FEDERAL STATE AUTONOMOUS EDUCATIONAL INSTITUTION
OF HIGHER EDUCATION
ITMO UNIVERSITY

Report

on learning practice No.1

«Analysis of univariate random variables»

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code

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[1]:
```
import pandas as pd
import numpy as np
from numpy.random import normal
import seaborn as sb
import scipy as sc
import scipy.stats as st
from scipy.optimize import curve_fit
from tqdm import tqdm
import time
import statsmodels.api as sm
from statsmodels.graphics.gofplots import qqplot
import matplotlib as mpl
import matplotlib.pyplot as plt
import math
import random
import warnings

pd.set_option("display.precision", 8)
```

https://www.kaggle.com/atulanandjha/national-stock-exchange-time-series?select=tcs_stock.csv

[2]:
```
source_data_path = "./../tcs_stock.csv"
```

1 Context

The National Stock Exchange of India Ltd. (NSE) is an Indian stock exchange located at Mumbai, Maharashtra, India. National Stock Exchange (NSE) was established in 1992 as a demutualized electronic exchange. It was promoted by leading financial institutions on request of the Government of India. It is India's largest exchange by turnover. In 1994, it launched electronic screen-based trading. Thereafter, it went on to launch index futures and internet trading in 2000, which were the first of its kind in the country.

This dataset represents stock exchange of high rated Indian company Tata Consultancy Services (TCS) from 01-01-2015 to 31-12-2015.

2 Data description

- Date - date on which data is recorded
- Symbol - NSE symbol of the stock
- Series - series of that stock | EQ - Equity
- Prev Close - last day close point
- Open - current day open point
- High - current day highest point
- Low - current day lowest point
- Last - the final quoted trading price for a particular stock, or stock-market index, during the most recent day of trading
- Close - closing point for the current day
- VWAP - volume-weighted average price is the ratio of the value traded to total volume traded over a particular time horizon
- Volume - the amount of a security that was traded during a given period of time.
- Turnover - total Turnover of the stock till that day
- Trades - number of buy or Sell of the stock.
- Deliverable - volumethe quantity of shares which actually move from one set of people to another set of people.
- %Deliverble - percentage deliverables of that stock

Dataset consists of 248 records with 15 features.

You can see features desription and example of data below.

[3]:
```
raw_df = pd.read_csv(source_data_path)
```

[4]:
```
raw_df.describe()
```

[4]:
```
         Prev Close          Open          High           Low  \
count  248.00000000  248.00000000  248.00000000  248.00000000
mean  2538.20745968 2542.17278226 2563.58044355 2514.40846774
std     86.82935866   87.60569877   90.59836765   82.95277758
min   2319.80000000 2319.40000000 2343.90000000 2315.25000000
25%   2495.31250000 2499.50000000 2518.90000000 2472.10000000
50%   2543.05000000 2548.50000000 2566.00000000 2520.00000000
75%   2592.00000000 2594.25000000 2615.75000000 2567.30000000
max   2776.00000000 2788.00000000 2812.10000000 2721.90000000

               Last         Close          VWAP          Volume  \
count  248.00000000  248.00000000  248.00000000  2.48000000e+02
mean  2538.03971774 2537.71794355 2538.43213710  1.17229615e+06
std     86.84930504   87.05781439   86.81305296  6.22063547e+05
min   2321.00000000 2319.80000000 2322.27000000  6.75820000e+04
25%   2497.50000000 2495.15000000 2496.66500000  7.82135250e+05
50%   2540.15000000 2541.47500000 2540.44500000  1.03102400e+06
75%   2593.42500000 2592.00000000 2592.60750000  1.39326625e+06
max   2785.10000000 2776.00000000 2763.04000000  4.83437100e+06
```

```
             Turnover          Trades  Deliverable Volume   %Deliverble
count  2.48000000e+02     248.00000000      2.48000000e+02  248.00000000
mean   2.97748865e+14   66873.60887097      7.96057548e+05    0.67033629
std    1.57644310e+14   28882.90678727      4.30991092e+05    0.09096782
min    1.66754978e+13    5197.00000000      3.40030000e+04    0.28830000
25%    1.95071574e+14   45476.25000000      4.87106500e+05    0.61085000
50%    2.63178319e+14   61449.50000000      7.00953000e+05    0.68560000
75%    3.55038967e+14   82066.75000000      9.94662750e+05    0.72605000
max    1.20643498e+15  211247.00000000      2.98913200e+06    0.89010000
```

[5]:
```
raw_df.head()
```

[5]:
```
         Date Symbol Series  Prev Close    Open     High      Low     Last  \
0  2015-01-01    TCS     EQ     2558.25  2567.0  2567.00  2541.00  2550.00
1  2015-01-02    TCS     EQ     2545.55  2551.0  2590.95  2550.60  2588.40
2  2015-01-05    TCS     EQ     2579.45  2581.0  2599.90  2524.65  2538.10
3  2015-01-06    TCS     EQ     2540.25  2529.1  2529.10  2440.00  2450.05
4  2015-01-07    TCS     EQ     2446.60  2470.0  2479.15  2407.45  2426.90

     Close     VWAP   Volume      Turnover  Trades  Deliverable Volume  \
0  2545.55  2548.51   183415  4.67434456e+13    8002               52870
1  2579.45  2568.19   462870  1.18874011e+14   27585              309350
2  2540.25  2563.94   877121  2.24888554e+14   43234              456728
3  2446.60  2466.90  1211892  2.98961536e+14   84503              714306
4  2417.70  2433.96  1318166  3.20836246e+14  101741              886368

   %Deliverble
0       0.2883
1       0.6683
2       0.5207
3       0.5894
4       0.6724
```

2.1 Step 1. Choose subsample with main variables for your further analysis.

For this lab you need subsample with 3-5 random variables, at least half of them should be described with continuous random variable type

