Network structure and visualization

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Lecture outline

1. Network cores
   - k-core decomposition
2. Mixing in networks
   - Assortative mixing
3. Network visualization
   - Simple geometric layouts
   - Force-directed layouts
   - Low dimensional embeddings
   - Adjacency matrix and permutations
4. Visualization tools

Typical network structure

Core-periphery structure of a network

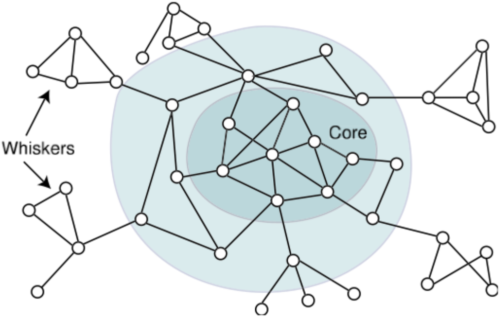
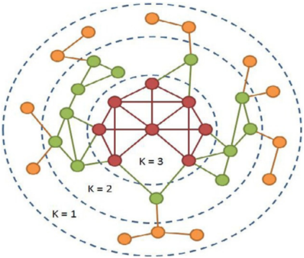


image from J. Leskovec, K. Lang, 2010

k-core decomposition

Definition

If from a given graph G = (V, E) recursively delete all vertices, and lines incident with them, of degree less than k, the remaining graph is the k-core.



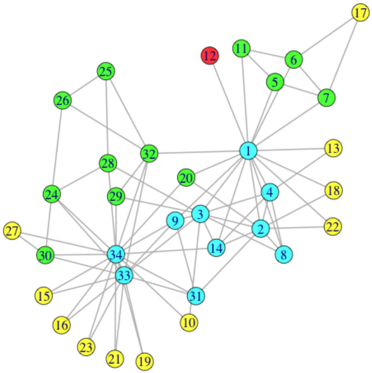
Every vertex in k-core has a degree $k_i \geq k$

$(k+1)$-core is always subgraph of k-core

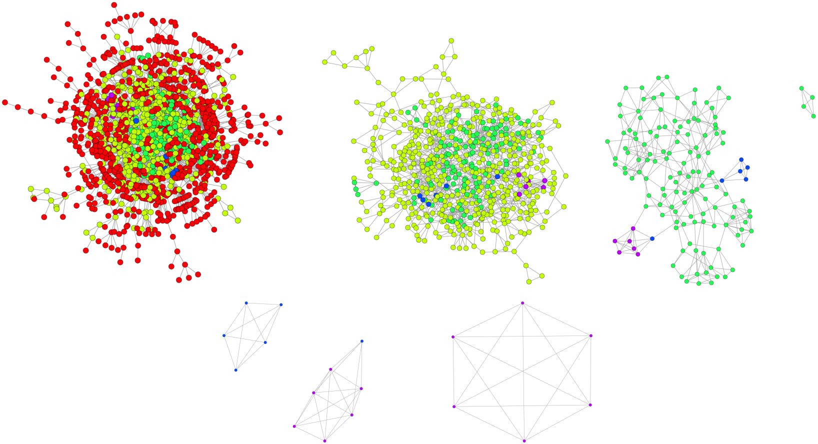
The core number of a vertex is the highest order of a core that contains this vertex

K-cores

Zachary karate club: 1,2,3,4 - cores



k-cores



k-cores: 1:1458, 2:594, 3:142, 4:12, 5:6

k-shells: 1:864-red, 2:452-pale green, 3:130-green, 5:6-blue, 6:6-purple

Mixing patterns

Network mixing patterns

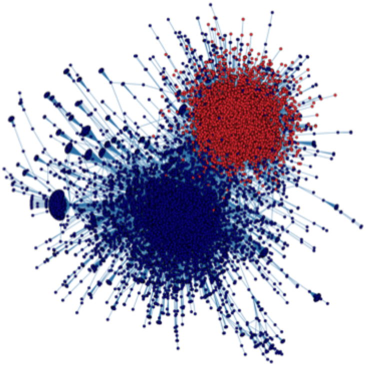
- **Assortative mixing**, "like links with like", attributed of connected nodes tend to be more similar than if there were no such edge
- **Disassortative mixing**, "like links with dislike", attributed of connected nodes tend to be less similar than if there were no such edge

