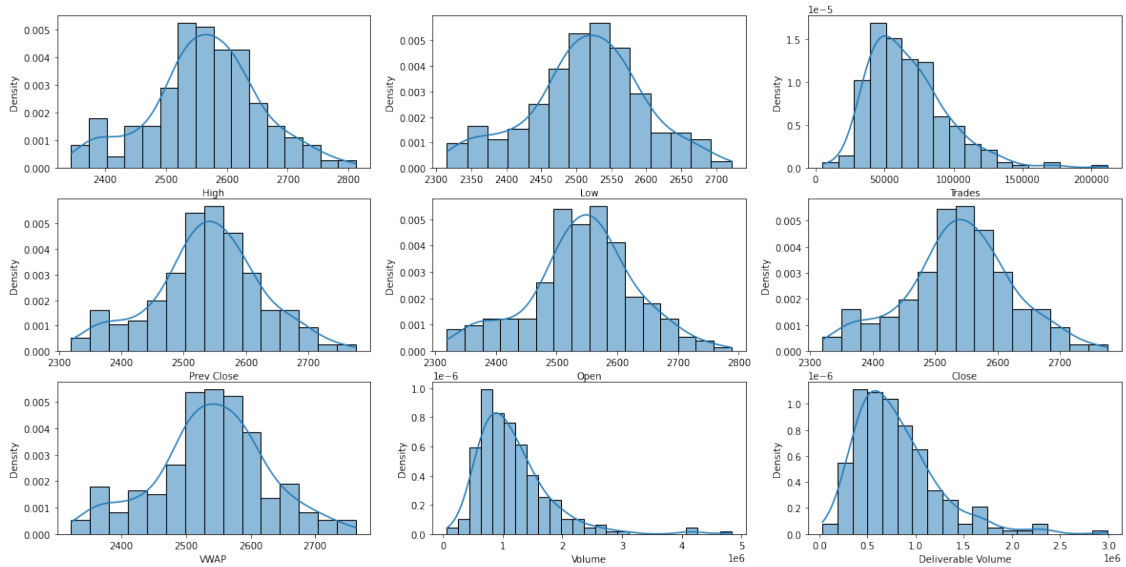
[4]:
```
      High      Low  Trades  Prev Close    Open    Close     VWAP   Volume  \
0  2567.00  2541.00    8002     2558.25  2567.0  2545.55  2548.51   183415
1  2590.95  2550.60   27585     2545.55  2551.0  2579.45  2568.19   462870
2  2599.90  2524.65   43234     2579.45  2581.0  2540.25  2563.94   877121
3  2529.10  2440.00   84503     2540.25  2529.1  2446.60  2466.90  1211892
4  2479.15  2407.45  101741     2446.60  2470.0  2417.70  2433.96  1318166

   Deliverable Volume      Turnover
0               52870  4.674345e+13
1              309350  1.188740e+14
2              456728  2.248886e+14
3              714306  2.989615e+14
4              886368  3.208362e+14
```

Distributions:
High - hypsecant with param 1.0910
Low - laplace with param 1.3288
Trades - gumbel_r with param 3.3611

Prev Close- laplace with parameter 7.3622
Open - laplace with parameter 8.493
Close - laplace with parameter 7.6574
VWAP - laplace with parameter 1.1143
Volume - johnsonsb with parameter 5.2133
Deliverable Volume - invgamma with parameter 9.9473

[5]:
```
fig, ax = plt.subplots(3, 3, figsize=(20,10))
sns.histplot(df['High'], ax=ax[0,0], kde=True, stat="density")
sns.histplot(df['Low'], ax=ax[0,1], kde=True, stat="density")
sns.histplot(df['Trades'], ax=ax[0,2], kde=True, stat="density")
```

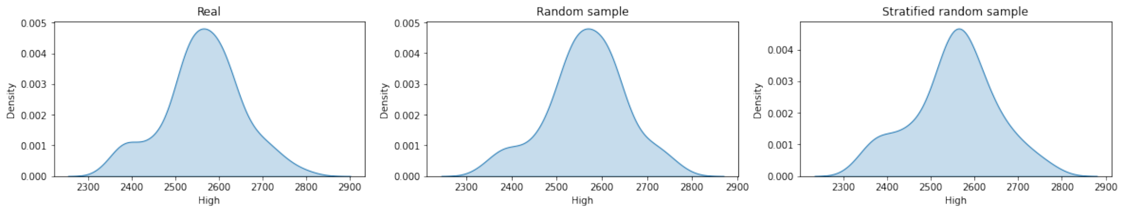
```python
sns.histplot(df['Prev Close'], ax=ax[1,0], kde=True, stat="density")
sns.histplot(df['Open'], ax=ax[1,1], kde=True, stat="density")
sns.histplot(df['Close'], ax=ax[1,2], kde=True, stat="density")
sns.histplot(df['VWAP'], ax=ax[2,0], kde=True, stat="density")
sns.histplot(df['Volume'], ax=ax[2,1], kde=True, stat="density")
sns.histplot(df['Deliverable Volume'], ax=ax[2,2], kde=True, stat="density")

plt.show()
```

