Informal Rungs on the Job Ladder: Theory and Evidence from Brazil*

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Abstract

This paper studies a labor market where heterogeneous workers climb a job ladder with informal and formal rungs. In this environment, the incidence of informal jobs in a worker's career is a function of her skill level and the economy's history of aggregate states. I estimate the model in Brazilian labor-force survey data, and show it successfully reproduces the observed heterogeneity and dynamics around informality. In equilibrium, informal jobs are less productive and are subject to higher layoff risk than their formal counterparts. However, workers rely on informal contracts not only to smooth transitions between employment and non-employment, but also to advance their careers through moves within and between jobs. According to the model, stronger enforcement of penalties against informal matches (i) increases unemployment and self-employment, (ii) dampens job-to-job transitions, (iii) reduces total output, and (iv) disproportionately hurts the low skilled.

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1 Introduction

Informal labor contracts are prevalent in many developing economies. Through them, companies and workers establish unofficial employment relationships outside applicable labor market regulations. In Brazil, for instance, the Labor Code entitles employees to generous mandated benefits and job-security provisions. In practice, however, only 55% of workers — the formally employed — get to enjoy such standards. The rest are distributed among unemployment (10%), self-employment (21%), and informal employment (14%).[1]

The prominence of the latter groups illustrates well the potential trade-off with mandated benefits. On the one hand, these provisions seek to preserve workers' welfare under difficult circumstances (e.g. unemployment). On the other, if set too high, they can end up protecting only some workers at the expense of many others. For this reason, in contexts like the Brazilian one, governments continuously need to ponder whether they should tighten their levels of labor law enforcement or just tolerate their existing informality rates. This paper revisits this question within a novel framework for informality.[2]

I consider a model of the labor market that features a series of job ladders with informal and formal rungs. Importantly, these ladders are skill specific and change with the business cycle. This characterization is motivated by three facts I document in the data.

First, informal labor contracts are widespread across demographic groups. They are more prevalent among the youngest and the least educated, but their incidence is high even among prime-aged workers with tertiary education. Second, informal jobs disproportionately account for inflows into employment. This pattern applies to a broad range of worker classes — including those rejoining employment after losing a

[1] I often allude to the distinction between an *informal* and a *formal* job. The former refers to an employment relationship between a firm and a worker that is not reported to the government, thereby circumventing corresponding labor regulations. Conversely, formal jobs represent standard dependent employment complying with applicable government mandates. The self-employment category refers to workers operating on their own and without employees in their main job. As a reference, note that employees, the unemployed, and the "solo self-employed" account for 90%, 3%, and 7% of workers in the US — see Boeri et al. (2020).

[2] In practice, governments can also choose to reassess their labor legislation standards. However, I do not explore this matter in this paper.

formal job — and is particularly salient during aggregate downturns. Third, transitions from an informal contract to a formal one — within or between jobs — or to a different informal job are frequent and are associated with significant increases in labor income.

Together, these facts suggest the range of individuals relying on informal jobs at some point in their careers is broader than what these contracts' static incidence indicates. In the data, workers seem to exploit informal jobs not only to smooth transitions between employment and non-employment, but also to advance their careers through moves within and between jobs. The job-ladder framework proposed in this paper accounts for such patterns.

The proposed environment extends the setup in Lise and Robin (2017) to introduce informal and formal jobs, self-employment, and firm-level productivity shocks.[3] In the model, heterogenous workers are matched pairwise to heterogenous firms. Each worker has an optimal firm she would like to be matched with. However, random search — off and on the job — induces workers to accept offers not only from their preferred matches, but also from a range of firm types. The economy therefore features imperfect sorting in equilibrium. Moreover, idiosyncratic and aggregate productivity shocks cause some of the ongoing matches to become unprofitable for the firm. As a result, each period, some firms would be better off destroying their matches, that is, firing their workers. In particular, the more mismatched a worker is to her current firm, the more likely she is to move into unemployment in the future.

In this context, I introduce a simple trade-off for informality. Whereas a formal contract imposes a fixed cost for the firm to fire a worker, an informal one avoids it but is subject to a detection probability and a corresponding fine. The former type of cost accounts for existing job-security provisions in Brazil, whereas the latter represents the expected penalties a firm faces from employing a worker informally. A direct implication is that the more vulnerable a match is to the arrival of negative shocks, the higher the cost for the firm to employ that worker formally. Hence, in equilibrium, firms allocate their less (more) productive and less (more) stable matches to informal (formal) contracts, while workers transit, through on-the-job search, from informal to

[3]The model accounts for the existence of a self-employment sector in the economy. This feature is important to match the levels of shares and transition rates targeted in the structural estimation of the model. The focus, however, is on explaining the heterogeneity and dynamics related to informal labor contracts. For a model aimed at explaining patterns related to self-employment in a similar context, see Narita (2020).

formal jobs across time.

Therefore, as observed in the data, workers in the model usually exit unemployment through an informal contract and gradually exploit informal and formal outside offers to climb the job ladder. For instance, upgrades in workers' formality statuses can arise between jobs when workers accept an outside offer, or within jobs when matches are hit by positive productivity shocks. Downgrades, on the other hand, are induced by formal workers falling off the ladder — losing their jobs due to a negative productivity shock or an exogenous separation — and rejoining it later through an informal contract.[4]

Moreover, because the characterization of job ladders in the model depends on the skill level of a worker and the history of aggregate states in the economy, the environment can account for (i) differences in the incidence of informal labor contracts across skill groups, (ii) states of the economy where mostly low-skill workers transit across informal rungs, or (iii) alternate states — e.g., a recession — where even high-skill workers rely on informal jobs to move up the job ladder.

The extent to which these patterns take place is an empirical matter. Therefore, I estimate the model using the simulated method of moments (SMM) and the Metropolis Hastings algorithm, which is a Markov chain Monte Carlo (MCMC) method. This technique has proved useful in the estimation of complex macroeconomic models and machine-learning applications. The targeted empirical moments include levels, standard deviations, and correlations with output of the unemployment rate, informality rate, self-employment rate, and associated transition rates.[5]

The model sucessfully reproduces the targeted characterization of the Brazilian economy. First, the evolution of unemployment is well traced along the business cycle. Second, informal contracts are widespread across worker types and are particularly prevalent among the low skilled. Third, informal employees fall off the ladder — lose their jobs — almost three times as frequently as formal ones. Fourth, although informal jobs account only for 18% of matches in the cross section, they are responsible for 58% of newly created matches each period. Moreover, among unemployed workers that previously held a formal job, this rate still amounts to 45%. Fifth, upgrades in workers' formality statuses are frequent both within and between jobs. Finally,

[4]Within-job downgrades in a worker's formality status are small in the data and are not accounted for in the model. See section 3 for a detailed discussion.

[5]Table 4 reports the targeted moments.

standard deviations, correlations with output, and non-targeted series are well fitted too.

Given the empirical plausibility of the framework, I use it to evaluate the effects of a policy often assessed in the literature: tightening labor law enforcement. In particular, I consider an increase in the per-period detection probability of informal matches from 3% to 10%. This policy makes employing workers informally costlier for firms, and therefore reduces the availability of informal contracts in the economy. Indeed, the results suggest that if detection had been set at 10% during the period between March 2002 and February 2016, the incidence of informal jobs in the cross section of matches would have dropped from 18% to 3%. The policy counterfactual, however, induces some negative unintended consequences as well. On average, unemployment and self-employment increase by 0.7 and 0.8 percentage points respectively, aggregate output falls by 1.0%, and the rate of job-to-job transitions decreases from 1.11% to 0.98%. Furthermore, the distribution of matched workers tilts toward the medium and high skilled. That is, the workers who are displaced into self-employment and unemployment are predominantly low skilled.

These results are consistent with available reduced-form evidence in the literature. Empirical studies assessing the effects of stronger enforcement in this context document a trade-off between reducing informality and increasing non-employment. In particular, Almeida and Carneiro (2012) implement an IV strategy and conclude stronger levels of enforcement cause both the incidences of formal employment and non-employment to rise. Similarly, Samaniego de la Parra (2017) finds random work-site inspections in Mexico generate larger transitions from informality to formality and from informality to non-employment at the establishment level. Finally, Ulyssea and Ponczek (2018) show negative labor-demand shocks induced by trade liberalization cause significant increases in informality in regions with weak enforcement, but significant increases in non-employment in regions with strong enforcement.

This paper contributes primarily to the large literature on theoretical frameworks for informality. Recent work in this area includes competitive frameworks such as Leal Ordóñez (2014) and Ulyssea (2018), and frictional environments of the labor market such as Bosch and Esteban-Pretel (2012), Haanwinckel and Soares (2020), and Meghir et al. (2015).[6] The latter, in particular, is the closest to the analysis implemented here, although key differences exist in our approaches.

Meghir et al. (2015) propose an equilibrium wage-posting model a la Burdett and Mortensen (1998), where heterogenous firms decide to open vacancies in the formal or the informal sector and homogeneous workers search off and on the job. As a result, this framework displays firms of equal productivity in both the formal and informal sectors and workers transiting into better jobs through on-the-job search across time. These patterns also hold in the environment proposed in this paper. Moreover, four differences induce a richer characterization of heterogeneity and dynamics. First, I allow firms to reassign their ongoing matches between formal and informal contracts every period. Thus, the environment accounts for within-job formality upgrades, which are important according to the data. Second, I consider heterogeneity not only on the firm side, but also on the worker side. Therefore, job ladders in the model are skill specific, and we have considerable flexibility to match the rich heterogeneity observed in the data.[7] Third, I account for aggregate shocks in the model. According to correlations in the data and empirical evidence from other papers — for example, Dix-Carneiro and Kovak (2019) and Ulyssea and Ponczek (2018) — the issue of transitional dynamics is probably central to the study of informal employment relationships in developing economies. This paper provides a framework to build on in this regard. Finally, I distinguish informal salaried employees from self-employed workers both in the model and data.

Importantly, these differences are relevant policy-wise. According to my model, a higher probability of detection for informal matches increases unemployment and decreases output. These implications are at odds with those in Meghir et al. (2015). Three elements explain the difference. First, because workers are heterogeneous in this framework, not all of them transit easily from informal to formal jobs with the counterfactual policy. This observation is particularly relevant the lower the skill of a worker. Second, because firms are also heterogeneous and there are positive skill-technology complementarities, some matches are no longer feasible in the counterfactual scenario. As a result, workers face job ladders with a smaller number of rungs — they

[6]For a recent review of the literature, see Ulyssea (2019).

[7]To be consistent with their homogeneity assumption, Meghir et al. (2015) restrict their analysis sample to workers with eight or less years of education.

take longer to enter the first tier and also to move up across jobs. Finally, tractability out of steady state allows me to incorporate pertinent dynamics to the analysis: the referred unintended consequences of increasing enforcement are particularly salient during aggregate downturns.

The rest of the paper is organized as follows. Section 2 documents the motivating empirical facts. Section 3 outlines the proposed search equilibrium environment. Section 4 estimates the model and discusses its fit to several empirical counterparts. Section 5 assesses the effects of tightening labor law enforcement in the model. Section 6 concludes.

2 Motivating Empirical Facts

In Brazil, 21% of employees in the private sector work in an informal job.[8] This incidence, although already substantial, understates the fraction of workers for whom this margin is relevant. Indeed, the evidence presented below suggests the range of workers relying on these jobs at some point in their careers is much wider: workers exploit informal contracts not only to smooth transitions between employment and non-employment, but also to advance their careers through moves within and between jobs. This characterization of the Brazilian labor market motivates the job-ladder framework proposed in this paper. Therefore, in this section, I briefly document the referred empirical patterns.

2.1 Data

I use data from the *Pesquisa Mensal de Emprego*, the labor-force survey for Brazil between March 2002 and February 2016.[9] This survey was monthly administered to a representative sample of households in the six largest cities of the country.[10] It collected socio-demographic and detailed labor information from respondents and had the same longitudinal structure as the US Current Population Survey. That is, households were

[8]I use the term *private sector* to distinguish employment at companies from public sector employment and self-employment.

[9]The dataset is publicly available at *www.ibge.gov.br*. I standardize survey rounds across time and build the panel dataset using codes from "Data Zoom", a site developed by the Department of Economics at PUC-Rio with support from FINEP: www.econ.puc-rio.br/datazoom/english/index.html.

[10]The cities are Belo Horizonte, Porto Alegre, Recife, Rio de Janeiro, Salvador, and Sao Paulo.

interviewed for four consecutive months in a first round; then, they were excluded from the sample for the next eight months; and finally, the households were brought back into the sample for four final months.

In subsection 2.2, I first characterize the incidence of informal contracts across workers' demographic groups. To do so, I rely on the pooled monthly rounds of the dataset. Subsection 2.3, on the other hand, exploits the longitudinal structure of the survey to identify workers' monthly transitions in employment and formality statuses.

