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At least and ignorance: A reply to Coppock & Brochhagen 2013*

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Abstract This commentary revisits Coppock & Brochhagen's 2013 account of speaker ignorance inferences associated with *at least*. The general principle that Coppock & Brochhagen propose to derive inferences about the speaker's information state, the Maxim of Interactive Sincerity, is shown to not fully derive the intended ignorance inferences. Amendments to Coppock & Brochhagen's proposal are discussed, but an account of the relevant inferences in terms of Gricean quantity implicature, as proposed in Büring 2008 and subsequent work, emerges as more parsimonious.

Keywords: ignorance inferences, *at least*, quantity implicature, inquisitive semantics

1 Introduction

Sentences with unembedded *at least* are known to give rise to inferences of speaker ignorance (e.g., Krifka 1999, Büring 2008). For example, sentence (1) is judged to convey speaker ignorance about the exact number of fallen apples.

(1) At least two apples fell.

Coppock & Brochhagen discard Büring's 2008 analysis of such ignorance inferences in terms of Gricean quantity implicature. They offer an alternative

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account within the framework of inquisitive semantics (Ciardelli, Groenendijk & Roelofsen 2011). Central to this account is a novel pragmatic principle, Coppock & Brochhagen's *Maxim of Interactive Sincerity*. I will show, however, that the Maxim of Interactive Sincerity does not actually derive the requisite ignorance inferences. Therefore, the utility of this principle is at present unclear, as is the rationale for preferring Coppock & Brochhagen's account to one in terms of Gricean quantity implicature.[1]

Section 2 outlines the accounts of Coppock & Brochhagen (2013) and Büring (2008); section 3 provides brief comparison of frameworks for the two accounts, after embedding Büring's analysis in a neo-Gricean setting; section 4 compares the inferences derived under the two accounts; section 5 examines the application of Coppock & Brochhagen's account to disjunction; section 6 briefly explores possible amendments to Coppock & Brochhagen's analysis; section 7 sums up the findings.

2 Two accounts of ignorance inferences with *at least*

Büring's and Coppock & Brochhagen's accounts posit the same semantic information content for sentence (1): the proposition that more than one apple fell, abbreviated here as **[2,...)**. Both accounts assume the Gricean quality inference in (2), that is, the inference that the speaker's information state s entails the semantic information content **[2,...)**.[2]

(2) $s \subseteq$ **[2,...)**

Coppock & Brochhagen subscribe to certain assumptions of inquisitive semantics (Ciardelli, Groenendijk & Roelofsen 2011). The denotation of a sentence ϕ, $[\![\phi]\!]$, is assumed to be a set of propositions. Coppock & Brochhagen propose that the denotation of an *at least* sentence is a plural set, that is, a set containing more than one proposition. The denotation of (1) is assumed to be the plural, in fact infinite, set of propositions in (3).

1 This commentary is solely concerned with Coppock & Brochhagen's proposal about ignorance inferences with *at least*. Their paper also makes important contributions regarding the focus sensitivity and syntactic distribution of *at least*, which are not discussed here. Also, to keep the exposition concise, the discussion will focus on cases where *at least* modifies a numeral; and while Coppock & Brochhagen discuss both *at least* and *at most*, attention will here be confined to *at least*.

2 The speaker's information state s is taken to be a proposition (a set of possible worlds), which can be thought of as the conjunction of all the propositions that the speaker believes to be true.

(3) $[\![\text{at least two apples fell}]\!] = \{\mathbf{[2,...)}, \mathbf{[3,...)}, \mathbf{[4,...)}, \ldots\}$

The semantic information content of a sentence ϕ, its truth-conditional meaning, is recovered as the union of all the propositions in $[\![\phi]\!]$. As intended, this assigns to (1) the semantic information content **[2,...)**.

Within the inquisitive semantics framework, an innovation that Coppock & Brochhagen 2013 introduce is a condition intended to derive ignorance inferences: the *Maxim of Interactive Sincerity* (*MIS*). This condition relates sentence denotations to the speaker's information state s. According to the MIS, conjoining the propositions in a plural set denotation with s must preserve plurality, yielding more than one non-contradictory proposition. Based on the auxiliary definition of *restriction* in (4), this can be stated as in (5).[3]

(4) Restriction
$Q/p := \{p \cap q\colon\ q \in Q \wedge p \not\subseteq \neg q\}$

(5) Maxim of Interactive Sincerity (MIS)
$\big|[\![\phi]\!]\big| > 1 \rightarrow \big|[\![\phi]\!]/s\big| > 1$

Given the plural set denotation in (3), the MIS derives for sentence (1) the condition on the speaker's information state s in (6).

