The maths behind Dobble / Spot It!

Rules

- 55 cards
- 8 symbols on each card
- only one common symbol between any two cards

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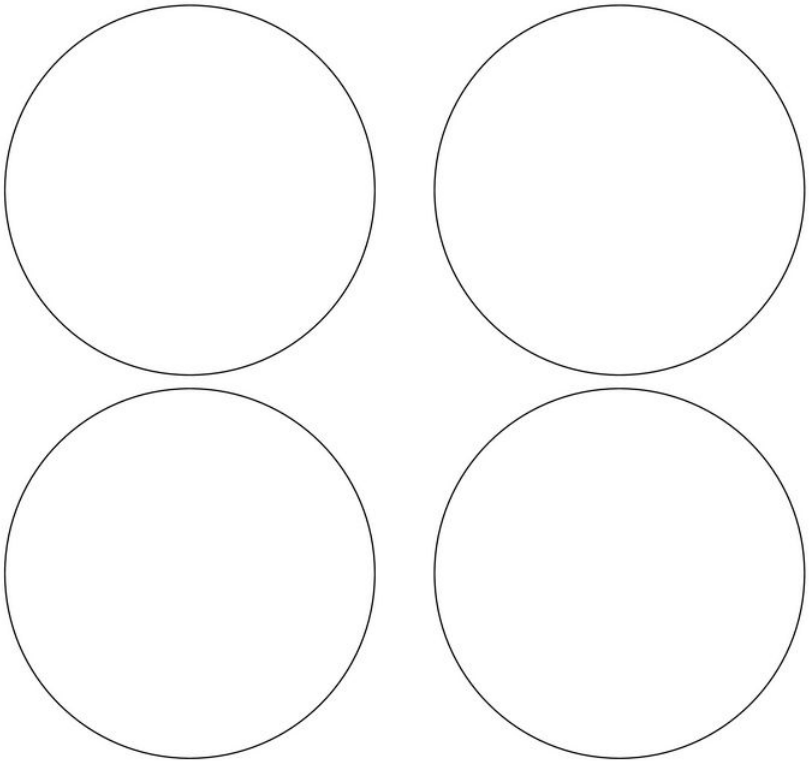
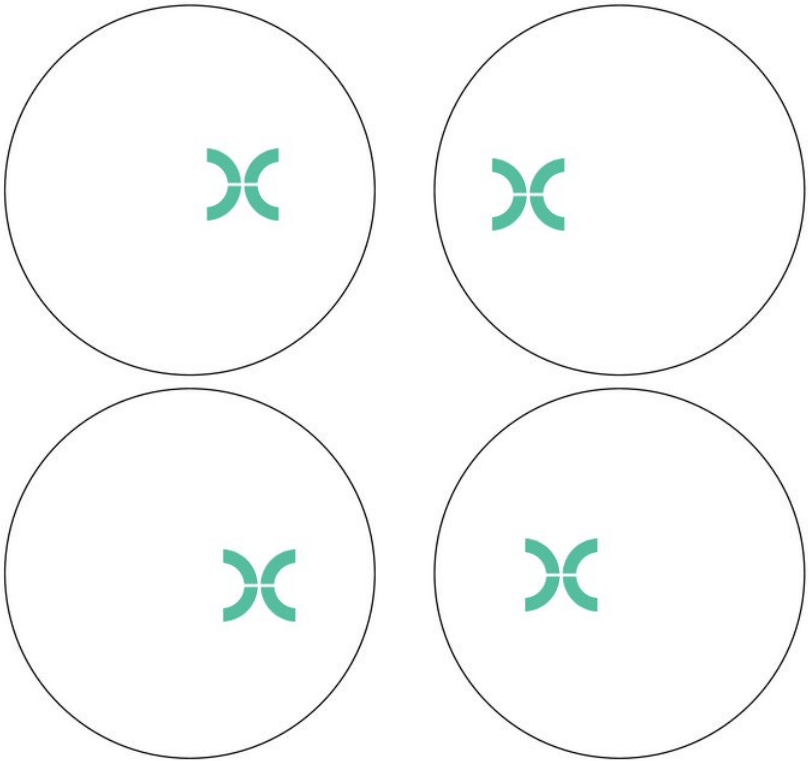
How can you make your own set of cards?

Trivial Approach



Trivial Approach



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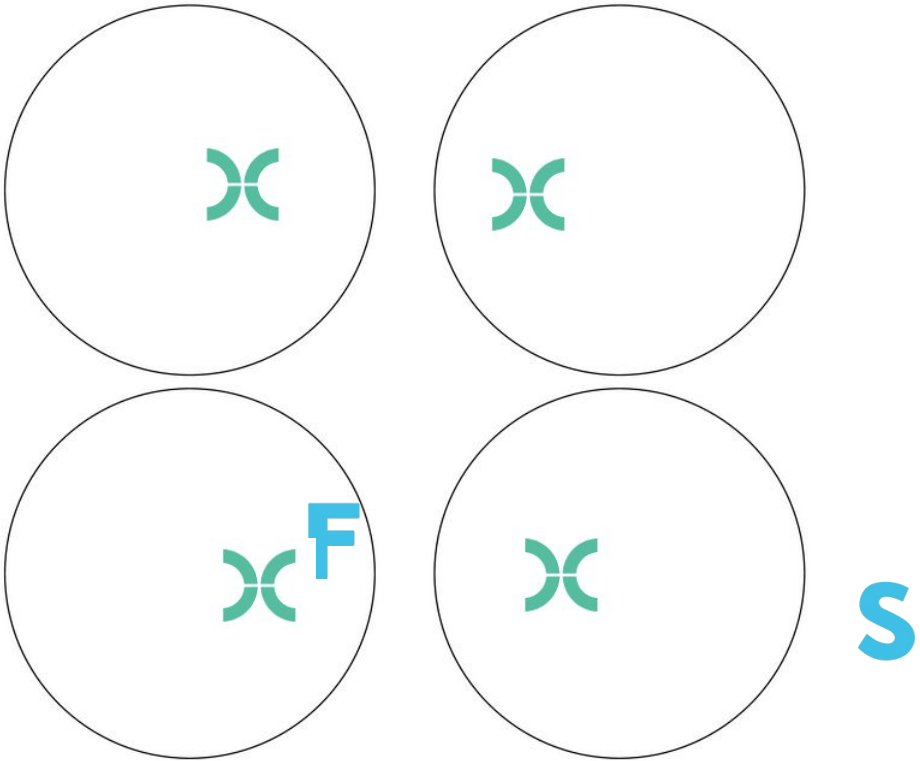
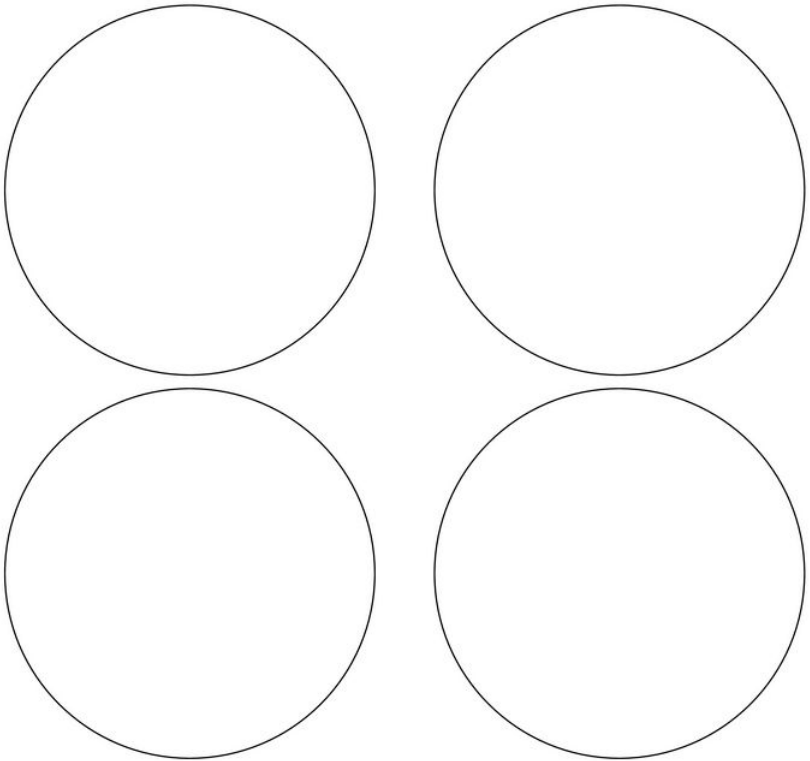


