Australian National University

Category Theory Report

Keeley Hoek (u5642917)

March 5, 2018

Everyone except Chris was present.

Material covered and comments pertaining to planning

Following Leinster I began by recalling the universal property of the free vector space on a set. In particular, I highlighted where (slightly secretly!) forgetful functors had to be present so that we get a diagram with objects and morphisms in the same category, and pointed out that this understanding will end up being clarifying and useful for what was to come.

I defined the general comma category $(P \Rightarrow Q)$, explaining what the objects and morphisms are. I then gave (Leinster's) examples of two specific cases:

- The slice category $\mathscr{A}/A$, and
- The comma category $A \Rightarrow G$ for $G : \mathscr{B} \to \mathscr{A}$ a functor and A an object of G.

I explained what the objects and morphisms become in both cases, and when ending with the morphisms for the second example I noted that we can recognise the more explicit diagram at the beginning of the talk as the diagram a morphism in this category must satisfy.

I was then more explicit than the text explaining that an initial object in this category is exactly an object satisfying the desired universal property, and that we of course have a completely analogous final object construction for final properties. I then gave the tensor product of vector spaces universal property/commutative diagram and explained that because we want some of our maps to be bilinear and others to be linear, it does not arise in *exactly* the same way we have encoded the free vector space universal property.

At this point I also *extremely briefly* mentioned that at this point I discovered (via Math.StackExchange exploration) the rabbit-hole of "internal hom functors", and that in our case they let one talk about bilinear maps as elements of a particular homset—but I just mentioned literally these words and offered no explanation (but I was excited and wanted to share!).

I then stated the main theorem of the section, the correspondence between adjunctions and natural transformations with components particular initial objects. The lemma preceding the theorem provided a "compact" example of adjunctions and initial objects interacting. I explained the structure of the proof of the theorem, separately obtaining uniqueness then existence. In particular I pointed out the use of an interesting technique in the existence proof; how two different diagrams, each giving rise to a morphism from the initial object under consideration, immediately implied equality of the morphisms in question and hence that the required naturality condition is satisfied.

If time remained, I planned to walk through the details of this the existence part of the theorem, at least explaining the first of the two times where the "two diagrams plus initial object" technique is used. Time didn't permit this—though the fact that everyone was timely allowed Josh and I to get closer to our 25 minutes than otherwise! The whole hour was quite action-packed.

After the hour expired it seems that quite a fraction of the class has to dash off to other classes! I was un-bound and Adwait and I talked more about the way in which universal properties may be encoded as initial objects in a comma category. We also talked about the specific universal property for the unit encoded in the equivalent definition of adjunction contained within the main theorem.

Evaluation

In summary, I largely followed Leinster and elaborated/riffed briefly on the points I thought were particularly important. My peers corrected erroneous diagrams when they were drawn and passed-over, which gave me confidence that they were following my exposition/the relevant parts of the text. I was a bit disappointed to not be able to elaborate on the proof of the main theorem—the idea was to then go back and expand the sketch in places which clarified the main ideas. This surprised me and beforehand I was confident that I'd have time to get at least a little further (expanding the details, at least up to the first use of "initiality").

I think this had a bit to do with not realising that the pacing of the first week of talks couldn't really be managed with with reduced timeframe—they were not *too* much longer, but were able to manage a more relaxed tone which promoted more discussion. I think that I should have deferred more auxiliary explanations, for instance the definition of the slice category and my tensor product side note, to the end if time permitted. This change and a faster pace would likely have allowed us to get a chunk further. Part of my choice of ordering was also due to a reluctance to deviate too much from the narrative of the text—I don't think doing this was as valuable as I thought it'd be, but I'm not sure. That said, I very much liked the coherent story I was able to tell. I thought that my decision to separately introduce universal properties as initial/final objects, going on to then emphasise that the application to describing adjoint functors was a specific example, helped to define the boundary between the two main ideas of the section.

We didn't pause for a discussion after I ended the last chunk of my talk; I polled the students and then yielded straight to Josh. In this way a block of discussions were saved to the end, where we were quite time constrained. I think this was a shame, in contrast to the experience I had in the first week where we were more free (of course, this was just a consequence of being lucky enough to have more time!). Perhaps we could informally devote some (perhaps teeny) fraction of our Monday meeting for this, but of course this would be a chronologically-impaired arrangement, and I think a bit of this was done this Monday anyway.