(6) $\big|\{\mathbf{[2,...)}, \mathbf{[3,...)}, \mathbf{[4,...)}, \ldots\}/s\big| > 1$

Coppock & Brochhagen effectively prove that (6) entails (7), where **[m]** stands for the proposition that exactly m apples fell. So the account correctly derives for (1) the intended inference that the speaker is ignorant about the exact number of apples that fell.

(7) $\forall m[s \not\subseteq \mathbf{[m]}]$

Coppock & Brochhagen consider the possibility that bare numeral sentences are semantically ambiguous between weak, one-sided, and strong, two-sided, meanings. If so, their account assigns to *Two apples fell*, for example, the two possible set denotations $\{\mathbf{[2,...)}\}$ and $\{\mathbf{[2]}\}$. Correspondingly,

3 In (4), Q and p are any set of propositions and any proposition, respectively. The restriction operator / maps Q and p to another set of propositions. (4) simplifies Coppock & Brochhagen's more complex definition of set restriction: $Q/p := \{\emptyset\}$ if $\{p \cap q\colon\ q \in Q\} = \{\emptyset\}$, and $Q/p := \{p \cap q\colon\ q \in Q \,\&\, p \not\subseteq \neg q\}$ otherwise. Since the case $\{p \cap q\colon\ q \in Q\} = \{\emptyset\}$ does not arise in the applications discussed here, adoption of the simplified definition is innocuous.

sentence (1) will in addition to (3) be assigned the possible two-sided set denotation in (8).

(8) $[\![\text{at least two apples fell}]\!] = \{\mathbf{[2]}, \mathbf{[3]}, \mathbf{[4]}, \ldots\}$

However, Coppock & Brochhagen suggest that this alternative set denotation supports the same pragmatic inferences as (3), and indeed the results for (3) reported above carry over to (8), as the reader is invited to verify.

Coppock & Brochhagen offer the account outlined above — call it the *CB analysis* — as an alternative to the account of Büring 2008. Under Büring's account, an utterance of (1) leads the listener to draw the two inferences in (9), that is, the inferences that the speaker's information state entails neither **[2]** nor **[3,...)**.

(9) a. $s \nsubseteq \mathbf{[2]}$
b. $s \nsubseteq \mathbf{[3,...)}$

Büring proposes that these inferences are Gricean quantity implicatures. Büring does not actually embed this proposal in a general or fully articulated theory of quantity implicature (see Section 3 below). However, given that both **[2]** and **[3,...)** are semantically stronger than **[2,...)**, the inferences in (9) can be attributed to the familiar rationale grounded in the Gricean maxims of quantity and quality (e.g., Gamut 1991, Sauerland 2004, Fox 2007, Geurts 2011).[4]

In conjunction with the quality inference (2), the quantity inferences in (9) entail the two possibility inferences in (10), that is, the inferences that the speaker's information state is compatible with both **[2]** and **[3,...)**.

(10) a. $s \nsubseteq \neg\mathbf{[2]}$
b. $s \nsubseteq \neg\mathbf{[3,...)}$

The quantity and possibility inferences taken together amount to a pair of ignorance inferences about **[2]** and **[3,...)**, i.e., $s \nsubseteq \mathbf{[2]}$ & $s \nsubseteq \neg\mathbf{[2]}$ and $s \nsubseteq$

4 In the terminology of Sauerland 2004, the inferences in (9) are *primary* quantity implicatures. In a familiar elaboration of the neo-Gricean approach (e.g., Gazdar 1979, Soames 1982, Horn 1989, Sauerland 2004, Fox 2007, Spector 2006, Geurts 2011, Schwarz 2016) a primary implicature $s \nsubseteq p$ is under certain conditions strengthened to the corresponding *secondary* implicature $s \subseteq \neg p$. This strengthening, however, is assumed to be preempted by the possibility inference $s \nsubseteq \neg p$ (Sauerland 2004). In the case at hand, the possibility inferences in (10) contradict, and can therefore be assumed to preempt, the potential secondary inferences $s \subseteq \neg\mathbf{[2]}$ and $s \subseteq \neg\mathbf{[3,...)}$.

[3,...) & $s \not\subseteq \neg$**[3,...)**. As intended, these imply that the speaker's information does not determine the exact number of fallen apples.

3 Comparison of frameworks

To derive quantity implicatures like those in (9) for sentences like (1), Büring suggested that the theory of quantity implicature applies to *at least* sentences in much the same way it applies to disjunctions. Büring in fact seemed to propose that at some level of syntactic description, *at least* sentences *are* disjunctions, hence that the analysis of (1) can be fully reduced to the analysis of a disjunctive paraphrase like *Two apples fell or more than two apples fell*. Coppock & Brochhagen argue that this suggestion is inconsistent with reasonable assumptions about the architecture of grammar. Also, given that Büring does not articulate a general theory of quantity implicature and merely characterizes the intended effect of such a theory for disjunctions, Coppock & Brochhagen rightly note that "Büring's implicature schema applies only to disjunctions, while the [Maxim of Interactive Sincerity] applies to all kinds of interactive content."

