Category Theory - Report for 28/2/18

Who Was There: Everyone except Chris (I think he had a tutor meeting with Vigleik).

What We Covered: I presented material from the representable section of Leinster, specifically most of chapter 4.1. We started off by defining what a hom functor H^A is and briefly talked about what a locally small category is. From there we defined what a representable functor is. This definition wasn't new for a lot of people because we talked about representable functors in algebraic geometry, specifically looking at Spec_R. We did some easy examples of representable functors including $H^{\mathbf{1}} : \mathrm{Set} \to \mathrm{Set}$ (where $\mathbf{1}$ is the one point set) which showed that $\mathrm{id}_{\mathrm{Set}}$ is representable. We also talked about how forgetful functors are usually representable and we did an example which showed that $\mathrm{Bilin}(U, V; -) : \mathrm{Vect}_k \to \mathrm{Set}$ is representable by some vector space T and T is the tensor product $U \otimes V$.

Next we proved a lemma that if $F : \mathcal{A} \to \mathcal{B}$ is adjoint to G then the functor $\mathcal{A}(A, G(-)) : \mathcal{B} \to \mathrm{Set}$ is representable. This was a fairly simple proof that involved a nice diagram and everyone seemed to follow the proof with no issues. We talked about how any set-valued functor with a left adjoint is representable and how this would follow from the lemma but we did not explicitly prove it. Next we talked about how given and $f : A' \to A$, we get an induced natural transformation from H^A to $H^{A'}$ and how this natural transformation is defined. We mentioned the contravarient-ness of this and then defined the functor $H^{\bullet} : \mathcal{A}^{\mathrm{OP}} \to [\mathcal{A}, \mathrm{Set}]$. It was mentioned that all of our formulations could be dualised to get H_A and $H_{\bullet}$, but we did not spend too much time properly defining these.

We defined a functor $\mathrm{Hom}_{\mathcal{A}} : \mathcal{A}^{\mathrm{OP}} \times \mathcal{A} \to \mathrm{Set}$ which maps $(A, B) \to \mathcal{A}(A, B)$ and we drew a diagram to understand how it was defined on maps. No one seemed to have any problems understanding this definition. We were running out of time at this point but we talking about what a generalised element of a category is and how sometimes it is useful to study the maps into an object rather than the objects itself. A lot of us are familiar with similar ideas to this because Jim talked about it a bunch in algebraic geometry. We ended but mentioning an example where instead of looking at loops in a topological space X, you can look at maps from S^1 into X and a (continuous) map $f : X \to Y$ sends loops in X to loops in Y.

Overall there were not too many difficulties in this presentation. I think it was partly due to most people already being familiar with the idea of a representable functor from algebraic geometry. There was not much discussion during the presentation and because we were short for time, we did not spend much time discussing afterwards. So if I could do it differently, I would aim to achieve less content in one session and I would use more time to discuss the ideas of what we were talking about. In terms of the actual presenting I think we had a good balance of being precise but not getting bogged down with details. There was nothing too technical to talk about in our examples and in the lemma we proved, the proof was very straightforward so got the point across nicely and accurately without needing to worry about being super precise.