## This work in under 60 seconds

## Selling points

- First methodology with theoretical guarantees
- Significantly faster than bootstrap and fully Bayesian approaches
- Easy to implement
- Easily adapted to new statistical tasks

## Set up

$$\begin{tikzcd}
 & \text{Data 1st half} \arrow[r] & \text{DNN} \arrow[r] & \{\hat{\phi}_j\}_{j=1}^K \arrow[d] \\
\text{Data} \arrow[ur] \arrow[dr] & & \text{Prior}: w_j \sim \Pi \arrow[r] & \sum_{j=1}^K w_j \hat{\phi}_j \text{[redacted]} \arrow[d] \\
 & \text{Data 2nd half} \arrow[rr] & & \text{Posterior } \Pi(..|\text{Data}) \text{[redacted]}
\end{tikzcd}$$

## Uncertainty quantification

Credible balls with suitably blown up radius give coverage

- $\log(n)$ in simulations
- $\log(n)^3$ in theory

## Conclusion

Empirical Bayesian deep neural networks provide a great way to do uncertainty quantification.

# Fast uncertainty quantification in Deep learning

## Introduction

We provide the first methodology for uncertainty quantification using deep neural networks and a theoretical study.

## Earlier results

- Johannes
- Suzuki
- Judith + Botond

## Assumptions

- True function is $\beta$-smooth
- Have $k_n = n^{1/(2\beta+d)}$ basis functions
- Have found a good (local) optimizer of the deep neural network
- Sparse deep neural networks[1]
- Near orthogonal basis functions[2]

[1]Needed in theory to get best rates

[2]Needed in theory to get control on radius of credible ball

## Simulation studies

Simulations were done using dense neural networks and gradient descent

**Regression** **Classification**

## Theoretical guarantees for regression

Denote $\epsilon_n = n^{\frac{-\beta}{2\beta+d}}$ and $\tilde{\epsilon_n} = \epsilon_n \log(n)^3$.

**Theorem 0.1.** *Let $\beta, M > 0$. Under some conditions*

- *The posterior contracts at near minimax rate:*
$$\limsup_{n\to\infty} \sup_{f_0 \in W_M^\beta} \mathbb{E}_{f_0}\left(\Pi\left(\|f - f_0\|_2 \geq M_n \tilde{\epsilon}_n | \mathcal{D}_n\right)\right) = 0,$$
*for all $M_n \to \infty$.*
- *The credible balls have uniform near optimal coverage: There exists $L_{\epsilon,\alpha}$ such that if $B(c_\alpha, R_\alpha)$ is an $\alpha$-credible ball, the ball $B = B(c_\alpha, L_{\epsilon,\alpha}\log(n)^3 R_\alpha)$ satisfies*
$$\liminf_n \inf_{f_0 \in W_M^\beta([0,1]^d)} \mathbb{P}_{f_0}^{(n)}(f_0 \in B) \geq 1 - \epsilon$$
- *The credible balls have uniform near optimal size:*
$$\liminf_n \inf_{f_0 \in W_M^\beta([0,1]^d)} \mathbb{P}_{f_0}^{(n)}(R_\alpha \leq C\epsilon_n) \geq 1 - \epsilon$$
*for some large enough $C > 0$.*