

Problem 1 Algorithms

10 Pts

- (a) An algorithm is defined as a (1) *finitely described*, (2) *deterministic*, (3) *effectively calculable* procedure that transform an input into an output.

Elaborate on these three properties in the definition. For each property, provide an example where it is *not* fulfilled.

- (b) Describe, in as much detail as possible, the steps to make a hard-boiled egg. Is your description an algorithm? Why or why not?

Problem 2 A Game of Guessing Numbers

10 Pts

Werner and Hannelore play a game called *Guess the Number*. The rules are as follows:

- (a) Werner and Hannelore agree on a positive integer $n \geq 1$.
- (b) Werner thinks of a secret number $x \in [1, n]$.¹
- (c) Hannelore tries to guess x by proposing numbers.
- (d) For each guess y , Werner responds with one of three statements:
 - “My number x is greater than y .”
 - “My number x is less than y .”
 - “Hit!” (if $y = x$)
- (e) The game ends when Hannelore guesses correctly.

Assume Werner plays honestly.

Hannelore uses the following strategy:

- She maintains two variables, a and b , initially set to 1 and n , respectively.
- In each round, Hannelore chooses a number $c \in [a, b]$.
- Based on Werner’s response, Hannelore updates a or b :
 - If “Hit!”, the game ends.
 - If “Less”, she sets $b = c - 1$.
 - If “Greater”, she sets $a = c + 1$.

¹The *brackets* $([])$ denote a *closed interval*, i.e. $1 \leq x \leq n$.

- (a) Provide a detailed example of this game for $n = 10$, showing Hannelore's strategy in action.
- (b) Prove that Hannelore's strategy always succeeds in a finite number of rounds.
Hint: Identify an invariant condition that holds at the beginning of each round.
 Your proof should address two key points:
 - (i) Hannelore makes "progress" in each round;
 - (ii) Hannelore cannot "miss" Werner's number.

Problem 3 First Steps in Python

10 Pts

- (a) Download and install Python (Version 3) on your computer.
- (b) Start the Python REPL and enter the following commands. For each, explain what happens and provide a brief interpretation:
 - (a) `a = 8 + 10`
 - (b) `help()`
`quit`
 - (c) `3 + 5 * 7 == a - 2`
 - (d) `a = 40`
`3 + 5 * 7 == a - 2`
 - (e) `print("KDP", str(a * 50 + 2*10 + 4 - 1) + ".\n")`
 - (f) `True or (False and True)`
 - (g) `if a - 4 <= 5:`
`print("Ja")`
`else:`
`print("Nein")`
 - (h) `2 * (4 +`
`5)`
 - (i) `for i in range(10):`
`print(2 * i + 1)`
 - (j) `exit()`
- (c) Download the file `mystery.py` from the course website and import it into Python using the command `from mystery import mystery`. What happens when you enter:


```
mystery("anna")
mystery("banane")
mystery("caesar")
```

Conduct further experiments and formulate a hypothesis about what the function "mystery" does.