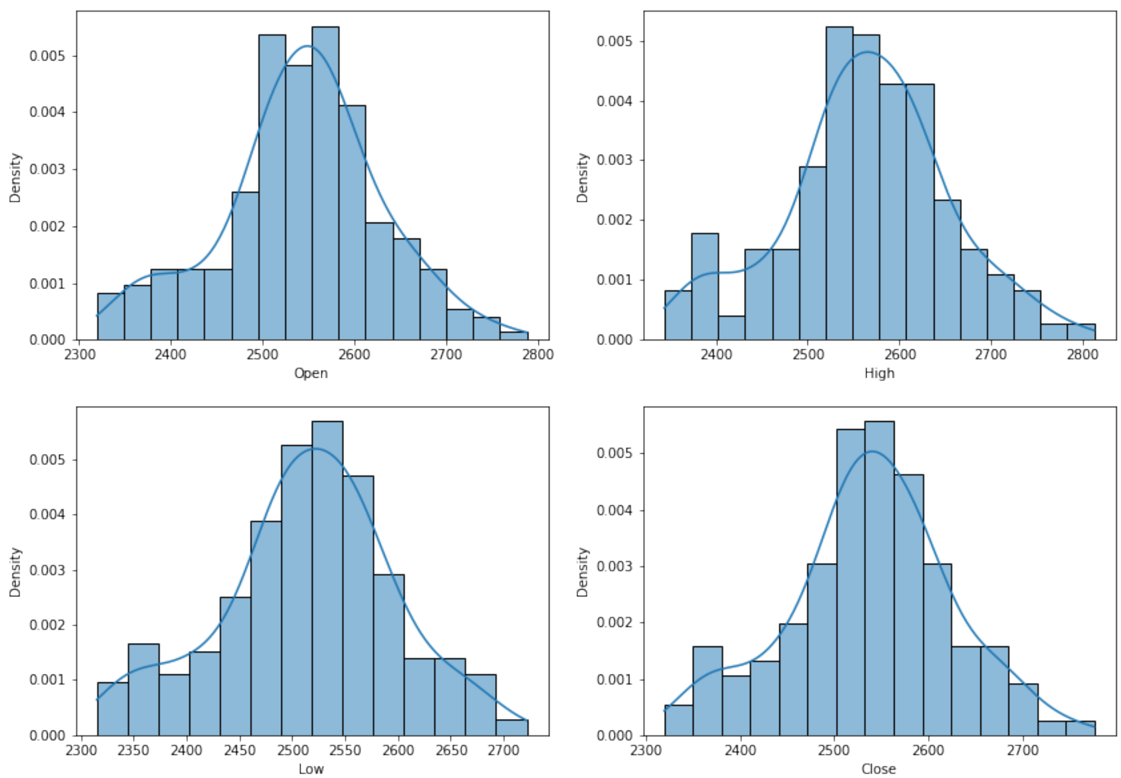
[6]:
```
features = ['Open', 'High', 'Low', 'Close']
df = raw_df[features]
df.head()
```

[6]:
```
     Open     High      Low    Close
0  2567.0  2567.00  2541.00  2545.55
1  2551.0  2590.95  2550.60  2579.45
```

2	2581.0	2599.90	2524.65	2540.25
3	2529.1	2529.10	2440.00	2446.60
4	2470.0	2479.15	2407.45	2417.70

2.2 Step 2. You need to make a non-parametric estimation of PDF in form of histogram and using kernel density function (or probability law in case of discrete RV).

[7]:
```python
# initialize figure canvas
fig, ax = plt.subplots(2, 2, figsize=(14,10))
sb.histplot(df['Open'], ax=ax[0,0], kde=True, stat="density")
sb.histplot(df['High'], ax=ax[0,1], kde=True, stat="density")
sb.histplot(df['Low'], ax=ax[1,0], kde=True, stat="density")
sb.histplot(df['Close'], ax=ax[1,1], kde=True, stat="density")

plt.show()
```



2.3 Step 3. You need to make an estimation of order statistics and represent them as "box with whiskers" plot.

we will find quintiles estimations

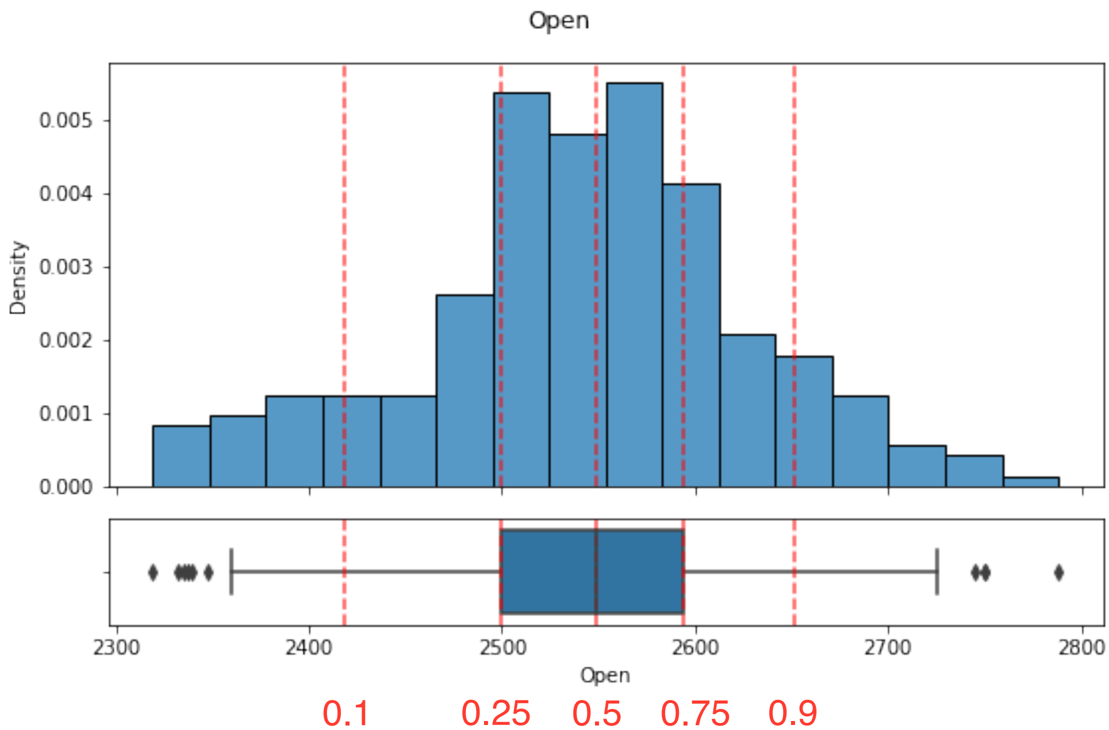
[8]:
```
def quantilies(array):
    return [
        np.quantile(array, 0.1),
        np.quantile(array, 0.25),
        np.quantile(array, 0.5),
        np.quantile(array, 0.75),
        np.quantile(array, 0.9)
    ]

results = {}
for feature in features:
    results[feature] = quantilies(df[feature].to_numpy())

pd.DataFrame.from_dict(results, orient='index',columns=['0.1', '0.25', '0.5',␣
↪'0.75', '0.9'])
```

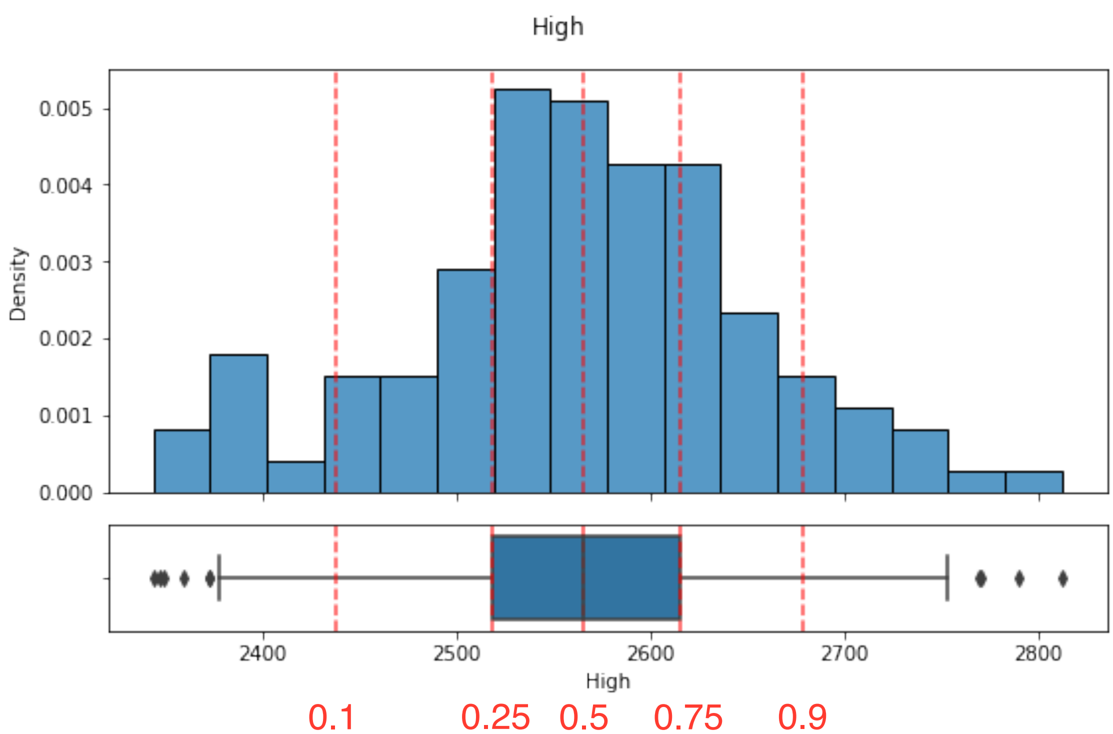
[8]:

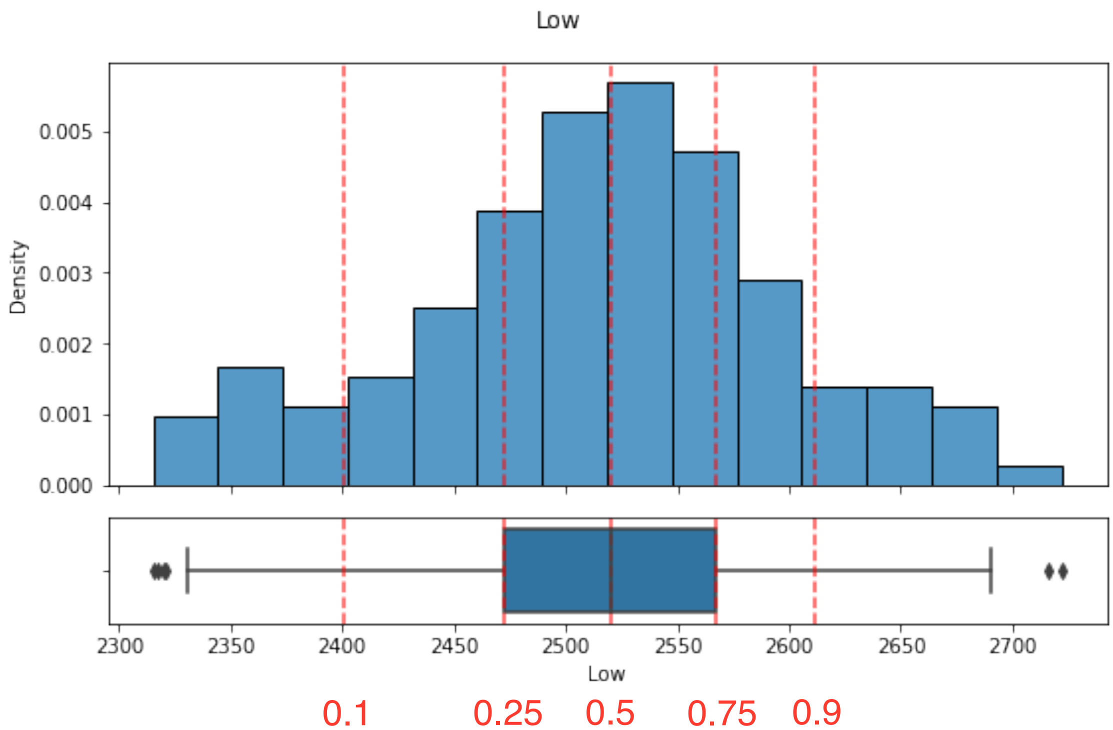
	0.1	0.25	0.5	0.75	0.9
Open	2418.690	2499.50	2548.500	2594.25	2651.160
High	2438.155	2518.90	2566.000	2615.75	2678.785
Low	2400.455	2472.10	2520.000	2567.30	2611.890
Close	2417.430	2495.15	2541.475	2592.00	2648.885

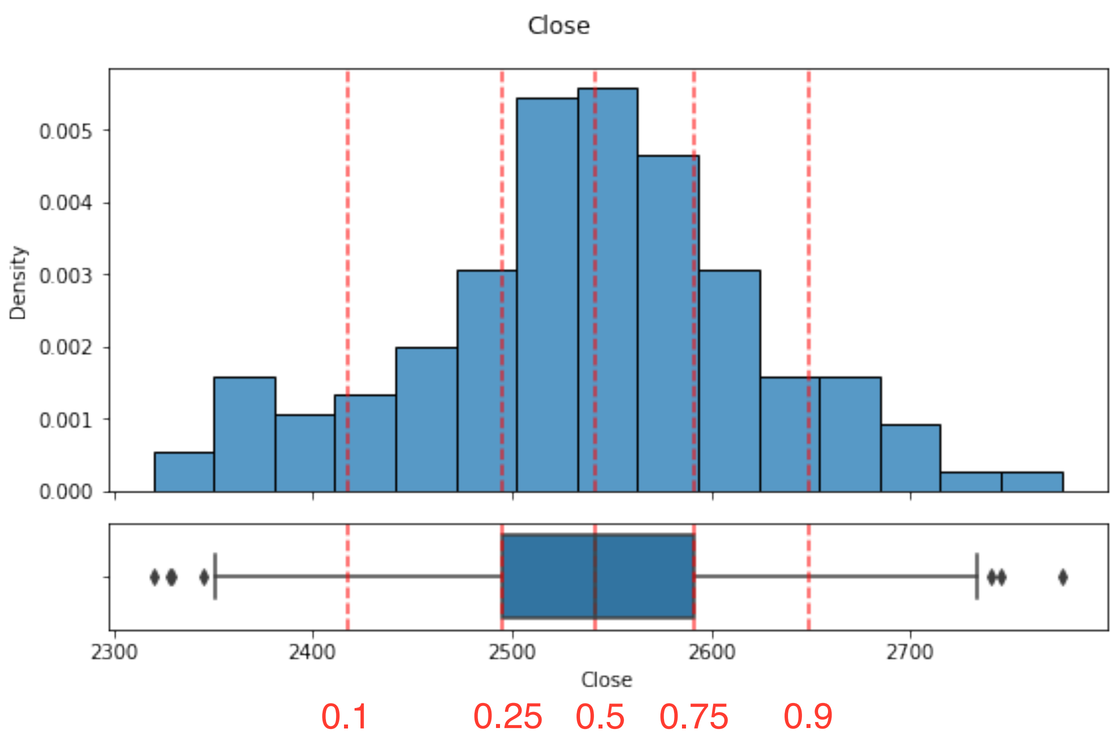
[9]:
```
def draw(label, method, df, width = 7):
    data = df[label]
    quantiles = quantilies(data.to_numpy())
    fig, ax = plt.subplots(2,1, figsize=(width, 5), sharex=True,␣
↪gridspec_kw={'height_ratios': [2,0.5]})
    fig.suptitle(label)

    sb.histplot(data, ax=ax[0], kde = False, stat=method)
    for q in quantiles:
        ax[0].axvline(q, color='r', linestyle='--', alpha=0.7)

    sb.boxplot(x=data, ax=ax[1])
    for q in quantiles:
        ax[1].axvline(q, color='r', linestyle='--', alpha=0.7)

    plt.tight_layout()
    plt.show()
```

