

# Network structure and visualization



NATIONAL RESEARCH  
UNIVERSITY

# Lecture outline

## 1 Network cores

- k-core decomposition

## 2 Mixing in networks

- Assortative mixing

## 3 Network visualization

- Simple geometric layouts
- Force-directed layouts
- Low dimensional embeddings
- Adjacency matrix and permutations

## 4 Visualization tools

# Typical network structure

Core-periphery structure of a network

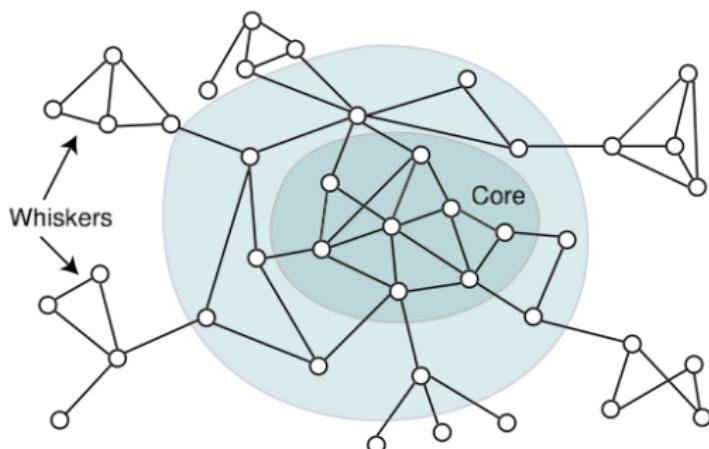
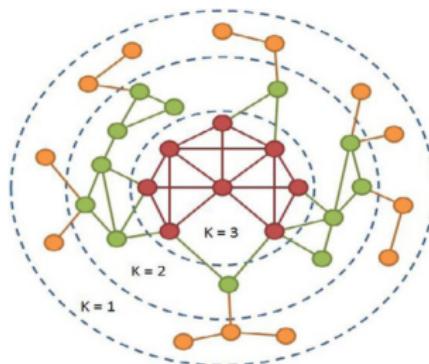


image from J. Leskovec, K. Lang, 2010

# k-core decomposition

## Definition

If from a given graph  $G = (V, E)$  recursively delete all vertices, and lines incident with them, of degree less than  $k$ , the remaining graph is the  $k$ -core.



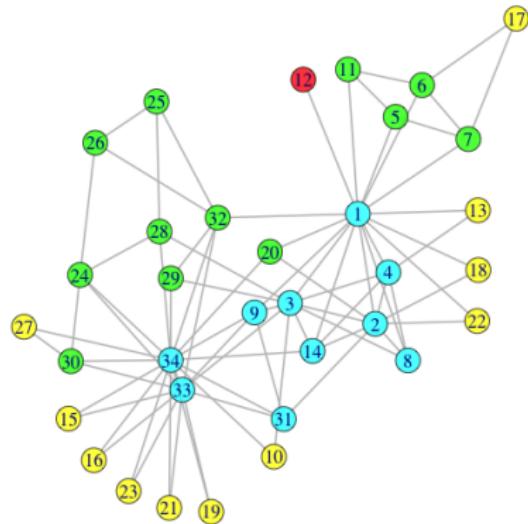
Every vertex in  $k$ -core has a degree  $k_i \geq k$

$(k + 1)$ -core is always subgraph of  $k$ -core

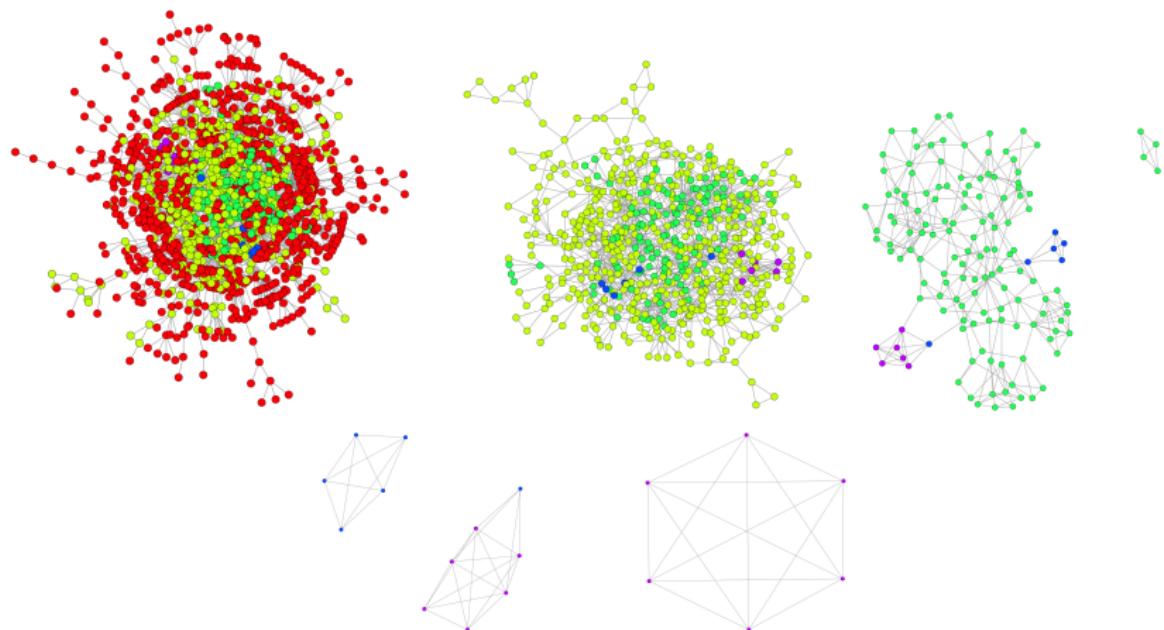
The core number of a vertex is the highest order of a core that contains this vertex