Inefficient Approach



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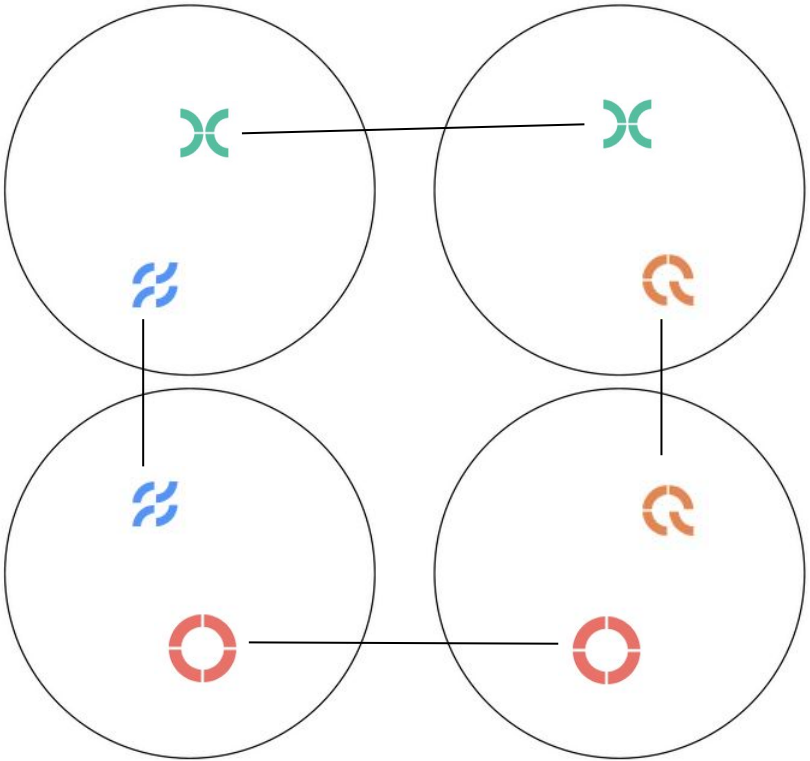
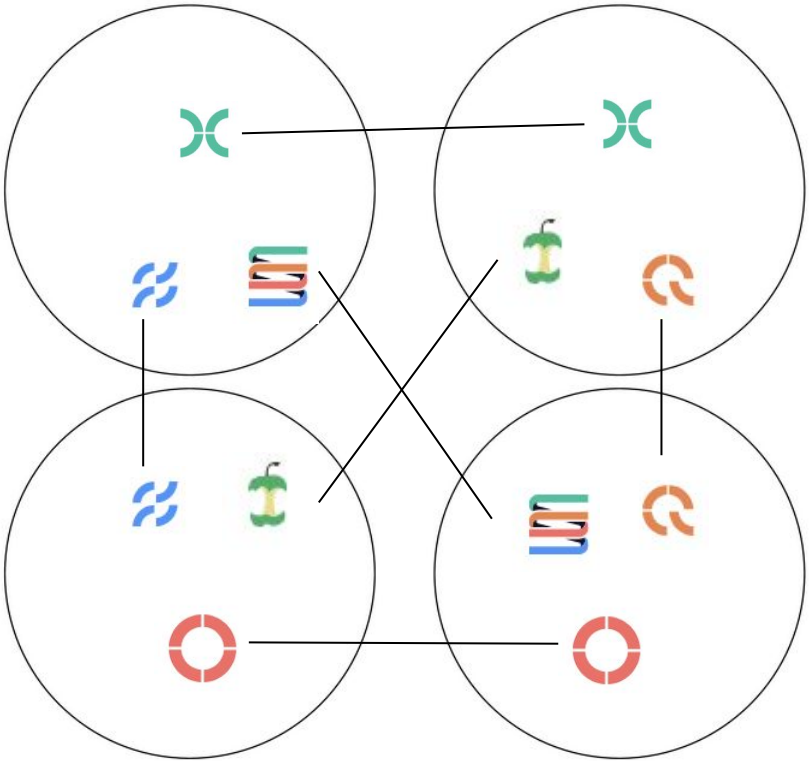


Inefficient Approach



Number of symbols needed?

Combination without repetition

For 4 cards = C(4,2) = 6

$$C(n,r) = \binom{n}{r} = \frac{n!}{(r!(n-r)!)}$$

n choose r

Number of symbols needed?

Combination without repetition

For 55 cards = C(55,2) = 1485

$$C(n,r) = \binom{n}{r} = \frac{n!}{(r!(n-r)!)}$$

n choose r

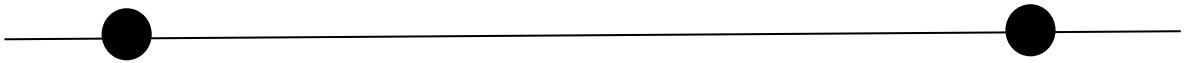
(Dobble only uses 57 symbols!)

