FEDERAL STATE AUTONOMOUS EDUCATIONAL INSTITUTION
OF HIGHER EDUCATION
ITMO UNIVERSITY

Report

on learning practice No.1

«Analysis of univariate random variables»

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code

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[1]:
```
import pandas as pd
import numpy as np
from numpy.random import normal
import seaborn as sb
import scipy as sc
import scipy.stats as st
from tqdm import tqdm
import time
import statsmodels.api as sm
import matplotlib as mpl
import matplotlib.pyplot as plt
import math
import random
import warnings
```

https://www.kaggle.com/atulanandjha/national-stock-exchange-time-series?select=tcs_stock.csv

[2]:
```
source_data_path = "./../tcs_stock.csv"
```

[3]:
```
raw_df = pd.read_csv(source_data_path)
```

0.1 Data description

- Date - date on which data is recorded
- Symbol - NSE symbol of the stock
- Series - series of that stock | EQ - Equity
- Prev Close - last day close point
- Open - current day open point
- High - current day highest point
- Low - current day lowest point
- Last - the final quoted trading price for a particular stock, or stock-market index, during the most recent day of trading
- Close - closing point for the current day
- VWAP - volume-weighted average price is the ratio of the value traded to total volume traded over a particular time horizon

[4]:
```
raw_df.head()
```

[4]:
```
         Date Symbol Series  Prev Close    Open     High      Low     Last  \
0  2015-01-01    TCS     EQ     2558.25  2567.0  2567.00  2541.00  2550.00
1  2015-01-02    TCS     EQ     2545.55  2551.0  2590.95  2550.60  2588.40
2  2015-01-05    TCS     EQ     2579.45  2581.0  2599.90  2524.65  2538.10
3  2015-01-06    TCS     EQ     2540.25  2529.1  2529.10  2440.00  2450.05
4  2015-01-07    TCS     EQ     2446.60  2470.0  2479.15  2407.45  2426.90

     Close     VWAP   Volume      Turnover  Trades  Deliverable Volume  \
0  2545.55  2548.51   183415  4.674345e+13    8002               52870
1  2579.45  2568.19   462870  1.188740e+14   27585              309350
2  2540.25  2563.94   877121  2.248886e+14   43234              456728
3  2446.60  2466.90  1211892  2.989615e+14   84503              714306
4  2417.70  2433.96  1318166  3.208362e+14  101741              886368

   %Deliverble
0       0.2883
1       0.6683
2       0.5207
3       0.5894
4       0.6724
```

0.2 Step 1. Choose subsample with main variables for your further analysis.

0.2.1 For this lab you need subsample with 3-5 random variables, at least half of them should be described with continuous random variable type

[5]:
```python
df = raw_df[['Open', 'High', 'Low', 'Close']]
df.head()
```

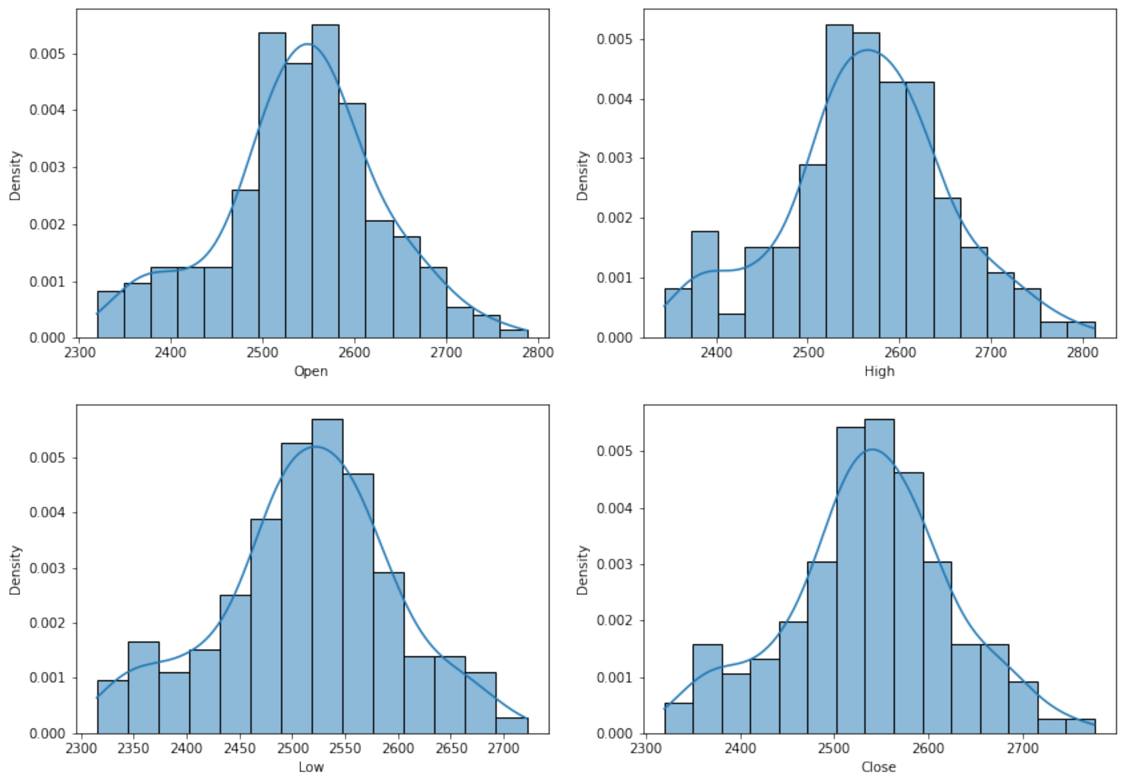
[5]:
```
     Open     High      Low    Close
0  2567.0  2567.00  2541.00  2545.55
1  2551.0  2590.95  2550.60  2579.45
2  2581.0  2599.90  2524.65  2540.25
3  2529.1  2529.10  2440.00  2446.60
4  2470.0  2479.15  2407.45  2417.70
```

