Reward-Punishment Symmetric Universal Intelligence

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Abstract. Can an agent's intelligence level be negative? We extend the Legg-Hutter agent-environment framework to include punishments and argue for an affirmative answer to that question. We show that if the background encodings and Universal Turing Machine (UTM) admit certain Kolmogorov complexity symmetries, then the resulting Legg-Hutter intelligence measure is symmetric about the origin. In particular, this implies reward-ignoring agents have Legg-Hutter intelligence 0 according to such UTMs.

1 Introduction

In their paper [10], Legg and Hutter write:

> "As our goal is to produce a definition of intelligence that is as broad and encompassing as possible, the space of environments used in our definition should be as large as possible."

So motivated, we investigate what would happen if we extended the universe of environments to include environments with rewards from $\mathbb{Q} \cap [-1, 1]$ instead of just from $\mathbb{Q} \cap [0, 1]$ as in Legg and Hutter's paper. In other words, we investigate what would happen if environments are not only allowed to reward agents but also to punish agents (a punishment being a negative reward).

We discovered that when negative rewards are allowed, this introduces a certain algebraic structure into the agent-environment framework. The main objection we anticipate to our extended framework is that it implies the negative intelligence of certain agents[3]. We would argue that this makes perfect sense when environments are capable of punishing agents: the intelligence level of a reinforcement learning agent should measure the degree to which that agent can extract large rewards on average across many environments. An agent who

[3] Thus, this paper falls under the broader program of advocating for intelligence measures having different ranges than the nonnegative reals. Alexander has advocated more extreme extensions of the range of intelligence measures [2] [3]; by contrast, here we merely question the assumption that intelligence never be negative, leaving aside the question of whether intelligence should be real-valued.

instead extracts large punishments on average across many environments should therefore have a negative intelligence level.

The structure of the paper is as follows:

- In Section 2, we give preliminary definitions.
- In Section 3, we introduce what we call the dual of an agent and of an environment, and prove some algebraic theorems about these.
- In Section 4, we show the existence of UTMs yielding Kolmogorov complexities with certain symmetries, and show that the resulting Legg-Hutter intelligence measures are symmetric too.
- In Section 5, we summarize and make concluding remarks, including remarks about how these ideas might be applied to certain other intelligence measures.

2 Preliminaries

In defining agent and environment below, we attempt to follow Legg and Hutter [10] as closely as possible, except that we permit environments to output rewards from $\mathbb{Q} \cap [-1, 1]$ rather than just $\mathbb{Q} \cap [0, 1]$ (and, accordingly, we modify which well-behaved environments to restrict our attention to).

Throughout the paper, we implicitly fix a finite set $\mathcal{A}$ of *actions*, a finite set $\mathcal{O}$ of *observations*, and a finite set $\mathcal{R} \subseteq \mathbb{Q} \cap [-1, 1]$ of *rewards* (so each reward is a rational number between -1 and 1 inclusive), with $|\mathcal{A}| > 1$, $|\mathcal{O}| > 0$, $|\mathcal{R}| > 0$. We assume that $\mathcal{R}$ has the following property: whenever $\mathcal{R}$ contains any reward r, then $\mathcal{R}$ also contains $-r$. We assume $\mathcal{A}$, $\mathcal{O}$, and $\mathcal{R}$ are mutually disjoint (i.e., no reward is an action, no reward is an observation, and no action is an observation). By $\langle\rangle$ we mean the empty sequence.

