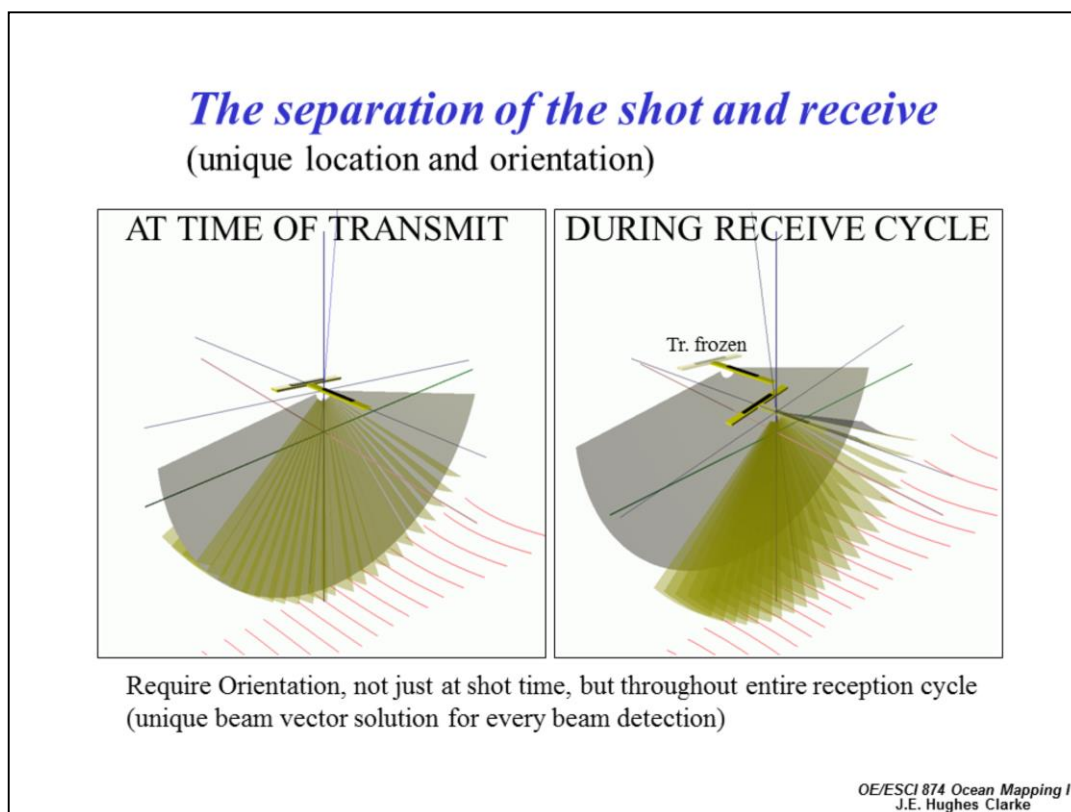


**The Full Calculation of Resulting Beam Vector
w.r.t.
Ship's Head at Transmit and the Local Level
*(the cone-to-cone intersection)***

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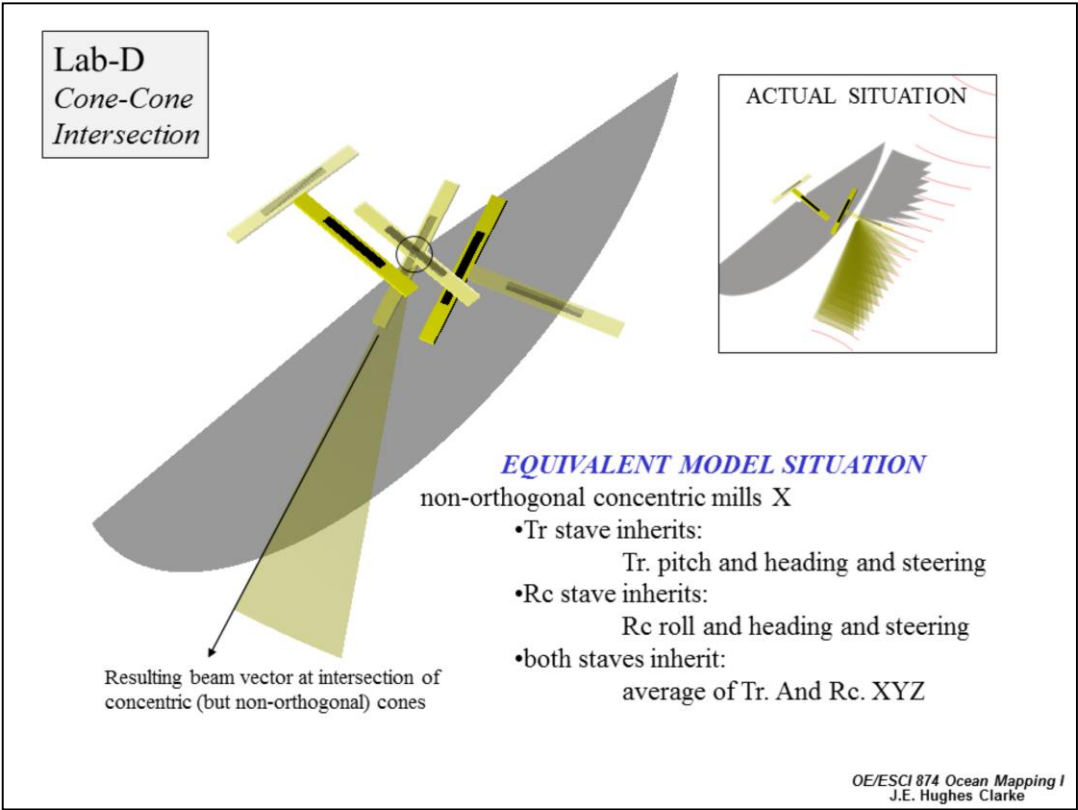
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As there is a finite time between transmission and reception, both the position and the orientation of the array will change in that interval. That interval can typically range from $\sim 1/10$ of second ($< 75\text{m}$ ranges) to 10's of seconds for deep water systems.

If a single location value is to be used, the average of the transmitter position at transmit and receiver position at the average received TWTT can reasonably be used. Strictly though, over the reception cycle, the receiver does propagate forward. But at typical vessel speeds ($\sim 4\text{m/s}$) the distance travelled, compared to the one way sound speed (750m/s) is a very small fraction (0.5%) which is smaller than the beam footprint. For example, the projected beam width of a 0.5° beam is 0.87% of Z (strictly slant range). Both these numbers are small compared to IHO Special Order horizontal accuracy requirements of 2m in 40m of water which corresponds to only 5% of Z .

NOTE that this excludes the physical separation of the transmitter and receiver acoustic centres (normally at least $\frac{1}{2}$ the physical array length). If bottom detection is attempted in the acoustic nearfield, that separation may be significantly larger than the forward propagation of the sonar over the ping cycle.

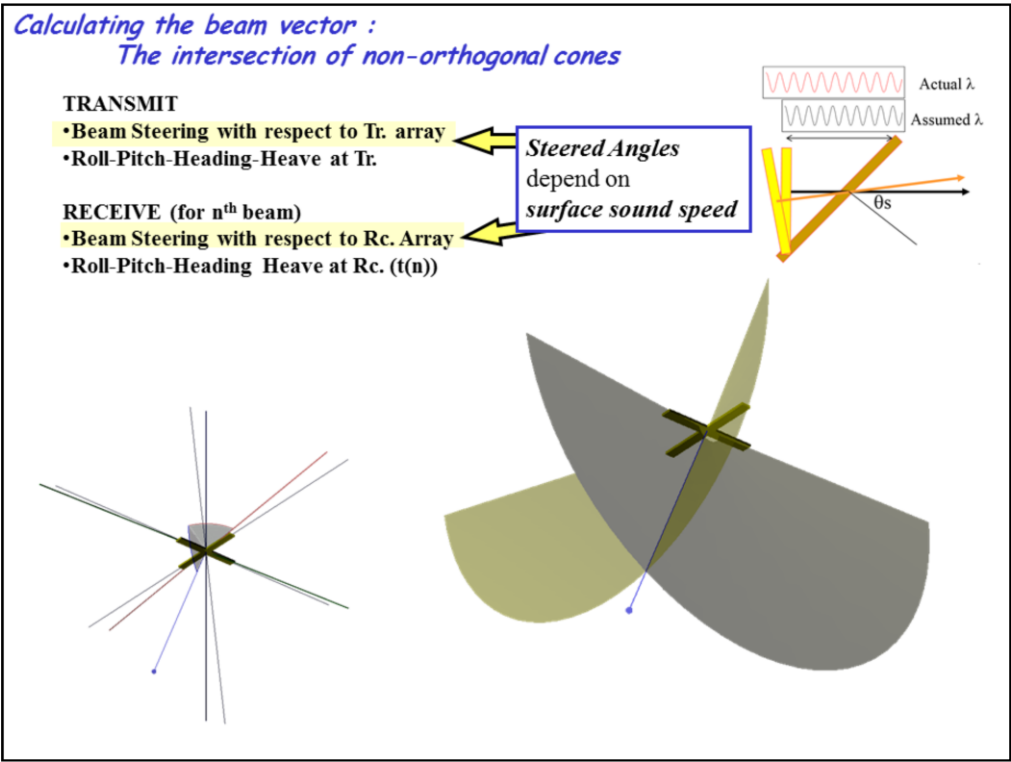


Equivalent Modeled Virtual Array.

To calculate a single beam vector fro the outgoing and incoming ray path, an equivalent array geometry needs to be defined in which the two acoustic centres are co-located.

The figure above illustrates the modeled beam vector solution. A virtual array location , half way in X,Y and Z from the acoustic centre at Tx and Rc. is calculated. The arrays, however, are no longer mutually orthogonal (even if the physically constructed arrays actually are). This is because time has elapsed.

The virtual concentric array model, adopts the Tx. Orientation for the transmitter and the Rc, orientations for the receiver.



For a given epoch, the relative alignment of the transmitter and receiver is precisely known and is usually as close to orthogonal as possible. This is known as it is either machined in the factory for a small sonar or precisely as-placed surveyed after installation on the hull. While their relative alignment is known at a single time, the geometry of the Mills Cross used to estimate the formed beam vector cannot assume that same relative misalignment. At the time of the receive, the receiver will no longer share the same orientation as the transmitter at time of transmit.

