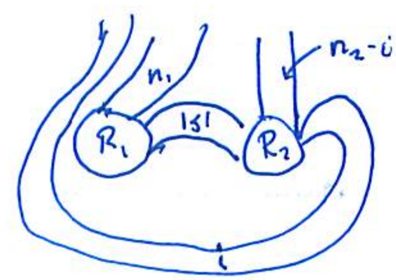
Decomposing Quadratic Tangle Reps

Suppose we have  with  $= \sigma_1$  and similarly  with σ_2.

For $s \leq 0$ (we just consider this case first)

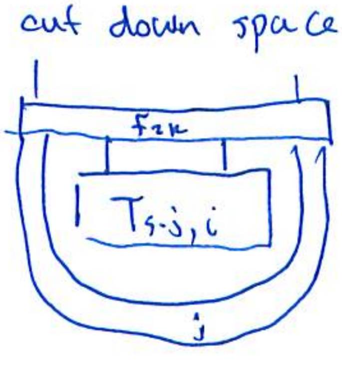
define $T_{s,i}$ = 

(not necessary, depends on parity n_1, n_2)

We want to decompose the weight 2K-box space.
All new lowest weight generators are uncapable, so we can cut down by f_{2K}, call space V_{2K}

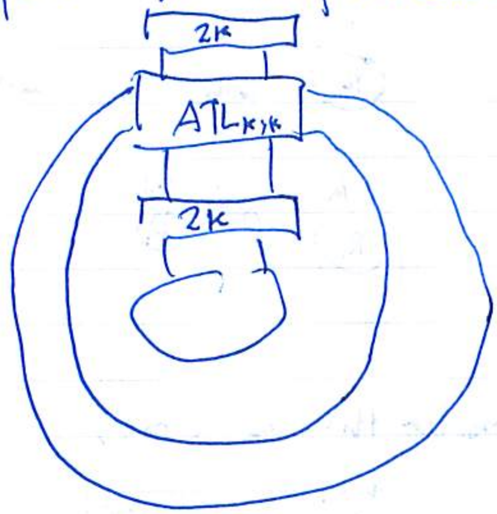
Assume (for easiness) that $2K = n+m+2s$ for some $s \leq 0$.

Then basis for cut down space is

$\Big\{ \hat{T}_{s-j,i} =$  $\Big\}$ $j = 0, \ldots,$ largest possible s
and $i = 0, \ldots, n+m+2(s-j)-1$
For a fixed j.

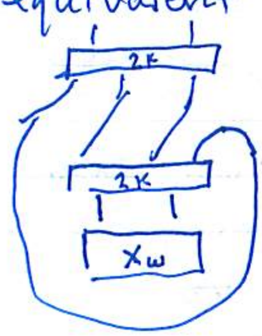
This space is representation

of

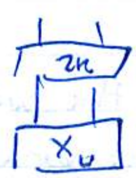


Being a new lowest weight 2k eigenvector

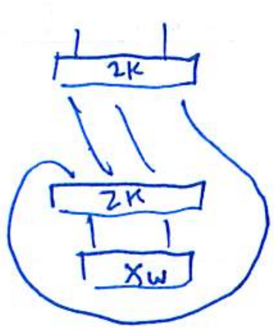
is equivalent to



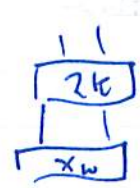
$= \omega$



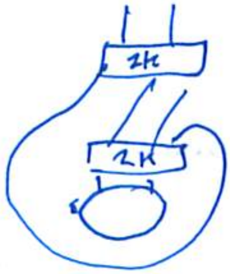
and

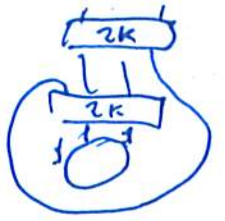


$= \omega^{-1}$



(Note these 2 conditions also imply uncappability).