Vertices can mix on any vertex attributes (age, sex, geography in social networks), unobserved attributes, vertex degrees

Examples:
assortative mixing - in social networks political beliefs, obesity, race
disassortative mixing - dating network, food web (predator/prey), economic networks (producers/consumers)

Assortative mixing

- Political polarization on Twitter: political retweet network ,red color - "right-learning" users, blue color - "left learning" users

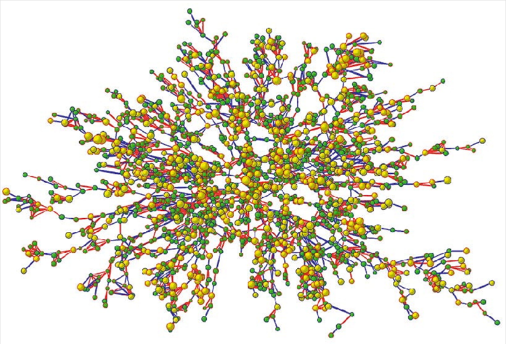


- Assortative mixing = homophily

Conover et al., 2011

Assortative mixing

- The Spread of Obesity in a Large Social Network over 32 Years



Node colors - person's obesity status: yellow denotes an obese person (body-mass index > 30) and green denotes a nonobese person.
Edge colors - relationship between them: purple denotes a friendship or marital tie and orange denotes a familial tie.

Christakis and Fowler, 2007

Assortativity measures

- **Discrete mixing** by categorical attribute (c_i -label: color, gender, ethnicity). How much more often do attributes match across edges than expected at random? Assortativity coefficient:

$$C = \frac{Q}{Q_{max}} = \frac{\sum_{ij}\left(A_{ij} - \frac{k_i k_j}{2m}\right)\delta(c_i, c_j)}{2m - \sum_{ij}\frac{k_i k_j}{2m}\delta(c_i, c_j)}$$

- **Mixing by scalar properties**, scalar value attribute (age, income, number of friends). Correlation of values across edges. Assortativity coefficient:

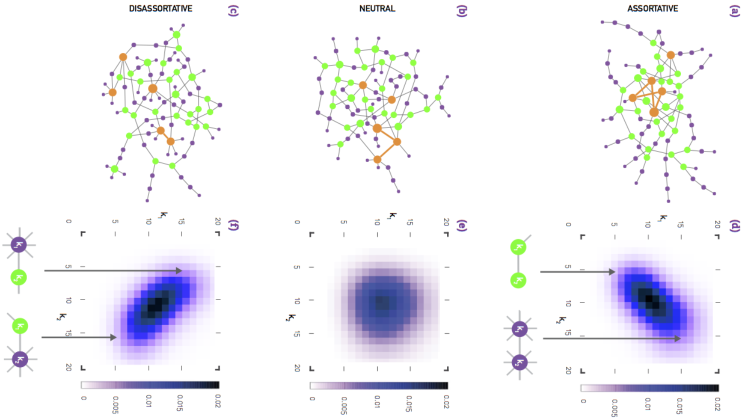
$$r = \frac{cov}{var} = \frac{\sum_{ij}\left(A_{ij} - \frac{k_i k_j}{2m}\right)x_i x_j}{\sum_{ij}\left(k_i\delta_{ij} - \frac{k_i k_j}{2m}\right)x_i x_j}$$

