MSRI Spring 2020 semester on **Quantum Symmetries**: summary

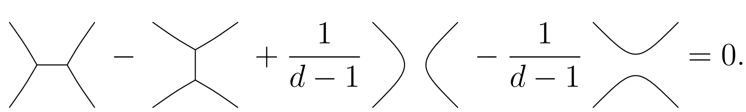
April 10, 2017

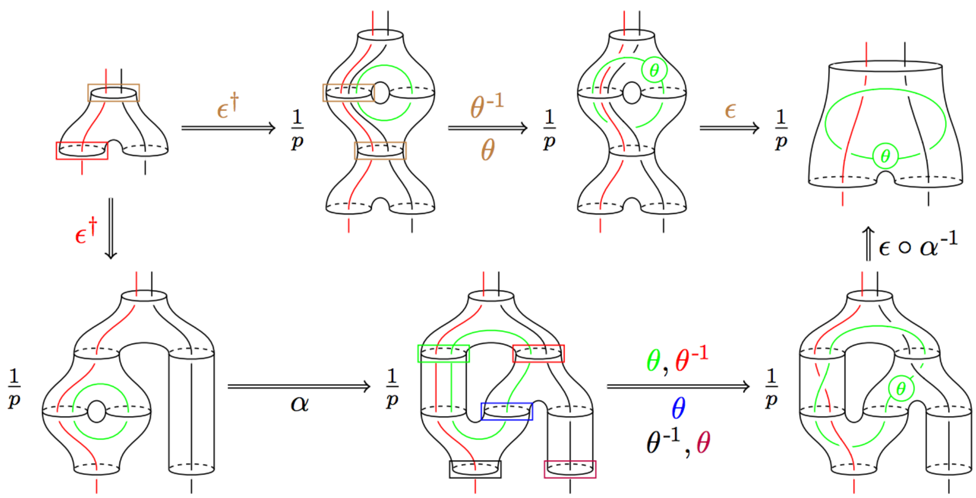
Symmetry, as formalized by group theory, is ubiquitous across mathematics and science. Classical examples include point groups in crystallography, Noether's theorem relating differentiable symmetries and conserved quantities, and the classification of fundamental particles according to irreducible representations of the Poincaré group and the internal symmetry groups of the standard model. However, in some quantum settings, the notion of a group is no longer enough to capture all symmetries. Important motivating examples include Galois-like symmetries of von Neumann algebras, anyonic particles in condensed matter physics, and deformations of universal enveloping algebras. The language of tensor categories provides a unified framework to discuss these notions of quantum symmetry.

Within the framework of studying the various guises of quantum symmetries, and their interactions, we will focus on following seven areas.

1. Tensor categories, fusion categories, module categories, and applications to representation theory.
2. Braided, symmetric, and modular tensor categories.
3. Hopf algebras, their actions on rings, and classification of semisimple and of pointed Hopf algebras.
4. Subfactors, planar algebras, and analytic properties of quantum symmetries.
5. Quantum invariants of knots and 3-manifolds, and local topological field theories.
6. Conformal nets, vertex algebras, and their representation theories.
7. Topological order and topological quantum computation.

A single skein relation simultaneously describes Q-systems in subfactor theory, the representation theory of quantum $SO(3)$, and the chromatic polynomial.





The study of tensor categories involves the interplay of representation theory, combinatorics, number theory, and low dimensional topology (from a string diagram calculation, describing the 3-dimensional bordism 2-category [arXiv:1411.0945]).