# K-cores

Zachary karate club: 1,2,3,4 - cores



## k-cores



k-cores: 1:1458, 2:594, 3:142, 4:12, 5:6

k-shells: 1:864-red, 2:452-pale green, 3:130-green, 4:6-blue, 5:6-purple

# Mixing patterns

## Network mixing patterns

- **Assortative mixing**, "like links with like", attributed of connected nodes tend to be more similar than if there were no such edge
- **Disassortative mixing**, "like links with dislike", attributed of connected nodes tend to be less similar than if there were no such edge

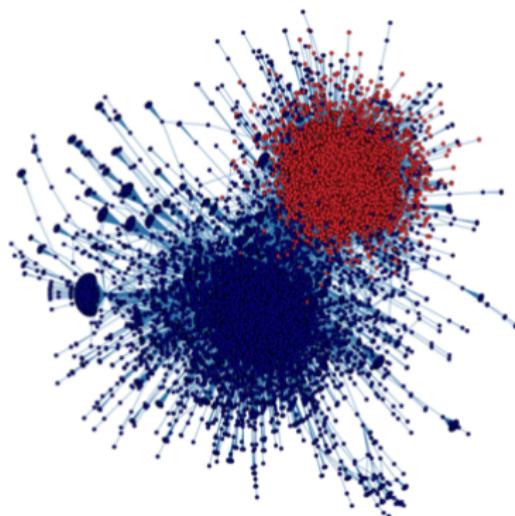
Vertices can mix on any vertex attributes (age, sex, geography in social networks), unobserved attributes, vertex degrees

### Examples:

assortative mixing - in social networks political beliefs, obesity, race  
disassortative mixing - dating network, food web (predator/prey),  
economic networks (producers/consumers)

# Assortative mixing

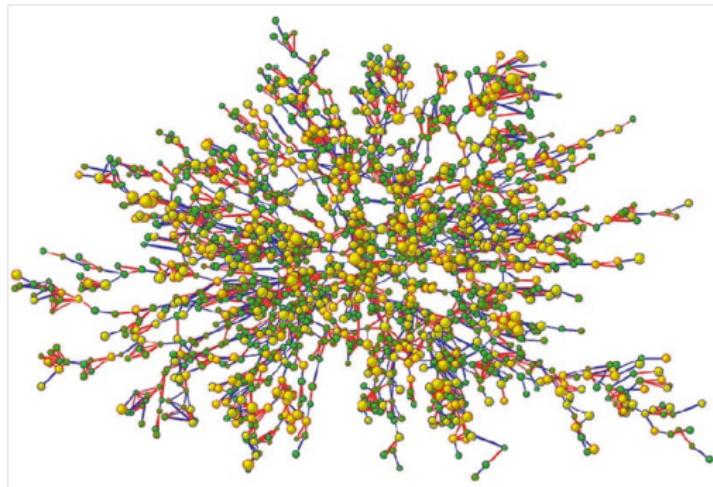
- Political polarization on Twitter: political retweet network , red color - "right-learning" users, blue color - "left learning" users



- Assortative mixing = homophily

# Assortative mixing

- The Spread of Obesity in a Large Social Network over 32 Years



Node colors - person's obesity status: yellow denotes an obese person (body-mass index  $> 30$ ) and green denotes a nonobese person.

Edge colors - relationship between them: purple denotes a friendship or marital tie and orange denotes a familial tie.

# Assortativity measures

- **Discrete mixing** by categorical attribute ( $c_i$  -label: color, gender, ethnicity). How much more often do attributes match across edges than expected at random? Assortativity coefficient:

$$C = \frac{Q}{Q_{max}} = \frac{\sum_{ij} \left( A_{ij} - \frac{k_i k_j}{2m} \right) \delta(c_i, c_j)}{2m - \sum_{ij} \frac{k_i k_j}{2m} \delta(c_i, c_j)}$$

- **Mixing by scalar properties**, scalar value attribute (age, income, number of friends). Correlation of values across edges. Assortativity coefficient:

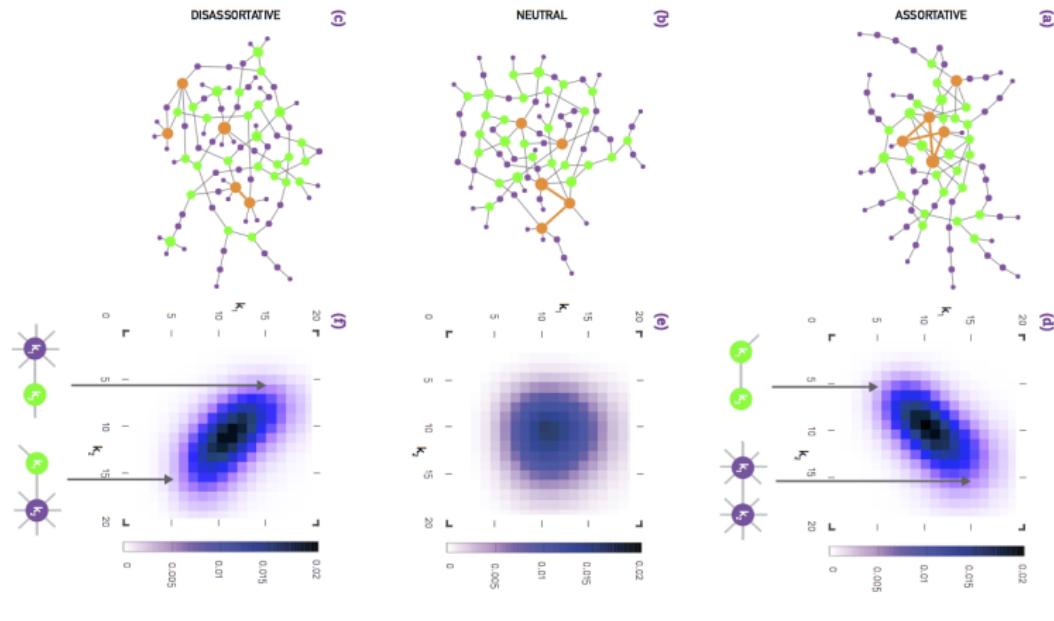
$$r = \frac{cov}{var} = \frac{\sum_{ij} \left( A_{ij} - \frac{k_i k_j}{2m} \right) x_i x_j}{\sum_{ij} \left( k_i \delta_{ij} - \frac{k_i k_j}{2m} \right) x_i x_j}$$

## Mixing by node degree

- Assortative mixing by node degree,  $x_i \leftarrow k_i - 1$

$$r = \frac{\sum_{ij} \left( A_{ij} - \frac{k_i k_j}{2m} \right) k_i k_j}{\sum_{ij} \left( k_i \delta_{ij} - \frac{k_i k_j}{2m} \right) k_i k_j}$$

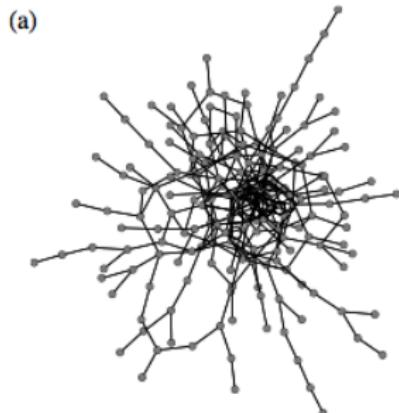
# Mixing by node degree



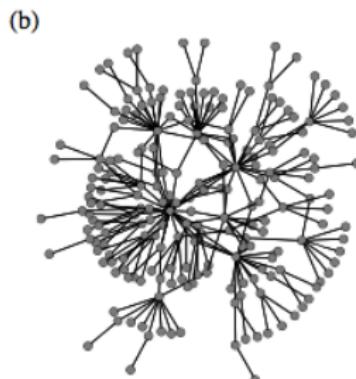
from A-L. Barabasi

# Mixing by node degree

- Assortative network: interconnected high degree nodes - core, low degree nodes - periphery
- Disassortative network: high degree nodes connected to low degree nodes, star-like structure



Assortative network



Disassortative network

# Network visualization



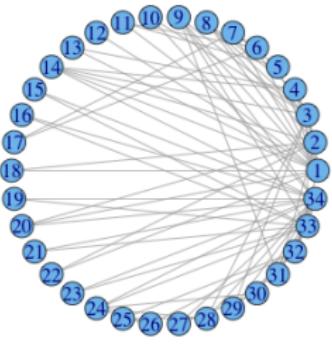
Network:

nodes (+node attributes)  
edges (+edge attributes)

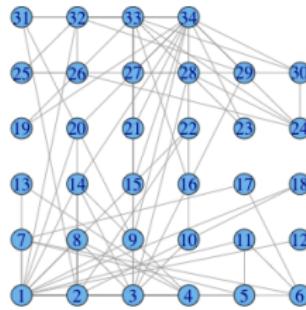
Graph (network) layout:

nodes coordinates (x,y)

