KEY FOR THE ASSIGNMENT

1. a. $A \cup (B \cap A') = A \cup B$

Proof.

$$\begin{aligned} A \cup (B \cap A') &= (A \cup B) \cap (A \cup A') && \text{by Distributive Property of } \cup \text{ over } \cap \\ &= (A \cup B) \cap U && \text{by Complement Law} \\ &= A \cup B && \text{by Identity Property} \end{aligned}$$

b. $(A \cap B) \cap (A' \cup B') = \varnothing$

Proof.

$$\begin{aligned} (A \cap B) \cap (A' \cup B') &= ((A \cap B) \cap A') \cup ((A \cap B) \cap B') && \text{by Distributive Property of } \cap \text{ over } \cup \\ &= ((B \cap A) \cap A') \cup ((A \cap B) \cap B') && \text{by Commutative Property of } \cap \\ &= (B \cap (A \cap A')) \cup (A \cap (B \cap B')) && \text{by Associative Property of } \cap \\ &= (B \cap \varnothing) \cup (A \cap \varnothing) && \text{by Complement Law} \\ &= \varnothing \cup \varnothing && \text{by Zero Property} \\ &= \varnothing && \text{by Identity Property} \end{aligned}$$

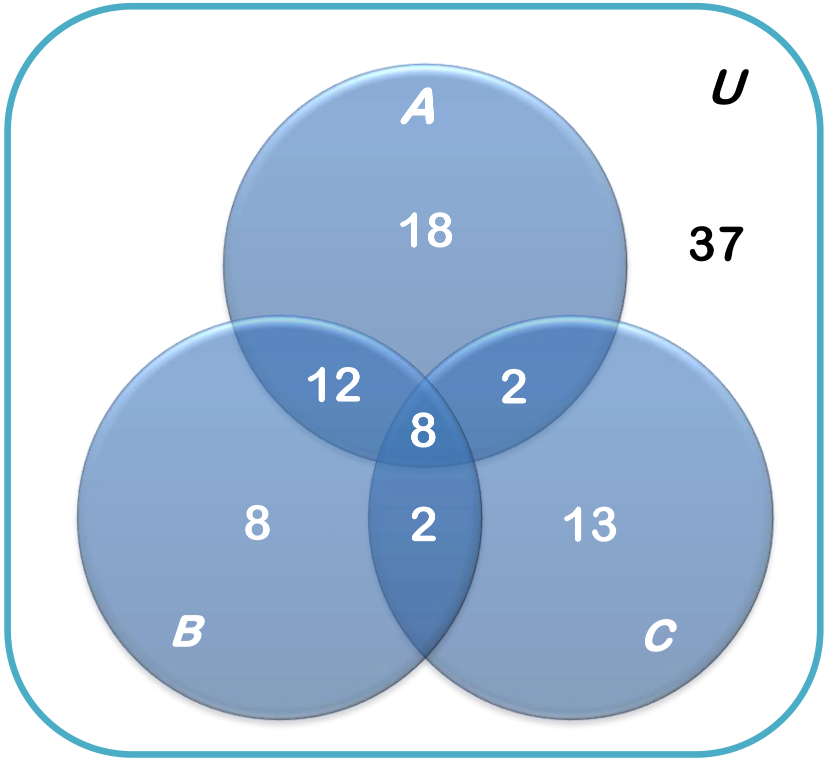
c. $(A \cup B) \cap (A' \cup B') = (A \cap B') \cup (A' \cap B)$

Proof.

$$\begin{aligned} (A \cup B) \cap (A' \cup B') &= ((A \cup B) \cap A') \cup ((A \cup B) \cap B') && \text{by Distributive Property of } \cap \text{ over } \cup \\ &= ((A \cap A') \cup (B \cap A')) \cup ((A \cap B') \cup (B \cap B')) && \text{by Distributive Property of } \cap \text{ over } \cup \\ &= (\varnothing \cup (B \cap A')) \cup ((A \cap B') \cup \varnothing) && \text{by Complement Law} \\ &= (B \cap A') \cup (A \cap B') && \text{by Identity Property} \\ &= (A \cap B') \cup (B \cap A') && \text{by Commutative Property of } \cup \\ &= (A \cap B') \cup (A' \cap B) && \text{by Commutative Property of } \cap \end{aligned}$$

2. a.

