Category Theory Talk Report

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March 2018

1 Attendance

Mitchell, Keeley, Josh, Likun, Yiming, Yossi, Chris, Adwait

2 Material Covered

In my presentation I proved the statement that if a locally small category $\mathscr{D}$ was complete, then the functor category $\mathscr{D}^{\mathscr{C}}$ was also complete (where $\mathscr{C}$ is a small category). To do this I revisited/introduced the definitions of: a category which has limits of shape $\mathbf{I}$ (for some small category $\mathbf{I}$); a category which has all limits; and finally, a complete category. At someone's request, I gave **Set**, and **Grp** as examples of complete categories. After stating these definitions I then proved the statement. Although I didn't actually realise that the proof of this statement was in the text, my proof was essentially identical to the proof of **Theorem 6.2.5** in Leinster.

3 Difficulties Encountered

Towards the end of the presentation, Keeley had a small problem determining where a certain collection of maps came from. This issue was simply a slight error in communication, and we resolved it quickly.

4 Post-Talk Discussion

The conversation after the presentation seemed quite fruitful. Having the extra time really allowed us to discuss a lot more so than in other weeks. We first discussed the style of proof - I believe Chris made a comment about how this was a quintessential example of a proof in category theory saying something to the effect of how once you understood all the mechanics of the things involved, the proof just falls out in the only way possible. After this we talked about the significance of this result - how it enables us to "compute limits pointwise". Finally we talked about why smallness (and local smallness) was important, and how/why things can break when we aren't particular about smallness.

5 Evaluation

Overall, I was quite happy with my talk. I don't think there were many ways I could have rearranged my presentation, since the proof had a rather obvious structure. However the proof in Leinster was presented in a slightly different order to my proof, and I believe my way was actually easier to digest in this particular format. Given that I now know that there is a section on this in the text (sorry), and given that we weren't pushed for time, if I were to give the talk again I would have talked more at the end about the implications of this particular result - namely I would have discussed how this result allows us to show that "limits commute with limits" as Leinster does. I think this is an interesting consequence, and while everyone will probably read the particular chapter that result is in, a class discussion would have been fun. Otherwise, I really quite enjoyed the discussion at the end, particularly the part about smallness. Finally, I'd again just like to say

how much nicer having the whole hour to give the talk was. It allowed us to unpack things where necessary, and meant we could get dug into discussion afterwards.