Logic: Introduction

- **LOGIC** is the study of reasoning and whether or not that reasoning is correct.

- **LOGIC** is used in
  - scientific investigation
  - mathematics
  - programming
  - algorithm analysis

Logic: Introduction

- **Three Main Areas in Logic**
  - **Propositional Logic**
    - *relationships of statements*
    - *literals and terms*
    - *use of conjunctions*
  - **Predicate Logic**
    - *content of statements*
    - *use of quantifiers*
  - **Fuzzy Logic**

Propositional Logic

- **OUTLINE**
  - **Propositions and Logical Connectives**
  - Syntax and Semantics of Propositional Logic
  - Rules of Inference
  - Laws of Equivalence
  - Method of Proof

Propositional Logic

- **Basic Terms and Concepts**

  DEFN: A **proposition** is a *declarative* sentence of which the *truth value is definitely known* or can be *validly determined*.
- ***Examples*:**

  1. Hello.
  2. When is our next exam?
  3. Dr. Albacea is the current ICS director.
  4. $2 + 3 > 5$
  5. $x+1$ is an even number.
  6. He is a CMSC 56 student.
  7. I will pass this course.

Propositional Logic

- **Assumptions:**

  - **Law of Excluded Middle**

    For every proposition P, either P is true or P is false.

  - **Law of Contradiction**

    For every proposition P, it is not the case that P is both true and false.

Propositional Logic

- ***Notation:***

  truth values: **T** *=true;* **F**= *false*
  propositions: ***P***, ***Q***, ***R***, *etc.*
  compound propositions: ***E***, ***F***, ***G***, ***H***, *etc.*

- ***Examples***:

  P: Today is a sunny day.
  Q: CMSC 56 is a prerequisite to CMSC 57.

Propositional Logic

- **LOGICAL CONNECTIVES** are used to modify or combine two or more propositions to form compound propositions (or sentences).
  - **NOT** ($\sim$)
  - **AND** ($\wedge$)
  - **OR** ($\vee$)
  - **IF-THEN** ($\rightarrow$)
  - **IF-AND-ONLY-IF** ($\leftrightarrow$)

Propositional Logic

- **NOT** ($\sim$)

~P means "**not P**"
means "the opposite or negation of P"

Example:
P: *The printer is working today*.
~P:

Rule:
The proposition ~P is true iff P is false.
The proposition ~P is false iff P is true.

Truth table:

P	**~P**
T	
F	

Propositional Logic

NOTE:
As a convention, propositional variables unmodified by a NOT connective should represent English statements expressing the *affirmative*, that is, they do not include words that express negation such as "not", "no" or "none".

Example:
"Sabrina is *not* a student."
is preferably represented by **~S**.
(that is, **S** represents "Sabrina is a student.")

Propositional Logic

- **AND** ($\wedge$)

$P \wedge Q$ means "**both P and Q**"

Example:
P: *It is raining right now*. Q: *It is cold right now*.
$P \wedge Q$:

Rule:
The **conjunction** $P \wedge Q$ is true iff both P and Q are true.

Truth table:

P	**Q**	**P ∧ Q**
T	T	
T	F	
F	T	
F	F	

Propositional Logic

- **OR** ($\vee$)

$P \vee Q$ means "**P or Q**", "**at least one of P and Q**"

Example:
P: *Michael is late for class*. Q: *Michael is sick today*.
$P \vee Q$:

Rule:
The **disjunction** $P \vee Q$ is true iff at least one of P or Q is true.

Truth table:

P	**Q**	**P ∨ Q**
T	T	
T	F	
F	T	
F	F	

Propositional Logic

- **IF-THEN** ($\rightarrow$)

$P \rightarrow Q$ means "**if P then Q**", "**P implies Q**", "**Q if P**"

- In $P \rightarrow Q$, P is called the **premise** and Q is called the **conclusion**.
- If $P \rightarrow Q$ is an **implication** then:
  - Its **inverse** is $\sim P \rightarrow \sim Q$
  - Its **converse** is $Q \rightarrow P$
  - Its **contrapositive** is $\sim Q \rightarrow \sim P$

Example:
P: I become UP president. Q: I will reduce tuition fees.
$P \rightarrow Q$:

Propositional Logic

- **IF-THEN** ($\rightarrow$)

  Rule:

  The **implication** or **conditional** $P \rightarrow Q$ is true iff P is false or Q is true.