It is hard to disagree with Coppock & Brochhagen's objections to Büring's specific proposal. However, Büring's particular treatment is not to be equated with the more general idea that the use of *at least* gives rise to the relevant quantity implicatures. In fact, recent literature presents natural neo-Gricean elaborations of the quantity based approach to *at least* that derives the quantity inferences in (9) and that is immune to Coppock & Brochhagen's criticism (Cummins & Katsos 2010, Schwarz & Shimoyama 2011, Schwarz 2013, Mayr 2013, Kennedy 2015, Schwarz 2016). At the heart of these elaborations is the neo-Gricean notion of a set of *alternatives*, a set of propositions that is determined by grammar and that serves as an input to the calculation of quantity implicatures (Horn 1972, Hirschberg 1991, Katzir 2007). Without implying that *at least* sentences are disjunctions at any syntactic level, the accounts in question capture the effect of Büring's analysis by positing suitable alternatives for the purposes of implicature calculation. The familiar Gricean rationale then yields the inferences that the speaker's beliefs do not entail alternatives that are semantically stronger than the asserted meaning (e.g., Gamut 1991, Sauerland 2004, Fox 2007, Geurts 2011). For sentence (1), this derives the intended quantity inferences (9) from the assumption that (1) has a pair of alternatives denoting the propositions **[2]** and **[3,...)**, given that these are semantically stronger than the asserted proposition **[2,...)**.

I will in the following refer to the neo-Gricean, quantity based, elaboration of Büring's account as the *Q analysis*. I note that, in terms of the underlying architecture of grammar assumed, the CB analysis and the Q analysis are quite similar. Central to both accounts is the assumption that the calculation of ignorance inferences in cases like (1) makes reference to a set of propositions — a set of alternatives for implicature calculation in one case, an inquisitive denotation of (1) in the other; as well, both accounts posit operations that take these sets as inputs to derive inferences about the speaker's beliefs — a standard Gricean algorithm in one case, the Maxim of Interactive Sincerity in the other.

It is not obvious, then, that framework considerations can provide an argument for one of the two accounts over the other.[5] At the same time, while Coppock & Brochhagen take both accounts to derive "ignorance inferences", they do not actually provide a thorough assessment and comparison of the inferences they derive. The remainder of this commentary demonstrates that the two accounts do not derive the same inferences, and it briefly explores possible consequences of this finding.

4 Comparison of inferences derived

Both the CB analysis and the Q analysis have been shown to derive for (1) an inference of speaker ignorance regarding the question how many apples fell. Granting this shared feature, what remains to be examined is the precise logical relation between the inferences derived in the two analyses. The following compares the complete pragmatic meanings of (1) under the two accounts, that is, the conjunction of the quality inference with the epistemic inference intended to introduce ignorance. Under the Q analysis, the complete pragmatic meaning of (1) is (11).

(11) Complete pragmatic meaning of (1) under the Q analysis:
$s \subseteq \mathbf{[2,...)}\ \&\ s \nsubseteq \mathbf{[2]}\ \&\ s \nsubseteq \mathbf{[3,...)}$

5 Conceivably, such an argument might refer to the different ways a sentence's semantic information content is encoded in the two frameworks. Under the Q analysis, semantic rules directly deliver the semantic information content, while in inquisitive semantics the rules instead output a set denotation, from which the semantic information content can be recovered. Inquisitive semantics allows for operations that map set denotations to set denotations, operations that do not have obvious counterparts under a Q analysis. Coppock & Brochhagen's exhaustification operation introduced in Section 5 below is a case in point.

As noted above, Coppock & Brochhagen establish that the condition in (6) (repeated below) entails (7) (also repeated below). In addition, it can be shown that (7) in conjunction with the quality inference (2) entails (6). Under the CB analysis, then, the complete pragmatic meaning of (1) is (12).

(6) $|\{\mathbf{[2,...)}, \mathbf{[3,...)}, \mathbf{[4,...)}, \ldots\}/s| > 1$

(7) $\forall m[s \not\subseteq \mathbf{[m]}]$

(12) Complete pragmatic meaning of (1) under the CB analysis:
$s \subseteq \mathbf{[2,...)} \,\&\, \forall m[s \not\subseteq \mathbf{[m]}]$

The logical relation between the two predicted complete pragmatic meanings is clarified by the equivalences in (13) and (14).

