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# A COMPUTATIONAL APPROACH TO THE ROBINSON PROJECTION

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## ABSTRACT

*The Robinson Projection is the most preferred World map projection in the atlas cartography. There are no analytical formulas except Robinson's look-up table for this projection. This deficiency has led a number of requests for the plotting formulas and cartographers have studied to derive analytical equations using different algorithms. In these works, different interpolation algorithms are applied to Robinson's table values and solutions are presented including some critics about the deformations on this projection. In this study, a summary of these computation algorithms is collected. The multiquadric interpolation method is suggested and applied to the Robinson's tabular coordinates. A series of numerical evaluations are presented then for the controversies and for comparison between these computation algorithms.*

## INTRODUCTION

Especially in atlas cartography the whole world is presented using different projection types in sense to show thematic information with physical and/or political features. The Robinson Projection was the first major map projection to be commissioned by a large private corporation; Rand McNally hired Robinson (who was a professor in the Geography Department at the University of Wisconsin in Madison from 1945 until he retired in 1980) to develop the projection because they were not satisfied with the ability of existing projections to create intuitively appealing depictions of the entire world. Rand McNally still makes extensive use of the Robinson projection, and the National Geographic Society uses it as well. From 1922 until 1988, world maps were produced using the Van der Grinten I projection. In 1988 NGS changed to the Robinson projection citing the need for a projection that reduced the amount of distortion. The adoption by the NGS of the Robinson Projection as a standard projection for its World Maps have led a number of requests for the plotting formulas.

The Robinson projection is highly unique. Unlike all other projections, Dr. Robinson did not develop this projection by developing new geometric formulas to convert latitude and longitude coordinates from the surface of the Model of the Earth to locations on the map. Instead, Dr. Robinson used a huge number of trial-and-error computer simulations to develop a table that allows a cartographer to look up how far above or below a Robinson map's equator a particular line of latitude will be located, and then to estimate (via an interpolation process) where along this line a particular longitude will fall.

For this reason cartographers have been studying to derive analytical equations using different algorithms for coding the Robinson projection. In these works, different interpolation algorithms are applied to Robinson's table values and solutions are presented including some critics about deformations on this projection. In this study, a summary of these computation algorithms is collected. The multiquadric interpolation method is suggested and applied to the Robinson's tabular coordinates. A series of numerical evaluations are presented then for the controversies and for comparison between these computation algorithms.

## ROBINSON PROJECTION

Prof. Dr. Arthur H. Robinson has developed this projection as a new grid for world maps in 1963 under the request of Rand McNally and Company. Its development and characteristics are published in 1974 [**18**]. The Robinson Projection is used in Rand McNally's *World Portrait Map* (1965), *International Atlas* (1965), *RandMark Wall Map Series, Goode's World Atlas* (1970) and Jones and Murphy's atlas of *Geography and World Affairs* (1971). In December 1988, a political World Map was published in the National Geographic magazine using the Robinson projection and the chief cartographer John B. Garver pointed out that "the Robinson projection is one of the best projections between 20 different projections selected for the optimal map grid design" [**9**]. As a recent study, Richard Capek, from the Charles University Czech Republic, arranged a hundred conventional projections into the sequence list in compliance with the distortion characterization Q, which is defined as the percentage ratio of the area represented in the map with permissible distortion to the area of the whole world. The Robinson Projection takes in the fourth place on top of this list [**5**].

The Robinson projection is in general a pseudo-cylindrical, neither a conformal nor an equal area projection. It is an uninterrupted projection where the poles are shown as lines being 0.53 times as long as the Equator and symmetric about the central meridian or the Equator. Parallels are straight parallel lines, equally spaced between latitudes 38° north and south. Space decreases beyond these limits. The Equator is as 0.8487 times long as the circumstance of a sphere of equal area. Central meridian is a straight line 0.5072 as long as the Equator. Other meridians are equally spaced elliptical arcs and concave toward the central meridian. It is pointed out that the scale is true along latitudes 38° north and south, being constant along any given latitude and same for the latitude of opposite sign [**18**], [**20**].

The continents on this projection appear in relatively correct size and location. Therefore it is called also "orthophanic" which means right appearing. Robinson gives, after empirical researches with a trial-and-error approach, tabular coordinates rather than mathematical formulas for his projection to yield the characteristics above [**1**], [**14**], [**18**], [**20**]. This tabular coordinates are shown in Table 1.

Table 1. *Tabular coordinates given by Robinson*

| i | $\varphi$ | A | B | $A^*$ | $B^*$ |
|---|---|---|---|---|---|
| 0 | 0 | 1.0000 | 0.0000 | 0.84870000 | 0.00000000 |
| 1 | 5 | 0.9986 | 0.0620 | 0.84751182 | 0.08384260 |
| 2 | 10 | 0.9954 | 0.1240 | 0.84479598 | 0.16768520 |
| 3 | 15 | 0.9900 | 0.1860 | 0.84021300 | 0.25152780 |
| 4 | 20 | 0.9822 | 0.2480 | 0.83359314 | 0.33537040 |
| 5 | 25 | 0.9730 | 0.3100 | 0.82578510 | 0.41921300 |
| 6 | 30 | 0.9600 | 0.3720 | 0.81475200 | 0.50305560 |
| 7 | 35 | 0.9427 | 0.4340 | 0.80006949 | 0.58689820 |
| 8 | 40 | 0.9216 | 0.4958 | 0.78216192 | 0.67047034 |
| 9 | 45 | 0.8962 | 0.5571 | 0.76060494 | 0.75336633 |
| 10 | 50 | 0.8679 | 0.6176 | 0.73658673 | 0.83518048 |
| 11 | 55 | 0.8350 | 0.6769 | 0.70866450 | 0.91537187 |
| 12 | 60 | 0.7986 | 0.7346 | 0.67777182 | 0.99339958 |
| 13 | 65 | 0.7597 | 0.7903 | 0.64475739 | 1.06872269 |
| 14 | 70 | 0.7186 | 0.8435 | 0.60987582 | 1.14066505 |
| 15 | 75 | 0.6732 | 0.8936 | 0.57134484 | 1.20841528 |
| 16 | 80 | 0.6213 | 0.9394 | 0.52729731 | 1.27035062 |
| 17 | 85 | 0.5722 | 0.9761 | 0.48562614 | 1.31998003 |
| 18 | 90 | 0.5322 | 1.0000 | 0.45167814 | 1.35230000 |

