FEDERAL STATE AUTONOMOUS EDUCATIONAL INSTITUTION
OF HIGHER EDUCATION
ITMO UNIVERSITY

REPORT

on learning practice No.2

«Analysis of multivariate random variables»

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code

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[1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sb
from scipy import stats as st
from sklearn.preprocessing import scale, PolynomialFeatures
from sklearn.decomposition import PCA
from sklearn.model_selection import train_test_split
from sklearn import linear_model as lm
from sklearn.metrics import mean_squared_error, r2_score
from tabulate import tabulate
import statsmodels.api as sm
```

[2]:
```
source_data_path = "./../tcs_stock.csv"
row_df = pd.read_csv(source_data_path)
row_df["Deliverble(%)"] = row_df['%Deliverble']
```

[3]:
```
feature_cols = ['Open', 'Low', 'Last', 'Close', 'Trades', 'Deliverble(%)']
target_col = 'High'

df = row_df[feature_cols + [target_col]]
df.head()
```

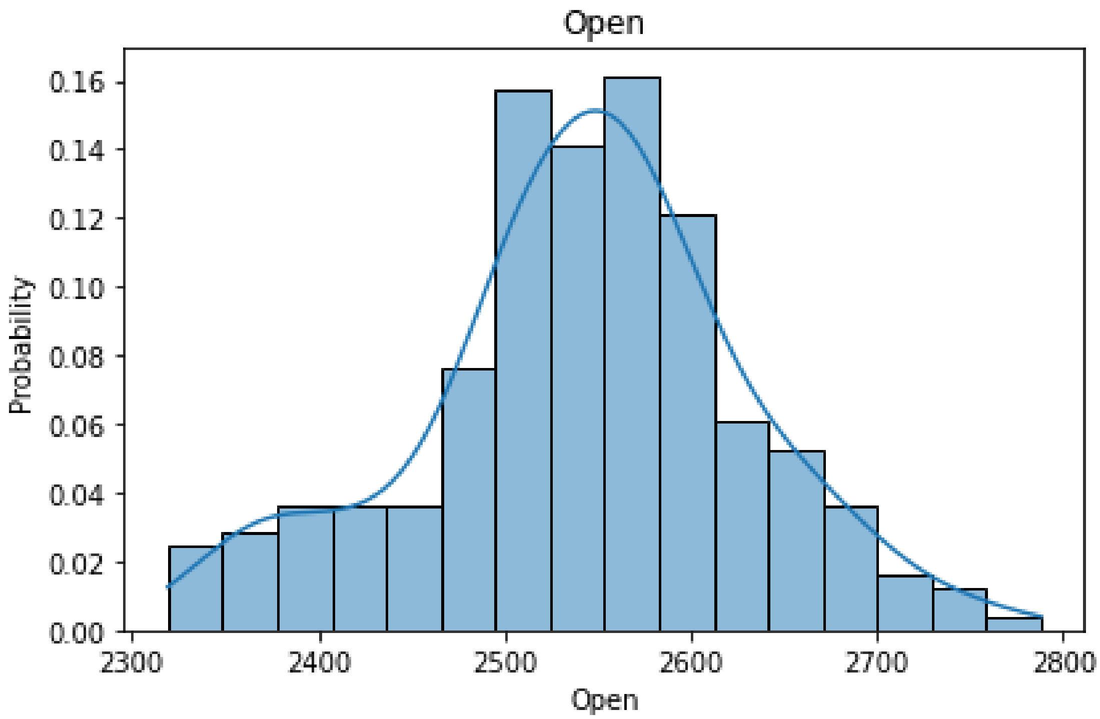
[3]:

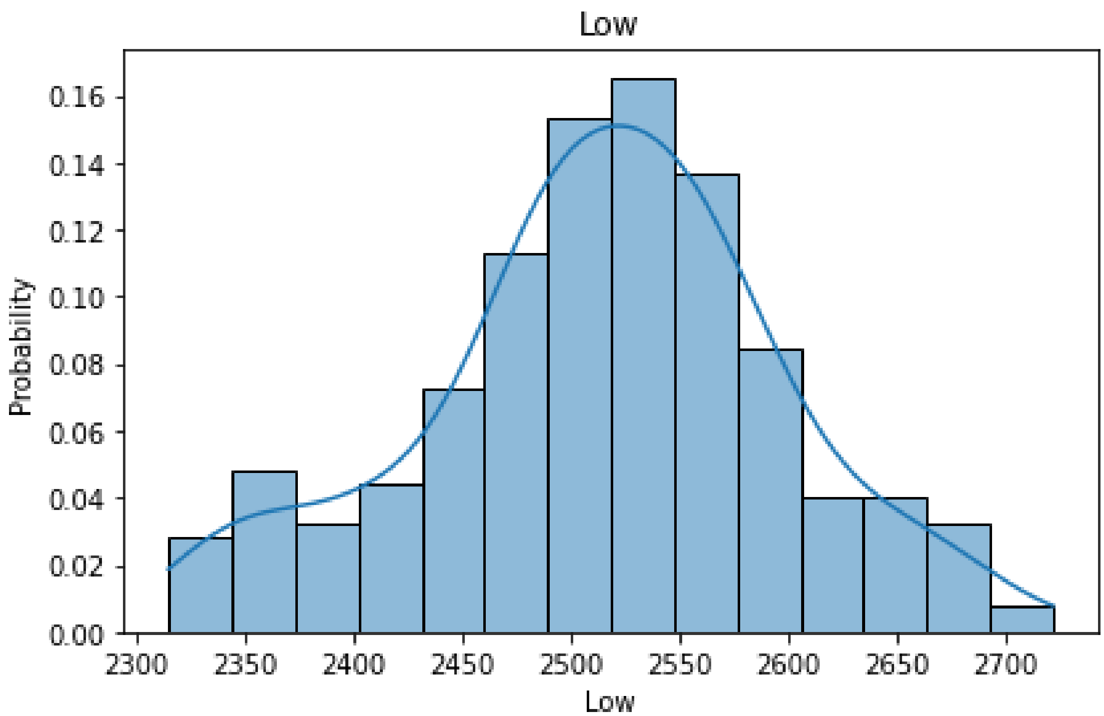
	Open	Low	Last	Close	Trades	Deliverble(%)	High
0	2567.0	2541.00	2550.00	2545.55	8002	0.2883	2567.00
1	2551.0	2550.60	2588.40	2579.45	27585	0.6683	2590.95
2	2581.0	2524.65	2538.10	2540.25	43234	0.5207	2599.90
3	2529.1	2440.00	2450.05	2446.60	84503	0.5894	2529.10
4	2470.0	2407.45	2426.90	2417.70	101741	0.6724	2479.15

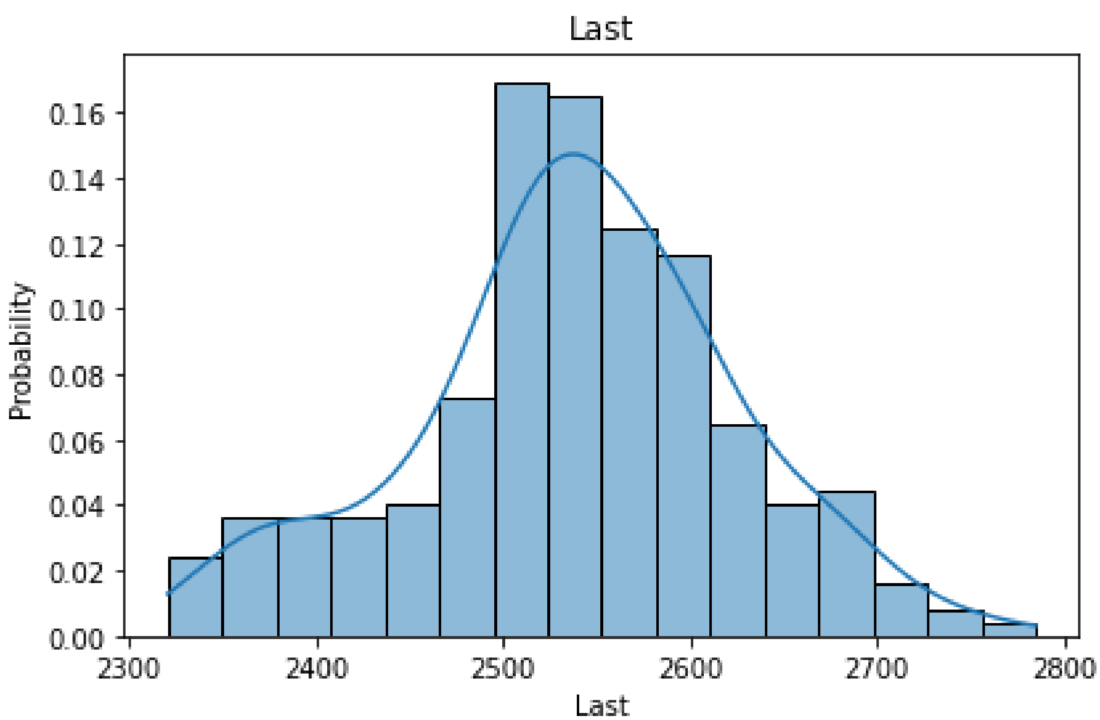
[4]:
```
features = df[feature_cols]
target = df[target_col]
```

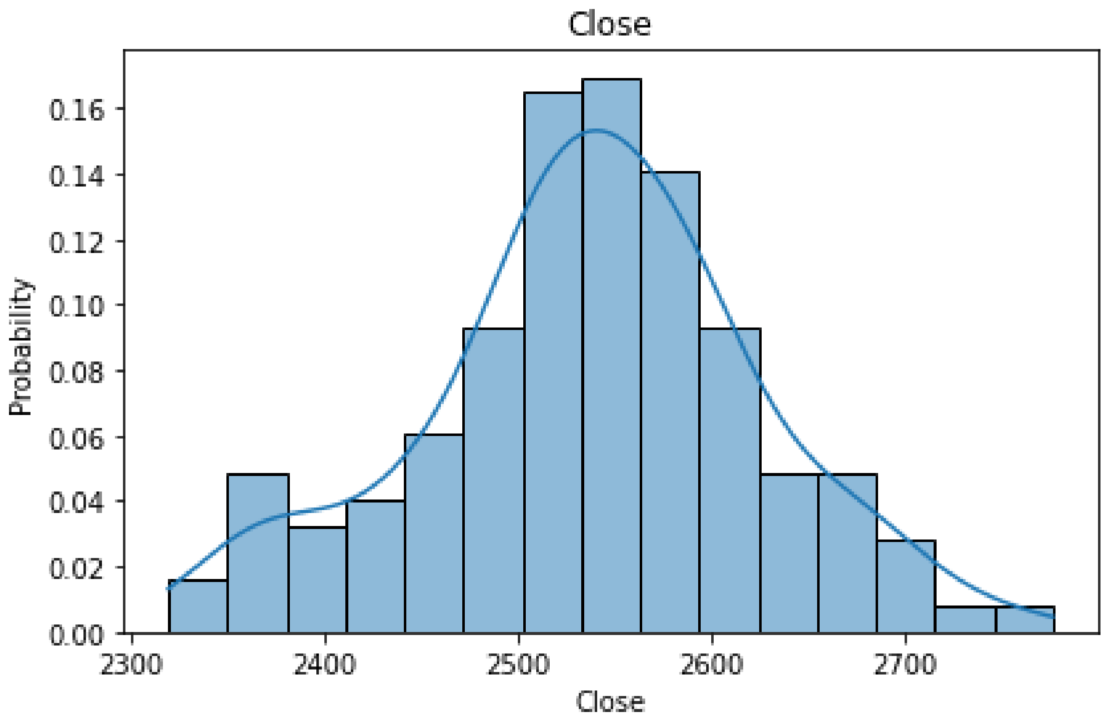
