Perhatikan tabel berikut:

Pendefrensialan $\longrightarrow$	
$F(x)$	$F'(x)$
$\longleftarrow$ Pengintegralan	
$3x^2$	$6x$
$3x^2 + 3$	$6x$
$3x^2 - 5$	$6x$
$3x^2 + 5$	$6x$

Jika konstanta 3,-5 dan 5 adalah C ,maka fungsi F(x) = 3 x^2 + C , dengan notasi integral dapat di tulis $\boxed{\int f(x)dx = F(x) + C}$

maka $\boxed{\int 6x\,dx = 3x^2 + C}$

1.2. Integral dari

a. $\int 4x\,dx$ = $2x^2 + C$

b. $\int 3x^2 dx$ = $x^3 + C$

c. $\int 4x^3 dx$ = $x^4 + C$

Dengan mengamati keteraturan atau pola fungsi di atas ,jika koefisien x adalah *a* dan pangkat dari x adalah *n*, maka secara umum dapat di simpulkan

$$\boxed{\int ax^n dx = \frac{a}{n+1} x^{n+1} + C}$$

dengan *n bilangan rasional dan* $n \neq -1$

Tentukan hasil dari :

a. $\int 2x^2 dx$

b. $\int 4x^5 \ dx$

c. $\int -2x^{-3} dx$

d. $\int x\sqrt{x} \ dx$

e. $\int 2 \ dx$

Jawab :

a. $\int 2x^2 dx = \int ax^n dx = \frac{a}{n+1} x^{n+1} + C = \frac{2}{2+1} x^{2+1} + C$

$= \boxed{\frac{2}{3} x^3 + C}$

b. $\int 4x^5 dx = \int ax^n dx = \frac{a}{n+1} x^{n+1} + C = \frac{4}{5+1} x^{5+1} + C$

$= \frac{4}{6} x^6 + C$

$= \boxed{\frac{2}{3} x^6 + C}$

c. $\int -2x^{-3}dx = \frac{-2}{-3+1}x^{-3+1}+C$

$= \boxed{x^{-2}+C}$

d. $\int x\sqrt{x}dx = \frac{1}{\frac{3}{2}+1}x^{\frac{3}{2}+1}+C$

$= \frac{1}{\frac{5}{2}}x^{\frac{5}{2}}+C$

$= \boxed{\frac{2}{5}x^{2}\sqrt{x}+C}$

e. $\int 2\ dx \quad \boxed{=2x+C}$

Tentukan integral-integral tak tentu dari :

a. $\int 4x\,dx$

b. $\int -4x^3\,dx$

c. $\int x^7\,dx$

d. $\int 6x^{11}\,dx$

e. $\int \frac{3}{x^4}\,dx$

f. $\int x^{\frac{2}{3}}\,dx$

g. $\int \frac{5}{x^{\frac{-3}{4}}}\,dx$

h. $\int \sqrt[5]{x^4}\,dx$

i. $\int \frac{2}{\sqrt[7]{x^2}}\,dx$

j. $\int -3x^{\frac{-2}{3}}\,dx$

Ingat Bilangan eksponen :

1. $\frac{1}{a^n} = a^{-n}$

$\frac{1}{x^3} = x^{-3}$

2. $\sqrt[m]{a^n} = a^{\frac{n}{m}}$

$\sqrt[5]{x^3} = x^{\frac{3}{5}}$

3. $\frac{a^m}{a^n} = a^{m-n}$

3.a $\frac{3}{x^5} = 3x^{-5}$

3.b $\frac{3x^5 - 6x^3}{3x^3} = 3^1.3^{-1}x^5.x^{-3} - 2.3^1.3^{-1}x^3.x^{-3}$

$= x^2 - 2x^0$

$= x^2 - 2$

4. $a^p.a^q = a^{p+q}$

4.a $x^2\sqrt[3]{x^2} = x^2x^{\frac{2}{3}} = x^{2+\frac{2}{3}} = x^{2\frac{2}{3}} = x^{\frac{8}{3}}$

4.b $x^3\sqrt[5]{x^2} = x^3x^{\frac{2}{5}} = x^{3+\frac{2}{5}} = x^{3\frac{2}{5}} = x^{\frac{17}{5}}$