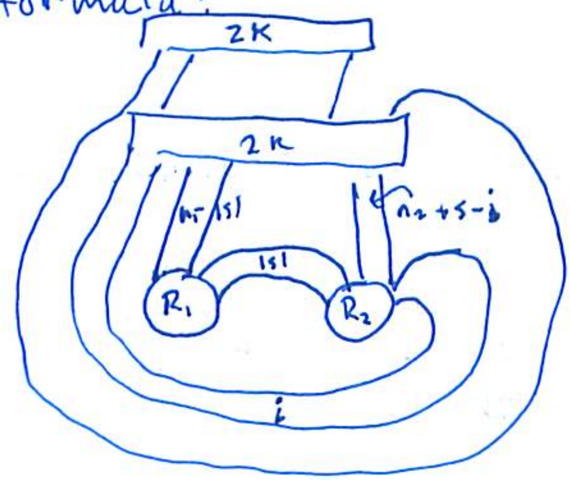
If we call $e =$ 

and $e^{-1} =$ 

(abusing notation, e^{-1} is not actually inverse of e, but it is on uncapable space)

Then we can compute action of e, e^{-1} on our basis $\hat{T}_{s-j,i}$.

We illustrate how to do this for $\hat{T}_{s,i}$ then give the general formula:

$e(\hat{T}_{s,i}) =$ 

$i+n+s$

Plug in TL diagrams

Now, ~~remove terms~~ in lower JW, only 3 diagrams give non-zero

an

$| \cdots |$

This gives the Formula

$$\rho(\hat{T}_{s,i}) = (-1)^{2K-i} \frac{[i]}{[2K]} (\sigma_1^{-1}\sigma_2) \hat{T}_{s-1,i-1} + (-1)^{n_2+s-i} \frac{[\overset{\text{Mod}(2K)}{n_1+s+i}]}{[2k]} \hat{T}_{s-1,i+1}$$

$$+ \hat{T}_{s,i+1}$$

Similarly

$$\rho^{-1}(\hat{T}_{s,i}) = (-1)^{i} \frac{[2K-i]}{[2K]} \sigma_1^{-1}\sigma_2 \hat{T}_{s-1,i-1} + (-1)^{n_1+s+i} [\overset{\text{Mod}(2K)}{n_2+s-i}] \hat{T}_{s-1,i}$$

$$+ \hat{T}_{s,i-1}$$

In general for $j>0$, Only 4-diagrams give non-zero, so

$$\rho(\hat{T}_{s-j,i}) = (-1)^{2K-j} \frac{[j]}{[2K]} \hat{T}_{s-j,i-1} + (-1)^{2K-j-i} \frac{[\overset{\text{Mod}(2K)}{j+i}]}{[2K]} (\sigma_1^{-1}\sigma^{-2}) \hat{T}_{s-j-1,i-1}$$

$$+ (-1)^{2K-(i+n_1+s)} \frac{[\overset{\text{Mod}(2K)}{i+n_1+s}]}{[2K]} \hat{T}_{s-j-1,i} + (-1)^{j} \frac{[2K-j]}{[2K]} \hat{T}_{s-j,i+1}$$

To obtain e^{-1}, one can simply switch the exponent of (-1) and the numerator of each term, so we have:

$$e^{-1}(\hat{T}_{s-j,i}) = (-1)^{j}\frac{[2K-j]}{[2K]}\hat{T}_{s-j,i-1} + (-1)^{j+i}\frac{[2K-(j+i)]}{[2K]}(\sigma_1^{-1}\cdot\sigma_2)\hat{T}_{s-j-1,i-1}$$

$$+ (-1)^{i+n_1+s}\frac{[2K-(i+n_1+s)]}{[2K]}\hat{T}_{s-j-1,i} + (-1)^{2K-j}\frac{[j]}{[2K]}\hat{T}_{s-j,i+1}$$

$$(-1)^{i+n_1+s}[n_2+s-i]$$

Now, we want to find soln to equations

$e(X_\omega) = \omega X_\omega$ and $e^{-1}(X_\omega) = \omega^{-1}(X_\omega)$ (such a soln will automaticly be uncapable)

Fix ω, and set

$$X_\omega = \sum_{0\le j\le \mathrm{Max}}\ \sum_{i=0,\dots 0,\, n_1+n_2+2s-j} c_{j,i}\,\hat{T}_{s-j,i}$$

We show that there is at most 1 solution for each ω and show how to find it.

