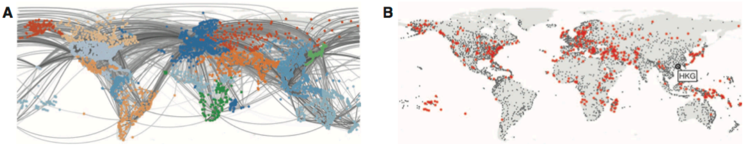
NATIONAL RESEARCH UNIVERSITY

Spreading phenomena in networks

Global contagion

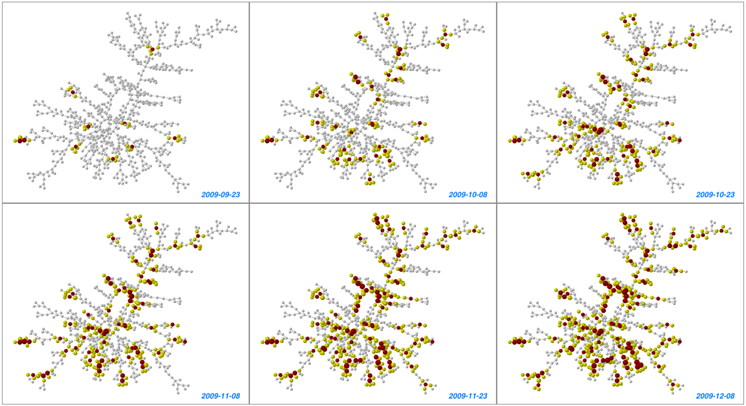
Outbreak of SARS in 2003, > 8000 cases, 10% fatality rate, 37 countries



Simulated model:
gray lines - passenger flow, red symbols epidemics location

D. Brockmann, D. Helbing, 2013

Flu contagion



Infected - red, friends of infected - yellow

N. Christakis, J. Fowler, 2010

Network epidemic model

- Given a network **G** of potential contacts
- Three states model: susceptible, infected, recovered states
- Probabilistic model (state of a node):

  $s_i(t)$ - probability that at t node i is susceptible

  $x_i(t)$ - probability that at t node i is infected

  $r_i(t)$ - probability that at t node i is recovered
- Model parameters:

  β - infection rate (probably to get infected on a contact in time δt)

  γ - recovery rate (probability to recover in a unit time δt)
- connected component - all nodes reachable
- network is undirected (matrix **A** is symmetric)
- if graph complete - fully mixing model
- Based upon models from mathematical epidemiology, W.O. Kermack and McKendrick, 1927

Probabilistic model

Two processes:

- Node infection:

$$P_{inf} \approx \beta s_i(t) \sum_{j \in \mathcal{N}(i)} x_j(t) \delta t$$

- Node recovery:

$$P_{rec} = \gamma x_i(t) \delta t$$

SI model

- SI Model

$$S \longrightarrow I$$

- Probabilities that node i: $s_i(t)$ - susceptible, $x_i(t)$ -infected at t

$$x_i(t) + s_i(t) = 1$$

- β - infection rate, probability to get infected in a unit time

$$x_i(t + \delta t) = x_i(t) + \beta s_i(t) \sum_j A_{ij} x_j(t) \delta t$$

- infection equations

$$\frac{dx_i(t)}{dt} = \beta s_i(t) \sum_j A_{ij} x_j(t)$$

SI model

SI Model

$$S \longrightarrow I$$

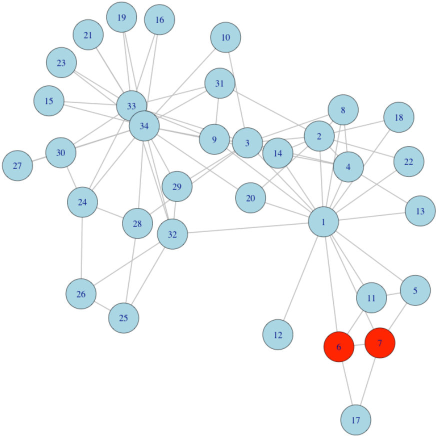
1. Every node at any time step is in one state $\{S, I\}$
2. Initialize c nodes in state I
3. On each time step each I node has a probability β to infect its nearest neighbors (NN), $S \rightarrow I$

