

Category theory Report
Chris Hone

Attendance: Adwait, Yossi, Josh, Yiming, Roger.

The topic of my talk was braided monoidal categories, with examples. I first gave the definition of a braided monoidal category, focusing on the braiding aspect. I defined a braiding, and stated one of the hexagon identities (and mentioned the other as similar). For this I also drew a picture of the hexagon identity using string diagrams. I made sure to highlight the difference between symmetric and non-symmetric braidings. I then gave the first examples of symmetric monoidal categories, which were various categories with the categorical product, as well as Vect with the tensor product.

I then went into the example of Braid, to show that an object of a braided monoidal category can be identified with a (monoidal) morphism from Braid into that category. I then went into significant detail for my main example, the category of crossed G-Sets, the comma category of G-Sets over the G-Set given by the conjugation action of G. I explained the monoidal structure on this category, and that this relied on our conjugation action being a group homomorphism.

I then defined the braiding, as described in <https://ncatlab.org/nlab/show/crossed+G-set>, and proved that this was a valid G-Set morphism (though notably I didn't prove the rest of the required properties). My final example was graded rings, with usual monoidal structure, with braiding given by swapping the elements but multiplying by a power of a fixed unit when the swap occurs.

This talk went for the full 50 minutes, which I think gave enough time to do a sufficiently complicated example in depth. One issue that occurred was the hexagon identity I wrote down initially was not correct, and had to be fixed later. There were also various times where I had to pause to think about the proofs, instead of having them already written. For both of these, it would have been a good idea to prepare more written material, to have a cleaner presentation. I also forgot to give the definition of a morphism of braided monoidal categories. I deliberately gave very algebraic examples of braided monoidal categories, and tried to put emphasis on non symmetric braidings. This was a personal choice, based on the (symmetric) monoidal categories we had been dealing with recently, and how all our non-symmetric braiding's thus far had a topological flavour.

There was not any discussion afterwards, though I wasn't sure of the cause.