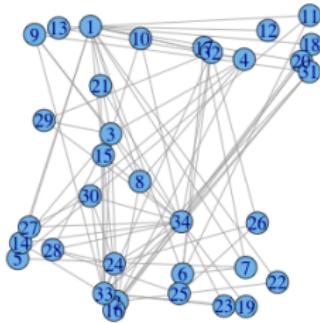
# Simple graph layouts



Circular layout

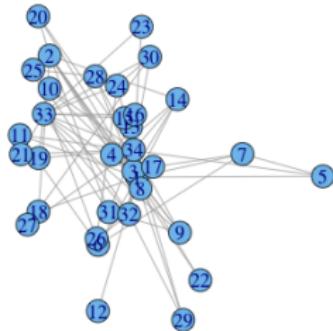


Grid layout

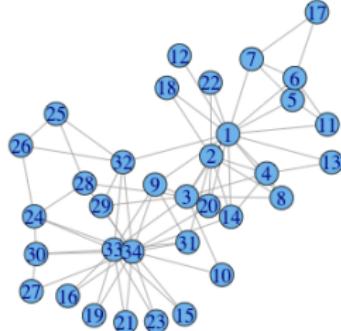


Random layout

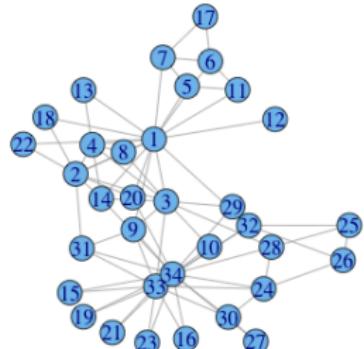
# Force-directed layouts



Spring based

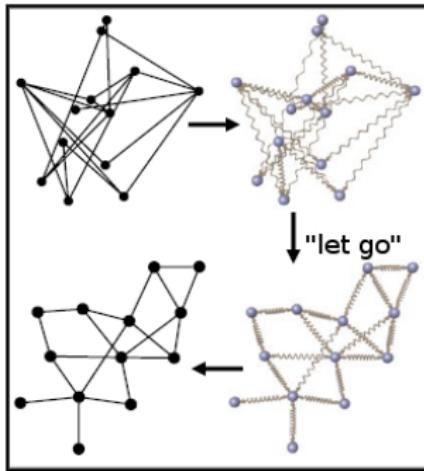


Kamada-Kawai



Fruchterman-Reingold

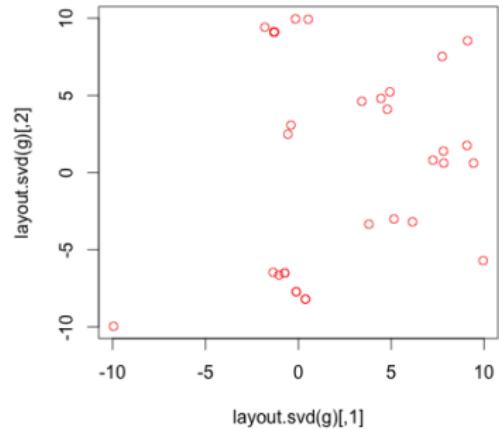
# Force-directed layouts



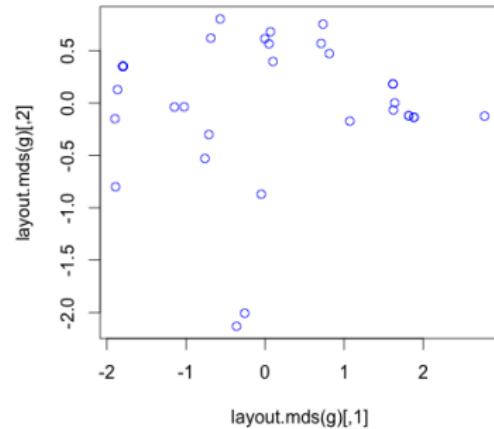
Kamada-Kawai model (stress minimization)

$$\text{stress}(X) = \sum_{i < j} w_{ij} (||X_i - X_j|| - d_{ij})^2$$

# Low dimensional embeddings



SVD (PCA)