Model dynamics:

$$I + S \quad \xrightarrow{\beta} \quad 2I$$

SI model

$\beta = 0.5$



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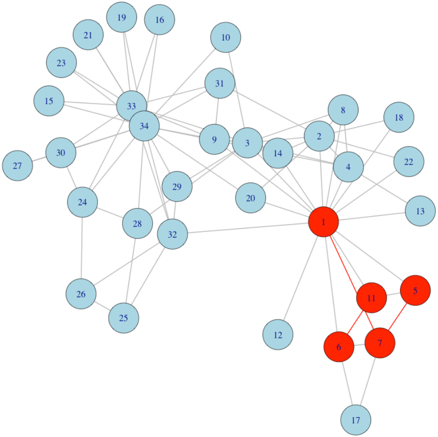

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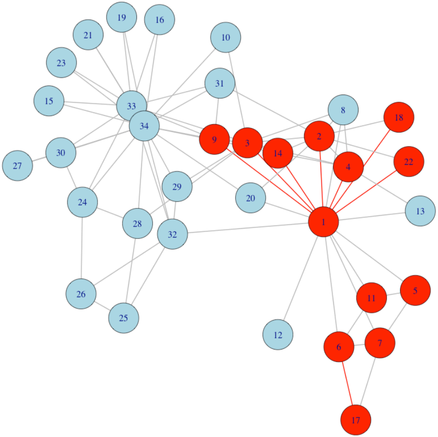

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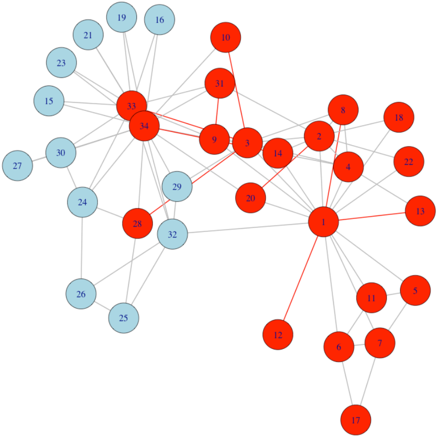

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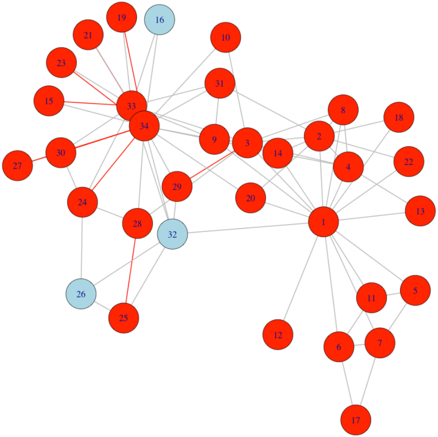

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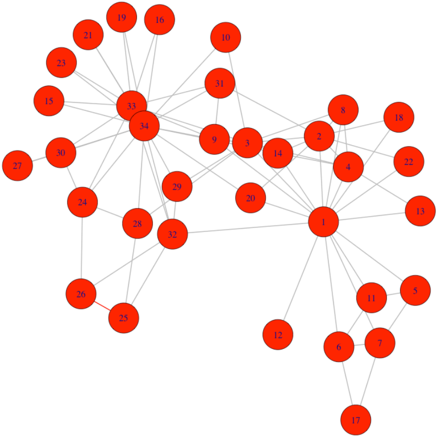

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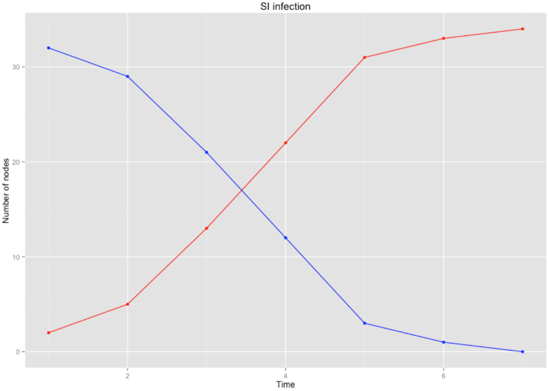
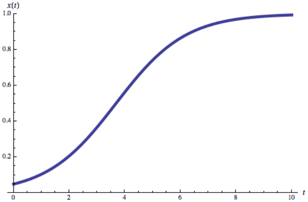