2.2 Fact 1: Informality is widespread across worker classes, and particularly prevalent among the youngest and least educated.

In Brazil, employees need to hold a job ID card (*Carteira de Trabalho*) signed by their current employer in order to be granted labor protection by the government. The survey directly asks respondents whether they hold such a document. Therefore, I follow the literature and use this question to distinguish between formal and informal employees in the data.

Figure A.1 in Appendix A plots the share of employees in the private sector who work in an informal job, across age and educational attainment groups. Among individuals with elementary school (high school) attainment, the share of employees at an informal job drops from 52% (29%) when they are between 16 and 20 years old to 20% (15%) when they are in their 30s. A similar trend applies to the other education groups. Simply put, informal contracts seem to be especially relevant for workers at early stages of their careers, and many of them transit eventually into a formal job. Nevertheless, even among workers in the age-education group with the smallest incidence of informality — those with a higher-education degree who are between 31 and 40 years old — 12% report being employed off the books. Informal contracts are therefore widespread across worker classes. Monthly transition rates presented below shed some light on why informality is so prevalent.

2.3 Fact 2: Informal jobs disproportionately account for inflows into employment.

Table 1 reports workers' monthly transition rates between non-employment and private-sector employment.[11] Average earnings of workers exiting or entering a job are also shown in the last column. Informal jobs are less productive, on average, as per differences in earnings levels. They also face higher termination risk. Specifically, 2.2% (0.9%) of informal (formal) workers transit into non-employment every month.

Table 1: Workers' Monthly Transitions between Employment and Non-employment and Exit/Entry Average Earnings

	Transition Rate (%)	Exit/Entry Avg. Earnings
I to NE	2.2	796
F to NE	0.9	1215
NE to I	3.1	776
NE to F	2.6	1102
f-NE to I	3.8	879
f-NE to F	5.0	1190

Source: *Pesquisa Mensal de Emprego*, March 2002 - February 2016. Employment statuses are formal (F), informal (I), non-employed (NE), and currently non-employed who report having held a formal job within the last year (f-NE). Earning levels are reported in 2016 Brazilian *Reais* units.

Informal contracts, however, disproportionately account for inflows into employment. That is, although they account for only 21% of jobs in the private-sector cross section, they are responsible for 54% (i.e., [3.1] / [3.1 + 2.6]) of total monthly transitions from non-employment into private-sector employment. More remarkably, many formal workers losing their jobs rejoin the private sector through informal contracts. Namely, among non-employed workers who report to have held a formal job within the last year and who return to work in a particular month, 43% (i.e. [3.8] / [3.8 + 5.0]) do so through an informal job. Moreover, Figure A.2 in Appendix A shows this rate is strongly countercyclical: as aggregate downturns arise, more workers rely

[11] Non-employment includes unemployment and non-participation.

on informal contracts to exit non-employment. This finding suggests the allocation of workers between formal and informal jobs in Brazil is far from static: many individuals oscillate between both types of jobs along their life cyle. This observation is consistent with the widespread incidence of informal contracts across worker classes documented in subsection 2.2.

2.4 Fact 3: Within- and between-job transitions from informality are frequent and induce improvements in earnings.

Table 2 reports monthly transition rates in formality status between and within jobs. Additionally, the average percent changes in earnings accompanying such moves are shown in the last column.

Table 2: Workers' Transitions in Formality Status between-Jobs and within-Job and Percent Change in Earnings

	Transition Rate (%)	Change in Earnings (%)
Between-Jobs		
I to I	1.8	+ 13
I to F	0.7	+ 30
F to I	0.2	− 8
F to F	0.7	+ 3
Within-Job		
I to F	2.2	+ 6
F to I	0.3	− 1

Source: *Pesquisa Mensal de Emprego*, March 2002 - February 2016. Employment statuses are formal (F) and informal (I). Between-jobs (within-job) transitions refer to the case in which the worker changes (keeps) her last-month job.

The evidence suggests transitions from informality are not infrequent relative to those taking place from formality. Moreover, they represent a driver of earnings growth. Indeed, the rate at which informal employees move into a different formal job has the same average magnitude as the one for formal employees (0.7%). Moreover, the former induce stronger upward revisions in earnings (+30% vs. +3%). Furthermore, workers also seem to exploit moves between informal jobs to climb up the job ladder: on average, 1.8% of informal employees move into a new informal job each month, experiencing an average earnings increase of 13%. Finally, upgrades in workers' formality status also take place within job. These within-job upgrades are, in fact, three times as large as those arising between jobs (2.2% vs. 0.7%) and are also associated with upward revisions in earnings (+6%).

In sum, informal contracts influence the way in which many workers in Brazil climb the job ladder. Any policy affecting the availability of these jobs should ponder its impact on this characterization. This project aims at providing a framework to implement this type of analysis.

3 Model

The model extends the environment in Lise and Robin (2017) to introduce informal and formal jobs, self-employment, and firm-level productivity shocks. Flows in employment and formality statuses arise from imperfect sorting (due to random search), on-the-job search, and shock arrivals. In equilibrium, the economy displays an endogenous productivity-stability characterization for job-worker matches that interacts with the trade-off between formal and informal contracts.

The framework's tractability relies on the match-level specification of job-worker surpluses. Hence, the model does not have a notion of firm size, and firms should be thought of as single-worker entities. I therefore use the terms *firm* and *job* interchangeably in the model discussion.

3.1 Environment

Time is discrete, and z denotes an aggregate productivity factor. The economy is populated by a measure one of risk-neutral, infinitely lived, heterogeneous workers. Workers differ in their skill endownment $x \in [0, 1]$, which is distributed according to an exogenous, time-invariant distribution $\ell(x)$. They are matched pairwise to heterogeneous jobs, which differ in terms of a productivity or technology index $y \in [0, 1]$. The distribution of jobs across technology levels is endogenously determined.

Matches are subject to stochastic shocks to both firm-level productivity (y) and aggregate productivity (z). They generate value added according to a function $p(x, y, z)$, which depends on the worker's skill, the job's idiosyncratic productivity, and the aggregate productivity factor.[12] The unemployed, on the other hand, generate value added according to a function $b(x)$, which depends exclusively on the worker's skill level. Finally, the model accounts for a self-employment state in which workers produce proportionally more than when unemployed. In particular, they generate value added $\mu b(x)$, where $\mu > 1$.

Jobs and workers are matched randomly in a frictional labor market. On the supply side, both unemployed and employed workers search, but on-the-job search is less intense. On the demand side, a measure v of vacancies at y-type jobs can be created every period subject to an exogenous cost function $c(v)$. The model adopts the sequential auction protocol in Postel-Vinay and Robin (2002). That is, firms are allowed to (i) make state-contingent offers to their contacted workers and (ii) respond to outside offers received by their incumbent employees. Incumbent and poaching firms engage in Bertrand competition for workers' services.

Each period, a $\delta \in [0, 1]$ fraction of matches and a $\delta_g \in [0, 1]$ fraction of self-employed jobs are exogenously destroyed. Similarly, a $\xi \in [0, 1]$ fraction of the unemployed move exogenously into self-employment.

Finally, firms and workers are allowed to establish formal or informal employment relationships. Both types of contracts entitle the worker to a specific wage, but the former implies a fixed cost $\gamma > 0$ the firm needs to pay to endogenously terminate a formal employment relationship, namely, to fire the formal worker. Informal contracts, conversely, avoid the firing cost, but they are subject to a detection probability $\kappa \in [0, 1]$. If detected, the firm needs to pay a fine equal to the value added generated by the match in the corresponding period $p(x, y, z)$.

3.2 Timing

Each period has six stages. First, an initial distribution of workers across employment states is carried from the previous period. This comprises the joint skill-technology densities of workers and firms in old (o) formal and informal matches, that is, $f_o(x, y)$ and $i_o(x, y)$; and the old skill densities of self-employed and unemployed workers, that

[12] The parametrization of this function allows for complementarities between worker and job types.

is, $g_o(x)$ and $u_o(x)$. These measures are such that the following identities hold:

$$\ell(x) = u_o(x) + g_o(x) + \int \Big[f_o(x, y) + i_o(x, y)\Big] dy \quad , \text{for all } x \in [0, 1], \tag{1}$$

and because ℓ is exogenous and time invariant, we can summarize the set of old densities by $\psi_o = \{f_o, i_o, g_o\}$.

In the second stage, shocks to aggregate (z) and idiosyncratic (y) productivity realize, and self-employment opportunities arise. In particular, a ξ fraction of workers who started the period unemployed transit exogenously into self-employment. As a result, densities (exogenously) change to $f(x, y)$, $i(x, y)$, and $g(x)$, and the aggregate state of the economy after shock realizations is represented by the duple (ψ, z).

Third, firms are allowed to adjust their labor force conditional on the realized aggregate state. In particular, firms can choose to reassign their ongoing informal matches to formal contracts at no cost, and they can also reassign their formal matches to informal contracts after paying the firing cost γ.[13] Simultaneously, firms are allowed to endogenously destroy matches that are no longer profitable to keep, given the realization of the idiosyncratic and aggregate productivity shocks. Finally, in this stage, we also assume a δ fraction of still-profitable old matches and a δ_g fraction of old self-employment jobs are exogenously destroyed. Therefore, let $\psi_m = \{f_m, i_m, g_m\}$ represent the set of updated densities after endogenous within-job transitions in formality statuses and endogenous and exogenous transitions into unemployment.

Fourth, workers search and firms post vacancies. In particular, the unemployed search with unity intensity, whereas the self-employed and matched workers search with intensity $\phi < 1$. The newly unemployed or newly self-employed are not allowed to search.[14] Hence, total search effort L exerted by workers is

$$L = \int \Big[u(x) \; + \; \phi \left[(1 - \delta)g_o(x) + \int \left(f_m(x, y) + i_m(x, y)\right) dy\right]\Big] \, dx. \tag{2}$$

On the demand side, a measure v of y-type jobs is created such that the marginal

[13]To reallocate a formal employee into an informal contract, the firm first needs to get the worker off of the official payroll. Hence, it needs to pay γ. This type of transition, however, does not occur in equilibrium given the characterization of (fixed) firing costs in this environment. These transitions are small in the data.

[14]This assumption is not quantitatively important, because the model is simulated at a weekly frequency. However, it simplifies the specification of the environment's value functions.

cost of creating a vacancy unit is equal to its expected value:

$$c'(v(y;\psi,z)) = q(\psi,z)\, J(y;\psi,z), \tag{3}$$

where $q(\psi,z)$ and $J(y;\psi,z)$ represent, respectively, the probability that a vacancy unit contacts a job seeker and the expected value from a contact for a y-type firm. Both objects are specified in detail below. Total vacancies in the economy are hence computed as $V(\psi,z) = \int v(y;\psi,z)dy$.

Fifth, meetings occur according to a constant-returns-to-scale function $M(L,V) = Lm(\theta)$, where $\theta = \frac{V}{L}$ represents the labor market tightness. Therefore, the unemployed contact vacancies with probability $\lambda = \frac{M}{L} = m(\theta)$, the employed do so with probability $\phi\lambda = \phi\, m(\theta)$, and a vacancy unit contacts job seekers with probability $q = \frac{M}{V} = \frac{m(\theta)}{\theta} = \frac{\lambda}{\theta}$. When a contact occurs, firms can offer a formal or informal contract to prospective workers. If unemployed or self-employed, we assume the contacted worker has zero bargaining power. If already employed, poaching and incumbent firms engage in Bertrand competition. At the end of this stage, a new (n) set of matches $\psi_n = \{f_n, i_n, g_n\}$ emerges.

Finally, in the sixth stage, production and detection of informal matches take place. In particular, I assume that at the end of the period, a κ fraction of informal matches are detected, and firms pay a fine equal to the value added of the match in the corresponding period $p(x,y,z)$.