Mathsy Approach

Geometry

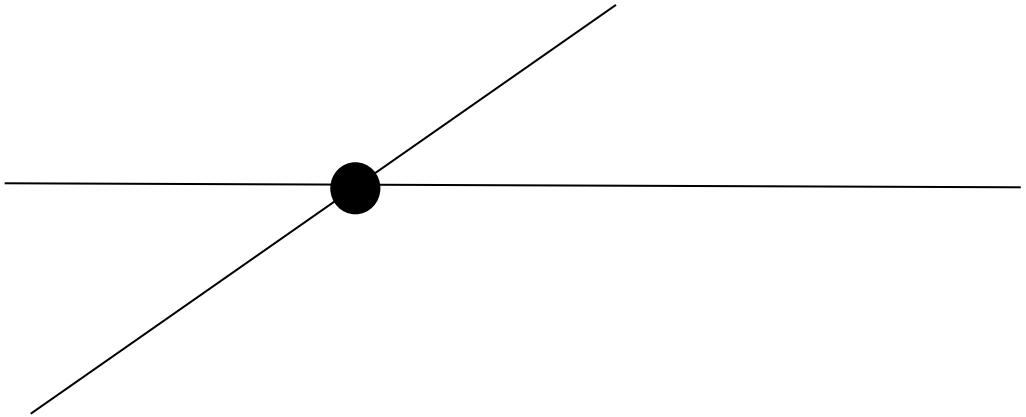
Euclidean geometry

For every two points, there always exists a line that goes through them



Projective geometry

For every two points, there always exists a line that goes through them
Every two lines, intersect at exactly one point



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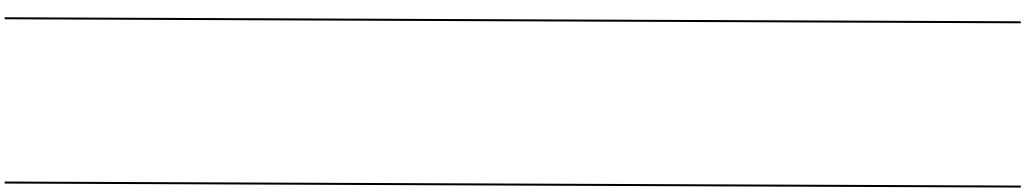

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For every two points, there always exists a line that goes through them
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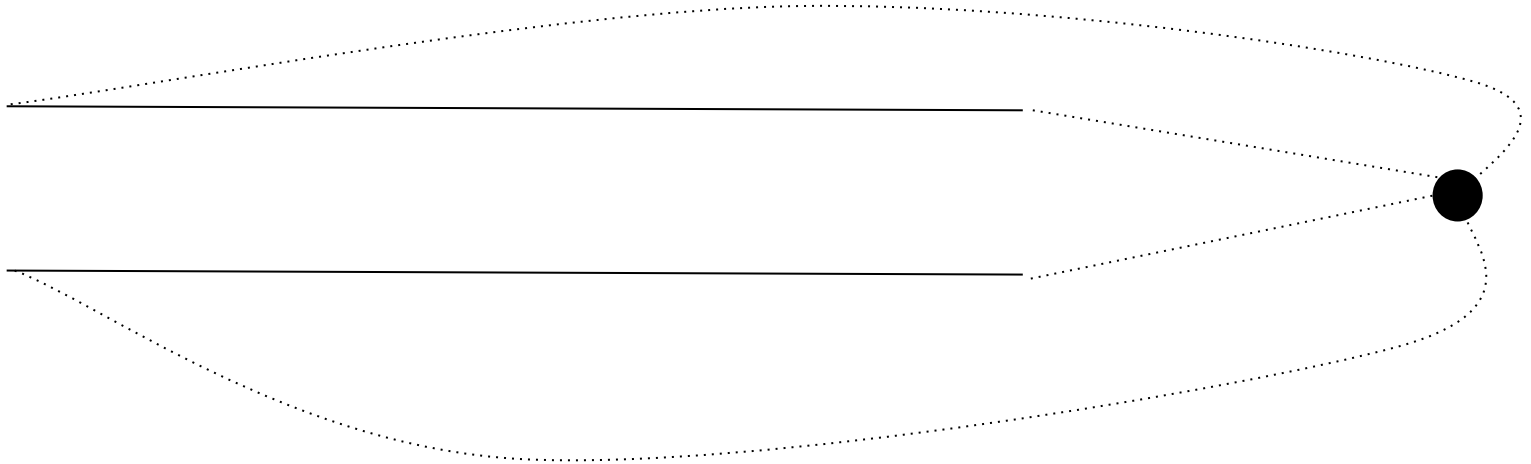
- Vanishing points at “infinity”, where parallel lines meet, wrap around



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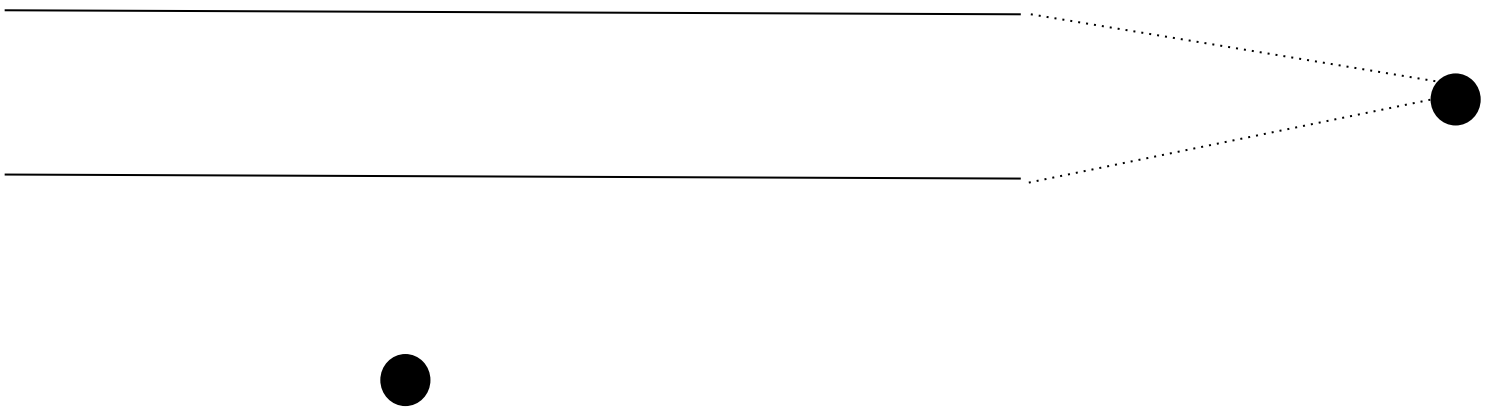