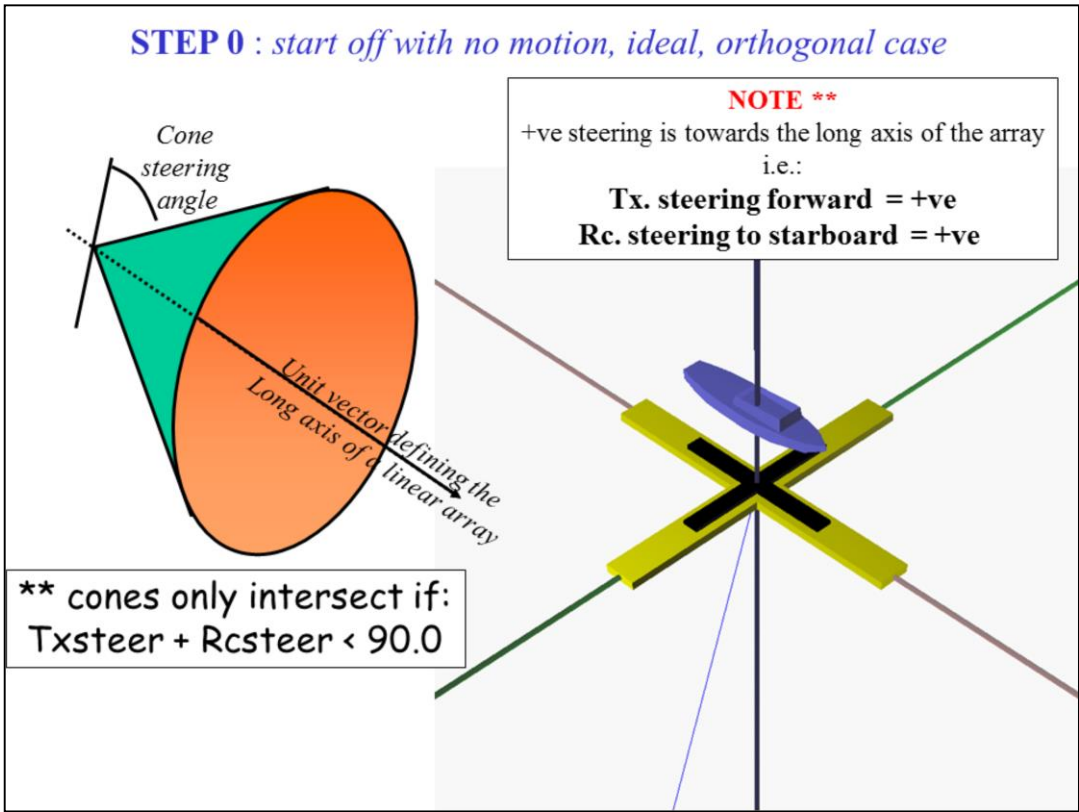
As a result, the beam vector has to modeled by calculating the intersection of non-orthogonal cones

To calculate the resultant beam vector, one must know all of:

- the orientation of the transmitter long axis at T_{tx} and
- The orientation of the receiver long axis at T_{rc} as well as
- the transit steering angle relative to the long axis of transmit array and
- The receiver steering angle (at the time of receive) relative to the long axis of the receiver array

Each of the steering angles, relative to the long axis of the transmitter and receiver, depend on an assumption of the sound speed at the array. During acquisition, a specific surface sound speed value is used on which the element-specific time delays are based. Thus if there is any reason to reapply a different surface sound speed, all the array-relative steering angles have to be adjusted.

The details of the cone-cone
intersection transformations
will be described in the
laboratory session.



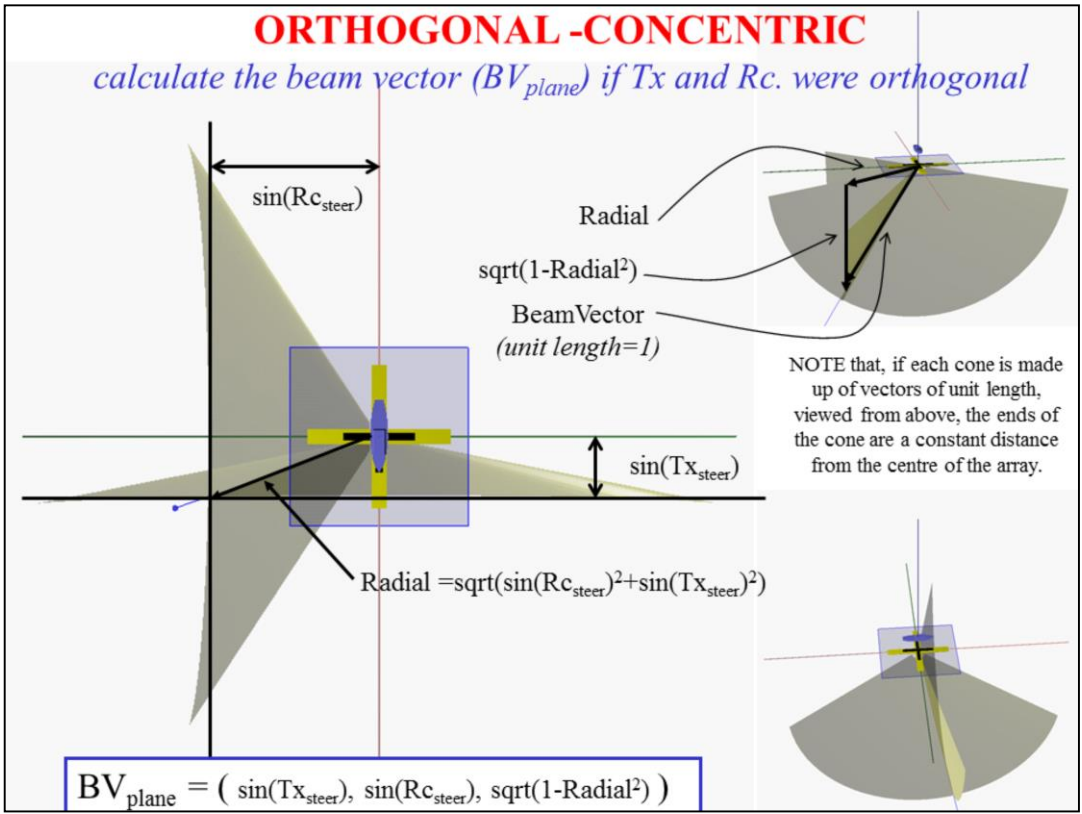
Why colocation?

We are concerned with establishing the point on the seafloor at which the conical maximum response axes of the transmitter and receiver intersect. In reality, the two cones will be distorted by refraction. If the two acoustic centres are not at the same location, then the refracted ray paths from each centre to the strike point will not be identical. If, however, they are concentric, they will share the same distortion. Under this special circumstance, we can ignore the refraction and just deal with a single vector which represents the initial ray vector as it leave/returns to the Mills cross.

Two cones from mutually orthogonal arrays, only intersect if the sum of the two steering angles is less than 90 degrees. For the case of a multibeam, the transmitter steering is generally less than 5 degrees (up to 15 though if heavy yaw stabilization is utilized). Thus receiver steering angles of up to 85 (75 for a heavy yaw stabilization condition) are allowable.

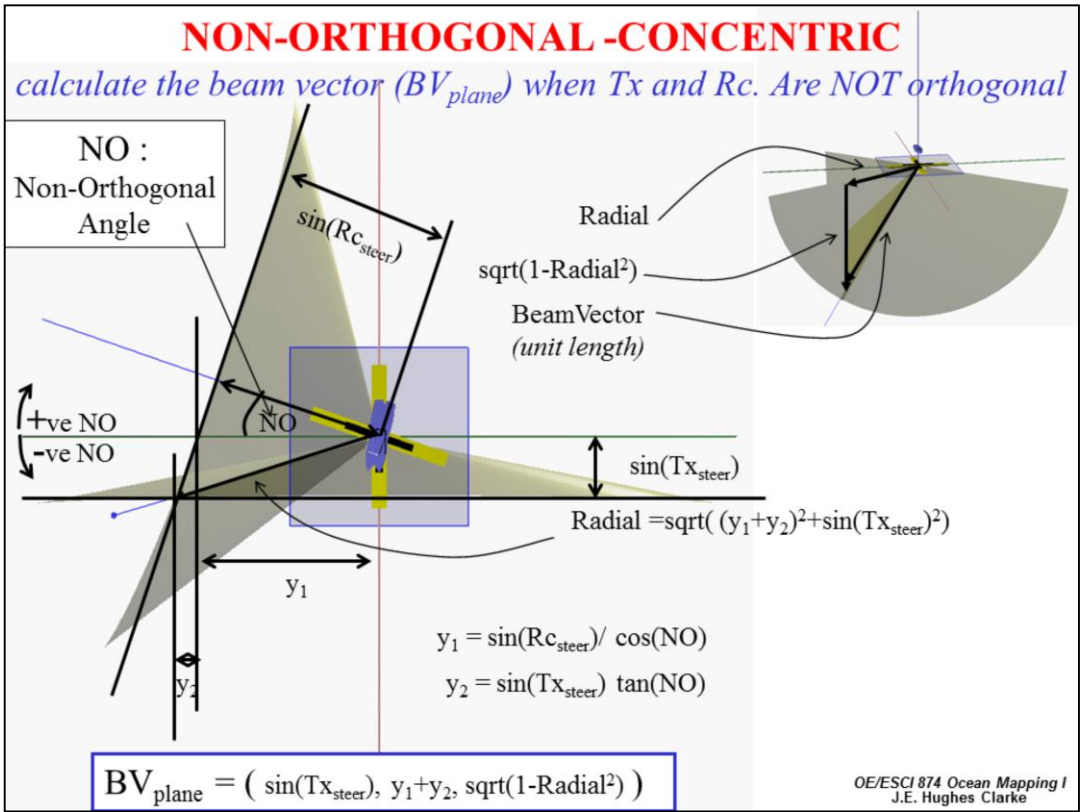
Sign Conventions:

Note the conventions used in the following calculations. The unit vector for the transmitter is 1,0,0 and that for the receiver is 0,1,0 Positive steering is defined as toward the principal vector orientation. Thus +ve forward for transmission, and positive starboard for reception. When unpacking third party sonar telegrams, the sign of the steering must be clarified as it is not a standard.



Formula for intersection of orthogonal arrays

Under this simplest geometry, the resulting beam vector can easily be calculated. Note that the vector is strictly relative to the plane containing the two arrays with the X axis aligned along the transmitter.



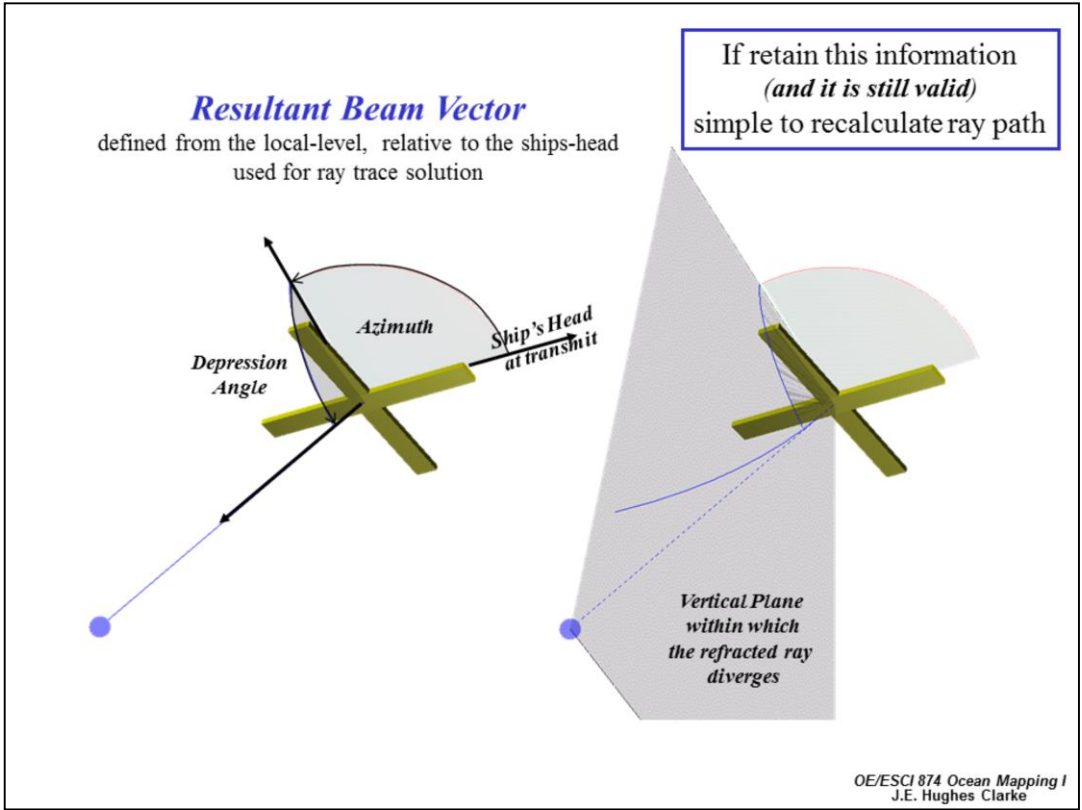
Formula for intersection of non-orthogonal arrays

As a small extension of the previous case, if the receiver is oriented at an angle away from 90 degrees to the transmitter, the formula is slightly different, but still easily calculated.