[10]:
```
for feature in features:
    draw(feature, 'density', df, 8)
```


Open
Density
0.005
0.004
0.003
0.002
0.001
0.000
2300
2400
2500
2600
2700
2800
Open
0.1
0.25
0.5
0.75
0.9


High
Density
0.005
0.004
0.003
0.002
0.001
0.000
2400
2500
2600
2700
2800
High
0.1
0.25
0.5
0.75
0.9


Low
Density
0.005
0.004
0.003
0.002
0.001
0.000
2300
2350
2400
2450
2500
2550
2600
2650
2700
Low
0.1
0.25
0.5
0.75
0.9


Close
Density
0.005
0.004
0.003
0.002
0.001
0.000
2300
2400
2500
2600
2700
Close
0.1
0.25
0.5
0.75
0.9

2.4 Step 4. Find one or several theoretical distributions that could describe your sample on a basis of non-parametric analysis results.

In this part, we will look for a suitable distribution for each of the selected features using scipy library.

Algorithm: We have collected all scipy distribution objects in variable ALL_DISTRIBUTIONS.

For each feature, we build a histogram and take the center values for the columns using Danoe'S formula. After we calculate SSE (sum of squared estimate of errors) using formula $SSE = \sum_i^n (y\_ - y_{pdf\_i})^2$. The distribution with the lowest SSE value is taken as the most suitable.

$ y_{pdf_i}$ are calculated usig PDF for each distribution. Correct PDF we get using scipy.fit_data.

[19]:
```
def danoes_formula(data):
    """
    DANOE'S FORMULA

    """
    N = len(data)
    skewness = st.skew(data)
    sigma_g1 = math.sqrt((6*(N-2))/((N+1)*(N+3)))
    num_bins = 1 + math.log(N,2) + math.log(1+abs(skewness)/sigma_g1,2)
    num_bins = round(num_bins)
    return num_bins

def fit_data(data):
    ## st.frechet_r,st.frechet_l: are disabled in current SciPy version
    ## st.levy_stable: a lot of time of estimation parameters
    ALL_DISTRIBUTIONS = [
        st.alpha,st.anglit,st.arcsine,st.beta,st.betaprime,st.bradford,st.
↪burr,st.cauchy,st.chi,st.chi2,st.cosine,
        st.dgamma,st.dweibull,st.erlang,st.expon,st.exponnorm,st.exponweib,st.
↪exponpow,st.f,st.fatiguelife,st.fisk,
        st.foldcauchy,st.foldnorm, st.genlogistic,st.genpareto,st.gennorm,st.
↪genexpon,
        st.genextreme,st.gausshyper,st.gamma,st.gengamma,st.genhalflogistic,st.
↪gilbrat,st.gompertz,st.gumbel_r,
        st.gumbel_l,st.halfcauchy,st.halflogistic,st.halfnorm,st.halfgennorm,st.
↪hypsecant,st.invgamma,st.invgauss,
        st.invweibull,st.johnsonsb,st.johnsonsu,st.ksone,st.kstwobign,st.
↪laplace,st.levy,st.levy_l,
        st.logistic,st.loggamma,st.loglaplace,st.lognorm,st.lomax,st.maxwell,st.
↪mielke,st.nakagami,st.ncx2,st.ncf,
        st.nct,st.norm,st.pareto,st.pearson3,st.powerlaw,st.powerlognorm,st.
↪powernorm,st.rdist,st.reciprocal,
```

```
        st.rayleigh,st.rice,st.recipinvgauss,st.semicircular,st.t,st.triang,st.
↪truncexpon,st.truncnorm,st.tukeylambda,
        st.uniform,st.vonmises,st.vonmises_line,st.wald,st.weibull_min,st.
↪weibull_max,st.wrapcauchy
    ]

    MY_DISTRIBUTIONS = [st.invgauss]

    # Calculae Histogram
    num_bins = danoes_formula(data)
    frequencies, bin_edges = np.histogram(data, num_bins, density=True)
    central_values = [(bin_edges[i] + bin_edges[i+1])/2 for i in␣
↪range(len(bin_edges)-1)]

    results = {}
    for distribution in tqdm(ALL_DISTRIBUTIONS):
        ## Get parameters of distribution
        params = distribution.fit(data)

        ## Separate parts of parameters
        arg = params[:-2]
        loc = params[-2]
        scale = params[-1]

        ## Calculate fitted PDF and error with fit in distribution
        pdf_values = [distribution.pdf(c, loc=loc, scale=scale, *arg) for c in␣
↪central_values]

        ## Calculate SSE (sum of squared estimate of errors)
        sse = np.sum(np.power(frequencies - pdf_values, 2.0))

        ## Build results and sort by sse
        results[distribution.name] = [float(sse), arg, round(loc, 3),␣
↪round(scale, 3)]

    res = pd.DataFrame.from_dict(results, orient='index',columns=['SSE', 'ARG',␣
↪'LOC', 'SCALE'])
    return res
```

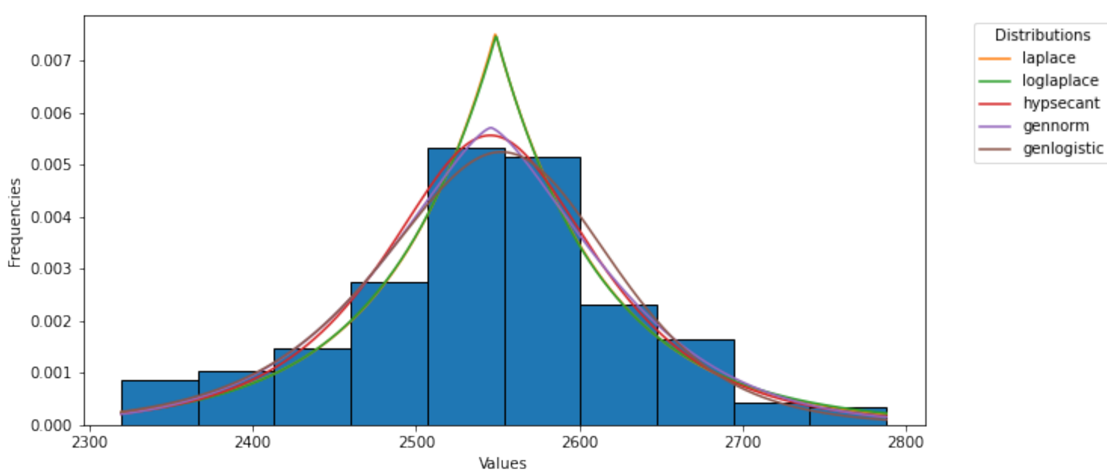
[20]:
```
def plot_histogram(data, results, n):
    ## n first distribution of the ranking
    N_DISTRIBUTIONS = results.sort_values(by=['SSE'])[:5]

    display(N_DISTRIBUTIONS)

    ## Histogram of data
```

```
    plt.figure(figsize=(10, 5))
    plt.hist(data, density=True, ec='black')
    plt.xlabel('Values')
    plt.ylabel('Frequencies')

    for distribution, values in N_DISTRIBUTIONS.iterrows():
        x_plot = np.linspace(min(data), max(data), 1000)
        y_plot = getattr(st.distributions, distribution).pdf(x_plot,␣
↪loc=values['LOC'], scale=values['SCALE'], *values['ARG'])
        plt.plot(x_plot, y_plot, label=distribution)

    plt.legend(title='Distributions', bbox_to_anchor=(1.05, 1), loc='upper␣
↪left')
    plt.show()
```