The x-axis of the plane coordinate system is the Equator and the y-axis corresponds to the central meridian. The projection equations are as follows;

$$x = 0.8487 \; RA\lambda$$
$$y = 1.3523 \; RB \qquad (1a)$$

where R is the radius of the sphere and λ the geographic longitude [**18**]. The variables A and B must be taken from the Table 1. But they must be interpolated for any arbitrary latitude in 5° intervals. After evaluating the constants in the projection equations, they can be written with the variables $A^*$ and $B^*$ as follows;

$$x = RA^*\lambda$$
$$y = RB^* \qquad (1b)$$

The variables $A^*$ and $B^*$ are also given in Table 1.

## COMPUTATION ALGORITHMS

A number of researchers presented different solutions calculating the plane coordinates of the Robinson projection for an arbitrary point using different interpolation algorithms between the table values. They studied on the derivation of an analytical and easily computable expression, which approximates best to the Robinson projection.

John P. Snyder, presented a second-order polynomial interpolation method using Stirling formulas [**21**]. The formulas for the Stirling interpolation may be written as

$$f(x_0+\alpha h) = f(x_0) + \frac{\alpha}{2}\left[\partial f(x_0+\frac{h}{2}) + \partial f(x_0-\frac{h}{2})\right] + \alpha^2 \frac{\partial^2 f(x_0)}{2!}$$
$$+\frac{\alpha(\alpha-1)(\alpha+1)}{2}\,\frac{\partial^3 f(x_0+\frac{h}{2}) + \partial^3 f(x_0-\frac{h}{2})}{3!} + \cdots \qquad (2a)$$

Here, $\partial$ is the difference operator and defined as [**6**].

$$\partial f(x) = f(x+\frac{h}{2}) - f(x-\frac{h}{2})$$
$$\partial^2 f(x) = \partial f(x+\frac{h}{2}) - \partial f(x-\frac{h}{2}) \qquad (2b)$$
$$\downarrow$$
$$\partial^n f(x) = \partial^{n-1} f(x+\frac{h}{2}) - \partial^{n-1} f(x-\frac{h}{2})$$

Frank Canters and Hugo Declair used a higher-order polynomial solving the coefficients with the least squares approximation method [**4**]. On the one hand, such an approximation procedure guarantees the minimization of the sum of the squares of the divergent from the reference values but on the other hand, it may not be possible to find the reference points (i.e. the tabular coordinates) exactly [**14**]. The polynomial equations of Canters & Declair [**4**] are given as follows:

$$x = R(A_0 + A_2\varphi^2 + A_4\varphi^4)\lambda$$
$$y = R(A_1 + A_3\varphi^3 + A_5\varphi^5) \qquad (3)$$

Here the coefficients are

$A_0$=0.8507, $A_2$= -0.1450, $A_4$= -0.0104, $A_1$=0.9642, $A_3$= -0.0013, $A_5$= -0.0129

Richardson examined the Robinson projection under a graphic computer program with the name Design Cad and made an area deformation analysis and printed the results [**17**].

Connected with the analytical projection principals a detailed examination of the Robinson projection was made by Beineke [1] ,[2]. His method is based on the approximation of the increase of empirical functions through the table values. In his study, with an effort to create an analytical expression in a simple form, Beineke suggested a new formulation, which resembles very close to the Robinson projection coordinates with his own words "An approximate Robinson Projection" [14].

$$x = (d + e\varphi^2 + f\varphi^4 + g\varphi^6)\frac{\lambda}{\pi} \qquad (4)$$

$$y = a\varphi + bs|\varphi|^c$$

Here, $s$ should be taken as $s = 1$ for $(\varphi \geq 0)$ or $s = -1$ for $(\varphi < 0)$) and the coefficients are

$$a = 0.96047 \quad b = -0.00857 \quad c = 6.41 \quad d = 2.6666$$

$$e = -0.367 \quad f = -0.150 \quad g = 0.0379$$

Beineke discussed the analytical approaches above and expressed that none of these computing methods are satisfactory as well. He also indicated that, the elegant solution is to use a curve which passes through all the reference points and which fits the grid construction of the projection sufficiently [1], [2].

Due to these reasons Bretterbauer dealt with the Robinson projection using cubic spline interpolation [3]. If the natural cubic spline polynomial interpolation method [7], [15], [19], [16] is applied to the Robinson projection, the following equations are given for an arbitrary latitude φ

$$\begin{aligned} A^* &= a(i)[\varphi - \varphi(i)]^3 + b(i)[\varphi - \varphi(i)]^2 + c(i)[\varphi - \varphi(i)] + d(i) \\ B^* &= e(i)[\varphi - \varphi(i)]^3 + f(i)[\varphi - \varphi(i)]^2 + g(i)[\varphi - \varphi(i)] + h(i) \end{aligned} \qquad (5)$$

φ(i) is the latitude in the (i) order on Table 1. This sequence will be calculated using

$$i = \mathrm{INT}(\mathrm{ABS}(\varphi)/5) \qquad (6)$$

for the relevant latitude φ.

