FEDERAL STATE AUTONOMOUS EDUCATIONAL INSTITUTION
OF HIGHER EDUCATION
ITMO UNIVERSITY

REPORT

on learning practice No.2

«Analysis of multivariate random variables»

Performed by:
Putnikov Semyon
Gabrielian Mikhail

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code

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[1]:
```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sb
from scipy import stats as st
from sklearn.preprocessing import scale, PolynomialFeatures
from sklearn.decomposition import PCA
from sklearn.model_selection import train_test_split
from sklearn import linear_model as lm
from sklearn.metrics import mean_squared_error, r2_score
from tabulate import tabulate
import statsmodels.api as sm
from statsmodels.stats.outliers_influence import variance_inflation_factor
```

[2]:
```python
source_data_path = "./../tcs_stock.csv"
row_df = pd.read_csv(source_data_path)
row_df["Deliverble(%)"] = row_df['%Deliverble']
```

[3]:
```python
feature_cols = ['Open', 'Low', 'Last', 'Close', 'Trades', 'Deliverble(%)']
target_col = 'High'

df = row_df[feature_cols + [target_col]]
df.head()
```

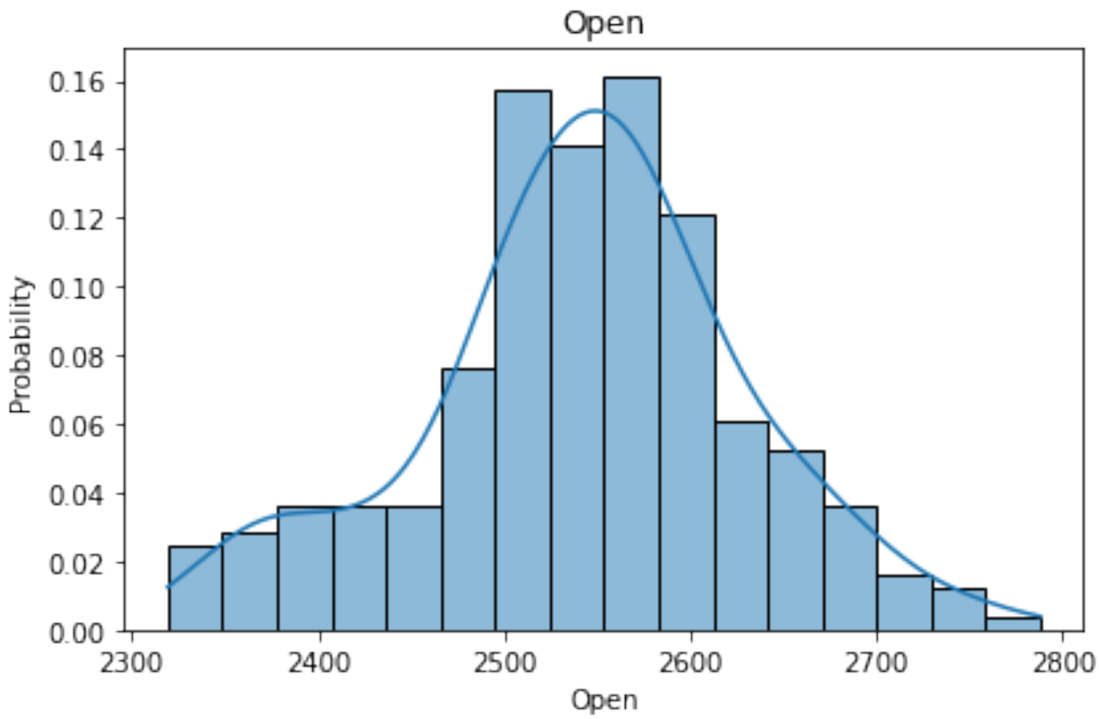
[3]:

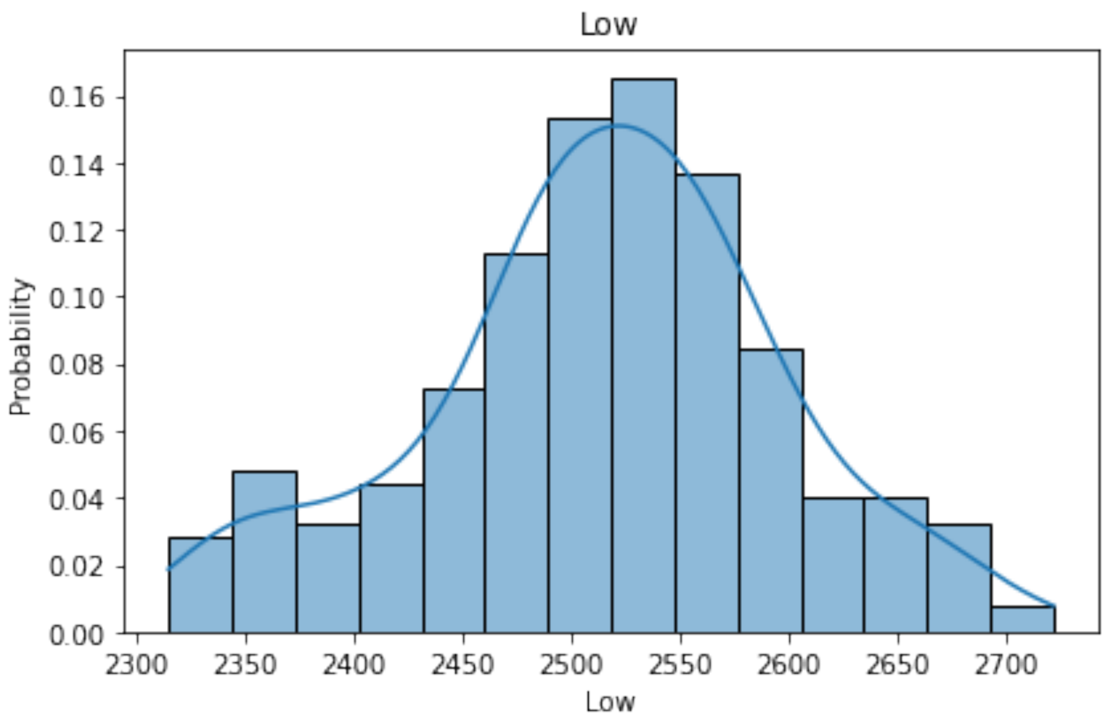
	Open	Low	Last	Close	Trades	Deliverble(%)	High
0	2567.0	2541.00	2550.00	2545.55	8002	0.2883	2567.00
1	2551.0	2550.60	2588.40	2579.45	27585	0.6683	2590.95
2	2581.0	2524.65	2538.10	2540.25	43234	0.5207	2599.90
3	2529.1	2440.00	2450.05	2446.60	84503	0.5894	2529.10
4	2470.0	2407.45	2426.90	2417.70	101741	0.6724	2479.15