SI model



1. growth rate of infections depends on λ_1
2. All nodes in connected component get infected $t \to \infty$
   $x_i(t) \to 1$

image from M. Newman, 2010

SIS model simulations

SIS Model

$$S \longrightarrow I \longrightarrow S$$

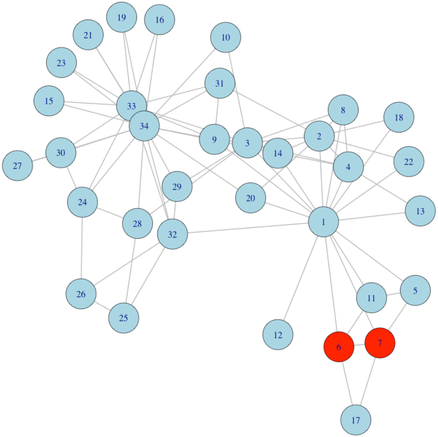
1. Every node at any time step is in one state $\{S, I\}$
2. Initialize c nodes in state I
3. Each node stays infected $\tau_\gamma = 1/\gamma$ time steps
4. On each time step each I node has a probability β to infect its nearest neighbors (NN), $S \to I$
5. After τ_γ time steps node recovers, $I \to S$

Model dynamics:

$$\begin{cases} I + S & \xrightarrow{\beta} \quad 2I \\ I & \xrightarrow{\gamma} \quad S \end{cases}$$

SIS model

$\beta = 0.5, \tau = 2$



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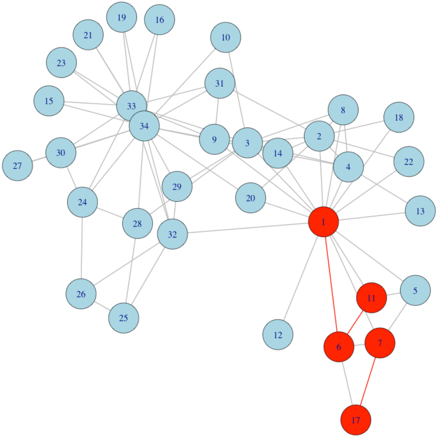

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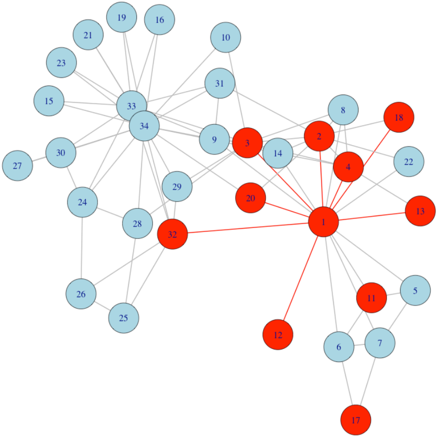

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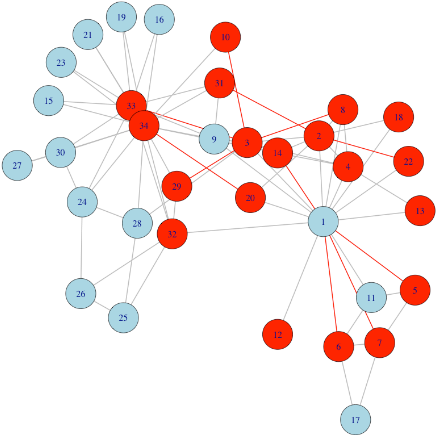

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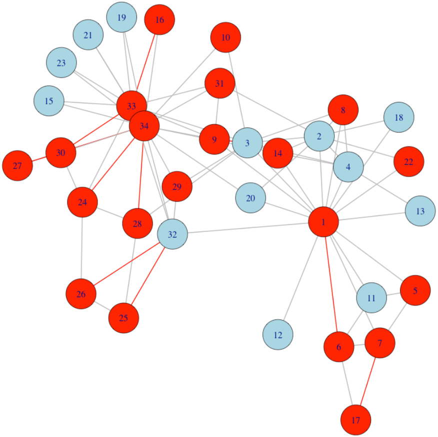

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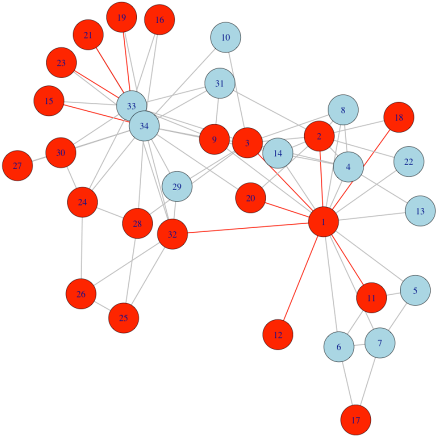

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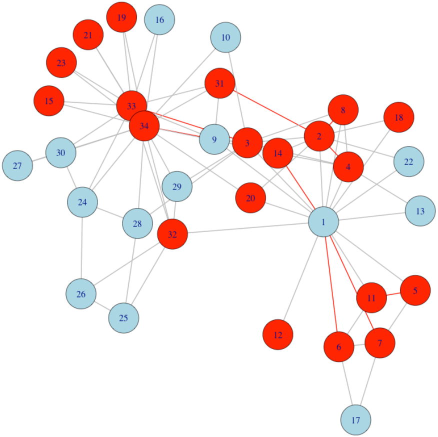