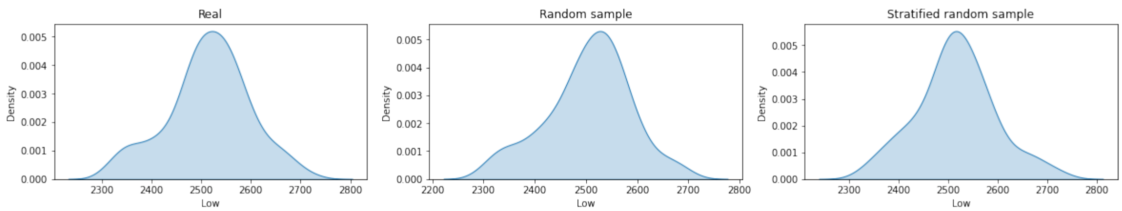


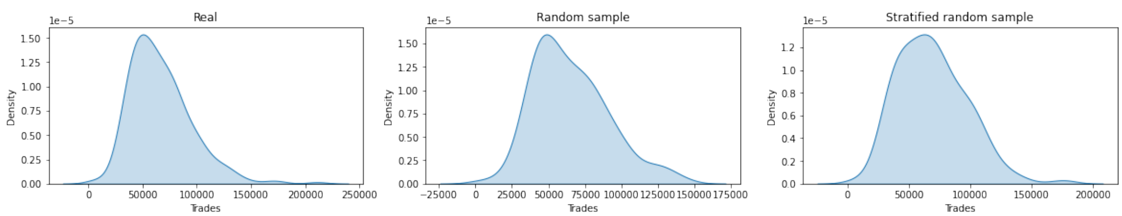
0.2 Step 2. Using univariate parametric distributions that were fitted in Lab#2 make sampling of chosen target variables. Use for this 2 different sampling methods.

[6]:
```python
labels =['High', 'Low', 'Trades']

for l in labels:
    fig, axes = plt.subplots(1, 3, figsize=(20, 3))
    sns.kdeplot(ax=axes[0], data=df, x=l, shade=True)

    # Random sample
    sample_data = df.sample(frac=.5)
    sns.kdeplot(ax=axes[1], data=sample_data, x=l, shade=True)

    # Stratified random sample
    mean = df[l].mean()

    sample_data_0 = df[df[l] > mean].sample(n=60, axis=0)
    sample_data_1 = df[df[l] <= mean].sample(n=60, axis=0)
```

```
sample_data = pd.concat([sample_data_0, sample_data_1])
sns.kdeplot(ax=axes[2], data=sample_data, x=l, shade=True)

axes[0].set_title('Real')
axes[1].set_title('Random sample')
axes[2].set_title('Stratified random sample')
plt.show()
```