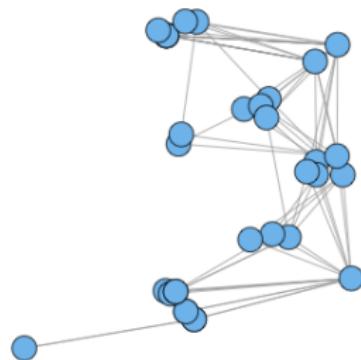


MDS multidimensional scaling

# Low dimensional embeddings

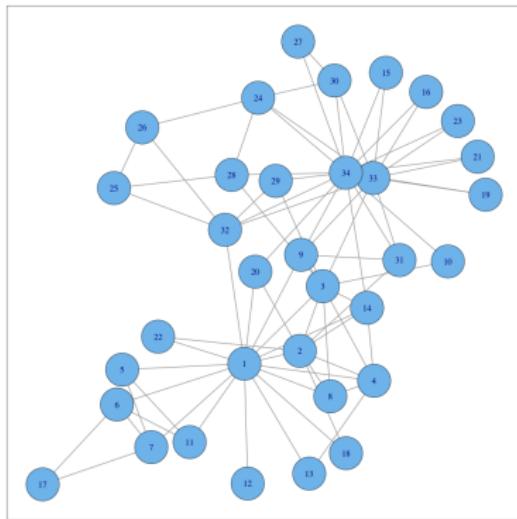
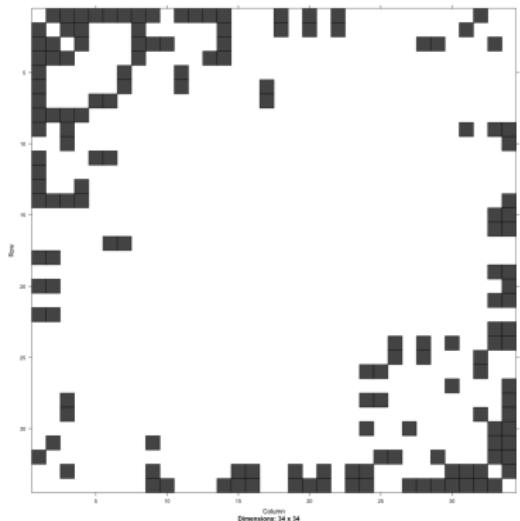


SVD (PCA)

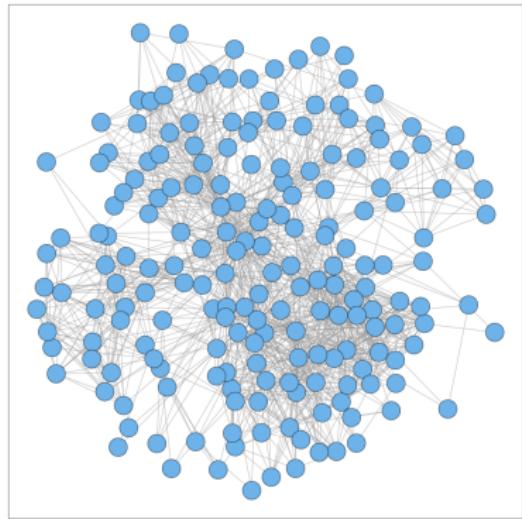
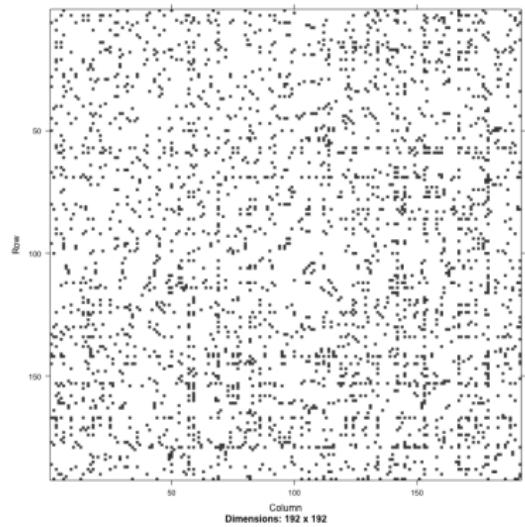