This is the more general case that will be used in the following steps. The only requirement is to establish that non-orthogonal angle (NO).

As with the previous example, the resulting beam vector is relative to the plane that contains the two arrays with the X- axis along the transmitter.

The main aim of the following steps is to obtain the instantaneous orientation (relative to the local level and north (or ships head at Tx)) of the plane that contains the transmit and receiver arrays vector. For this we need to know the instantaneous orientation of the two arrays at Tx and Rx times.

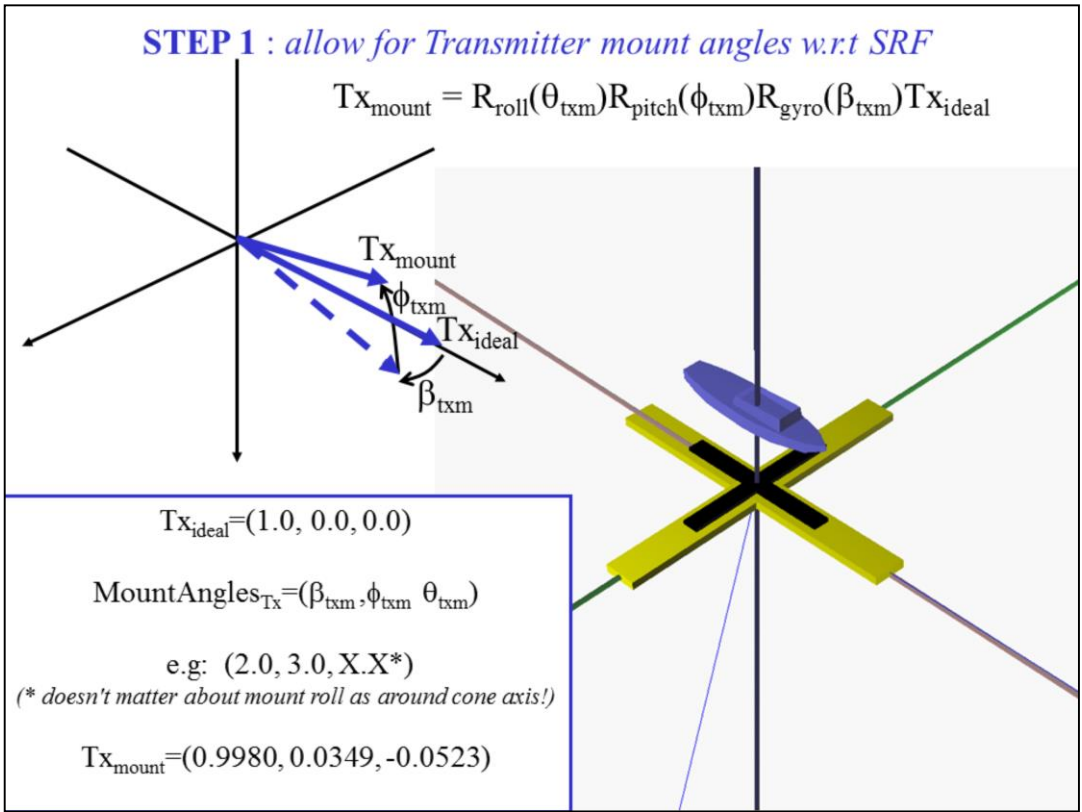


Azimuth and Depression angle of beam vector

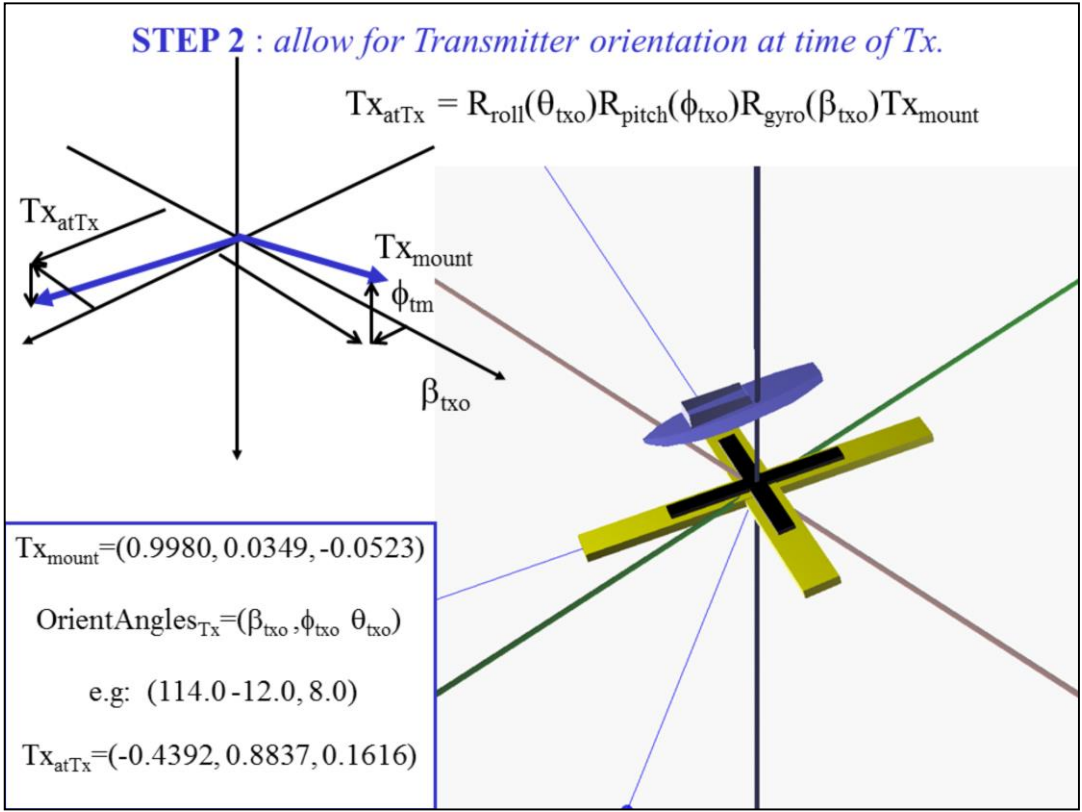
The end result of this calculation, will be to obtain the beam vector relative to the local level (depression angle) and either north or the ship's head at transmit (azimuth).

The depression angle (together with the array depth, TWTT and sound speed profile), will be used to undertake the ray trace calculation. This will all take place in a vertical plane oriented along the beam azimuth orientation.

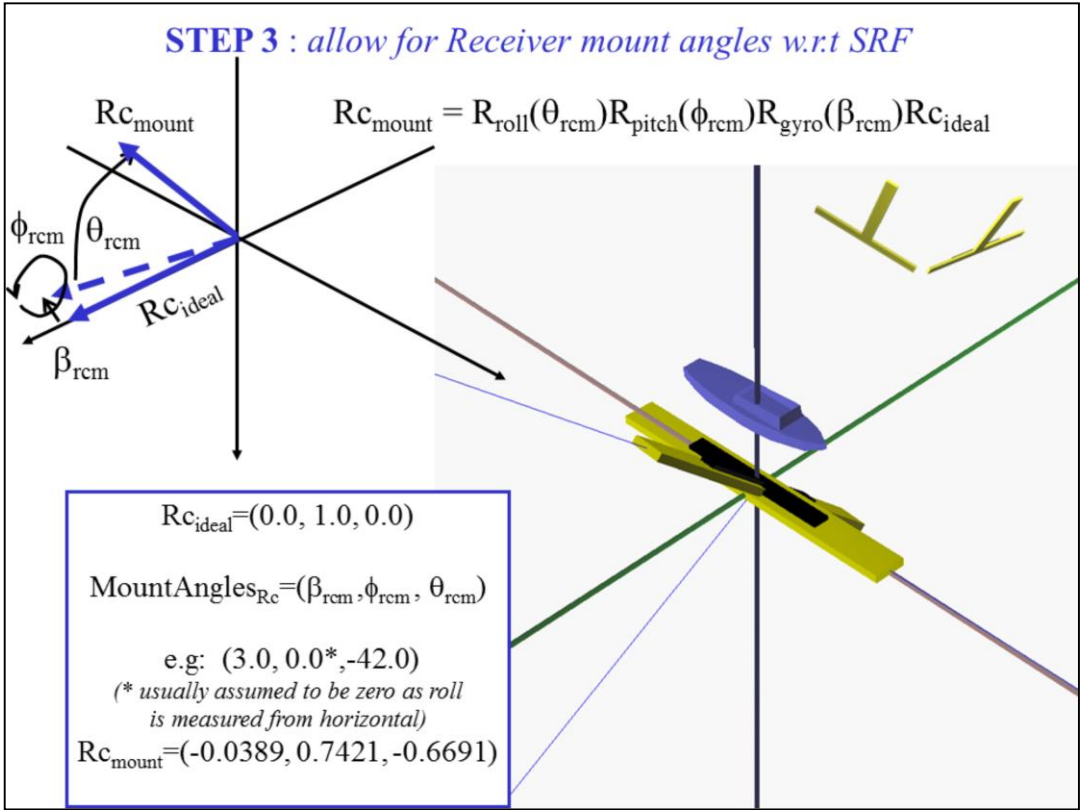
The ray trace solution will provide a depth solution that is located a radial distance out from the virtual array location in the direction of the beam azimuth. Ultimately, the location of the virtual array will have to be fixed in a geographic (or across-along referenced) coordinate system, to provide a final geo-referenced bottom detection.



Starting with an ideal Tx vector of (1,0,0).
Establishing the as-installed Tx vector accounting for the Tx mounting angles relative to the SRF.

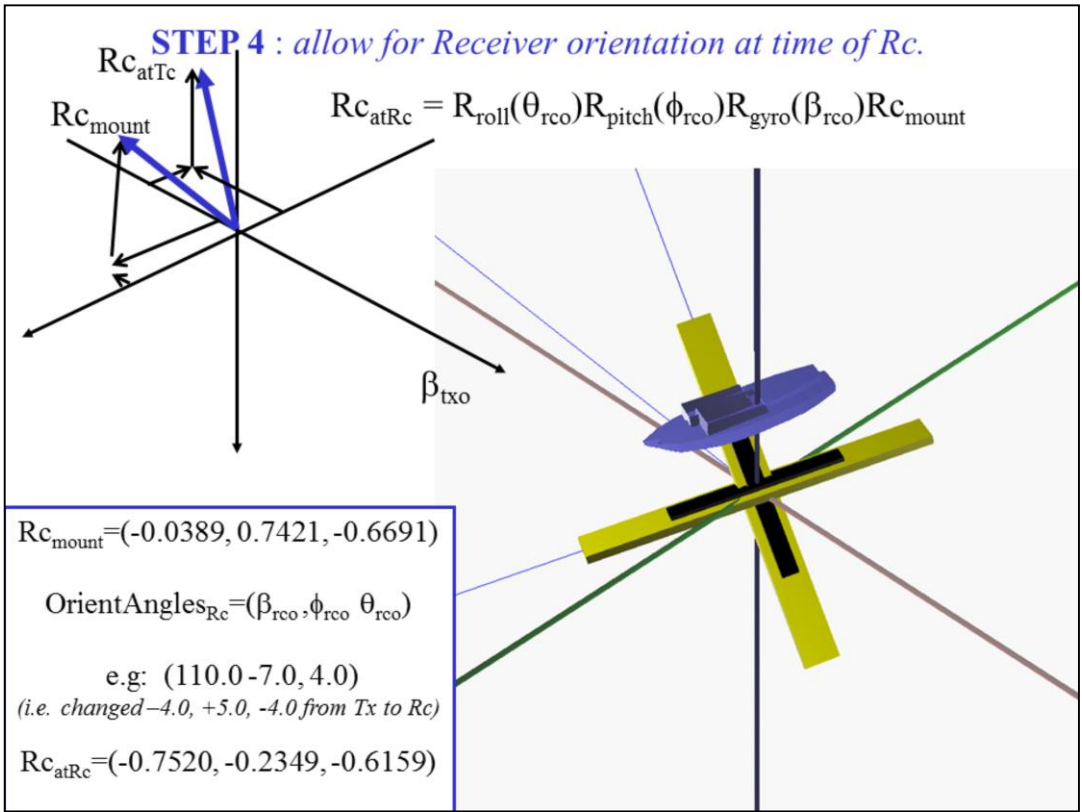


Establishing the Tx vector accounting for the instantaneous SRF orientation at the time of Tx