0.1 Step 1. You need to make a non-parametric estimation of PDF in form of histogram and using kernel density function for MRV (or probability law in case of discrete MRV)

[5]:
```
for col in feature_cols:
    fig, ax = plt.subplots(tight_layout=True)
    sb.histplot(df[col], ax=ax, kde=True, stat="probability")
    plt.title(col)
    plt.show()
```

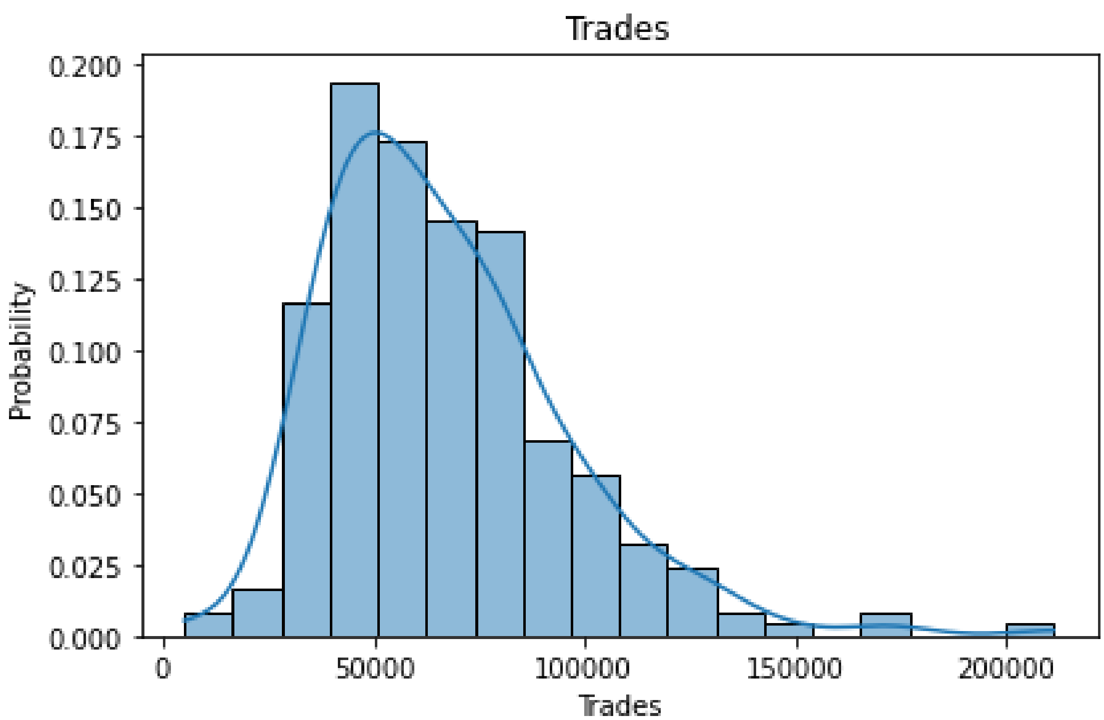



Low
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Probability
2300
2350
2400
2450
2500
2550
2600
2650
2700
Low

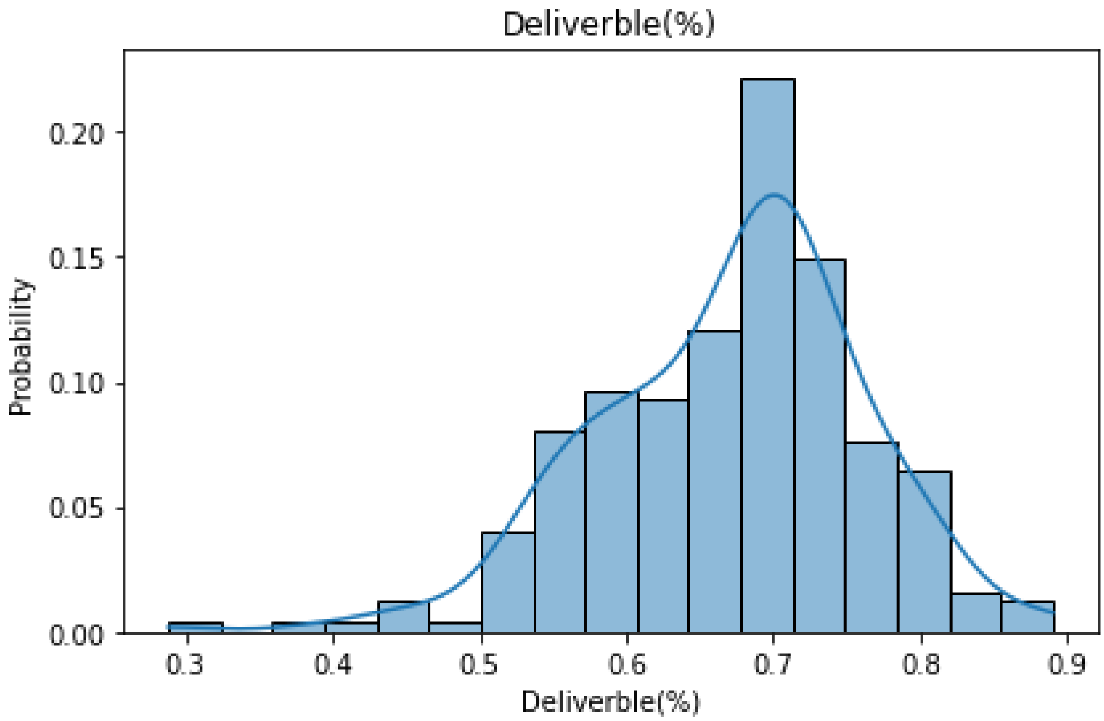

Last
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Probability
2300
2400
2500
2600
2700
2800
Last



Close
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Probability
2300
2400
2500
2600
2700
Close



Trades
