

$$\square\square\square \quad c_1 \cancel{X} + c_2 H + c_3 I + c_4 \cancel{X} + c_5 H + c_6 \cancel{X} = 0.$$

Diagram illustrating a sequence of 20 boxes (1 to 20) arranged in two rows. The top row contains boxes 1 through 10, and the bottom row contains boxes 11 through 20. A blue line connects the 5th box of the top row to the 5th box of the bottom row.

$$\gamma = \gamma^{12} \cup \quad \gamma = -v^6 \gamma \quad 0 = d \quad 0 = 0$$

~~$x = -v^6$~~

$$c_1 H + c_2 \cancel{X} - v^{-6} c_3 I + c_4 L + c_5 \cancel{X} + v^{-12} c_6 \cancel{X} = 0.$$

$$c_1 I + c_2 X - v^{-6} c_3 H + c_4 \approx + c_5 X + v^{-12} c_6 \quad (=0).$$

$$(A1) -v^{-6}c_2 - v^{-6}c_3 + c_4 + c_5 + v^{-12}c_6 + c_4 = 0.$$

Age Group	Percentage
18-24	18%
25-34	22%
35-44	15%
45-54	12%
55-64	10%
65-74	8%
75-84	5%
85+	3%

$$(IHX) \quad v^{-3} \text{X} - v^{-1} \text{H} + v \text{I} + \alpha (\text{X} + v^{-4}) (+v^4 \text{X}) = 0.$$

$$\begin{aligned} \gamma &= v^{12} \nu & \gamma &= -v^6 \Upsilon & 0 &= d \\ v^{-3} \cancel{X} - v^{-1} \cancel{H} + v \cancel{I} + \alpha (\cancel{X} + v^{-4}) (\cancel{I} + v^4 \cancel{I}) &= 0. \end{aligned} \quad (2)$$

$$v^{-3} \gamma - v^{-1} \nu + \alpha (\gamma + v^{-4} \nu + v^4 \cancel{0}) = 0 \quad \text{or} \quad [5] \cancel{0} + \alpha (d + \{8\}) \cancel{0} = 0$$

$$v^{-3} \gamma - v^{-1} \Upsilon + v \gamma + \alpha (\gamma + v^{-4} \Upsilon) = 0 \quad \text{or} \quad -\{2\} \Upsilon + (b - [5]\alpha) \Upsilon = 0$$

$$\cancel{0} = b \cancel{0}$$

$$\Upsilon = t \Upsilon$$

$$b = -\frac{\alpha(d + \{8\})}{[5]}$$

$$t = \frac{b - [5]\alpha}{\{2\}}$$

$$d = -\frac{\{2\}[\lambda+5][\lambda-6]}{[\lambda][\lambda-1]}$$

$$b = \frac{\{2\}[\lambda+2]\{2\}[\lambda-3][3]}{[1]}$$

$$\alpha = -\frac{[\lambda][\lambda-1]}{[1]}$$

$$t = \{1\} \left((v + v^{-1}) \frac{w^2}{v} + (v^4 - v^2 - 1 - v^{-2} + v^{-4}) + (v + v^{-1}) \frac{v}{w^2} \right)$$

$$[k\lambda + l] := w^k v^l - w^{-k} v^{-l}$$

$$\{k\lambda + l\} := w^k v^l + w^{-k} v^{-l}$$

$$+ v^2 \left(v^{-3} \cancel{X} - v^{-1} \cancel{H} + v \cancel{I} + \alpha (\cancel{X} + v^{-4} b) (\cancel{I} + v^4 \cancel{I}) \right)$$

$$+ v^{-2} \left(v^{-3} \cancel{X} - v^{-1} \cancel{I} + v t \cancel{I} + \alpha (\cancel{X} + v^{-4} \cancel{H} + v^4 b \cancel{I}) \right)$$

$$+ \left(v^{-3} \cancel{X} - v^{-1} \cancel{H} + v \cancel{I} + \alpha (b \cancel{X} + v^{-4} \cancel{H} + v^4 \cancel{I}) \right)$$

$$[3] \cancel{I} + (t + [1]\alpha) v^{-3} \cancel{X} - (v t + v^{-2} [4]\alpha) \cancel{H} + (v^{-1} t + v^2 [4]\alpha) \cancel{I}$$

$$+ \alpha b \cancel{X} + \alpha b v^{-2} \cancel{I} + \alpha b v^2 \cancel{I} = 0.$$

$$\gamma = v^{12} \nu \quad \gamma = -v^6 \gamma$$

$$v^{-3} \cancel{\gamma} - v^{-1} \gamma + \alpha (\cancel{\gamma} + v^{-4}) (\gamma + v^4 \cancel{\gamma}) = 0.$$