(13) $s \subseteq \mathbf{[2,...)} \,\&\, s \not\subseteq \mathbf{[2]} \,\&\, s \not\subseteq \mathbf{[3,...)}$ iff
$s \subseteq \mathbf{[2,...)} \,\&\, \exists m, n[m \neq n \,\&\, m = 2 \,\&\, s \not\subseteq \neg\mathbf{[m]} \,\&\, s \not\subseteq \neg\mathbf{[n]}]$

(14) $s \subseteq \mathbf{[2,...)} \,\&\, \forall m[s \not\subseteq \mathbf{[m]}]$ iff
$s \subseteq \mathbf{[2,...)} \,\&\, \exists m, n[m \neq n \,\&\, s \not\subseteq \neg\mathbf{[m]} \,\&\, s \not\subseteq \neg\mathbf{[n]}]$

The equivalences in (13) and (14) establish that (11) is strictly stronger than (12). The pragmatic meaning derived under the Q analysis entails the pragmatic meaning derived under the CB analysis, but not vice versa. Under the Q analysis, but not the CB analysis, the complete pragmatic meaning of (1) entails $s \not\subseteq \neg\mathbf{[2]}$ in addition to $s \not\subseteq \mathbf{[2]}$. Hence only the Q analysis derives an ignorance inference about **[2]**. In fact, while (12) entails that there is some number m such that the speaker is ignorant about **[m]**, there is no particular number m such that (12) entails the speaker's ignorance about **[m]**. For that matter, (12) does not entail an ignorance inference about any proposition, that is, does not entail any inference of the form $s \not\subseteq p \,\&\, s \not\subseteq \neg p$. Instead, (12) encodes ignorance only in the sense of entailing that the speaker fails to know the exhaustive answer (in the sense of Groenendijk & Stokhof 1984) to a relevant question, viz., the question how many apples fell.

Yet the additional inference $s \not\subseteq \neg\mathbf{[2]}$ derived under the Q analysis is in accordance with intuitions: (1) is not merely perceived to imply that the speaker fails to know the exact number of fallen apples, but moreover is judged to convey that the speaker cannot exclude that that number is two, and hence fails to know whether that number is two. Accordingly, a use of (1) would be infelicitous in a scenario where the speaker believes that more than two apples fell, even if she had no further beliefs about the number of fallen apples. Under the unembellished CB analysis, this judgment is not accounted for.

The next section shows that the CB analysis also does not fully derive attested ignorance inferences for disjunctions, a classic test case for speaker ignorance inferences (e.g., Gazdar 1979, Simons 2000, Zimmermann 2000, Sauerland 2004, Alonso-Ovalle 2006, Katzir 2007, Geurts 2011, Meyer 2013).

5 Applying Coppock & Brochhagen's account to disjunction

Matrix disjunctions are known to introduce ignorance inference about the disjuncts (e.g., Gazdar 1979). For example, (15) carries the ignorance inferences in (16), implying the the speaker is uncertain about whether Ann snored and also about whether Bill snored. Likewise the three-part disjunctions in (17) is judged to give rise to the three speaker ignorance inferences in (18), one for each disjunct.

(15) Ann or Bill snores.

(16) a. $s \nsubseteq \mathbf{a} \;\&\; s \nsubseteq \neg\mathbf{a}$
b. $s \nsubseteq \mathbf{b} \;\&\; s \nsubseteq \neg\mathbf{b}$

(17) Ann or Bill or Chris snores.

(18) a. $s \nsubseteq \mathbf{a} \;\&\; s \nsubseteq \neg\mathbf{a}$
b. $s \nsubseteq \mathbf{b} \;\&\; s \nsubseteq \neg\mathbf{b}$
c. $s \nsubseteq \mathbf{c} \;\&\; s \nsubseteq \neg\mathbf{c}$

Such ignorance inferences with disjunctions are well-studied and derived in familiar neo-Gricean accounts in terms of quantity implicature (e.g., Sauerland 2004, Spector 2006). In the following, I will extend the use of the term *Q analysis* to include (some version of) these accounts of ignorance inferences with disjunctions. The Q analysis assumes that the alternatives for quantity implicature with disjunctions include the propositions expressed by the individual disjuncts as well as all possible disjunctions of these propositions. For the two-part disjunction in (15), the Q analysis accordingly yields the quantity inferences $s \nsubseteq \mathbf{a}$ and $s \nsubseteq \mathbf{b}$; in conjunction with the quality inference $s \subseteq \mathbf{a} \cup \mathbf{b}$, these quantity inferences entail the possibility inferences $s \nsubseteq \neg\mathbf{a}$ and $s \nsubseteq \neg\mathbf{b}$, and hence the ignorance inferences in (16). For the three-part disjunction in (17), the Q analysis delivers the inferences $s \nsubseteq \mathbf{a} \cup \mathbf{b}$, $s \nsubseteq \mathbf{a} \cup \mathbf{c}$, and $s \nsubseteq \mathbf{b} \cup \mathbf{c}$, as well as $s \nsubseteq \mathbf{a}$, $s \nsubseteq \mathbf{b}$, and $s \nsubseteq \mathbf{c}$; in conjunction with the quality inference $s \subseteq \mathbf{a} \cup \mathbf{b} \cup \mathbf{c}$, these entail the possibility inferences $s \nsubseteq \neg\mathbf{a}$, $s \nsubseteq \neg\mathbf{b}$, and $s \nsubseteq \neg\mathbf{c}$, hence the ignorance inferences in (18).