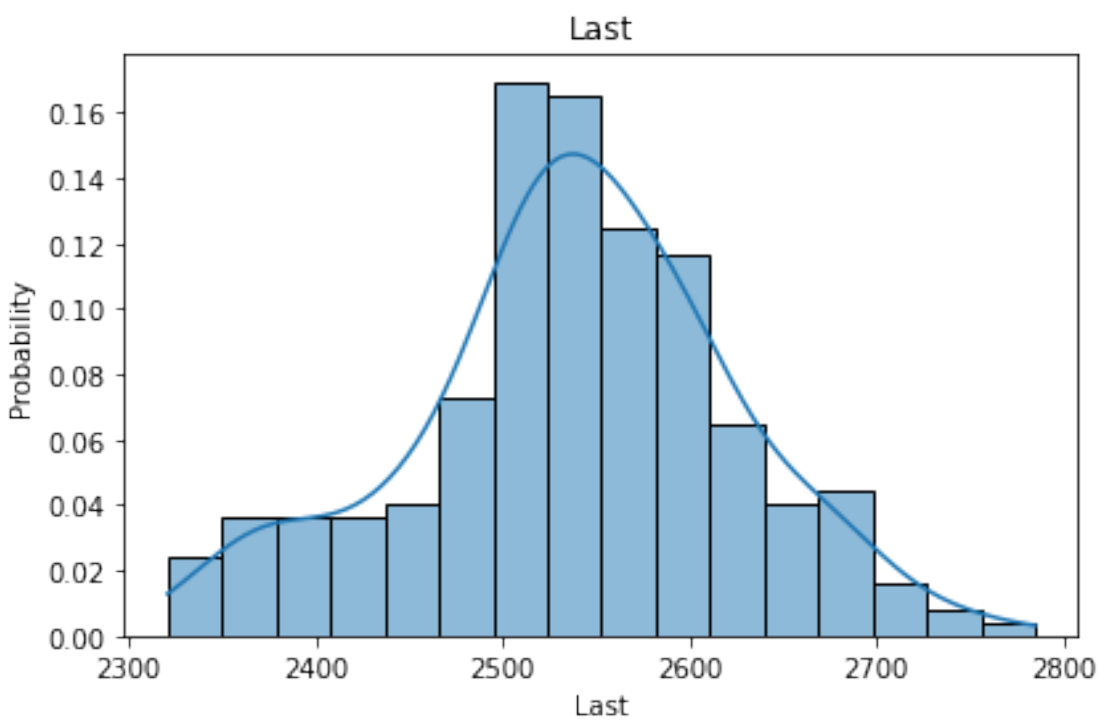
[4]:
```python
features = df[feature_cols]
target = df[target_col]
```

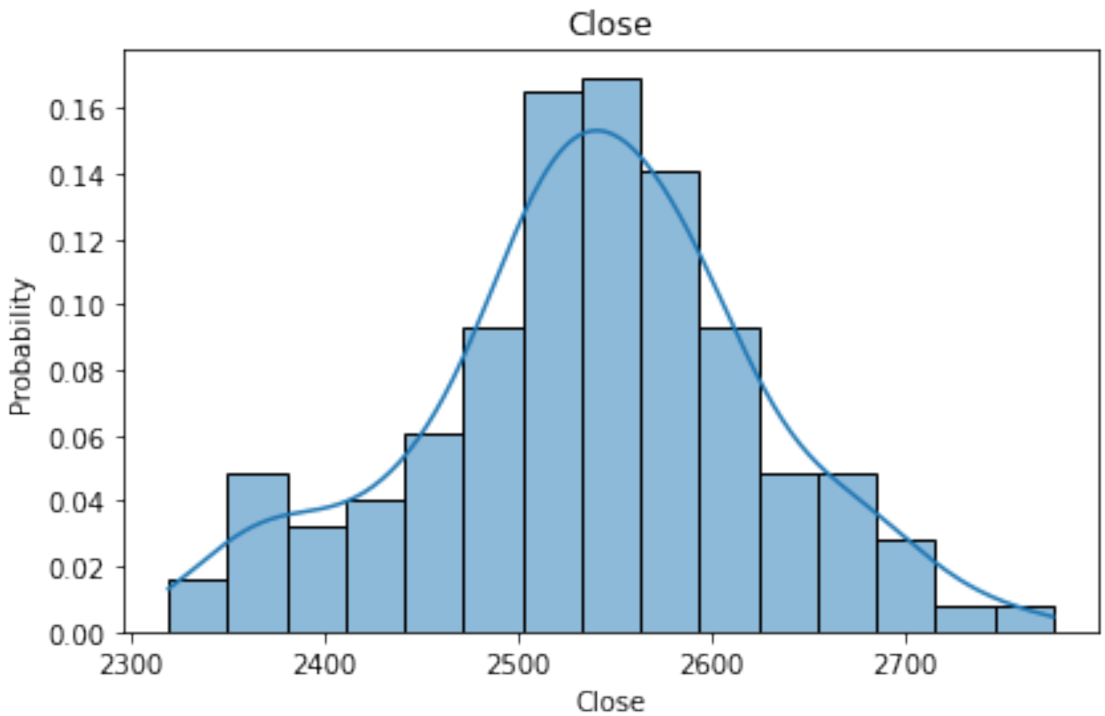
0.1 Step 1. You need to make a non-parametric estimation of PDF in form of histogram and using kernel density function for MRV (or probability law in case of discrete MRV)