0.3 Step 2. You need to make a non-parametric estimation of PDF in form of histogram and using kernel density function (or probability law in case of discrete RV).

(for each variable)

[6]:
```python
# initialize figure canvas
fig, ax = plt.subplots(2, 2, figsize=(14,10))
sb.histplot(df['Open'], ax=ax[0,0], kde=True, stat="density")
sb.histplot(df['High'], ax=ax[0,1], kde=True, stat="density")
sb.histplot(df['Low'], ax=ax[1,0], kde=True, stat="density")
```

```python
sb.histplot(df['Close'], ax=ax[1,1], kde=True, stat="density")

plt.show()
```



0.4 Step 3. You need to make an estimation of order statistics and represent them as "box with whiskers" plot.

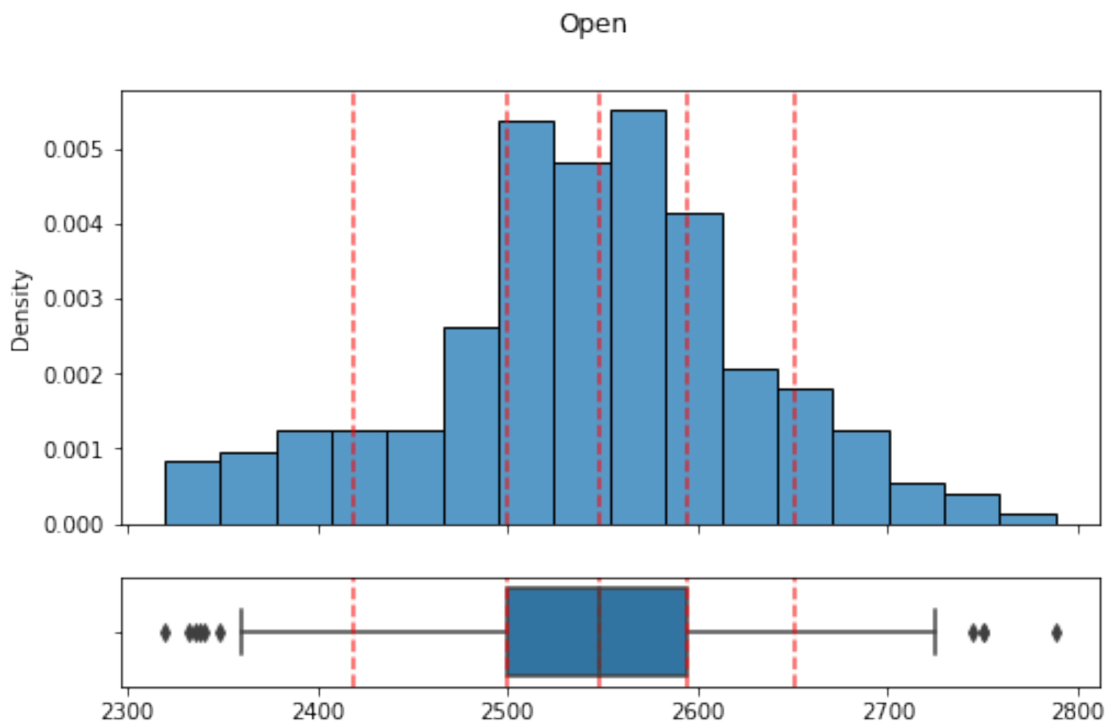
we will find quintiles estimations

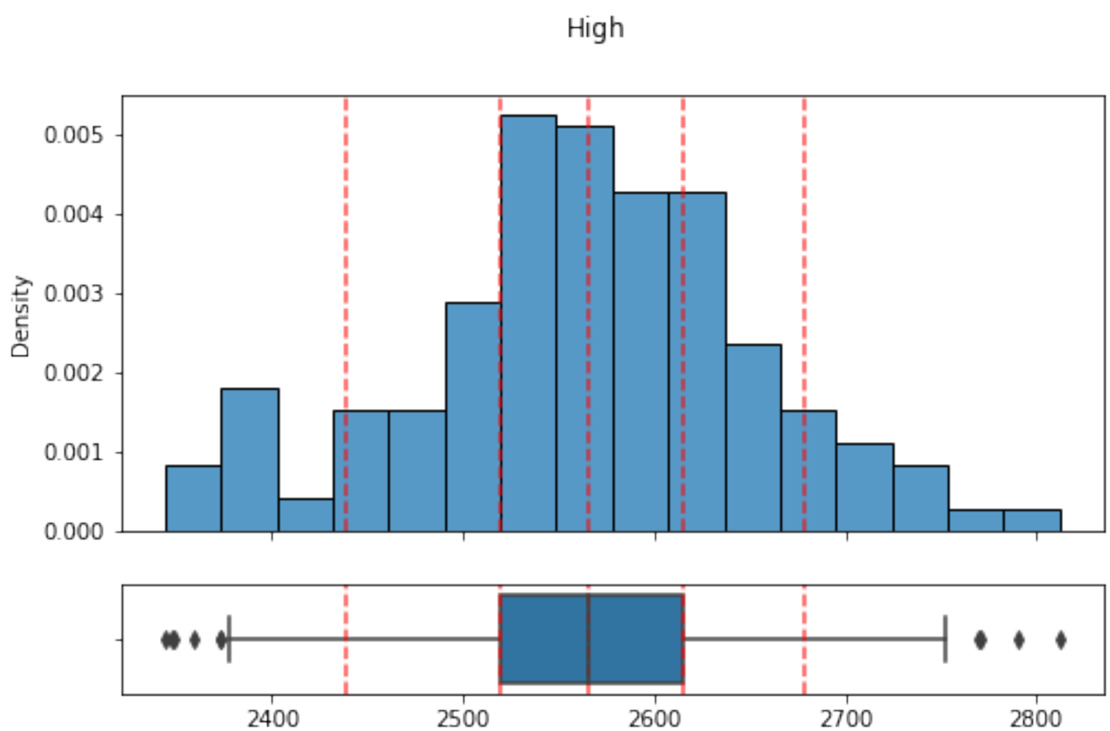
[7]:
```python
def quantilies(array):
    return [
        np.quantile(array, 0.1),