Mixing by node degree

- Assortative mixing by node degree, $x_i \leftarrow k_i - 1$

$$r = \frac{\sum_{ij}\left(A_{ij} - \frac{k_i k_j}{2m}\right) k_i k_j}{\sum_{ij}\left(k_i \delta_{ij} - \frac{k_i k_j}{2m}\right) k_i k_j}$$

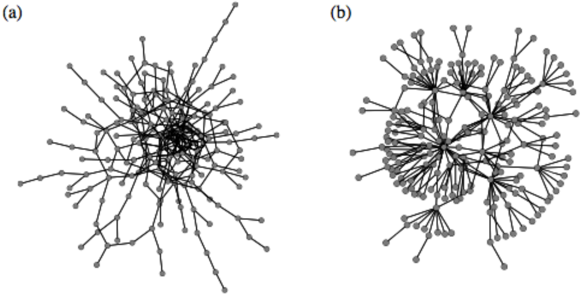
Mixing by node degree



from A-L. Barabasi

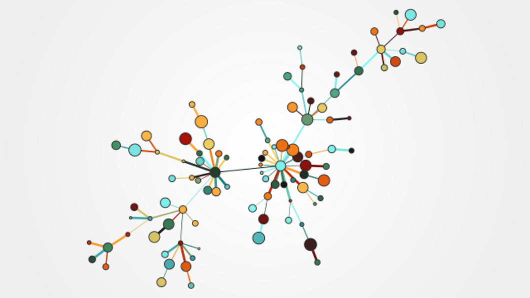
Mixing by node degree

- Assortative network: interconnected high degree nodes - core, low degree nodes - periphery
- Disassortative network: high degree nodes connected to low degree nodes, star-like structure



Assortative network Disassortative network

Network visualization



Network:

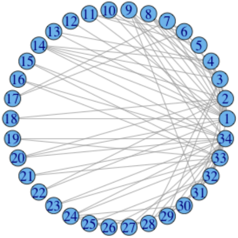
nodes (+node attributes)

edges (+edge attributes)

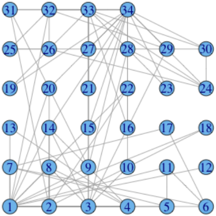
Graph (network) layout:

nodes coordinates (x,y)

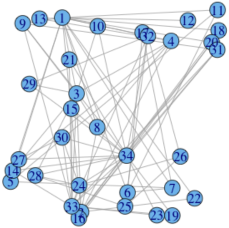
Simple graph layouts



Circular layout

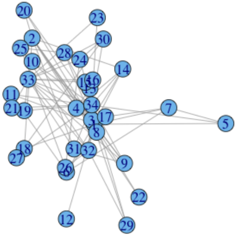


Grid layout

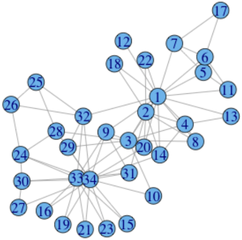


Random layout

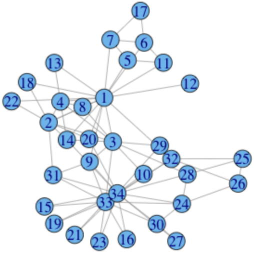
Force-directed layouts



Spring based

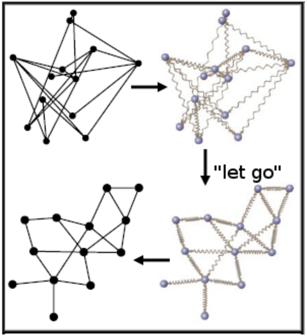


Kamada-Kawai



Frutcherman-Reingold

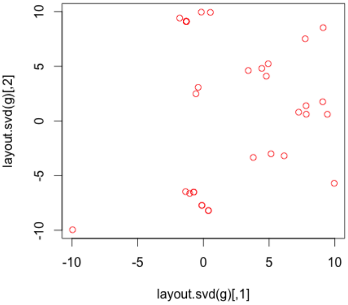
Force-directed layouts

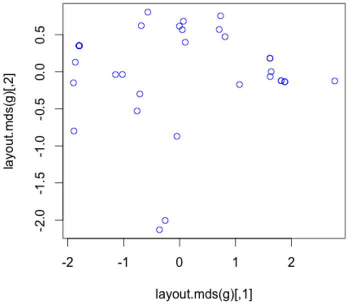


Kamada-Kawai model (stress minimization)

