**CS3431-A19 Wong**

**Assignment 5: Functional Dependencies and Normalization**

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**Problem 1 (50 Points)**

For the relational schema given below and its corresponding functional dependencies (FDs):

R(A, B, C, D, E ) S = { AC 🡪 B, BC 🡪 E, C🡪 A, E 🡪 D } answer the following questions:

1. (10 Points) find all candidate keys of the relation through an exhaustive set of attribute closures. Specify when an attribute set closure is trivial.

**A+ = trivial**

**B+ = trivial**

***C+ = {A, B, C, D, E}***

**D+ = trivial**

**E+ = {D, E}**

**AB+ = trivial**

***AC+ = {A, B, C, D, E}***

**AD+ = trivial**

**AE+ = {A, D, E}**

***BC+ = {A, B, C, D, E}***

**BD+ = trivial**

**BE+ = {B, D, E}**

***CD+ = {A, B, C, D, E}***

***CE+ = {A, B, C, D, E}***

**DE+ = trivial**

***ABC+ = {A, B, C, D, E}***

**ABD+ = trivial**

**ABE+ = {A, B, E, D}**

***ACD+ = {A, B, C, D, E}***

***ACE+ = {A, B, C, D, E}***

**ADE+ = trivial**

***BCD+ = {A, B, C, D, E}***

***BCE+ = {A, B, C, D, E}***

***CDE+ = {A, B, C, D, E}***

***ABCD+ = {A, B, C, D, E}***

***ABCE+ = {A, B, C, D, E}***

***ACDE+ = {A, B, C, D, E}***

***BCDE+ = {A, B, C, D, E}* ABDE+ = trivial**

**ABCDE+ = trivial**

**The only candidate key (minimal super key) is C**

**All attribute closures that cover all 5 attributes contain C, and if any attribute is removed from that closure besides C then nothing changes**

1. (5 Points) Given the keys you defined in step a, find the FDs in order that they appear in S that violate BCNF.

**AC 🡪 B - Does not violate BCNF (AC is super key)**

**BC 🡪 E - Does not violate BCNF (BC is super key)**

**C🡪 A - Does not violate BCNF (C is super key)**

**E 🡪 D – Violates BCNF because E is not a super key**

1. (20 Points) Decompose the relations to satisfy BCNF always using the left-most FD violation. For example, if AC 🡪 B is in BCNF but the other three FDs are in violation, you would use BC 🡪 E for the decomposition. Specify which FD is used to make the decomposition. If there is multi-step decomposition, then indicate each step along with which FD is used for the decomposition.

**Decomposing E -> D**

**E+ = {D, E}**

**R1 = {D, E} R2 = {A, B, C, E}**

**Closures for R1:**

**E+ = {D,E}**

**D+ = trivial**

**DE+ = trivial**

**Key = {E}**

**SR1 = {E->D}**

**2 attribute relation R1 is in BCNF**

**R2 = {A, B, C, E}**

**Closures for R2:**

**A+ = trivial**

**B+ = trivial**

***C+ = ABCE***

**E+ = trivial**

**AB+ = trivial**

**AC+ = ABCE**

**AE+ = trivial**

**BC+ = ABCE**

**BE+ = trivial**

**CE+ = ACE**

**ABC+ = ABCE**

**ABE+ = ABE**

**ACE+ = ABCE**

**BCE+ = ABCE**

**ABCE+ = trivial**

**C->ABE**

**All other FDs that can be made (i.e. AC->BE) are redundant**

**Key = {C}**

**SR2 = {C->ABE}**

**R2 in BCNF because SR2 = C->ABE & C is a candidate key**

**Final = (DE)(ABCE)**

1. (5 Points) If the FDs in S are not in 3NF, calculate a minimal basis for the FDs. If the FDs in S are already in 3NF, explain why.