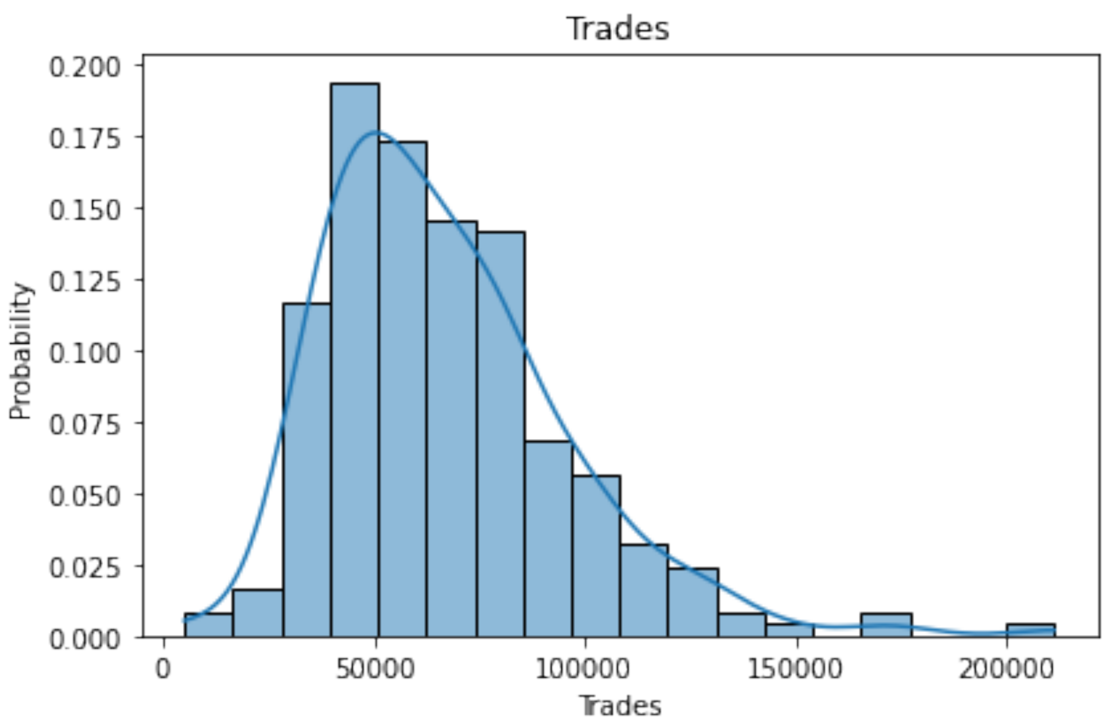
[5]:
```
for col in feature_cols:
    fig, ax = plt.subplots(tight_layout=True)
    sb.histplot(df[col], ax=ax, kde=True, stat="probability")
    plt.title(col)
    plt.show()
```

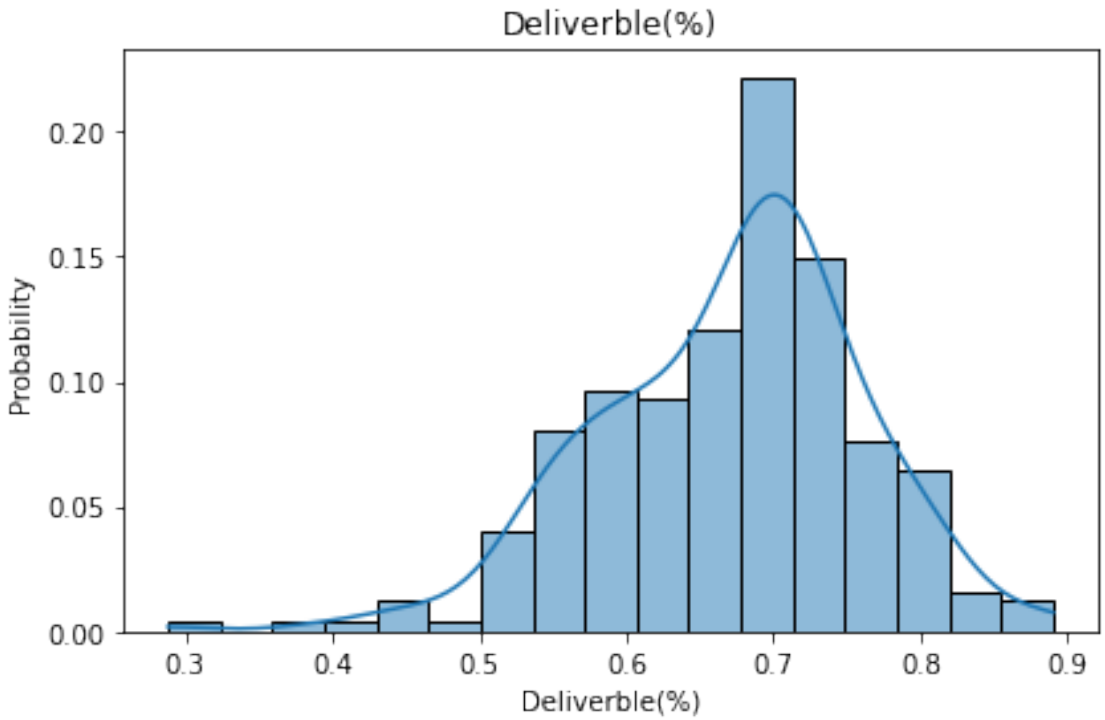



Low
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Probability
2300
2350
2400
2450
2500
2550
2600
2650
2700
Low


Last
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
Probability
2300
2400
2500
2600
2700
2800
Last


Close
Probability
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
2300
2400
2500
2600
2700
Close


Trades
Probability
0.000
0.025
0.050
0.075
0.100
0.125
0.150
0.175
0.200
0
50000
100000
150000
200000
Trades



0.2 Step 2. You need to make an estimation of multivariate mathematical expectation and variance.

[6]:
```
df.mean()
```

[6]:
```
Open               2542.172782
Low                2514.408468
Last               2538.039718
Close              2537.717944
Trades            66873.608871
Deliverble(%)         0.670336
High               2563.580444
dtype: float64
```

