Here are all the 1-parameter families of (super) semi-simple Lie algebras, in ascending order of interest.

Global conventions: there are three homogeneous parameters, α, β, and γ. For convenience, we set $\alpha+\beta+\gamma=3$, except for points on the line at infinity ($\alpha+\beta+\gamma=0$).

Triv. Equation: $\alpha=6^{(\times 3)}$. Parameter: β. Symmetry: $\beta\mapsto -3-\beta$.

These usually correspond to 0-dimensional Lie groups.

β	(α,β,γ)	Other names
0	$(6,0,-3)$	$\mathfrak{sl}_2$ $[l=3]$, $\mathfrak{osp}_n$ $[n=4]$, Subexcpt $[n=8]$
2	$(6,2,-5)$	Excpt $[\mu=5]$
3	$(6,3,-6)$	$\mathfrak{sl}_n$ $[n=1]$, $\mathfrak{osp}_n$ $[n=0]$
$\frac{9}{2}$	$(6,\frac{9}{2},-\frac{15}{2})$	Subexcpt $[n=-1]$
9	$(6,9,-12)$	$\mathfrak{osp}_n$ $[n=1]$, Subexcpt $[n=0]$
∞	$(0,1,-1)$	$D(2,1,\alpha)$ $[\alpha=-1]$

$\mathfrak{sl}_2$. Equation: $\alpha=-3^{(\times 3)}$. Parameter: $l=\beta-3$. Symmetry: $l\mapsto -l$.

l	(α,β,γ)	Other names
l	$(-3,3+l,3-l)$	
0	$(-3,3,3)$	$\mathfrak{sl}_n$ $[n=2]$,Subexcpt $[n=-4]$
1	$(-3,4,2)$	Excpt $[\mu=2]$
$\frac{3}{2}$	$(-3,\frac{9}{2},\frac{3}{2})$	$\mathfrak{osp}_n$ $[n=-2]$
3	$(-3,6,0)$	$\mathfrak{osp}_n$ $[n=4]$, Triv $[\beta=0]$
9	$(-3,12,-6)$	$\mathfrak{osp}_n$ $[n=3]$
12	$(-3,15,-9)$	Subexcpt $[n=4]$
∞	$(0,1,-1)$	$D(2,1,\alpha)$ $[\alpha=-1]$

$\mathfrak{sl}_n$. Equation: $\alpha=3^{(\times 3)}$. Parameter: $n=\frac{6}{\beta}$. Symmetry: $n\mapsto -n$.

n	(α,β,γ)	Other names
n	$(3,\frac{6}{n},\frac{-6}{n})$	
0	$(0,1,-1)$	$D(2,1,\alpha)$ $[\alpha=-1]$
1	$(3,6,-6)$	Triv $[\beta=3]$, $\mathfrak{osp}_n$ $[n=0]$
2	$(3,3,-3)$	$\mathfrak{sl}_2$ $[l=0]$, Subexcpt $[n=-4]$
3	$(3,2,-2)$	Excpt $[\mu=2]$
4	$(3,\frac{3}{2},\frac{-3}{2})$	$\mathfrak{osp}_n$ $[n=6]$
6	$(3,1,-1)$	
∞	$(3,0,0)$	$\mathfrak{osp}_n$ $[n=\infty]$

$\mathfrak{osp}_n$. Equation: $\beta - \gamma = 3^{(\times 6)}$. Parameter: $n = 2 - \frac{6}{\gamma}$.
Another unusual parameter is $a = n - 4$ (so $n = a + 4$).

n	(α, β, γ)	Other names
n	$(12, 3n-12, -6)/(n-2)$	
-6	$(-6, 15, 3)/4$	Subexcpt $[n = 14]$
-4	$(-2, 4, 1)$	$\mathfrak{osp}_n$ $[n = 5]$
-2	$(-6, 9, 3)/2$	$\mathfrak{sl}_2$ $[l = \frac{3}{2}]$
-1	$(-4, 5, 2)$	Excpt $[\mu = 4]$
0	$(-6, 6, 3)$	Triv $[\beta = 3]$, $\mathfrak{sl}_n$ $[n = 1]$
1	$(-12, 9, 6)$	Triv $[\beta = 9]$, Subexcpt $[n = 0]$
2	$(2, 0, 1)$	$D(2, 1, \alpha)$ $[\alpha = 2]$
3	$(12, -3, -6)$	$\mathfrak{sl}_2$ $[l = 9]$
4	$(6, 0, -3)$	$\mathfrak{sl}_2$ $[l = 3]$, Triv $[\beta = 0]$
5	$(4, 1, -2)$	$\mathfrak{osp}_n$ $[n = -4]$
6	$(6, 3, -3)/2$	$\mathfrak{sl}_n$ $[n = 4]$
7	$(12, 9, -6)/5$	Subexcpt $[n = -8]$
8	$(2, 2, -1)$	Excpt $[\mu = 1]$
10	$(6, 9, -3)/4$	Subexcpt $[n = -10]$
12	$(6, 12, -3)/5$	Subexcpt $[n = 32]$
∞	$(0, 3, 0)$	$\mathfrak{sl}_n$ $[n = \infty]$

Excpt. Equation: $\alpha = 2^{(\times 3)}$. Parameter: $\mu = -\beta$. Symmetry: $\mu \mapsto -1 - \mu$.
Alternate parameters include Deligne and Gross' $\nu = 1/\mu$, $\eta = (\mu + 1)/\mu$ ($\mu = 1/(\eta - 1)$), and $a = 2\eta - 4 = 2(1 - \mu)/\mu$. Note: μ is called λ by Deligne.