MDS multidimensional scaling

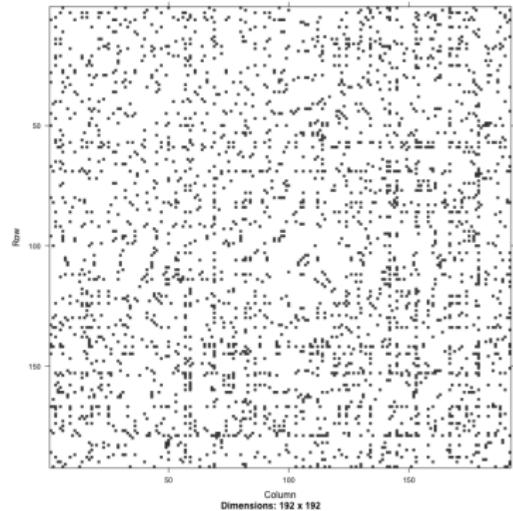
# Sparse matrix visualization



# Sparse matrix visualization



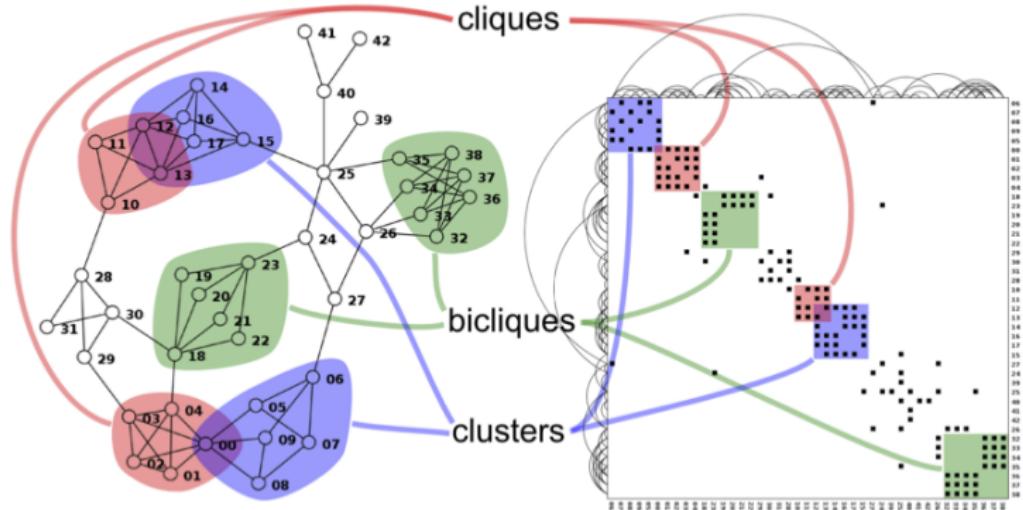
# Sparse matrix visualization



## Matrix permutations (bandwidth reduction)

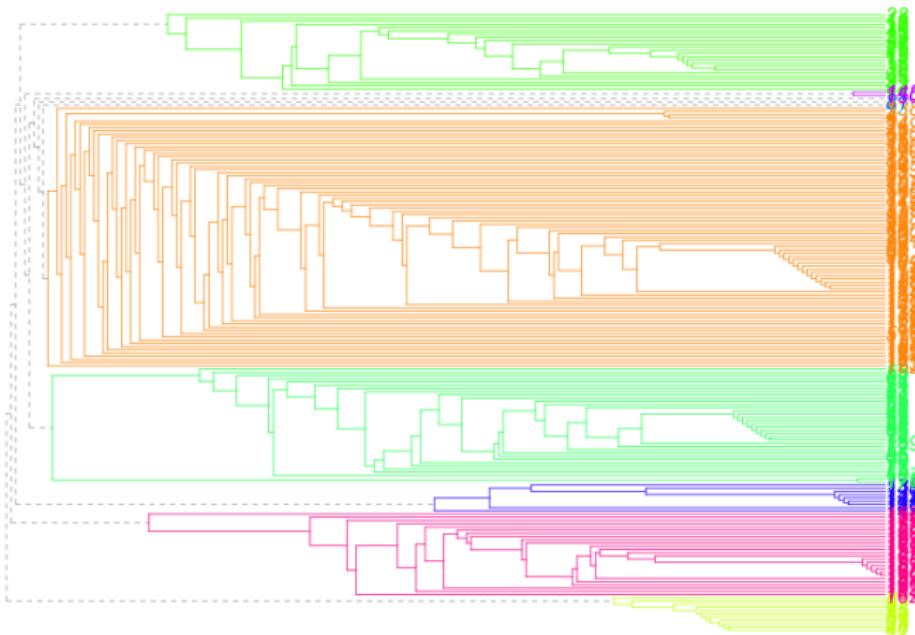
- Minimum degree ordering
- Reverse Cuthill-McKee ordering

# Sparse matrix visualization

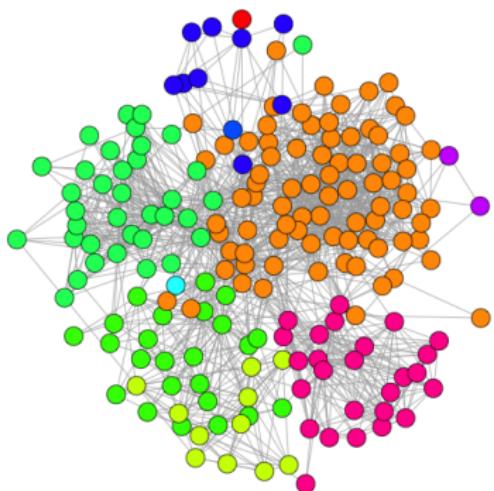
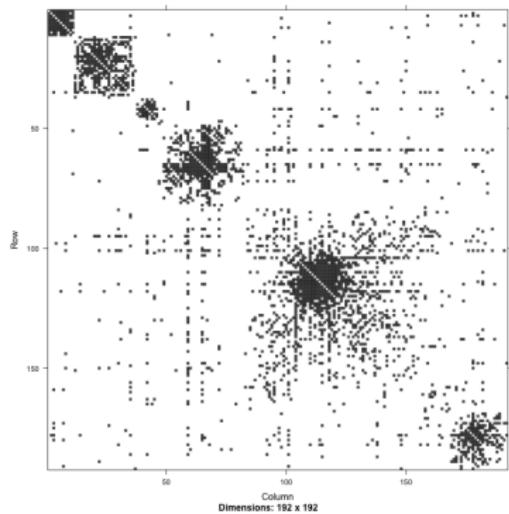