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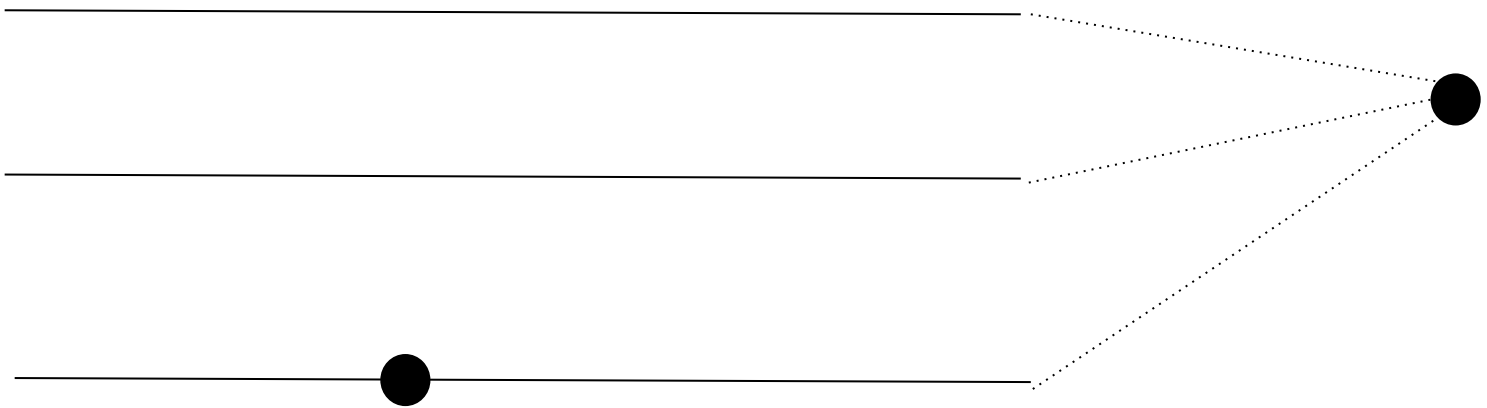


Finite Geometries

Finite Plane:

- discrete, finite number of points / lines

Finite Projective Planes

can be mapped to dobble cards

Finite Projective Planes

Finite Plane:

- discrete, finite number of points / lines

Projective Plane:

- Vanishing points, extra “points at infinity”, where parallel lines meet

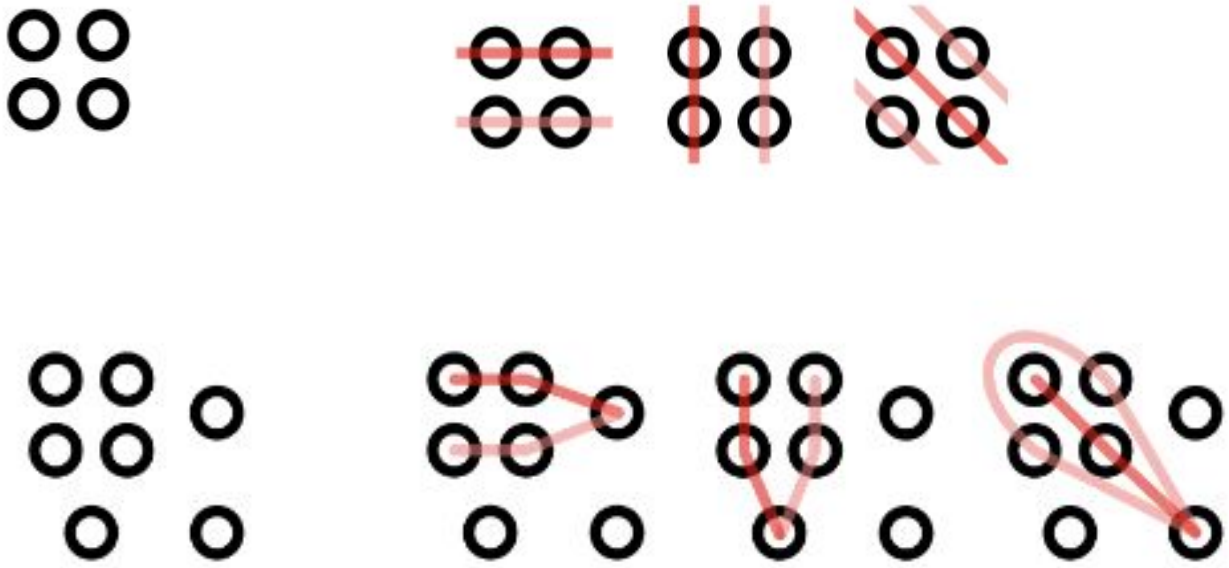
Order of the Projective Plane: n

- each line contains n+1 points, each point is on n+1 lines
- n^2 + n + 1 total number of lines and also total number of points