[21]:
```
for feature_name in df.keys():
    data = df[feature_name]
    with warnings.catch_warnings():
        warnings.filterwarnings('ignore')
        res = fit_data(data)
        plot_histogram(data, res, 5)
        first_value = next(iter(res.keys()))
        time.sleep(1)
```

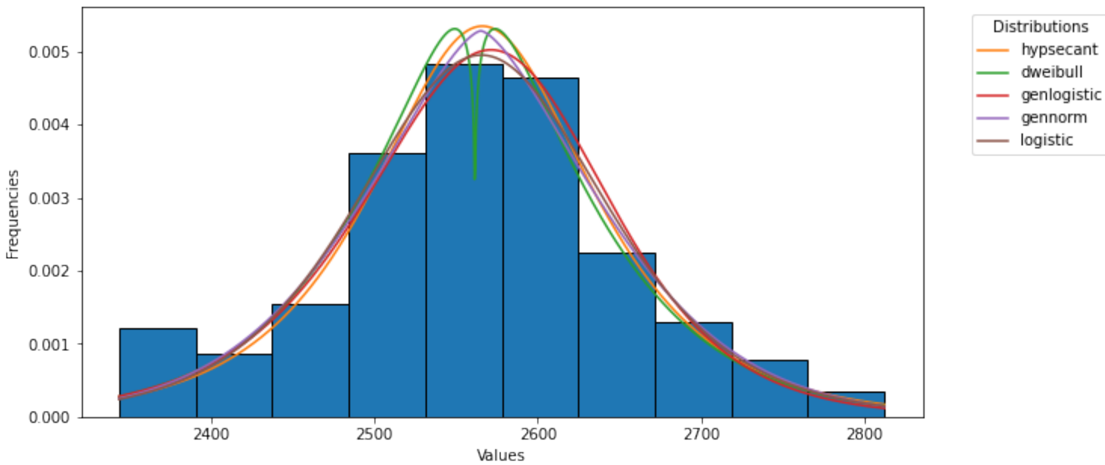
```
100%|                                          |
86/86 [00:08<00:00,  9.73it/s]
```

	SSE	ARG	LOC	SCALE
laplace	0.00000085	()	2548.500	66.478
loglaplace	0.00000086	(38.169396159952825,)	-1.390	2550.390
hypsecant	0.00000122	()	2545.465	57.223
gennorm	0.00000126	(1.4014591878045362,)	2545.773	96.095
genlogistic	0.00000143	(0.7390392202666065,)	2565.231	43.053



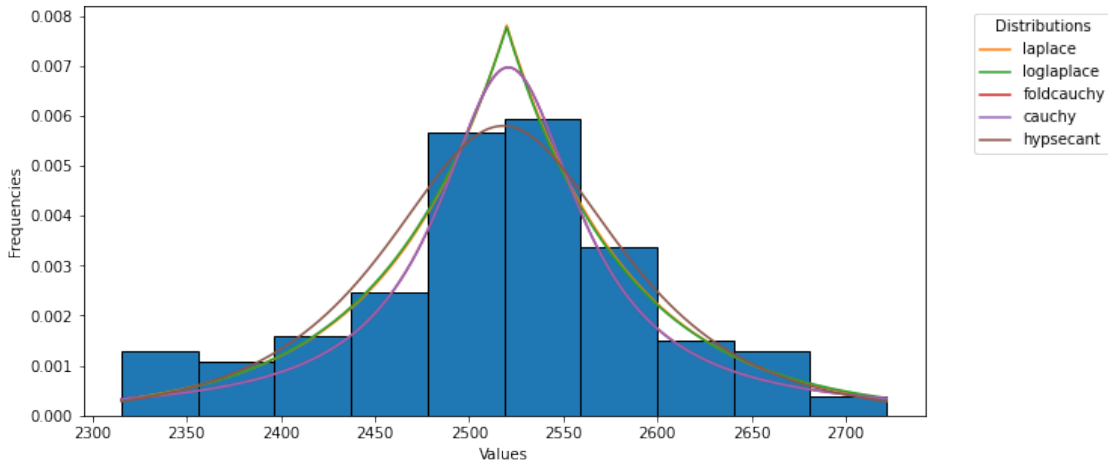
```
100%|                                        |
86/86 [00:09<00:00,  9.24it/s]
```

	SSE	ARG	LOC	SCALE
hypsecant	0.00000109	()	2566.114	59.518
dweibull	0.00000112	(1.1490674893116348,)	2561.555	72.897
genlogistic	0.00000117	(0.7962286855589259,)	2581.887	46.169
gennorm	0.00000121	(1.486995842398066,)	2565.162	104.718
logistic	0.00000121	()	2565.381	50.480



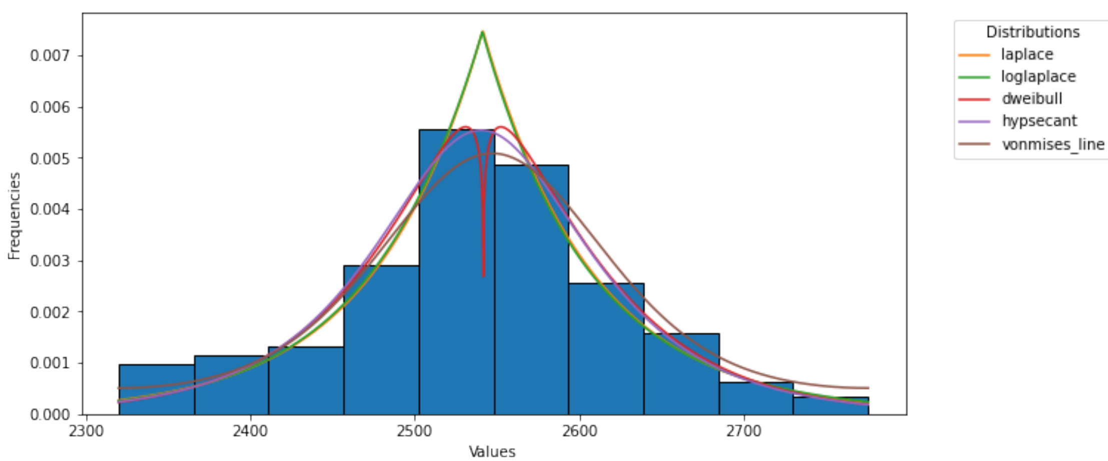

```
100%|                                        |
86/86 [00:09<00:00,  9.23it/s]
```