Coppock & Brochhagen follow Roelofsen & van Gool 2010 in assigning to a disjunction a set denotation containing a proposition for each disjunct. The disjunctive sentences in (15) and (17) are accordingly assigned the set denotations in (19). Since the information content associated with a set denotation is assumed to be given by the disjunction of its members, (15) and (17) encode the standard weak information contents $\mathbf{a} \cup \mathbf{b}$ and $\mathbf{a} \cup \mathbf{b} \cup \mathbf{c}$, respectively.[6]

(19) a. $[\![$Ann or Bill snores$]\!] = \{\mathbf{a}, \mathbf{b}\}$
b. $[\![$Ann or Bill or Chris snores$]\!] = \{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$

Coppock & Brochhagen use disjunction to illustrate the inquisitive semantics framework. However, while they note that the MIS is a fully general principle that is applicable to any set of propositions, they do not explore the application of the MIS to disjunction.

The MIS applied to (19a) yields the condition about the speaker's information state in (20). (Note that the conjunct $s \cap \mathbf{a} \neq s \cap \mathbf{b}$ ensures that $\mathbf{a}$ and $\mathbf{b}$ are not equivalent relative to the speaker's information state s and hence that $\{\mathbf{a}, \mathbf{b}\}/s$ is not reduced to a singleton.)

(20) $s \nsubseteq \neg\mathbf{a} \,\&\, s \nsubseteq \neg\mathbf{b} \,\&\, s \cap \mathbf{a} \neq s \cap \mathbf{b}$

The condition in (20) does not by itself entail the ignorance inferences in (16). Even in conjunction with the quality inference in (21), (20) does not entail these ignorance inferences, as the conjunction of (20) and (21) is consistent with both $s \subseteq \mathbf{a}$ and $s \subseteq \mathbf{b}$.

(21) $s \subseteq \mathbf{a} \cup \mathbf{b}$

However, independently of the MIS, Coppock & Brochhagen also propose that set denotations are subject to an exhaustification operation that derives standard instances of scalar implicature, and in particular derives so-called exclusive disjunction meanings. Coppock & Brochhagen take the content of this strengthening to be determined by the so-called Question under Discussion (Roberts 1996/2012), also a set of propositions. Exhaustification conjoins

6 Preceding the inquisitive semantics literature that Coppock & Brochhagen credit for the proposal, the idea that disjunctions denote sets of propositions given by the disjuncts is employed in Simons 2005, Alonso-Ovalle 2006, and Aloni 2007.

propositions in a set denotation with the negations of propositions in the Question under Discussion, provided such conjunction is non-contradictory. In the cases considered here, the Question under Discussion can be identified with the very set that serves as the denotation of the asserted sentence. Exhaustification then derives an exclusive meaning for (15) by virtue of mapping (19a) to $\{\mathbf{a} \cup \neg\mathbf{b}, \mathbf{b} \cap \neg\mathbf{a}\}$. The corresponding information content, the disjunction of the propositions in this set, has the intended exclusivity entailment $\neg(\mathbf{a} \cap \mathbf{b})$.

Assuming exhaustification, the quality inference in (21) is to be supplemented with the epistemic exclusivity inference in (22). Together with this inference, (20) and (21) entail (23), hence the intended ignorance inferences in (16).[7]

(22) $s \subseteq \neg(\mathbf{a} \cap \mathbf{b})$

(23) $s \nsubseteq \mathbf{a} \;\&\; s \nsubseteq \mathbf{b}$

So the familiar ignorance inferences attested for disjunctive sentences like (19) can be derived under the CB analysis, but only if supplemented with the exclusivity inference. This contrasts with the Q analysis, where ignorance inferences arise independently of exclusivity (e.g., Sauerland 2004, Spector 2006, Fox 2007, Geurts 2011). This contrast invites one to consider disjunctions that are interpreted without exhaustification, that is, disjunctions that do not receive an exclusive interpretation. The extension of sentence (15) shown in (24) is a case in point.