3.3 Value functions

3.3.1 Unemployed workers

Consider an x-type worker unemployed during the production stage of a particular period with value function denoted by $B(x;\psi,z)$. First, in the current period, the worker generates value added according to $b(x)$. Next period, with probability ξ, she transits into self-employment and does not get to search. $G(x;\psi',z')$, the value function for self-employment, represents the corresponding continuation value in this case. With probability $1-\xi$, however, she does not transit into self-employment and searches for a vacancy in the labor market. She contacts one with probability $\lambda(\psi',z')$ and is offered a particular value. This offer depends, in principle, on the job type she contacts. However, due to zero bargaining power, the unemployed are always offered their reservation value by firms willing to hire them, namely, $B(x;\psi',z')$. Therefore,[15]

$$B(x;\psi,z) = b(x) \; + \; \frac{1}{1+r}\, E\left[\, B(x;\psi',z') + \xi\,\left[G(x;\psi',z') - B(x;\psi',z')\right] \,\middle|\, \psi, z \right]. \quad (4)$$

3.3.2 Self-employed workers

Consider now $G(x;\psi,z)$, the value function for self-employment, and the corresponding surplus function $S_g(x;\psi,z) = G(x;\psi,z) - B(x;\psi,z)$. A self-employed worker generates value added $\mu b(x)$ during the production stage. Next period, she transits into unemployment with probability δ_g. Otherwise, she searches for a vacancy, and because of zero bargaining power, the worker is offered the value of staying self-employed if contacted. Hence,[16]

$$G(x;\psi,z) = \mu b(x) \; + \; \frac{1}{1+r}\, E\left[\, G(x;\psi',z') - \delta_g\, S_g(x;\psi',z') \,\middle|\, \psi, z \right]. \quad (5)$$

Then, combining equations (4) and (5), we get

$$S_g(x;\psi,z) = B(x;\psi,z) - G(x;\psi,z) = \frac{(1+r)(\mu-1)}{r+\delta_g+\xi} b(x). \quad (6)$$

In words, equation (6) implies we can compute the surplus of self-employment for an x-type worker independently from the aggregate state of the economy. The same holds for the value functions of unemployment and self-employment. Abusing notation, I henceforth denote the value functions of unemployment and self-employment $B(x)$ and $G(x)$ and the surplus function of self-employment $S_g(x)$. We will achieve a similar result for surpluses of informal and formal matches, and this is what will provide tractability to the model.

3.3.3 Informal and formal matches

Let $P_j(x,y;\psi,z)$ for $j = \{i, f\}$ represent the value of an x-y match that operates under an informal or a formal contract. Note as well that because no unemployed worker is

[15] Complete derivations are in Appendix B.1.
[16] Complete derivations are in Appendix B.2.

willing to form a match unless she gets a value of at least $B(x)$, considering the corresponding surplus functions $S_j(x, y; \psi, z) = P_j(x, y; \psi, z) - B(x)$ will be helpful. Because firms get a fraction of the match surplus, they are willing to keep any match providing non-negative surplus. Conversely, they are better off destroying any match generating a negative surplus. This is the endogenous source of transitions into unemployment in the economy. As already mentioned, I also allow for exogenous separations: every period, a δ fraction of matches with non-negative surpluses is exogenously destroyed. In other words, only a $(1 - \delta)$ fraction of matches with non-negative surpluses survive the separations stage.

Moreover, to properly compute the probability that a match survives the separation and within-job-reallocation stage, we need to account for the fact that firms have the option to reassign their workers between formal and informal contracts every period. We focus first on the case of a match operating informally. Define $P(x, y'; \psi', z') = \max\{P_i(x, y'; \psi', z'), P_f(x, y'; \psi', z')\}$. An informal worker can be reallocated to a formal contract at zero transition cost. Therefore, the probability that an informal match survives the next period's separation and within-job-reallocation stage can be represented by $(1 - \delta)\ E_{y;\psi,z}\, \mathbb{1}\Big\{P(x, y'; \psi', z') \geq B(x)\Big\}$.[17] Taking this result into account, Appendix B.3 specifies in detail the value function $P_i(x, y; \psi, z)$.

An informal match generates value added according to $p(x, y, z)$, which is seized with probability κ at the end of the period. Next period, if the match is destroyed during the separation and within-job-reallocation stage, the continuation value for the match is equal to $B(x)$, that is, what the worker would get from unemployment. If, on the other hand, the match survives, the worker searches on the job. With probability $[1 - \phi\, \lambda(\psi', z')]$, she does not contact a vacancy and stays at the firm under a formal or an informal contract, with continuation value represented by the object $P(x, y'; \psi', z')$. With probability $\phi\, \lambda(\psi', z')$, however, the worker does get an outside offer. In that case, the incumbent (y') and the poaching $(\widetilde{y})$ firms engage in Bertrand competition for the worker's labor services.

The poaching firm can offer the worker a formal or an informal contract. We can consider then two cases. If the match value with the incumbent firm is larger, the worker stays and the continuation value for the match is just $P(x, y'; \psi', z')$. If, on

[17]Note I consider y' to account for idiosyncratic productivity shocks.

the other hand, the match value with the poaching firm is larger, the informal worker leaves for a value equal to the second-highest offer — in this case, what the incumbent firm could have offered her at most, namely, $P(x, y'; \psi', z')$.

Therefore, between-employer Bertrand competition induces the continuation value of the informal match to be independent of whether the worker is poached. This result considerably simplifies the informal match value function and, after some re-arrangement, implies the following:[18]

$$\begin{aligned} S_i(x, y; \psi, z) = (1-\kappa)\, p(x, y, z) - \frac{r + \delta_g + \xi\mu}{r + \delta_g + \xi} b(x) \\ + \frac{1-\delta}{1+r} \sum_{z'} \sum_{y'} S(x, y'; \psi', z')^{+} \pi_y(y, y') \pi_z(z, z'), \end{aligned} \tag{7}$$

where π_y and π_z represent the transition matrices for firm-level and aggregate productivity, and I define $S(x, y'; \psi', z') = P(x, y'; \psi', z') - B(x)$, and $h^{+} = \max\{h, 0\}$.

Appendix B.4 presents the analogous procedure for the case of formal matches. A formal worker can be reallocated to an informal contract, but the firm would first need to pay γ — the cost to terminate formal contracts. Because the firing cost is fixed, within-job downgrades in formality status do not occur in equilibrium. Therefore, a formal match survives next period's separation and within-job-reallocation stage with probability $(1-\delta)\ E_{y;\psi,z}\, \mathbb{1}\Big\{P_f(x, y'; \psi', z') \geq B(x) - \gamma\Big\}$. In the specification of the value function, I distinguish between an exogenous and an endogenous separation: only the latter induces the payment of firing costs. Proceeding similarly as before, we get[19]

$$\begin{aligned} S_f(x, y; \psi, z) = p(x, y, z) - \frac{r + \delta_g + \xi\mu}{r + \delta_g + \xi} b(x) - \frac{\gamma}{1+r} \\ + \frac{1-\delta}{1+r} \sum_{z'} \sum_{y'} \mathbb{1}\Big\{S_f(x, y'; \psi', z') + \gamma \geq 0\Big\} \Big(S_f(x, y'; \psi', z') + \frac{\gamma}{1-\delta}\Big) \pi_y(y, y') \pi_z(z, z'). \end{aligned} \tag{8}$$

From equations (7) and (8) we can conclude that informal and formal matches' surpluses depend on the aggregate state of the economy (ψ, z) only through the ag-

[18]Complete derivation is reported in Appendix B.3.
[19]Complete derivation is reported in Appendix B.4.

gregate productivity shock z. This result implies we can solve for the surplus of every potential match, that is, every possible combination of x and y, for any realization of the aggregate productivity shock without having to track how the distribution of workers changes across employment states. Moreover, having characterized these surplus functions and provided a particular sequence of aggregate shocks and some initial distribution of workers across employment states, we can recover the whole dynamics for unemployment, informal employment, formal employment, self-employment, vacancies, and related transition rates, as shown in Appendices B.5 to B.8. Similar to Lise and Robin (2017), this result is what makes the framework tractable.

4 Estimation

4.1 Parametrization

The first column in Table 3 presents the parametrization of the model, which follows Lise and Robin (2017). In particular, I consider evenly spaced 15-point grids for skill endowment x and firm-level technology y, both within the $(0, 1)$ support. The distribution of workers across skill levels $\ell(x)$ is assumed to be a beta distribution with parameters β_1 and β_2. Aggregate productivity z follows an AR(1) process with persistence parameter ρ_z and unconditional variance σ. I implement the Rouwenhorst (1995) method to discretize this process along a 15-point grid with support defined by σ and to characterize the corresponding transition matrix π_z.

The assumed technology function is quadratic and allows for complementarities between worker and job types. These complementarities are assumed to be positive or negative depending on the estimated sign for p_6. Home production $b(x)$ is defined as the fraction $\iota \in [0, 1]$ of the value added an x-type worker would generate with her optimal firm under the median shock realization. The cost function to create vacancies is assumed to be strictly increasing and strictly convex with parameters $c_0 > 0$ and $c_1 > 0$. Finally, the matching function is specified as a constant-returns-to-scale Cobb Douglas function with efficiency factor $\alpha > 0$ and labor elasticity $\omega \in [0, 1]$.

Table 3: Parametrization [estimated parameters in blue font]

$\ell(x)$ beta distribution	$\beta_1 = 2.0$ and $\beta_2 = 2.0$
$log\ z' = \rho_z\ log\ z\ +\ \sigma\sqrt{1-\rho^2}\ \nu'$ Similarly for idiosyncratic shock	$\nu' \sim \mathcal{N}(0,1)$, $\rho_z =\ 0.9987$ and $\sigma =\ 0.0658$. $\rho_y =\ 0.9974$
$p(x,y,z) = z(p_1 + p_2\,x + p_3\,y$ $+p_4\,x^2 + p_5\,y^2 + p_6\,xy)$	$p_1 = 0.0482$, $p_2 =\ 0.413$, $p_3 =\ 0.017$ $p_4 =\ -4.34$, $p_5 =\ -5.26$, $p_6 =\ 9.33$
$b(x) = \iota\big(x, y^*(x, z_{50th}), z_{50th}\big)$	$\iota = 0.69$
$c(v) = [c_0 v^{1+c_1}]/[1+c_1]$	$c_0 = 0.028$ and $c_1 = 0.084$
$M(L,V) = min\{\alpha\, L^{\omega}\, V^{1-\omega},\ L,\ V\}$	$\alpha = 0.497$ and $\omega = 0.5$
Additional	$\delta =\ 0.00119$ (weekly), $r = 0.05$ (annual), $\phi =\ 0.083$
Self-Employment	$\delta_g = 0.0218$; , $\xi = 0.0244$, $\mu = 1.083$
Informal/Formal contracts	$\gamma = 0.0264$ and $\kappa =\ 0.03$

4.2 Simulation

First, given exogenously defined $p(x,y,z)$, $b(x,z)$, δ, δ_g, ξ, μ, r, γ, κ, π_y, and π_z, I can characterize the surplus functions of self-employment and of informal and formal matches for all worker and firm types and all possible realizations of the aggregate shock. In particular, I set $S_g(x)$ according to equation (6) and establish initial guesses $S_i^0(x,y,z)$ and $S_f^0(x,y,z)$ for all x, y, and z. Then, I iterate the system defined by equations (7) and (8) until the change in the surplus functions is lower than some established tolerance level.

Second, given the characterization of surpluses and all exogenous objects defined in Table 3, I can compute the equilibrium distribution of workers across employment states for any level of the aggregate productivity shock (e.g., the median realization). Computationally, I need to start from some initial guesses for the measures of unemployed workers $u^0(x)$, self- employed workers $g^0(x)$, informal matches $i^0(x,y)$, and formal matches $f^0(x,y)$, solve the whole model given the considered shock realization, and iterate until the four objects reach a resting point — again, given some tolerance level.

Finally, to compute the dynamics of the model, I need to simulate it along T periods. In particular, given an initial distribution of workers across employment states (e.g., the equilibrium distribution under the median aggregate schock) and some exogenous sequence of aggregate productivity shocks $\{z_t\}_{t=1}^{T}$, I can compute the evolving measures $\{u_t(x),\, g_t(x),\, i_t(x,y),\, f_t(x,y)\,\}_{t=1}^{T}$ and the corresponding transition rates.

4.3 Estimation Algorithm

The model contains 24 parameters, some of which cannot be identified with the available data. For instance, I do not have vacancy data for Brazil. Hence, I fix the parameters related to the vacancy-creation cost function — c_0 and c_1 — and the matching function — α and ω — to the values considered by Lise and Robin (2017). Both cases correspond to standard characterizations of these functions in the literature. Additionally, I set the discount rate r to be 5% (annually) and define workers to be distributed across skill levels according to a symmetric beta distribution with parameters $\beta_1 = \beta_2 = 2$.

The parameters δ_g and ξ are directly identified by the transition rates from unemployment to self-employment and from self-employment to unemployment, respectively. I therefore calibrate them to match the observed average monthly values of 8.2% and 9.1% in the data. That is, I set $\delta_g = 0.0218$ and $\xi = 0.0244$ at a weekly frequency. The trade-off of informality in the model is determined by two parameters, γ and κ. I set the weekly detection probability of informal matches κ to be equal to 3% and let γ be free for estimation.