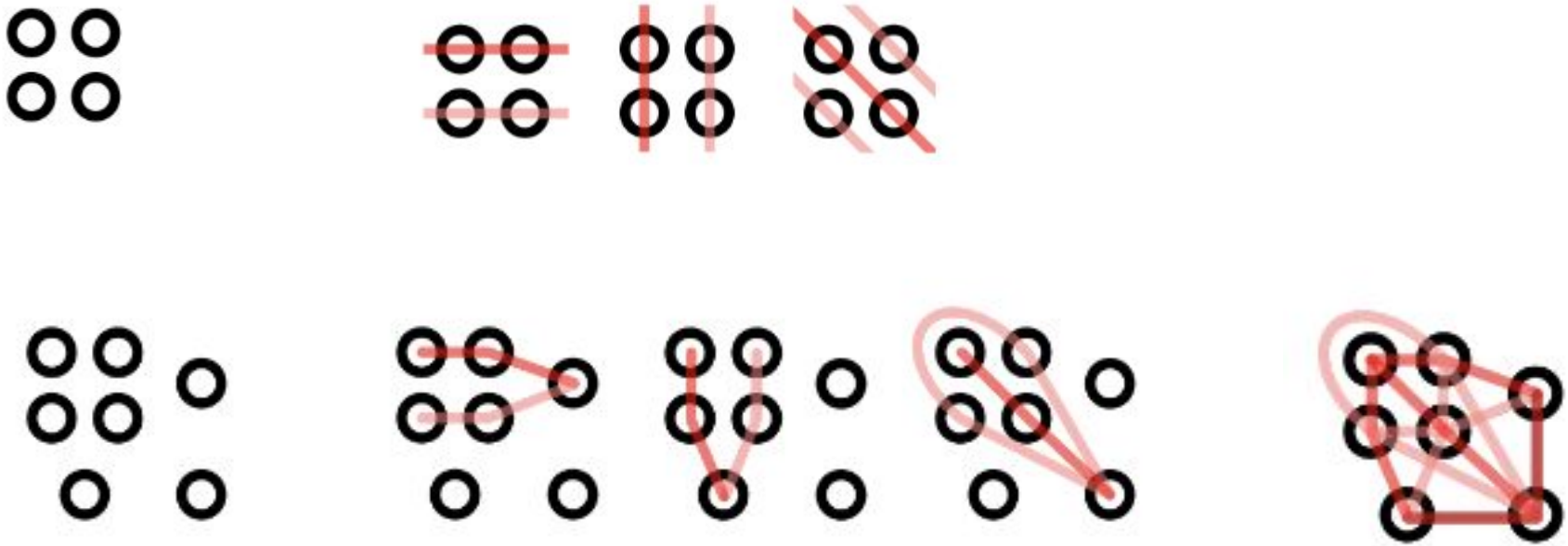
Start with n x n points

Add n+1 sets of parallel lines

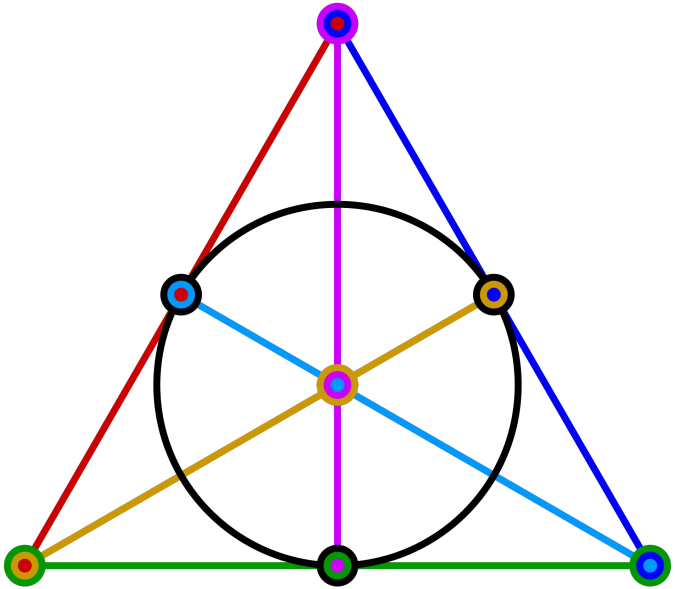
Add n + 1 vanishing points at “infinity”

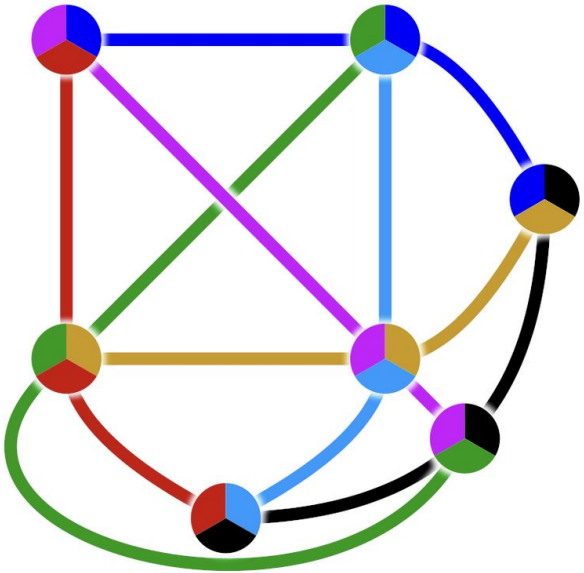


Connect the vanishing point with an extra line

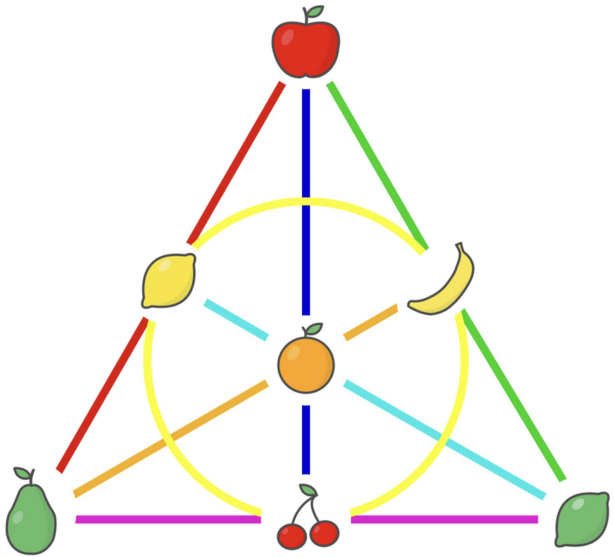


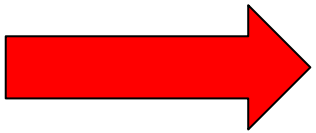
Finite Projective Planes (order n=2)

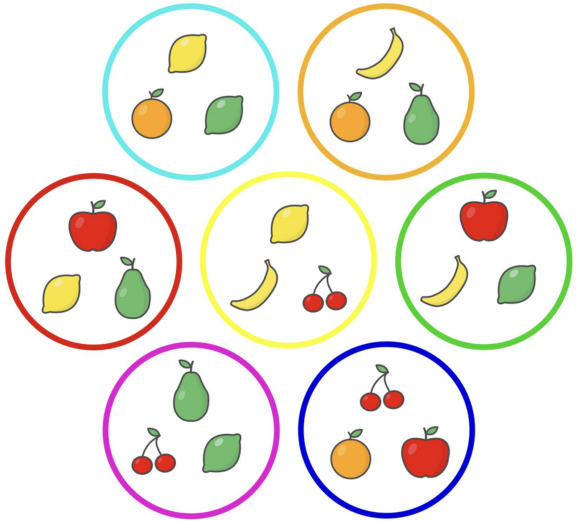




7 points (symbols), 7 lines (cards)