        np.quantile(array, 0.25),
        np.quantile(array, 0.5),
        np.quantile(array, 0.75),
        np.quantile(array, 0.9)
    ]

open_v = df['Open'].to_numpy()
open_v_quantiles = quantilies(open_v)

high = df['High'].to_numpy()
```

```
high_quantiles = quantilies(high)

last = df['Low'].to_numpy()
last_quantiles = quantilies(last)

close = df['Close'].to_numpy()
close_quantiles = quantilies(close)

print('Open: ', open_v_quantiles)
print('High: ', high_quantiles)
print('Low: ', last_quantiles)
print('Close: ', close_quantiles)
```

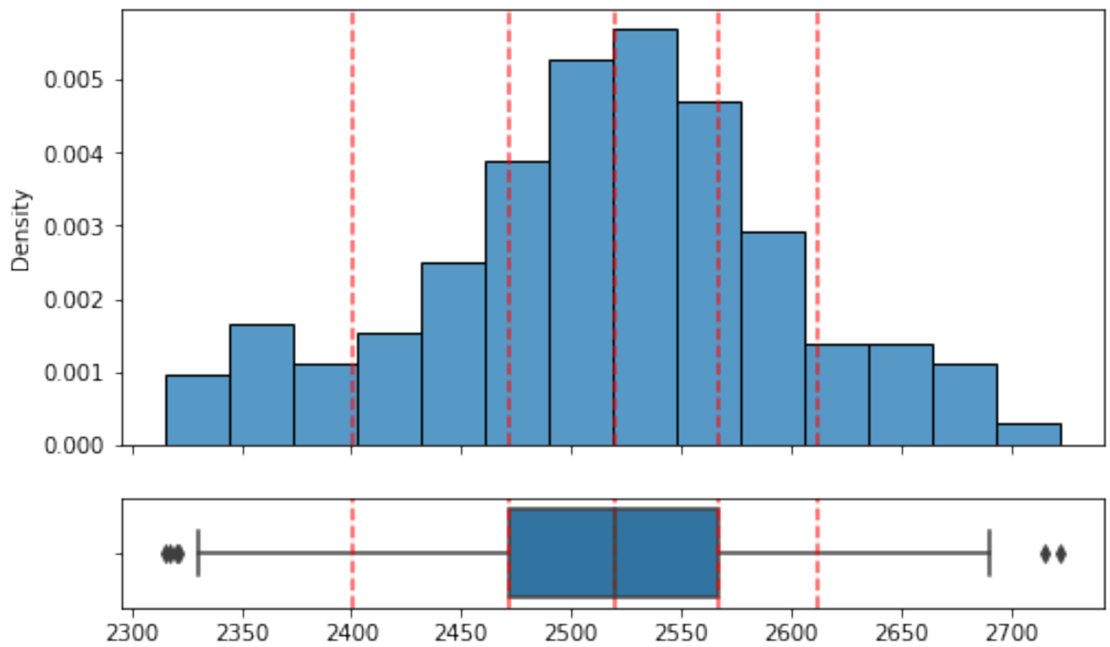
```
Open:  [2418.69, 2499.5, 2548.5, 2594.25, 2651.16]
High:  [2438.1549999999997, 2518.9, 2566.0, 2615.75, 2678.785]
Low:  [2400.455, 2472.1000000000004, 2520.0, 2567.2999999999997,
2611.8900000000003]
Close:  [2417.43, 2495.1499999999996, 2541.4750000000004, 2592.0, 2648.885]
```