from M.J. McGuffin

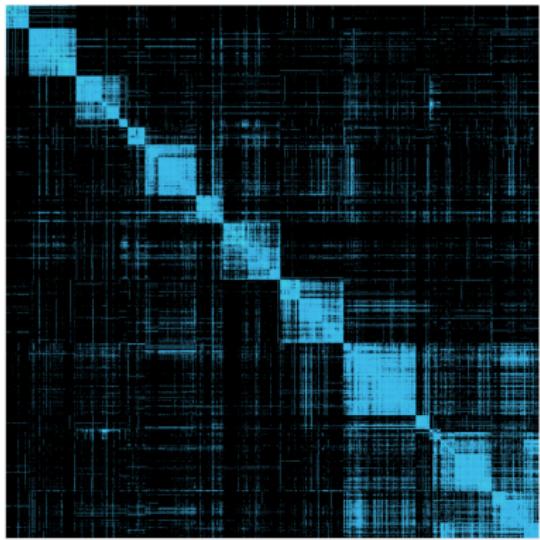
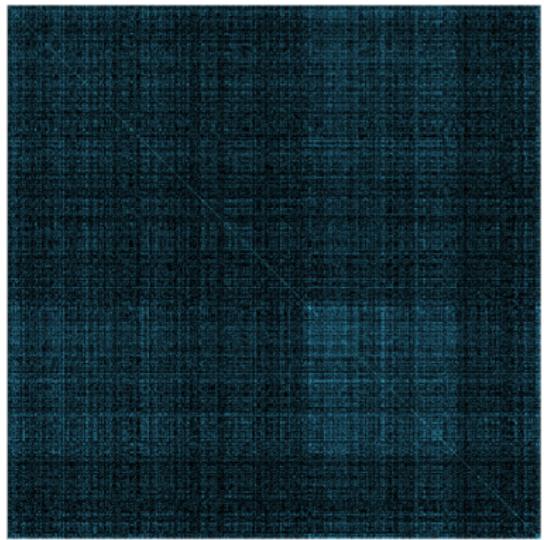
# Hierarchical clustering



# Sparse matrix visualization



# Sparse matrix visualization



# Visualization tools

- Graphviz (<http://www.graphviz.org>)
- Gephi (<http://www.gephi.org>)
- yEd (<http://www.yworks.com>)
- Visone (<http://www.visone.net>)
- Pajek (<http://pajek.imfm.si>)

yEd

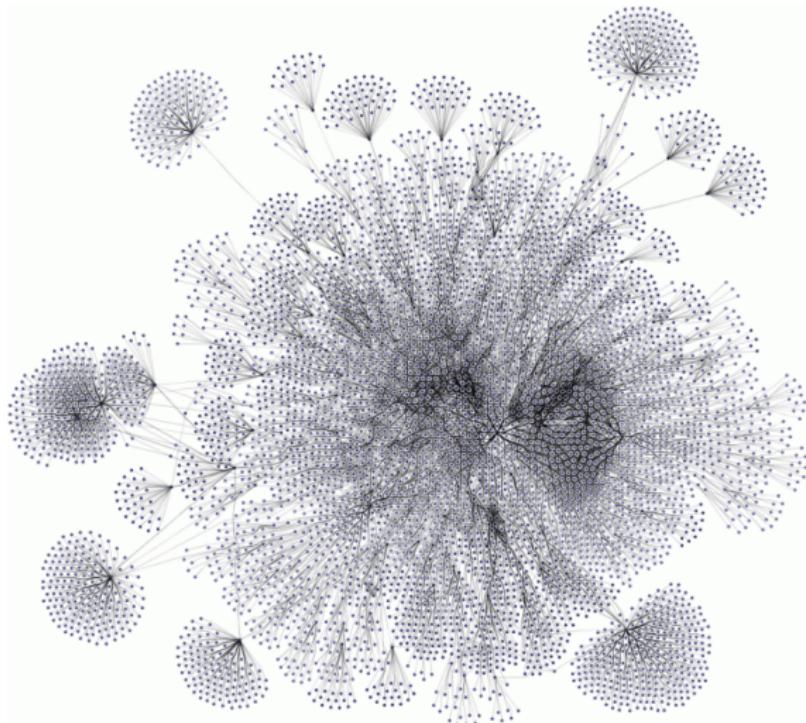
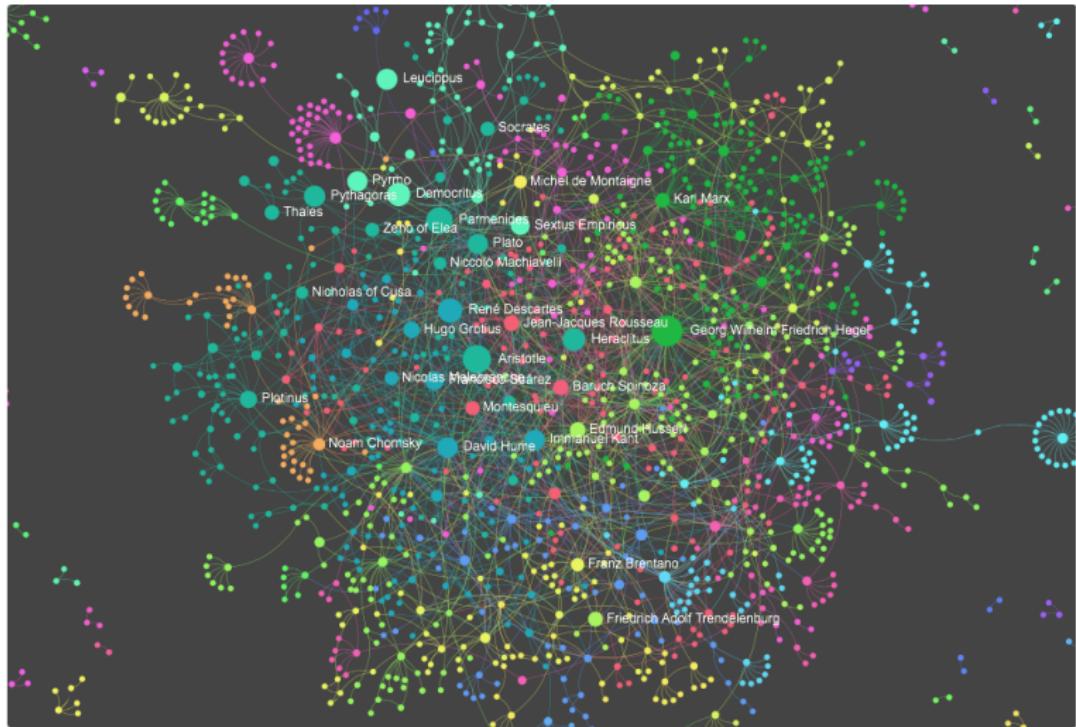
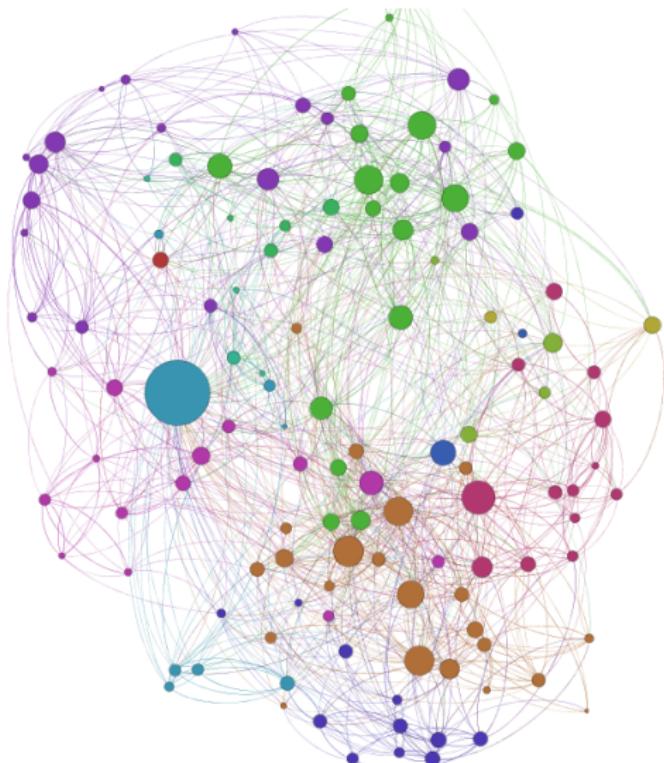