[7]:
```
df.var()
```

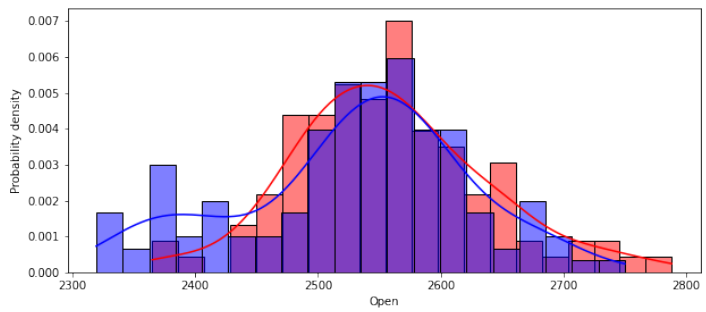
[7]:
```
Open             7.674758e+03
Low              6.881163e+03
Last             7.542802e+03
Close            7.579063e+03
Trades           8.342223e+08
Deliverble(%)    8.275145e-03
High             8.208064e+03
```

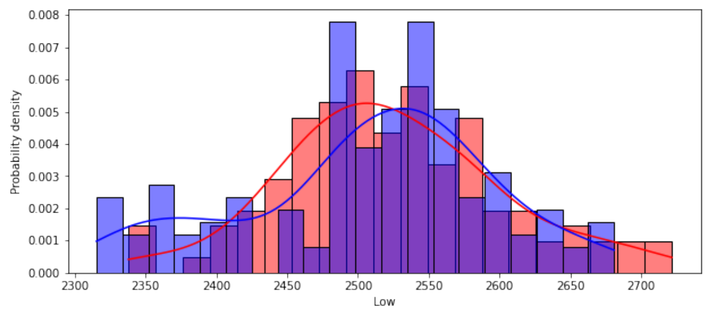
```
dtype: float64
```

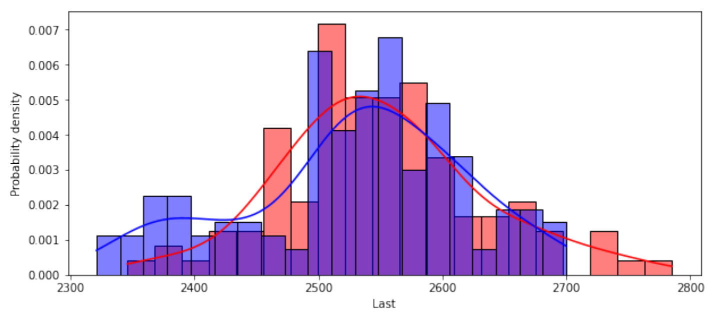
0.3 Step 3. You need to make a non-parametric estimation of conditional distributions, mathematical expectations and variances.

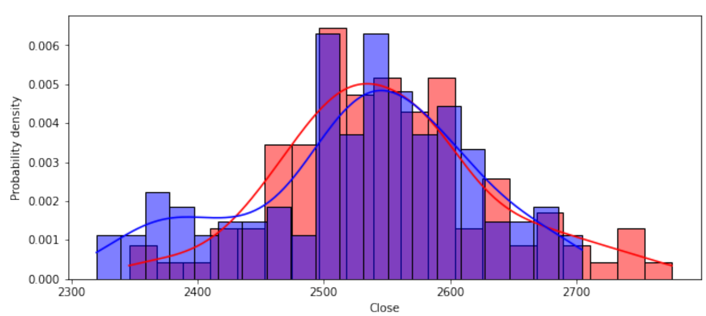
[8]:
```
condition_more = df['Trades'] >= 66000
condition_less = df['Trades'] < 66000
```

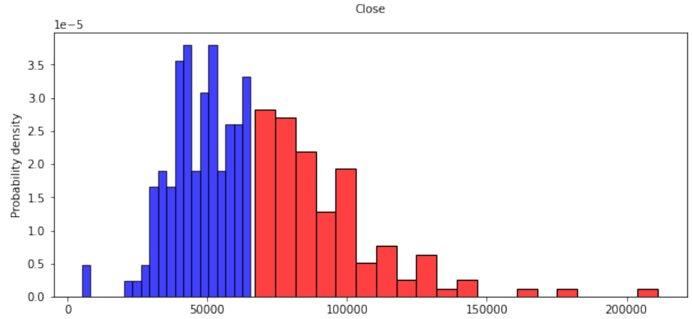
[9]:
```
fig, ax = plt.subplots(6, 1, figsize=(10,30))
for i in range(0,6):

    col=feature_cols[i]

    isKde=True

    if(col=="Trades"):
        isKde=False

    sb.
 ↪histplot(df[condition_more][col],stat="density",bins=20,common_norm=False,color="red",kde=i
    sb.
 ↪histplot(df[condition_less][col],stat="density",bins=20,common_norm=False,color="blue",kde=
    ax[i].set(ylabel="Probability density")

plt.show()
```

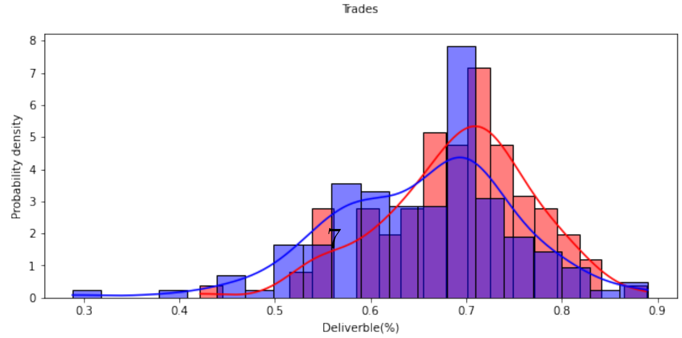

Probability density
Open


Probability density
Low


Probability density
Last


Probability density
Close


1e−5
Probability density
Trades


Probability density
Deliverble(%)