Definition 1 *(Agents, environments, etc.)*

1. *By $(\mathcal{ORA})^*$ we mean the set of all finite sequences starting with an observation, ending with an action, and following the pattern "observation, reward, action, ...". We include $\langle\rangle$ in this set.*
2. *By $(\mathcal{ORA})^*\mathcal{OR}$ we mean the set of all sequences of the form $s \frown o \frown r$ where $s \in (\mathcal{ORA})^*$, $o \in \mathcal{O}$ and $r \in \mathcal{R}$ ($\frown$ denotes concatenation).*
3. *By an* agent*, we mean a function π with domain $(\mathcal{ORA})^*\mathcal{OR}$, which assigns to every sequence $s \in (\mathcal{ORA})^*\mathcal{OR}$ a $\mathbb{Q}$-valued probability measure, written $\pi(\bullet|s)$, on $\mathcal{A}$. For every such s and every $a \in \mathcal{A}$, we write $\pi(a|s)$ for $(\pi(\bullet|s))(a)$. Intuitively, $\pi(a|s)$ is the probability that agent π will take action a in response to history s.*
4. *By an* environment*, we mean a function μ with domain $(\mathcal{ORA})^*$, which assigns to every $s \in (\mathcal{ORA})^*$ a $\mathbb{Q}$-valued probability measure, written $\mu(\bullet|s)$, on $\mathcal{O} \times \mathcal{R}$. For every such s and every $(o, r) \in \mathcal{O} \times \mathcal{R}$, we write $\mu(o, r|s)$ for $(\mu(\bullet|s))(o, r)$. Intuitively, $\mu(o, r|s)$ is the probability that environment μ will issue observation o and reward r to the agent in response to history s.*

5. *If π is an agent, μ is an environment, and $n \in \mathbb{N}$, we write $V^\pi_{\mu,n}$ for the expected value of the sum of the rewards which would occur in the sequence $(o_0, r_0, a_0, \ldots, o_n, r_n, a_n)$ randomly generated as follows:*
   (a) *$(o_0, r_0) \in \mathcal{O} \times \mathcal{R}$ is chosen randomly based on the probability measure $\mu(\bullet|\langle\rangle)$.*
   (b) *$a_0 \in \mathcal{A}$ is chosen randomly based on the probability measure $\pi(\bullet|o_0, r_0)$.*
   (c) *For each $i > 0$, $(o_i, r_i) \in \mathcal{O} \times \mathcal{R}$ is chosen randomly based on the probability measure $\mu(\bullet|o_0, r_0, a_0, \ldots, o_{i-1}, r_{i-1}, a_{i-1})$.*
   (d) *For each $i > 0$, $a_i \in \mathcal{A}$ is chosen randomly based on the probability measure $\pi(\bullet|o_0, r_0, a_0, \ldots, o_{i-1}, r_{i-1}, a_{i-1}, o_i, r_i)$.*
6. *If π is an agent and μ is an environment, let $V^\pi_\mu = \lim_{n\to\infty} V^\pi_{\mu,n}$. Intuitively, V^π_μ is the expected total reward which π would extract from μ.*

Note that it is possible for V^π_μ to be undefined. For example, if μ is an environment which always issues reward $(-1)^n$ in response to the agent's nth action, then V^π_μ is undefined for every agent π. This would not be the case if rewards were required to be ≥ 0, so this is one way in which allowing punishments complicates the resulting theory.

In order to define Legg-Hutter-style intelligence measures, it is, in any case, necessary to restrict attention to certain well-behaved environments. Legg and Hutter (in their Section 3.2) restrict attention to environments μ that never give total reward more than 1, from which it follows that $V^\pi_\mu \leq 1$. This implication is less clear when environments can punish the agent, so we take a different approach: rather than limit the total reward the environment can give and use that to force V^π_μ to be bounded, we will cut the middleman and directly assume that V^π_μ is bounded.

Definition 2 *An environment μ is* well-behaved *if μ is computable and the following condition holds: for every agent π, V^π_μ exists and $-1 \leq V^\pi_\mu \leq 1$.*

Note that reward-space $[0, 1]$ can be transformed into punishment-space $[-1, 0]$ either via $r \mapsto -r$ or via $r \mapsto r - 1$. An advantage of $r \mapsto -r$ is that it preserves well-behavedness of environments (we prove this below in Corollary 7)[4].