Jawaban :

a. $\int 4x\,dx = 4x + C$

b. $\int -4x^3\,dx = -x^4 + C$

c. $\int x^7\,dx = \frac{1}{7+1} x^{7+1} + C$

$= \frac{1}{8} x^8 + C$

d. $\int 6x^{11}\,dx = \frac{6}{11+1} x^{11+1} + C$

$= \frac{6}{12} x^{12} + C$

$= \frac{1}{2} x^{12} + C$

e. $\int \frac{3}{x^4}\,dx = \int 3x^{-4}\,dx$

$= \frac{3}{-4+1} x^{-4+1} + C$

$= -x^{-3} + C$

f. $\int x^{\frac{2}{3}}\,dx = \frac{1}{\frac{2}{3}+1} x^{\frac{2}{3}+1} + C$

$= \frac{1}{\frac{5}{3}} x^{1\frac{2}{3}} + C$

$= \frac{3}{5} x \sqrt[3]{x^2} + C$

g. $\int \frac{5}{x^{-\frac{3}{4}}}\,dx = \int 5x^{\frac{3}{4}}dx$

$= \frac{5}{\frac{3}{4}+1}x^{\frac{3}{4}+1} + C$

$= \frac{5}{\frac{7}{4}}x^{1\frac{3}{4}} + C$

$= \boxed{\frac{20}{7}x\sqrt[4]{x^3} + C}$

h. $\int \sqrt[5]{x^4}\,dx = \int x^{\frac{4}{5}}dx$

$= \frac{5}{\frac{4}{5}+1}x^{\frac{4}{5}+1} + C$

$= \frac{5}{\frac{9}{5}}x^{1\frac{4}{5}} + C$

$= \boxed{\frac{25}{9}x\sqrt[5]{x^4} + C}$

i. $\int \frac{2}{\sqrt[7]{x^2}}\,dx = \int 2x^{-\frac{2}{7}}dx$

$= \frac{2}{-\frac{2}{7}+1}x^{-\frac{2}{7}+1} + C$

$= \frac{2}{\frac{5}{7}}x^{\frac{5}{7}} + C$

$= \boxed{\frac{14}{5}\sqrt[7]{x^5} + C}$

j. $\int -3x^{\frac{-2}{3}}\,dx = \frac{-3}{-\frac{2}{3}+1}x^{-\frac{2}{3}+1} + C$

$= \frac{-3}{\frac{1}{3}}x^{\frac{1}{3}} + C$

$= \boxed{-9\sqrt[3]{x^1} + C}$

1.3. Menentukan Rumus Dasar Integral :

Perhatikan kasus berikut :

Kasus.1 $\int 2\, dx$ = 2x + C

Jika 2 = *a maka* $\int 2\, dx$ = 2x + C dapat ditulis menjadi

1.a $\int a\, dx = ax + C$ Jika *a = 1 maka*

1.b $\int dx = x + C$

Kasus.2

2.a $\int ax^n dx = \frac{a}{n+1} x^{n+1} + C$ Jika *a = 1 maka*

2.b $\int x^n dx = \frac{1}{n+1} x^{n+1} + C$

Kasus.3

$$\int 4x^3 dx = \tfrac{4}{3+1} x^{3+1} + C = x^4 + C$$

$$4\int x^3 dx = 4(\tfrac{1}{3+1} x^{3+1}) + C = 4(\tfrac{1}{4} x^4) + C$$

$$= x^4 + C$$

Kesimpulan kasus 3

$$\int 4x^3 dx = 4\int x^3 dx$$

Jika 4 = k dan $x^3 = f(x)$ maka dapat disimpulkan

3.a $\int kf(x)dx = k\int f(x)dx$

Contoh :

$$\int 20\, x^4 dx = 20 \int x^4 dx$$

$$= 20[(\tfrac{1}{4+1})x^{4+1}] + C$$

$$= 20\, [(\tfrac{1}{5})x^5] + C$$

$$= \boxed{4x^5 + C}$$

3.b $\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$

$= F(x) \pm G(x) + C$ C = C1+C2+…+Cn

Contoh.1 :

$$\int (4x^3 + 4)dx = \int 4x^3 dx + \int 4dx$$

$$= 4\int x^3 dx + 4\int dx$$

$$= 4[(\tfrac{1}{3+1})x^{3+1} + C_1] + 4(x + C_2)$$

$$= x^4 + 4C_1 + 4x + 4C_2$$

$$= x^4 + 4x + 4C_1 + 4C_2$$

$$= x^4 + 4x + C$$

Contoh.2 :

$$\int (3x^3 - 2x)dx = \int 3x^3 dx - \int 2x dx$$

$$= \tfrac{3}{4} x^4 - x^2 + C$$

Contoh.3 :

$$\int (x-2)^2 dx = \int (x^2 - 4x + 4)dx$$

$$= \tfrac{1}{3} x^3 - 2x^2 + 4x + C$$

Contoh.4:

$$\int \frac{x^2 + 2x}{x} dx = \int (x+2)dx$$

$$= \boxed{\tfrac{1}{2} x^2 + 2x + C}$$

Tentukan hasil integral tak tentu berikut !