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
Probability
0
50000
100000
150000
200000
Trades



0.2 Step 2. You need to make an estimation of multivariate mathematical expectation and variance.

[6]:
```
df.mean()
```

[6]:
```
Open               2542.172782
Low                2514.408468
Last               2538.039718
Close              2537.717944
Trades            66873.608871
Deliverble(%)         0.670336
High               2563.580444
dtype: float64
```

[7]:
```
df.var()
```

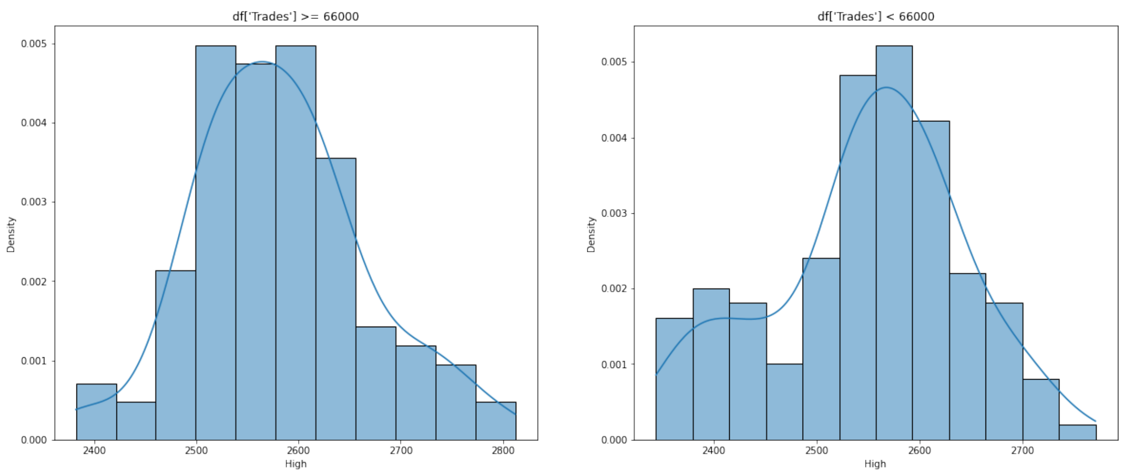
[7]:
```
Open              7.674758e+03
Low               6.881163e+03
Last              7.542802e+03
Close             7.579063e+03
Trades            8.342223e+08
Deliverble(%)     8.275145e-03
High              8.208064e+03
```