Table 2. *The cubic spline coefficients for* $A^*$

| i | d(i) | c(i) | b(i) | a(i) |
|---|---|---|---|---|
| 0 | 0.84870000 | -0.00017530 | 0.00000000 | -0.00000249 |
| 1 | 0.84751182 | -0.00036231 | -0.00003740 | 0.00000025 |
| 2 | 0.84479598 | -0.00071788 | -0.00003371 | -0.00000121 |
| 3 | 0.84021300 | -0.00114546 | -0.00005180 | 0.00000322 |
| 4 | 0.83359314 | -0.00142198 | -0.00000350 | -0.00000488 |
| 5 | 0.82578510 | -0.00182334 | -0.00007677 | 0.00000002 |
| 6 | 0.81475200 | -0.00258932 | -0.00007643 | 0.00000140 |
| 7 | 0.80006949 | -0.00324874 | -0.00005546 | -0.00000222 |
| 8 | 0.78216192 | -0.00396977 | -0.00008875 | 0.00000408 |
| 9 | 0.76060494 | -0.00455092 | -0.00002748 | -0.00000461 |
| 10 | 0.73658673 | -0.00517168 | -0.00009667 | 0.00000282 |
| 11 | 0.70866450 | -0.00592662 | -0.00005432 | 0.00000079 |
| 12 | 0.67777182 | -0.00641080 | -0.00004252 | 0.00000082 |
| 13 | 0.64475739 | -0.00677444 | -0.00003021 | -0.00000203 |
| 14 | 0.60987582 | -0.00722903 | -0.00006071 | -0.00000694 |
| 15 | 0.57134484 | -0.00835697 | -0.00016487 | 0.00001487 |
| 16 | 0.52729731 | -0.00889021 | 0.00005822 | 0.00001059 |
| 17 | 0.48562614 | -0.00751340 | 0.00021714 | -0.00001448 |

For the inverse solution $B^*$ should be calculated from

$$B^* = \frac{y}{R} \tag{7}$$

and then using the coefficients *k(i), l(i), m(i)* and *n(i)* the latitude can be obtained from

$$\varphi = k(i)\left[B^* - B^*(i)\right]^3 + l(i)\left[B^* - B^*(i)\right]^2 + m(i)\left[B^* - B^*(i)\right] + n(i) \tag{8}$$

The cubic spline coefficients *k(i), l(i), m(i)* and *n(i)* are given in [**3**].

Table 3. *The cubic spline coefficients for* $B^*$

| i | h(i) | g(i) | f(i) | e(i) |
|---|---|---|---|---|
| 0 | 0.00000000 | 0.01676852 | 0.00000000 | -0.00000000 |
| 1 | 0.08384260 | 0.01676851 | -0.00000000 | 0.00000000 |
| 2 | 0.16768520 | 0.01676854 | 0.00000001 | -0.00000000 |
| 3 | 0.25152780 | 0.01676845 | -0.00000003 | 0.00000001 |
| 4 | 0.33537040 | 0.01676880 | 0.00000010 | -0.00000003 |
| 5 | 0.41921300 | 0.01676749 | -0.00000036 | 0.00000011 |
| 6 | 0.50305560 | 0.01677238 | 0.00000134 | -0.00000042 |
| 7 | 0.58689820 | 0.01675411 | -0.00000499 | -0.00000059 |
| 8 | 0.67047034 | 0.01666002 | -0.00001383 | -0.00000047 |
| 9 | 0.75336633 | 0.01648669 | -0.00002084 | -0.00000079 |
| 10 | 0.83518048 | 0.01621931 | -0.00003264 | -0.00000071 |
| 11 | 0.91537187 | 0.01583940 | -0.00004335 | -0.00000068 |
| 12 | 0.99339958 | 0.01535457 | -0.00005362 | -0.00000087 |
| 13 | 1.06872269 | 0.01475283 | -0.00006673 | -0.00000123 |
| 14 | 1.14066505 | 0.01399341 | -0.00008515 | -0.00000070 |
| 15 | 1.20841528 | 0.01308909 | -0.00009571 | -0.00000894 |
| 16 | 1.27035062 | 0.01146158 | -0.00022979 | -0.00001547 |
| 17 | 1.31998003 | 0.00800345 | -0.00046184 | 0.00003079 |

The geographical longitude is derived [**3**].

$$\lambda = \frac{x}{RA^*} \tag{9}$$

## MULTIQUADRIC INTERPOLATION METHOD: DEFINITION AND CONCEPT

As an alternative computation algorithm, Ipbuker applied multiquadric interpolation method to the Robinson projection [**14**]. According to Hardy, an irregular (mathematically undefined) surface may be approximated to any desired degree of exactness by the summation of regular (mathematically defined) surfaces, particularly in quadric forms. Hardy preferred to call this method as multiquadric analysis [**11**], [**12**].

By the multiquadric interpolation method a single function Z=f(x,y) is defined for the data set using all the reference points. This function can be written as the summation of the second-degree equations, of which coefficients should be determined. Hardy developed this method in 1968 with the derivation of equations for topography and other irregular surfaces [**10**], [**11**], [**13**]. Beyond the digital terrain models, this is applied to solve different surveying problems where an irregular surface should be defined with a single function using all reference points [**14**], [**22**], [**23**], [**24**].