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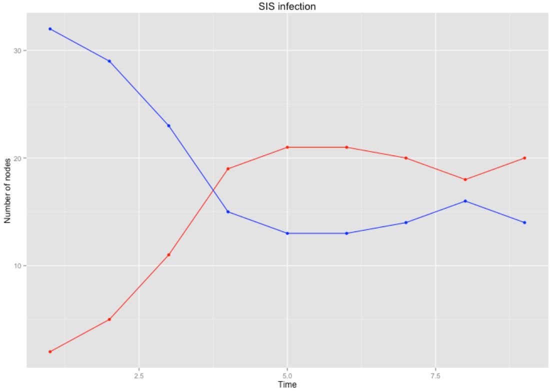
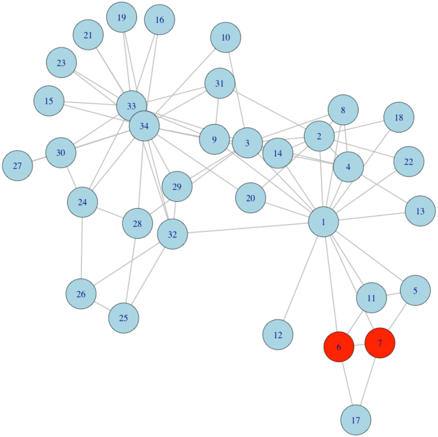


SIS model

$\beta = 0.2, \tau = 2$



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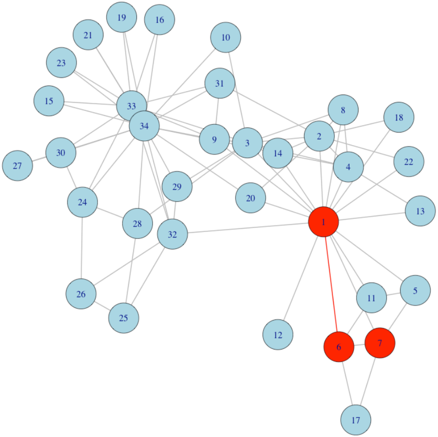
$\beta = 0.2, \tau = 2$



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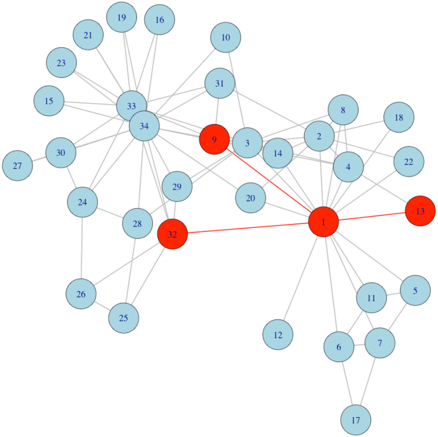

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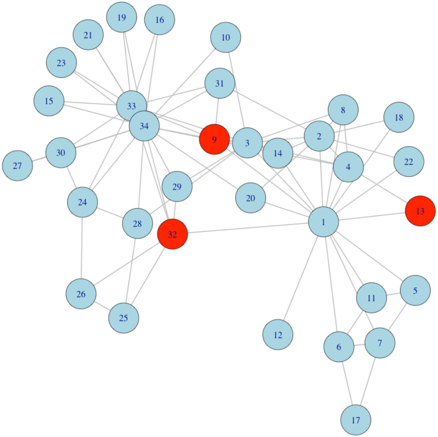

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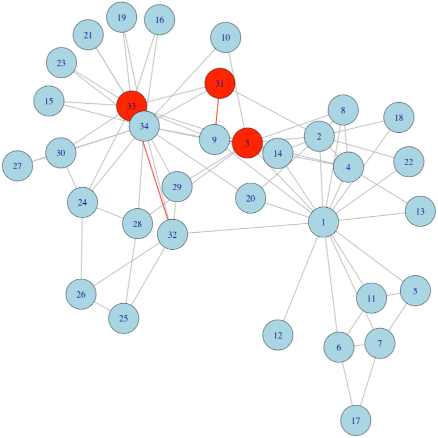