I then estimate the remaining 14 parameters in the model through the simulated method of moments (SMM). In particular, I minimize the distance between 24 data moments and their model analogues. To do so, I use the Metropolis–Hastings algorithm, which is a Markov chain Monte Carlo (MCMC) method for classical estimators as proposed by Chernozhukov and Hong (2003). This approach draws parameter candidates from a proposal distribution, simulates the entire model, and accepts or rejects jumps in the parameter space based on how likely the sample is given the data moments. The method is therefore computationally intensive, because it requires solving the model thousands of times in parallel processors. On the other hand, it works well in applications where the objective function of the extremum estimator is not smooth.[20]

Table 4: Targeted Moments for Estimation

	Moments	**Data**	**Model**
1)	E[U]	9.28	9.32
2)	E[share I]	20.51	18.48
3)	E[share G]	15.56	9.44
4)	E[I to U]	2.82	3.08
5)	E[F to U]	1.29	1.06
6)	E[U to I]	4.03	6.72
7)	E[U to F]	3.5	4.95
8)	E[share I of (U to E)]	53.89	57.82
9)	E[f-U to I]	9.41	10.28
10)	E[f-U to F]	11.96	12.95
11)	E[share I (f-U to E)]	43.88	44.65
12)	E[I to F - within]	2.25	1.75
13)	E[I to I - between]	1.81	0.48
14)	E[I to F - between]	0.75	0.58
15)	E[F to I - between]	0.23	0.26
16)	E[F to F - between]	0.74	0.7
17)	E[G to I]	0.49	0.53
18)	E[G to F]	0.31	0.4
19)	sd[GDP]	3.58	3.64
20)	sd[U]	7.8	8.46
21)	sd[share I (f-U to E)]	4.83	3.95
22)	autocorr[GDP]	96.32	94.95
23)	corr[U]	-77.25	-97.13
24)	corr[share I (f-U to E)]	-68.7	-82.63

Note: All data moments (except for GDP) are based on series computed from the *Pesquisa Mensal de Emprego*, March 2002 - February 2016. The considered series are unemployment (U), the incidence of informal jobs in the private sector (share I), the incidence of the self-employed in the workforce (share G), gross domestic product (GDP), and the various transition rates introduced in Tables 1 and 2, except for within-job downgrades in formality status. Moreover, I also target the share of new matches accounted for by informal contracts, both among all the unemployed and among the unemployed who have held a formal job just before their current unemployment spell.

The targeted moments (reported in Table 4) comprise averages, standard deviations, and correlations with output of time series generated from the *Pesquisa Mensal*

[20]See Lise (2012) and Jarosch (2015) for implementations of this technique in similar environments.

de Emprego. These series cover the period between March 2002 and February 2016, and they are seasonally adjusted and detrended before computing the targeted moments. To generate the model counterparts, I simulate the model for 120 quarters and consider a random sequence of shocks. The model is simulated at a weekly frequency, but it replicates the survey structure to aggregate series at the monthly and quarterly frequencies.

4.4 Results: Fit and Parameter Estimates

The fit of the model to the targeted characterization of the Brazilian economy is quite good. Simulated moments related to the *levels* of the unemployment rate, the incidence of informal contracts in the cross section of matches, and the associated transition rates are consistent with the evidence presented in section 2.

First, informal employees fall off the ladder, that is, lose their jobs almost three times as frequently as formal ones. Second, although informal jobs account only for 18% of matches in the cross section, they are responsible for 58% of newly created matches each period. Moreover, among unemployed workers who previously held a formal job, this rate still amounts to 45%. Third, upgrades in workers' formality statuses are frequent both within and between jobs. In particular, each month, 0.58% (0.7%) of informal (formal) employees transit into a new formal job, whereas 1.75% of informal employees are reallocated into a formal contract within the same job. Only moves between informal jobs are poorly matched by the model. The corresponding transition rate is 1.8% in the data, but only 0.5% in the estimated model.

The framework also matches the standard deviations and correlations with output of the unemployment rate and of the share of previously formal unemployed workers who return to work through an informal contract. Importantly, moments related to the latter time series inform how the role of informal jobs changes across the business cycle. As in the data, both series are countercyclical in the model. In particular, as aggregate downturns arise, a larger fraction of workers rely on informal contracts to shorten their unemployment spells. The standard deviation and autocorrelation of GDP are also well fitted in the simulations.

Table C.3 in Appendix C shows the fit of the framework to non-targeted moments is also reasonably good. Notably, these moments include the standard deviations and correlations with output of all the transition rates discussed in section 2 — recall I only target their levels in the estimation. The only exceptions are the correlation of the rate at which formal employees transit into unemployment ("F to U") and the correlation of within-job formality upgrades ("I to F - within"). The former result is expected. The "F to U" rate is procyclical in the data probably because good economic conditions increase voluntary quits into unemployment. In this environment, however, I do not allow the value-added function for unemployed workers to depend on the aggregate productivity shock. As a result, positive aggregate shocks do not induce voluntary quits into unemployment. This assumption is important for γ to be interpreted as a firing cost: endogenous transitions of employees into unemployment always correspond to layoffs in this environment. The second exception, on the other hand, suggests within-job upgrades in the current version of the estimated model mainly come as a result of idiosyncratic shocks. This implication is potentially more important, and I plan to explore its behavior in future implementations of the estimation algorithm.

Figures C.1 and C.2 provide further evidence in support of the fit of the model to the data. In particular, I first pin down the sequence of aggregate shocks in the model that best replicate the quarterly GDP series observed in Brazil between 2004 and 2015. Then, I simulate the model considering the fitted sequence of shocks along 48 quarters. Figure C.1 plots the resultant model-generated GDP series, and Figure C.2 reports the rates of unemployment at different duration levels. These rates are well traced along the observed business cycles for Brazil, even when these rates are not directly targeted by the estimation algorithm.

The estimated parameter values are similar to other results in the literature, particularly those of Lise and Robin (2017) . Although these parameters are not mapped one-to-one to moments in the data, I provide heuristic arguments for their identification. The rate of exogenous separations δ is identified by the frequency at which workers — particularly formal employees — transit from employment into unemployment. ι, μ, and ϕ are identified by the rate at which the unemployed, the self-employed, and already matched workers move into new (formal or informal) matches. γ is identified by the incidence of informal matches in the cross section and by the frequency of transitions in formality statuses. The aggregate shock parameters ρ_z and σ are identified by the autocorrelation and standard deviations of GDP. The firm-level shock parameter ρ_y is identified by the level of transitions in the data in excess of those generated by the aggregate shock and exogenous separations.

Identification for the parameters of the quadratic value-added function $p(x, y, z)$ comes from separation rates, transitions in formality statuses taking place within and between jobs, and from two restrictions I impose on the minimization problem for estimation, namely, that the incidence of informal contracts must decrease with the workers' skill level and with the firms' technology factor, as observed in the data.[21] Because $p(x, y, z)$ determines the quality of matches between worker and firm types, and this characterization influences the trade-off of informality, these restrictions inform the characterization of complementarities between worker and firm types in the estimation.

4.5 Mechanisms in the Model

Search frictions in the model induce imperfect sorting in equilibrium. Workers are willing to work not only at their preferred jobs, but also at a range of job types. Figure 1 depicts this outcome. The dashed black line represents the optimal job type $y^*(x, z_{50th})$ for each x-type worker; whereas the red and blue solid lines in the upper panel (henceforth, contour lines) represent x-y pairs generating zero surplus given the 10th (i.e. "bad") and 90th (i.e. "good") percentile realizations of the aggregate productivity shock, respectively.[22] Because surpluses increase toward high-type matches (i.e., toward the upper-right corner of the graph, as shown by the lower panel), x-y pairs within the contour lines imply positive surpluses. Therefore, contour lines define the set of acceptable or feasible matches under particular shock realizations. These matches are x-y pairs that, if contacted under the considered shock realization, are effectively formed. Imperfect sorting then is represented by the fact that matches do not need to be on the dashed line to be feasible.

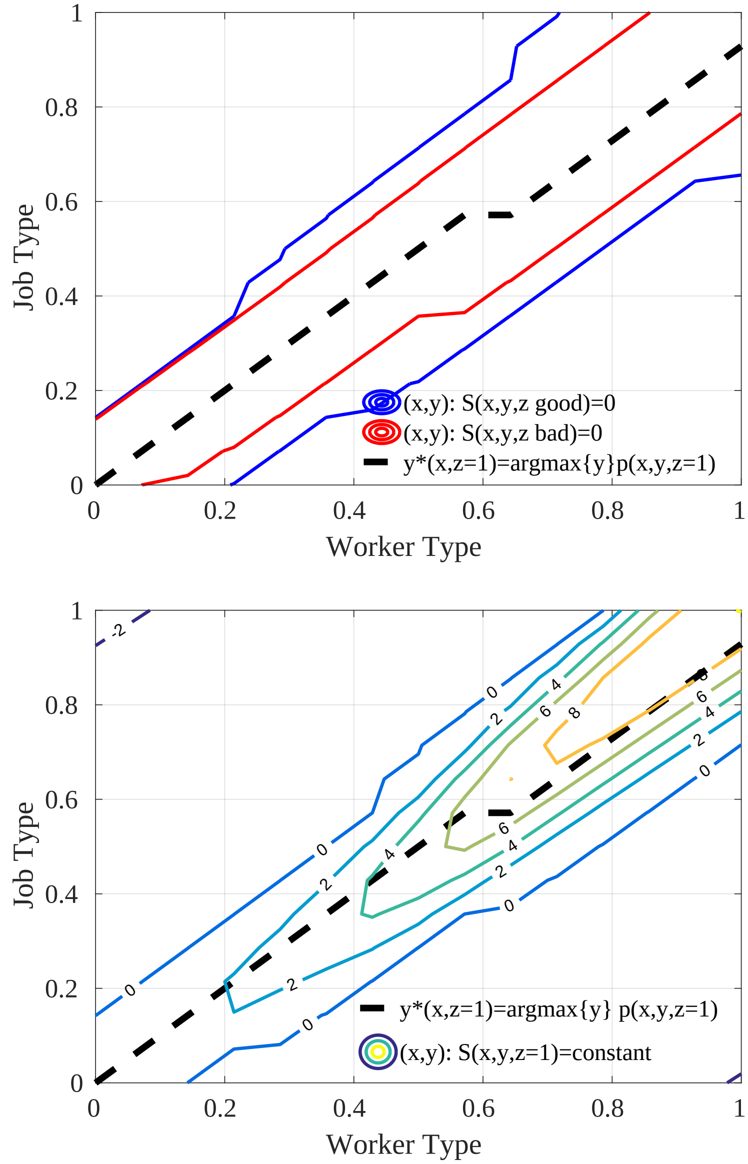
Furthermore, recall that in the case of informal matches, the object $S(x, y, z)$ determines separation decisions as well. Then, Figure 1 implies that if the economy transits from the 90th percentile to the 10th percentile aggregate shock realization, that is, from the blue to the red contour line, every existing informal match in the area

[21] To be precise, I first divide the 15-point grids for skill endowment x and firm-level technology y into three groups, each with five grid categories. Then, I require the incidence of informal matches to be monotonically decreasing along these three groups.

[22] The positive slope of the dashed black line results from positive complementarities between worker and job types, as defined by the estimated p_6 parameter.

between both lines is endogenously destroyed. They are no longer feasible under the newly realized productivity shock. In this sense, the closer a match is to the contour line, the more sensible it is to aggregate shocks — and also to firm-level shocks. The same reasoning applies to a formal match, but incorporating the need to pay γ.

Figure 1: Surpluses and Matches Feasibility Sets



The trade-off between formal and informal contracts interacts with this productivity-stability characterization for matches.[23] Figure 2 illustrates this aspect. The yellow area in both panels represents x-y pairs for which the surplus under a formal contract is higher than under an informal one. To interpret this outcome, recall formal contracts introduce costs upon layoffs, whereas informal ones induce a detection probability (and corresponding fine) every period. Therefore, the lower the survival probability of a match, the more relatively expensive the formal contract option becomes. In other words, matches close enough to (far away enough from) the contour lines are assigned to informal (formal) contracts by firms.