	SSE	ARG	LOC	SCALE
laplace	0.00000133	()	2520.000	64.048
loglaplace	0.00000142	(39.18516955081586,)	-1.370	2521.370
foldcauchy	0.00000224	(55.16844084327158,)	-0.529	45.703
cauchy	0.00000224	()	2520.831	45.691
hypsecant	0.00000244	()	2518.103	54.922




```
100%|                                        |
86/86 [00:09<00:00,  9.34it/s]
```

	SSE	ARG	LOC	SCALE
laplace	0.00000077	()	2541.475	66.768
loglaplace	0.00000085	(37.95089813785944,)	-1.380	2542.570
dweibull	0.00000094	(1.1350707272595078,)	2541.716	69.791
hypsecant	0.00000108	()	2540.403	57.462
vonmises_line	0.00000142	(1.1541932774700352,)	2547.157	72.843



Let's summarize the calculations above in mathematical terms. For each feature (Open, High, Low, Close) we calculated histogram and PDFs of distibutions. Graphics contains legend with SSE value. As mentioned earlier, the minimum SSE value shows a suitable distribution

Looking at the legends and tables, we can conclude that all the features following distributions: * Open - Laplace distribution * High - Hyperbolic secant distribution * Low - Laplace distribution *

Close - Laplace distribution

Laplace distribution PDF: $f(x) = \frac{1}{2b}exp(-\frac{|x-\mu|}{b})$

Hyperbolic secant distribution PDF: $f(x) = \frac{1}{2}sech(\frac{\pi}{2}x)$

PDF of this distribution has no parameters, but can be shifted and/or scaled using the loc and scale parameters. Specifically if we talk about scipy realization, hypsecant.pdf(x, loc, scale) is identically equivalent to hypsecant.pdf(y) / scale with $y = \frac{(x-loc)}{scale}$.

2.5 Step 5. Estimate parameters of chosen distributions using methods of maximum likelihood and least squares method.

as we can see above, our Hyperbolic secant distribution has two parameters: loc and scale

[14]:
```
def estimate_params(data, var, distribution):
    res = list(getattr(st, distribution).fit(data[var]))
    return [round(i,3) for i in res]

def estimate_params_ls(data, var, distribution, bins=90):
    min_val, max_val = data[var].min(), data[var].max()
    dists, _edges = np.histogram(data[var], bins=bins, density=True)
    xs = np.linspace(min_val, max_val, num=bins)
    def f(xdata, loc, scale):
        return getattr(st.distributions, distribution).pdf(xdata, loc=loc,␣
 ↪scale=scale)

    p, _ = curve_fit(f, xs, dists, p0=estimate_params(data, var, distribution),␣
 ↪maxfev=2000)
    return [round(i,3) for i in p]
```

[15]:
```
features_with_dist = {'Open': 'laplace', 'High': 'hypsecant', 'Low': 'laplace',␣
 ↪'Close': 'laplace'}
report = pd.DataFrame()

for var, distribution in features_with_dist.items():
    res = [[var, 'Maximum likelihood', estimate_params(df, var, distribution)],
           [var,'Least squares', estimate_params_ls(df, var, distribution)]]
    report = report.append(res, ignore_index=True)

report.columns = ["Feature", "Method", "Distribution args"]
report = report.set_index(['Feature', 'Method']).sort_index()
report
```

[15]:

Feature	Method	Distribution args
Close	Least squares	[2544.977, 75.884]
	Maximum likelihood	[2541.475, 66.768]

```
High     Least squares        [2569.022, 59.844]
         Maximum likelihood   [2566.114, 59.518]
Low      Least squares        [2522.225, 75.037]
         Maximum likelihood     [2520.0, 64.048]
Open     Least squares        [2552.261, 73.267]
         Maximum likelihood     [2548.5, 66.478]
```

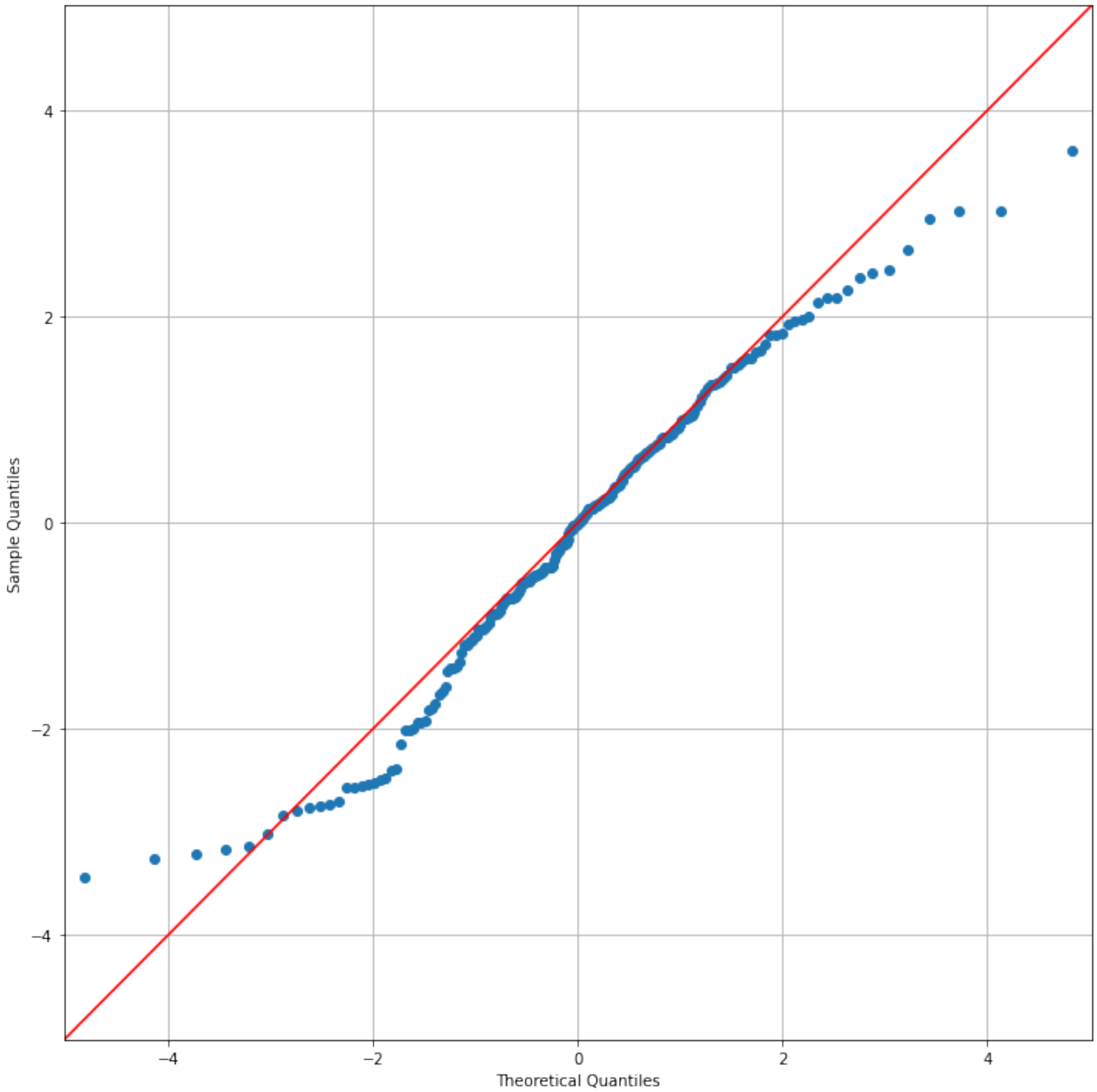
Let's take a closer look at the table an find out what parameters have distributions:

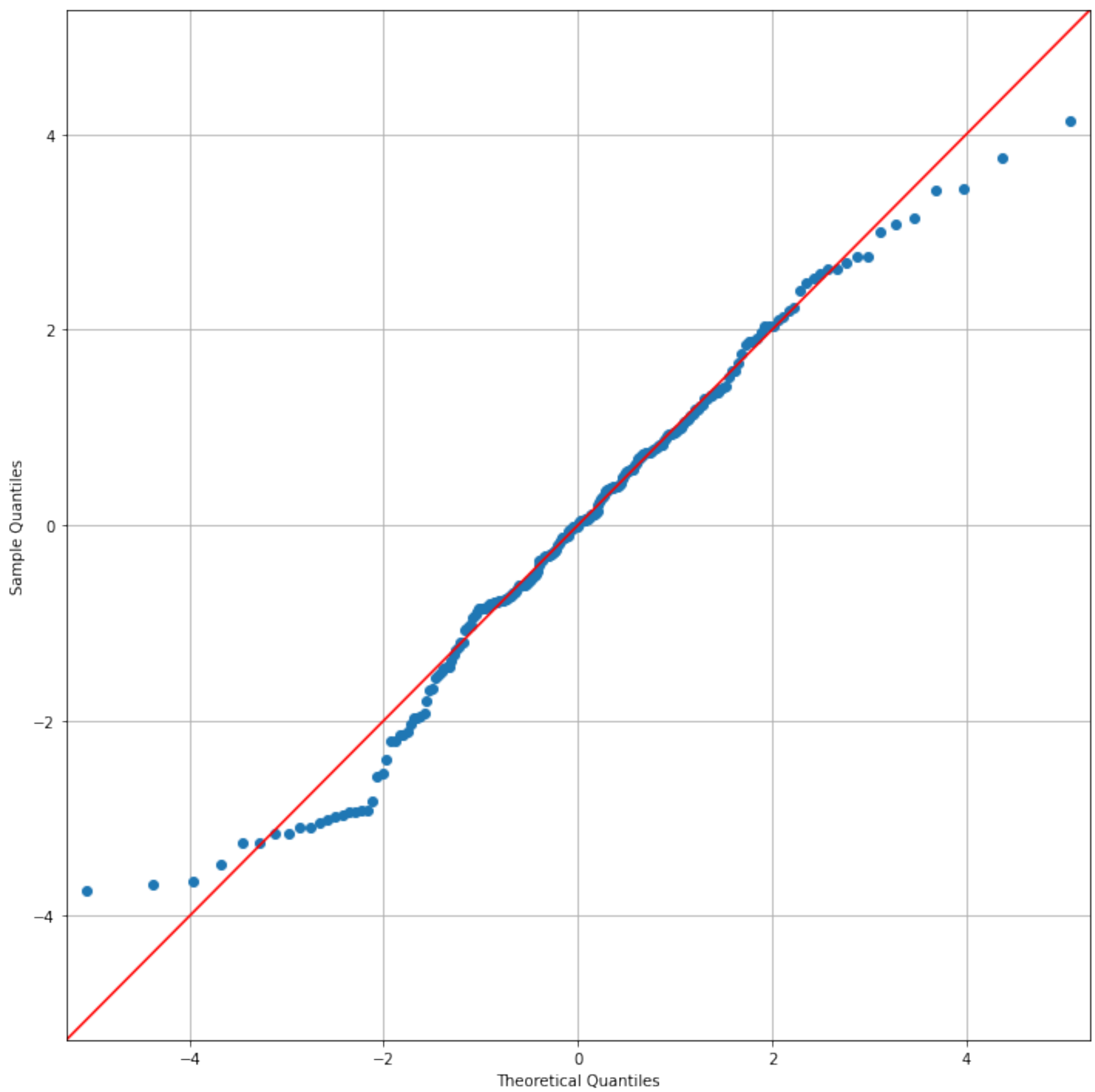
For Close, Low, Open
1st number in column **Distribution args** is μ
2st number in column **Distribution args** is b

For High
1st number in column **Distribution args** is loc
2st number in column **Distribution args** is scale

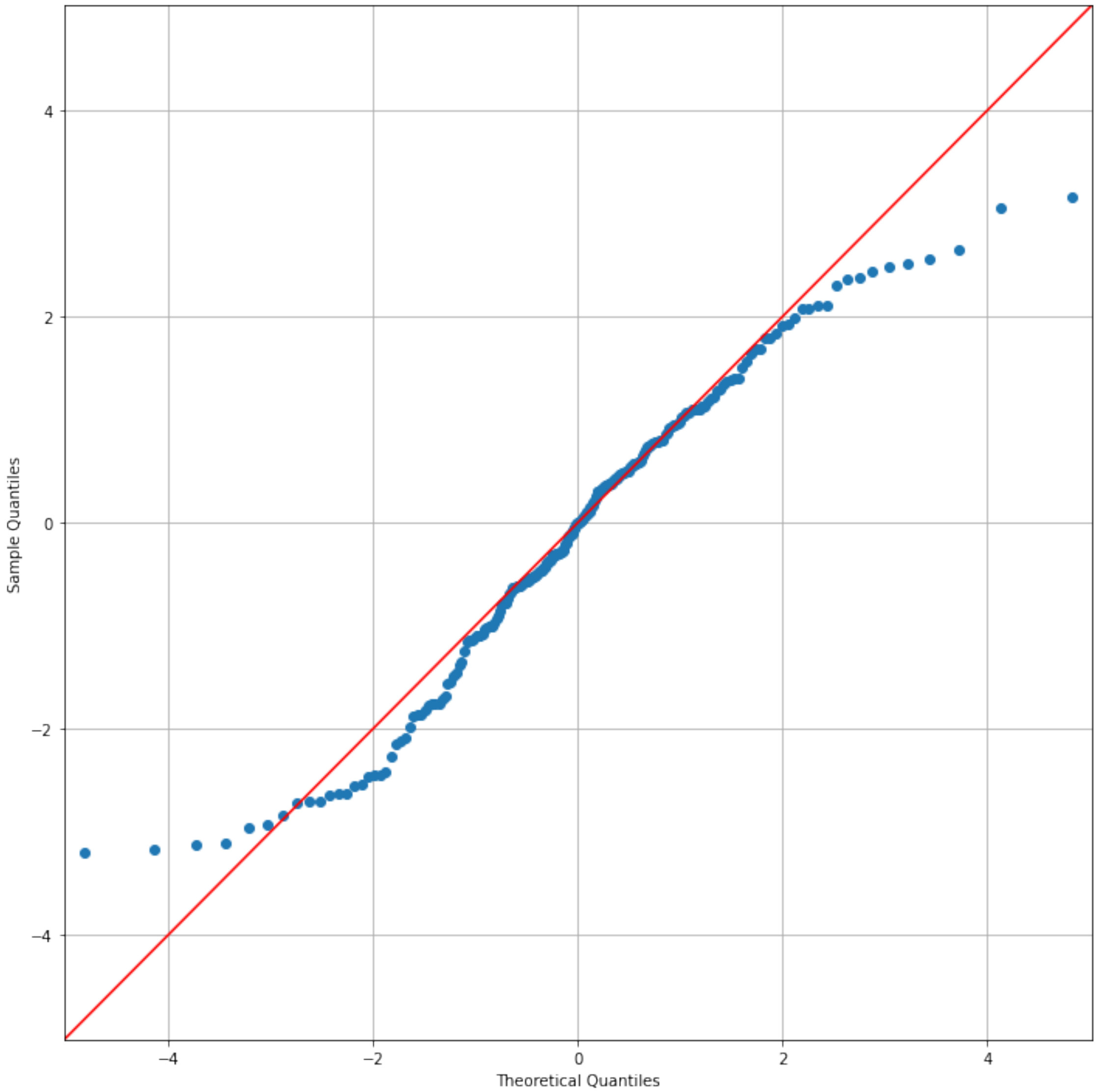
2.6 Step 6. Validate your estimated parameters using QQ biplots.