[4] It is worth mentioning another difference between these two transforms. The hypothetical agent AI_μ with perfect knowledge of the environment's reward distribution would not change its behavior in response to $r \mapsto r - 1$ (nor indeed in response to any positive linear scaling $r \mapsto ar + b$, $a > 0$), but it would generally change its behavior in response to $r \mapsto -r$. Interestingly, this behavior invariance with respect to $r \mapsto r - 1$ would not hold if AI_μ were capable of "suicide" (deliberately ending the environmental interaction): one should never quit a slot machine that always pays between 0 and 1 dollars, but one should immediately quit a slot machine that always pays between -1 and 0 dollars. The agent AIXI also changes behavior in response to $r \mapsto r - 1$, and it was recently argued [12] that this can be interpreted in terms of suicide/death: AIXI models its environment using a mixture distribution over a countable class of semimeasures, and AIXI's behavior can be interpreted as treating the complement of the domain of each semimeasure as death.

3 Dual Agents and Dual Environments

In the Introduction, we promised that by allowing environments to punish agents, we would reveal algebraic structure not otherwise present. The key to this additional structure is the following definition.

Definition 3 *(Dual Agents and Dual Environments)*

1. *For each sequence s, let $\overline{s}$ be the sequence obtained by replacing every reward r in s by $-r$.*
2. *Suppose π is an agent. We define a new agent $\overline{\pi}$, the* dual *of π, as follows: for each $s \in (\mathcal{ORA})^*\mathcal{OR}$, for each action $a \in \mathcal{A}$,*
$$\overline{\pi}(a|s) = \pi(a|\overline{s}).$$
3. *Suppose μ is an environment. We define a new environment $\overline{\mu}$, the* dual *of μ, as follows: for each $s \in (\mathcal{ORA})^*$, for each observation $o \in \mathcal{O}$ and reward $r \in \mathcal{R}$,*
$$\overline{\mu}(o, r|s) = \mu(o, -r|\overline{s}).$$

Lemma 4 *(Double Negation)*

1. *For each sequence s, $\overline{\overline{s}} = s$.*
2. *For each agent π, $\overline{\overline{\pi}} = \pi$.*
3. *For each environment μ, $\overline{\overline{\mu}} = \mu$.*

Proof. Follows from the fact that for every real number r, $--r = r$. □

Theorem 5 *Suppose μ is an environment and π is an agent. Then*
$$V_{\overline{\mu}}^{\overline{\pi}} = -V_{\mu}^{\pi}$$
(and the left-hand side is defined if and only if the right-hand side is defined).

Proof. By Definition 1 part 6, it suffices to show that for each $n \in \mathbb{N}$, $V_{\overline{\mu},n}^{\overline{\pi}} = -V_{\mu,n}^{\pi}$. For that, it suffices to show that for every $s \in ((\mathcal{ORA})^*) \cup ((\mathcal{ORA})^*\mathcal{OR})$, the probability X of generating s using π and μ (as in Definition 1 part 5) equals the probability X' of generating $\overline{s}$ using $\overline{\pi}$ and $\overline{\mu}$. We will show this by induction on the length of s.

Case 1: s is empty. Then $X = X' = 1$.

Case 2: s terminates with an action. Then $s = t \frown a$ for some $t \in (\mathcal{ORA})^*\mathcal{OR}$. Let Y (resp. Y') be the probability of generating t (resp. $\overline{t}$) using π and μ (resp. $\overline{\pi}$ and $\overline{\mu}$). We reason:

$$\begin{aligned}
X &= \pi(a|t)Y && \text{(Definition of } X\text{)} \\
&= \pi(a|\overline{\overline{t}})Y && \text{(Lemma 4)} \\
&= \overline{\pi}(a|\overline{t})Y && \text{(Definition of } \overline{\pi}\text{)} \\
&= \overline{\pi}(a|\overline{t})Y' && \text{(By induction, } Y = Y'\text{)} \\
&= X'. && \text{(Definition of } X'\text{)}
\end{aligned}$$

