

Frequentist coverage guarantees for Empirical Bayesian deep neural networks

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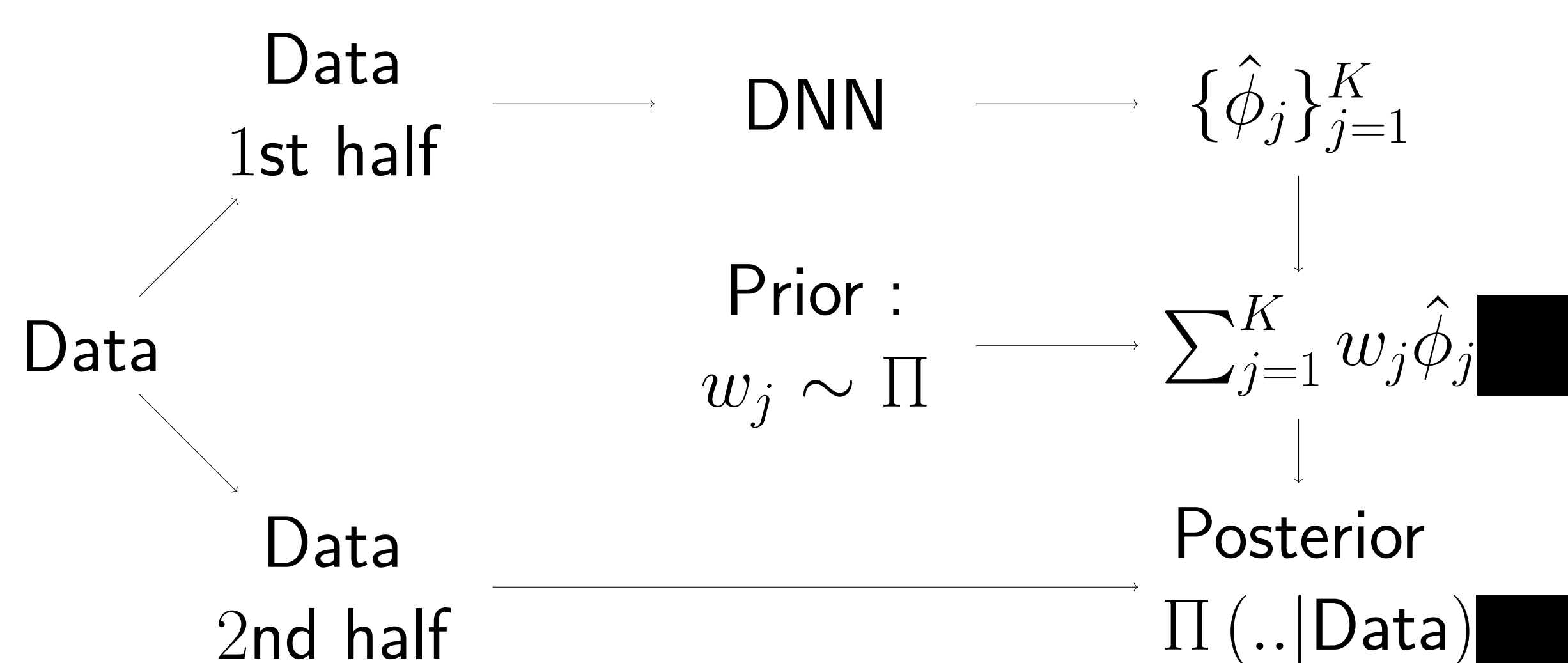
Introduction

We provide the first methodology for uncertainty quantification using deep neural networks with theoretical guarantees.

Selling points

- First methodology with theoretical guarantees
- Significantly faster than bootstrap and fully Bayesian approaches
- Easy to implement
- Easily adapted to new statistical tasks

Set up



Earlier results on convergence rates

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Assumptions

- True function is β -smooth
- Have $k_n = n^{1/(2\beta+d)}$ basis functions
- Have found a good (local) optimizer of the deep neural network
- Sparse deep neural networks*
- Near orthogonal basis functions*

* Needed in theory to get the best bounds

Simulation studies

Regression

Classification

Theoretical guarantee for regression

Denote $\epsilon_n = n^{-\frac{\beta}{2\beta+d}}$ and $\tilde{\epsilon}_n = \epsilon_n \log(n)^3$.

Theorem 1. Let $\beta, M > 0$. Under some conditions

- The posterior contracts at near minimax rate:

$$\limsup_{n \rightarrow \infty} \sup_{f_0 \in W_M^\beta} \mathbb{E}_{f_0} (\Pi(\|f - f_0\|_2 \geq M_n \tilde{\epsilon}_n | \mathcal{D}_n)) = 0,$$

for all $M_n \rightarrow \infty$.

- The credible balls have uniform near optimal coverage: There exists $L_{\epsilon, \alpha}$ such that if $B(c_\alpha, R_\alpha)$ is an α -credible ball, the ball $B = B(c_\alpha, L_{\epsilon, \alpha} \log(n)^3 R_\alpha)$ satisfies

$$\liminf_n \inf_{f_0 \in W_M^\beta([0,1]^d)} \mathbb{P}_{f_0}^{(n)}(f_0 \in B) \geq 1 - \epsilon$$

- The credible balls have uniform near optimal size:

$$\liminf_n \inf_{f_0 \in W_M^\beta([0,1]^d)} \mathbb{P}_{f_0}^{(n)}(R_\alpha \leq C \epsilon_n) \geq 1 - \epsilon$$

for some large enough $C > 0$.

Conclusion

Empirical Bayesian deep neural networks provide a great way to do uncertainty quantification.