[10]:
```
df[condition_more].mean()
```

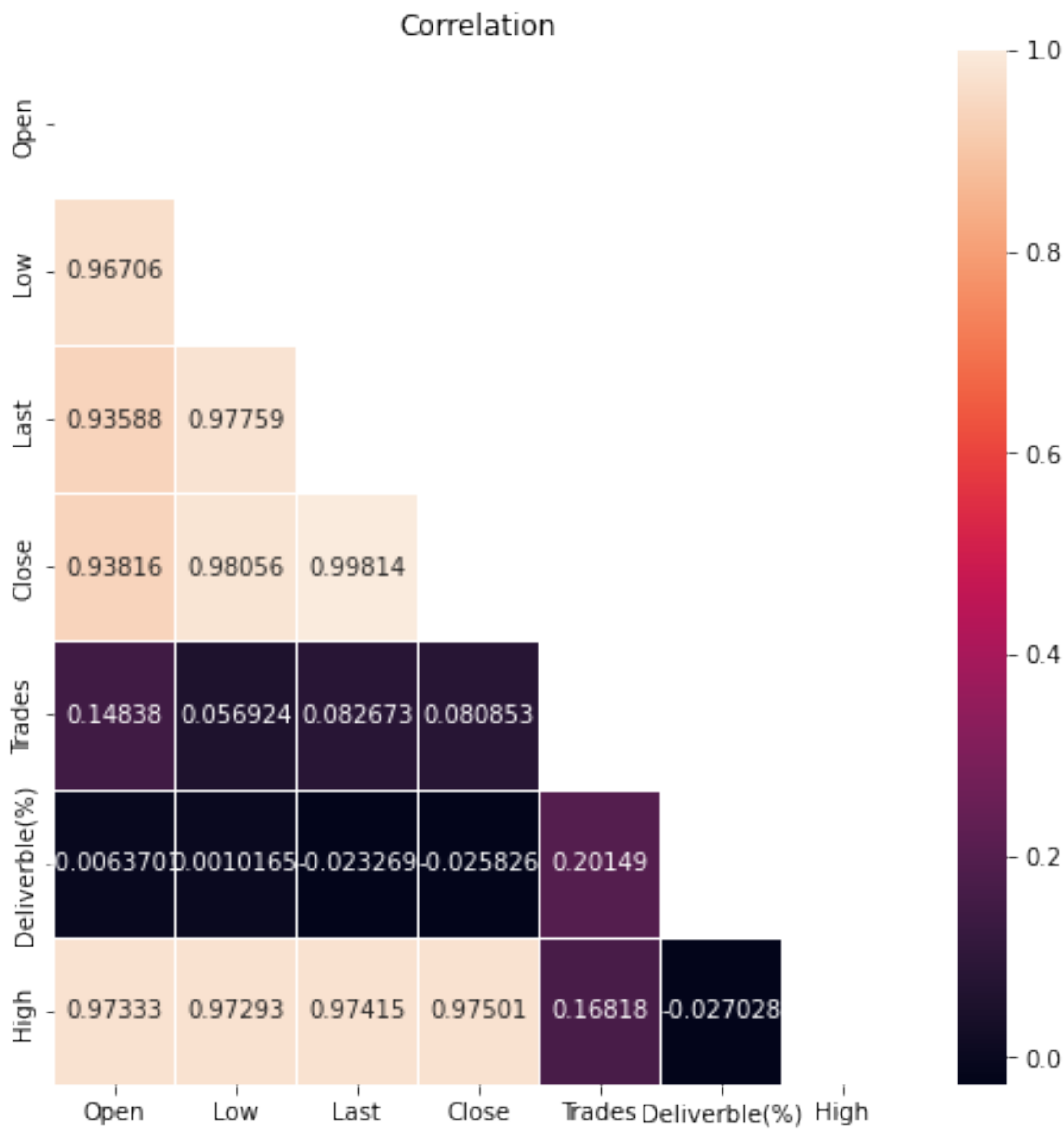
[10]:
```
Open              2558.012500
Low               2523.619444
Last              2550.018056
Close             2549.429630
Trades           92309.546296
Deliverble(%)        0.691087
High              2582.467130
dtype: float64
```

[11]:
```
df[condition_more].var()
```

[11]:
```
Open             6.191144e+03
Low              5.887515e+03
Last             6.700395e+03
Close            6.871748e+03
Trades           5.982571e+08
Deliverble(%)    6.254171e-03
High             7.031163e+03
dtype: float64
```

0.4 Step 4. You need to make an estimation of pair correlation coefficients, confidence intervals for them and significance levels.

[12]:
```
fig, ax = plt.subplots(figsize=(8,8))

corr = df.corr()
mask = np.triu(np.ones_like(corr, dtype=bool))
sb.heatmap(corr, mask=mask, annot=True, ax=ax, vmax=1, fmt='.5g', linewidths=.5)

plt.title('Correlation')
plt.show()
```



[13]:
```
def _estimate_correlation(x, y):
    return st.pearsonr(x, y)

def _estimate_confidence_intervals(cor, x, y, alpha = 0.05):
    coeff = np.arctanh(cor)

    std = 1/np.sqrt(x.size-3)
    z = st.norm.ppf(1-alpha/2)
    return coeff-z*std, coeff+z*std
```

[14]:
```
tab = [["Pair", "Correlation coeff", "p-value", "Low of conf int", "High of␣
↪conf int"]]
df_cls = ['High', 'Open', 'Low', 'Last', 'Close', 'Trades', 'Deliverble(%)']
for i, x_col_name in enumerate(df_cls):
    j = i + 1

    if j >= len(df_cls):
        break

    for k in range(j, len(df_cls)):
        y_col_name = df_cls[k]
        x = df[x_col_name]
        y = df[y_col_name]

        cor, p = _estimate_correlation(x, y)

        low, high = _estimate_confidence_intervals(cor, x, y)
        tab.append([f'{x_col_name} - {y_col_name}',round(cor,3),round(p,␣
↪3),round(low,3),round(high,3)])
print(tabulate(tab, headers="firstrow", tablefmt="grid"))
```

```
+------------------------+---------------------+-----------+-------------------+
--------------------+
| Pair                   |   Correlation coeff |   p-value |   Low of conf int |
High of conf int |
+========================+=====================+===========+===================+
====================+
| High - Open            |               0.973 |     0     |             2.027 |
2.277 |
+------------------------+---------------------+-----------+-------------------+
--------------------+
| High - Low             |               0.973 |     0     |             2.019 |
2.27  |
+------------------------+---------------------+-----------+-------------------+
--------------------+
| High - Last            |               0.974 |     0     |             2.043 |
2.293 |
+------------------------+---------------------+-----------+-------------------+
--------------------+
| High - Close           |               0.975 |     0     |             2.06  |
2.31  |
+------------------------+---------------------+-----------+-------------------+
--------------------+
| High - Trades          |               0.168 |     0.008 |             0.045 |
0.295 |
+------------------------+---------------------+-----------+-------------------+
--------------------+
```

