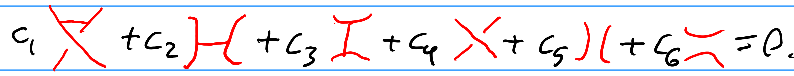
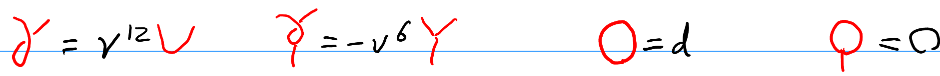
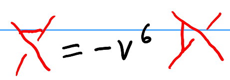
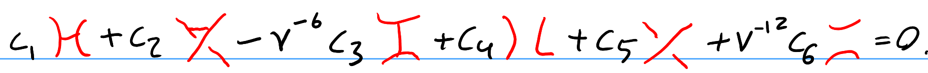
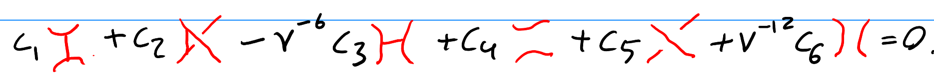
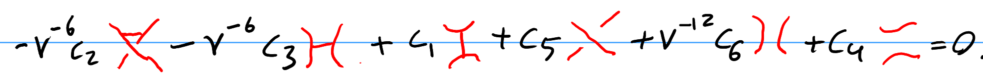


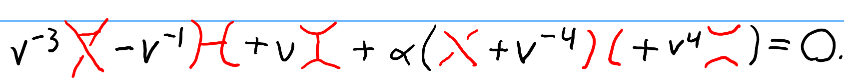


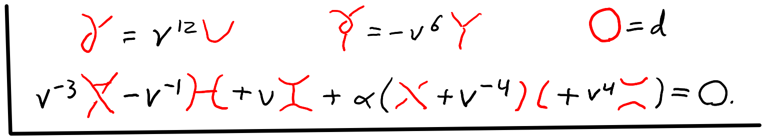




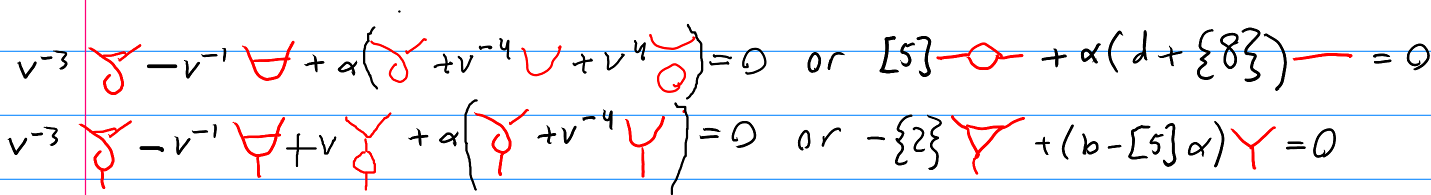


(A1) 

(IHX) 

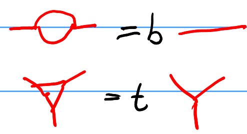


[illegible]



[illegible]

[illegible]



$$b = -\frac{\alpha\,(d + \{8\})}{[5]}$$

$$t = \frac{b - [5]\alpha}{\{2\}}$$

[illegible]

$$d = -\frac{\{2\}[\lambda+5][\lambda-6]}{[\lambda][\lambda-1]}$$

$$[k\lambda + l] := w^k v^l - w^{-k} v^{-l}$$

$$\{k\lambda + l\} := w^k v^l + w^{-k} v^{-l}$$

$$b = \frac{\{\lambda+2\}\{\lambda-3\}[3]}{[1]}$$

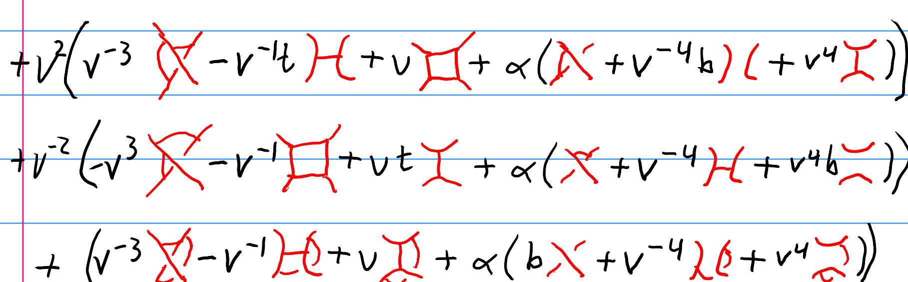
$$\alpha = -\frac{[\lambda][\lambda-1]}{[1]}$$

$$t = \{1\}\left((v+v^{-1})\frac{w^2}{v} + (v^4 - v^2 - 1 - v^{-2} + v^{-4}) + (v+v^{-1})\frac{v}{w^2}\right)$$