(24) Ann or Bill snores — perhaps both do.

The *perhaps both do* continuation in (24) can be judged as consistent with the preceding disjunction. If so, the disjunction is apparently interpreted inclusively, hence without exhaustification. Under the CB analysis, this predicts that the disjunction also need not be associated with an ignorance inference: as noted above, the inference derived by the MIS alone is consistent with both $s \subseteq \mathbf{a}$ and $s \subseteq \mathbf{b}$. However, intuitions about (24) indicate that there the ignorance inference persists. Just like (15), (24) suggests that the speaker

7 It is unclear to me whether Coppock & Brochhagen intend the MIS to apply to exhaustified denotations. If so, then (20) should be replaced with the stronger inference that the MIS delivers for $\{\mathbf{a} \cup \neg\mathbf{b}, \mathbf{b} \cap \neg\mathbf{a}\}$. However, this replacement would not actually strengthen the overall pragmatic meaning given by (20), (21), and (22), as the reader is invited to verify. The same comment applies to the discussion below of the three-part disjunction in (17).

knows neither whether Ann snored nor whether Bill did. Apparently, then, the continuation *perhaps both do* does not remove the ignorance inferences about the disjuncts alongside the exclusivity inference. Given this, a quantity based analysis of ignorance inferences with disjunctions seems preferable to one based on an analysis in terms of the MIS.

The difference between the CB analysis and the Q analysis becomes yet more pronounced in relation to three-part disjunctions such as (17). The MIS applied to (19b) delivers the inferences in (25a). Unsurprisingly, given the above findings about (15), the conjunction of (25a) with the quality inference in (25b) is not strong enough to entail any of the ignorance inferences in (18). Moreover, in this case even the addition of an exclusivity inference is insufficient to derive ignorance. Exhaustification derives the exclusivity inference by virtue of mapping (19b) to $\{\mathbf{a}\cap\neg(\mathbf{b}\cup\mathbf{c}), \mathbf{b}\cap\neg(\mathbf{a}\cup\mathbf{c}), \mathbf{c}\cap\neg(\mathbf{a}\cup\mathbf{b})\}$, ensuring that the information content of (17) entails $\neg(\mathbf{a}\cap\mathbf{b})\cap\neg(\mathbf{a}\cap\mathbf{c})\cap\neg(\mathbf{b}\cap\mathbf{c})$. This adds to (25a) and (25b) the additional epistemic inference (25c). However, even the conjunction of all the inferences in (25) fails to entail any of the ignorance inferences in (18): while this conjunction entails $s \nsubseteq \mathbf{a}$, $s \nsubseteq \mathbf{b}$, and $s \nsubseteq \mathbf{c}$, it fails to entail any of the possibility inferences $s \nsubseteq \neg\mathbf{a}$, $s \nsubseteq \neg\mathbf{b}$, and $s \nsubseteq \neg\mathbf{c}$.

(25) a. $(s \nsubseteq \neg\mathbf{a} \,\&\, s \nsubseteq \neg\mathbf{b} \,\&\, s\cap\mathbf{a} \neq s\cap\mathbf{b}) \vee$
$(s \nsubseteq \neg\mathbf{a} \,\&\, s \nsubseteq \neg\mathbf{c} \,\&\, s\cap\mathbf{a} \neq s\cap\mathbf{c}) \vee$
$(s \nsubseteq \neg\mathbf{b} \,\&\, s \nsubseteq \neg\mathbf{c} \,\&\, s\cap\mathbf{b} \neq s\cap\mathbf{c})$

b. $s \subseteq \mathbf{a}\cup\mathbf{b}\cup\mathbf{c}$

c. $s \subseteq \neg(\mathbf{a}\cap\mathbf{b})\cap\neg(\mathbf{a}\cap\mathbf{c})\cap\neg(\mathbf{b}\cap\mathbf{c})$

For example, the information state $s = (\neg\mathbf{a})\cap(\mathbf{b}\cup\mathbf{c})\cap\neg(\mathbf{b}\cap\mathbf{c})$ meets all of the conditions in (25), and yet $s \subseteq \neg\mathbf{a}$ ensures that (18a) is false. Likewise, states can be constructed to demonstrate that the conditions in (25) do not entail (18b) or (18c), either. It is apparent, then, that even assuming exhaustification, the MIS falls short of deriving ignorance inferences for disjunctions in the general case.[8]

8 Note that the discussion of *at least* in Section 4 did not take into account the possibility of exhaustification. But this omission is inconsequential under Coppock & Brochhagen's assumptions. For bare numerals, exhaustification is intended to derive the strong, two-sided meanings that might alternatively be attributed to a lexical ambiguity. For *Two apples fell*, for example, exhaustification maps the set denotation {**[2,...)**} to {**[2]**}. (This assumes that the Question under Discussion for exhaustification is in this case not given by the set denotation {**[2,...)**}, but by the set of propositions corresponding to the interrogative *How many apples*

6 Quantity inferences in inquisitive semantics?

The CB analysis has been shown to fall short of fully deriving attested inferences about the speaker's information state associated with *at least* sentences and disjunctions, inferences that are captured correctly under the Q analysis. I will briefly examine some conceivable revisions to the CB analysis that could be entertained to address these shortcomings.