μ	η	(α, β, γ)	Other names
μ		$(2, -\mu, 1 + \mu)$	
5		$(2, -5, 6)$	Triv $[\beta = 2]$
4		$(2, -4, 5)$	$\mathfrak{osp}_n$ $[n = -1]$
3	$\frac{4}{3}$	$(2, -3, 4)$	$\mathfrak{sl}_2$ $[l = 1]$
2	$\frac{3}{2}$	$(2, -2, 3)$	$\mathfrak{sl}_n$ $[n = 3]$
$\frac{3}{2}$	$\frac{5}{3}$	$(2, -\frac{3}{2}, \frac{5}{2})$	G_2, Subexcpt $[n = -7]$
1	2	$(2, -1, 2)$	$\mathfrak{osp}_n$ $[n = 8]$
$\frac{2}{3}$	$\frac{5}{2}$	$(2, -\frac{2}{3}, \frac{5}{3})$	F_4
$\frac{1}{2}$	3	$(2, -\frac{1}{2}, \frac{3}{2})$	E_6
$\frac{1}{3}$	4	$(2, -\frac{1}{3}, \frac{4}{3})$	E_7, Subexcpt $[n = 56]$
$\frac{1}{5}$	6	$(2, -\frac{1}{5}, \frac{6}{5})$	E_8
0	∞	$(2, 0, 1)$	Subexcpt $[n = -16]$
∞		$(0, -1, 1)$	$D(2, 1, \alpha)$ $[\alpha = -1]$

Subexcpt. Equation: $\alpha-\beta-2\gamma=0^{(\times 6)}$. Parameter: n.
Another parameter is $a=(n-8)/6$ (so $n=6a+8$).

n	a	(α,β,γ)	t	Other names
n		$(n-8,n+16,-12)$	$2n-4$	
-16	-4	$(2,0,1)$	3	Excpt $[\mu=0]$
-10	-3	$(3,-1,2)$	4	$\mathfrak{osp}_n$ $[n=10]$
-8	$-\frac{8}{3}$	$(4,-2,3)$	5	$\mathfrak{osp}_n$ $[n=7]$
-7	$-\frac{5}{2}$	$(5,-3,4)$	6	G_2, Excpt $[\mu=\frac{3}{2}]$
-4	-2	$(1,-1,1)$	1	$\mathfrak{sl}_n$ $[n=2]$,$\mathfrak{sl}_2$ $[l=0]$
-1	$-\frac{3}{2}$	$(3,-5,4)$	2	Triv $[\beta=9/2]$
0	$-\frac{4}{3}$	$(2,-4,3)$	1	Triv $[\beta=9]$, $\mathfrak{osp}_n$ $[n=1]$
2	-1	$(-1,3,-2)$	0	$D(2,1,\alpha)$ $[\alpha=?]$
4	$-\frac{2}{3}$	$(-1,5,-3)$	1	$\mathfrak{sl}_2$ $[l=12]$
8	0	$(0,2,-1)$	1	$\mathfrak{sl}_2^{\oplus 3}$, $\mathfrak{sl}_2$ $[l=3]$, Triv $[\beta=0]$, $\mathfrak{osp}_n$ $[n=4]$
14	1	$(1,5,-2)$	4	$\mathfrak{osp}_n$ $[n=-6]$
20	2	$(1,3,-1)$	3	$\mathfrak{sl}_n$ $[n=6]$
32	4	$(2,4,-1)$	5	$\mathfrak{osp}_n$ $[n=12]$
56	8	$(4,6,-1)$	9	E_7, Excpt $[\mu=\frac{1}{3}]$

$F3_4$. Equation: $2\alpha-\beta-2\gamma=0^{(\times 6)}$. Parameter: n.

a	n	(α,β,γ)	t	Other names
a		$(a,2a+4,-2)$	$3a+2$	
$-\frac{5}{3}$		$(5,-2,6)$	9	F_4, Excpt $[\mu=\frac{2}{3}]$
$-\frac{8}{5}$	-16	$(4,-2,5)$	7	$\mathfrak{osp}_n$ $[n=9]$
$-\frac{3}{2}$		$(3,-2,4)$	5	$\mathfrak{osp}_n$ $[n=7]$
$-\frac{4}{3}$		$(2,-2,3)$	3	$\mathfrak{sl}_n$ $[n=3]$, Excpt $[\mu=2]$
$-\frac{6}{5}$		$(3,-4,5)$	4	$\mathfrak{sl}_2$ $[l=\frac{3}{4}]$
-1		$(1,-2,2)$	1	Triv $[\beta=3]$,$\mathfrak{osp}_n$ $[n=0]$,$\mathfrak{sl}_n$ $[n=1]$
$-\frac{4}{5}$		$(2,-6,5)$	1	Triv $[\beta=15]$
$-\frac{2}{3}$		$(1,-4,3)$	0	$D(2,1,\alpha)$ $[\alpha=?]$
$-\frac{1}{2}$		$(-1,6,-4)$	1	$\mathfrak{sl}_2$ $[l=15]$
0	16	$(0,2,-1)$	1	$\mathfrak{sl}_2^{\oplus 4}$, $\mathfrak{sl}_2$ $[l=3]$, Triv $[\beta=0]$, $\mathfrak{osp}_n$ $[n=4]$
1	42	$(1,6,-2)$	5	$\mathfrak{osp}_n$ $[n=-8]$
2	70	$(1,4,-1)$	4	$\mathfrak{sl}_n$ $[n=8]$
4	128	$(2,6,-1)$	7	$\mathfrak{osp}_n$ $[n=16]$
∞		$(1,2,0)$	3	Excpt $[\mu=0]$