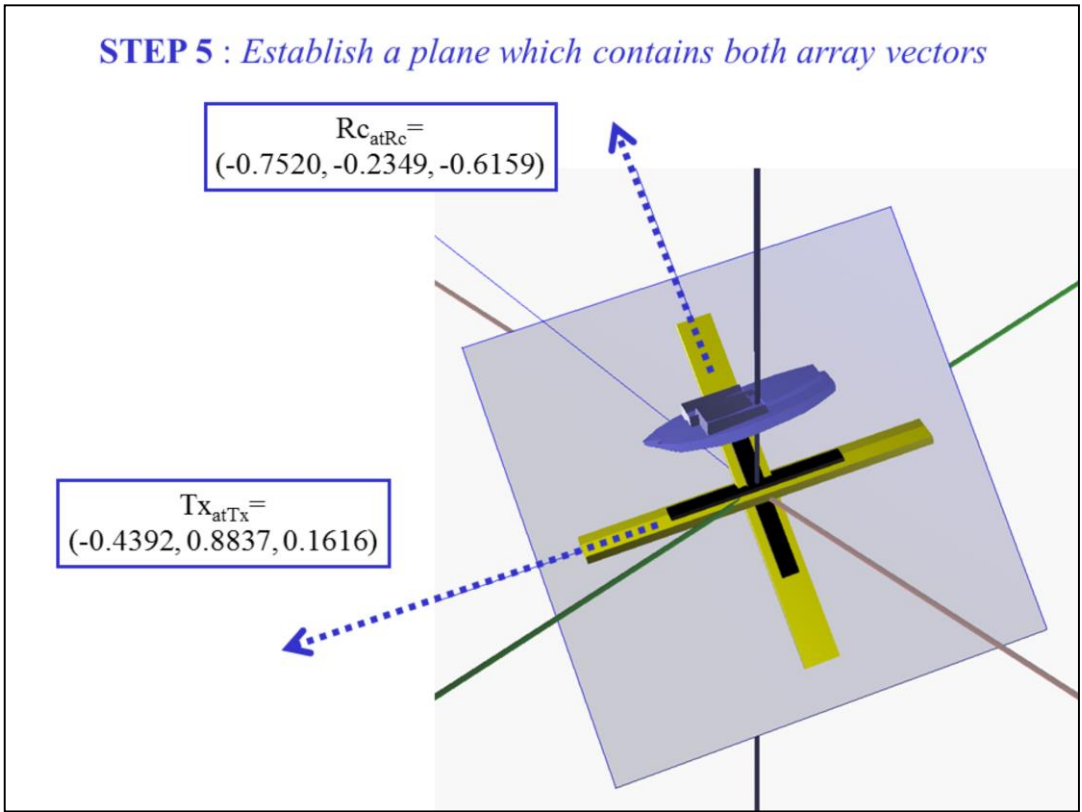
Starting with an ideal Rx vector of (0,1,0).

Establishing the as-installed Rx vector accounting for the Rx mounting angles relative to the SRF.



Establishing the Rx vector accounting for the instantaneous SRF orientation at the time of Rx.

Note that the orientation of the SRF used has changed from time of Tx to time of Rx.

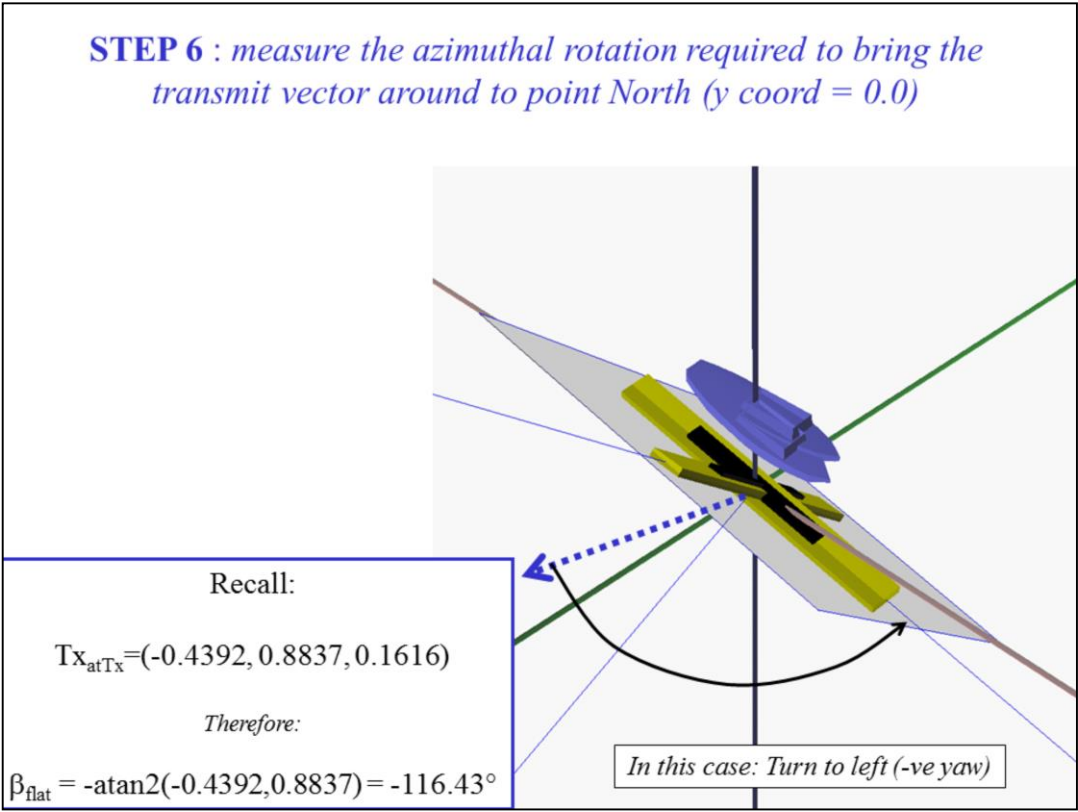


Now that we have the real-world orientation of the Tx and Rx vectors, we need to establish the orientation of the plane that contains the two vectors. This is necessary so that we can utilize the calculation of the intersection of non-orthogonal cones (see earlier slide).

That resulting vector is relative to the plane. In order to establish the beam vector orientation with respect to the real world, we need to know the orientation of that plane.

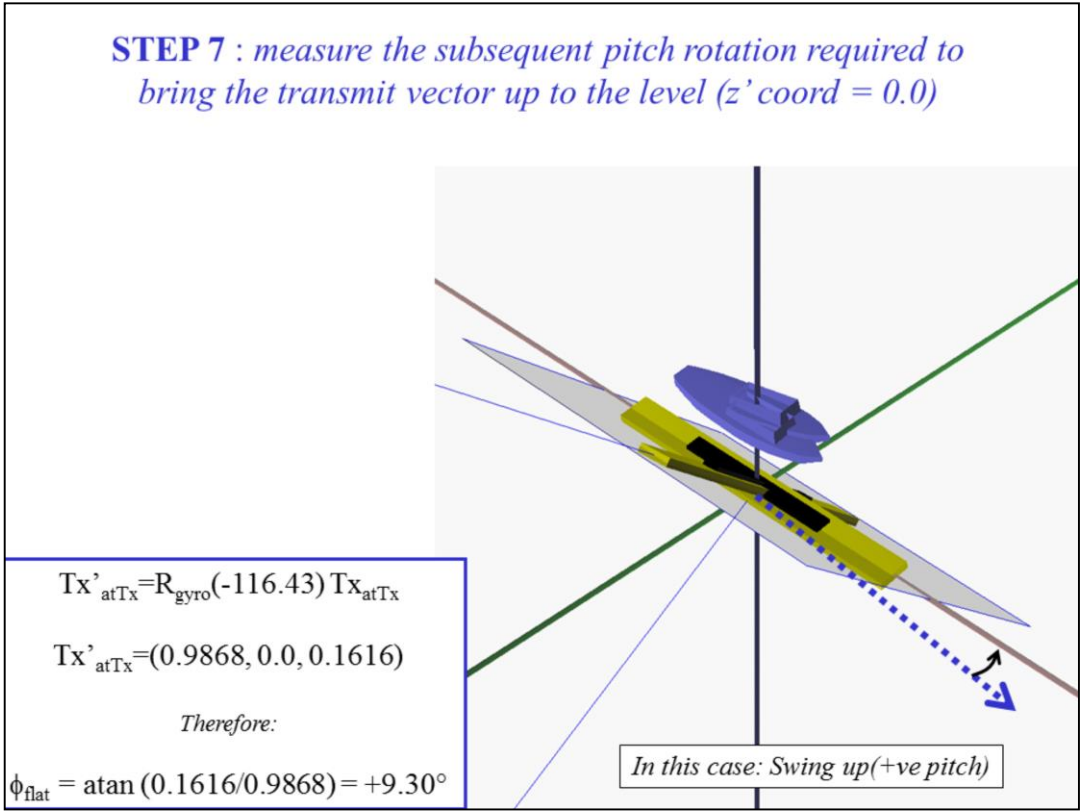
Steps 5-9 demonstrate how to calculate the three rotations required to bring the plane back to the local level and the how to estimate the non-orthogonality angle (NO). Step 10 calculates the beam vector relative to the coplanar coordinate frame and step 11 actually applies those rotations in reverse.

Step 5' and 9' shows a computationally more efficient way of getting NO and the required transformation from the coplanar orientation to the local level.



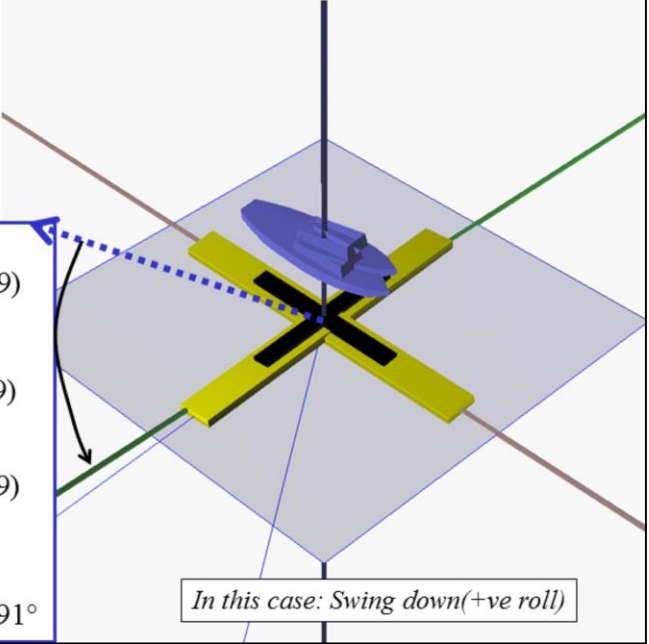
To move from the coplanar coordinate system to the real-world coordinate sytem (local level), we need to calculate three rotations.