[8]:
```
def draw(title, method, data, quantiles, width = 7):
    fig, ax = plt.subplots(2,1, figsize=(width, 5), sharex=True,␣
 ↪gridspec_kw={'height_ratios': [2,0.5]})
    fig.suptitle(title)

    sb.histplot(data, ax=ax[0], kde = False, stat=method)
    for q in quantiles:
        ax[0].axvline(q, color='r', linestyle='--', alpha=0.7)

    sb.boxplot(x=data, ax=ax[1])
    for q in quantiles:
        ax[1].axvline(q, color='r', linestyle='--', alpha=0.7)

draw('Open', 'density', open_v, open_v_quantiles, 8)
draw('High', 'density', high, high_quantiles, 8)
draw('Low', 'density', last, last_quantiles, 8)
draw('Close', 'density', close, close_quantiles)

plt.tight_layout()
plt.show()
```


Open
Density
0.005
0.004
0.003
0.002
0.001
0.000
2300
2400
2500
2600
2700
2800

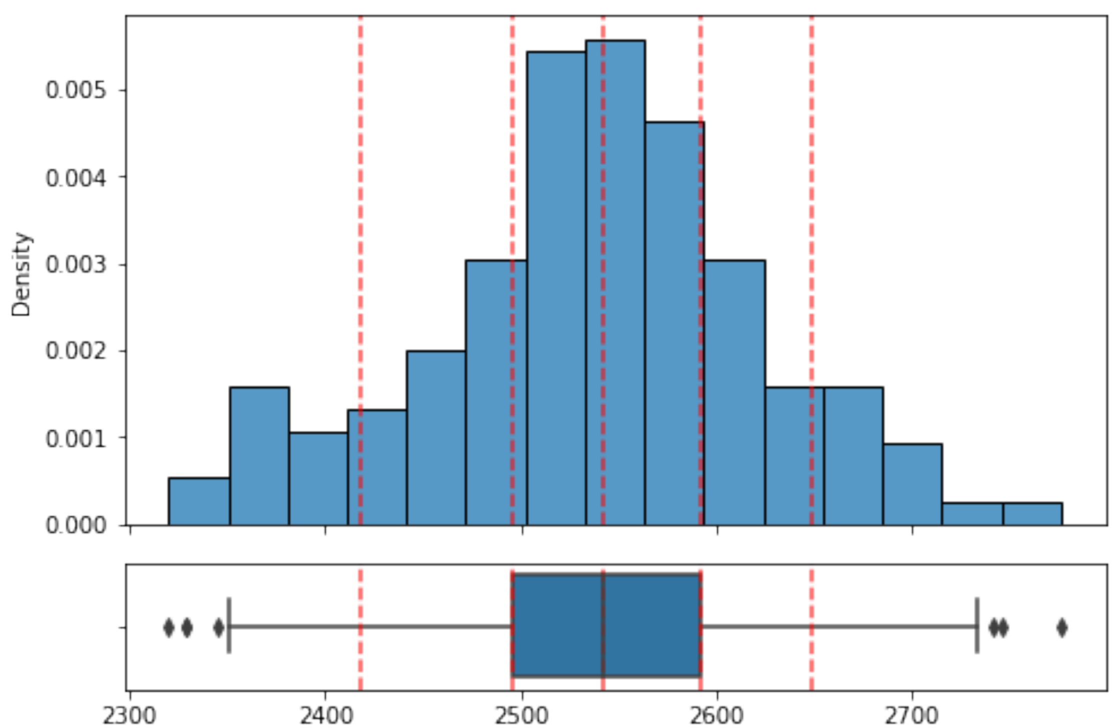

High
Density
0.005
0.004
0.003
0.002
0.001
0.000
2400
2500
2600
2700
2800

Low



Density
0.005
0.004
0.003
0.002
0.001
0.000
2300
2350
2400
2450
2500
2550
2600
2650
2700

Close



Density
0.005
0.004
0.003
0.002
0.001
0.000
2300
2400
2500
2600
2700

0.5 Step 4. Find one or several theoretical distributions that could describe your sample on a basis of non-parametric analysis results.