In a general Cartesian coordinate form, a multiquadric surface may be expressed as series as follows;

$$\sum_{j=1}^{n} c_j q(x_j, y_j, x, y) = Z \tag{10}$$

Here Z is a function of x and y as the summation of a single class of quadric surfaces. The unknowns $c_j$ are just coefficients and expressed as the slope and algebraic sign of the second-order term. Multiquadric surface may be defined also as the summation of the series of hyperboloids as follows;

$$\sum_{j=1}^{n} c_j \left[(x_j - x)^2 + (y_j - y)^2 + k\right]^{1/2} = Z \tag{11}$$

Here k is a constant. This constant is taken as zero and the expression is expanded to a representation of n linear equations with n unknowns as follows;

$$\sum_{j=1}^{n} c_j \left[(x_j - x_i)^2 + (y_j - y_i)^2\right]^{1/2} = Z_i \quad , \; i = 1,2,\cdots,n \tag{12}$$

Here, each element of the multiquadric coefficient matrix $\underline{\mathbf{A}}$ is

$$\left[(x_j - x_i)^2 + (y_j - y_i)^2\right]^{1/2} = a_{ij} \tag{13}$$

If the unknown coefficients $c_j$ are defined as a n-dimensional vector in the following form

$$\underline{c} = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix}^T \tag{14}$$

and the variables $Z_I$ which are known from the position of the reference points are shown as

$$\underline{Z} = \begin{bmatrix} Z_1 & Z_2 & \cdots & Z_n \end{bmatrix}^T \tag{15}$$

so the system of equations which is represented in (13) reduces to

$$\underline{\mathbf{A}}\,\underline{\mathbf{c}} = \underline{\mathbf{Z}} \tag{16}$$

and the unknown's $c_j$ obtained from the matrix equation

$$\underline{\mathbf{c}} = \underline{\mathbf{A}}^{-1}\,\underline{\mathbf{Z}} \tag{17}$$

Once the coefficients $c_j$ have been computed, the multiquadric surface is formed. Consequently the variable $Z_i$ can be calculated for an arbitrary point $(x_i,y_i)$ using equation (12) **[14]**, **[22]**, **[23]**, **[24]**.

$$\sum_{j=0}^{18} p_j \left[(\lambda_j - \lambda_i)^2 + (\varphi_j - \varphi_i)^2\right]^{1/2} = A_i^* \tag{18a}$$

$$\sum_{j=0}^{18} q_j \left[(\lambda_j - \lambda_i)^2 + (\varphi_j - \varphi_i)^2\right]^{1/2} = B_i^* \tag{18b}$$

After solving the unknown coefficients $p_j$ and $q_j$ the Robinson projection coordinates for an arbitrary point can be calculated using the following equations

$$x_i = R\lambda_i \sum_{j=1}^{18} p_j \left|5j - \varphi_i\right| \tag{19a}$$

$$y_i = R \sum_{j=1}^{18} q_j \left|5j - \varphi_i\right|$$

(19b)

The coefficients $p_j$ and $q_j$ are given in Table 4. A world map drawn using multiquadric method is shown in Figure 1.

Table 4. *Multiquadric coefficients*

| i | φ | p | q | m | n |
|---|---|---|---|---|---|
| 0 | 0 | 0.40711579454 | 0.91083562255 | 0.47371661113 | 1.07729625255 |
| 1 | 5 | -0.00875326537 | -0.00000589975 | -0.00911028522 | -0.00012324928 |
| 2 | 10 | -0.01069796348 | 0.00000564852 | -0.01113479305 | -0.00032923415 |
| 3 | 15 | -0.01167039606 | -0.00000557909 | -0.01214704697 | -0.00056627609 |
| 4 | 20 | -0.00680782592 | 0.00000555879 | -0.00708577740 | -0.00045168290 |
| 5 | 25 | -0.01847822803 | -0.00000001291 | -0.01923282436 | -0.00141388769 |
| 6 | 30 | -0.02090931959 | -0.00000546138 | -0.02176345915 | -0.00211521349 |
| 7 | 35 | -0.01847842619 | -0.00154708482 | -0.01957843209 | -0.00083658786 |
| 8 | 40 | -0.02090971277 | -0.00387351841 | -0.02288586729 | 0.00073523299 |
| 9 | 45 | -0.01410147990 | -0.00619324913 | -0.01676092031 | 0.00349045186 |
| 10 | 50 | -0.02236858853 | -0.00930492848 | -0.02731224791 | 0.00502041018 |
| 11 | 55 | -0.01701955610 | -0.01239340212 | -0.02386224240 | 0.00860101415 |
| 12 | 60 | -0.01215649454 | -0.01549814705 | -0.02119239013 | 0.01281238969 |
| 13 | 65 | -0.01069792545 | -0.01937169560 | -0.02327513775 | 0.01794606372 |
| 14 | 70 | -0.02090967766 | -0.02401844414 | -0.04193330922 | 0.02090220870 |
| 15 | 75 | -0.03160740722 | -0.03331171624 | -0.07123235442 | 0.02831504310 |
| 16 | 80 | 0.01361549135 | -0.07051393824 | -0.06423048161 | 0.11177176318 |
| 17 | 85 | 0.04425022432 | -0.09917388904 | -0.10536278437 | 0.28108668066 |
| 18 | 90 | 0.60843116534 | 0.24527101656 | 1.00598851957 | -0.45126573496 |

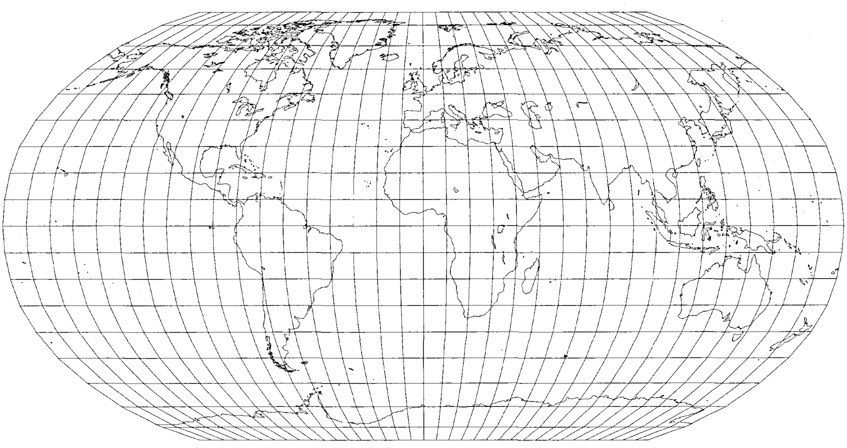

Fig.1. Robinson Projection with Shorelines, 10° Graticule, Central Meridian 0°, Drawn Using Multiquadric Equations