This shows the yaw rotation required.



Once the x axes are aligned in azimuth. This shows the next step which is a pitch rotation required to bring the two x axes together.

STEP 8 : *measure the subsequent roll rotation required to bring the receive vector onto the level (z' coord = 0.0)*



Recall from STEP 4:
 $R_{c_{atRc}} = (-0.7520, -0.2349, -0.6159)$

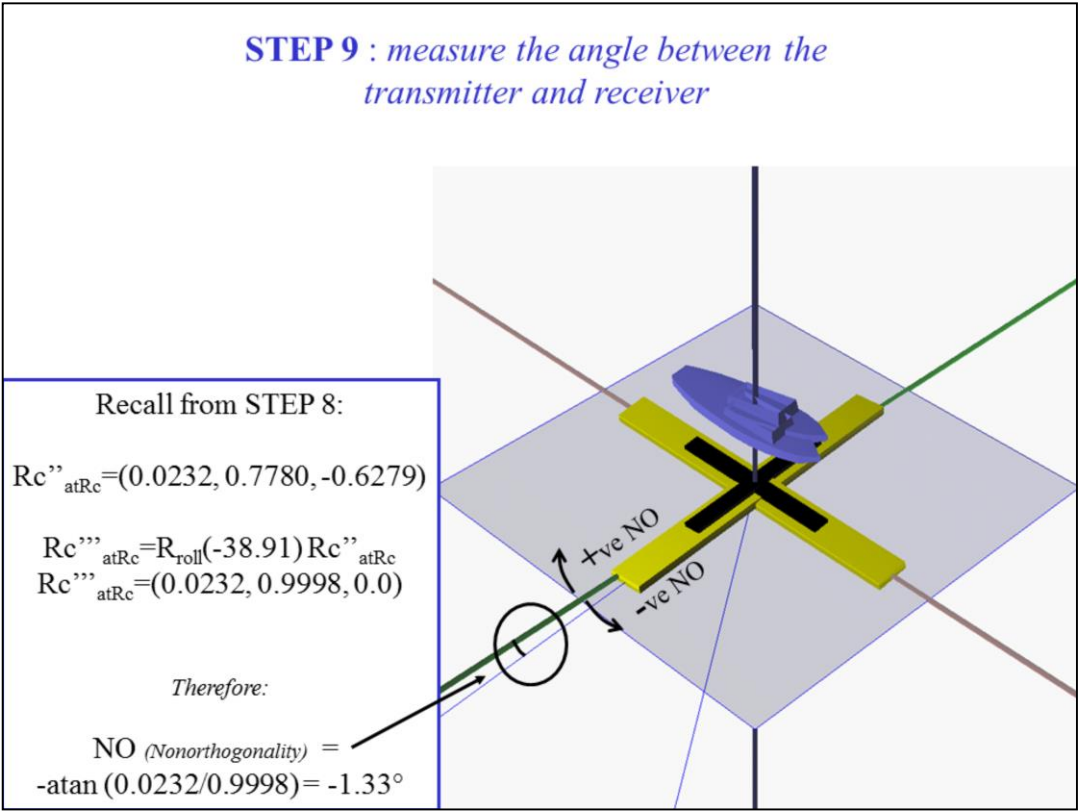
$R_{c'_{atRc}} = R_{gyro}(-116.43) R_{c_{atRc}}$
 $R_{c'_{atRc}} = (0.1243, 0.7780, -0.6159)$

$R_{c''_{atRc}} = R_{pitch}(9.30) R_{c'_{atRc}}$
 $R_{c''_{atRc}} = (0.0232, 0.7780, -0.6279)$

Therefore:
 $\theta_{flat} = -atan(-0.6279/0.7780) = 38.91^\circ$

And, now that the x axes align, the last rotation is about that x axis to bring the y axes together. This is a roll rotation.

The three successive rotations calculated, allow one to take a vector in the coplanar coordinate system and represent it in the local level real –world coordinate system



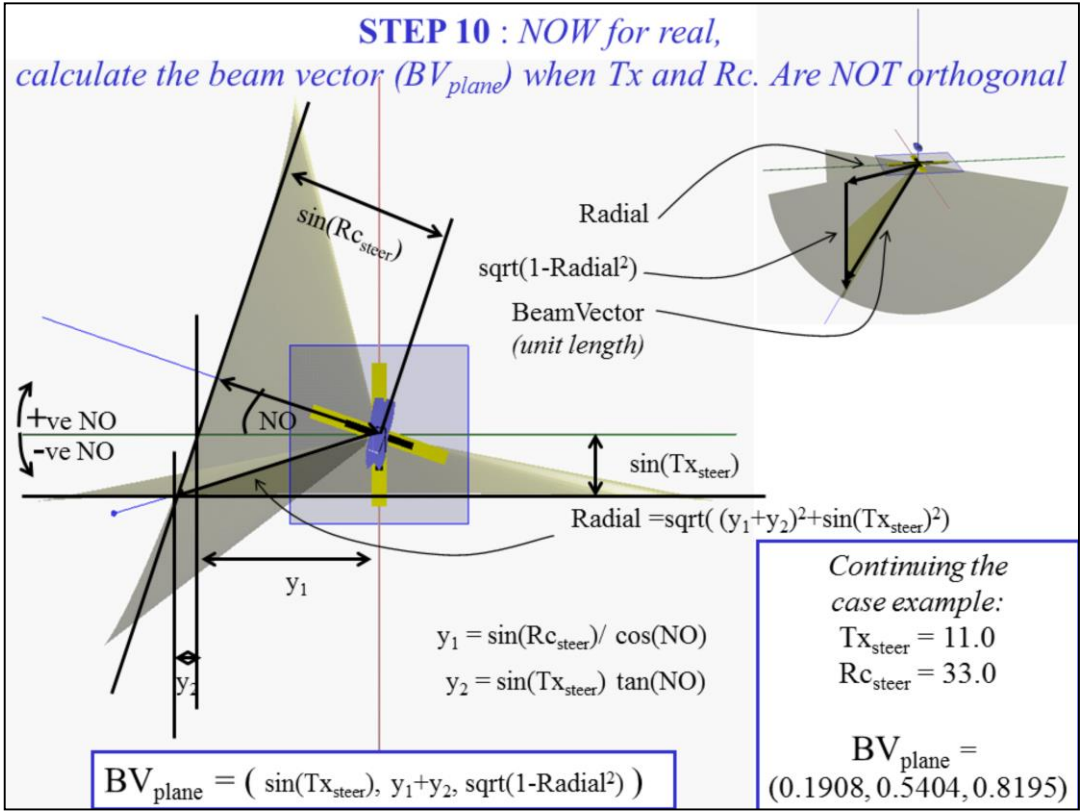
Once the two array vectors have been brought into the x-y plane, one can now calculate the non-orthogonality angle.

STEP 10 : *now that we have the Tx-Rc plane in the local level,
Work out the beam vector w.r.t. that plane*

We have to work out the
cone-to-cone
Intersection

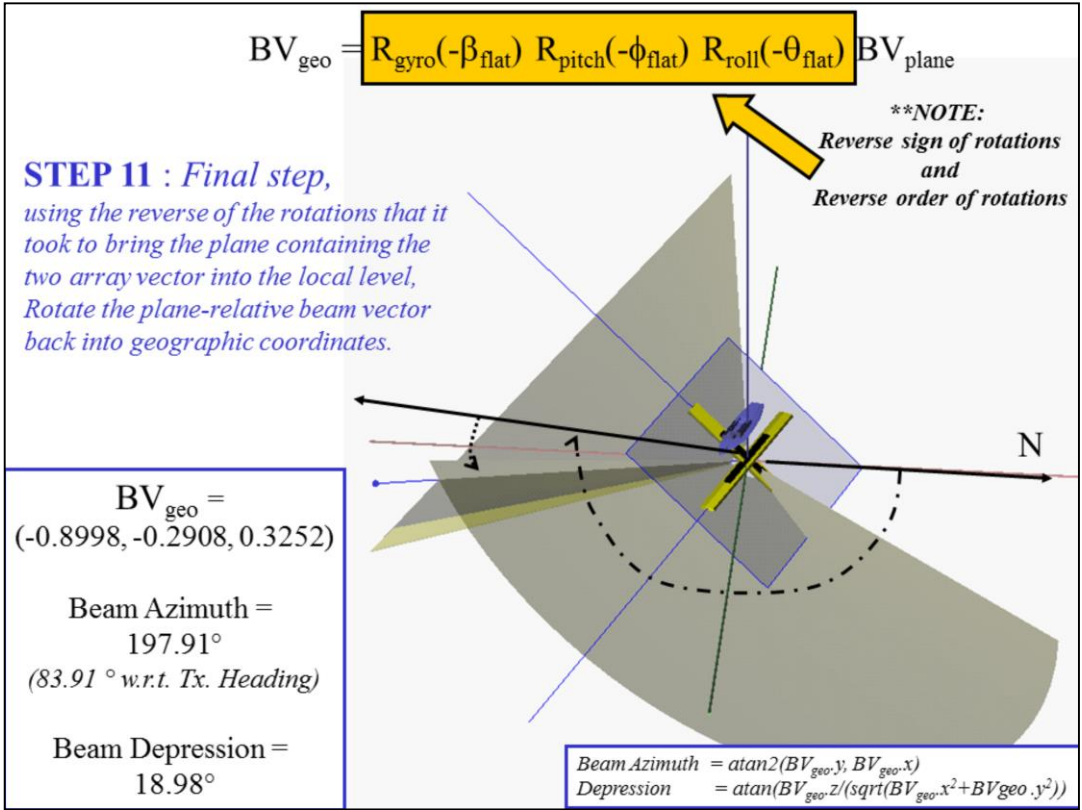
Case example:
 $Tx_{steer} = 11.0$
 $Rc_{steer} = 33.0$

Now that we have the two array vectors in the x-y plane, and knowing the angle between them (90+NO), we can proceed to calculate the beam vector relative to that plane, using the transmit and receiver steering angles.



This is the same calculation as before. In this case we now are using the specific worked example steering directions (11 deg for T_x , 33 deg for R_x).

This produces a specific beam vector in that coplanar coordinate system. The next step is to rotate it back into real world local level coordinate system.



This applies the three rotations in the reverse order (and reverse sign) to go back from the coplanar coordinate system to the local level coordinate system.

The worked example results are provided to check your calculations.