$$\gamma - \cancel{\gamma} = \frac{[67]}{[27][37]} (\cancel{\gamma} - \gamma) + [\lambda][\lambda-1] (\gamma - \cancel{\gamma}).$$

$$d = - \frac{[23][\lambda+5][\lambda-6]}{[\lambda][\lambda-1]}$$

$$b = \frac{[\lambda+2][\lambda-3][3]}{[1]}$$

$$\alpha = - \frac{[\lambda][\lambda-1]}{[1]}$$

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$$(cross) \quad \gamma - \cancel{\gamma} = \frac{[67]}{[27][37]} (\cancel{\gamma} - \gamma) + [\lambda][\lambda-1] (\gamma - \cancel{\gamma}).$$

$$\gamma = \beta \cancel{\gamma} \Rightarrow \gamma = \cancel{\gamma} = 0.$$

$$IHX \Rightarrow \beta v^{-3} \cancel{\gamma} - v^{-1} \gamma + \alpha (\beta + v^{-4}) \cancel{\gamma} = 0$$

$$\gamma (v - \beta v^{-1}) = - \frac{[\lambda][\lambda-1] (\beta v^2 + v^{-2})}{[1]}$$

$$cross \Rightarrow \gamma = \frac{v^2 - 1 + v^{-2}}{[1]} (\beta - \beta^{-1}) + [\lambda][\lambda-1]$$

$$\beta = -1 \quad \gamma = [\lambda][\lambda-1]$$

$$\beta = w^2 \quad \gamma = \frac{[\lambda][\lambda+2]}{[1]}$$

$$\beta = \frac{v^2}{w^2} \quad \gamma = \frac{[\lambda-1][\lambda-3]}{[1]}$$

$$(a) \quad v \leftrightarrow v^{-1} \quad w \leftrightarrow w$$

$$\cancel{\gamma} \leftrightarrow \gamma$$

$$(b) \quad v \leftrightarrow v \quad w \leftrightarrow \frac{v}{w}$$

$$\cancel{\gamma} \leftrightarrow \gamma$$

$$\text{Under (b), } [\lambda] \leftrightarrow -[\lambda-1]$$

$$\gamma = v^{12} \nu \quad \gamma = -v^6 \gamma$$

$$v^{-3} (\gamma - v^{-1}) (-v \gamma + \alpha (\gamma + v^{-4}) (-v^4 \gamma)) = 0.$$

$$\gamma (-\gamma) = \frac{\{6\}}{\{2\} \{3\}} (\gamma - \gamma) + [\gamma] [\gamma - 1] (\gamma - \gamma).$$

$$d = -\frac{\{2\} \{3\} [\gamma + 5] [\gamma - 6]}{[\gamma] [\gamma - 1]}$$

$$b = \frac{\{2\} \{3\} \{2\} \{3\} \{3\}}{[\gamma]}$$

$$\alpha = -\frac{[\gamma] [\gamma - 1]}{[\gamma]}$$

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The characteristic equation is then

$$(\gamma + 1)(w^{-1} \gamma - w)(\frac{w}{v} \gamma - \frac{v}{w}) = v^{18} p \gamma + v^9 q \gamma$$

Capping off gives

$$(v^{12} + 1)(w^{-1} v^{12} - w)(\frac{w}{v} v^{12} - \frac{v}{w}) = v^{18} p d$$

$$p = \frac{\{6\}(-[\gamma - 6])[\gamma + 5]}{d} = \frac{[\gamma] [\gamma - 1] \{6\}}{\{2\}}$$

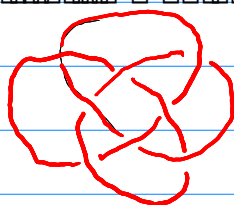
$$(-v^6 + 1)(-w^{-1} v^6 - w)(-\frac{w}{v} v^6 - \frac{v}{w}) = v^9 q b$$

$$q = \frac{-\{3\} \cdot -\{2\} \cdot -\{2\}}{b} = -[1]$$

so we find

$$\frac{1}{v} \gamma - (\frac{w^2}{v} - \frac{1}{v} + \frac{v}{w^2}) \gamma - (\frac{w^2}{v} - v + \frac{v}{w^2}) (\gamma + 1) = v^{18} \frac{[\gamma] [\gamma - 1] \{6\}}{\{2\}} \gamma - v^9 [1] \gamma.$$