Case 3: s terminates with a reward. Then $s = t \frown o \frown r$ for some $t \in (\mathcal{ORA})^*$. Let Y (resp. Y') be the probability of generating t (resp. $\bar{t}$) using π and μ (resp. $\overline{\pi}$ and $\overline{\mu}$). We reason:

$$\begin{aligned} X &= \mu(o, r|t)Y && \text{(Definition of } X\text{)} \\ &= \mu(o, --r|\bar{\bar{t}})Y && \text{(Lemma 4)} \\ &= \overline{\mu}(o, -r|\bar{t})Y && \text{(Definition of } \overline{\mu}\text{)} \\ &= \overline{\mu}(o, -r|\bar{t})Y' && \text{(By induction, } Y = Y'\text{)} \\ &= X'. && \text{(Definition of } X'\text{)} \end{aligned}$$

□

Corollary 6 *For every agent π and environment μ,*

$$V_{\overline{\mu}}^{\pi} = -V_{\mu}^{\overline{\pi}}$$

(and the left-hand side is defined if and only if the right-hand side is defined).

Proof. If neither side is defined, then there is nothing to prove. Assume the left-hand side is defined. Then

$$\begin{aligned} V_{\overline{\mu}}^{\pi} &= V_{\overline{\mu}}^{\overline{\overline{\pi}}} && \text{(Lemma 4)} \\ &= -V_{\mu}^{\overline{\pi}}, && \text{(Theorem 5)} \end{aligned}$$

as desired. A similar argument holds if we assume the right-hand side is defined. □

Corollary 7 *For every environment μ, μ is well-behaved if and only if $\overline{\mu}$ is well-behaved.*

Proof. We prove the $\Rightarrow$ direction, the other is similar. Since μ is well-behaved, μ is computable, so clearly $\overline{\mu}$ is computable. Let π be any agent. Since μ is well-behaved, $V_{\mu}^{\overline{\pi}}$ is defined and $-1 \leq V_{\mu}^{\overline{\pi}} \leq 1$. By Corollary 6, $V_{\overline{\mu}}^{\pi} = -V_{\mu}^{\overline{\pi}}$ is defined, implying $-1 \leq V_{\overline{\mu}}^{\pi} \leq 1$. By arbitrariness of π, this shows $\overline{\mu}$ is well-behaved. □

4 Symmetric Intelligence

By definition, agent $\overline{\pi}$ acts exactly as agent π would act if π were confused into thinking that punishments were rewards and rewards were punishments. Whatever ingenuity π applies to maximize rewards, $\overline{\pi}$ applies that exact same ingenuity to maximize punishments. Thus, if Υ is an intelligence function (intended to measure how well an agent extracts rewards in some aggregate sense across the whole infinite space of all well-behaved environments), then Υ ought to satisfy the equation:

$$\Upsilon(\overline{\pi}) = -\Upsilon(\pi).$$

Not every intelligence function satisfies the above equation, but the above equation should be considered desirable when we invent ways of measuring intelligence: all else being equal, an intelligence function which satisfies the above equation should be considered better than an intelligence function which does not. In this section, we will show that the universal intelligence measure proposed by Legg and Hutter satisfies the above equation, provided a background UTM and encoding are suitably chosen.

We write 2^* for the set of finite binary strings. We write $f :\subseteq A \to B$ to indicate that f has codomain B and that f's domain is some subset of A.

Definition 8 *(Prefix-free universal Turing machines)*

1. *A partial computable function $f :\subseteq 2^* \to 2^*$ is* prefix-free *if the following requirement holds: $\forall p, p' \in 2^*$, if p is a strict initial segment of p', then $f(p)$ and $f(p')$ are not both defined.*
2. *A* prefix-free universal Turing machine *(or* PFUTM*) is a prefix-free partial computable function $U :\subseteq 2^* \to 2^*$ such that the following condition holds. For every prefix-free partial computable function $f :\subseteq 2^* \to 2^*$, $\exists y \in 2^*$ such that $\forall x \in 2^*$, $f(x) = U(y \frown x)$. In this case, we say y is a* computer program for f in programming language U.

Environments do not have domain $\subseteq 2^*$, and they do not have codomain 2^*. Rather, their domain and codomain are $(\mathcal{ORA})^*$ and the set of $\mathbb{Q}$-valued probability measures on $\mathcal{O} \times \mathcal{R}$, respectively. Thus, in order to talk about their Kolmogorov complexities, one must encode said inputs and outputs. This low-level detail is usually implicit, but we will need (in Theorem 11) to distinguish between different kinds of encodings, so we must make the details explicit.