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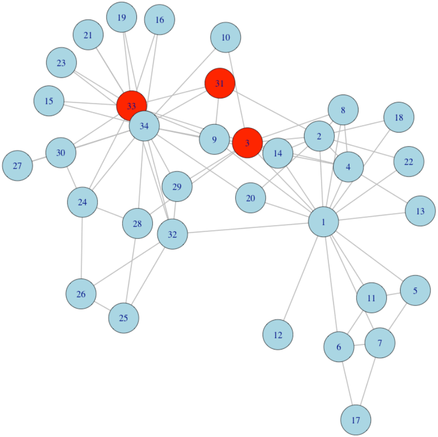

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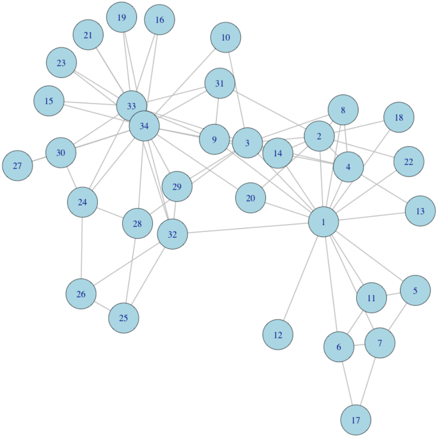

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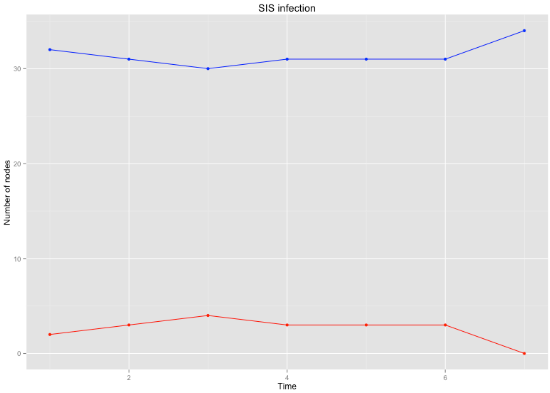
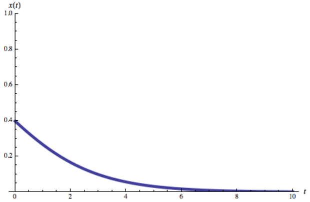


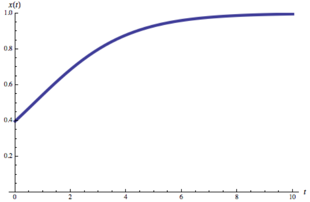
SIS model

Epidemic threshold R_0:

- if $\frac{\beta}{\gamma} < R_0$ - infection dies over time



- if $\frac{\beta}{\gamma} > R_0$ - infection survives and becomes epidemic



SIR model simulation

SIR Model

$$S \longrightarrow I \longrightarrow R$$

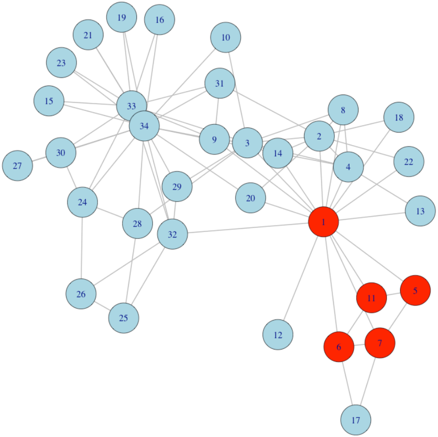
1. Every node at any time step is in one state $\{S, I, R\}$
2. Initialize c nodes in state I
3. Each node stays infected $\tau_\gamma = 1/\gamma$ time steps
4. On each time step each I node has a prabability β to infect its nearest neighbours (NN), $S \to I$
5. After τ_γ time steps node recovers, $I \to R$
6. Nodes R do not participate in further infection propagation

Model dynamics:

$$\begin{cases} I + S & \xrightarrow{\beta} & 2I \\ I & \xrightarrow{\gamma} & R \end{cases}$$

SIR model

$\beta = 0.5, \tau = 2$



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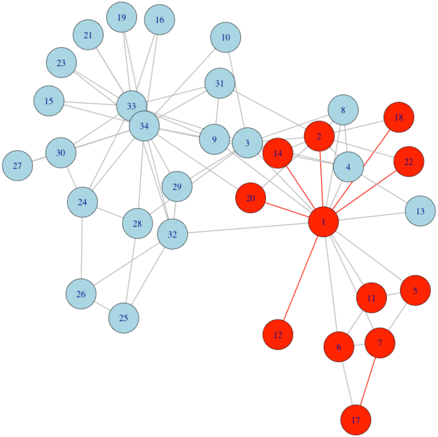