In case of inverse computation, at first $B^*$ should be calculated from

$$B_i^* = \frac{y_i}{R} \tag{20}$$

and corresponding $A^*$ values are obtained using

$$A_i^* = R\sum_{j=0}^{18} m_j \left|B_j^* - B_i^*\right| \tag{21}$$

The longitude $\lambda_i$ is easily determined from

$$\lambda_i = \frac{x_i}{RA_i^*} \tag{22}$$

Then the latitude $\varphi_i$ may be calculated from

$$\varphi_i = \sum_{j=0}^{18} n_j \left[ \left(A_j^* - A_i^*\right)^2 + \left(B_j^* - B_i^*\right)^2 \right]^{1/2} \tag{23}$$

The coefficients $m_j$ and $n_j$ are computed and also given in Table 5.

## DEFORMATIONS ON THE ROBINSON PROJECTION

The Robinson projection is examined according to the deformations using the above algorithms. The scale along the meridian ($h$), the scale along the parallel ($k$), the area scale ($p$) and the maximum angular distortion ($\varpi$) are given as follows [**8**].

$$h = \frac{1}{R}\sqrt{\left(\frac{\partial x}{\partial \varphi}\right)^2 + \left(\frac{\partial y}{\partial \varphi}\right)^2} \tag{24}$$

$$k = \frac{1}{R\cos\varphi}\sqrt{\left(\frac{\partial x}{\partial \lambda}\right)^2 + \left(\frac{\partial y}{\partial \lambda}\right)^2} \tag{25}$$

$$p = \frac{1}{R^2 \cos\varphi}\left(\frac{\partial x}{\partial \varphi}\frac{\partial y}{\partial \lambda} - \frac{\partial x}{\partial \lambda}\frac{\partial y}{\partial \varphi}\right) \tag{26}$$

$$\varpi = 2\arctan\sqrt{\frac{h^2 + k^2}{4p} - \frac{1}{2}} \tag{27}$$

These distortion values that are computed using the equations suggested by Canters & Declair, Beineke, Bretterbauer and Ipbuker are given in Table 5, 6, 7 and 8 respectively. It is needed the partial derivatives of the functions given for each method to compute the deformation values mentioned above.

Table 5. *Deformations using the method of Canters & Declair*

| φ | λ | h | k | ω | φ | λ | h | k | ω |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0.964 | 0.851 | 7.17 | 60 | 0 | 0.882 | 1.358 | 24.53 |
| 0 | 30 | 0.964 | 0.851 | 7.17 | 60 | 30 | 0.901 | 1.358 | 26.24 |
| 0 | 60 | 0.964 | 0.851 | 7.17 | 60 | 60 | 0.956 | 1.358 | 30.73 |
| 0 | 90 | 0.964 | 0.851 | 7.17 | 60 | 90 | 1.041 | 1.358 | 36.83 |
| 0 | 120 | 0.964 | 0.851 | 7.17 | 60 | 120 | 1.149 | 1.358 | 43.64 |
| 0 | 150 | 0.964 | 0.851 | 7.17 | 60 | 150 | 1.275 | 1.358 | 50.64 |
| 0 | 180 | 0.964 | 0.851 | 7.17 | 60 | 180 | 1.413 | 1.358 | 57.55 |
| 30 | 0 | 0.958 | 0.935 | 1.38 | 85 | 0 | 0.643 | 5.521 | 104.62 |
| 30 | 30 | 0.962 | 0.935 | 5.18 | 85 | 30 | 0.708 | 5.521 | 104.72 |
| 30 | 60 | 0.972 | 0.935 | 10.07 | 85 | 60 | 0.875 | 5.521 | 105.02 |
| 30 | 90 | 0.990 | 0.935 | 14.98 | 85 | 90 | 1.097 | 5.521 | 105.52 |
| 30 | 120 | 1.014 | 0.935 | 19.85 | 85 | 120 | 1.349 | 5.521 | 106.20 |
| 30 | 150 | 1.044 | 0.935 | 24.65 | 85 | 150 | 1.615 | 5.521 | 107.05 |
| 30 | 180 | 1.079 | 0.935 | 29.37 | 85 | 180 | 1.891 | 5.521 | 108.05 |

Table 6. *Deformations using Beineke's method*

| φ | λ | h | k | ω | φ | λ | h | k | ω |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0.960 | 0.849 | 7.08 | 60 | 0 | 0.890 | 1.358 | 24.05 |
| 0 | 30 | 0.960 | 0.849 | 7.08 | 60 | 30 | 0.911 | 1.358 | 25.99 |
| 0 | 60 | 0.960 | 0.849 | 7.08 | 60 | 60 | 0.972 | 1.358 | 31.00 |
| 0 | 90 | 0.960 | 0.849 | 7.08 | 60 | 90 | 1.065 | 1.358 | 37.66 |
| 0 | 120 | 0.960 | 0.849 | 7.08 | 60 | 120 | 1.184 | 1.358 | 44.99 |
| 0 | 150 | 0.960 | 0.849 | 7.08 | 60 | 150 | 1.321 | 1.358 | 52.43 |
| 0 | 180 | 0.960 | 0.849 | 7.08 | 60 | 180 | 1.471 | 1.358 | 59.68 |
| 30 | 0 | 0.959 | 0.939 | 1.18 | 85 | 0 | 0.496 | 5.611 | 113.74 |
| 30 | 30 | 0.962 | 0.939 | 4.79 | 85 | 30 | 0.550 | 5.611 | 113.80 |
| 30 | 60 | 0.971 | 0.939 | 9.34 | 85 | 60 | 0.685 | 5.611 | 113.96 |
| 30 | 90 | 0.986 | 0.939 | 13.91 | 85 | 90 | 0.864 | 5.611 | 114.24 |
| 30 | 120 | 1.007 | 0.939 | 18.45 | 85 | 120 | 1.065 | 5.611 | 114.61 |
| 30 | 150 | 1.033 | 0.939 | 22.94 | 85 | 150 | 1.279 | 5.611 | 115.09 |
| 30 | 180 | 1.064 | 0.939 | 27.36 | 85 | 180 | 1.498 | 5.611 | 115.66 |

