Category theory presentation report

Statement of the Yoneda lemma

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Attendance:

All were present, Zach ~15 mins late.

Presentation Summary:

In my presentation, I gave the statement of the yoneda lemma, explained all the terms involved, and claimed that the isomorphism was natural, when viewed as a functor from the category A to Set. I also gave the example of when both functors were representable functors.

I then explained some of the philosophy behind the yoneda lemma, which was primarily that presheaves are objects we should care about, since we understand explicitly the maps from representable functors into them, and used the previous example to support this line of reasoning, in that we completely understand representable functors and the maps between them in terms of the original category.

Building on this example led to the description of the yoneda embedding, and then the idea that since the embedding is full and faithful, we can gain understanding of our category A by considering the category of Set valued presheaves on A. As a basic example of this is that if we are interested in groups, then we can view our group (technically its opposite group, though I ignored this point) as a full subcategory within the category of functors from G to Set, which are just group actions on sets.

I then gave a more concrete example, calculating the endomorphisms of the forgetful functor from Ring to Set. Identifying this functor as representable then gave the result that all endomorphisms of this functor are just given by polynomials with integer coefficients.

Confusions and difficulties:

I had no questions during my talk, or any discussion afterward, but when I asked of a natural transformation of the forgetful functor, it took quite a while to get a response, which may be indicative of difficulty in understanding my presentation, or just the unfamiliarity of the audience with this kind of question. It was hard to assess what difficulties people had, since no questions were asked, and no real discussion occurred.

Evaluation of my presentation:

I think my presentation of the material was overall okay, though I feel like I made the mistake of going too fast. I think the content I wanted to cover was good for thirty minutes, though I only ended up taking twenty. I think that I could have gone slower especially on the concrete example, to ensure that everyone understood what was going on, possibly emphasised further the precise way that polynomials give rise to the endomorphisms of the functor.

Another issue that I only realised in hindsight was that I only gave the definition for covariant hom functors, and didn't distinguish between this and its dual when talking about the philosophy of the yoneda lemma.

For another talk like this in the future, I think I would ask more questions of the audience to assess their understanding during my talk, go slower with the content, and try to encourage more discussion at the end.