Section 4 established that the unembellished CB analysis fails to derive for sentence (1) the attested possibility inference $s \not\subseteq \neg\mathbf{[2]}$. To be sure, this leaves open the possibility that the missing inference is captured under a suitably amended version of the CB analysis. Under what may be the most obvious amendment, the CB analysis is supplemented with a version of neo-Gricean quantity implicature. Capitalizing on the similarity of the frameworks highlighted in section 3, this could be done within inquisitive semantics by letting the propositions in a set denotation supply alternatives for the purpose of implicature calculation. Specifically, as stated in (26), suppose that an utterance of ϕ supports the inference that there is no subset of $[\![\phi]\!]$ such that the disjunction of its members is (i) semantically stronger than the disjunction of the members of $[\![\phi]\!]$ itself, and (ii) entailed by the speaker's information state.

(26) Inquisitive Quantity Implicature (IQ)
$s \not\subseteq \bigcup A$, for any $A \subseteq [\![\phi]\!]$ such that $\bigcup A \subset \bigcup[\![\phi]\!]$

Assuming the set denotation in (3), IQ derives for (1) the inference that the speaker's information state does not entail the disjunction of the propositions in $\{\mathbf{[3,...)}, \mathbf{[4,...)}, \ldots\}$, that is, the inference $s \not\subseteq \mathbf{[3,...)}$. In conjunction with the quality inference $s \subseteq \mathbf{[2,...)}$, this yields the missing possibility inference $s \not\subseteq \neg\mathbf{[2]}$. This amended version of the CB analysis — call it the *CB-IQ analysis* — thereby matches the effect of the Q analysis of *at least* presented in sections 2 and 3.[9]

fell?.) Similarly, for the *at least* case in (1), Coppock & Brochhagen take exhaustification to map the set of weak, one-sided, propositions in (3) to the set of strong, two-sided, propositions in (8). As already noted in Section 2, under the CB analysis, the move from (3) to (8) leaves the information content and pragmatic inferences unaltered. Exhaustification, then, cannot be credited for delivering the missing possibility inference identified in Section 4.

9 Applied to the two-sided set denotation (8), IQ derives stronger ignorance inferences. For example, IQ would deliver both $s \not\subseteq \mathbf{[4]}$ and $s \not\subseteq \mathbf{[2]} \cup \mathbf{[3]} \cup \mathbf{[5,...)}$, which in conjunction with the quality inference $s \subseteq \mathbf{[2,...)}$ entail an ignorance inference about $\mathbf{[4]}$. More generally, relative to (8), IQ derives an inference of total speaker ignorance about the number of fallen apples,

The CB-IQ analysis also derives the intended ignorance inferences for disjunctions from the set denotations introduced in section 5. For the two-part disjunction in (15), IQ yields the quantity inferences $s \nsubseteq \mathbf{a}$ and $s \nsubseteq \mathbf{b}$; for the three-part disjunction in (17), IQ delivers the inferences $s \nsubseteq \mathbf{a} \cup \mathbf{b}$, $s \nsubseteq \mathbf{a} \cup \mathbf{c}$, and $s \nsubseteq \mathbf{b} \cup \mathbf{c}$, as well as $s \nsubseteq \mathbf{a}$, $s \nsubseteq \mathbf{b}$, and $s \nsubseteq \mathbf{c}$. These are the very inferences already listed in section 5 in sketching the Q analysis of disjunction. As noted there, these inferences in conjunction with the relevant quality inferences entail the ignorance inferences in (16) and (18). Notably, just like under the Q analysis, these results under the CB-IQ analysis are not dependent on exhaustification, which in view of the discussion in section 5 is a welcome result. The CB-IQ analysis, then, succeeds at matching all the intended effects of the Q analysis presented above.