Definition 9 *By an* RL-encoding *we mean a computable function $\sqcap : (\mathcal{ORA})^* \cup M \to 2^*$ (where M is the set of $\mathbb{Q}$-valued probability-measures on $\mathcal{O} \times \mathcal{R}$) such that for all $x, y \in (\mathcal{ORA})^* \cup M$ (with $x \neq y$), $\sqcap(x)$ is not an initial segment of $\sqcap(y)$. We say $\sqcap$ is* suffix-free *if for all $x, y \in (\mathcal{ORA})^* \cup M$ (with $x \neq y$), $\sqcap(x)$ is not a terminal segment of $\sqcap(y)$. We write $\ulcorner x \urcorner$ for $\sqcap(x)$.*

Definition 10 *(Kolmogorov Complexity) Suppose U is a PFUTM and $\sqcap$ is an RL-encoding.*

1. *For each computable environment μ, the* Kolmogorov complexity of μ given by $U, \sqcap$, *written $K_U^{\sqcap}(\mu)$, is the smallest $n \in \mathbb{N}$ such that there is some computer program of length n, in programming language U, for some function $f :\subseteq 2^* \to 2^*$ such that for all $s \in (\mathcal{ORA})^*$, $f(\ulcorner s \urcorner) = \ulcorner \mu(\bullet|s) \urcorner$.*
2. *We say U is* symmetric in its $\sqcap$-encoded-environment cross-section *(or simply that U is* $\sqcap$-symmetric*) if $K_U^{\sqcap}(\mu) = K_U^{\sqcap}(\overline{\mu})$ for every computable environment μ.*

Theorem 11 *For every suffix-free RL-encoding $\sqcap$, there exists a $\sqcap$-symmetric PFUTM.*

Proof. Let U_0 be a PFUTM, we will modify U_0 to obtain a $\sqcap$-symmetric PFUTM. For readability's sake, write POS for 0 and NEG for 1. Thinking of U_0 as a programming language, we define a new programming language U as follows. Every program in U must begin with one of the keywords POS or NEG. Outputs of U are defined as follows.

- $U(\text{POS} \frown x) = U_0(x)$.
- To compute $U(\text{NEG} \frown x)$, find $s \in (\mathcal{ORA})^*$ such that $x = y \frown \ulcorner s \urcorner$ for some y (if no such s exists, diverge). Note that s is unique by suffix-freeness of $\sqcap$. If $U_0(y \frown \ulcorner \overline{s} \urcorner) = \ulcorner m \urcorner$ for some $\mathbb{Q}$-valued probability-measure m on $\mathcal{O} \times \mathcal{R}$, then let $U(\text{NEG} \frown x) = \ulcorner \overline{m} \urcorner$ where $\overline{m}(o, r) = m(o, -r)$. Otherwise, diverge.
  - Informally: If x appears to be an instruction to plug s into computer program y to get a probability measure $\mu(\bullet|s)$, then instead plug $\overline{s}$ into y and flip the resulting probability measure so that the output ends up being the flipped version of $\mu(\bullet|\overline{s})$, i.e., $\overline{\mu}(\bullet|s)$.

By construction, whenever $\text{POS} \frown y$ is a U-computer program for a function f satisfying $f(\ulcorner s \urcorner) = \ulcorner \mu(\bullet|s) \urcorner$, $\text{NEG} \frown y$ is an equal-length U-computer program for a function g satisfying $g(\ulcorner s \urcorner) = \ulcorner \overline{\mu}(\bullet|s) \urcorner$, and vice versa. It follows that U is $\sqcap$-symmetric. □

The proof of Theorem 11 proves more than required: any PFUTM can be modified to make a $\sqcap$-symmetric PFUTM if $\sqcap$ is suffix-free. In some sense, the construction in the proof of Theorem 11 works by eliminating bias: reinforcement learning itself is implicitly biased in its convention that rewards be positive and punishments negative. We can imagine a pessimistic parallel universe where RL instead follows the opposite convention, and the RL in that parallel universe is no less valid than the RL in our own. To be unbiased in this sense, a computer program defining an environment should specify which of the two RL conventions it is operating under (hence the POS and NEG keywords).