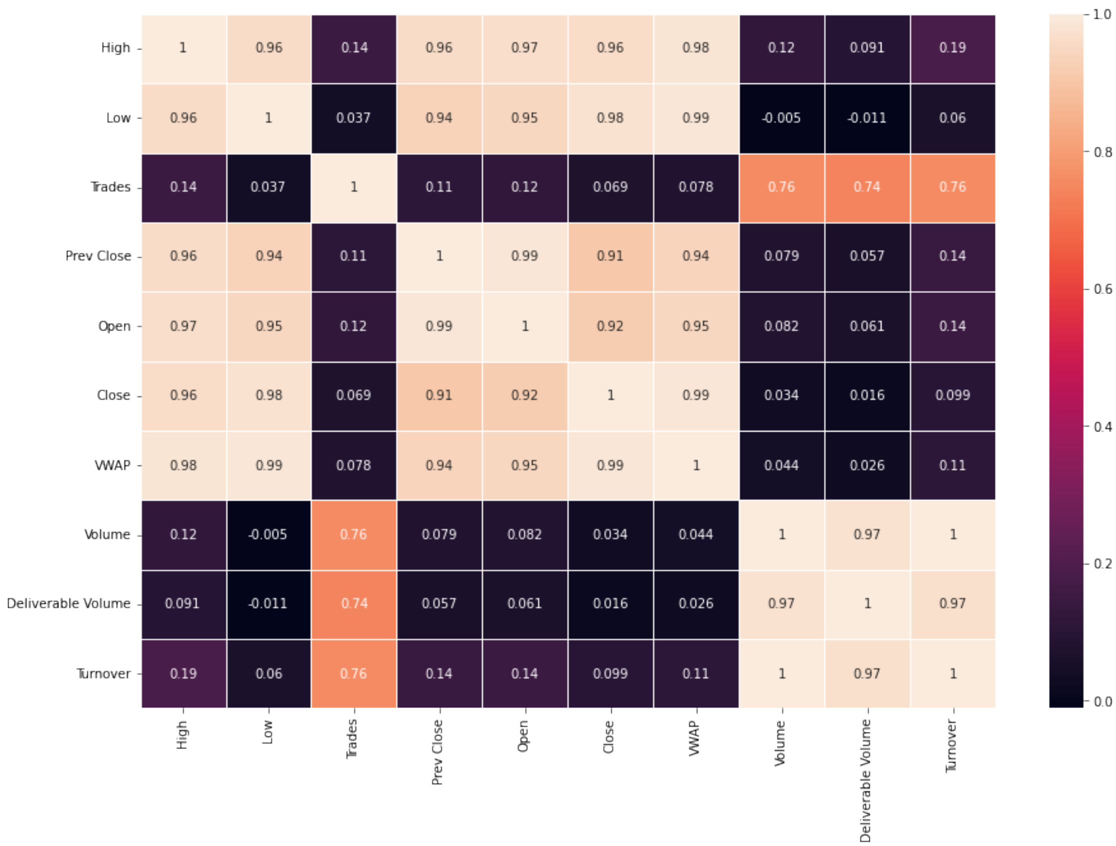




0.3 Step 3. Estimate relations between predictors and chosen target variables. At least, they should have significant correlation coefficients.

[7]:
```
predictors = df[['Prev Close', 'Open', 'Close', 'VWAP', 'Volume', 'Deliverable␣
↪Volume']]
targets = df[['High', 'Low', 'Trades']]
```

[8]:
```
plt.figure(figsize=(15, 10))
sns.heatmap(df.corr(method='spearman'), annot=True, linewidths=.5)
plt.show()
```



We have got troubles with correlation of Volume and Deliverable Volume. So we will remove this predictors from dataset. Other predictors with ban correlation will be used, because we have small number of columns ¯\( )/¯

Another point - use Trades feature as target. But it has a problem - only Turnover has a great correlation with it. We can check out make decision after all.

[9]:
```
df = row_df[['High', 'Low', 'Trades', 'Prev Close', 'Open', 'Close', 'VWAP',␣
↪'Turnover']]
```

0.4 Step 4. Build a Bayesian network for chosen set of variables. Choose its structure on the basis of multivariate analysis and train distributions in nodes using chosen algorithm.

[10]:
```
df_transformed = df.copy()
discretizer = KBinsDiscretizer(n_bins=10, encode='ordinal', strategy='kmeans')
df_discretized = discretizer.fit_transform(df.values[:])
```

[11]:
```
df_transformed[:] = df_discretized
df_transformed
```

[11]:

```
     High  Low  Trades  Prev Close  Open  Close  VWAP  Turnover
0     4.0  5.0     0.0         5.0   5.0    5.0   5.0       0.0
1     5.0  5.0     1.0         5.0   5.0    5.0   5.0       1.0
2     5.0  5.0     1.0         5.0   6.0    4.0   5.0       2.0
3     3.0  3.0     3.0         4.0   4.0    2.0   3.0       3.0
4     2.0  2.0     4.0         2.0   3.0    2.0   2.0       3.0
..    …    …       …           …     …      …     …         …
243   2.0  2.0     0.0         2.0   2.0    2.0   2.0       1.0
244   2.0  2.0     1.0         2.0   2.0    2.0   3.0       5.0
245   2.0  3.0     1.0         2.0   2.0    2.0   2.0       2.0
246   2.0  2.0     2.0         2.0   2.0    2.0   2.0       2.0
247   2.0  2.0     1.0         2.0   2.0    2.0   2.0       1.0

[248 rows x 8 columns]
```

[12]:

```
blacklist = [(x, y) for x in df_transformed.columns.to_list() for y in␣
 ↪['Turnover', 'Trades'] if x != y]
```

From the structure of the Baess network, we excluded the parameters of Turnover, Trades based on the correlation analysis. These parameters are weakly correlated with the rest, so there is no point in considering them.