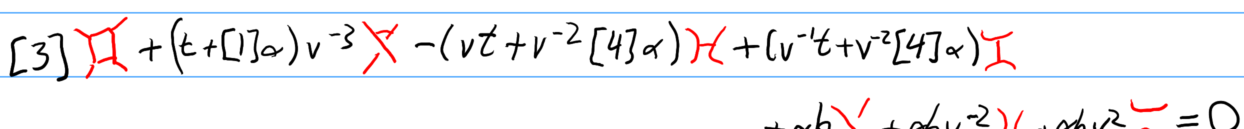
[illegible]

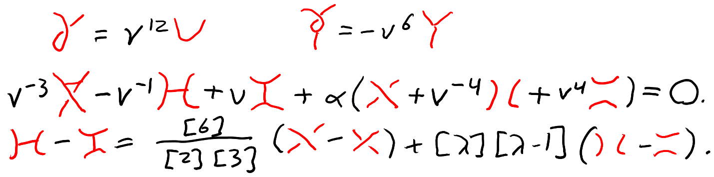
[illegible]

[illegible]



[illegible]

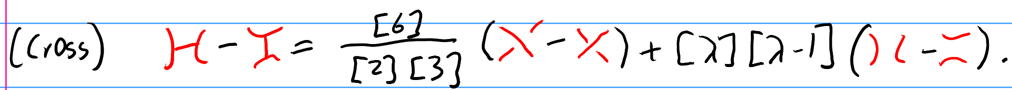


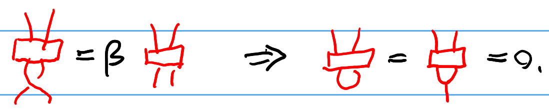


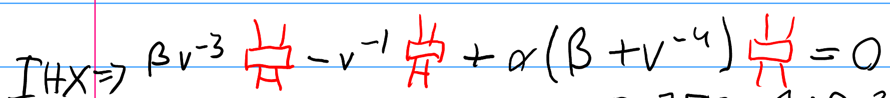
$$d = -\frac{\{2\}[\lambda+5][\lambda-6]}{[\lambda][\lambda-1]}$$

$$b = \frac{\{\lambda+2\}\{\lambda-3\}[3]}{[1]}$$

$$\alpha = -\frac{[\lambda][\lambda-1]}{[1]}$$







$$\gamma\left(v - \beta v^{-1}\right) = -\frac{[\lambda][\lambda-1](\beta v^2 + v^{-2})}{[1]}$$

$$\text{Cross} \Rightarrow \gamma = \frac{v^2 - 1 + v^{-2}}{[1]}\left(\beta - \beta^{-1}\right) + [\lambda][\lambda-1]$$

$\beta = -1$ $\quad\gamma = [\lambda][\lambda-1]$

$\beta = w^2$ $\quad\gamma = \frac{[\lambda]\{\lambda+2\}}{[1]}$

$\beta = \frac{v^2}{w^2}$ $\quad\gamma = -\frac{[\lambda-1]\{\lambda-3\}}{[1]}$

(a) $v \leftrightarrow v^{-1}$ $\quad w \leftrightarrow w$

(b) $v \leftrightarrow v$ $\quad w \leftrightarrow \frac{v}{w}$

Under (b), $[\lambda] \leftrightarrow -[\lambda-1]$

$\gamma' = v^{12}\cup \qquad \gamma = -v^{6}\,Y$

$v^{-3} - v^{-1} + v + \alpha(+ v^{-4})(+ v^{4}) = 0.$

$ - = \frac{[6]}{[2][3]}(-) + [\lambda][\lambda-1](\ -).$

$d = -\frac{\{2\}[\lambda+5][\lambda-6]}{[\lambda][\lambda-1]}$

$b = \frac{\{\lambda+2\}\{\lambda-3\}[3]}{[1]}$

$\alpha = -\frac{[\lambda][\lambda-1]}{[1]}$

The characteristic equation is then

$$(+ 1)(w^{-1} - w)(\tfrac{w}{v} - \tfrac{v}{w}) = v^{18}p + v^{9}q$$

Capping off gives

$$(v^{12}+1)(w^{-1}v^{12}-w)(\tfrac{w}{v}v^{12}-\tfrac{v}{w}) = v^{18}pd$$

$$p = \frac{\{6\}(-[\lambda-6])[\lambda+5]}{d} = \frac{[\lambda][\lambda-1]\{6\}}{\{2\}}$$