Definition 12 *Let W be the set of all well-behaved environments. Let $\overline{W} = \{\overline{\mu} : \mu \in W\}$.*

Lemma 13 *$W = \overline{W}$.*

Proof. By Corollary 7. □

Definition 14 *For every PFUTM U, RL-encoding $\sqcap$, and agent π, the* Legg-Hutter universal intelligence of π given by $U, \sqcap$, *written $\Upsilon_U^{\sqcap}(\pi)$, is*

$$\Upsilon_U^{\sqcap}(\pi) = \sum_{\mu \in W} 2^{-K_U^{\sqcap}(\mu)} V_\mu^\pi.$$

The sum defining $\Upsilon_U^{\sqcap}(\pi)$ is absolutely convergent because the summands in absolute value are dominated by the summands defining Chaitin's constant (this is why we insist on the UTM being prefix-free). Thus, by a well-known theorem

from elementary calculus, the sum does not depend on which order the $\mu \in W$ are enumerated.

Legg-Hutter intelligence has been accused of being subjective because of its UTM-sensitivity [11] [7]. More optimistically, its UTM-sensitivity could be considered a feature, reflecting the fact that there are many kinds of intelligence. It could be used to measure intelligence in various contexts, by choosing UTMs appropriate for those contexts. One could even use it to measure, say, chess intelligence, by choosing a UTM where chess-related environments are easiest to program.

Theorem 15 *(Symmetry about the origin) For every RL-encoding $\sqcap$, every $\sqcap$-symmetric PFUTM U, and every agent π,*

$$\Upsilon_U^{\sqcap}(\overline{\pi}) = -\Upsilon_U^{\sqcap}(\pi).$$

Proof. Compute:

$$\begin{aligned}
\Upsilon_U^{\sqcap}(\overline{\pi}) &= \sum_{\mu \in W} 2^{-K_U^{\sqcap}(\mu)} V_\mu^{\overline{\pi}} && \text{(Definition 14)} \\
&= -\sum_{\mu \in W} 2^{-K_U^{\sqcap}(\mu)} V_{\overline{\mu}}^{\pi} && \text{(Corollary 6)} \\
&= -\sum_{\mu \in W} 2^{-K_U^{\sqcap}(\overline{\mu})} V_{\overline{\mu}}^{\pi} && (U \text{ is } \sqcap\text{-symmetric}) \\
&= -\sum_{\mu \in \overline{W}} 2^{-K_U^{\sqcap}(\mu)} V_{\mu}^{\pi} && \text{(Change of variables)} \\
&= -\sum_{\mu \in W} 2^{-K_U^{\sqcap}(\mu)} V_{\mu}^{\pi} && \text{(Lemma 13)} \\
&= -\Upsilon_U^{\sqcap}(\pi). && \text{(Definition 14)}
\end{aligned}$$

□

The above desideratum, that $\Upsilon(\overline{\pi}) = -\Upsilon(\pi)$, applies to numerical intelligence measures. If one is merely interested in binary intelligence comparators (such as those in [1]), the desideratum can be weakened into a non-numerical comparator form: If π is more intelligent than ρ, then $\overline{\pi}$ should be less intelligent than $\overline{\rho}$. This recalls an observation Socrates made: "Don't you think the ignorant person would often involuntarily tell the truth when he wished to say falsehoods, if it so happened, because he didn't know; whereas you, the wise person, if you should wish to lie, would always consistently lie?" [13]. The following corollary addresses this desideratum.

