Category Theory Presentation Report

Yossi Bokor

14^th March 2018

1 Attendance

Keeley, Chris, Josh, Yiming, Adwait, Roger.

2 Material covered

I set out to show that given any category that has small products and equalisers, this category has all small limits. At the start, I thought this would be best achieved by proving the result that *if $\mathcal{B}$ has all small products and equalisers then $\mathcal{B}$ has all small limits.* One direction of the theorem is trivial, which I commented on, and then began to prove the other direction. I did so by showing a method through which, given any functor $F : \mathcal{A} \to \mathcal{B}$ we can construct a universal cone, or limit, over F. I restricted to the case where $\mathcal{A}$ is a small category. I began by showing how to construct a cone over the functor F using the existence of small products and equalisers, and then showed this cone was a universal cone, and hence a limit.

Initially I had planned on providing specific example of this construction, using Set, and then provide a statement of a theorem I came across whilst preparing for the presentation. The theorem is the following:

Theorem 1. Let $F : \mathcal{J}?C$ be a diagram in C. Then an object $X \in C$ is a limit of F if and only if there is a natural isomorphism

$$C(C, X) \cong \lim_{I \in \mathcal{J}} C(C, F(I))$$

where the limit on the right is in Set and hence exists.

I did not have enough time to provide the explicit example with Set as Adwait finished later into the hour than expected, so I only ended up providing a statement of Theorem 1.

3 Issues

Whilst proving the *if $\mathcal{B}$ has all small products and equalisers then $\mathcal{B}$ has all small limits,* there was some difficulties in picturing exactly how the construction worked, which we navigated by constructing and example where $\mathcal{B}$ had 3 objects and 1 non-identity morphism. Other than this, there were no issues I detected, or that were brought to my attention.

4 Discussion

Scott came in the last few minutes of the meeting time, and used possible discussion time to talk about the structure of the weekly presentation and time constraints, as well as about the structure of the assignments.

5 Reflection

I think I was a bit ambitious with the amount of material I wanted to cover, particularly with including the statement of the extra theorem. Had I had about another 5 minutes, I think I would have been able to cover the example of Set. I would have liked to discuss how this would make the construction of more general limits easier/possible, but did not have the opportunity.