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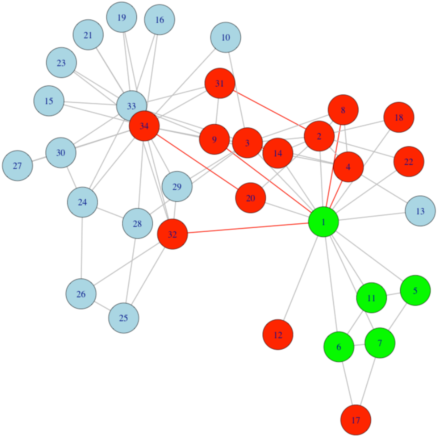

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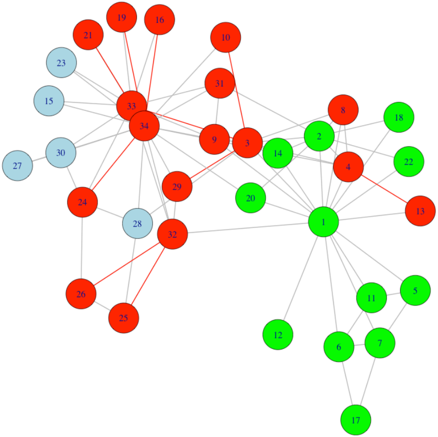

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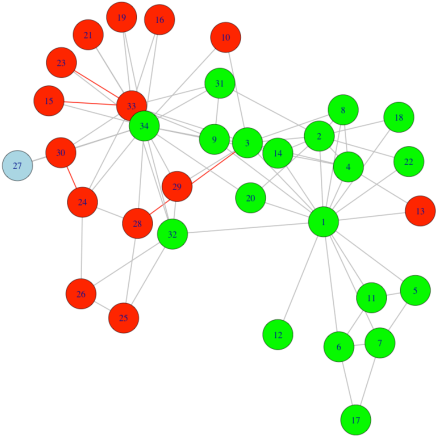

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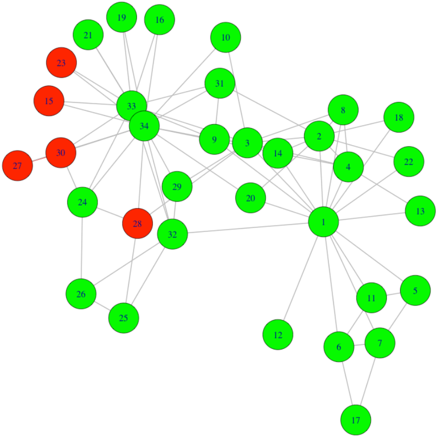

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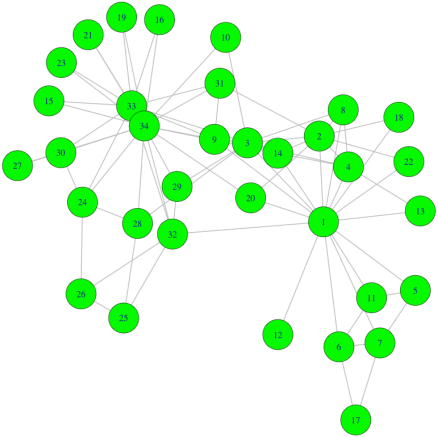
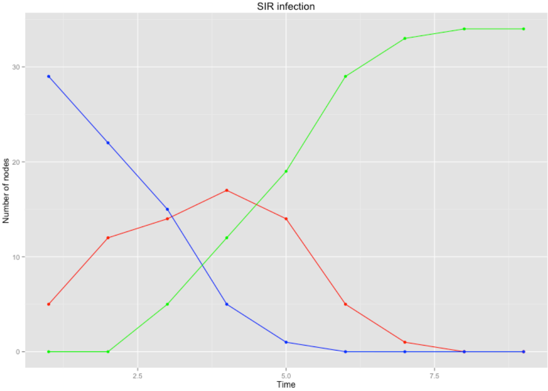
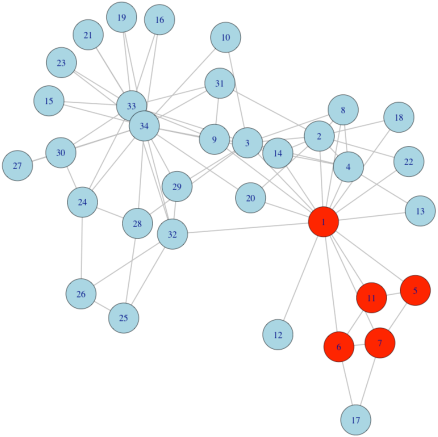