[16]:
```
i = 0
for col, distribution in features_with_dist.items():
    fig, ax = plt.subplots(figsize=(10, 10), sharey=True)
    qqplot(df[col], getattr(st.distributions, distribution), fit=True, ax=ax,␣
 ↪line="45")
    ax.grid()
    i += 1

    plt.tight_layout()
    plt.show()
```

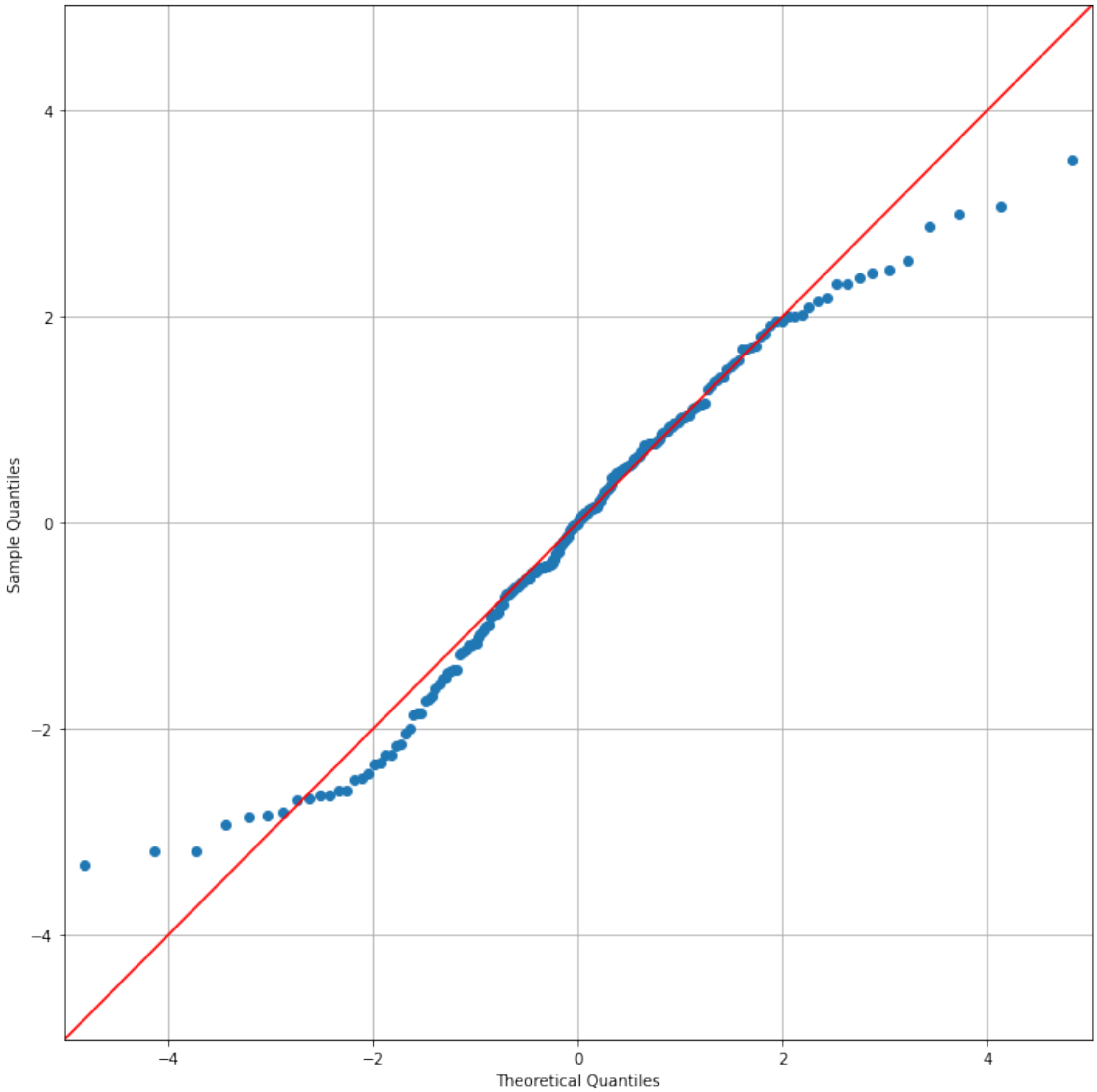

Sample Quantiles
Theoretical Quantiles
4
2
0
−2
−4
−4
−2
0
2
4



4
2
0
−2
−4
Sample Quantiles
−4
−2
0
2
4
Theoretical Quantiles



Sample Quantiles
Theoretical Quantiles
4
2
0
−2
−4
−4
−2
0
2
4



The points from all plots seem to fall about a straight line. So we can conclude that the theoretical distribution is close to the real distribution of the data.

2.7 Step 7. Estimate correctness of fitted distributions using at least 2 statistical tests.

At step 4, we fitted the distributions for each of the features using the SSE value. Now we will proof it using statistical tests.

[17]:
```
features_with_dist
```

[17]:
```
{'Open': 'laplace', 'High': 'hypsecant', 'Low': 'laplace', 'Close': 'laplace'}
```

In the next section we are perfoming Kolmogorov-Smirnov and Cramér-von Mises statistical tests.

[18]:
```
results={}
for col, dst in features_with_dist.items():
    data = df[col]
    a, b = getattr(st.distributions, distribution).fit(data)
    l1 = getattr(st.distributions, distribution)(a, b)

    ks_p_value = st.kstest(data, l1.cdf).pvalue
    cm_p_value = sc.stats.cramervonmises(data, l1.cdf).pvalue

    results[col]=[round(ks_p_value, 3), round(cm_p_value, 3)]

pd.DataFrame.from_dict(results, orient='index',columns=['Kolmogorov-Smirnov␣
↪test', 'Cramér-von Mises test'])
```

[18]:

	Kolmogorov-Smirnov test	Cramér-von Mises test
Open	0.295	0.598
High	0.734	0.646
Low	0.496	0.512
Close	0.478	0.705

In all cases, the p-value takes on rather high values. This means that at step 4 we have chosen the distributions well.
For Open, Low, Close it is Laplace.
For High it Hyperbolic secant distribution.