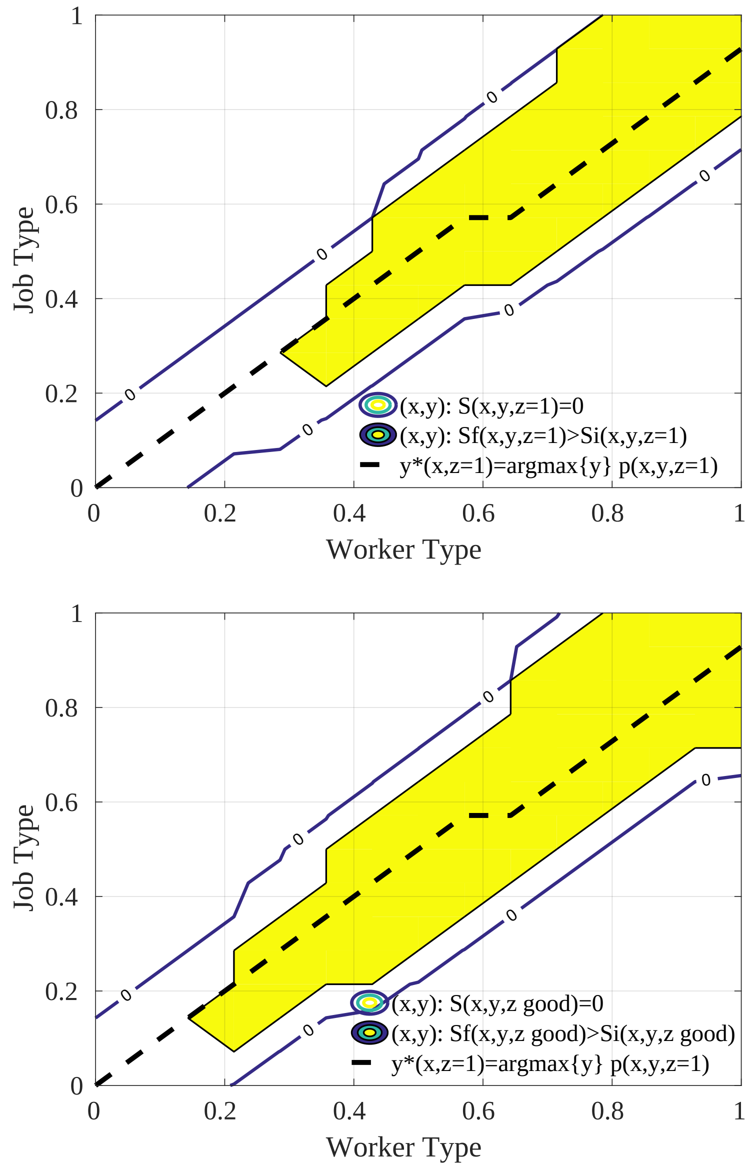
As expected, when we consider better productivity shock realizations (the upper panel considers the 50th percentile shock realization, and the lower panel shows the 90th percentile case), more matches are suitable to operate formally. Observe, however, that even at the 90th percentile shock realization, some workers can find jobs only in the informal sector. Also, on the other side and depending on the level of aggregate productivity, some firms are willing to hire workers only under informal contracts. Nevertheless, informality is widespread across all worker and job categories. This characterization of formal and informal matches is consistent with the documented empirics.

Note as well that on-the-job search introduces job ladders in this environment. Unemployed workers accept in principle any job within the feasibility set and gradually move toward their preferred firm through on-the-job search. In other words, workers transit from the contours (i.e. poorly stable matches) of the feasibility set towards the dashed line (i.e. highly stable matches). That is, workers move from informal into formal jobs across time.

The above discussion deals with the characterization of the surpluses across aggregate productivity shock realizations. By contrast, Figures C.3 and C.4 plot the equilibrium joint skill-technology densities of matched workers under the median-level shock realization. In particular, Figure C.3 reports the distribution of all active (formal and informal) matches, and Figure C.4 distinguishes between informal (upper panel) and formal (lower panel) matches. In equilibrium, 18% of the matches are informal and are concentrated around the contour lines. On the other hand, formal matches are concentrated along the dashed line.

[23]The matches' productivity-stability characterization is an outcome already available in Lise and Robin (2017).

Figure 2: Allocation of matches across formal and informal contracts



5 Counterfactual Analysis

Having established that the model is empirically plausible, I use it to evaluate the effects of a policy often assessed in the literature: increasing labor law enforcement.

In particular, I simulate the economy for 48 quarters using the sequence of aggregate shocks in the model that best replicate the quarterly GDP series observed in Brazil between 2004 and 2015. This sequence is the one I pinned down to construct Figures C.1 and C.2; see section 4.4 for a detailed discussion. The first column in Table 5 reports moments from the simulation that considers the baseline parameters.[24] The second column, on the other hand, reports moments from a simulation that considers an increase in the detection probability of informal matches — κ — from 3% to 10%.

Table 5: Baseline and Counterfactual

Moments	Baseline ($\kappa = 3\%$)	Counterfactual ($\kappa = 10\%$)
E[share I]	18.02	2.54
E[U]	9.34	10.00
E[share G]	9.38	10.21
E[J to J - between]	1.11	0.98
E[GDP]	1.00	0.99
E[Share of Matches, $x \in (0, 0.33]$]	14.39	13.19
E[Share of Matches, $x \in (0.33, 0.66]$]	58.63	59.43
E[Share of Matches, $x \in (0.66, 1)$]	26.98	27.39

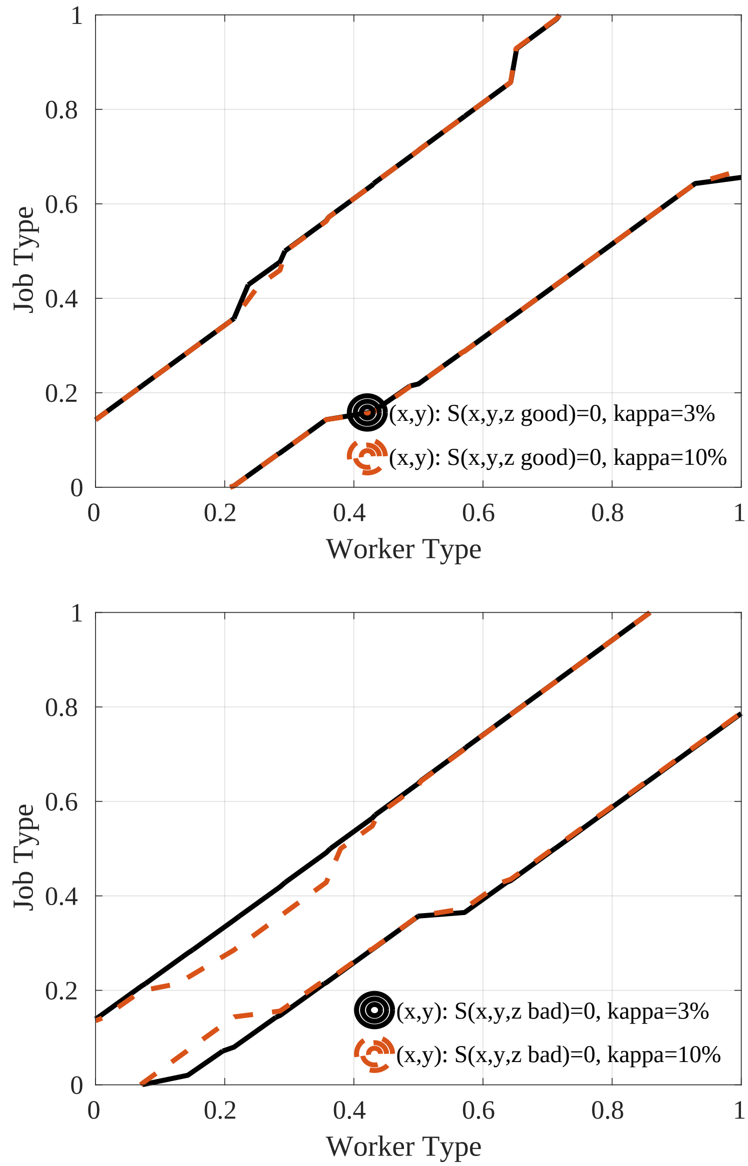
The counterfactual policy makes employing workers informally costlier for firms, and therefore reduces the availability of informal contracts in the economy. Indeed, the results suggest that if detection had been set at 10% during the period between March 2002 and February 2016, the incidence of informal jobs in the cross section of matches would have dropped from 18% to 3%. The policy counterfactual, however, induces some negative unintended consequences as well. On average, unemployment and self-employment increase by 0.7 and 0.8 percentage points respectively, aggregate output falls by 1.0%, and the rate of job-to-job transitions decreases from 1.11% to 0.98%. Furthermore, the distribution of matched workers tilts toward the medium- and high-skill workers. As reported in the last three rows in Table 5, low-skill workers account for 13.2% of matches in the economy under the counterfactual policy, whereas they represent 14.4% of matches in the baseline scenario.

These unintended consequences are particularly salient under bad realizations of the aggregate productivity shock. Figure 3 shows how the set of feasible matches changes between the baseline and counterfactual scenarios, under the 90th percentile

[24] Note these moments are slightly different than the ones reported in Table 4, because here I am considering a different sequence of aggregate productivity shocks and a smaller number of periods.

(good) aggregate shock and the 10th percentile (bad) aggregate shock. The feasibility set does not change much under the good shock. Conversely, under the bad shock, the feasibility set shrinks among the low skilled.

Figure 3: Feasibility Sets under Baseline and Counterfactual, Good Shock (Upper Panel) and Bad Shock (Lower Panel)



6 Conclusions

In this paper, I develop and estimate a model of the labor market that features a series of job ladders with informal and formal rungs. Importantly, these ladders are skill specific and change with the business cycle. In the model, matches can operate under formal or informal employment contracts. Formal contracts impose firing costs on firms, whereas informal ones induce a penalty fee if the employment relationship is detected. This characterization induces firms to allocate their less (more) productive and less (more) stable matches to informality (formality), while workers transit, through on-the-job search, from informal to formal jobs across time.

The framework successfully reproduces the observed heterogeneity and dynamics around informality in Brazil. First, informal employees fall off the ladder, that is, lose their jobs almost three times as frequently as formal ones. Second, although informal jobs account only for 18% of matches in the cross section, they are responsible for 58% of newly created matches each period. Moreover, even among unemployed workers who held a formal job in the past, 45% return to work through an informal contract. Third, this pattern is strongly countercyclical: as aggregate downturns arise, more workers rely on informal contracts to exit unemployment. Fourth, upgrades in workers' formality statuses are frequent both within and between jobs. In particular, each month, 0.58% (0.7%) of informal (formal) employees transit into a new formal job, whereas 1.75% of informal employees are reallocated into a formal contract within the same job. Therefore, as it is well documented in the literature, informal jobs are less productive and are subject to higher layoff risk than their formal counterparts in this environment. However, workers exploit informal contracts not only to smooth transitions between employment and non-employment, but also to advance their careers through moves within and between jobs.

Counterfactual analysis indicates accounting for these features is relevant policy-wise. According to the model, stronger enforcement of penalties against informal matches increases unemployment and self-employment, dampens job-to-job transitions, reduces total output, and disproportionately hurts the low skilled. These implications are at odds with those in Meghir et al. (2015), who also consider a framework with search on the job for informality. Three elements explain the difference. First, because workers are heterogeneous in this framework, not all of them transit easily from informal to formal jobs with the counterfactual policy. This observation is particularly relevant the lower the skill of a worker. Second, because firms are also heterogeneous and there are positive skill-technology complementarities, some matches are no longer feasible in the counterfactual scenario. As a result, workers face job ladders with a smaller number of rungs — they take longer to enter the first tier and also to move up across jobs. Finally, tractability out of steady state allows me to incorporate pertinent dynamics to the analysis: the referred unintended consequences of increasing enforcement are particularly salient during aggregate downturns.

Recent empirical work suggests the issue of transitional dynamics is central to the study of informal employment relationships in developing economies. Dix-Carneiro and Kovak (2019), for example, study the regional labor market effects of a unilateral trade liberalization (UTL) episode in Brazil. They document strong medium-run increases in non-employment among regions that were highly exposed to the implemented tariff cuts. Most importantly, they find the longer-run employment recovery in these markets took place entirely through informal jobs. Similarly, Ulyssea and Ponczek (2018) show negative labor-demand shocks induced by UTL caused significant increases in informality among regions with weak enforcement, but significant increases in non-employment among regions with strong enforcement. This paper provides a framework to build on in this regard.