Let U be the set of the 100 respondents (students) in the survey.
A be the set of students who preferred MATH 1
B be the set of students who preferred MATH 2
C be the set of students who preferred NASC 3



b. 20 students preferred MATH 1 but not MATH 2.

c. 20 students preferred MATH 2 but not NASC 3.

d. 13 students preferred NASC 3 only.

e. 37 students did not prefer any of the three courses.

3. We know that if we have two sets, A_1 and A_2,

$$n(A_1 \cup A_2) = n(A_1) + n(A_2) - n(A_1 \cap A_2) = \sum_{i=1}^{2} n(A_i) - \sum_{1 \le i < j \le 2} n(A_i \cap A_j)$$

And if we have three sets, A_1, A_2, and A_3,

$$n(A_1 \cup A_2 \cup A_3) = n(A_1) + n(A_2) + n(A_3) - n(A_1 \cap A_2) - n(A_1 \cap A_3) - n(A_2 \cap A_3) + n(A_1 \cap A_2 \cap A_3) = \sum_{i=1}^{3} n(A_i) - \sum_{1 \le i < j \le 3} n(A_i \cap A_j) + \sum_{1 \le i < j < m \le 3} n(A_i \cap A_j \cap A_m).$$

We can generalize the formula for the cardinality of the union of k sets, $k \ge 4$, into

$$n(A_1 \cup A_2 \cup \ldots \cup A_k) = \sum_{i=1}^{k} n(A_i) - \sum_{1 \le i < j \le k} n(A_i \cap A_j) + \sum_{1 \le i < j < m \le k} n(A_i \cap A_j \cap A_m) - \ldots + (-1)^{k-1} n(A_1 \cap A_2 \cap \ldots \cap A_k)$$

where

$\sum_{i=1}^{k} n(A_i)$ is the sum of the cardinalities of all the k sets,

$\sum_{1 \le i < j \le k} n(A_i \cap A_j)$ is the sum of the cardinalities of all the $\frac{k(k-1)}{2}$ intersections of any pair of distinct sets,

$\sum_{1 \le i < j < m \le k} n(A_i \cap A_j \cap A_m)$ is the sum of the cardinalities of all the $\frac{k(k-1)(k-2)}{6}$ intersections of any group of three distinct sets, and

$n(A_1 \cap A_2 \cap \ldots \cap A_k)$ is the cardinality of the intersection of all the k sets.

4. $(A \cap B) \times C = (A \times C) \cap (B \times C)$

Proof.

If $x \in (A \cap B) \times C$, $\quad x = (u, v)$ where $u \in A \cap B$ and $v \in C$.
$u \in A \cap B$ means that $u \in A$ and $u \in B$.

Since $u \in A$ and $v \in C$, we can say that $x = (u, v)$ is an element of $A \times C$.
Moreover, since $u \in B$ and $v \in C$, $x = (u, v)$ is also an element of $B \times C$.

Hence, $x \in (A \times C) \cap (B \times C)$.

On the other hand, if $x \in (A \times C) \cap (B \times C)$, we can say that x is an element of both $A \times C$ and $B \times C$.
By saying that $x \in A \times C$, we mean that $x = (u, v)$ where $u \in A$ and $v \in C$.

And since $x = (u, v)$ is also an element of $B \times C$, u must also be an element of B.
Thus, it follows that $u \in A \cap B$; and therefore, $x = (u, v)$ is also an element of $(A \cap B) \times C$.

So now, by the definition of equal sets, we can conclude that $(A \cap B) \times C = (A \times C) \cap (B \times C)$.