[9]:
```
def danoes_formula(data):
    """
    DANOE'S FORMULA
    https://en.wikipedia.org/wiki/Histogram#Doane's_formula
    """
    N = len(data)
    skewness = st.skew(data)
    sigma_g1 = math.sqrt((6*(N-2))/((N+1)*(N+3)))
    num_bins = 1 + math.log(N,2) + math.log(1+abs(skewness)/sigma_g1,2)
    num_bins = round(num_bins)
    return num_bins

def fit_data(data):
    ## st.frechet_r,st.frechet_l: are disabled in current SciPy version
    ## st.levy_stable: a lot of time of estimation parameters
    ALL_DISTRIBUTIONS = [
        st.alpha,st.anglit,st.arcsine,st.beta,st.betaprime,st.bradford,st.
 ↪burr,st.cauchy,st.chi,st.chi2,st.cosine,
        st.dgamma,st.dweibull,st.erlang,st.expon,st.exponnorm,st.exponweib,st.
 ↪exponpow,st.f,st.fatiguelife,st.fisk,
        st.foldcauchy,st.foldnorm, st.genlogistic,st.genpareto,st.gennorm,st.
 ↪genexpon,
        st.genextreme,st.gausshyper,st.gamma,st.gengamma,st.genhalflogistic,st.
 ↪gilbrat,st.gompertz,st.gumbel_r,
        st.gumbel_l,st.halfcauchy,st.halflogistic,st.halfnorm,st.halfgennorm,st.
 ↪hypsecant,st.invgamma,st.invgauss,
        st.invweibull,st.johnsonsb,st.johnsonsu,st.ksone,st.kstwobign,st.
 ↪laplace,st.levy,st.levy_l,
        st.logistic,st.loggamma,st.loglaplace,st.lognorm,st.lomax,st.maxwell,st.
 ↪mielke,st.nakagami,st.ncx2,st.ncf,
        st.nct,st.norm,st.pareto,st.pearson3,st.powerlaw,st.powerlognorm,st.
 ↪powernorm,st.rdist,st.reciprocal,
        st.rayleigh,st.rice,st.recipinvgauss,st.semicircular,st.t,st.triang,st.
 ↪truncexpon,st.truncnorm,st.tukeylambda,
        st.uniform,st.vonmises,st.vonmises_line,st.wald,st.weibull_min,st.
 ↪weibull_max,st.wrapcauchy
    ]

    MY_DISTRIBUTIONS = [st.invgauss]

    ## Calculae Histogram
    num_bins = danoes_formula(data)
```

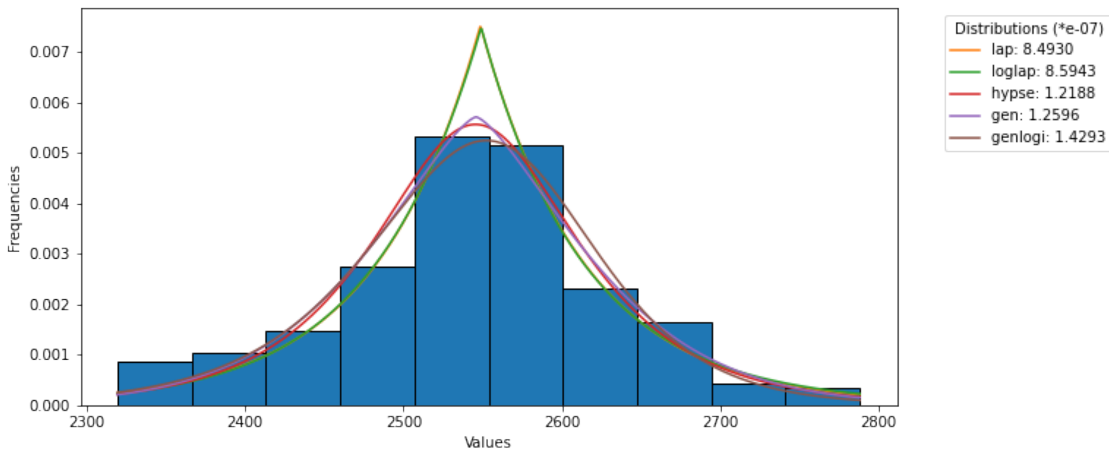
```
    frequencies, bin_edges = np.histogram(data, num_bins, density=True)
    central_values = [(bin_edges[i] + bin_edges[i+1])/2 for i in␣
↪range(len(bin_edges)-1)]

    results = {}
    for distribution in tqdm(ALL_DISTRIBUTIONS):
        ## Get parameters of distribution
        params = distribution.fit(data)

        ## Separate parts of parameters
        arg = params[:-2]
        loc = params[-2]
        scale = params[-1]

        ## Calculate fitted PDF and error with fit in distribution
        pdf_values = [distribution.pdf(c, loc=loc, scale=scale, *arg) for c in␣
↪central_values]

        ## Calculate SSE (sum of squared estimate of errors)
        sse = np.sum(np.power(frequencies - pdf_values, 2.0))

        ## Build results and sort by sse
        results[distribution] = [sse, arg, loc, scale]

    results = {k: results[k] for k in sorted(results, key=results.get)}
    return results
```