SIR model



SIR model

$\beta = 0.2, \tau = 2$



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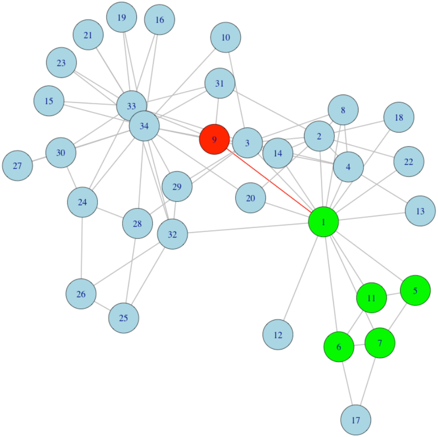
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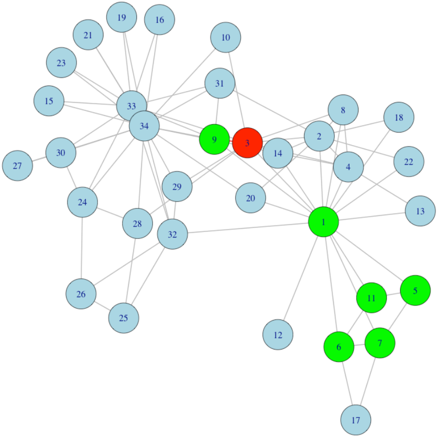

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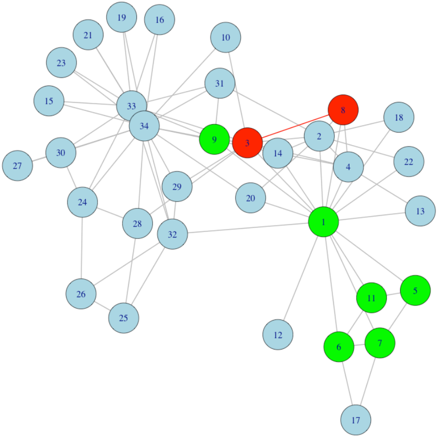

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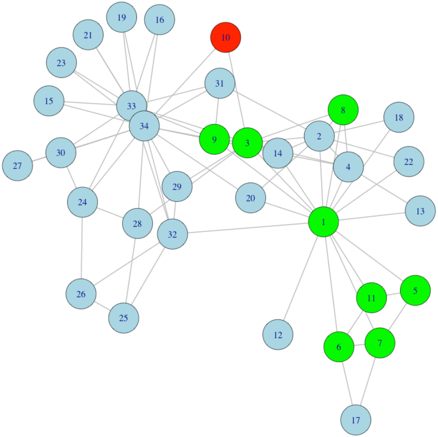

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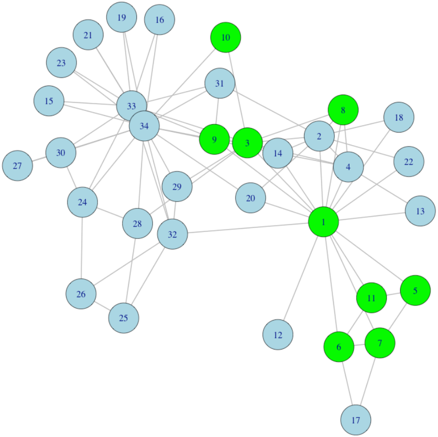

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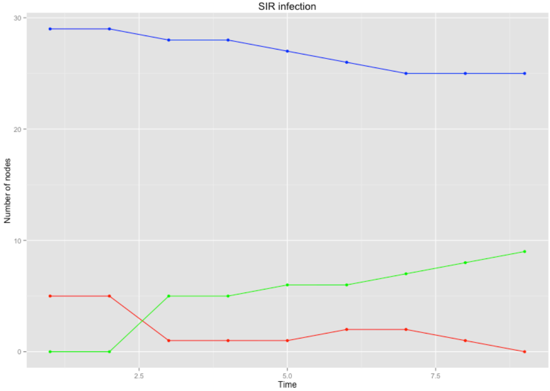
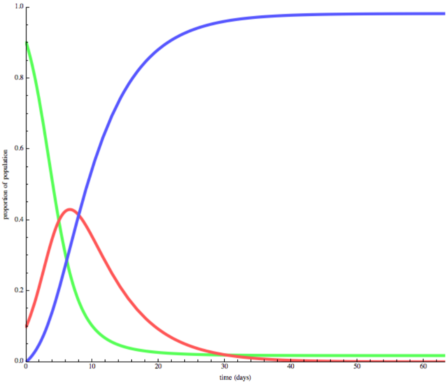