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A Motivating Facts: Additional Figures and Tables

Figure A.1: Informality across the worker's lifecycle

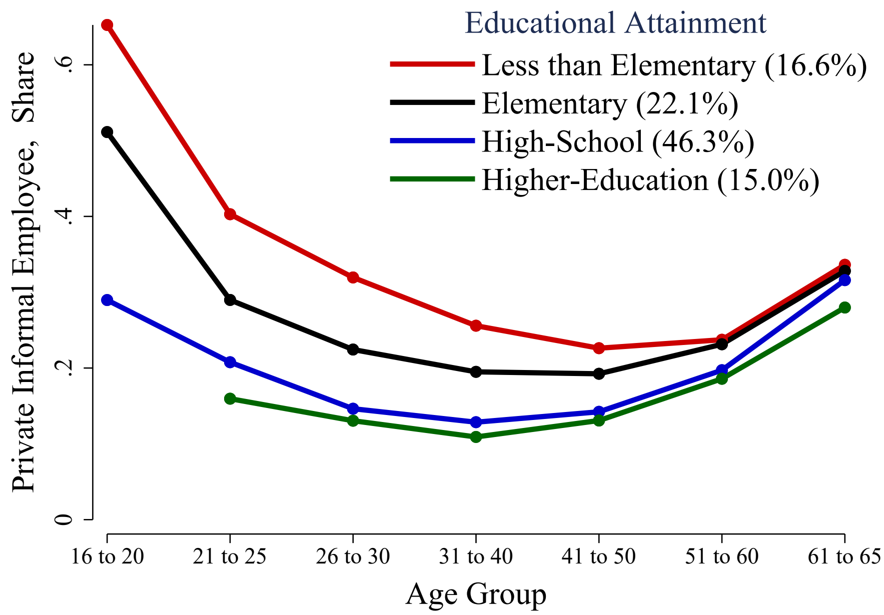
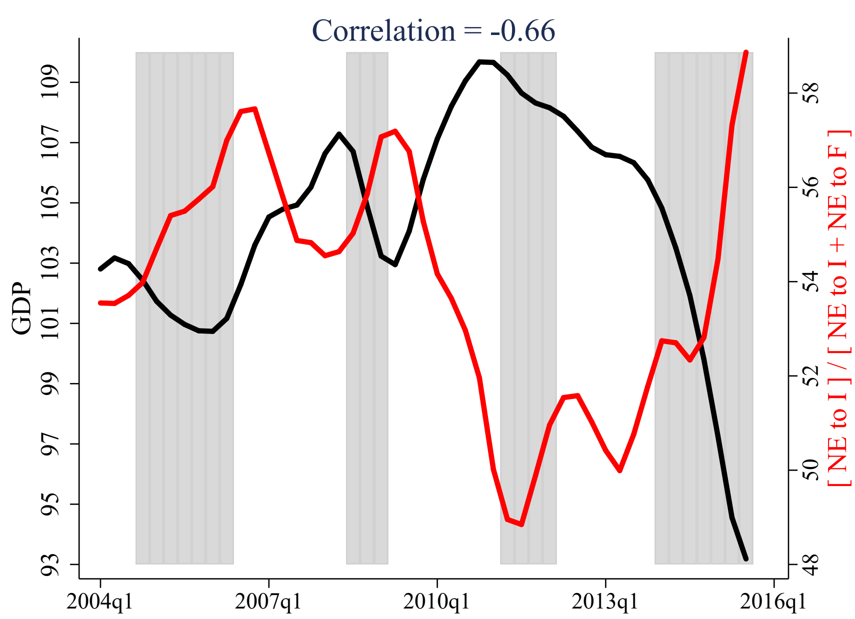
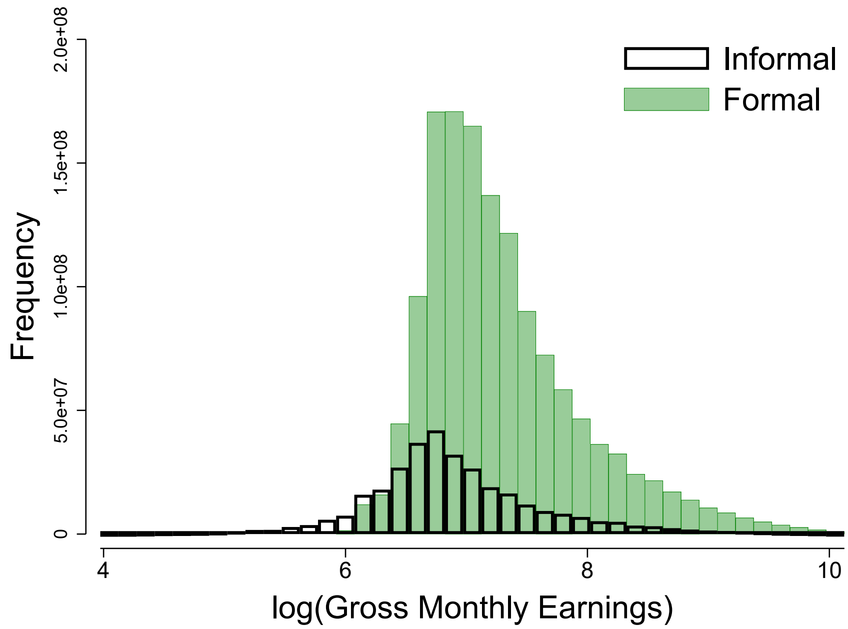


Figure A.2: [Cyclical] GDP and NE to I / (NE to I + NE to F)



Source: *Pesquisa Mensal de Emprego*, March 2002 - February 2016. NE to I / (NE to I + NE to F) represents the share of inflows from non-employment into private sector employment accounted for by informal contracts.

Figure A.3: Log Monthly Earnings, Histograms



Source: *Pesquisa Mensal de Emprego*, March 2002 - February 2016. I trim the 0.5 percent bottom and top before plotting each distribution.

Table A.1: Monthly Transition Rates, by Demographic Group

	No High School Diploma Young		No High School Diploma Prime-Age		High School Graduates Young		High School Graduates Prime-Age		All	
Into NE										
I to NE	3.5	665	2.2	758	3.0	729	1.3	1175	2.2	796
F to NE	2.0	900	0.9	1054	1.3	1032	0.6	1692	0.9	1215
From NE										
NE to I	4.3	648	3.3	728	3.3	762	2.1	1003	3.1	776
NE to F	1.9	871	2.0	1019	3.2	971	2.8	1482	2.6	1102
f-NE to I	5.6	742	4.9	809	4.1	844	2.6	1105	3.8	879
f-NE to F	4.6	903	4.8	1090	5.3	1014	4.9	1574	5.0	1190
Between-Jobs										
I to I	3.2	+15	2.1	+7	2.1	+18	1.1	+8	1.8	+13
I to F	0.8	+26	0.6	+23	1.2	+39	0.5	+29	0.7	+30
F to I	0.5	-7	0.2	-10	0.3	-4	0.1	-8	0.2	-8
F to F	1.1	+2	0.7	+3	1.1	+4	0.5	+4	0.7	+3
Within-Job										
I to F	2.1	+6	1.7	+5	3.8	+7	2.0	+5	2.2	+6
F to I	0.8	-2	0.3	-1	0.5	-2	0.2	-1	0.3	-1

Source: *Pesquisa Mensal de Emprego*, March 2002 - February 2016. Employment statuses are formal (F), informal (I), non-employed (NE), and currently non-employed who report having held a formal job within the last year (f-NE). Between-jobs (within-job) transitions refer to the case in which the worker changes (keeps) her last-month job. Earning levels are reported in 2016 Brazilian *Reais* units.

Table A.2: 3-month Transition Rates, by Demographic Group

	No High School Diploma				High School Graduates				All	
	Young		Prime-Age		Young		Prime-Age			
Into NE										
I to NE	10.3	667	6.6	777	8.8	776	4.1	1264	6.7	828
F to NE	6.2	902	3.1	1055	4.5	1039	2.1	1700	3.1	1226
From NE										
NE to I	13.0	645	9.7	715	10.6	755	7.1	1037	9.7	781
NE to F	6.3	863	7.0	1007	11.6	964	9.4	1431	9.1	1082
f-NE to I	16.9	748	13.6	792	12.7	837	8.7	1156	12.1	895
f-NE to F	13.7	910	17.0	1086	19.4	1008	15.8	1560	17.2	1169
Between-Jobs										
I to I	11.5	+8	7.8	+4	8.5	+7	4.3	+3	7.2	+6
I to F	3.7	+21	3.0	+19	5.8	+24	2.6	+19	3.5	+19
F to I	2.3	-5	1.1	-3	1.2	0	0.4	-9	0.9	-5
F to F	4.7	+4	3.0	+4	4.8	+4	2.3	+3	3.1	+3
Within-Job										
I to F	5.3	+4	4.6	+4	8.9	+6	5.2	+4	5.5	+5
F to I	1.4	+1	0.6	-1	1.0	-2	0.4	0	0.6	-1

Source: *Pesquisa Mensal de Emprego*, March 2002 - February 2016. Employment statuses are formal (F), informal (I), non-employed (NE), and currently non-employed who report having held a formal job within the last year (f-NE). Between-jobs (within-job) transitions refer to the case in which the worker changes (keeps) her last-month job. Earning levels are reported in 2016 Brazilian *Reais* units.

B Additional Material for Model Section

B.1 Unemployed Workers

$$
\begin{aligned}
B(x;\psi,z) &= b(x) \;+\; \frac{1}{1+r}\,E\Bigg[\,\xi\,G(x;\psi',z') +\; (1-\xi)\,\Big[1-\lambda(\psi',z')\Big]\,B(x;\psi',z') \\
&\quad +\; (1-\xi)\,\lambda(\psi',z')\int max\Bigg\{B(x;\psi',z'),W_B(x,y;\psi',z')\Bigg\}\frac{v(y;\psi',z')}{V(\psi',z')}dy\;\Bigg|\;\psi,z\Bigg] \\
&= b(x) \;+\; \frac{1}{1+r}\,E\Bigg[\,B(x;\psi',z') + \xi\,\big[G(x;\psi',z') - B(x;\psi',z')\big]\;\Bigg|\;\psi,z\Bigg]
\end{aligned}
$$

B.2 Self-Employed Workers

$$
\begin{aligned}
G(x;\psi,z) &= \mu b(x) \;+\; \frac{1}{1+r}\,E\Bigg[\,\delta_g\,B(x;\psi',z') \;+\; (1-\delta_g)\;\Big[1-\phi\,\lambda(\psi',z')\Big]\;G(x;\psi',z') \\
&\quad + (1-\delta_g)\;\;\phi\,\lambda(\psi',z')\int max\Big\{G(x;\psi',z'),W_D(x,y;\psi',z')\frac{v(y;\psi',z')}{V(\psi',z')}dy\;\Bigg|\;\psi,z\Bigg] \\
&= \mu b(x) \;+\; \frac{1}{1+r}\,E\Big[\,G(x;\psi',z') - \delta_g\,S_g(x;\psi',z')\;\Big|\;\psi,z\,\Big]
\end{aligned}
$$

B.3 Informal Match Surplus

Let $P(x,y;\psi,z) \coloneqq \max\big\{P_f(x,y;\psi,z), P_i(x,y;\psi,z)\big\}$; and, $\chi^i \coloneqq (1-\delta)\mathbb{1}\big\{P(x,y';\psi',z') \geq B(x)\big\}$. Therefore, an informal match survives next period's third stage with probability $E\left[\,\chi^i \mid y;\psi,z\right]$.

$$
\begin{aligned}
P_i(x,y;\psi,z) = \;& \big(\,1\;\; - \kappa\;\big) \quad p(x,y,z) \quad + \quad \frac{1}{1+r}\,E\Bigg[\;\Big(1-\chi^i\Big)\;B(x) \\
&+\;\chi^i\;\Big[1-\phi\,\lambda(\psi',z')\Big]\;P(x,y';\psi',z') \\
&+\;\chi^i\;\;\phi\,\lambda(\psi',z')\int max\Big\{P(x,y';\psi',z'), W_P(x,y',\widetilde{y};\psi',z')\Big\}\frac{v(\widetilde{y};\psi',z')}{V(\psi',z')}d\widetilde{y}\;\Bigg|\;y;\psi,z\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
P_i(x,y;\psi,z) = \;& (1-\kappa)\;p(x,y,z)\;+\;\frac{1}{1+r}\,E\Bigg[\;\Big(1-\chi^i\Big)\;\Big[B(x)\Big] \\
&+\;\chi^i\;\Big[P(x,y';\psi',z')\Big]\qquad\Big|\;\;y,\psi,z\Bigg]
\end{aligned}
$$

$$
P_i(x,y;\psi,z) = \;(1-\kappa)\;p(x,y,z)\;+\;\frac{1}{1+r}\,E\Bigg[\;B(x)\;\;+\;\;\chi^i\Bigg]\;\Bigg|\;y,\psi,z\Bigg]
$$

Substracting Equation (4)

$$
\begin{aligned}
S_i(x,y;\psi,z) = \;& (1-\kappa)\;p(x,y,z)\;\;-\;\;b(x)\;\;-\frac{\xi}{1+r}S_g(x) \\
&+\;\;\frac{1-\delta}{1+r}\,E\Bigg[\;S(x,y';\psi',z')^{+}\;\;\Big|\;\;y,\psi,z\Bigg]
\end{aligned}
$$

B.4 Formal Match Surplus

Note $\max\{P_f(x,y;\psi,z), P_i(x,y;\psi,z)-\gamma\} = P_f(x,y;\psi,z)$; and, let $\chi^f \coloneqq \mathbb{1}\{P_f(x,y';\psi',z') \geq B(x)\ -\gamma\ \}$. Therefore, a formal match survives next period's third stage with probability $(1-\delta)\, E\left[\ \chi^f \mid y;\psi,z\right]$.

