COGS543 Assignment 1

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Question 1:

(a) $\lambda f \lambda g \lambda h \lambda x.(f(g(hx))) = (\lambda f.(\lambda g.(\lambda h.(\lambda x.(f(g(hx)))))))$

(b) $xxxx = (((xx)x)x)$

(c) $\lambda x.x\lambda y.y = (\lambda x.(x\lambda y.(y))))$

(d) $\lambda x.(x\lambda y.yxx)x = (\lambda x.((x\lambda y.((yx)x)))x))$

Question 2:

(a) $(xy) = xy$

(b) $(x(yz)) = x(yz)$

(c) $((xy)z) = xyz$

(d) $(\lambda x.x) = \lambda x.x$

(e) $(\lambda y.(\lambda x.x)) = \lambda y.\lambda x.x$

(f) $(\lambda z.(x(\lambda y.(yz)))) = \lambda z.x\lambda y.yz$

(g) $(x(\lambda z.(\lambda y.(yz)))) = x\lambda z.\lambda y.yz$

(h) $(x(\lambda x.x)) = x\lambda x.x$

(i) $((\lambda y.(\lambda x.x))(\lambda x.x)) = \lambda y.\lambda x.x\lambda x.x$

(j) $(((\lambda y.(\lambda x.x))(\lambda x.x))(xy)) = \lambda y.\lambda x.x\lambda x.x(xy)$

(k) $((x(yz))((xy)z)) = x(yz)(xyz)$

(l) $(\lambda x.(\lambda y.(\lambda z.((xz)(yz))))) = \lambda x.\lambda y.\lambda z.xz(yz)$

(m) $(((ab)(cd))((ef)(gh))) = ab(cd)(ef(gh))$

(n) $(\lambda x.((\lambda y.(yx))(\lambda v.v)z)u)(\lambda w.w) = \lambda x.\lambda y.yx\lambda v.vzu\lambda w.w$

Question 3:

(a) $(\lambda f.fx)g = gx$

(b) $(\lambda f.fx)ga = gxa$

(c) $(\lambda f.fx)(ga) = gax$

(d) $(\lambda f\lambda x.fx)ga = ga$

(e) $(\lambda x\lambda y\lambda z.x(yz))f = \lambda y.\lambda z.f(yz)$

(f) $(\lambda x.mx)j = mj$

(g) $(\lambda y.yj)m = mj$

(h) $(\lambda x.\lambda y.y(yx))jm = m(mj)$

(i) $(\lambda y.yj)(\lambda x.mx) = mj$

(j) $(\lambda x.xx)(\lambda y.yyy) = (\lambda y.yyy)\cdots(\lambda y.yyy)$ (infinite loop)

Question 4:

M_1	M_2
$D = \{d_1, d_2, d_3\}$	$D = \{d_1, d_2, d_3\}$
$F = \{d_1, d_2, d_3\}$	$F = \{d_1, d_2, d_3\}$
$E = \{\langle d_1, d_1\rangle, \langle d_2, d_2\rangle, \langle d_3, d_3\rangle\}$	$E = \{\langle d_1, d_1\rangle, \langle d_2, d_2\rangle\}$

Model 1 makes the notation true, and model 2 makes it false.

Question 5:

There are some models where $\forall x.Fx \to \exists y.Gxy \models \exists x.Fx$ does not hold. According to the model, $M = \langle D = \{d_1, d_2, d_3\}, F = \{d_1, d_2, d_3\}, G = \emptyset\rangle$, the notation $\forall x.Fx$ is true, but $\exists y.Gxy$ is false. Therefore, the truth value of the left side is false. However, for the same model, the right side $\exists x.Fx$ is true, so there is a model that makes one true but the other false. Another way of showing P does not imply Q is to check the possible truth values of the components of the notation.

$\forall x.Fx$	$\exists y.Gxy$	$\forall x.Fx \to \exists y.Gxy$	$\exists x.Fx$
1	1	1	1
1	0	0	1
0	1	1	1/0
0	0	0	1/0

When we look at the last three rows of the table, the truth value of P does not always match the truth value of Q. Therefore, this implication does not hold.