$$\gamma, \gamma, \gamma, \gamma, \gamma, \gamma.$$



$\delta_{1,3}$

$$\gamma = v^{12}v \quad \gamma = -v^6\gamma$$

$$v^{-3} \cancel{X} - v^{-1} \cancel{H} + v \cancel{I} + \alpha(\cancel{X} + v^{-4}) \cancel{I} + v^4 \cancel{I} = 0.$$

$$\cancel{H} - \cancel{I} = \frac{\{6\}}{\{2\}\{3\}} (\cancel{X} - \cancel{X}) + \{ \lambda \} \{ \lambda - 1 \} (\cancel{I} - \cancel{I}).$$

$$d = - \frac{\{2\}\{\lambda+5\}\{\lambda-6\}}{\{ \lambda \} \{ \lambda - 1 \}}$$

$$b = \frac{\{ \lambda + 2 \} \{ \lambda - 3 \} \{ \lambda - 1 \}}{\{ \lambda \}}$$

$$\alpha = - \frac{\{ \lambda \} \{ \lambda - 1 \}}{\{ \lambda \}}$$

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$$\frac{1}{v} \cancel{X} - \left(\frac{w^2}{v} - \frac{1}{v} + \frac{v}{w^2} \right) \cancel{X} - \left(\frac{w^2}{v} - v + \frac{v}{w^2} \right) \cancel{X} + v \cancel{X} = v^6 \frac{\{ \lambda \} \{ \lambda - 1 \} \{ 6 \}}{\{ 2 \}} d + v^3 \{ 1 \} b d$$

$$\text{Hopf} = v d \left(v^{-12} \left(\frac{w^2}{v} - \frac{1}{v} + \frac{v}{w^2} \right) + d \left(\frac{w^2}{v} - v + \frac{v}{w^2} \right) - v^{13} + v^6 \frac{\{ \lambda \} \{ \lambda - 1 \} \{ 6 \}}{\{ 2 \}} + v^3 \{ 1 \} b \right)$$

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(%i98) hopf : v*d*(v^(-12)*(w^2/v - 1/v + v/w^2) + d*(w^2/v - v + v/w^2) - v^13
+ v^6*qlam(1,6)*qlam(1,-1)*qtwo(6)/qtwo(2) + v^3*qint(1)*b);

(%o98) (v (-v^2
-1/(v^2)) (w/(v^6)-(v^6)/w) (v^5 w-1/(v^5 w)) ((-v^2-1/(v^2)) (w/(v^6)-(v^6)/w) (v^5 w

(%i92) factor(ratexpand(hopf));

- ((v^4+1) (w-v^6) (w+v^6) (v^5 w-1) (v^5 w+1) (v^22 w^8-v^12 w^8-v^10 w^8+w^8
-v^24 w^6-v^22 w^6+v^20 w^6-v^16 w^6+2 v^12 w^6-v^8 w^6+v^4 w^6-v^2 w^6-w^6+v^26 w^4
+2 v^24 w^4-v^20 w^4+v^18 w^4+v^16 w^4-2 v^14 w^4-2 v^12 w^4+v^10 w^4+v^8 w^4-v^6 w^4
+2 v^2 w^4+w^4-v^26 w^2-v^24 w^2+v^22 w^2-v^18 w^2+2 v^14 w^2-v^10 w^2+v^6 w^2-v^4 w^2
-v^2 w^2+v^26-v^16-v^14+v^4))/(v^24 (w-1)^2 w^2 (w+1)^2 (w-v)^2 (w+v)^2)

(%i93) factor(ratexpand(hopf/d));

(v^22 w^8-v^12 w^8-v^10 w^8+w^8-v^24 w^6-v^22 w^6+v^20 w^6-v^16 w^6+2 v^12 w^6
-v^8 w^6+v^4 w^6-v^2 w^6-w^6+v^26 w^4+2 v^24 w^4-v^20 w^4+v^18 w^4
+v^16 w^4-2 v^14 w^4-2 v^12 w^4+v^10 w^4+v^8 w^4-v^6 w^4+2 v^2 w^4+w^4
-v^26 w^2-v^24 w^2+v^22 w^2-v^18 w^2+2 v^14 w^2-v^10 w^2+v^6 w^2-v^4 w^2
-v^2 w^2+v^26-v^16-v^14+v^4)/(v^12 (w-1) w^2 (w+1) (w-v) (w+v))

(%i94)

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One more small consequence: the eigenvector with eigenvalue -1 is

$$\frac{1}{v} \cancel{X} - \{2\lambda - 1\} \cancel{I} + v \cancel{X} - \frac{\{ \lambda \} \{ \lambda - 1 \}}{\{ 2 \}} \cancel{X} + \frac{\{ 1 \}}{\{ 3 \}} \cancel{I}$$