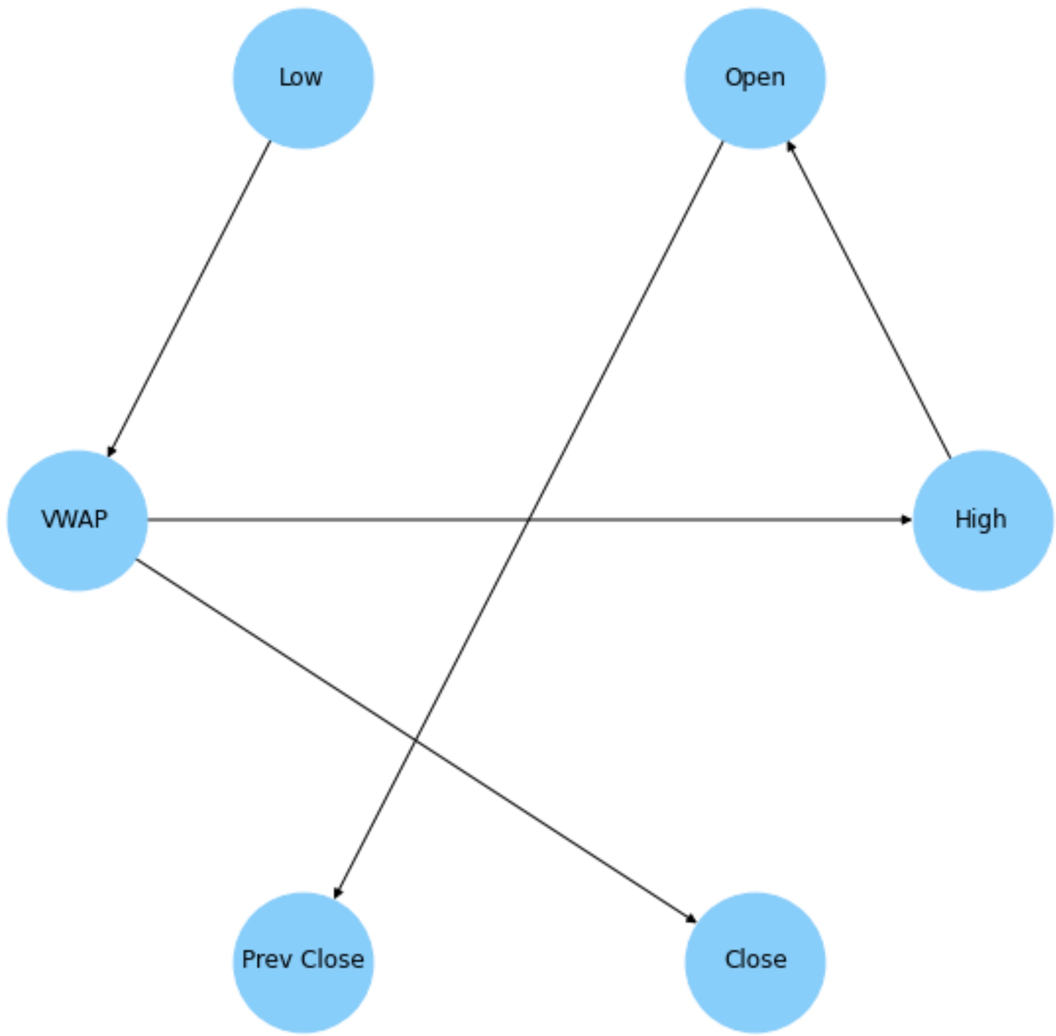
0.5 K2Score

[13]:

```
hc_search = HillClimbSearch(data=df_transformed)
model_k2 = hc_search.estimate(scoring_method=K2Score(df_transformed),␣
 ↪black_list=blacklist, show_progress=False)
```

[14]:

```
figure, ax = plt.subplots(1, 1, figsize=(10, 10))

graph = nx.DiGraph()
graph.add_edges_from(model_k2.edges())
positions = nx.layout.circular_layout(graph)
nx.draw(graph, positions, with_labels=True, node_color='lightskyblue',␣
 ↪node_size=5000)

plt.show()
```



[15]:
```
def accuracy_params_restoration(bn: BayesianNetwork, data: pd.DataFrame):
    bn.fit(data)
    result = pd.DataFrame(columns=['Parameter', 'accuracy'])
    bn_infer = VariableElimination(bn)
    for j, param in enumerate(data.columns):
        accuracy = 0
        test_param = data[param].copy()
        test_data = data.drop(columns=param)
        evidence = test_data.to_dict('records')
        predicted_param = []
        for element in evidence:
            prediction = bn_infer.map_query(variables=[param],␣
↪evidence=element, show_progress=False)
```

```
            predicted_param.append(prediction[param])
        accuracy = accuracy_score(test_param.values, predicted_param)
        result.loc[j,'Parameter'] = param
        result.loc[j, 'accuracy'] = accuracy
    return result
```

[16]:

```
accuracy_params_restoration(BayesianNetwork(model_k2.edges()),
                            df_transformed[['High','Low','Prev␣
 ↪Close','Open','Close','VWAP']].sample(frac=0.3))
```

[16]:

	Parameter	accuracy
0	High	0.743243
1	Low	0.810811
2	Prev Close	0.72973
3	Open	0.77027
4	Close	0.756757
5	VWAP	0.891892

0.6 Step 5. Build a Bayesian network for the same set of variables but using 2 chosen algorithms for structural learning.

