

# Reward-Punishment Symmetric Universal Intelligence

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# Motivation

- Measuring intelligence is a key step toward AGI
- The Legg-Hutter universal intelligence measure is one approach, but it depends on choice of UTM.
- Leike/Hutter (2015): “What are other desirable properties of a UTM?”
- We propose a symmetry constraint on UTMs

# Dual Agents and Environments

Throughout,  $\pi$  and  $\mu$  are an RL agent and environment, resp. We explicitly allow environments to give negative rewards (punishments).

Definition:  $\bar{\pi}$  is the agent which acts as  $\pi$  would act if  $\pi$  mistook rewards for punishments and punishments for rewards.

Definition:  $\bar{\mu}$  is the environment which responds as  $\mu$  would respond if  $\mu$  mistook rewards for punishments and punishments for rewards.

# Kolmogorov Complexity Symmetry

Definition: Assume a prefix-free UTM  $U$  (and, implicitly, a suitable encoding of RL). Let  $K_U$  be the corresponding Kolmogorov Complexity function.  $U$  is *symmetric* if

$$K_U(\mu) = K_U(\bar{\mu})$$

for every RL environment  $\mu$ .

# Universal Intelligence Symmetry

For every RL agent  $\pi$  and UTM  $U$ , let  $\Upsilon_U(\pi)$  denote the Legg-Hutter universal intelligence of  $\pi$  given by  $U$ .

Theorem (symmetry about the origin): If  $U$  is symmetric then

$$\Upsilon_U(\bar{\pi}) = -\Upsilon_U(\pi).$$

Corollary: If  $U$  is symmetric and  $\pi$  ignores rewards then  $\Upsilon_U(\pi) = 0$ .

# “But why should intelligence be symmetric?”

- In our initial submission, we simply took it for granted that, all else equal, an intelligence measure  $\Upsilon$  that satisfies  $\Upsilon(\bar{\pi}) = -\Upsilon(\pi)$  is better than one which does not.
- Reviewers correctly pointed out this demands justification.
- We will give an informal justification under the assumption that, like Legg-Hutter,  $\Upsilon$  measures intelligence as average performance averaged across many environments.

# Justification Step 1: Weak Symmetry

- Assume  $\Upsilon$  measures intelligence as average performance.
- Say  $\Upsilon$  is *weak symmetric* if: whenever  $\Upsilon(\pi) \neq 0$  then  $\Upsilon(\pi) \neq \Upsilon(\bar{\pi})$ .

Weak symmetry is a reasonable/natural requirement: Say  $\Upsilon(\pi) > 0$ . This should mean  $\pi$  is intelligent:  $\pi$  uses ingenuity to get positive rewards. By def.,  $\bar{\pi}$  uses that same ingenuity to obtain *punishments*. So it would be strange for  $\bar{\pi}$  to get the *exact* same average rewards as  $\pi$ !

(We don't state weak symmetry as absolute law, we merely opine it's reasonable/natural)

# Justification Step 2: Weak Symmetry implies Symmetry

Let  $\pi$  be any agent. Assume  $\Upsilon$  is weak symmetric and measures intelligence as average performance.

Let  $\rho$  be an agent who, at the start of every environment, flips a coin and thereafter plays as  $\pi$  if HEADS,  $\bar{\pi}$  if TAILS.

Since  $\Upsilon$  measures avg. performance,  $\Upsilon(\rho) = \frac{\Upsilon(\pi) + \Upsilon(\bar{\pi})}{2}$ .

Define  $\rho'$  the same but swap HEADS and TAILS.  $\rho$  seems indistinguishable from  $\rho'$  so  $\Upsilon(\rho) = \Upsilon(\rho')$ . Swapping HEADS and TAILS is the same as swapping  $\pi$  and  $\bar{\pi}$ , thus  $\rho' = \bar{\rho}$ . Thus  $\Upsilon(\rho) = \Upsilon(\bar{\rho})$ . By weak symmetry,  $\Upsilon(\rho) = 0$ .

Thus  $\Upsilon(\bar{\pi}) = -\Upsilon(\pi)$ !



# “But why should intelligence be symmetric?”: Conclusion

We argued if  $\Upsilon$  measures intelligence as average performance, it's natural/reasonable to expect Weak Symmetry: whenever  $\Upsilon(\pi) \neq 0$  then  $\Upsilon(\pi) \neq \Upsilon(\bar{\pi})$ .

- (Not absolute law, just our opinion.)

We argued Weak Symmetry implies Symmetry:  $\Upsilon(\bar{\pi}) = -\Upsilon(\pi)$ .

- Therefore  $\Upsilon(\bar{\pi}) = -\Upsilon(\pi)$  is natural/reasonable to expect.
- (Not absolute law, just our opinion.)

# Existence of symmetric UTM's

- Our theorem, “If  $U$  is symmetric then  $\Upsilon_U(\bar{\pi}) = -\Upsilon_U(\pi)$ ”, raises the question: do symmetric UTM's  $U$  exist?

Theorem: Any prefix-free UTM can be transformed into a symmetric UTM under a mild technical assumption on how RL is encoded.

# Inherent Bias in RL

- RL is inherently biased because of its arbitrary convention that positive rewards are good and negative rewards are bad.
- Imagine a parallel universe where RL is formalized with positive rewards bad, negative rewards good. RL would work just as well.
- Our existence proof works by eliminating this bias: valid programs are required to state which RL convention they're written for.

# Whether to Use Absolute Values

- In a true-false IQ test, 0% is as hard to get as 100%.
- So then, if an agent consistently manages to get environments to punish it, shouldn't we assume the agent is intelligent (but loves punishment)?
- Shouldn't we use  $|\Upsilon_U(\pi)|$  instead of  $\Upsilon_U(\pi)$  to measure  $\pi$ 's intelligence?

# Whether to use Absolute Values (cont'd)

Shouldn't we use  $|\Upsilon_U(\pi)|$  instead of  $\Upsilon_U(\pi)$  to measure  $\pi$ 's intelligence?

- Our stance:
  - $\Upsilon_U(\pi)$  measures intelligence as average performance.
  - $|\Upsilon_U(\pi)|$  measures intelligence as ability to consistently extremize rewards (whether consistently positive or consistently negative).
  - We consider them equally valid intelligence measures. They measure different aspects of intelligence.

# Abs Values History: Plato's "Lesser Hippias"

- Whether to take absolute values is an ancient debate.
- In Plato's "Lesser Hippias", Socrates presents what initially seems like a compelling argument in favor of taking absolute values.

SOCRATES: "Which of the two then is a better runner? He who runs slowly voluntarily, or he who runs slowly involuntarily?" Etc. etc. etc...



# Abs Values History: Socrates' Evil Twist

- From what initially seems like a pro-abs-values argument, Socrates uses the same logic to defend the ludicrous position that it's better to be intentionally evil than unintentionally evil.

(An interesting AGI safety question. Is an intentionally evil AGI better or worse than an unintentionally evil one?)

The dialogue ends with poor Hippias hopelessly confused. Better not to take sides on the abs-value question.



# Summary

- Symmetric UTMs make symmetric Legg-Hutter intelligence measures.
- We argued symmetry is a reasonable intelligence-measure axiom.
- Symmetric UTMs can be built by eliminating a certain RL bias.
- This could narrow the space of UTMs, advancing AGI development.
- $|\Upsilon_U(\pi)|$  is an alternative to  $\Upsilon_U(\pi)$  (they measure different things).
- The  $|\Upsilon_U(\pi)|$  vs.  $\Upsilon_U(\pi)$  question goes back millennia.

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