Corollary 16 *For every RL-encoding $\sqcap$, every $\sqcap$-symmetric PFUTM U, for all agents π and ρ, if $\Upsilon_U^{\sqcap}(\pi) > \Upsilon_U^{\sqcap}(\rho)$ then $\Upsilon_U^{\sqcap}(\overline{\pi}) < \Upsilon_U^{\sqcap}(\overline{\rho})$.*

Proof. By Theorem 15 and basic algebra. □

The following corollary shows that with suitable choice of encoding and UTM, Legg-Hutter universal intelligence satisfies another obvious desideratum.

Corollary 17 *Let $\sqcap$ be an RL-encoding, let U be a $\sqcap$-symmetric PFUTM and suppose π is an agent which ignores rewards (by which we mean that $\pi(\bullet|s)$ does not depend on the rewards in s). Then $\Upsilon^{\sqcap}_U(\pi) = 0$.*

Proof. The hypothesis clearly implies $\pi = \overline{\pi}$. Thus by Theorem 15, $\Upsilon^{\sqcap}_U(\pi) = -\Upsilon^{\sqcap}_U(\pi)$, forcing $\Upsilon^{\sqcap}_U(\pi) = 0$. □

Corollary 17 illustrates why it is appropriate, for purposes of Legg-Hutter universal intelligence, to choose a $\sqcap$-symmetric PFUTM[5]. Consider an agent π_a which always blindly repeats a fixed action $a \in \mathcal{A}$. For any particular environment μ, where π_a earns some total reward r by blind luck, that total reward ought to be cancelled out by $\overline{\mu}$, where the exact same blind luck becomes blind misfortune and π_a earns total reward $-r$ (Corollary 6). But if $K^{\sqcap}_U(\mu) \neq K^{\sqcap}_U(\overline{\mu})$, then the different weights $2^{-K^{\sqcap}_U(\mu)} \neq 2^{-K^{\sqcap}_U(\overline{\mu})}$ would prevent these two outcomes from cancelling each other.

We conclude this section with an exercise, suggesting how the techniques of this paper can be used to obtain other structural results.

Exercise 18 *(Permutations)*

1. *For each permutation $P : \mathcal{A} \to \mathcal{A}$ of the action-space, for each sequence s, let Ps be the result of applying P to all the actions in s. For each agent π, let $P\pi$ be the agent defined by $P\pi(a|s) = \pi(Pa|Ps)$. For each environment μ, let $P\mu$ be the environment defined by $P\mu(o, r|s) = \mu(o, r|Ps)$. Show that in general $V^{\pi}_{\mu} = V^{P\pi}_{P^{-1}\mu}$ and $V^{P\pi}_{\mu} = V^{\pi}_{P\mu}$.*
2. *Say that a PFUTM U is* $\sqcap$-permutable *if $K^{\sqcap}_U(\mu) = K^{\sqcap}_U(P\mu)$ for every computable environment μ and permutation $P : \mathcal{A} \to \mathcal{A}$. Show that if $\sqcap$ is suffix-free then any given PFUTM can be transformed into a $\sqcap$-permutable PFUTM.*
3. *Show that if U is a $\sqcap$-permutable PFUTM, then $\Upsilon^{\sqcap}_U(P\pi) = \Upsilon^{\sqcap}_U(\pi)$ for every agent π and permutation $P : \mathcal{A} \to \mathcal{A}$.*
4. *Modify this exercise to apply to permutations of the observation-space.*

5 Conclusion

By allowing environments to punish agents, we found additional algebraic structure in the agent-environment framework. Using this, we showed that certain Kolmogorov complexity symmetries yield Legg-Hutter intelligence symmetry.

In future work it would be interesting to explore how these symmetries manifest themselves in other Legg-Hutter-like intelligence measures [5] [6] [8]. The

[5] Thus providing one answer to Leike and Hutter [11] who asked: "But what are other desirable properties of a UTM?"

precise strategy we employ in this paper is not directly applicable to prediction-based intelligence measurement [9] [3] [4]. However, a higher-level idea still applies: a predictor who intentionally mis-predicts should be considered less intelligent than a 0-intelligence blind guesser.

A high-level takeaway of this paper is that if intelligence is supposed to measure how much reward an agent extracts in aggregate across many environments, including environments capable of punishment, then such intelligence measures should sometimes output negative values.

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