$$P_f(x,y;\psi,z) = \quad p(x,y,z) \ + \ \frac{1}{1+r}\, E\Bigg[\ \Big(1-\chi^f\Big)\ \Big[B(x)\ -\gamma\ \Big]$$

$$+\ \delta\ \chi^f\ B(x)$$

$$+\ (1-\delta)\chi^f\ \Big[1-\phi\,\lambda(\psi',z')\Big]\ P_f(x,y';\psi',z')$$

$$+\ (1-\delta)\chi^f\ \ \phi\,\lambda(\psi',z')\int max\Big\{P_f(x,y';\psi',z'), W_P(x,y',\widetilde{y};\psi',z')\Big\}\frac{v(\widetilde{y};\psi',z')}{V(\psi',z')}d\widetilde{y}\,\Big|\, y;\psi,z\Bigg]$$

$$P_f(x,y;\psi,z) = \quad p(x,y,z) + \frac{1}{1+r}\, E\Bigg[\ \Big(1-\chi^f\Big)\ \Big[B(x)-\gamma\Big]$$

$$+\ \ \delta\ \chi^f\ B(x)$$

$$+\ \ (1-\delta)\ \chi^f\ \Big[P_f(x,y';\psi',z')\Big]\qquad\Big|\ \ y,\psi,z\Bigg]$$

$$P_f(x,y;\psi,z) = \quad p(x,y,z) + \frac{1}{1+r}\, E\Bigg[B(x)-\gamma$$

$$+\,(1-\delta)\ \chi^f\ \Big[P_f(x,y';\psi',z') - B(x) + \frac{\gamma}{1-\delta}\Big]\ \Big|\ \ y,\psi,z\Bigg]$$

Substracting Equation (4)

$$S_f(x,y;\psi,z) = \ p(x,y,z)\ \ -\ \ b(x)\ \ -\ \ \frac{\gamma}{1+r}\ \ -\frac{\xi}{1+r}S_g(x)$$

$$+\ \ \frac{1-\delta}{1+r}\, E\Bigg[\mathbb{1}\Big\{S_f(x,y';\psi',z')+\gamma\ \geq\ 0\Big\}\Big(S_f(x,y';\psi',z')+\frac{\gamma}{1-\delta}\Big)\ \Big|\ \ y,\psi,z\Bigg]$$

B.5 Updated Densities after Shocks' Realizations, ψ (Stage 2)

Given $\psi_o = \{f_o, i_o, g_o\}$, $\pi_y(y, y')$, and ξ, we can derive $\psi = \{f, i, g\}$

$$f(x,y) = \int \underbrace{f_o(x,\widetilde{y})\,\pi_y(\widetilde{y},y)}_{\substack{\text{measure of formal x-workers} \\ \text{that transit from } \widetilde{y} \text{ into } y}} \, d\widetilde{y}$$

$$i(x,y) = \int i_o(x,\widetilde{y})\,\pi_y(\widetilde{y},y)\, d\widetilde{y}$$

$$g(x) = g_o(x) + \underbrace{u_o(x)\,\xi}_{\substack{\text{U-to-SE} \\ \text{transitions}}}$$

B.6 Updated Densities after within-job contract reassignments and separations, ψ_m (Stage 3)

Given g_o, $\psi = \{f, i, g\}$, δ, γ, z, and surplus functions, we can derive $\psi_m = \{f_m, i_m, g_m\}$

$$
\begin{aligned}
i_m(x,y) \;=\;& \underbrace{i(x,y)}_{\substack{\text{measure of in-}\\ \text{formal matches}\\ \text{between } x \text{ and } y}} \underbrace{\mathbb{1}\left\{S_i(x,y;z) \;\geq\; S_f(x,y;z)\right\}}_{\text{not reallocated}} \underbrace{\mathbb{1}\left\{S_i(x,y;z) \geq 0\right\}(1-\delta)}_{\text{not separated}} \\
&+ \underbrace{f(x,y)}_{\substack{\text{measure of}\\ \text{formal mat-}\\ \text{ches btw}\\ x \text{ and } y}} \underbrace{\mathbb{1}\left\{S_i(x,y;z) - \gamma \;>\; S_f(x,y;z)\right\}}_{\substack{\text{reallocated [does not}\\ \text{happen in equilibrium]}}} \underbrace{\mathbb{1}\left\{S_i(x,y;z) - \gamma \geq -\gamma\right\}(1-\delta)}_{\text{not separated}}
\end{aligned}
$$

$$
\begin{aligned}
f_m(x,y) \;=\;& i(x,y)\,\mathbb{1}\left\{S_i(x,y;z) \;<\; S_f(x,y;z)\right\}\mathbb{1}\left\{S_f(x,y;z) \geq 0\right\}(1-\delta) \\
&+ f(x,y)\,\mathbb{1}\left\{S_i(x,y;z) - \gamma \;\leq\; S_f(x,y;z)\right\}\mathbb{1}\left\{S_f(x,y;z) \geq -\gamma\right\}(1-\delta)
\end{aligned}
$$

$$
g_m(x) \;=\; g_o(x)\,(1-\delta) + [g(x) - g_o(x)]
$$

B.7 Expected value from a contact for a y-type firm (Stage 4)

Given $u,\ f_m,\ i_m,\ g_o, \phi,\ z$, and surplus functions, we can compute

$$
\begin{aligned}
J(y; u, f_m, i_m, , g_o, z) \quad &= \underbrace{\int S(x, y; z)^{+} \frac{u(x)}{L(u, f_m, i_m, g_o)}\, dx}_{\text{EV from contact w/ unemployed worker}} \\
&+ \underbrace{\int \Big[S(x, y; z) - S_g(x; z)\Big]^{+} \frac{\phi(1-\delta) g_o(x)}{L(u, f_m, i_m, g_o)}\, dx}_{\text{EV from contact w/ self-employed worker}} \\
&+ \underbrace{\int\int \Big[S(x, y; z) - S_f(x, y'; z)\Big]^{+} \ \frac{\phi f_m(x, y')}{L(u, f_m, i_m, g_o)} \ \ dx\, dy'}_{\text{EV from contact w/ formal worker in another firm } y'} \\
&+ \underbrace{\int\int \Big[S(x, y; z) - S_i(x, y'; z)\Big]^{+} \ \frac{\phi\, i_m(x, y')}{L(u, f_m, i_m, g_o)} \ dx\, dy'}_{\text{EV from contact w/ informal worker in another firm } y'}
\end{aligned}
$$

such that we can recover $v(y; u, f_m, i_m, g_o, z)$ for all $y \in [0, 1]$. Computation of L and V is straightforward.

B.8 Updated Densities after Search and Matching, ψ_n (Stage 5)

Let η denote the array $(u,\ f_m,\ i_m,\ g_o,\ z)$. Given η, $v(y;\eta)$, ϕ, and surpluses, we can derive $\psi_n = \{f_n, i_n, g_n\}$. In particular, for all $x \in [0,1]$ and $y \in [0,1]$:

$$
\begin{aligned}
f_n(x,y) = \quad & \underbrace{f_m(x,y)\Big[1-\phi\lambda(\eta)\Big] + f_m(x,y)\,\phi\,\lambda(\eta)\int \mathbb{1}\big\{S_f(x,y;z) \geq S(x,y',z)\big\}\frac{v(y',\eta)}{V(\eta)}\ dy'}_{\substack{\text{incumbent formal } x\text{-workers at } y\text{-firms that stay} \\ \text{[not contacted + contacted but not poached]}}} \\
& + \underbrace{u(x)\ \lambda(\eta)\ \frac{v(y,\eta)}{V(\eta)}\ \mathbb{1}\big\{S(x,y;z) \geq 0\big\}\ \mathbb{1}\big\{S_i(x,y;z)\ \leq\ S_f(x,y;z)\big\}}_{\text{unemployed } x\text{-workers hiblack by } y\text{-firms through a formal contract}} \\
& + \underbrace{(1-\delta)g_o(x)\ \phi\lambda(\eta)\ \frac{v(y,\eta)}{V(\eta)}\ \mathbb{1}\big\{S(x,y;z) \geq S_g(x;z)\big\}\ \mathbb{1}\big\{S_i(x,y;z)\ \leq\ S_f(x,y;z)\big\}}_{\text{self-employed } x\text{-workers hiblack by } y\text{-firms through a formal contract}} \\
& + \underbrace{\phi\lambda(\eta)\frac{v(y,\eta)}{V(\eta)}\ \mathbb{1}\big\{S(x,y;z) \geq 0\big\}\ \mathbb{1}\big\{S_i(x,y;z) \leq S_f(x,y;z)\big\}\int f_m(x,y')\ \mathbb{1}\big\{S_f(x,y;z) > S_f(x,y';z)\big\}\ dy'}_{\text{formal } x\text{-workers at } y'\text{-firms poached by } y\text{-firms through a formal contract}} \\
& + \underbrace{\phi\lambda(\eta)\frac{v(y,\eta)}{V(\eta)}\ \mathbb{1}\big\{S(x,y;z) \geq 0\big\}\ \mathbb{1}\big\{S_i(x,y;z) \leq S_f(x,y;z)\big\}\int i_m(x,y')\ \mathbb{1}\big\{S_f(x,y;z) > S_i(x,y';z)\big\}\ dy'}_{\text{informal } x\text{-workers at } y'\text{-firms poached by } y\text{-firms through a formal contract}}
\end{aligned}
$$

$$
\begin{aligned}
i_n(x,y) = \quad & \underbrace{i_m(x,y)\Big[1-\phi\lambda(\eta)\Big] + i_m(x,y)\,\phi\,\lambda(\eta)\int \mathbb{1}\big\{S_i(x,y;z) \geq S(x,y',z)\big\}\frac{v(y',\eta)}{V(\eta)}\ dy'}_{\substack{\text{incumbent informal } x\text{-workers at } y\text{-firms that stay} \\ \text{[not contacted + contacted but not poached]}}} \\
& + \underbrace{u(x)\ \lambda(\eta)\ \frac{v(y,\eta)}{V(\eta)}\ \mathbb{1}\big\{S(x,y;z) \geq 0\big\}\ \mathbb{1}\big\{S_i(x,y;z)\ >\ S_f(x,y;z)\big\}}_{\text{unemployed } x\text{-workers hiblack by } y\text{-firms through an informal contract}} \\
& + \underbrace{(1-\delta)g_o(x)\ \phi\lambda(\eta)\ \frac{v(y,\eta)}{V(\eta)}\ \mathbb{1}\big\{S(x,y;z) \geq S_g(x;z)\big\}\ \mathbb{1}\big\{S_i(x,y;z)\ >\ S_f(x,y;z)\big\}}_{\text{self-employed } x\text{-workers hiblack by } y\text{-firms through an informal contract}} \\
& + \underbrace{\phi\lambda(\eta)\frac{v(y,\eta)}{V(\eta)}\ \mathbb{1}\big\{S(x,y;z) \geq 0\big\}\ \mathbb{1}\big\{S_i(x,y;z) > S_f(x,y;z)\big\}\int f_m(x,y')\ \mathbb{1}\big\{S_i(x,y;z) > S_f(x,y';z)\big\}\ dy'}_{\text{formal } x\text{-workers at } y'\text{-firms poached by } y\text{-firms through an informal contract}} \\
& + \underbrace{\phi\lambda(\eta)\frac{v(y,\eta)}{V(\eta)}\ \mathbb{1}\big\{S(x,y;z) \geq 0\big\}\ \mathbb{1}\big\{S_i(x,y;z) > S_f(x,y;z)\big\}\int i_m(x,y')\ \mathbb{1}\big\{S_i(x,y;z) > S_i(x,y';z)\big\}\ dy'}_{\text{informal } x\text{-workers at } y'\text{-firms poached by } y\text{-firms through an informal contract}}
\end{aligned}
$$

$$g_n(x) \;= g_m(x) - \underbrace{(1-\delta)g_o(x)\,\phi\lambda(\eta)\int \mathbb{1}\big\{S(x,y;z) \geq S_g(x;z)\big\}\,\frac{v(y,\eta)}{V(\eta)}\,dy}_{\text{self-employed } x\text{-workers hiblack}}$$