0.6.1 K2 algorithm implemented above!

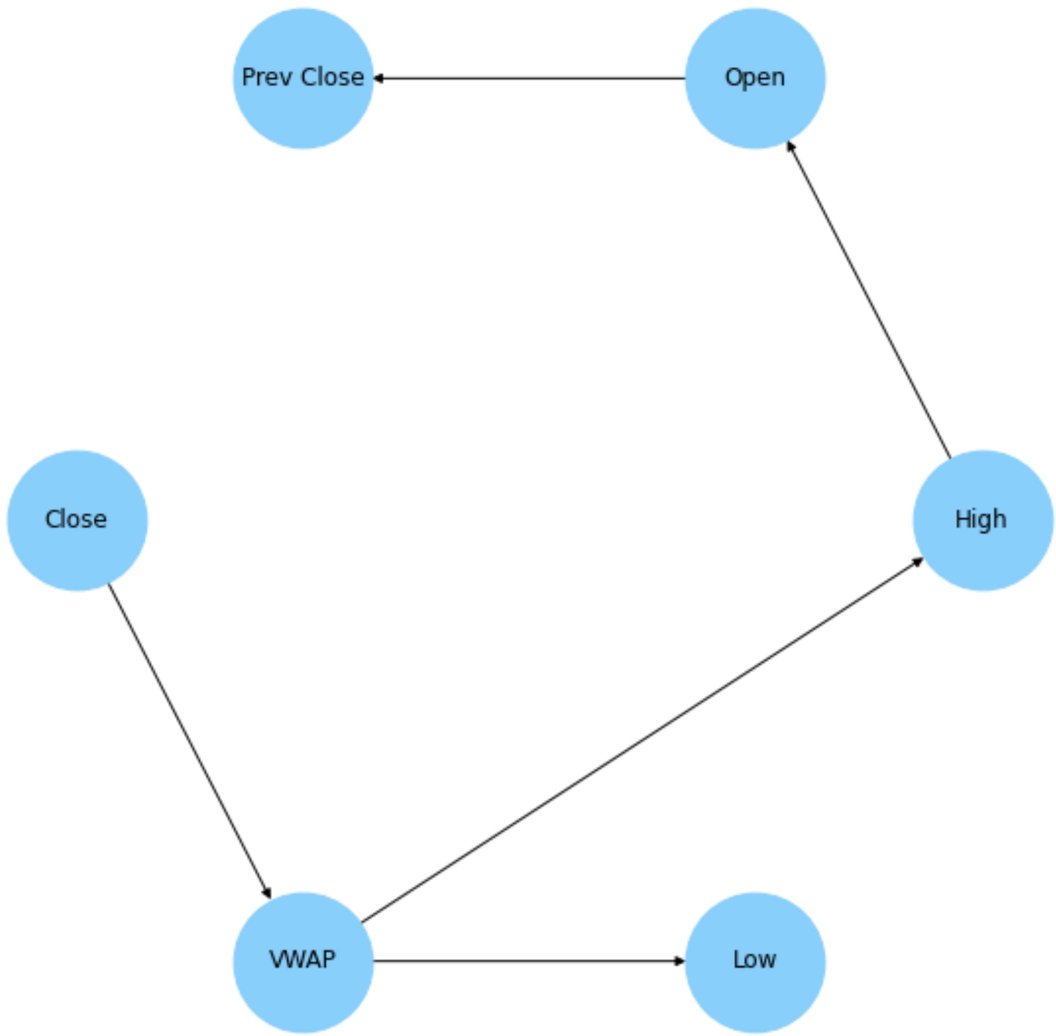
0.7 BicScore with stratified random sample

[17]:

```
model_bic = hc_search.estimate(scoring_method=BicScore(df_transformed.
 ↪sample(frac=.8)), black_list=blacklist, show_progress=False)
```

[18]:

```
figure, ax = plt.subplots(1, 1, figsize=(10, 10))

graph = nx.DiGraph()
graph.add_edges_from(model_bic.edges())
positions = nx.layout.circular_layout(graph)
nx.draw(graph, positions, with_labels=True, node_color='lightskyblue',␣
 ↪node_size=5000)

plt.show()
```