The partial derivatives can be taken for the equations suggested by Canters & Declair as follows;

$$\frac{\partial y}{\partial \varphi} = A_1 + 3A_3\varphi^2 + 5A_5\varphi^4 \qquad \frac{\partial y}{\partial \lambda} = 0$$
$$\frac{\partial x}{\partial \varphi} = \lambda(2A_2\varphi + 4A_4\varphi^3) \qquad \frac{\partial x}{\partial \lambda} = A_0 + A_2\varphi^2 + A_4\varphi^4 \tag{28}$$

for Beineke's equations

$$\frac{\partial y}{\partial \varphi} = a + bsc\varphi^{c-1} \qquad \frac{\partial y}{\partial \lambda} = 0$$
$$\frac{\partial x}{\partial \varphi} = (2e\varphi + 4f\varphi^3 + 6g\varphi^5)\lambda / \pi \qquad \frac{\partial x}{\partial \lambda} = (d + e\varphi^2 + f\varphi^4 + g\varphi^6)/\pi \tag{29}$$

and for Bretterbauer's cubic spline method

$$\frac{\partial y}{\partial \varphi} = 3e(i)[\varphi - \varphi(i)]^2 + 2f(i)[\varphi - \varphi(i)] + g(i)$$
$$\frac{\partial x}{\partial \varphi} = \lambda\left(3a(i)[\varphi - \varphi(i)]^2 + 2b(i)[\varphi - \varphi(i)] + c(i)\right)$$
$$\frac{\partial y}{\partial \lambda} = 0 \tag{30}$$
$$\frac{\partial x}{\partial \lambda} = a(i)[\varphi - \varphi(i)]^3 + b(i)[\varphi - \varphi(i)]^2 + c(i)[\varphi - \varphi(i)] + d(i)$$

If the multiquadric method is applied, the following partial derivatives should be used

$$\frac{\partial y}{\partial \varphi} = R\lambda\sum_{j=0}^{18} p_j \frac{\varphi - 5j}{\left(25j^2 - 10j\varphi + \varphi^2\right)^{1/2}}$$
$$\frac{\partial y}{\partial \lambda} = R\sum_{j=0}^{18} p_j \left(25j^2 - 10j\varphi + \varphi^2\right)^{1/2} \tag{31}$$

$$\frac{\partial x}{\partial \varphi}=R\sum_{j=0}^{18} q_j \frac{\varphi-5j}{\left(25j^2-10j\varphi+\varphi^2\right)^{1/2}} \qquad \frac{\partial x}{\partial \lambda}=0 \qquad [\mathbf{14}].$$

Table 7. *Deformations using cubic spline method*

| φ | λ | h | k | ω | φ | λ | h | k | ω |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0.961 | 0.849 | 7.10 | 60 | 0 | 0.880 | 1.356 | 24.58 |
| 0 | 30 | 0.961 | 0.849 | 7.11 | 60 | 30 | 0.901 | 1.356 | 26.45 |
| 0 | 60 | 0.961 | 0.849 | 7.13 | 60 | 60 | 0.960 | 1.356 | 31.30 |
| 0 | 90 | 0.961 | 0.849 | 7.17 | 60 | 90 | 1.052 | 1.356 | 37.80 |
| 0 | 120 | 0.961 | 0.849 | 7.23 | 60 | 120 | 1.169 | 1.356 | 44.99 |
| 0 | 150 | 0.961 | 0.849 | 7.29 | 60 | 150 | 1.303 | 1.356 | 52.32 |
| 0 | 180 | 0.961 | 0.849 | 7.38 | 60 | 180 | 1.451 | 1.356 | 59.50 |
| 30 | 0 | 0.961 | 0.941 | 1.22 | 85 | 0 | 0.459 | 5.572 | 115.97 |
| 30 | 30 | 0.964 | 0.941 | 4.83 | 85 | 30 | 0.511 | 5.572 | 116.02 |
| 30 | 60 | 0.973 | 0.941 | 9.42 | 85 | 60 | 0.643 | 5.572 | 116.17 |
| 30 | 90 | 0.989 | 0.941 | 14.02 | 85 | 90 | 0.817 | 5.572 | 116.42 |
| 30 | 120 | 1.010 | 0.941 | 18.60 | 85 | 120 | 1.012 | 5.572 | 116.76 |
| 30 | 150 | 1.037 | 0.941 | 23.12 | 85 | 150 | 1.217 | 5.572 | 117.19 |
| 30 | 180 | 1.068 | 0.941 | 27.57 | 85 | 180 | 1.428 | 5.572 | 117.70 |

Here, the coefficients $p_j$ and $q_j$ should be taken from Table 4.