However, the CB-IQ analysis can be questioned on conceptual grounds. While the Q analysis provides parallel, quantity based, derivations of ignorance inferences with *at least* and disjunctions, the CB-IQ analysis appeals to a notion of quantity implicature *in addition* to the MIS. All other things being equal, a quantity based account that steers clear of additional principles like the MIS seems preferable on the grounds of theoretical parsimony. Note also that under the CB-IQ analysis, the MIS has no role to play in the derivation of ignorance inferences which disjunctions, as these follow from IQ and quality inferences alone. So, while Coppock & Brochhagen emphasize the general applicability of the MIS, presently the only known case where its application might do any work seems to be the one case for which it was specifically designed, viz., the case of ignorance inferences with superlative modifiers like *at least*. So it seems appropriate to question the role of MIS in a general theory of epistemic inferences.

It is therefore worth observing that in a further revision of the CB analysis within the inquisitive semantics framework, call it the *IQ analysis*, the MIS can be eliminated altogether. This can be done without losing an account of ignorance inferences with *at least*, viz., by simultaneously revising the set denotations of *at least* sentences. Abandoning (3) and (8), suppose that (1) instead has the set denotation in (27).

(27) $[\![$at least two apples fell$]\!] = \{\mathbf{[2]}, \mathbf{[3,\ldots)}\}$

modulo the belief that it was more than one. As discussed in Schwarz 2016, this seems too strong. For example, a speaker could use (1) felicitously even while believing that the number of fallen apples is not four. The adoption of IQ, therefore, is not consistent with maintaining the two-sided set denotation in (8). By the same token, since exhaustification maps (3) to (8), it would have to be assumed that IQ cannot apply to the output of exhaustification.

This revision preserves the semantic information content of sentence (1) encoded by (3) and (8), as the disjunction the two propositions in (27) is again **[2,...)**. Moreover, IQ derives the quantity inferences $s \not\subseteq$ **[2]** and $s \not\subseteq$ **[3,...)**. In conjunction with the quality inference $s \subseteq$ **[2,...)**, these entail the possibility inferences $s \not\subseteq \neg$**[2]** and $s \not\subseteq \neg$**[3,...)**, hence the intended ignorance inferences about **[2]** and **[3,...)**. Within the inquisitive semantics framework, then, the IQ analysis closely mimics the workings of Q analysis, deriving the very same inferences.[10,11]

7 Conclusion

Coppock & Brochhagen present an intriguing new approach to ignorance inferences with *at least*. Central to this approach is their Maxim of Interactive Sincerity, a novel pragmatic principle couched in the framework of inquisitive semantics. But close inspection of its predictions reveals that the Maxim of Interactive Sincerity does not actually derive all the desired inferences for *at least* sentences, and also falls short of deriving ignorance inferences for

10 As a reviewer points out, however, the adoption of (27) is not consistent with Coppock & Brochhagen's assumption about exhaustification. Relative to the Question under Discussion given by the interrogative *How many apples fell?*, exhaustification would map (27) to {**[2]**, **[3]**}. This derives the information content **[2]** $\cup$ **[3]**, predicting that (1) can be read as conveying that exactly two or three apples fell. As Mayr (2013) notes in a somewhat different context, this sort of interpretation seems unavailable.

11 Coppock & Brochhagen report that Pruitt & Roelofsen (2011) propose a condition similar to the MIS, called the *Maxim of Attentive Sincerity* (*MAS*), which requires the speaker's information state to be compatible with each proposition in the set denotation of an asserted sentence. In conjunction with the quality inference, the MAS derives the intended ignorance inference from (27) (given that the members of (27) are mutually exclusive). But like the MIS, the MAS does not derive the intended ignorance inference from the set denotations for disjunctions given in (19). In addition, Coppock & Brochhagen themselves observe that applied to $\{\mathbf{a}, \mathbf{a} \cap \mathbf{b}\}$, the MAS merely derives the inference $s \not\subseteq \neg(\mathbf{a} \cap \mathbf{b})$, which is consistent with $s \subseteq \mathbf{a} \cap \mathbf{b}$; given the set denotations Coppock & Brochhagen posit, the MAS thereby fails to derive attested ignorance inference for sentences like *At least Ann snores.* For other set denotations, the MAS delivers inferences that seem too strong (cf. Schwarz 2016). Applied to the infinite set denotation in (3), the MAS requires that the speaker's information state not place any upper bound on the number of fallen apples. Applied to the alternative, two-sided, set denotation in (8), the MAS derives an even stronger inference, viz., an implication of total speaker ignorance regarding the number of fallen apples, modulo the assumption that it was more than one. More generally, applied to sets of mutually exclusive propositions, the MAS has the very same effect as the IQ, described in footnote 9, deriving inferences of total speaker ignorance, modulo the spreader's belief that the asserted semantic information content holds true.

disjunctions. In the end, an account in terms of Gricean quantity implicature of the kind proposed in previous work, whether couched in inquisitive semantics or otherwise, emerges as a more parsimonious analytical option.

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