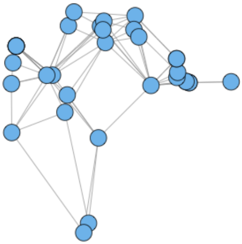
$$stress(X) = \sum_{i<j} w_{ij}(||X_i - X_j|| - d_{ij})^2$$

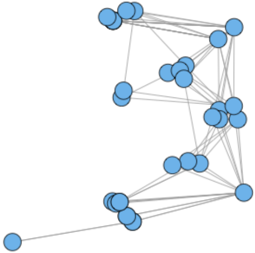
Low dimensional embeddings





SVD (PCA) MDS multidimensional scaling

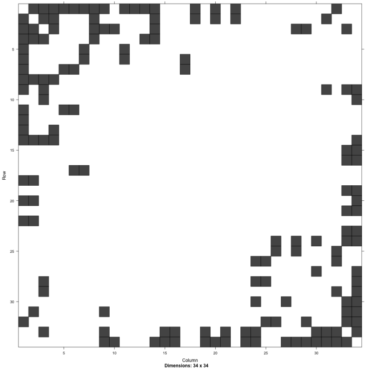


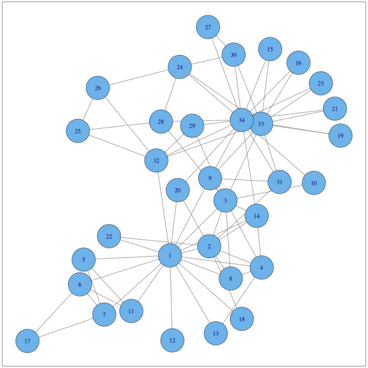


SVD (PCA)