High - Deliverble(%)	-0.027	0.672	-0.152	0.098
Open - Low	0.967	0	1.92	2.17
Open - Last	0.936	0	1.579	1.829
Open - Close	0.938	0	1.597	1.848
Open - Trades	0.148	0.019	0.024	0.275
Open - Deliverble(%)	-0.006	0.92	-0.132	0.119
Low - Last	0.978	0	2.115	2.365
Low - Close	0.981	0	2.187	2.437
Low - Trades	0.057	0.372	-0.068	0.182
Low - Deliverble(%)	0.001	0.987	-0.124	0.126
Last - Close	0.998	0	3.364	3.614
Last - Trades	0.083	0.194	-0.042	0.208

```
| Last - Deliverble(%)    |                  -0.023 |     0.715 |                 -0.148 |
0.102 |
+------------------------+---------------------+-----------+-------------------+
--------------------+
| Close - Trades          |                   0.081 |     0.204 |                 -0.044 |
0.206 |
+------------------------+---------------------+-----------+-------------------+
--------------------+
| Close - Deliverble(%)   |                  -0.026 |     0.686 |                 -0.151 |
0.099 |
+------------------------+---------------------+-----------+-------------------+
--------------------+
| Trades - Deliverble(%)  |                   0.201 |     0.001 |                  0.079 |
0.329 |
+------------------------+---------------------+-----------+-------------------+
--------------------+
```

We have each pair of variables and their Pearson correlation in coefficients with corresponding p-values and confidence intervals. The null hypothesis for the correlation test is that the variables have a zero correlation coefficient. An alternative hypothesis is a nonzero value. If the p value is less than the significance level ($= 0.05$), then the correlation coefficient differs significantly from zero, and if it is greater than the significance level, we do not reject the null hypothesis, and the correlation coefficient does not differ significantly from zero.

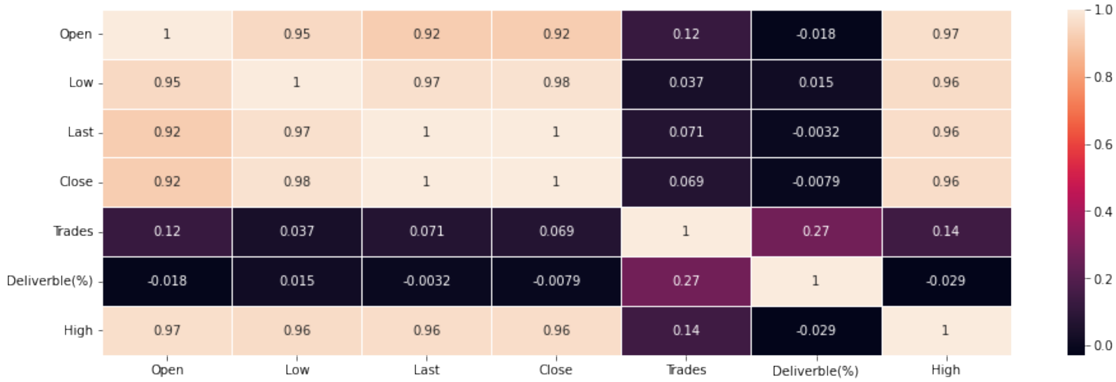
Thus, we can guarantee the dependence of the High ',' Open ',' Low ',' Last ',' Close 'values among themselves. The 'Trades', 'Deliverble (%)' values are interdependent, but independent with respect to the rest of the selected features.

0.5 Step 5. Choose a task formulation for regression. Estimate multivariate correlation (target -predictors).

Let's pose a regression problem. We have that Trades and Deliverble (%) practically do not depend on other parameters and it would be inappropriate to use them as a target. Another option: we can take one of the dependent features and build a regression for it. Yes, in this case, the accuracy of the model decreases, but this is better than predicting the behavior of independent variables.

Let's set the task: Predict the behavior of the High feature based on the rest presented.

[15]:
```
plt.figure(figsize=(16, 5))
sb.heatmap(df.corr(method='spearman'), annot=True, linewidths=.5)
plt.show()
```



Use PCA method and see how many variables we need to take for regression.

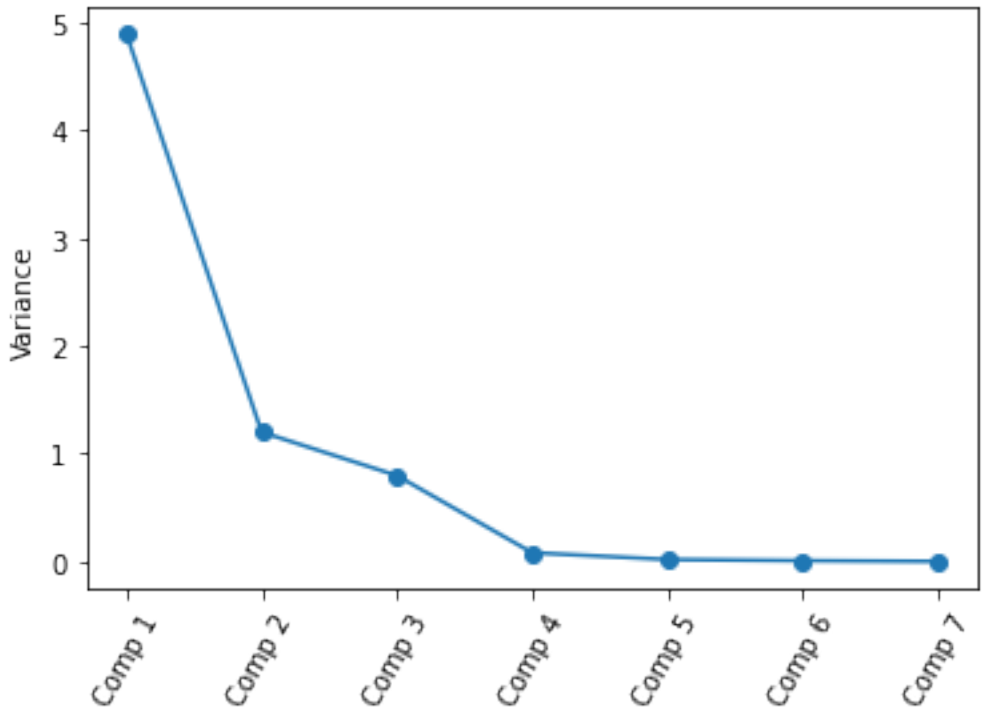
[16]:
```
#Standardize a dataset
std_df = scale(df)
std_df = pd.DataFrame(std_df, index=df.index, columns=df.columns)
```