[10]:
```
def plot_histogram(data, results, n):
    ## n first distribution of the ranking
    N_DISTRIBUTIONS = {k: results[k] for k in list(results)[:n]}

    ## Histogram of data
    plt.figure(figsize=(10, 5))
    plt.hist(data, density=True, ec='black')
    plt.xlabel('Values')
    plt.ylabel('Frequencies')

    ## Plot n distributions
    for distribution, result in N_DISTRIBUTIONS.items():
        sse, arg, loc, scale = result
        x_plot = np.linspace(min(data), max(data), 1000)
        y_plot = distribution.pdf(x_plot, loc=loc, scale=scale, *arg)
        plt.plot(x_plot, y_plot, label=str(distribution)[32:-34] + ": " +␣
↪str(sse)[0:6])

    plt.legend(title='Distributions (*e-07)', bbox_to_anchor=(1.05, 1),␣
↪loc='upper left')
```

```
    plt.show()
```

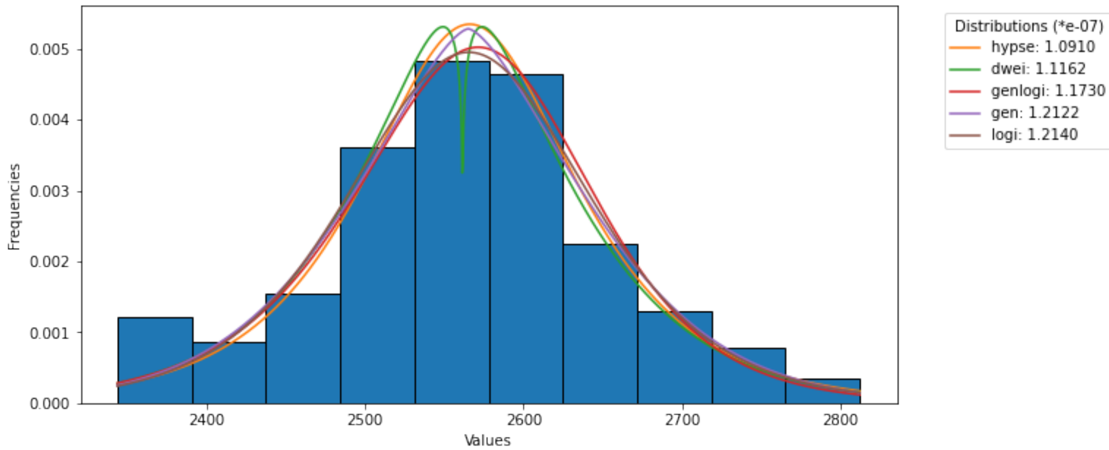
[11]:
```
for feature_name in df.keys():
    data = df[feature_name]
    with warnings.catch_warnings():
        warnings.filterwarnings('ignore')
        results = fit_data(data)
        plot_histogram(data, results, 5)
        first_value = next(iter(results.keys()))
        print("Best distribution for '{}' feature is '{}'".format(feature_name,␣
↪first_value.name))
        time.sleep(1)
```

```
100%|
            | 86/86 [00:10<00:00,  8.05it/s]
```




```
Best distribution for 'Open' feature is 'laplace'
```

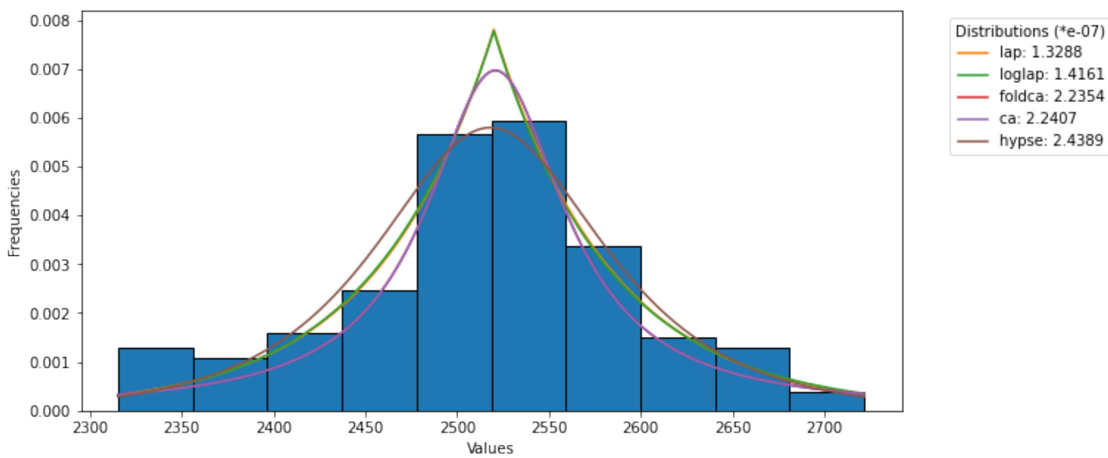
```
100%|
            | 86/86 [00:11<00:00,  7.71it/s]
```



Best distribution for 'High' feature is 'hypsecant'