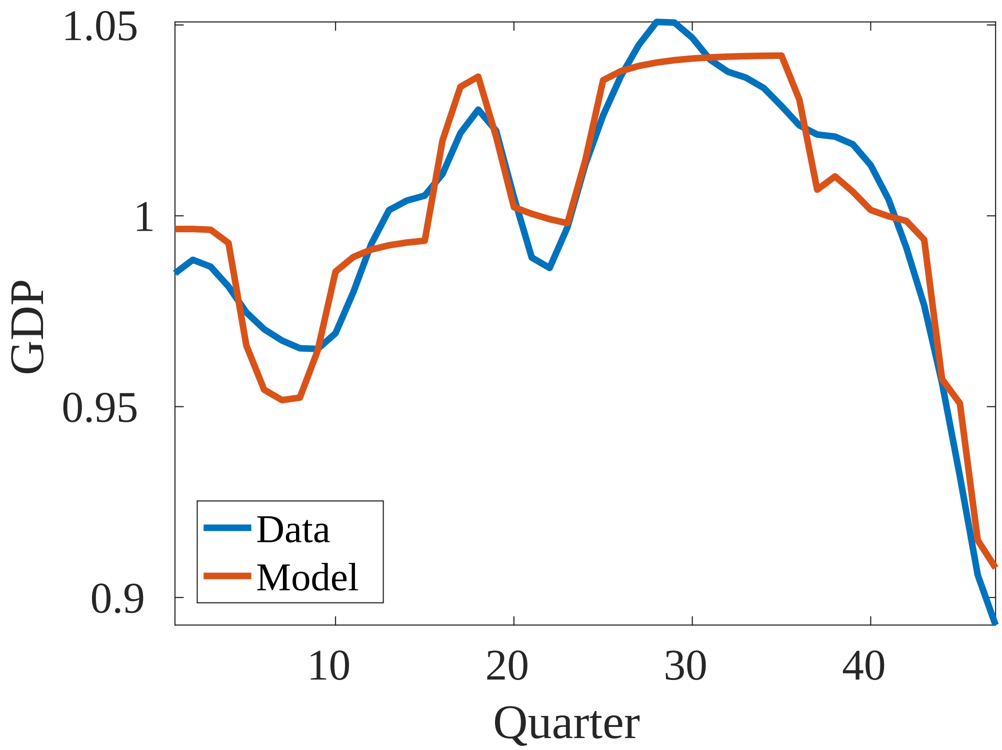
C Additional Material for Estimation Section

Table C.3: Fit to Non-Targeted Moments

	Moments	Data	Model
25)	E[U5+]	6.91	7.11
26)	E[U18+]	4.1	4.03
27)	E[U31+]	2.47	2.39
28)	E[J to J]	1.31	1.1
29)	sd[U5+]	9.38	10.24
30)	sd[U18+]	12.79	12.77
31)	sd[U31+]	15.91	13.92
32)	sd[J to J]	11.89	10.2
33)	sd[share I]	4.13	2.68
34)	sd[I to U]	4.6	10.51
35)	sd[F to U]	5.1	2.8
36)	sd[U to I]	7.4	11.15
37)	sd[U to F]	12.54	13.88
38)	sd[share I of (U to E)]	−57.05	−66
39)	sd[f-U to I]	7.45	7.78
40)	sd[f-U to F]	12.21	13.63
41)	sd[I to F - within]	7.5	16
42)	sd[I to I - between]	8.88	13.53
43)	sd[I to F - between]	12.66	16.64
44)	sd[F to I - between]	13.24	9.07
45)	sd[F to F - between]	16.69	13.12
46)	corr[J to J]	79.38	89.96
47)	corr[I to U]	−10.75	0.81
48)	corr[F to U]	72.14	−83.62
49)	corr[U to I]	50.83	81.62
50)	corr[U to F]	93.87	93.17
51)	corr[share I of (U to E)]	−57.05	−66
52)	corr[f-U to I]	51.85	87.38
53)	corr[f-U to F]	86.31	92.91
54)	corr[I to F - within]	83.01	−6.92
55)	corr[I to I - between]	64.36	59
56)	corr[I to F - between]	77.61	88.51
57)	corr[F to I - between]	76.85	91.53
58)	corr[F to F - between]	75.51	93.36

Note: All data moments are based on series computed from the *Pesquisa Mensal de Emprego*, March 2002 - February 2016. The considered series are the share of workers that are unemployed for at least N weeks (UN+) for $N = \{5, 18, 31\}$, job-to-job transitions (J to J), the incidence of informal jobs in the private sector (share I), and the various transition rates introduced in Tables 1 and 2, except for within-job downgrades in formality status.

Figure C.1: GDP, given
Fitted Sequence of Aggregate Shocks



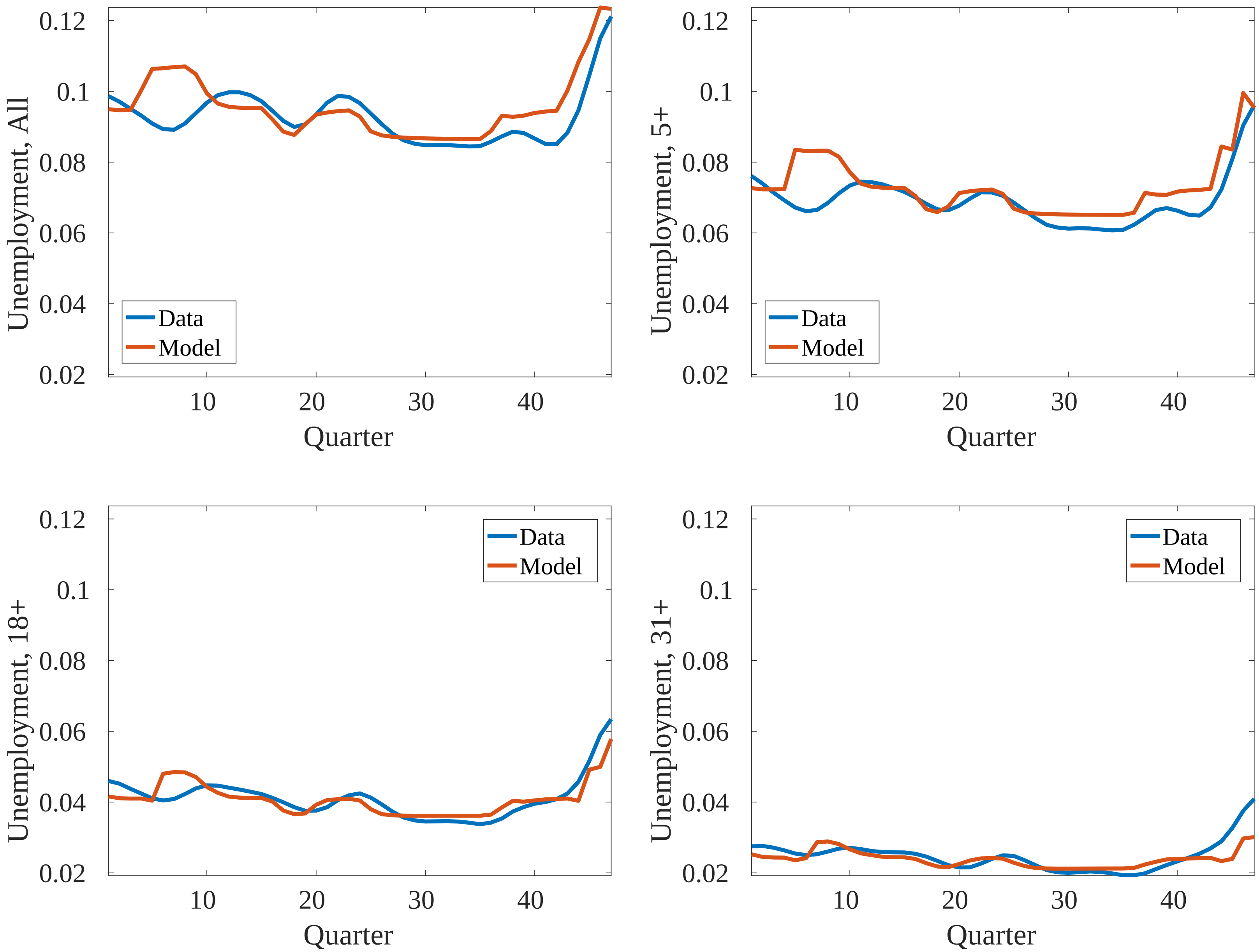


Figure C.2: Fit for Unemployment
at different duration (weeks) horizons

Figure C.3: Equilibrium Distribution of Matches, z_{50th}

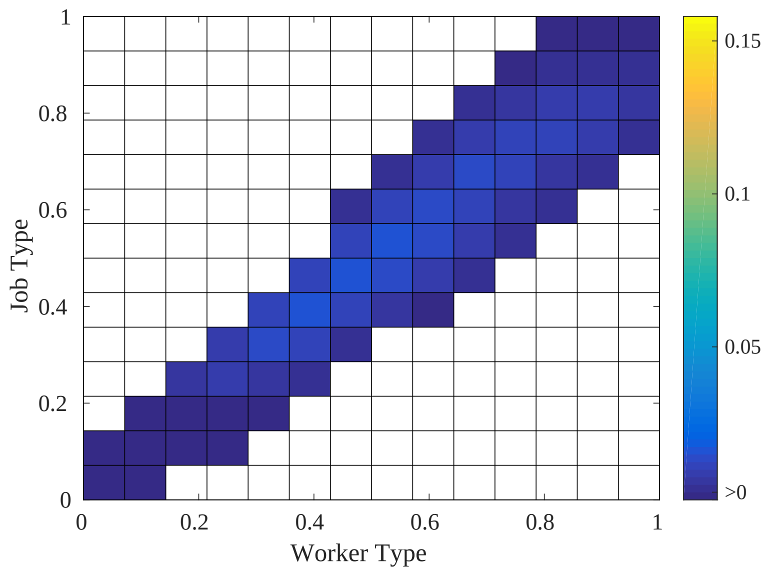
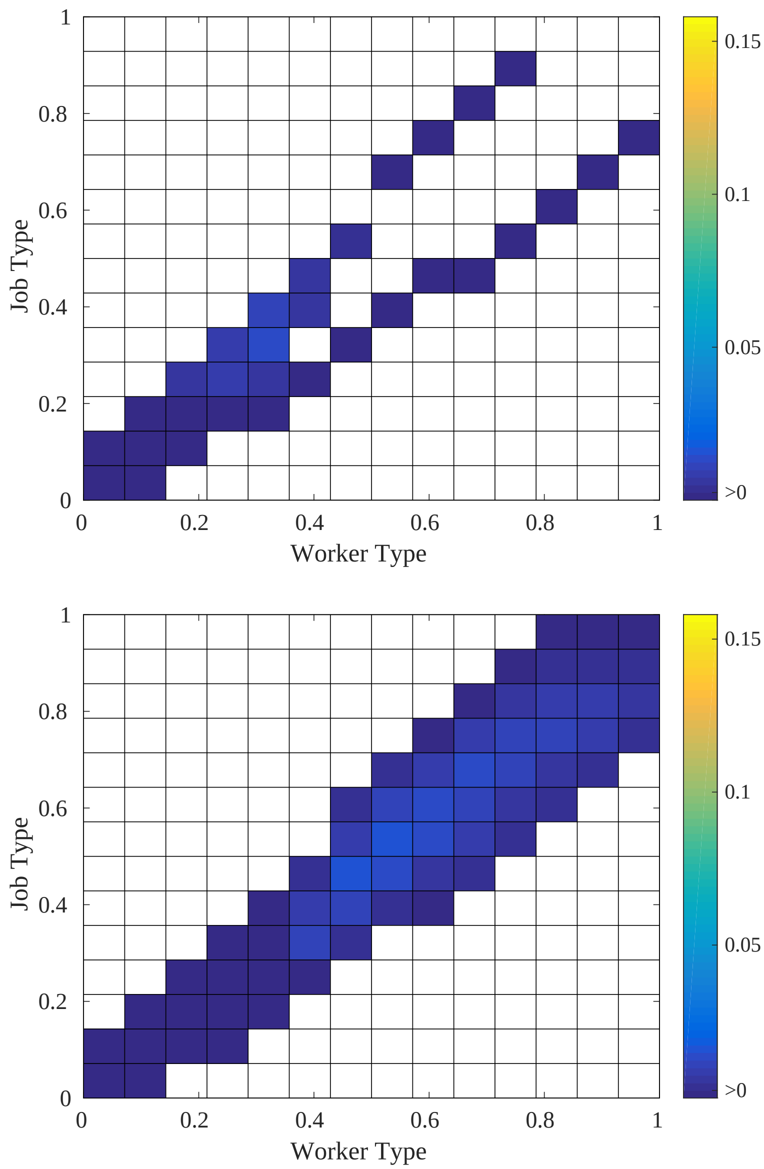


Figure C.4: Equilibrium Distribution of Informal Matches (Upper Panel) and Formal Matches (Lower Panel), z_{50th}



D Additional Material for Counterfactual Analysis Section

Figure D.1: GDP and Unemployment,
Baseline and Counterfactual