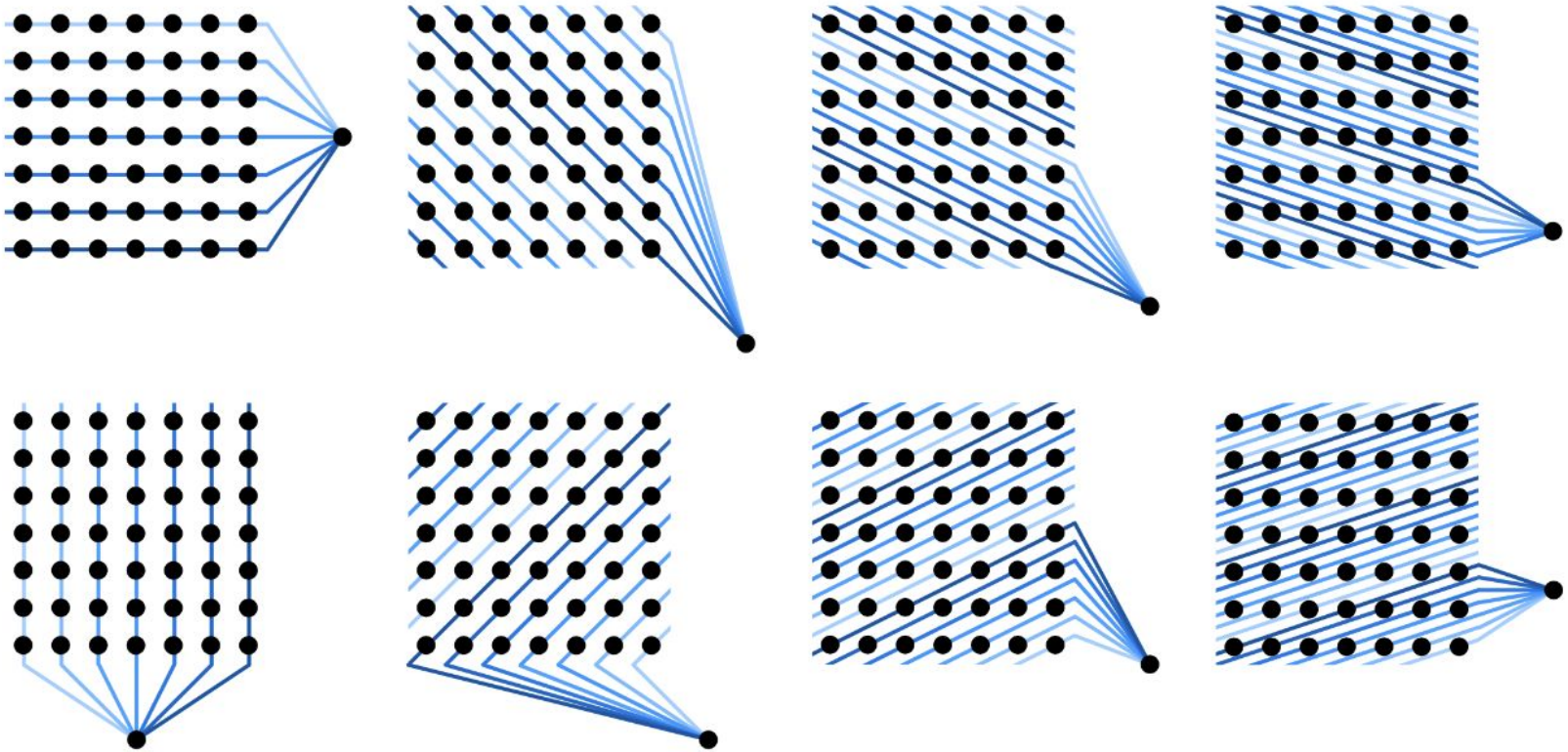




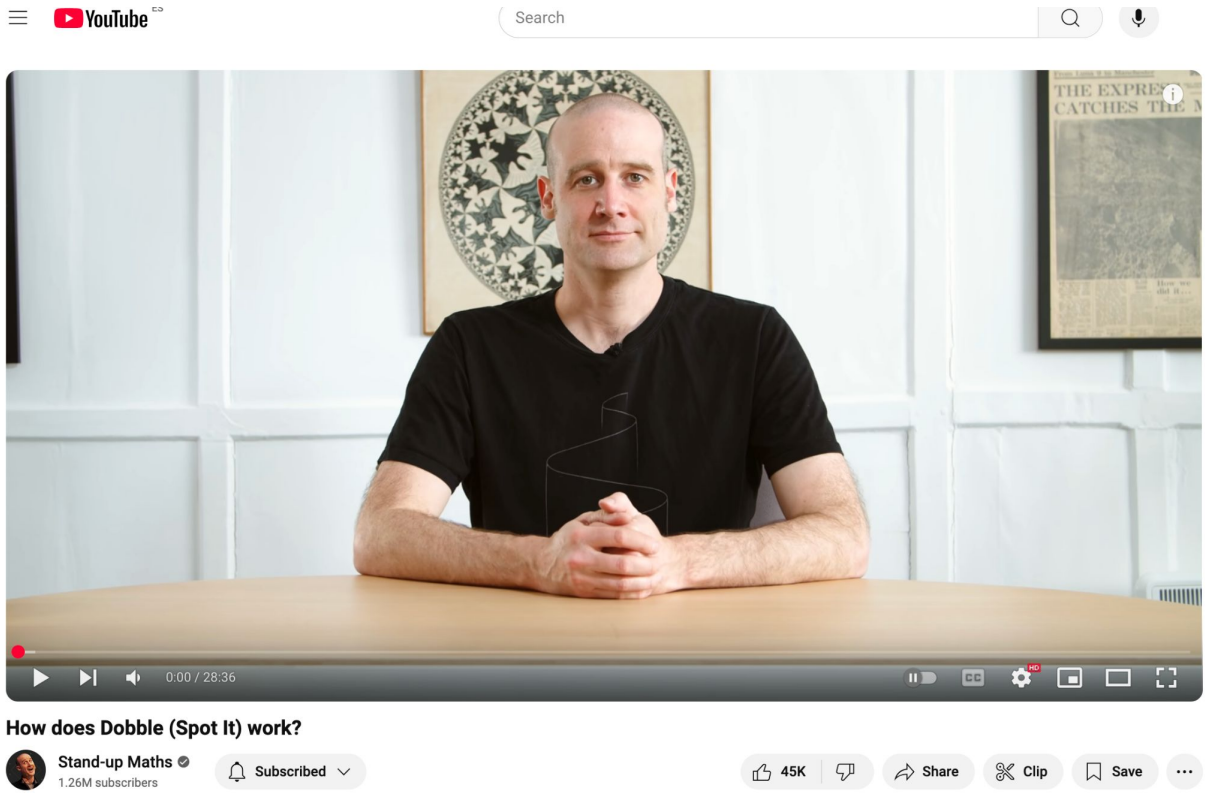


Dobble =
Finite Projective Plane (order=7)