```
dtype: float64
```

0.3 Step 3. You need to make a non-parametric estimation of conditional distributions, mathematical expectations and variances.

[8]:
```
condition_more = df['Trades'] >= 66000
condition_less = df['Trades'] < 66000
```

[9]:
```
fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(20,8))

sb.histplot(df[condition_more]['High'], ax=ax1, kde=True, stat="density")
ax1.set_title("df['Trades'] >= 66000")

sb.histplot(df[condition_less]['High'], ax=ax2, kde=True, stat="density")
ax2.set_title("df['Trades'] < 66000")

plt.show()
```



[10]:
```
df[condition_more].mean()
```

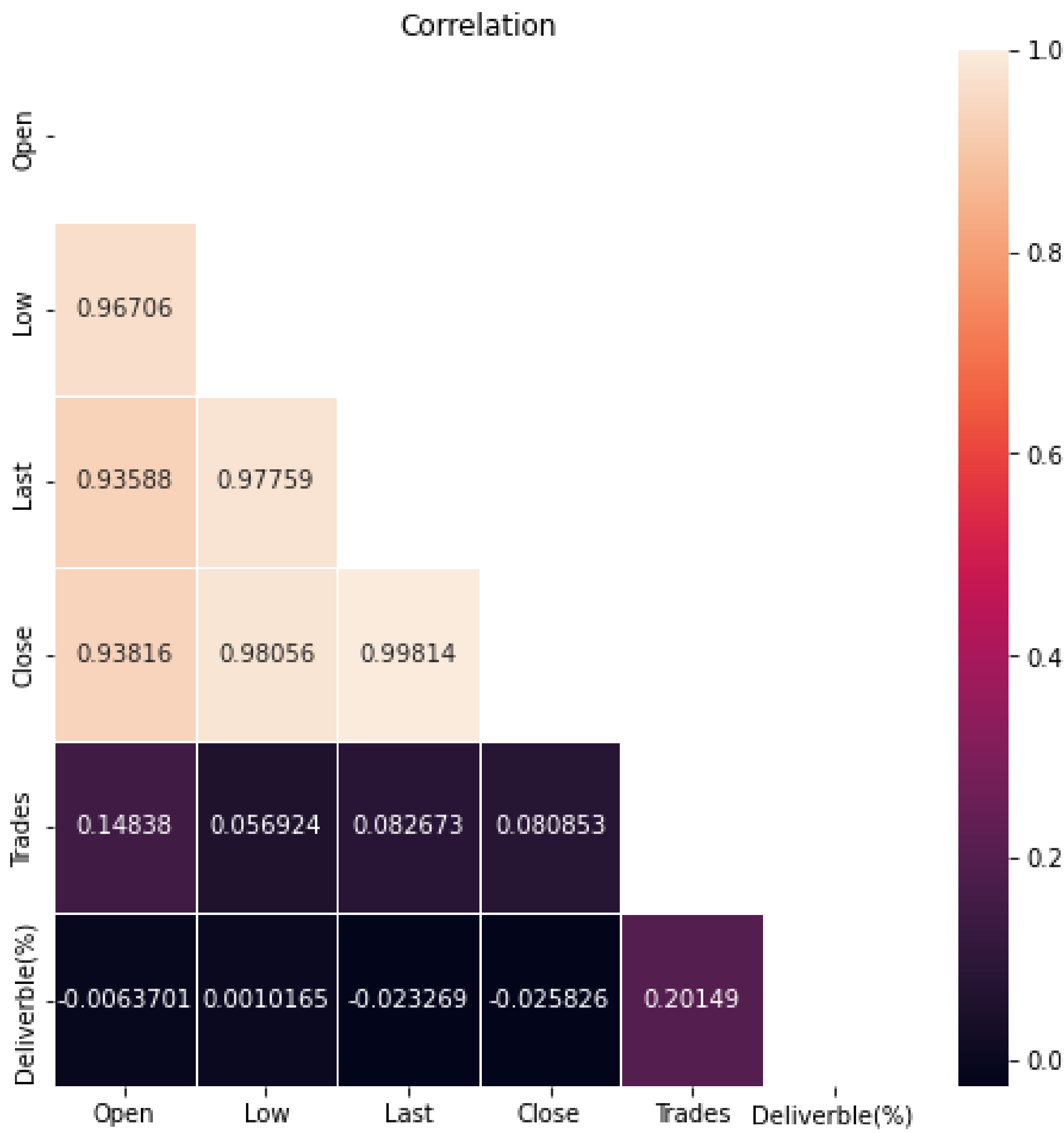
[10]:
```
Open              2558.012500
Low               2523.619444
Last              2550.018056
Close             2549.429630
Trades           92309.546296
Deliverble(%)        0.691087
High              2582.467130
dtype: float64
```

[11]:
```
df[condition_more].var()
```

[11]:
```
Open             6.191144e+03
Low              5.887515e+03
Last             6.700395e+03
Close            6.871748e+03
Trades           5.982571e+08
Deliverble(%)    6.254171e-03
High             7.031163e+03
dtype: float64
```

0.4 Step 4. You need to make an estimation of pair correlation coefficients, confidence intervals for them and significance levels.

[12]:
```
fig, ax = plt.subplots(figsize=(8,8))

corr = features.corr()
mask = np.triu(np.ones_like(corr, dtype=bool))
sb.heatmap(corr, mask=mask, annot=True, ax=ax, vmax=1, fmt='.5g', linewidths=.5)

plt.title('Correlation')
plt.show()
```



[13]:
```python
def _estimate_correlation(x, y):
    return st.pearsonr(x, y)

def _estimate_confidence_intervals(cor, x, y, alpha = 0.05):
    coeff = np.arctanh(cor)

    std = 1/np.sqrt(x.size-3)
    z = st.norm.ppf(1-alpha/2)
    return coeff-z*std, coeff+z*std
```

[14]:
```
tab = [["Pair", "Correlation coeff", "Low border of conf interval", "High␣
↪border of conf interval"]]
for i, x_col_name in enumerate(feature_cols):
    j = i + 1

    if j >= len(feature_cols):
        break

    for k in range(j, len(feature_cols)):
        y_col_name = feature_cols[k]
        x = df[x_col_name]
        y = df[y_col_name]

        cor, p = _estimate_correlation(x, y)