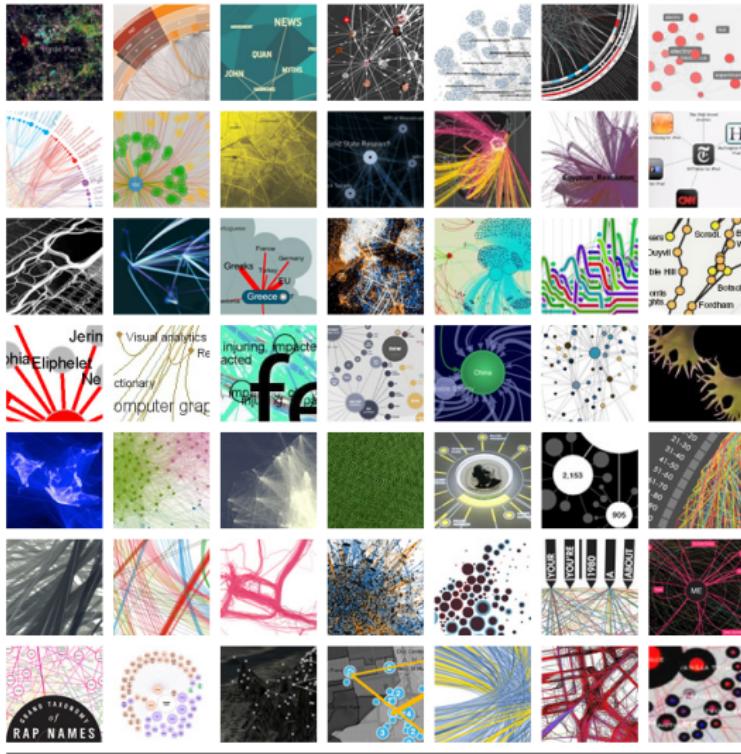
image from [www.yworks.com](http://www.yworks.com)

# Gephi





# Visual complexity



<http://www.visualcomplexity.com>