Basically apply e, collect terms and same for ω^{-1}!

We see that for $j=0$, the action of e shows $C_{0,i} = \omega C_{0,i-1}$. Set $C_{0,0}=1$. Then $C_{0,i} = \omega^i$

Now we claim that if $C_{j,i}$ are all determined for all i $\{C_{j+1,i}\}_i$ are also uniquely determed.

Using e, we see that

$$\omega C_{j+1,i} = (-1)^{2K-(i+n_1+s)} \frac{[i+n_1+s]}{[2K]} C_{j,i} + (-1)^{2K-j-i-1} \frac{[j+i+1]}{[2K]} (C_{j,i+1})(\sigma_i^{-1}\sigma_2)$$
$$+ (-1)^{2K-j-1} \frac{[j+1]}{[2K]} C_{j+1,i+1} + (-1)^{j+1} \frac{[2K-j-1]}{[2K]} C_{j+1,i-1}$$

We have a similar (But independent) eqn for e^{-1}.

$$\omega^{-1} C_{j+1,i} = (-1)^{i+n_1+s} \frac{[2K-(i+n_1+s)]}{[2K]} C_{j,i} + (-1)^{j+i+1} \frac{[2K-j-i-1]}{[2K]} C_{j,i+1} (\sigma_1^{-1}\cdot\sigma_2)$$
$$+ (-1)^{j+1} \frac{[2K-j-1]}{[2K]} C_{j+1,i+1} + (-1)^{2K-j-1} \frac{[j+1]}{[2K]} C_{j+1,i-1}.$$

Now since we have assumed $c_{j,i}$ is known for all i,

We have the 2x2 symmetric matrix

$$A_{j+1,i} = \begin{bmatrix} (-1)^{2K-j-1}\dfrac{[j+1]}{[2K]} & (-1)^{j+1}\dfrac{[2K-j-1]}{[2K]} \\ (-1)^{j+1}\dfrac{[2K-j-1]}{[2K]} & (-1)^{2K-j-1}\dfrac{[j+1]}{[2K]} \end{bmatrix}$$

and $$A_{j+1,i}\begin{bmatrix} c_{j+1,i+1} \\ c_{j+1,i-1} \end{bmatrix} = \begin{bmatrix} w\, c_{j+1,i} + \text{something known} \\ w^{-1} c_{j+1,i} + \text{something known} \end{bmatrix}$$

But note $A_{j+1,i}$ is invertible for $j+1 \neq K$ (this situation is disallowed by geometry of our situation, namely $j \leq K$). with easy inverse

This solves $c_{j+1,i+1}$ AND $c_{j+1,i-1}$ in terms of $c_{j+1,i}$

Now, to solve for $c_{j+1,i}$ we have 2 equations (linear) relating $c_{j+1,i}$ and $c_{j+1,i+1}$,

Namely one from

Top

$$A_{j+1,i}\begin{bmatrix} c_{j+1,i+1} \\ c_{j+1,i-1} \end{bmatrix} = \begin{bmatrix} w\, c_{j+1,i} + \cdots \\ w^{-1} c_{j+1,i} + \cdots \end{bmatrix}$$

and $$A_{j+1,i+1}\begin{bmatrix} c_{j+1,i+2} \\ c_{j+1,i} \end{bmatrix} = \begin{bmatrix} w\, c_{j+1,i+1} + \cdots \\ w^{-1} c_{j+1,i+1} + \cdots \end{bmatrix}$$

Bottom

Writing down the $\cdots$ inductively, these eqns ~~are~~ seem to be independent, i.e. they solve for $c_{j+1,i}$.

Note this system is over determined, might not exist soln...

Since $c_{0,i} = w^i$ are fixed, rest will follow.

(A) Questions

1) → Can you get a computer to solve these guys explicity?

2) Is this basis we're using (cutdown by f_{2k}) bad for planar algebras somehow?

Note I solved eqn's for first couple of j by hand, not so bad.