        low, high = _estimate_confidence_intervals(cor, x, y)
        tab.append([f'{x_col_name} - {y_col_name}',cor,low,high])
print(tabulate(tab, headers="firstrow", tablefmt="grid"))
```

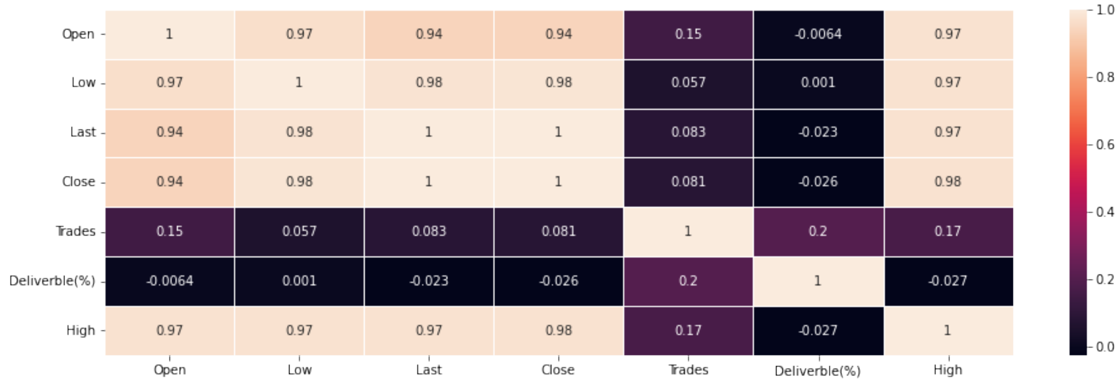
```
+-------------------------+----------------------+------------------------------+
--------------------------------+
| Pair                    |   Correlation coeff |   Low border of conf interval |
High border of conf interval |
+=========================+======================+==============================+
================================+
| Open - Low              |          0.967063   |                      1.91963  |
2.17006   |
+-------------------------+----------------------+------------------------------+
--------------------------------+
| Open - Last             |          0.935876   |                      1.57853  |
1.82896   |
+-------------------------+----------------------+------------------------------+
--------------------------------+
| Open - Close            |          0.938159   |                      1.59724  |
1.84768   |
+-------------------------+----------------------+------------------------------+
--------------------------------+
| Open - Trades           |          0.148377   |                      0.0242634 |
0.274698  |
+-------------------------+----------------------+------------------------------+
--------------------------------+
| Open - Deliverble(%)    |         -0.00637008 |                     -0.131588  |
0.118847  |
+-------------------------+----------------------+------------------------------+
--------------------------------+
| Low - Last              |          0.977593   |                      2.11492  |
2.36535   |
```

Low - Close	0.980559	2.18665	2.43708
Low - Trades	0.0569236	-0.0682323	0.182203
Low - Deliverble(%)	0.00101649	-0.124201	0.126234
Last - Close	0.998138	3.36381	3.61424
Last - Trades	0.0826734	-0.0423549	0.20808
Last - Deliverble(%)	-0.0232686	-0.14849	0.101945
Close - Trades	0.080853	-0.0441876	0.206247
Close - Deliverble(%)	-0.0258261	-0.151049	0.0993856
Trades - Deliverble(%)	0.201487	0.079064	0.329499

0.5 Step 5. Choose a task formulation for regression. Estimate multivariate correlation (target -predictors).

Predict values of ‘High’ feature

[15]:
```
plt.figure(figsize=(16, 5))
sb.heatmap(df.corr(method='pearson'), annot=True, linewidths=.5)
plt.show()
```



Use PCA method and see how many variables we need to take for regression.

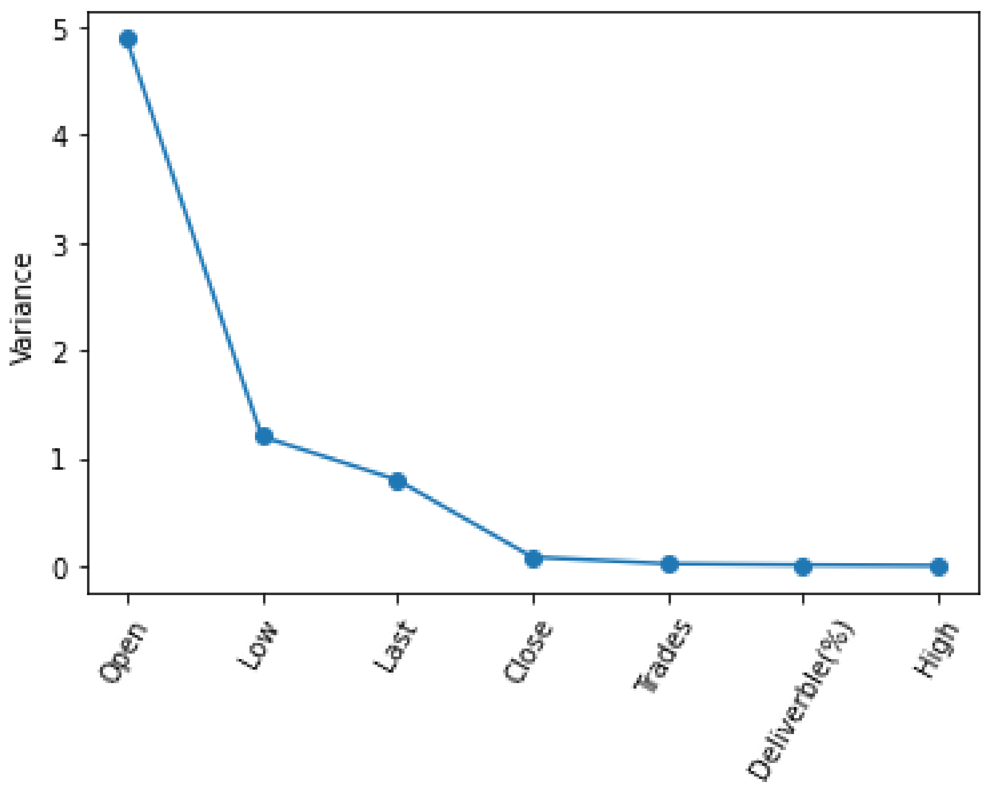
[16]:
```
#Standardize a dataset
std_df = scale(df)
std_df = pd.DataFrame(std_df, index=df.index, columns=df.columns)
```

[17]:
```
std_df.head()
```

[17]:

	Open	Low	Last	Close	Trades	Deliverble(%)	High
0	0.283970	0.321211	0.137992	0.090146	-2.042407	-4.208179	0.037820
1	0.100965	0.437173	0.581031	0.480330	-1.363023	-0.022430	0.302709
2	0.444101	0.123712	0.000696	0.029144	-0.820119	-1.648263	0.401696
3	-0.149525	-0.898812	-1.015180	-1.048754	0.611609	-0.891524	-0.381355
4	-0.825503	-1.291997	-1.282272	-1.381388	1.209639	0.022732	-0.933805

[18]:
```
pca = PCA().fit(std_df)
y = np.std(pca.transform(std_df), axis=0)**2
x = np.arange(len(y)) + 1
plt.plot(x, y, "o-")
plt.xticks(x, df.columns, rotation=60)
plt.ylabel("Variance")
plt.show()
```



I think that 3 features is ok. Let's use Open, Close, Low

0.6 Step 6. Build regression model and make an analysis of multicollinearity and regularization (if needed).

[33]:
```
train, test = train_test_split(df[['High', 'Open', 'Close', 'Low']].copy())