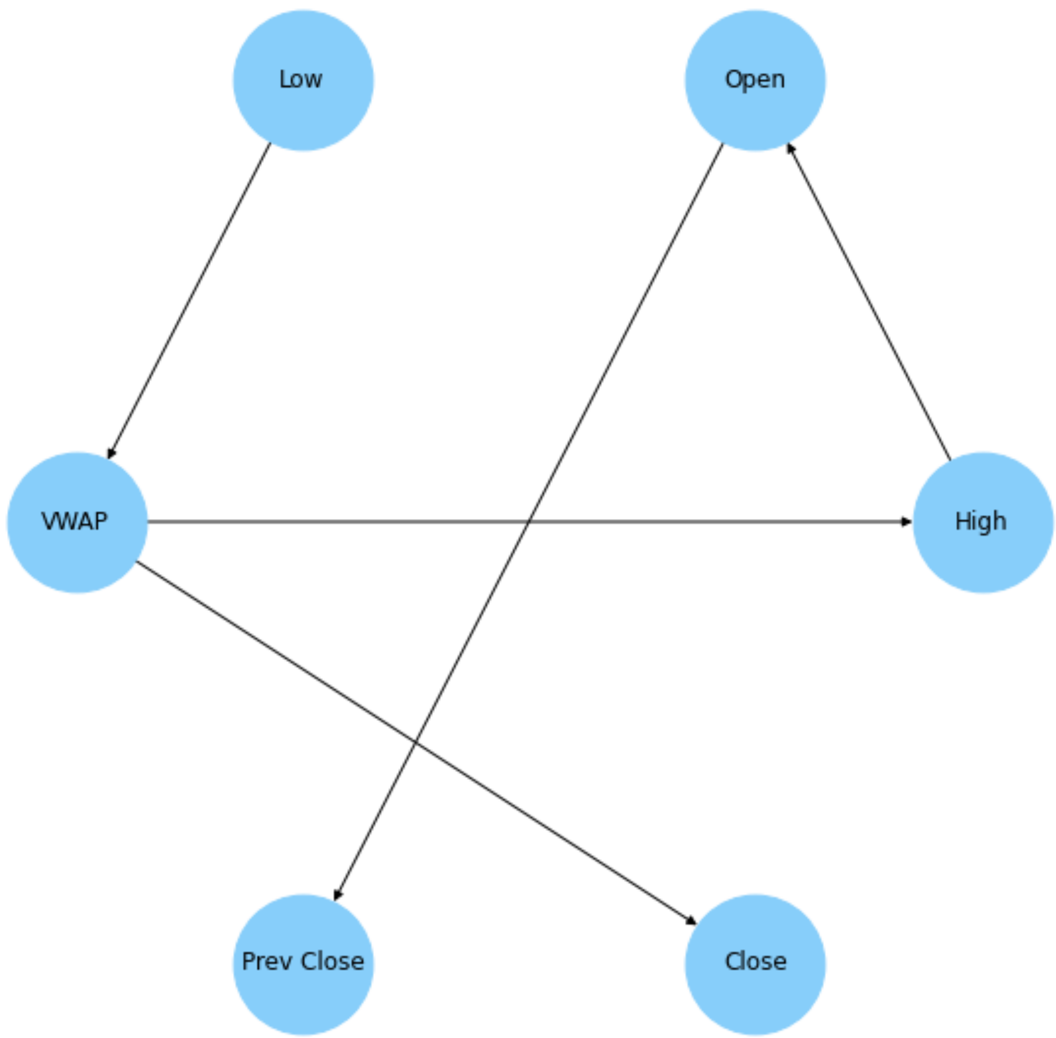


0.8 BicScore with random sampling

[19]:
```
mean = df_transformed[1].mean()

sample_data_0 = df_transformed[df_transformed[1] > mean].sample(n=80, axis=0)
sample_data_1 = df_transformed[df_transformed[1] <= mean].sample(n=80, axis=0)
sample_data = pd.concat([sample_data_0, sample_data_1])

model_bic_2 = hc_search.estimate(scoring_method=BicScore(sample_data),
    black_list=blacklist, show_progress=False)
```

[20]:
```
figure, ax = plt.subplots(1, 1, figsize=(10, 10))
```

```
graph = nx.DiGraph()
graph.add_edges_from(model_bic_2.edges())
positions = nx.layout.circular_layout(graph)
nx.draw(graph, positions, with_labels=True, node_color='lightskyblue',␣
↪node_size=5000)

plt.show()
```



Step 6. Analyze a quality of sampled target variables from the point of view of problem statement (e.g. prediction, gap filling, synthetic generation).

[21]:
```
def sampling (bn: DAG, data: pd.DataFrame, n: int = 100):
    bn_new = BayesianModel(bn.edges())
    bn_new.fit(data)
```

```
    sampler = BayesianModelSampling(bn_new)
    sample = sampler.forward_sample(size=n)
    return sample
```

[22]:
```
def draw_comparative_hist (parametr: str, original_data: pd.DataFrame,␣
 ↪data_sampled: pd.DataFrame, axes=None):
    final_df = pd.DataFrame()

    df1 = pd.DataFrame()
    df1[parametr] = original_data[parametr]
    df1['Data'] = 'Original data'
    df1['Probability'] = df1[parametr].apply(lambda x: (df1.
 ↪groupby(parametr)[parametr].count()[x])/original_data.shape[0])

    df2 = pd.DataFrame()
    df2[parametr] = data_sampled[parametr]
    df2['Data'] = 'Synthetic data'
    df2['Probability'] = df2[parametr].apply(lambda x: (df2.
 ↪groupby(parametr)[parametr].count()[x])/data_sampled.shape[0])
    final_df = pd.concat([df1, df2])

    sns.barplot(ax=axes, x=parametr, y="Probability", hue="Data", data=final_df)
```

[23]:
```
sample_K2 = sampling(model_k2, df_transformed, df_transformed.shape[0])
sample_Bic = sampling(model_bic, df_transformed, df_transformed.shape[0])
```

```
HBox(children=(IntProgress(value=0, max=6), HTML(value='')))
```

```
HBox(children=(IntProgress(value=0, max=6), HTML(value='')))
```

0.9 Quality

[24]:
```
print(classification_report(df_transformed['High'], sample_K2['High']))
```

```
              precision    recall  f1-score   support

         0.0       0.08      0.12      0.10         8
         1.0       0.05      0.07      0.06        14
         2.0       0.00      0.00      0.00        21
         3.0       0.20      0.16      0.18        49
         4.0       0.16      0.12      0.14        50
         5.0       0.21      0.25      0.23        44
         6.0       0.07      0.07      0.07        30
         7.0       0.05      0.06      0.06        16
         8.0       0.00      0.00      0.00        12
```

```
         9.0       0.00      0.00      0.00         4

    accuracy                           0.12       248
   macro avg       0.08      0.09      0.08       248
weighted avg       0.13      0.12      0.12       248
```