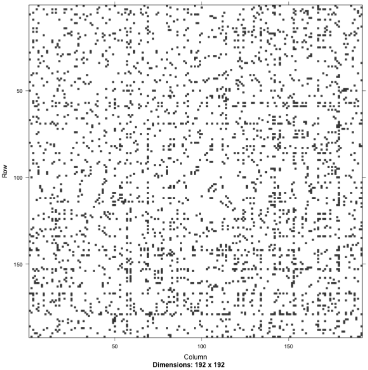
MDS multidimensional scaling

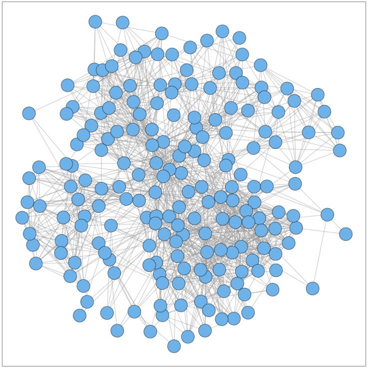
Sparse matrix visualization



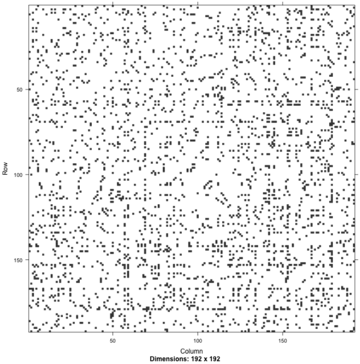


Sparse matrix visualization





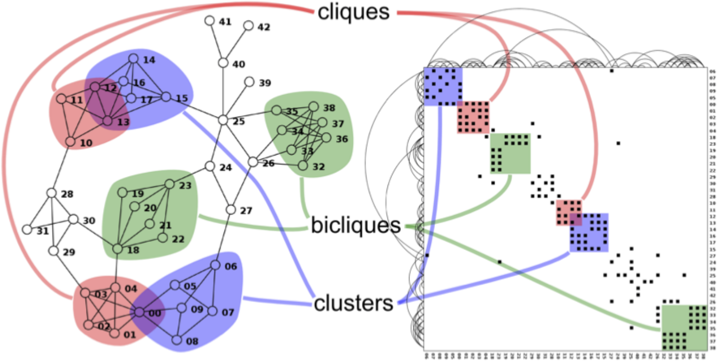
Sparse matrix visualization



Matrix permutations (bandwidth reduction)

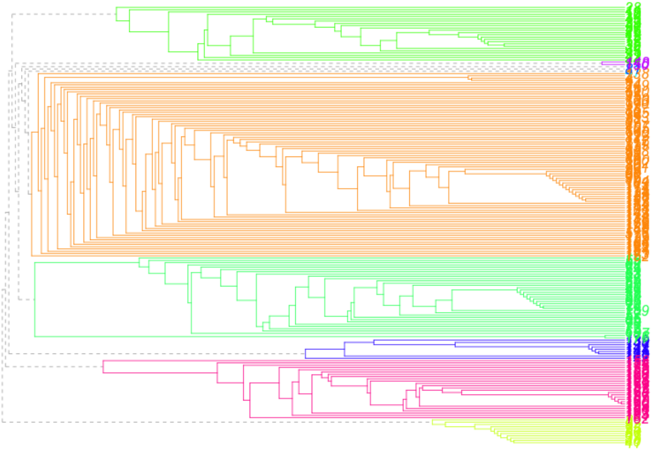
- Minimum degree ordering
- Reverse Cuthill-McKee ordering

Sparse matrix visualization

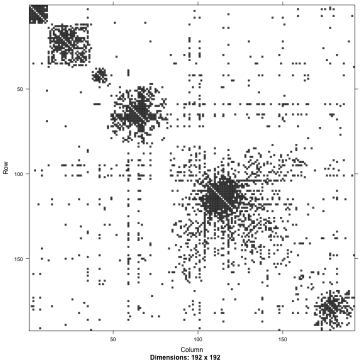


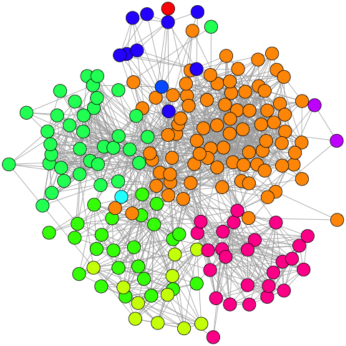
from M.J. McGuffin

Hierarchical clustering

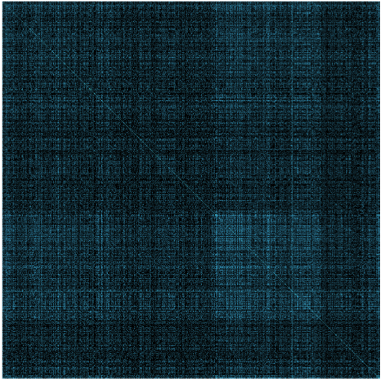


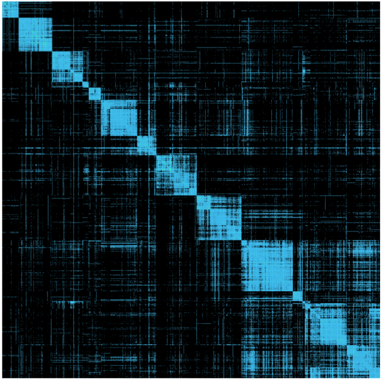
Sparse matrix visualization





Sparse matrix visualization





Visualization tools

- Graphviz (http://www.graphviz.org)
- Gephi (http://www.gephi.org)
- yEd (http://www.yworks.com)
- Visone (http://www.visone.net)
- Pajek (http://pajek.imfm.si)