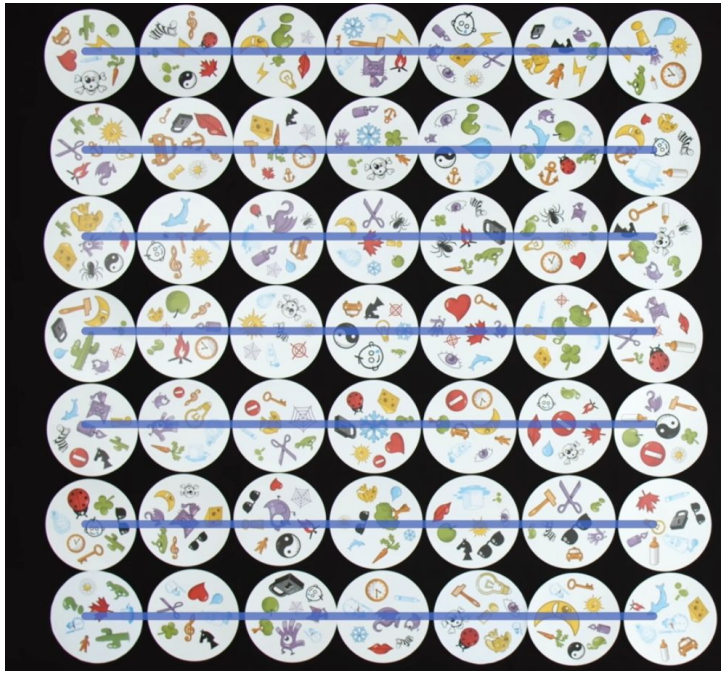
Projective Finite Plane of order: 7

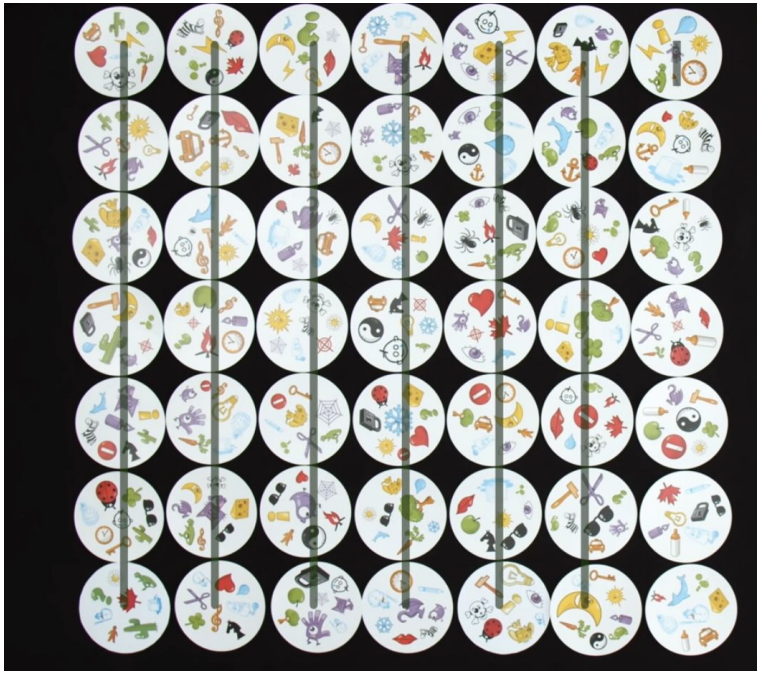


Stand-up Maths youtube channel - Steve Mould

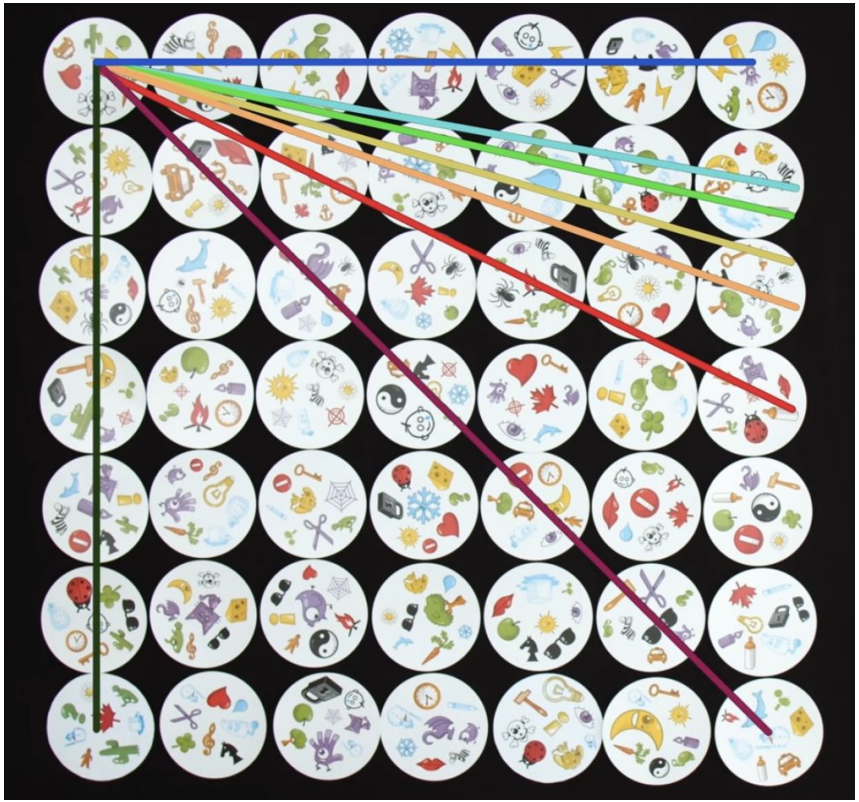


Projective Finite Plane of order: 7

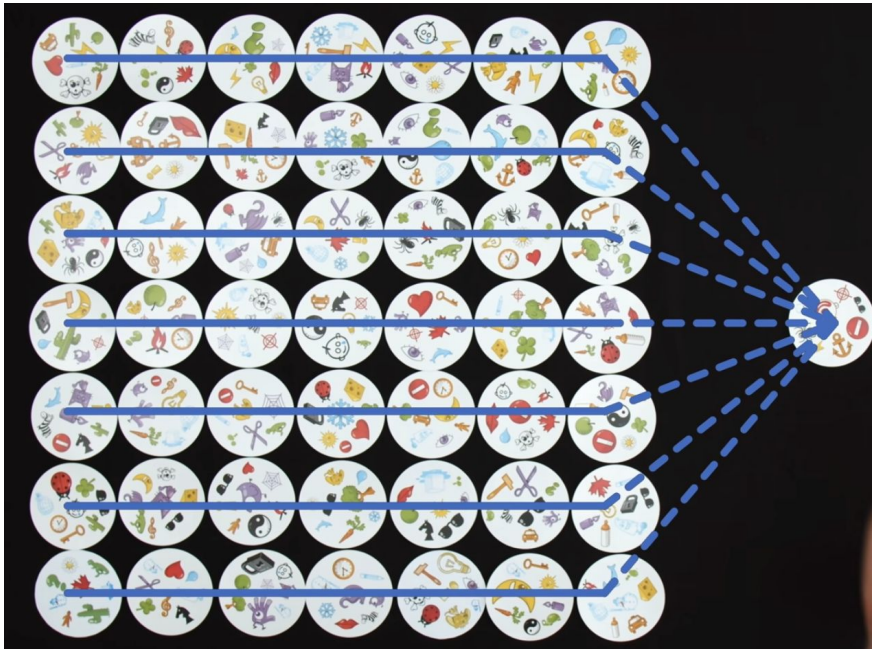


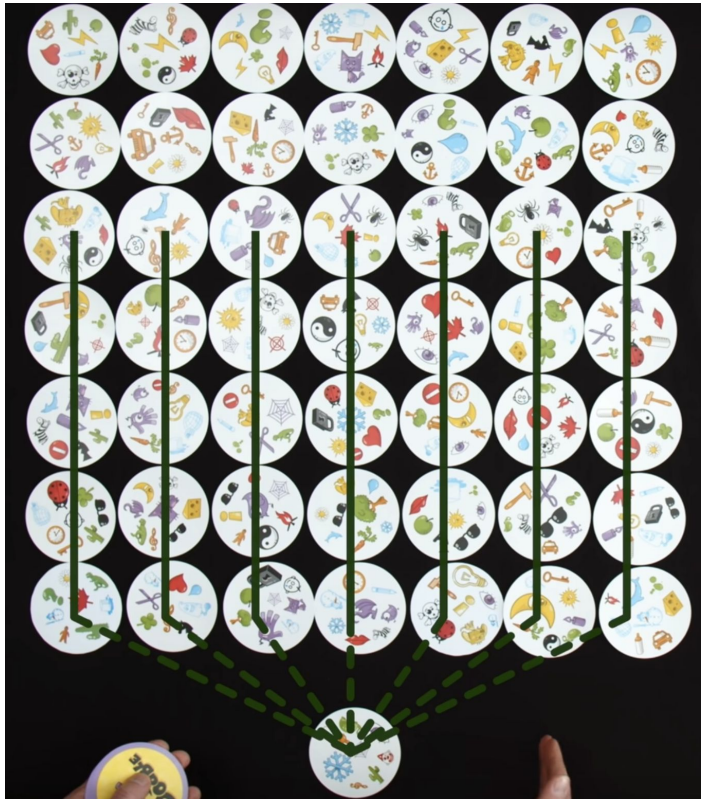


8 set of 7 lines (56)

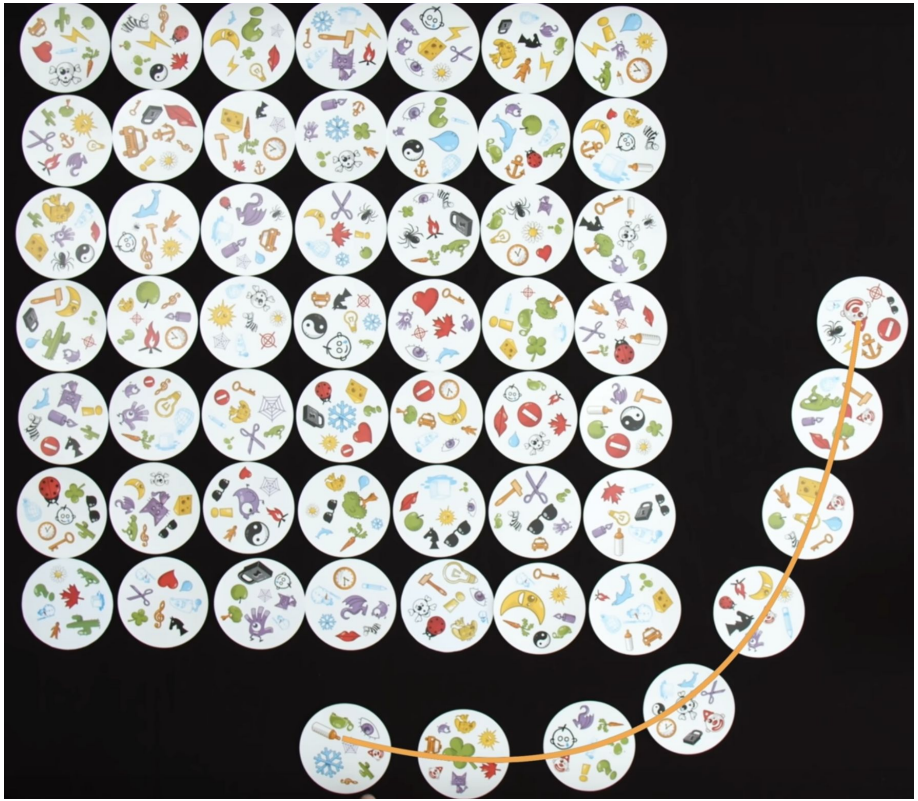


Vanishing points at “infinity”

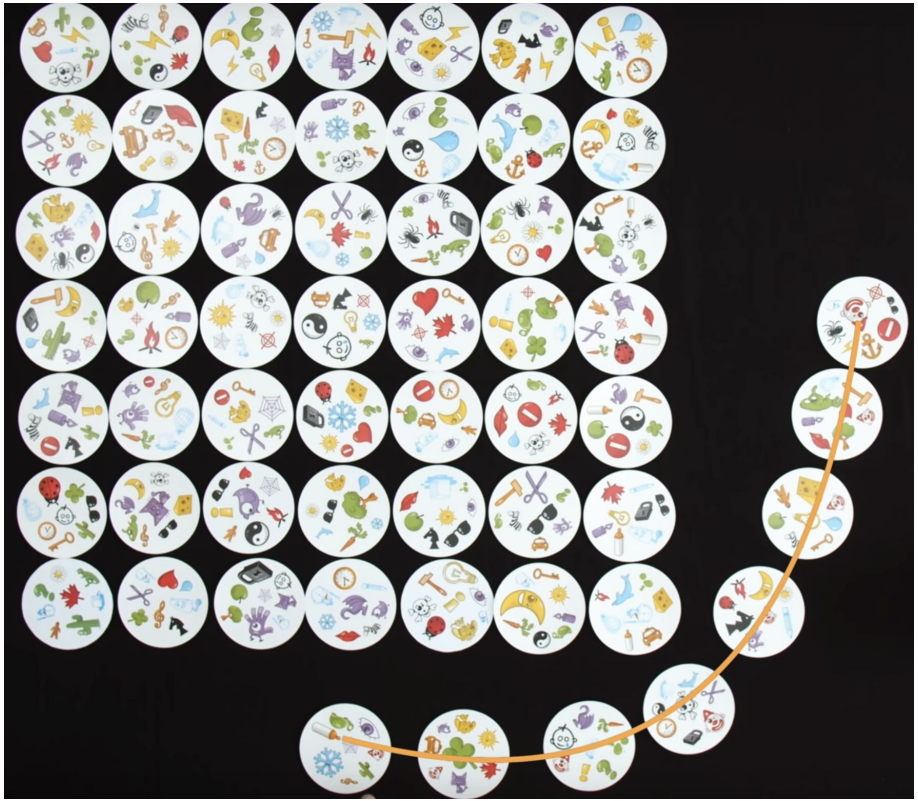




All 8 vanishing points - extra line at “infinity”



8 symbols, 8 lines, 49+8 = 57 cards



Point (Images), Lines (Cards)

Points (57)
7*7 grind =49
+ 8 vanishing points

Lines (57)
8 sets of 7 rows/columns = 56
+ 1 line connecting vanishing points

57 vs 55 cards?

- Why are the extra 2 cards missing?

- Why?
  - Manufacturing optimization?
    - 5 X 11 grid used for printing standard cards (52 plus 2 jokers and 1 advertising card)
    - Reusing the same machine

“While they [kids] may think they are picking out the crazy clown common to both cards,
I know that they are actually identifying the unique line that passes through points in the projective plane of order seven”

Marcus du Sautoy - Around the World in 80 Games