Category Theory Report

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1   Attendance

Everyone were present. Keeley left about 10 minutes early.

2   Material

After Chris gave the statement of Yoneda Lemma, I presented the proof of Yoneda Lemma, following the proof in the book. More specifically, I gave the definition of the isomorphism maps $(\hat{x})$ and $(\tilde{x})$ between $\mathrm{Nat}(H_A, X)$ and $X(A)$. I also talked a little about why there is only one sensible map. Then I went into details of the proof, namely, showing the naturalality square of $(\tilde{x})$ commutes, and $\hat{\tilde{x}} = x$, $\tilde{\hat{x}} = x$. I wasn't intended to give an explicit proof of showing that the isomorphism is natural, as it contains too much details. So what I did was drawing the naturality diagram and talked about the general idea of how to prove it.

3   Difficulties and discussions

There was a little confusion at the start, because Chris stated the dual version of the Yoneda Lemma, and we used different notation. This didn't trouble us for long. There was also a little confusion when I stated that there are only one way to define $(\hat{x})$ and $(\tilde{x})$. So I spent some time explaining these two maps. We didn't meet any difficulty or confusion for the actual proof.

After the talk, we discussed about the philosophical view that Yoneda lemma provides. We guessed that the general idea might be something like "You are what you see.". But no one thinks themselves really understand it and was hoping for Scott to explain it on Friday. We also discussed a little about how a natural transformation $H_A \to X$ is determined by its value at 1_A. We were a little confused about this statement and tried to work through an example, but ran out of time.

4   Evaluation

Overall I am satisfied about the talk. I think the important part of the proof is to understand two "canonical" isomorphisms, on which I devoted most time to

make sure everyone understand it. Once the notations and definitions are well-understood, checking the commutativity of some diagrams just follows naturally, which is, I think, not tricky but a dirty work. I spent a fair bit amount of time on explaining the details of the proofs, since it is easy to get lost with lots of notations and arrows floating around. I feel that I might too much time on the technical details on the proof, but since my job is to present the proof and make everyone understand, I believe that some technical details are inevitable.

One thing that I missed was discussion with Chris beforehand. It would certainly be much better to have the same notation and version of the Yoneda Lemma throughout the whole session. Also, Chris gave some explicit examples for Yoneda Lemma, so it would be very helpful if I have talked about what is the actual isomorphism is for these examples.

I believe I managed time relatively well. I avoided the most heavy part of the details and left five minutes for discussion. Maybe another way of delivering the talk is to give the general idea of the proof, talk about the examples, and let people work out details by themselves. But I believe that understanding details are necessary for understanding the proof and it should worth time explaining.