train_feature = train[['Open', 'Close', 'Low']]
test_feature = test[['Open', 'Close', 'Low']]

train_target = train['High']
test_target = test['High']
```

[20]:
```
tab = [["Type", "MSE", "R2", "Coeff"]]

regression = lm.LinearRegression()
regression.fit(train_feature, train_target)
predicted = regression.predict(test_feature)
tab.append(["Least Squares model", str(mean_squared_error(test_target,␣
↪predicted)), str(r2_score(test_target, predicted)), str(regression.coef_)])

regression = lm.Lasso(alpha=0.0001, random_state=5)
```

```
regression.fit(train_feature, train_target)
predicted = regression.predict(test_feature)
tab.append(["Lasso model", str(mean_squared_error(test_target, predicted)),␣
↪str(r2_score(test_target, predicted)), str(regression.coef_)])

regression = lm.Ridge(random_state=1)
regression.fit(train_feature, train_target)
predicted = regression.predict(test_feature)
tab.append(["Ridge model", str(mean_squared_error(test_target, predicted)),␣
↪str(r2_score(test_target, predicted)), str(regression.coef_)])

print(tabulate(tab, headers="firstrow", tablefmt="grid"))
```

```
+---------------------+---------+----------+-------------------------------------
---+
| Type                |     MSE |       R2 | Coeff
|
+=====================+=========+==========+=====================================
===+
| Least Squares model | 154.324 | 0.973688 | [ 0.61183695  0.73816119
-0.32282928] |
+---------------------+---------+----------+-------------------------------------
---+
| Lasso model         | 154.324 | 0.973688 | [ 0.61183662  0.73816111
-0.32282885] |
+---------------------+---------+----------+-------------------------------------
---+
| Ridge model         | 154.325 | 0.973688 | [ 0.61182267  0.73813709 -0.3227891
] |
+---------------------+---------+----------+-------------------------------------
---+
```

0.7 Step 7. Analyze the quality of regression model (distribution of residuals, determination coefficient).

R^2 score can be find at the end of Step 6

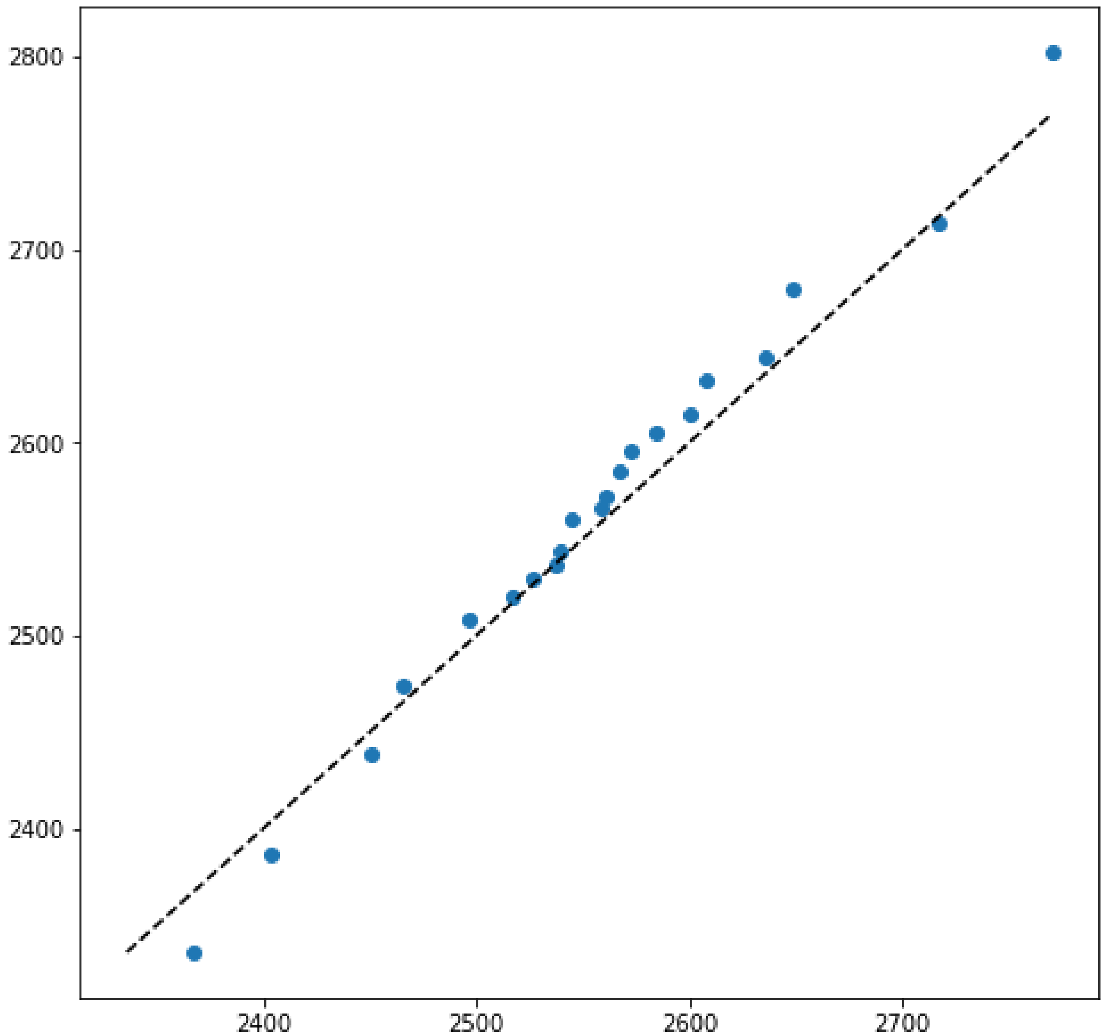
[21]:
```
X = df[['Open', 'Close', 'Low']]
Y = df['High']
```

[22]:
```
def _draw_qq_plot(predicted, predicted_all):
    percs = np.linspace(0, 100, 21)
    qn_first = np.percentile(predicted, percs)
    qn_second = np.percentile(predicted_all, percs)