100%|

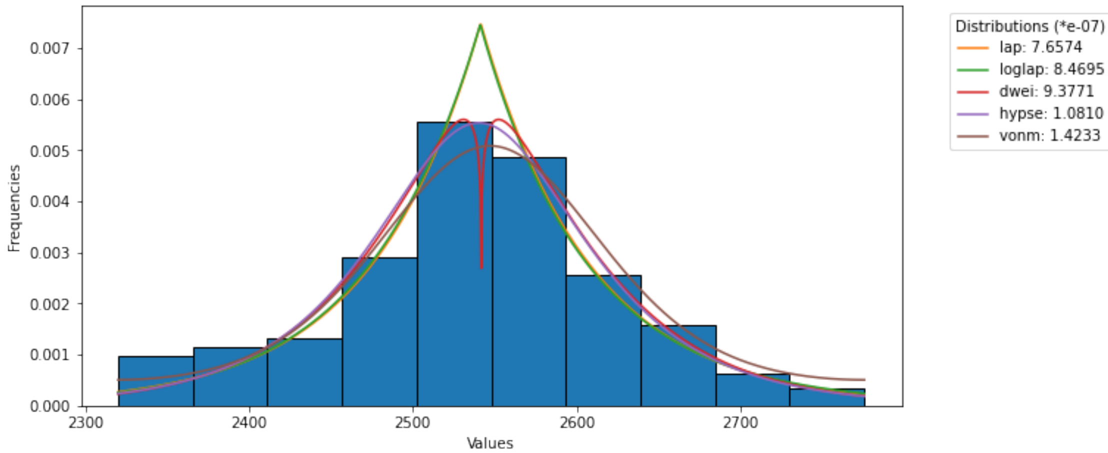
| 86/86 [00:11<00:00, 7.65it/s]



Best distribution for 'Low' feature is 'laplace'

100%|

| 86/86 [00:10<00:00, 7.92it/s]




```
Best distribution for 'Close' feature is 'laplace'
```

0.6 Step 5. Estimate parameters of chosen distributions using methods of maximum likelihood and least squares method.

[100]:
```python
def estimate_params(data, var, distribution):
    return list(getattr(st, distribution).fit(data[var]))

def estimate_params_ls(data, var, distribution, bins=90):
    min_val, max_val = data[var].min(), data[var].max()
    dists, _edges = np.histogram(data[var], bins=bins, density=True)
    xs = np.linspace(min_val, max_val, num=bins)
    def f(xdata, loc, scale):
        return getattr(st.distributions, distribution).pdf(xdata, loc=loc,␣
↪scale=scale)

    p, _ = scipy.optimize.curve_fit(f, xs, dists,
                                    p0=estimate_params(data, var, distribution),
                                    maxfev=2000)
    return p
```

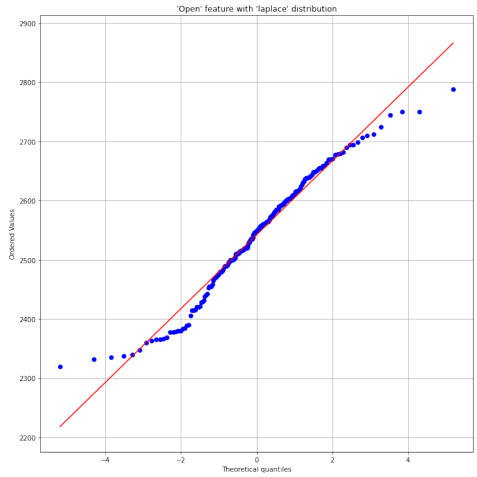
[101]:
```python
var_check = {'Open': 'laplace', 'High': 'hypsecant', 'Low': 'laplace', 'Close':␣
↪'laplace'}

for var, distribution in var_check.items():
    print(f'{var} parameters using MLE: {estimate_params(df, var,␣
↪distribution)}\n')
    print(f'{var} parameters using LS: {estimate_params_ls(df, var,␣
↪distribution)}\n')
```

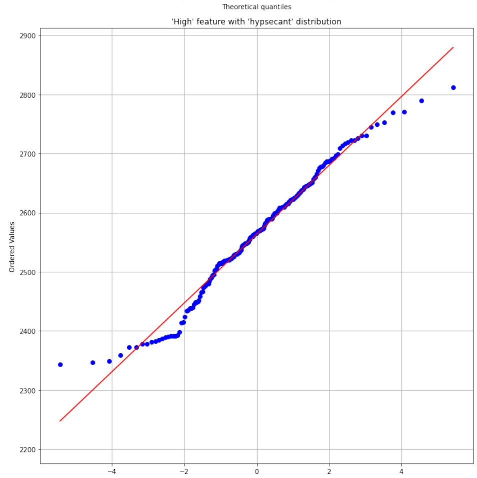
```
Open parameters using MLE: [2548.5, 66.47842741935482]
```

