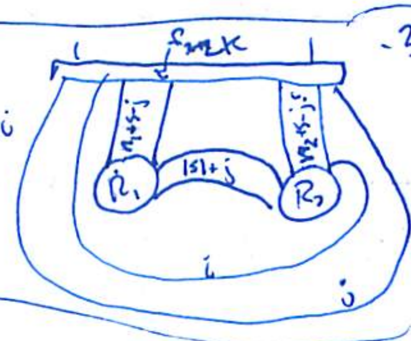
New Eqn, same as old but correct Mod 2K mistake

$$wC_{j+1,i} = (-1)^{i+n_1+s}\frac{[(i+n_1+s)\bmod(2K-2j)]}{[2K]}C_{j,i} + (-1)^{j+1+i}C_{j,i+1}\cdot(\sigma_1^{-1}\cdot\sigma_2)\frac{[2K-(j+i+1)]}{[2K]}$$
$$+ (-1)^{j+1}\frac{[j+1]}{[2K]}C_{j+1,i+1} + (-1)^{j+1}\frac{[2K-(j+1)]}{[2K]}C_{j+1,i-1}$$

$$w^{-1}C_{j+1,i} = (-1)^{i+n_1+s}\frac{[2K-((i+n_1+s)\bmod(2K-2j))]}{[2K]}C_{j,i} + (-1)^{j+i+1}\sigma_1^{-1}\sigma_2 C_{j,i+1}\cdot\frac{[j+i+1]}{[2K]}$$
$$+ (-1)^{j+1}\frac{[2K-(j+1)]}{[2K]}C_{j+1,i+1} + (-1)^{j+1}\frac{[j+1]}{[2K]}C_{j+1,i-1}$$

Recall: $X_w = \sum_j \sum_i C_{j,i}$



$2K = n_1+n_2+2s$, $s \leq 0$ is supposed to be weight 2K eigenvect with e.v. w

set

$$B_j = \frac{(-1)^{j+1}}{[2K]}\begin{bmatrix}[j+1] & [2K-(j+1)] \\ [2K-(j+1)] & [j+1]\end{bmatrix}$$

Then set

$$G_{j,i} = \begin{bmatrix}(-1)^{i+n_1+s}[i+n_1+s \bmod (2K-2j)] & (-1)^{j+i+1}\cdot(\sigma_1^{-1}\cdot\sigma_2)[2K-(j+i+1)] \\ (-1)^{i+n_1+s}[2K-(i+n_1+s \bmod 2K-2j)] & (-1)^{j+i+1}\cdot(\sigma_1^{-1}\sigma_2)[j+i+1]\end{bmatrix}$$

Then we have Eqn

$$B_j \begin{bmatrix} c_{j+1,\,i+1} \\ c_{j+1,\,i-1} \end{bmatrix} = c_{j+1,\,i} \begin{bmatrix} \omega \\ \omega^{-1} \end{bmatrix} + G_{j,i} \begin{bmatrix} c_{j,i} \\ c_{j,\,i+1} \end{bmatrix}$$

Then B_j is inventible and We have

$$B_j^{-1} = \frac{[2K]\,(-1)^{j+1}}{[j+1]^2 - [2K-(j+1)]^2} \begin{bmatrix} [j+1] & -[2K-(j+1)] \\ -[2K-(j+1)] & [j+1] \end{bmatrix}$$

Set $D^{i,j} := B_j^{-1} \cdot G_{j,i}$ 2×2 matrix

Then above eqn reads

$$\begin{bmatrix} c_{j+1,\,i+1} \\ c_{j+1,\,i-1} \end{bmatrix} = \frac{c_{j+1,\,i} \cdot [2K](-1)^{j+1}}{[j+1]^2 - [2K-(j+1)]^2} \begin{bmatrix} \omega[j+1] - \bar{\omega}[2K-(j+1)] \\ \bar{\omega}[j+1] - \omega[2K-(j+1)] \end{bmatrix} + D^{i,j} \begin{bmatrix} c_{j,i} \\ c_{j,\,i+1} \end{bmatrix}$$

Set $\alpha_j^w = \dfrac{(-1)^{j+1} \cdot [2k] \cdot (w[j+1] - \bar{w}[2k-(j+1)])}{[j+1]^2 - [2k-(j+1)]^2}$

We see $\alpha_j^w \neq 0$

Then we have 2 eqn relating $c_{j+1,i}$ and $c_{j+1,i+1}$.

The first, from top row of what we've just written is

1) $c_{j+1,i+1} = \alpha_j^w \cdot c_{j+1,i} + D_{11}^{i,j} \cdot c_{j,i} + D_{12}^{i,j} \cdot c_{j,i+1}$

The second comes from bottom row of eqn. obtained by adding 1 to i

2) $c_{j+1,i} = \overline{\alpha_j^w} \cdot c_{j+1,i+1} + D_{21}^{i+1,j} \cdot c_{j,i+1} + D_{22}^{i+1,j} \cdot c_{j,i+2}$

Note: reduced mod $2k-2j$

Adding 1) + $\frac{1}{\alpha_j^w}$ 2) gives

$$\left(\frac{1}{\alpha_j^w} - \alpha_j^w\right) c_{j+1,i} = (D_{11}^{i,j}) \cdot c_{j,i} + (D_{1,2}^{i,j} + D_{2,1}^{i+1,j}) c_{j,i+1} + (D_{22}^{i+1,j}) c_{j,i+2}$$

This gives the final formula

Mod (2K-2j)

$$\boxed{c_{j+1,i} = \frac{\overline{\alpha_j^\omega}}{1-|\alpha_j^\omega|^2}\left[D_{11}^{i,j}\cdot c_{j,i} + \left(D_{12}^{i,j} + D_{21}^{i+1,j}\right)c_{j,i+1} + D_{22}^{i+1,j}\cdot c_{j,i+2}\right]}$$

Thus $c_{j+1,i}$ is determined by $c_{j,i}$, $c_{j,i+1}$ and $c_{j,i+2}$

Since $c_{0,i} = \omega^i$, this implies all coefficents are uniquely determined (possibly over determined)

We record ~~the matrix D^{ij} on the next page.~~
Easy to compute

→ Notice its symetries

Can we simplify, perhaps as rational functions of q?

→ Yes → To Come