

# Seminari Informal de Matemàtiques de Barcelona

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**Speaker:** David Ruiz-Baños.  
**Universitat:** Department of Mathematics, University of Oslo).

**Data:** dilluns, 7 d'abril de 2014.  
**Horari:** 17:45, *coffee break*; 18:00, xerrada.  
**Lloc:** Aula IMUB (al terrat), Facultat de Matemàtiques de la UB.

**Títol:** Stochastic Differential Equations.

**Resum:**

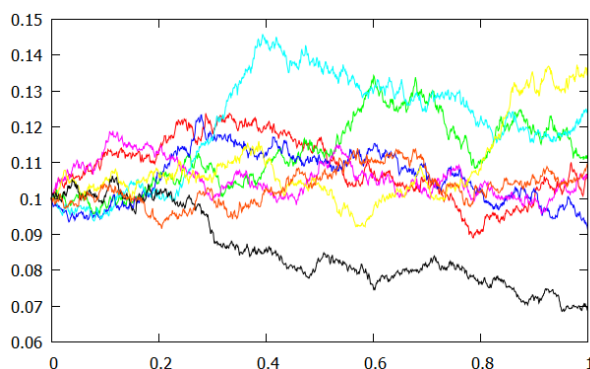
In this talk we want to explain what a *stochastic differential equation* (SDE) is. As an idea, we will recall what a (deterministic) *ordinary differential equation* (ODE) is and then we will add some "noise" to it. Such equations are widely used to study and model phenomena in nature such as for instance: the "random" movement of a particle in a fluid due to collisions with the molecules of the fluid, variable uncertainty, macroeconomic dynamics, etc. The solution of an SDE is therefore a *stochastic process*, i.e. think of it as a function whose image is not a real number, but a random variable. It is a big area of research to study the solutions of SDE's and the densities of such solutions. It is very difficult to say something about the densities.

An SDE looks typically like

$$dx(t) = f(t, x(t))dt + \text{"noise"}, \quad t_0 \leq t \leq a, \quad x(t_0) = x_0 \in \mathbb{R} \quad (1)$$

where the initial condition  $x(t_0) = x_0$  is typically taken as a *deterministic* point  $x_0 \in \mathbb{R}$  that we have observed or might also be taken as  $x(t_0) = Z$  where  $Z$  is a random variable (e.g. normal distributed).

Finally, we will mention some properties of SDE's, like... For example, you know that the solution when  $f$  is Lipschitz exists and it is unique for ODE's (Picard's theorem). In the case of SDE's even if  $f$  is very "ugly" a unique solution also exists! Meaning that, the "noise" somehow regularizes  $x(t)$ .



**Figura 1:** Eight realizations of a *geometric Brownian motion*

## Referències

- [1] Øksendal, Bernt K. *Stochastic Differential Equations: An Introduction with Applications*. Berlin: Springer (2003).

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**Qui som?** El SIMBa és un seminari jove organitzat per estudiants de doctorat de matemàtiques. Està dirigit a estudiants de doctorat, de màster i, fins i tot, dels darrers cursos de grau. El nostre objectiu és donar a conèixer la recerca que estem fent, així com adquirir coneixements d'altres àrees de les matemàtiques diferents de la pròpia. Més informació a [www.ub.edu/simba](http://www.ub.edu/simba).