[25]:
```python
print(classification_report(df_transformed['Low'], sample_K2['Low']))
```

```
              precision    recall  f1-score   support

         0.0       0.13      0.23      0.17        13
         1.0       0.00      0.00      0.00        10
         2.0       0.00      0.00      0.00        17
         3.0       0.06      0.04      0.05        25
         4.0       0.19      0.14      0.16        56
         5.0       0.23      0.25      0.24        61
         6.0       0.08      0.09      0.09        34
         7.0       0.00      0.00      0.00        15
         8.0       0.00      0.00      0.00        13
         9.0       0.00      0.00      0.00         4

    accuracy                           0.12       248
   macro avg       0.07      0.07      0.07       248
weighted avg       0.12      0.12      0.12       248
```

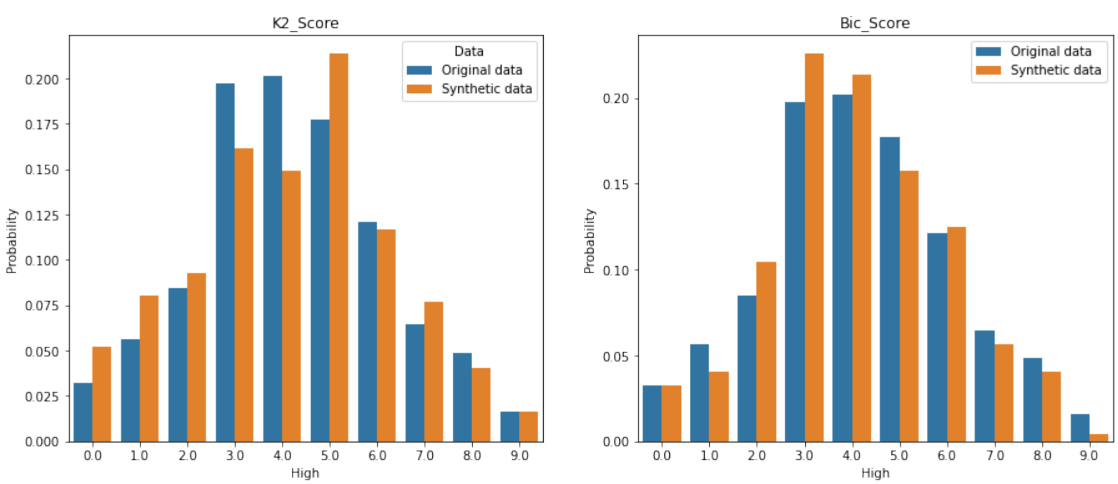
[26]:
```python
print(classification_report(df_transformed['High'], sample_Bic['High']))
```

```
              precision    recall  f1-score   support

         0.0       0.00      0.00      0.00         8
         1.0       0.10      0.07      0.08        14
         2.0       0.08      0.10      0.09        21
         3.0       0.16      0.18      0.17        49
         4.0       0.19      0.20      0.19        50
         5.0       0.21      0.18      0.19        44
         6.0       0.16      0.17      0.16        30
         7.0       0.00      0.00      0.00        16
         8.0       0.10      0.08      0.09        12
         9.0       0.00      0.00      0.00         4

    accuracy                           0.15       248
   macro avg       0.10      0.10      0.10       248
weighted avg       0.14      0.15      0.14       248
```

[27]:
```python
print(classification_report(df_transformed['Low'], sample_Bic['Low']))
```

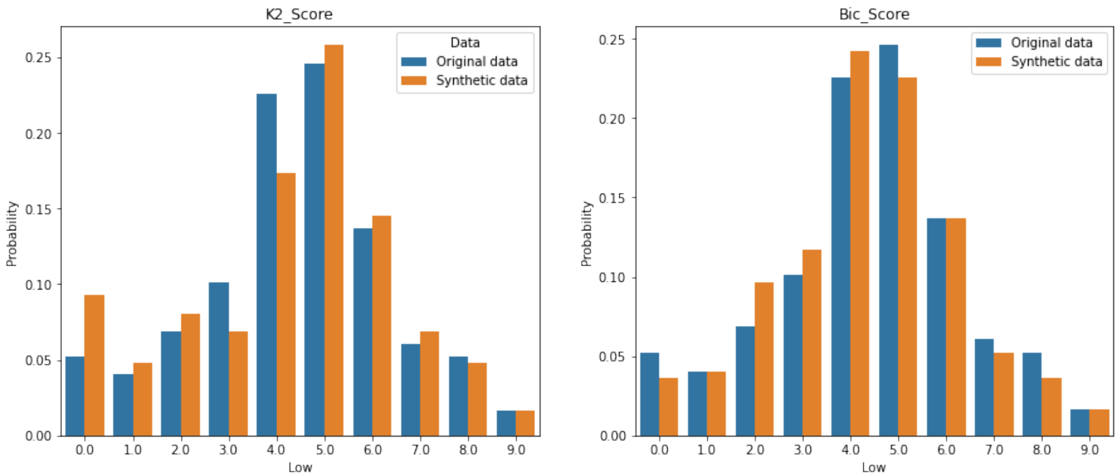
```
              precision    recall  f1-score   support

         0.0       0.00      0.00      0.00        13
         1.0       0.00      0.00      0.00        10
         2.0       0.08      0.12      0.10        17
         3.0       0.10      0.12      0.11        25
         4.0       0.15      0.16      0.16        56
         5.0       0.30      0.28      0.29        61
         6.0       0.18      0.18      0.18        34
         7.0       0.00      0.00      0.00        15
         8.0       0.00      0.00      0.00        13
         9.0       0.25      0.25      0.25         4

    accuracy                           0.15       248
   macro avg       0.11      0.11      0.11       248
weighted avg       0.15      0.15      0.15       248
```