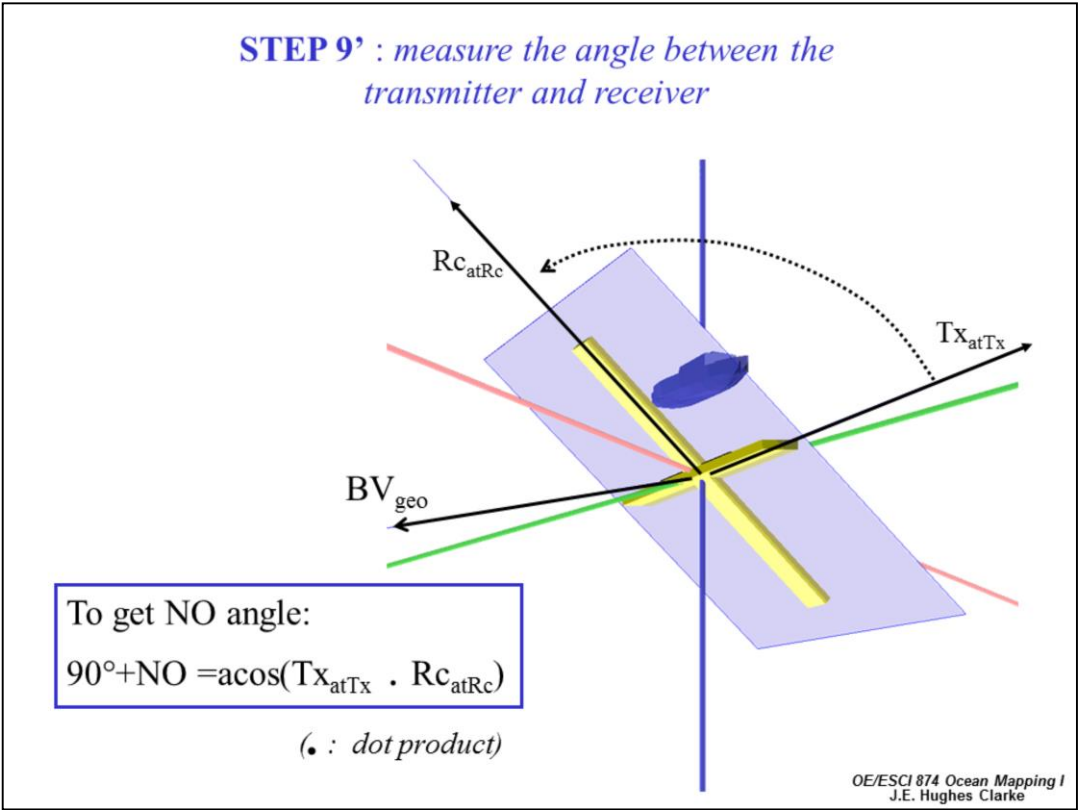
Two equivalent approaches....

Clunky mode : easy to visualise....
STEPS 5-9 and 11

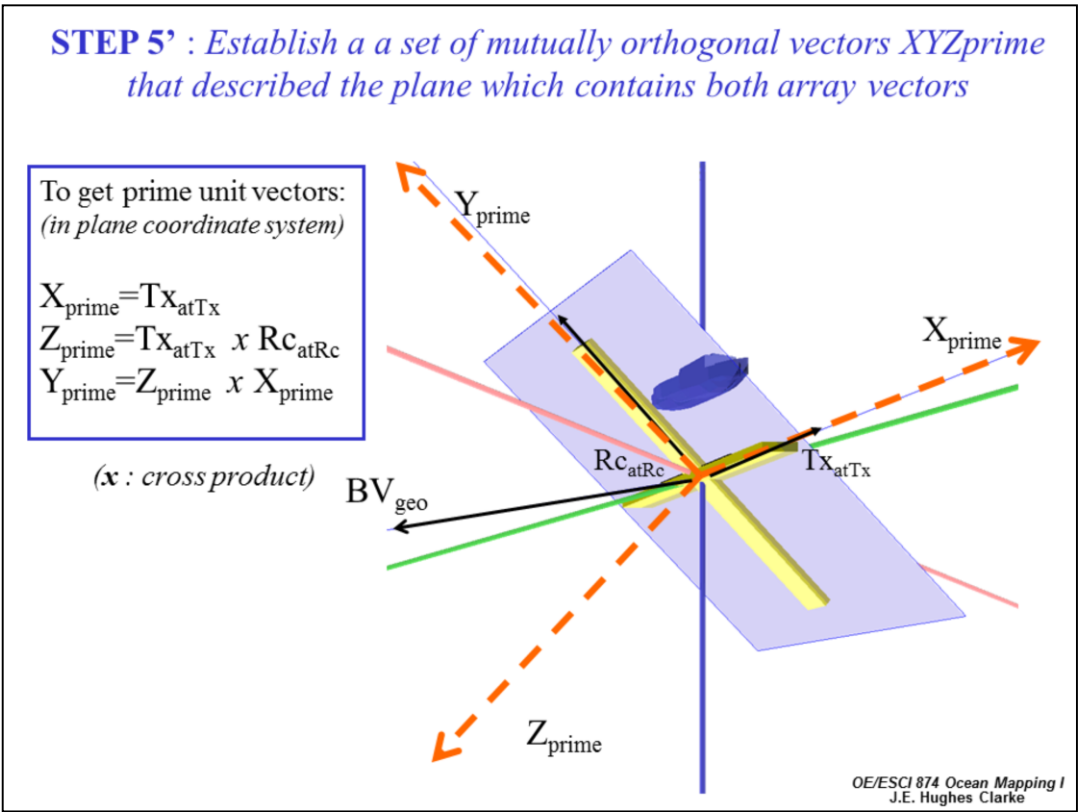
Elegant mode : faster to compute.
See over ...

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The following three steps are more computationally efficient (but less intuitive to describe). They result in the identical result.



Taking advantage of the fact that the dot product automatically provides the angle between two vectors. This is $90^\circ + NO$.



And an alternate way to translate from the local level to the coplanar coordinate systems is to establish a set of mutually orthogonal vectors in the real world system that align with the transmitter (X axis), the plane containing the Tx and Rx (which defines the Y axis), and the Z axis which is mutually orthogonal.

Two steps are involved that utilize a cross product (which provides a mutually orthogonal vector from two input vectors).

This establishes a set of mutually orthogonal vectors that describe the coplanar coordinate frame in the real world coordinates.

and replacing STEPS 6, 7, 8

Rows of equivalent rotation matrix
(to go from plane to geographic)
are coordinates of primed unit vectors
in original coordinate system (XYZ_{prime}).

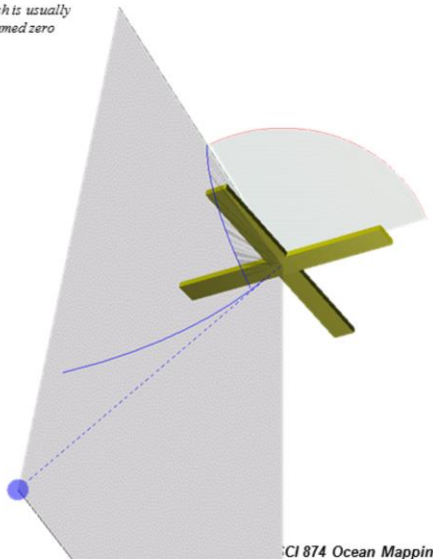
Inverse (transpose) of this matrix will bring
•plane-relative vector (BV_{plane})
•into geographic coordinates (BV_{geo})

$$\begin{aligned} BV_{\text{geo}.x} &= BV_{\text{plane}.x} X_{\text{prime}.x} + BV_{\text{plane}.y} X_{\text{prime}.x} + BV_{\text{plane}.z} X_{\text{prime}.x} \\ BV_{\text{geo}.y} &= BV_{\text{plane}.x} X_{\text{prime}.y} + BV_{\text{plane}.y} X_{\text{prime}.y} + BV_{\text{plane}.z} X_{\text{prime}.y} \\ BV_{\text{geo}.z} &= BV_{\text{plane}.x} X_{\text{prime}.z} + BV_{\text{plane}.y} X_{\text{prime}.z} + BV_{\text{plane}.z} X_{\text{prime}.z} \end{aligned}$$

What it says above..... It works!

To do the full calculation you need **14 angle** measurements:

- Transmit Mount Angles (β, ϕ, θ)
Tx rolls is actually not necessary
- Transmit Mount Angles (β, ϕ, θ)
Rx pitch is usually assumed zero
- Transmit Orientation (β, ϕ, θ)
- Receive Orientation (β, ϕ, θ)
- Transmit Steering
- Receive Steering



However, you go about implementing the calculation, the thing to realise is that it is necessary to have 14 angles to come up with the final beam vector.

Lots of opportunity to make a mistake:

- Are the Tx and Rx arrays well enough defined in SRF coordinates?
- Is the SRF motion at Tx and Rx time correctly measured (latency)?
- Is the IMU to SRF alignment correctly applied?
- Are the steering angle signs correctly implemented?
- Are the steering angle correct for the actual sound speed at the array?

Worked Example to check the logic:

Tx Mount Angles : (2.0, 3.0, 0.0)

Rc Mount Angles : (3.0, 0.0, -42.0)

Tx Orientation : (114.0 -12.0, 8.0)

Rc Orientation : (110.0 -7.0, 4.0)

(i.e. changed -4.0, +5.0, -4.0 from Tx to Rc)

Cone Steering:

Tx Steering : 11

Rc Steering : 33

Answer :

Azimuth : 197.91°

Depression - 18.98°

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The worked number provided should allow you to check the logic of your function. Once it is working, you can then apply it to the specific values you have for the beam that you are trying to register.

A better (*more complicated though*)
way of calculating the beam vector.

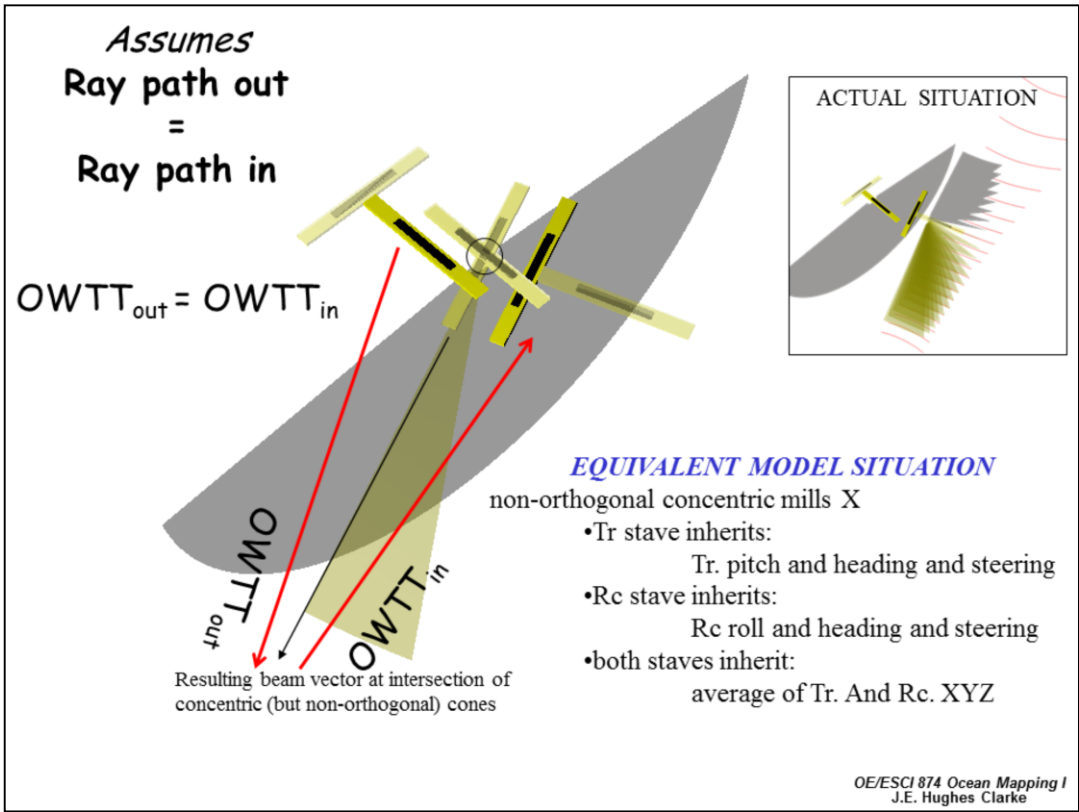
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The previous worked explanation assumed that the outbound and inbound paths of the ray path are the same.

For the case, however, of very strong transmit steering (as for example commonly done when undertaking yaw –stabilization with multiple sectors) ,this assumption is not quite adequate.

The following slides explain a newer better way of doing the calculation.

Unfortunately, the only published version of this method, involves iterations and approximations and thus makes it very hard to implement. Thus, for this course, the co-located method is used for the lab.