**AC** 🡪 **B - FD in 3NF (AC is super key)**

**BC** 🡪 **E - FD in 3NF (BC is super key)**

**C**🡪 **A - FD in 3NF (C is super key)**

**E** 🡪 **D – FD not in 3NF (E is not super key and FD is not prime)**

**Minimal basis calculations:**

**Single attribute on the right side – S = { AC** 🡪 **B, BC** 🡪**E, C**🡪 **A, E** 🡪 **D}**

**Remove non-minimal FDs:**

**Remove AC** 🡪 **B, AC+ = trivial**

**Remove BC** 🡪**E, BC+ = ABC**

**Remove C**🡪 **A, C+ = trivial**

**Remove E** 🡪 **D, E+ = trivial**

**S remains unchanged**

**Remove unnecessary left side attributes:**

**Remove A🡪B from AC 🡪 B, C+ = ABCDE**

**Remove C->B, A+ = trivial**

**Remove B->E from BC** 🡪**E, C+ = ABCDE**

**Remove C->E, B+ = trivial**

**Since XY->Z and X->Y implies X->Z**

**AC->B becomes C->B, BC->E becomes C->E**

**S = { C->B, C->E, C->A, E->D }**

**Minimal basis = {C->ABE, E->D}**

1. (10 points) Decompose the relation R to satisfy 3NF

**Single attribute on the right side – S = { AC** 🡪 **B, BC** 🡪**E, C**🡪 **A, E** 🡪 **D }**

**Remove non-minimal FDs:**

**Remove AC** 🡪 **B, AC+ = trivial**

**Remove BC** 🡪**E, BC+ = ABC**

**Remove C**🡪 **A, C+ = trivial**

**Remove E** 🡪 **D, E+ = trivial**

**S remains unchanged**

**Remove unnecessary left side attributes:**

**Remove A🡪B from AC 🡪 B, C+ = ABCDE**

**Remove C->B, A+ = trivial**

**Remove B->E from BC** 🡪**E, C+ = ABCDE**

**Remove C->E, B+ = trivial**

**Minimal basis = {C->ABE, E->D}**

**(ABCE)(DE)**

**Remove subsets:**

**Schema = (ABCE)(DE)**

**Problem 2 (50 Points)**

For the relational schema given below and its corresponding functional dependencies (FDs):

R(A, B, C, D, E ) S = { A 🡪 E, BCE 🡪 A, D 🡪 C } answer the following questions:

1. (10 Points) find all candidate keys of the relation R through an exhaustive set of attribute closures. Specify when an attribute set closure is trivial.

**A = E**

**B = trivial**

**C = trivial**

**D = CD**

**E =trivial**

**AB = ABE**

**AC =trivial**

**AD = ACDE**

**AE = trivial**

**BC = trivial**

**BD = BCD**

**BE = trivial**

**CD =trivial**

**CE =trivial**

**DE = CDE**

**ABC = ABCE**

***ABD = ABCDE***

**ABE = trivial**

**ACD = ACDE**

**ACE = trivial**

**ADE = ACDE**

**BCD = trivial**

**BCE = ABCE**

***BDE = ABCDE***

**CDE = trivial**

**ABCD =ABCDE**

**ABCE = trivial**

**ABDE = ABCDE**

**ACDE = trivial**

**BCDE =ABCDE**

**Candidate keys are {ABD} and {BDE}**

1. (2 points) List the dependencies, in the order given in S, that violate **BCNF**.

**A->E, Violates BCNF (A not a super key)**

**BCE -> A, Violates BCNF (BCE is not a super key)**

**D-> C, Violates BCNF (D is not a super key)**

1. (20 points) If R is not in **BCNF**, provide decomposition into multiple relations where each one is in BCNF. For each decomposition step, use the left-most FD violation following the FD order given in S. For example, if A 🡪 E is in BCNF but the other two FDs are in violation, then you would use BCE 🡪 A for the decomposition. Make sure to specify which FD is used to make the decomposition.

**Decomposing A 🡪 E**

**A+ = AE**

**Therefore, R1(A, E) and R2(A, B, C, D)**

**R1(A, E) is in BCNF because it is a two-attribute relation**

**R2(A, B, C, D) with D 🡪 C because it is the only FD that does not contain E**

**A+ = trivial AB+ =trivial BD+ = BCD ABCD+ = trivial**

**B+ = trivial AC+ = trivial CD+ = trivial**

**C+ = Trivial AD+ = ACD ABC+ = trivial**

**D+ = CD BC+ = trivial ABD+ = ABCD**

**ABD is candidate key of R2**

**Only FD we have is D->C**

**So R3(C, D) and R4(A, B, D)**

**R3 is a two-attribute relation so it is in BCNF**

**With no FDs remaining (since C is not in R4) all closures of R4 will be trivial**

**Thus R4 is also in BCNF.**

**(ABD)(CD)(AE)**

1. (5 points) Why is R NOT in 3NF? Explain your answer.

**R is not in 3NF because in the functional dependency D 🡪 C, D is not a super key and the FD is not prime, C is not part of either candidate key: ABD or BDE.**

1. (8 points) Calculate a minimal basis for R.

**A->E BCE -> A D-> C**

**All FDs have singular RHS**

**All FDs violate BCNF, Calculate closures for them**

**A+ = AE**

**BCE+ = ABCE**

**D+ = CE**

**Remove A 🡪 E, Calculate A+ with Other two FDs**

**A+ = trivial**

**Remove BCE 🡪 A, Calculate BCE+**

**BCE+ = trivial**

**Remove D 🡪 C, Calculate D+**

**D+ = trivial**

**The only FD with more than one left side attribute is BCE 🡪 A.**

**Remove B. With original FDs, CE+ = trivial**

**Remove C. With original FDs, BE+ = trivial**

**Remove E. With original FDs, BC+ = trivial**

**Because A is not in any of the three closures, no attributes can be removed.**

**The minimal basis is the same as the original FDs: {A->E, BCE -> A, D-> C}**

1. (5 points) Decompose the relation R to satisfy 3NF.

**Minimal Key = {A->E, BCE -> A, D-> C}**

**R1(A, E) ,R2(A, B, C, E), R3(C, D)**

**Neither Candidate Key, ABD or BDE, appears in the listed relations so we need to add one of them as R4**

**Thus, the final decomposition is R1(A, E), R2(B, C, E), R3(D), R4(A, B, D)**