Table 8. *Deformations using multiquadric method*

| φ | λ | h | k | ω | φ | λ | h | k | ω |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0.961 | 0.849 | 7.10 | 60 | 0 | 0.879 | 1.356 | 24.65 |
| 0 | 30 | 0.961 | 0.849 | 7.12 | 60 | 30 | 0.899 | 1.356 | 26.50 |
| 0 | 60 | 0.961 | 0.849 | 7.16 | 60 | 60 | 0.959 | 1.356 | 31.32 |
| 0 | 90 | 0.961 | 0.849 | 7.23 | 60 | 90 | 1.050 | 1.356 | 37.79 |
| 0 | 120 | 0.961 | 0.849 | 7.33 | 60 | 120 | 1.166 | 1.356 | 44.95 |
| 0 | 150 | 0.961 | 0.849 | 7.45 | 60 | 150 | 1.300 | 1.356 | 52.26 |
| 0 | 180 | 0.962 | 0.849 | 7.60 | 60 | 180 | 1.449 | 1.356 | 59.41 |
| 30 | 0 | 0.961 | 0.941 | 1.20 | 85 | 0 | 0.470 | 5.572 | 115.25 |
| 30 | 30 | 0.964 | 0.941 | 4.80 | 85 | 30 | 0.521 | 5.572 | 115.30 |
| 30 | 60 | 0.973 | 0.941 | 9.36 | 85 | 60 | 0.653 | 5.572 | 115.45 |
| 30 | 90 | 0.988 | 0.941 | 13.93 | 85 | 90 | 0.827 | 5.572 | 115.71 |
| 30 | 120 | 1.009 | 0.941 | 18.47 | 85 | 120 | 1.022 | 5.572 | 116.05 |
| 30 | 150 | 1.035 | 0.941 | 22.97 | 85 | 150 | 1.228 | 5.572 | 116.49 |
| 30 | 180 | 1.066 | 0.941 | 27.39 | 85 | 180 | 1.440 | 5.572 | 117.02 |

The main characteristics of the Robinson projection are the ratio of the length of the central meridian ($S_2$) to the length of equator ($S_1$), the ratio of the length of the poles ($S_3$) to the length of equator ($S_1$) and the ratio of the length of the equator ($S_1$) to the length of the circumstance of the sphere ($S_4$=628.31853 mm). These values are calculated for each method and presented in Table 9.

Table 9. *Comparison of the projection characteristics (R=100)*

| Method | Length of Equator ($S_1$) (mm) | Length of Central Meridian ($S_2$) (mm) | Length of Poles ($S_3$) mm) | $S_2/S_1$ | $S_3/S_1$ | $S_1/S_4$ |
|---|---|---|---|---|---|---|
| Canters & Declair | 534.5106 | 277.2318 | 269.933 | 0.51866 | 0.5050 | 0.8507 |
| Beineke | 533.3200 | 270.7564 | 283.435 | 0.50768 | 0.5315 | 0.8488 |
| Cubic Spline | 533.2539 | 270.4600 | 283.798 | 0.50719 | 0.5322 | 0.8487 |
| Multiquadric | 533.2539 | 270.4600 | 283.798 | 0.50719 | 0.5322 | 0.8487 |

Richardson calculated the area scale values, but they differ from Robinson's values significantly. These area scale values are computed according to the four algorithms and presented with the Robinson's and Richardson's values together in Table 10.

Table 10. F(%)=(F-1)x100 *Comparison of the area deformation values ($\lambda$=0)*

| φ | Robinson | Richardson | Canters&Declair | Beineke | Cubic Spline | Multiquadric |
|---|---|---|---|---|---|---|
| 0 | -19.55 | -18.46 | -17.976 | -18.475 | -18.460 | -18.459 |
| 5 | -19.23 | -18.26 | -17.772 | -18.249 | -18.263 | -18.263 |
| 10 | -18.96 | -17.58 | -17.159 | -17.569 | -17.583 | -17.583 |
| 15 | -17.31 | -16.43 | -16.129 | -16.420 | -16.428 | -16.428 |
| 20 | -14.45 | -14.77 | -14.667 | -14.784 | -14.770 | -14.771 |
| 25 | -11.21 | -12.46 | -12.753 | -12.635 | -12.465 | -12.459 |
| 30 | -7.24 | -9.61 | -10.353 | -9.941 | -9.591 | -9.612 |
| 35 | -3.06 | -6.32 | -7.419 | -6.666 | -6.242 | -6.313 |
| 40 | 2.16 | -2.54 | -3.879 | -2.765 | -2.537 | -2.614 |
| 45 | 7.85 | 1.59 | 0.378 | 1.822 | 1.609 | 1.512 |
| 50 | 14.78 | 6.45 | 5.527 | 7.180 | 6.491 | 6.368 |
| 55 | 23.06 | 12.20 | 11.854 | 13.450 | 12.127 | 12.003 |
| 60 | 31.89 | 19.31 | 19.856 | 20.891 | 19.254 | 19.103 |
| 65 | 45.14 | 28.96 | 30.459 | 30.014 | 28.957 | 28.727 |
| 70 | 65.63 | 43.00 | 45.561 | 41.927 | 42.967 | 42.720 |
| 75 | 94.95 | 65.34 | 69.678 | 59.363 | 65.552 | 61.592 |
| 80 | 135.53 | 100.38 | 116.525 | 90.753 | 99.413 | 94.592 |
| 85 | 227.96 | 152.57 | 255.11603 | 178.55149 | 155.50868 | 161.608 |

The crucial point here is the closeness of the values obtained due to the cubic spline interpolation proposed by Bretterbauer and the values given by Richardson. If it is considered that Richardson obtained these values by graphically, it can be said that the projection geometry would be expressed most rightly by the numerical way. The values obtained from the equations of Beineke differ as much as latitudes grew and the insufficiency of the equations of Canters & Declair in practical limits appears here too.

Another numerical exercise has been carried out to obtain the correct latitude value on which the area scale is preserved by using the coordinates calculated with the multiquadric method. The areas of sequentially zones which have 10 degrees latitude interval on sphere (F) are calculated and the areas of consecutive overall of 0°.5 interval ratio graticules are taken from the projection coordinates and the reciprocal area values to each zones which have 10 degree interval are given in Table 11 (R=100 accepted). It can be seen that the minimum deformation is around the 43° latitude on the Robinson projection.