[17]:
```
std_df.head()
```

[17]:

	Open	Low	Last	Close	Trades	Deliverble(%)	High
0	0.283970	0.321211	0.137992	0.090146	-2.042407	-4.208179	0.037820
1	0.100965	0.437173	0.581031	0.480330	-1.363023	-0.022430	0.302709
2	0.444101	0.123712	0.000696	0.029144	-0.820119	-1.648263	0.401696
3	-0.149525	-0.898812	-1.015180	-1.048754	0.611609	-0.891524	-0.381355
4	-0.825503	-1.291997	-1.282272	-1.381388	1.209639	0.022732	-0.933805

[18]:
```
pca = PCA().fit(std_df)
y = np.std(pca.transform(std_df), axis=0)**2
x = np.arange(len(y)) + 1
plt.plot(x, y, "o-")
plt.xticks(x, [f"Comp {i+1}" for i in range(df.columns.size)], rotation=60)
plt.ylabel("Variance")
plt.show()
```



This graph shows that the first 3 features have a high value of the explained variance. By the 4th feature, this value goes close to 3. So I consider the use of the first three features for regression reasonable.

0.6 Step 6. Build regression model and make an analysis of multicollinearity and regularization (if needed).

[19]:
```
train, test = train_test_split(df[['High', 'Open', 'Close', 'Low']].copy())

train_feature = train[['Open', 'Close', 'Low']]
test_feature = test[['Open', 'Close', 'Low']]

train_target = train['High']
test_target = test['High']
```

[20]:
```
tab = [["Type", "MSE", "R2", "Coeff"]]

lin_regression = lm.LinearRegression()
lin_regression.fit(train_feature, train_target)
predicted = lin_regression.predict(test_feature)
tab.append(["Least Squares model", str(mean_squared_error(test_target,␣
 ↪predicted)), str(r2_score(test_target, predicted)), str(lin_regression.
 ↪coef_)])
```

```
print(tabulate(tab, headers= "firstrow", tablefmt="grid"))
```

```
+---------------------+---------+----------+------------------------------------
---+
| Type                |     MSE |       R2 | Coeff
|
+=====================+=========+==========+====================================
===+
| Least Squares model | 239.105 | 0.971949 | [ 0.64429794  0.73402532
-0.35690893] |
+---------------------+---------+----------+------------------------------------
---+
```

[44]:
```
R2 = r2_score(test_target,predicted)
adj_r2 = 1 - (1-R2)*(df.size-1)/(df.size-1-1)
print(f"Adj R^2: {round(adj_r2,3)}")

subsample_X=df[['Open', 'Close', 'Low']]
vif_data=pd.DataFrame()
vif_data["features"]=subsample_X.columns
vif_data["VIF"]=[variance_inflation_factor(subsample_X.values, i) for i in␣
↪range(len(subsample_X.columns))]

display(vif_data)
```

```
Adj R^2: 0.939
```

	features	VIF
0	Open	13575.748555
1	Close	22872.331108
2	Low	43408.655006

VIF test shows that we have multicollinearity in a set of multiple regression variables. Multicollinearity produce estimates of the regression coefficients that are not statistically significant. That's why we will take only one regression variable and rebuild model.

[22]:
```
train_feature = train[['Open']]
test_feature = test[['Open']]

train_target = train['High']
test_target = test['High']
```

[23]:
```
tab = [["Type", "MSE", "R2", "Coeff"]]

lin_regression = lm.LinearRegression()
lin_regression.fit(train_feature, train_target)
predicted = lin_regression.predict(test_feature)
```

```
tab.append(["Least Squares model", str(mean_squared_error(test_target,␣
↪predicted)), str(r2_score(test_target, predicted)), str(lin_regression.
↪coef_)])

print(tabulate(tab, headers= "firstrow", tablefmt="grid"))
```

```
+---------------------+---------+----------+-------------+
| Type                |     MSE |       R2 | Coeff       |
+=====================+=========+==========+=============+
| Least Squares model | 518.857 | 0.939129 | [0.9939338] |
+---------------------+---------+----------+-------------+
```

Due to high R^2 result we still have significant regression model.

0.7 Step 7. Analyze the quality of regression model (distribution of residuals, determination coefficient).

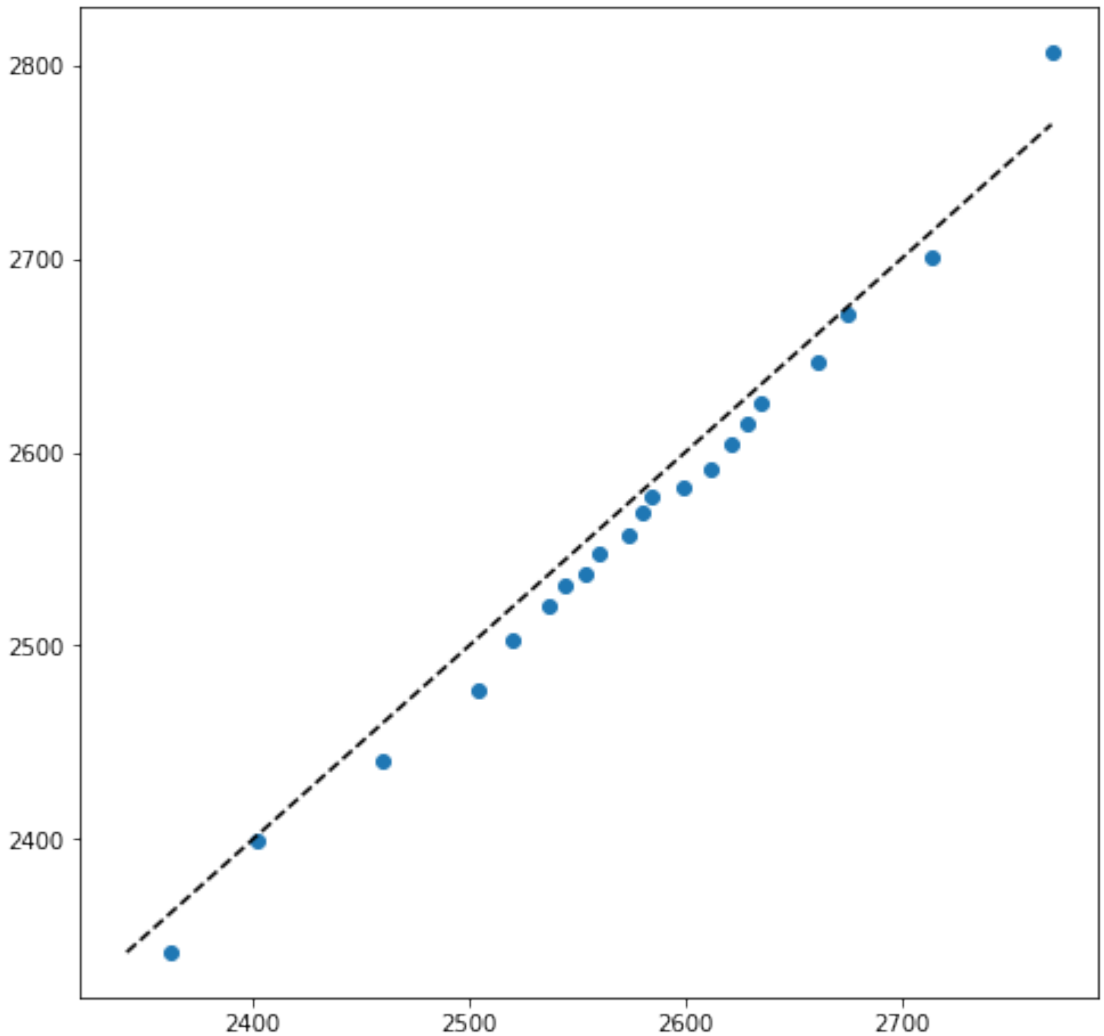
R^2 score can be find at the end of Step 6