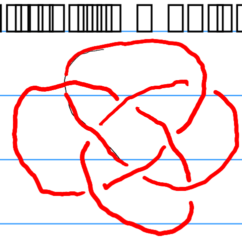
$$(-v^{6}+1)(-w^{-1}v^{6}-w)(-\tfrac{w}{v}v^{6}-\tfrac{v}{w}) = v^{9}qb$$

$$q = \frac{-[3]\cdot-\{\lambda-3\}\cdot-\{\lambda+2\}}{b} = -[1]$$

so we find

$$\frac{1}{v} - \left(\frac{w^2}{v} - \frac{1}{v} + \frac{v}{w^2}\right) - \left(\frac{w^2}{v} - v + \frac{v}{w^2}\right) + v = v^{18}\frac{[\lambda][\lambda-1]\{6\}}{\{2\}} - v^{9}[1].$$

8_{18}:



$$= v^{12}\qquad = -v^{6}$$

$$v^{-3} - v^{-1} + v + \alpha(+ v^{-4}) (+ v^{4}) = 0.$$

$$ - = \frac{[6]}{[2][3]}(-) + [\lambda][\lambda-1](-).$$

$$d = -\frac{\{2\}[\lambda+5][\lambda-6]}{[\lambda][\lambda-1]}$$

$$b = \frac{\{\lambda+2\}\{\lambda-3\}[3]}{[1]}$$

$$\alpha = -\frac{[\lambda][\lambda-1]}{[1]}$$

$$\frac{1}{v} - \left(\frac{w^2}{v} - \frac{1}{v} + \frac{v}{w^2}\right) - \left(\frac{w^2}{v} - v + \frac{v}{w^2}\right) + v = v^6 \frac{[\lambda][\lambda-1]\{6\}}{\{2\}} d + v^3 [1] b d$$

$$\mathrm{Hopf} = vd\left(v^{-12}\left(\frac{w^2}{v} - \frac{1}{v} + \frac{v}{w^2}\right) + d\left(\frac{w^2}{v} - v + \frac{v}{w^2}\right) - v^{13} + v^6 \frac{[\lambda][\lambda-1]\{6\}}{\{2\}} + v^3[1]b\right)$$

```
(%i90) hopf : v*d*(v^(-12)*(w^2/v - 1/v + v/w^2) + d*(w^2/v - v + v/w^2) - v^13
+ v^6*qlam(1,0)*qlam(1,-1)*qtwo(6)/qtwo(2) + v^3*qint(1)*b);
```

(%o90) $\Big(v\,(-v^2$

$-1/(v^2))\ (w/(v^6)-(v^6)/w)\ (v^5\,w-1/(v^5\,w))\ \Big(((-v^2-1/(v^2))\ (w/(v^6)-(v^6)/w)\ (v^5\,w$

```
(%i92) factor(ratexpand(hopf));
```

(%o92)
$$-\big((v^4+1)(w-v^6)(w+v^6)(v^5w-1)(v^5w+1)(v^{22}w^8-v^{12}w^8-v^{10}w^8+w^8 -v^{24}w^6-v^{22}w^6+v^{20}w^6-v^{16}w^6+2v^{12}w^6-v^8w^6+v^4w^6-v^2w^6-w^6+v^{26}w^4 +2v^{24}w^4-v^{20}w^4+v^{18}w^4+v^{16}w^4-2v^{14}w^4-2v^{12}w^4+v^{10}w^4+v^8w^4-v^6w^4 +2v^2w^4+w^4-v^{26}w^2-v^{24}w^2+v^{22}w^2-v^{18}w^2+2v^{14}w^2-v^{10}w^2+v^6w^2-v^4w^2 -v^2w^2+v^{26}-v^{16}-v^{14}+v^4)\big)\Big/\Big(v^{24}(w-1)^2w^2(w+1)^2(w-v)^2(w+v)^2\Big)$$

```
(%i93) factor(ratexpand(hopf/d));
```

(%o93)
$$(v^{22}w^8-v^{12}w^8-v^{10}w^8+w^8-v^{24}w^6-v^{22}w^6+v^{20}w^6-v^{16}w^6+2v^{12}w^6 -v^8w^6+v^4w^6-v^2w^6-w^6+v^{26}w^4+2v^{24}w^4-v^{20}w^4+v^{18}w^4 +v^{16}w^4-2v^{14}w^4-2v^{12}w^4+v^{10}w^4+v^8w^4-v^6w^4+2v^2w^4+w^4 -v^{26}w^2-v^{24}w^2+v^{22}w^2-v^{18}w^2+2v^{14}w^2-v^{10}w^2+v^6w^2-v^4w^2 -v^2w^2+v^{26}-v^{16}-v^{14}+v^4)/(v^{12}(w-1)w^2(w+1)(w-v)(w+v))$$

```
(%i94)
```

One more small consequenel: the eigenvector with eigenvalue -1 is

$$\frac{1}{v} - \{2\lambda-1\} + v - \frac{[\lambda][\lambda-1]}{\{2\}} + \frac{[1]}{[3]}$$