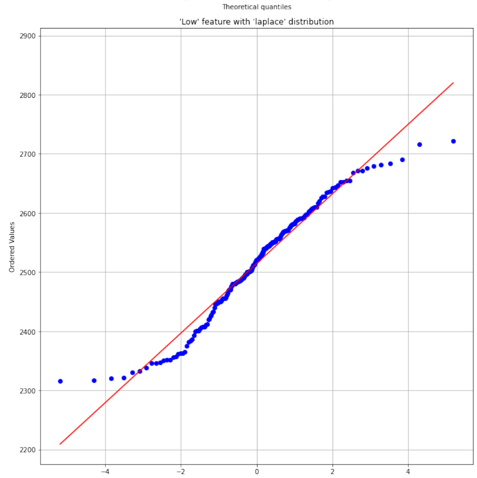
```
Open parameters using LS: [2552.26134837   73.26727496]

High parameters using MLE: [2566.1139624858956, 59.51842636604291]

High parameters using LS: [2569.02210113   59.84409266]

Low parameters using MLE: [2520.0, 64.04838709677419]

Low parameters using LS: [2522.22531967   75.0366586 ]

Close parameters using MLE: [2541.4750000000004, 66.76834677419355]

Close parameters using LS: [2544.97725667   75.88376731]
```

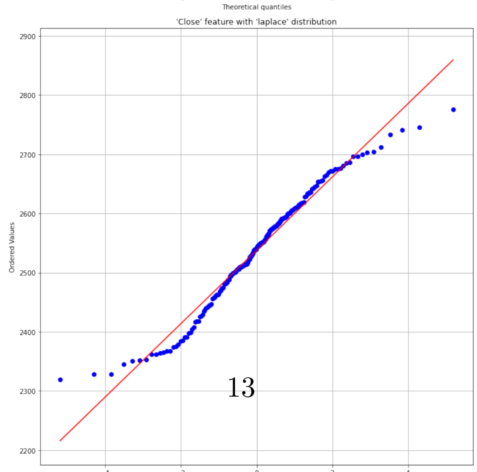
0.7 Step 6. Validate your estimated parameters using QQ biplots.

[13]:
```python
columns = ['Open', 'High', 'Low', 'Close']
distributions = [laplace, hypsecant, laplace, laplace]

fig, ax = plt.subplots(len(columns), 1, figsize=(10, 10*len(columns)),␣
 ↪sharey=True)

for distribution, col, i in zip(distributions, columns, range(0, len(columns))):
    feature = df[col]

    percs = np.linspace(0,100,21)
    qn_feature = np.percentile(feature, percs)
    x = np.linspace(np.min(feature), np.max(feature))
    st.probplot(feature, dist = distribution, plot=ax[i])
    ax[i].set_title(f"'{col}' feature with '{distribution.name}' distribution")
    ax[i].grid()

plt.tight_layout()
plt.show()
```


'Open' feature with 'laplace' distribution
Ordered Values
Theoretical quantiles
2200
2300
2400
2500
2600
2700
2800
2900
−4
−2
0
2
4


'High' feature with 'hypsecant' distribution
Ordered Values
Theoretical quantiles
2200
2300
2400
2500
2600
2700
2800
2900
−4
−2
0
2
4


'Low' feature with 'laplace' distribution
Ordered Values
Theoretical quantiles
2200
2300
2400
2500
2600
2700
2800
2900
−4
−2
0
2
4


'Close' feature with 'laplace' distribution
Ordered Values
Theoretical quantiles
2200
2300
2400
2500
2600
2700
2800
2900
−4
−2
0
2
4

The points from all plots seem to fall about a straight line. So we can conclude that the theoretical distribution is close to the real distribution of the data.

0.8 Step 7. Estimate correctness of fitted distributions using at least 2 statistical tests.

[43]:
```
columns = ['Open', 'High', 'Low', 'Close']
dist = [st.laplace, st.hypsecant, st.laplace, st.laplace]

for col, dst in list(zip(columns, dist)):
    data = df[col]
    a, b = dst.fit(data)
    l1 = dst(a, b)

    ks_p_value = st.kstest(data, l1.cdf).pvalue
    cm_p_value = st.cramervonmises(data, l1.cdf).pvalue

    print(f"Result for '{col}'")
    print("Kolmogorov-Smirnov test: p-value = ", ks_p_value)
    print("Cramér-von Mises test: p-value = ", cm_p_value)
    print("")
```

```
Result for 'Open'
Kolmogorov-Smirnov test: p-value =  0.2945892478928587
Cramér-von Mises test: p-value =  0.5980876041242682

Result for 'High'
Kolmogorov-Smirnov test: p-value =  0.8395566093601524
Cramér-von Mises test: p-value =  0.8947212156051837

Result for 'Low'
Kolmogorov-Smirnov test: p-value =  0.4962830863565647
Cramér-von Mises test: p-value =  0.5120218278609918

Result for 'Close'
Kolmogorov-Smirnov test: p-value =  0.4782554079997128
Cramér-von Mises test: p-value =  0.7048860495172959
```

As we can see here p-value > 0.05 for all described features. Then we can say that theoretical distribution and real distribution of our data is quite similar.

[ ]:
```
```