[24]:
```
X = df[['Open']]
Y = df['High']
```

[31]:
```
def _draw_qq_plot(predicted, predicted_all):
    percs = np.linspace(0, 100, 21)
    qn_first = np.percentile(predicted, percs)
    qn_second = np.percentile(predicted_all, percs)

    plt.figure(figsize=(8,8))

    min_qn = np.min([qn_first.min(), qn_second.min()])
    max_qn = np.min([qn_first.max(), qn_second.max()])
    x = np.linspace(min_qn, max_qn)

    plt.plot(qn_first, qn_second, ls="", marker="o", markersize=6)
    plt.plot(x,x,color="k", ls="--")
    plt.show
```

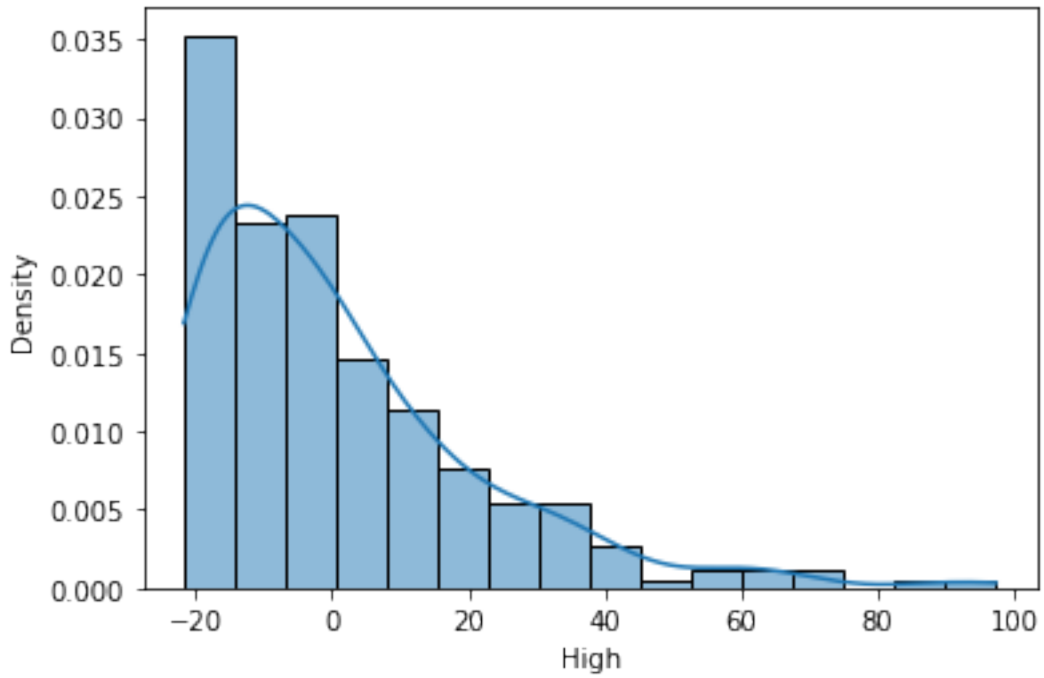
[45]:
```
regression = lm.LinearRegression()
regression.fit(train_feature, train_target)
predicted = regression.predict(test_feature)

print('R^2 score =', r2_score(test_target, predicted))

predicted_all = regression.predict(X)
_draw_qq_plot(predicted, predicted_all)
```

```
R^2 score = 0.9391285811668861
```



R^2 show a good quality of our model as a qqplot

[40]:
```
residuals = Y - regression.predict(X)
ax = sb.histplot(residuals, kde = 'True', stat="density")
```



[41]:
```
st.kstest(residuals, 'norm', args=(residuals.mean(), residuals.var()))
```

[41]: KstestResult(statistic=0.4794403465426097, pvalue=6.1324498635471015e-53)

Residuals are not distributed normally. We have built a regression model. It gives a similar result. But in order for us to guarantee the accuracy of the regression model, it is necessary that the residuals are normally distributed. Thus, the constructed model is not accurate. This happened due to the choice of features that are highly dependent on each other.

[29]:
```
mod = sm.OLS(train_target, train_feature)
res = mod.fit()
print(res.conf_int(0.01))
```

```
             0         1
Open  1.006641  1.009673
```

[30]:
```
residuals.describe()
```

[30]:
```
count    248.000000
mean       0.694914
std       20.814582
min      -21.642530
25%      -14.648954
50%       -4.144588
75%        9.349721
```

```
max        97.257732
Name: High, dtype: float64
```