    plt.figure(figsize=(8,8))
```

```
    min_qn = np.min([qn_first.min(), qn_second.min()])
    max_qn = np.min([qn_first.max(), qn_second.max()])
    x = np.linspace(min_qn, max_qn)

    plt.plot(qn_first, qn_second, ls="", marker="o", markersize=6)
    plt.plot(x,x,color="k", ls="--")
    plt.show
```

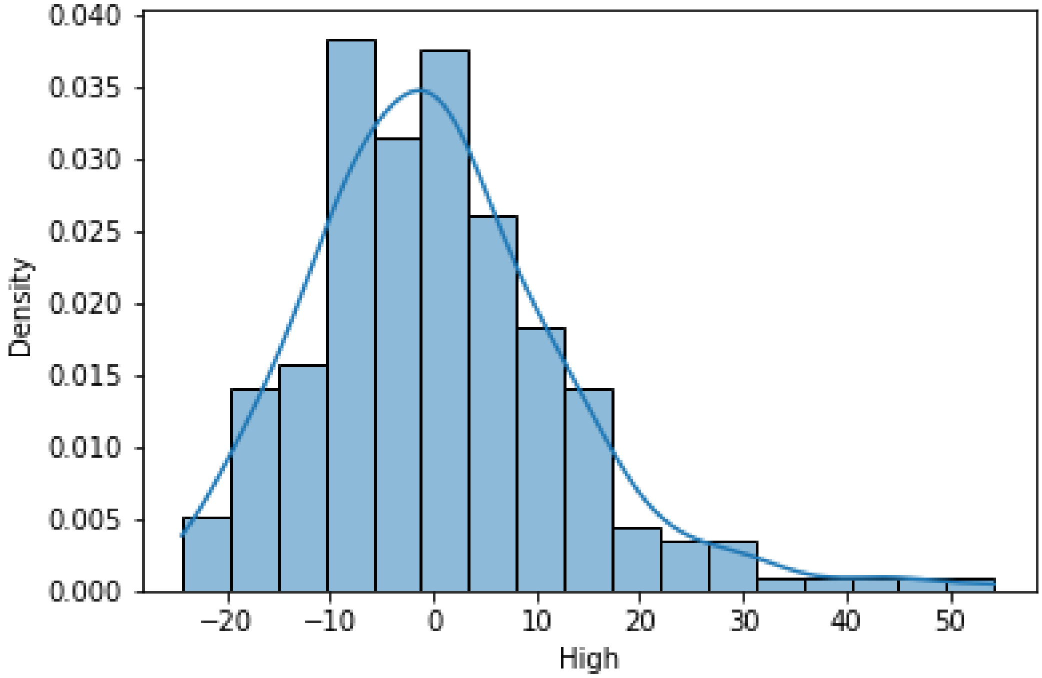
[34]:
```
regression = lm.LinearRegression()
regression.fit(train_feature, train_target)
predicted = regression.predict(test_feature)

print('R^2 score =', r2_score(test_target, predicted))

predicted_all = regression.predict(X)
_draw_qq_plot(predicted, predicted_all)
```

```
R^2 score = 0.9697279864152346
```



R^2 show a good quality of our model as a qqplot

[35]:
```
residuals = Y - regression.predict(X)
ax = sb.histplot(residuals, kde = 'True', stat="density")
```



[25]:
```
st.kstest(residuals, 'norm', args=(residuals.mean(), residuals.var()))
```

[25]:
```
KstestResult(statistic=0.4353543408175741, pvalue=2.886743745829084e-43)
```

Residuals are not distributed normally.

[26]:
```
mod = sm.OLS(train_target, train_feature)
res = mod.fit()
print(res.conf_int(0.01))
```

```
              0         1
Open   0.510829  0.731068
Close  0.617991  0.902066
Low   -0.573751 -0.176412
```

[37]:
```
residuals.describe()
```

[37]:
```
count    248.000000
mean       0.380230
std       12.471677
min      -24.312439
25%       -8.113857
50%       -0.971024
75%        6.655066
max       54.325712
```

```
Name: High, dtype: float64
```