a. $\int (2x - 1)dx$

b. $\int (2 - 3x)^2 \, dx$

c. $\int \frac{x - 2}{\sqrt{x}} \, dx$

d. $\int \frac{(x - 2)^2}{x\sqrt{x}} \, dx$

e. $\int x(\sqrt{x} + 2) \, dx$

a. $\int (2x-1)dx \quad = \quad \int 2x\,dx - \int dx$

$$= \quad x^2 - x + C$$

b. $\int (2-3x)^2\,dx \quad = \quad \int (4-12x+9x^2)dx$

$$= \quad 4x - 6x^2 + 3x^3 + C$$

c. $\int \frac{x-2}{\sqrt{x}}\,dx$

$$= \quad \int (x^{\frac{1}{2}} - 2x^{-\frac{1}{2}})dx$$

$$= \quad \int x^{\frac{1}{2}}dx - \int 2x^{-\frac{1}{2}}dx$$

$$= \quad \tfrac{2}{3}x\sqrt{x} - 4\sqrt{x} + C$$

d. $$\int \frac{(x-2)^2}{x\sqrt{x}}\,dx = \int \left(\frac{x^2-4x+4}{x\sqrt{x}}\right)dx$$

$$= \int \left(x^{\frac{1}{2}} - 4x^{-\frac{1}{2}} + 4x^{-1\frac{1}{2}}\right)dx$$

$$= \tfrac{2}{3}x\sqrt{x} - 8\sqrt{x} - \frac{8}{\sqrt{x}} + C$$

e. $$\int x(\sqrt{x}+2)\,dx = \int (x\sqrt{x}+2x)dx$$

$$= \int \left(x^{1\frac{1}{2}} + 2x\right)dx$$

$$= \tfrac{2}{5}x^2\sqrt{x} + x^2 + C$$

1.4. Integral substitusi

Jika $u = g(x)$ dengan g adalah fungsi yang mempunyai turunan

Maka $f(u) = f(g(x))$

Turunan $u =$ du Turunan $g(x) =$ $g'(x)$

$$\int f(u)du = \int f(g(x))g'(x)$$

$$\int f(u)du = F(u) + C$$

$$\int f(g(x))g'(x) = F(g(x)) + C$$

$$\boxed{\int f(u)du = \int f(g(x))g'(x) = F(u) + C = F(g(x)) + C}$$

Contoh :

Carilah hasil integral dari $\int (2x-5)(x^2-5x+14)^6\,dx$

Jawab :

$\int (2x-5)(x^2-5x+14)^6\,dx \;=\; \int (\boldsymbol{x^2-5x+14})^6(2x-5)\,dx$

Missal $u =$

maka turunan $u \longrightarrow du = \quad (2x-5)dx$

$= \int (x^2-5x+14)^6(2x-5)\,dx$

$= \int u^6\,du$

$= \frac{1}{7}u^7 + C$

$= \frac{1}{7}(x^2-5x+14)^7 + C$

Contoh :

Tentukan integral dari $\int \sqrt{x^3+4}.x^2dx$

Jawab :

$$\int \sqrt{x^3+4}.x^2dx$$

Misal $u = x^3 + 4$, maka $du = 3x^2dx \longrightarrow x^2dx = \frac{1}{3}du$

Jadi, $\int \sqrt{\underbrace{x^3+4}_{u}}.\underbrace{x^2dx}_{\frac{1}{3}du}$

$$= \int \sqrt{u}\ \tfrac{1}{3}du$$

$$= \tfrac{1}{3}\int u^{\frac{1}{2}}du$$

$$= \frac{1}{3}\,\frac{2}{3}u^{\frac{3}{2}} + C$$

.

$$= \tfrac{2}{9}(x^3+4)^{\frac{3}{2}} + C = \boxed{\tfrac{2}{9}(x^3+4)\sqrt{x^3+4} + C}$$

Contoh :

Tentukan integral dari $\displaystyle\int \frac{(3x^2-4)dx}{\sqrt{(x^3-4x)^3}}$

Jawab :

Misal $u = x^3 - 4x$ $\quad du = (3x^2-4)dx \longrightarrow dx = \dfrac{du}{(3x^2-4)}$

$$= \int \frac{(3x^2-4)}{\sqrt{(x^3-4x)^3}}dx$$

$$\int \frac{(3x^2-4)}{\sqrt{(x^3-4x)^3}}dx \qquad \int \frac{du}{\sqrt{u^3}}$$

$$= \int \frac{\cancel{(3x^2-4)}}{\sqrt{u^3}} \; \frac{du}{\cancel{(3x^2-4)}}$$

$$= \int \frac{du}{\sqrt{u^3}} = \int u^{-\frac{3}{2}}du = \frac{1}{-\frac{3}{2}+1}u^{-\frac{3}{2}+1} + C$$

$$= -2u^{-\frac{1}{2}} + C$$

$$= \frac{-2}{\sqrt{u}} + C$$

$$= \frac{-2}{\sqrt{(x^3-4x)}} + C$$