SIR model

Epidemic threshold R_0:
$\frac{\beta}{\gamma} > R_0$ - infection survives and becomes epidemic



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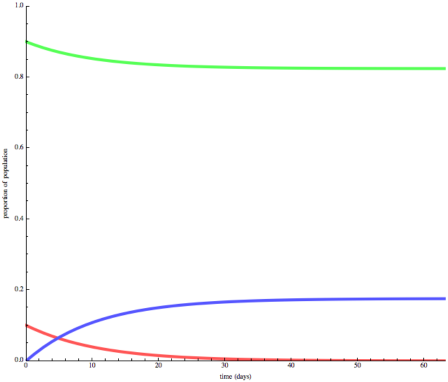
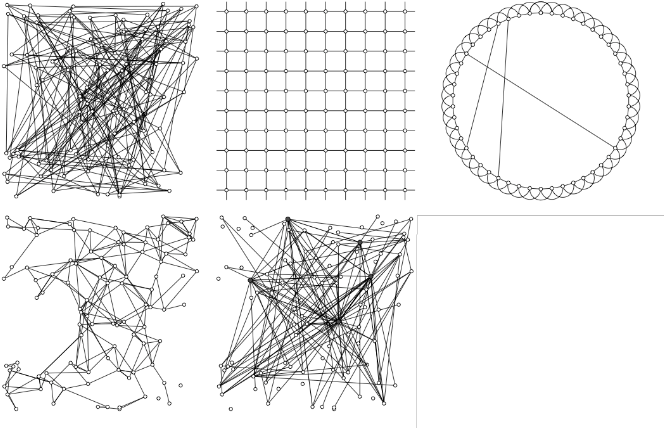


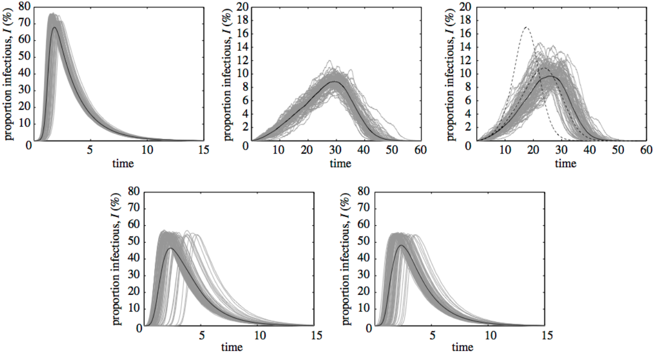
5 Networks, SIR



Networks: 1) random, 2) lattice, 3) small world, 4) spatial, 5) scale-free

image from Keeling et al, 2005

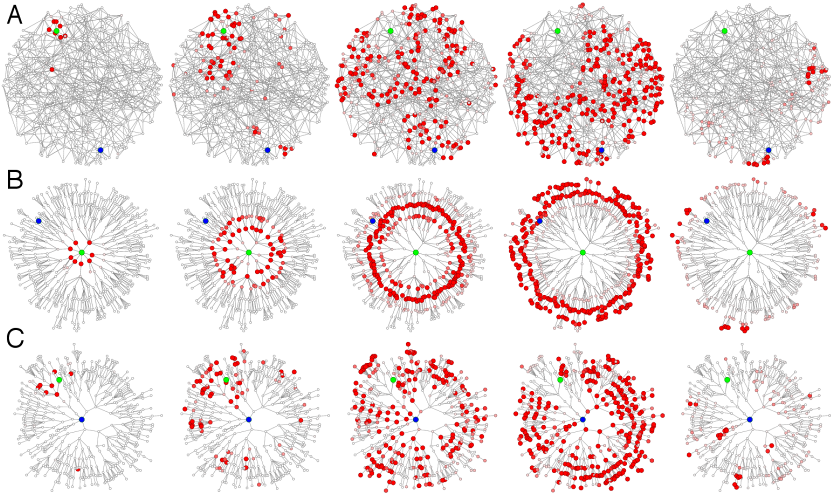
5 Networks, SIR



Networks: 1) random, 2) lattice, 3) small world, 4) spatial, 5) scale-free

Keeling et al, 2005

Effective distance



Social contagion

Social contagion phenomena refer to various processes that depend on the individual propensity to adopt and diffuse knowledge, ideas, information.

- Similar to epidemiological models:
  - "susceptible" - an individual who has not learned new information
  - "infected" - the spreader of the information
  - "recovered" - aware of information, but no longer spreading it
- Two main questions:
  - if the rumor reaches high number of individuals
  - rate of infection spread

Mem diffusion

Mem diffusion on Twitter