Better honouring the true ensonification geometry

The transmitter geometry is completely defined by :

- Its location (horizontal position and depth at time of transmission)
- Orientation of the ship’s reference frame (at time of transmission of specific sector)
- Transmitter mount orientation in the ship’s reference frame
- Transmitter array-relative fore-aft steering angle

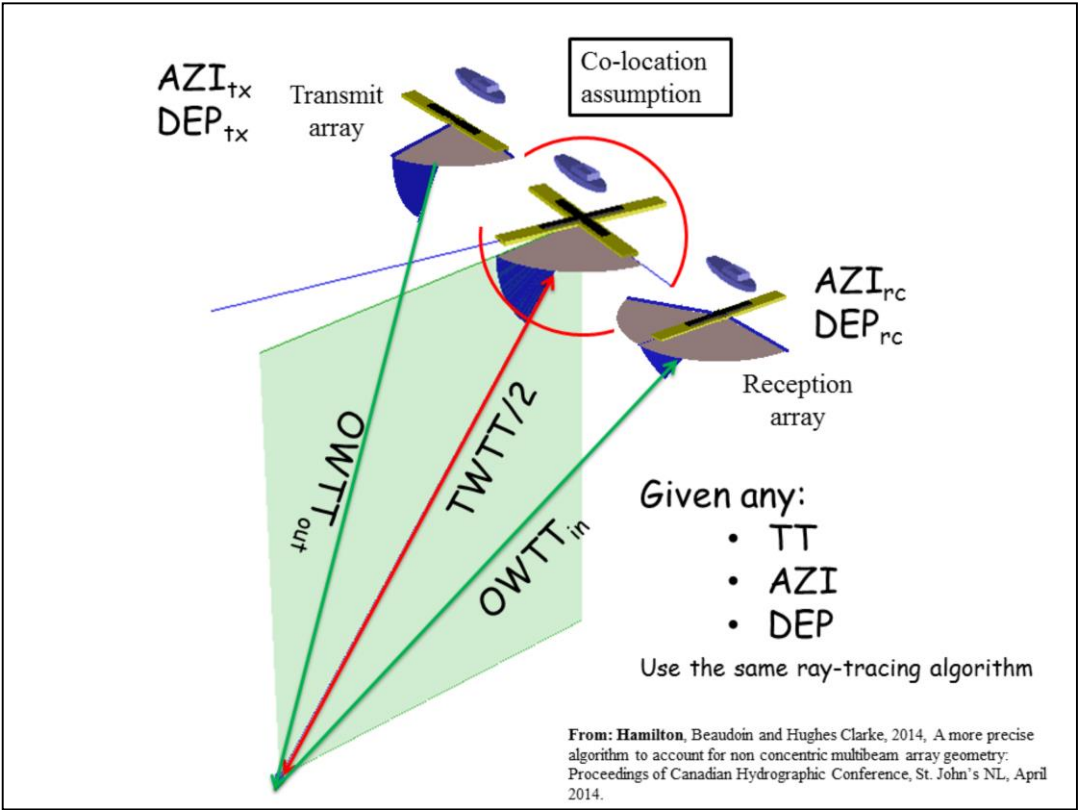
Similarly the receiver geometry is completely defined by :

- Its location (horizontal position and depth at time of reception)
- Orientation of the ship’s reference frame (at time of reception of bottom echo)
- Mount orientation of the receiver in the ship’s reference frame
- Receiver array-relative port-starboard steering angle

For the purposes of modeling an equivalent singular starting vector, a location is selected midpoint (in horizontal and depth) between the transmission and reception acoustic centres. At that location, a beam vector is defined reflecting the intersection of the two concentric, but non-orthogonal steering cones.

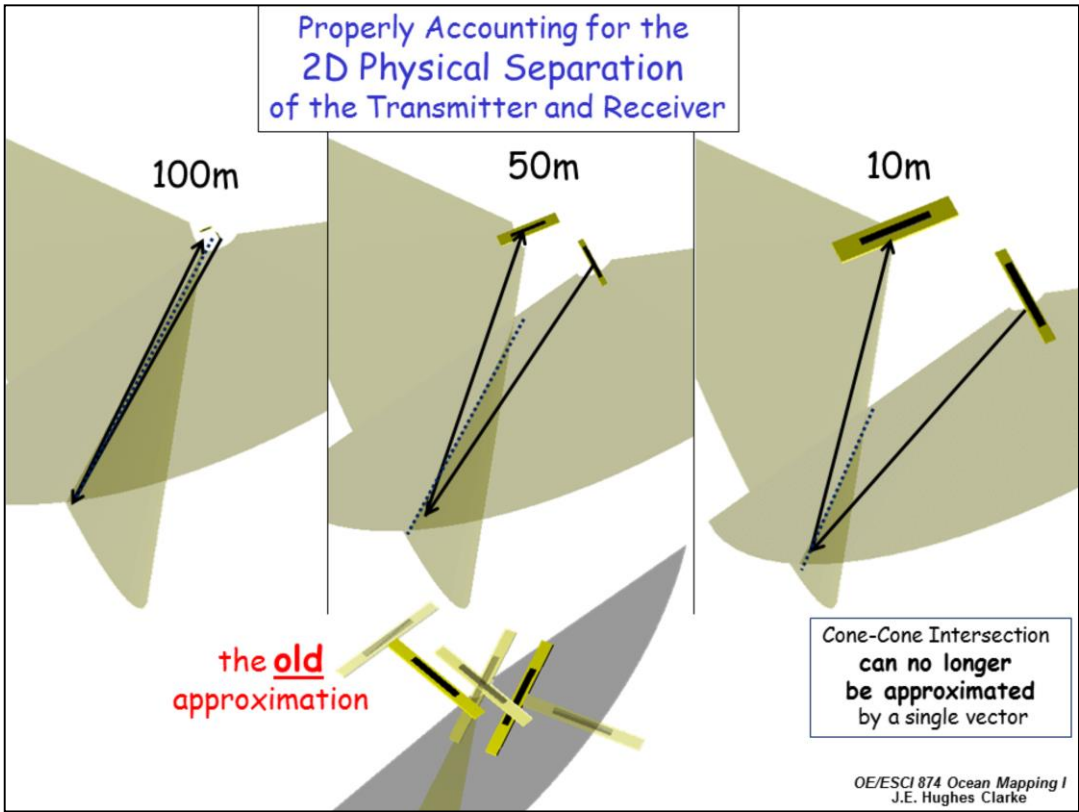
This approximation is especially convenient, as the ray path in and outbound can now be handled as a reciprocal ray path with exactly half the two-way travel time consumed in each path.

What is the impact of ignoring the fact that there are really two ray paths, not one?



At the end of this lecture the impact of using a the co-located transmitter and receiver will be discussed.

Properly accounting for the separation of the transmit and receiver becomes important when the non-concentric path length is significantly different than the concentric geometry. When that is the case, the ray trace methodology described in the next section has to be applied either from the transmitter or the receiver using respectively the azimuth and depression (and one-way travel time) from that sensor.

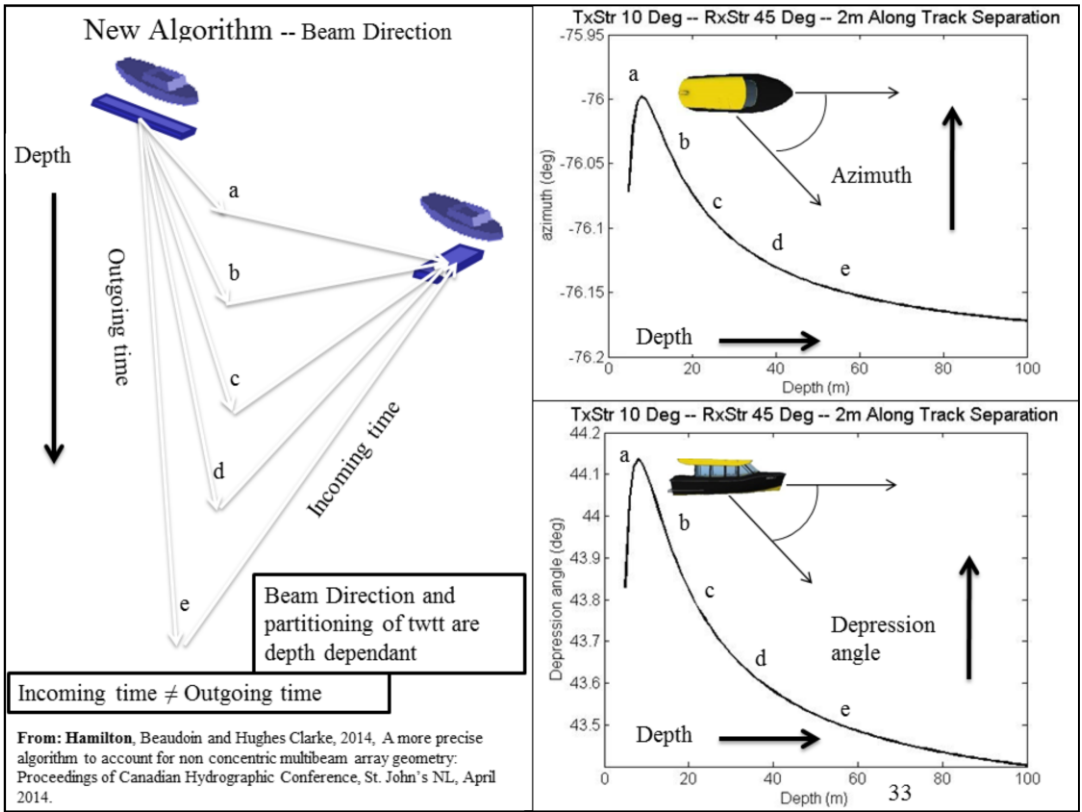


Note that the model situation assuming an equivalent virtual co-located transmitter and receiver is only a convenient approximation. For the case when the acoustic centre displacements is actually a significant fraction of the target distance, that separation has to be accounted for.

The forward propagation of the receiver over the shot cycle is never a significant separation as it grows only at vessel propagation velocities (typically $\sim 4\text{m/s}$) whereas the sound is travelling at one-way travel time speed of $\sim 750\text{m/s}$. Thus the opening angle is only $\arctan(4/750)$ which is $\sim 0.3^\circ$. The range consequence of this is only $1 - \cos(0.3^\circ) = 0.014\%$.

It is the physical mounting distance between the transmitter and receiver that needs to be accounted for. This has to be at least half the length of one of the arrays and is usually more. As longer arrays (to accommodate narrower beamwidths) have become the norm, the separations have grown and thus the minimum range for which the separation becomes significant has grown.

This more correct geometry is particularly important when there is very strong transmit steering (when it intersects the receiver cone at an oblique angle). This now occurs more frequently with multi-sector yaw stabilization.