[28]:
```
fig, axes = plt.subplots(1, 2, figsize=(15, 6))

draw_comparative_hist('High', df_transformed, sample_K2, axes=axes[0])
draw_comparative_hist('High', df_transformed, sample_Bic, axes=axes[1])

axes[0].set_title('K2_Score')
axes[1].set_title('Bic_Score')
plt.legend()
plt.show()
```



[29]:
```
fig, axes = plt.subplots(1, 2, figsize=(15, 6))

draw_comparative_hist('Low', df_transformed, sample_K2, axes=axes[0])
```

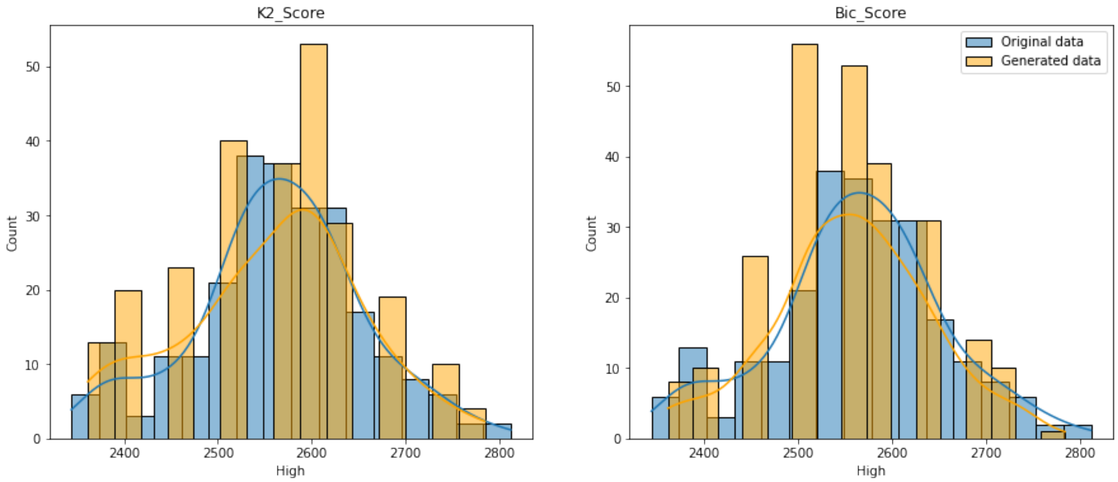
```
draw_comparative_hist('Low', df_transformed, sample_Bic, axes=axes[1])

axes[0].set_title('K2_Score')
axes[1].set_title('Bic_Score')
plt.legend()
plt.show()
```



[30]:
```
for in_blacklist in list(set(df_transformed.columns) - set(sample_K2.columns)):
    sample_K2[in_blacklist] = df_transformed[in_blacklist].copy()
    sample_Bic[in_blacklist] = df_transformed[in_blacklist].copy()
```

[31]:
```
sample_K2[sample_K2.columns] = discretizer.
 ↪inverse_transform(sample_K2[sample_K2.columns].values)
sample_Bic[sample_K2.columns] = discretizer.
 ↪inverse_transform(sample_Bic[sample_K2.columns].values)
```

[32]:
```
fig, axes = plt.subplots(1, 2, figsize=(15, 6))

sns.histplot(df['High'], label='Original data', kde=True, ax=axes[0])
sns.histplot(sample_K2['High'], label='Generated data', kde=True,␣
 ↪color='orange', ax=axes[0])

sns.histplot(df['High'], label='Original data', kde=True, ax=axes[1])
sns.histplot(sample_Bic['High'], label='Generated data', kde=True,␣
 ↪color='orange', ax=axes[1])

axes[0].set_title('K2_Score')
axes[1].set_title('Bic_Score')
plt.legend()
plt.show()
```



[33]:
```
fig, axes = plt.subplots(1, 2, figsize=(15, 6))

sns.histplot(df['Prev Close'], label='Original data', kde=True, ax=axes[0])
sns.histplot(sample_K2['Prev Close'], label='Generated data', kde=True,␣
 ↪color='orange', ax=axes[0])

sns.histplot(df['Prev Close'], label='Original data', kde=True, ax=axes[1])
sns.histplot(sample_Bic['Prev Close'], label='Generated data', kde=True,␣
 ↪color='orange', ax=axes[1])

axes[0].set_title('K2_Score')
axes[1].set_title('Bic_Score')
plt.legend()
plt.show()
```