Table 11. *Comparison of the areas of the zones using multiquadric method*

| $\varphi_1$-$\varphi_2$ | f (mm$^2$) | F (mm$^2$) | f/F |
|---|---|---|---|
| 0°-10° | 8927 | 10911 | 0.82 |
| 10°-20° | 8850 | 10579 | 0.84 |
| 20°-30° | 8678 | 9926 | 0.87 |
| 30°-40° | 8376 | 8972 | 0.93 |
| 37°50′-38°20′ | 275 | 288 | 0.96 |
| 42°50′-43°20′ | 267 | 267 | 1.00 |
| 40°-50° | 7884 | 7744 | 1.02 |
| 50°-60° | 7121 | 6282 | 1.13 |
| 60°-70° | 6009 | 4629 | 1.30 |
| 70°-80° | 4508 | 2835 | 1.59 |

## CONCLUSION

Cubic spline interpolation seems as the suitable mathematical tool among others for the calculation of the Robinson projection coordinates. This interpolation method was applied by Bretterbauer and then approved by Beineke. Recently, rather than the complex algorithms, simple and easily applicable algorithms have been preferred. The equations for "*approximate Robinson Projection*" suggested and named by Beineke yield satisfactory results for practical use. It produces quite efficacious results when compared with the cubic spline method. The interpolation algorithm used by Snyder is easy use and appropriate for practical purposes as making small-scale world maps.

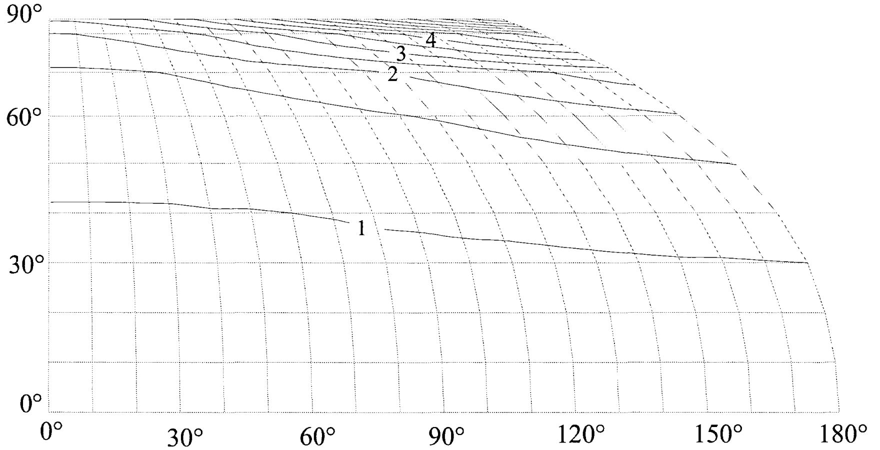


Fig.2. Isolines for area deformation

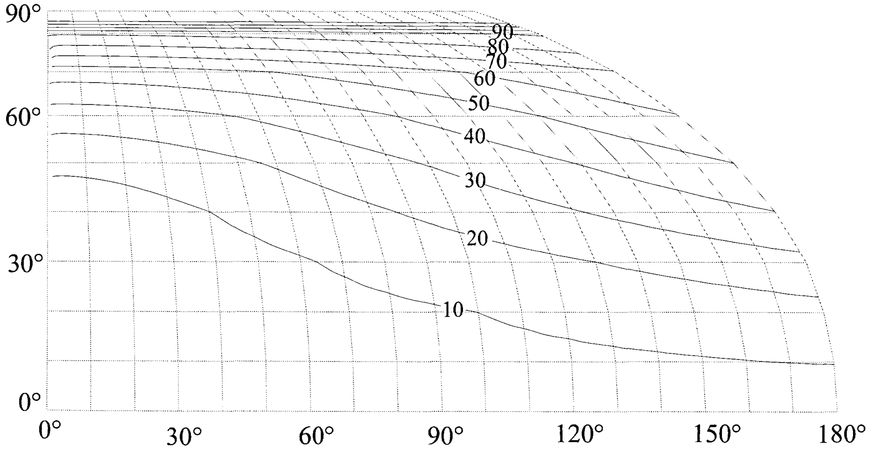


Fig.3. Isolines for maximum angular deformation

Beyond coding the projection, when the Robinson Projection is examined in terms of the deformations, quite interesting and specific results are obtained as a supplementary objective. For example; we know that the Robinson Projection was designed according to the 38° standard parallel and is equidistant along this latitude [18]. But the numerical results are contradictory with this assumption. Beineke has realized that the projection is equidistant on the 43° latitude, rather than the 38° latitude.

When we look at the Figure 2, the projection could not be equivalent along a demonstrated latitude because of the equations also depend on the geographical longitude $\lambda$. The preservation of scale on an arbitrary point or along an arbitrary direction depends on the both of the geographical quantities i.e. latitude and longitude in accordance with the construction of the projection. This is the fact in the zone between 38°- 48° latitudes. Beineke was right when he was pointing to the central latitude 43° of the zone. But in my opinion, the assertion becomes significant when it is answered to the question on which latitude the scale deformation coefficient equals 1 along the prime meridian ($\lambda_o$=0°).

Richardson reported that he has interpolated this value as 43°.14 using on screen digitized coordinates under a graphic program namely Design Cad. He also interpolated the latitude on which the maximum angular deformation equals to zero as 32°.85 [**17**]. I calculated the latitude on the central meridian on which the area scale is true as 43°.15972 using the multiquadric method. I also obtained the latitude on the central meridian on which the maximum angular deformation equals to zero as 32°.905.

Under these numerical results, it can be said that the Robinson projection preserves the area scale on a latitude zone between 42°50′ - 43°20′ in practical limits. Multiquadric equations seem as the best algorithm, coding the Robinson projection. But the work with series of coefficients may be regarded as a disadvantage. In that case, the equations suggested by Beineke can be taken as the plotting formulas for the Robinson projection in small scale mapping.

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