yEd

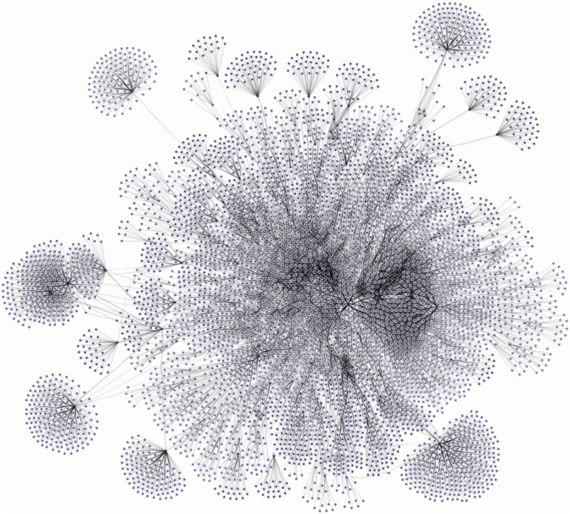
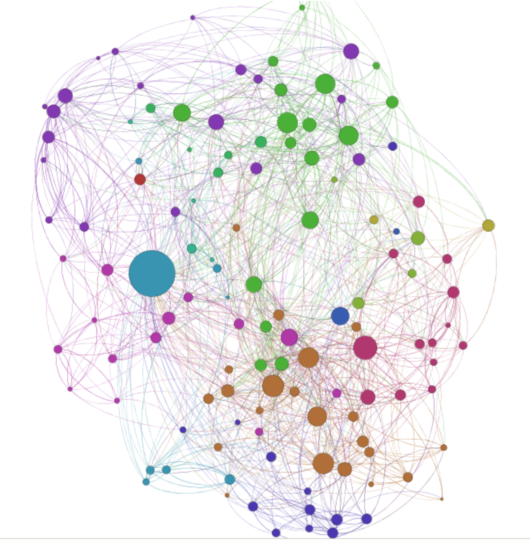


image from www.yworks.com

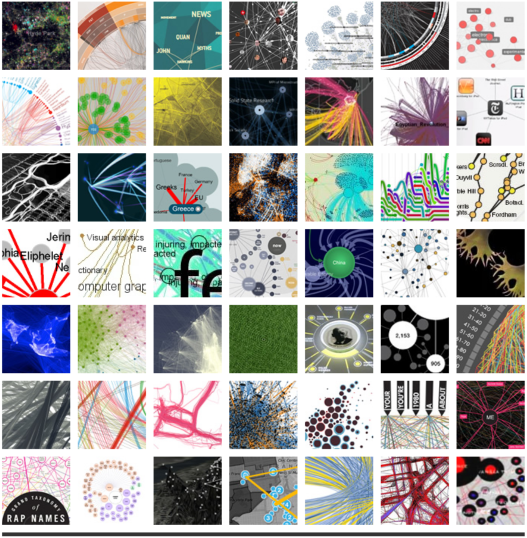
Gephi

Leucippus
Socrates
Pyrrho
Pythagoras
Democritus
Michel de Montaigne
Karl Marx
Thales
Parmenides
Zeno of Elea
Sextus Empiricus
Plato
Niccolò Machiavelli
Nicholas of Cusa
René Descartes
Jean-Jacques Rousseau
Hugo Grotius
Heraclitus
Georg Wilhelm Friedrich Hegel
Aristotle
Baruch Spinoza
Plotinus
Montesquieu
Edmund Husserl
Noam Chomsky
David Hume
Immanuel Kant
Franz Brentano
Friedrich Adolf Trendelenburg

Gephi



Visual complexity



http://www.visualcomplexity.com