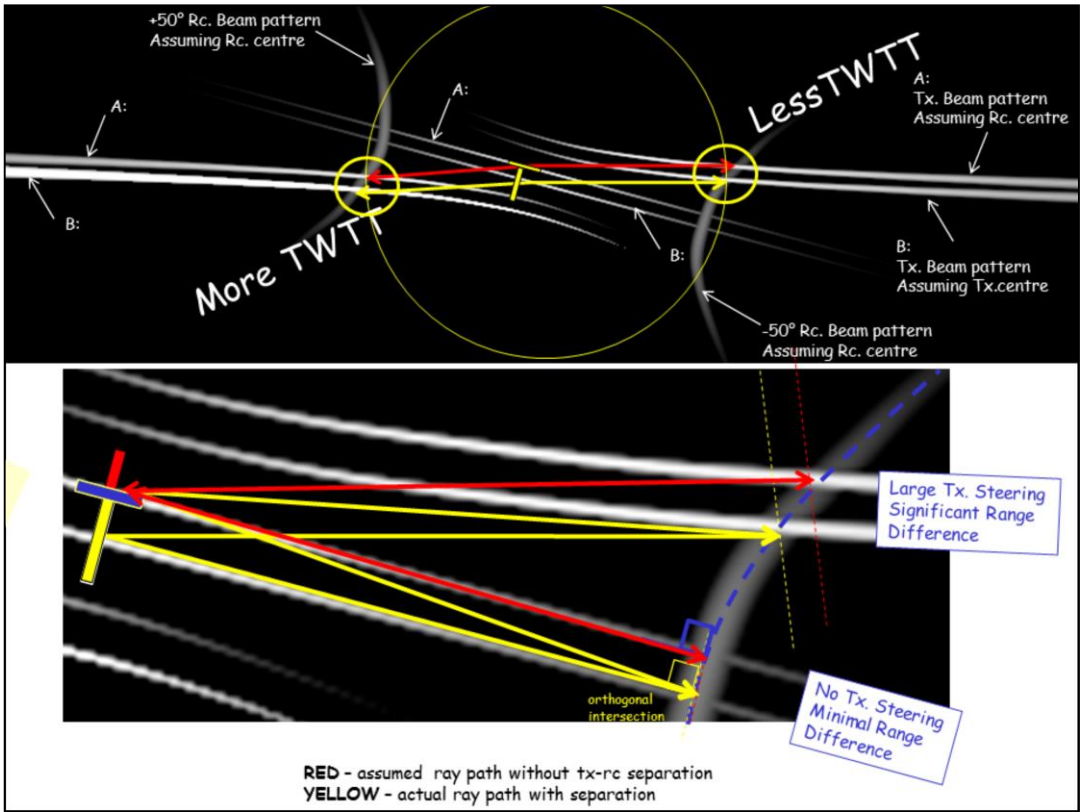


The difficulty is that as the depth of the seabed below the sonar increases, the direction at which the beam leaves the transmit array changes. In addition, the separated arrays also means that the partitioning of the TWTT into a $OWTT_{out}$ from the Tx array to the seabed, and a $OWTT_{in}$ from the seabed to the Rx array varies with depth.

So the direction and OWTT from the Tx array to the seabed are both depth dependant, but to know the depth we need to calculate a direction and OWTT.

What this means is that we will have to make an initial guess at the depth and employ an iterative strategy to find the correct depth. Once the correct depth is found we will end up with the Azimuth and depression angle of the beam, and the properly partitioned OWTT (in and out) so that a ray-trace can be performed to find the across track and along track coordinates of the sounding.

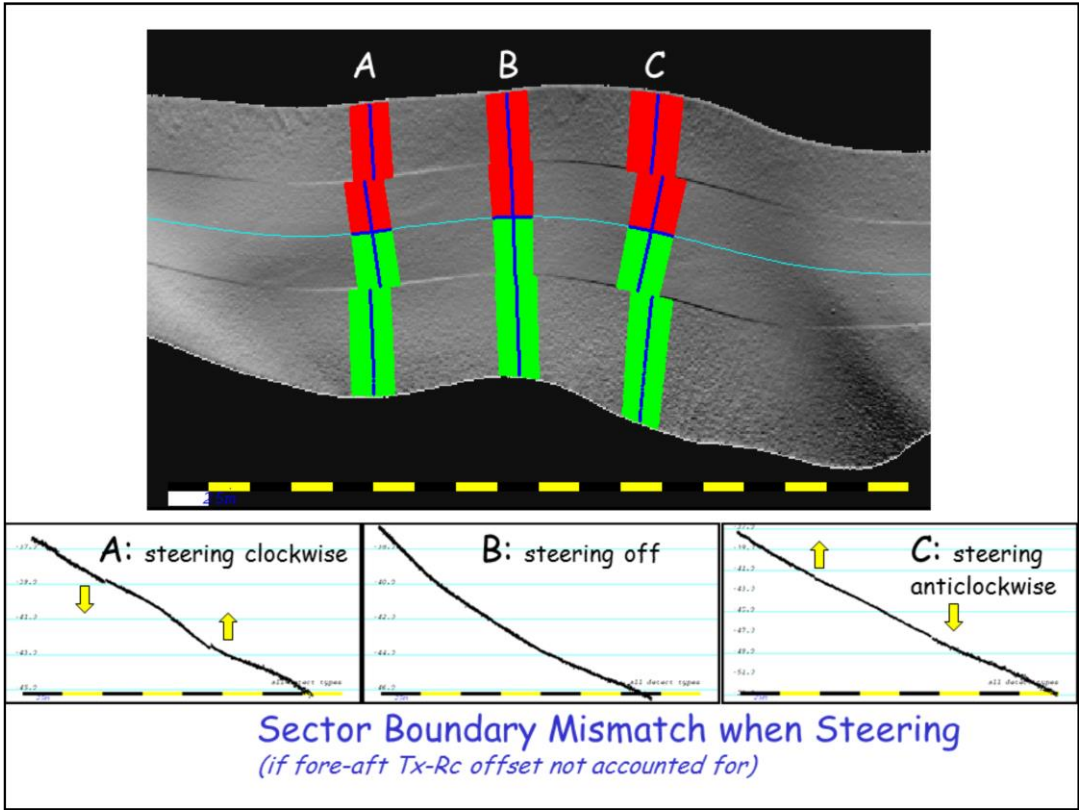
The advantage of this will be that we can perform a vector from just one of the transducers and not have to go about calculating the average horizontal and vertical positions.



The example illustrated above show the seabed projection of transmit and receiver beam footprints for an single swath EM710 (3 sector) installation . The receiver footprint for one beam on either side, located at the sector boundary, is illustrated. Two transmit beam footprints are drawn illustrating the projection of the transmitter beam pattern either co-located with the receiver or displaced forward.

When no yaw steering is implemented, (the case for the central sector) the consequences are minimal as the projection of the receive beam pattern is almost parallel to the ship's track at broadside and thus orthogonal to the transmit beam pattern. But for the strongly yaw-stabilized case, the receive pattern is intercepted, obliquely rather than orthogonally. If the transmit beam pattern is in fact ~ 1m further forward/aft than modelled, the intercept is significantly closer on one side (and the reverse of the other side).

A result of this is that, unless properly accounted for, there will be an artificial inter-sector depth offset at the sector boundary, where the sense and sign of the sector offset correlates with the direction of yaw steering.



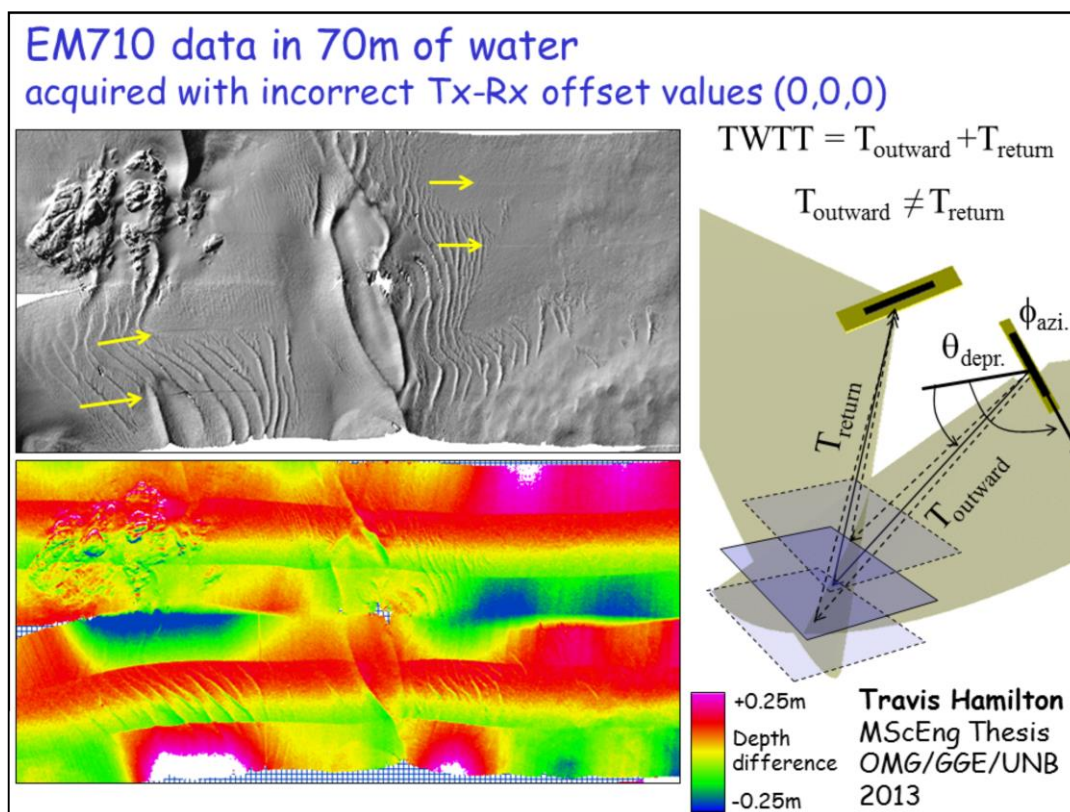
If the along-track separation offset between the receiver and transmitter is not included in the refraction solution, there will be an error that grows with transmitter steering. During yaw stabilization the transmitter steering of the outer sectors is changing rapidly between large (max usually 10°) values.

A single sector system can only pitch compensate and thus the error is not usually noticed as it only results in a minor across track bowing (smile or frown) of the whole swath and there is no sector boundary so that, even if there is a small error, it changes only slowly within the single sector without abrupt changes in magnitude.

For multi-sector systems, the transmit steering can change abruptly from sector to sector. As a result, this small error shows up as an abrupt step in the seafloor at the sector boundary. This is only seen where there is strong transmit steering.

For an unaccounted forward offset of the transmitter, if the steering is clockwise (example A above), the steps will be down to port. If the steering is anticlockwise (example C above) the steps will be down to starboard. When the yaw stabilization is not being used (example B above) there is no significant sector offset.

While the sector offsets are small (typically $\sim 0.1\%$ of Z), now that the sounder bottom detection noise is also small (typically $\sim 0.1\%Z$) this shows up in sun-illuminated terrain models. As it is systematic, it should be modeled and removed.



Effect of Ignoring the Transmitter-Receiver offset in the beam geometry:

The figures here show an example of an EM710 (three sector, single swath) which was operated with an incorrect configuration. The offset between the transmitter and receiver was not correctly entered (all zeros used). The system used a 2m transmitter and a 1m receiver. The separation was 1.3m in the fore-aft direction and ~0.6m in the port-starboard direction. As a result the ray trace solution applied by SIS revealed a sector boundary problem. This can be seen as abrupt small (~<0.25m in 70m of water) steps at the sector boundaries when heavy yaw stabilization is applied.

A full geometric solution: To fix the problem a new calculation needs to be performed that accounts for the actual separation. A proprietary version of this algorithm is built into the SIS real time software. The results illustrated above are derived from a new independent algorithm developed by Travis Hamilton at UNB as part of his Master's Thesis (2013).

The method combines both the mount offsets and the propagation of the receiver since the transmit time. An exact analytical solution is strictly only possible for a homogenous water mass. Therefore, the solution is achieved by iteration. An initial estimate of the depth solution is obtained ignoring the offset. A flat seafloor at that depth is then used to project the transmit and receive cones from the offset locations to get a beam azimuth. The ray trace model used is an approximation using the harmonic mean and the surface sound speed, with the depression angle adjusted accordingly.. The depth is then iterated until the sum of the outgoing and returning travel time equals the observed two-way travel time (TWTT).

At that point, the outgoing travel time is used (which is *not* exactly half the TWTT). And a new azimuth and depression angle is calculated that is unique to the position of the transmitter.

At that point a final full ray trace is used. The resulting solution removes the sector boundary steps. The depth difference map illustrates how that correction was distributed. The central sector was always tilted and the error in the outer sector is directly proportional to the amount of yaw steering applied.