  Truth table:

P	**Q**	**P → Q**
T	T	
T	F	
F	T	
F	F	

Propositional Logic

- **IF-AND-ONLY-IF** ($\leftrightarrow$)

  $P \leftrightarrow Q$ means "**P iff Q**", "**if P then Q and if Q then P**"

  Example:

  P: Jude has a fever. Q: Jude's temperature is at least 38C

  $P \leftrightarrow Q$:

  Rule:

  The **biconditional** or **equivalence** $P \leftrightarrow Q$ is true iff both P and Q have the same truth values.

  Truth table:

P	**Q**	**P ↔ Q**
T	T	
T	F	
F	T	
F	F	

Propositional Logic

- **Additional examples**:
  - Given:

    V: *Mia is watching tv.*

    S: *Mia is studying in her room.*

    E: *Mia is eating out with her friends.*
  - **V ∨ S → ~E**
  - **E ↔ ~S**

Propositional Logic

- **Additional examples**:
  - Jonna will watch a movie if she has no exam tomorrow.

    Let ***W****: Jonna will watch a movie.*

    E*: Jonna has an exam tomorrow.*
  - Harry is taking neither Arithmancy nor Divination classes.

    Let ***A****: Harry is taking Arithmancy classes.*

    D*: Harry is taking Divination classes..*

Propositional Logic

- **Other terms**:
  - A **literal** is a single propositional variable or its negation (e.g. P, ~Q).
  - A **term** is a group of two or more literals combined by a connective (e.g. $P \wedge Q$, $P \rightarrow$ ~R).

Propositional Logic

- **OUTLINE**
  - Propositions and Logical Connectives
  - **Syntax and Semantics of Propositional Logic**
  - Rules of Inference
  - Laws of Equivalence
  - Method of Proof

Propositional Logic

- **SYNTAX & SEMANTICS of Propositional Logic**
  - **Syntax**
    - T and F are valid propositions.
    - If P and Q are valid propositions, then the following are also valid propositions:
      $\sim P$, $P \wedge Q$, $P \vee Q$, $P \rightarrow Q$, and $P \leftrightarrow Q$.
    - All propositions generated from a finite number of the above are valid propositions.

Propositional Logic

- **SYNTAX & SEMANTICS of Propositional Logic**
  - **Syntax**

    Remarks on **grouping symbols**
    - Used to reduce ambiguity.

      Example:

      $[\sim P \wedge (Q \vee S)] \rightarrow (R \leftrightarrow Q)$

      is very much different from

      $\sim(P \wedge Q) \vee [S \rightarrow (R \leftrightarrow Q)]$.
    - Without grouping symbols, the connectives must be evaluated in the following order:

      $\sim \ \wedge \ \vee \ \rightarrow \ \leftrightarrow$

Propositional Logic

- **SYNTAX & SEMANTICS of Propositional Logic**
  - **Syntax**
    - ***Example***:

      The proposition $P \wedge \sim Q \vee R \rightarrow S \leftrightarrow U \vee W$ may be evaluated in the following order:

      a)

      b)

      c)

      d)

      e)

      and therefore can be rewritten, with grouping symbols, as

      $[((P \wedge (\sim Q)) \vee R) \rightarrow S] \leftrightarrow (U \vee W)$

Propositional Logic

- **SYNTAX & SEMANTICS of Propositional Logic**
  - **Syntax**
    - Similar connectives may be evaluated from left to right.

      Example:

      For the proposition $P \wedge R \wedge S$

      $P \wedge R$ can be evaluated first followed by $\wedge S$
    - Some grouping symbols may be omitted.

      Example:

      The proposition $\{\sim[P \wedge (\sim Q)]\}$

      may be rewritten simply as $\sim(P \wedge \sim Q)$.

Propositional Logic

- **SYNTAX & SEMANTICS of Propositional Logic**
  - **Semantics**
    - To determine truth value of (compound) proposition:
      - assign truth values to each component propositional variable
      - use the rules of each connective for determining the truth value of the proposition.

Propositional Logic

- **SYNTAX & SEMANTICS of Propositional Logic**
  - **Semantics**

    Examples:

    Suppose P=F(alse), Q=F(alse) and R=T(rue).

    What are the resulting truth values of the following sentences?

    1. $P \vee Q$
    2. $\sim P \wedge R$
    3. $(P \vee Q) \rightarrow \sim R$

Propositional Logic

- **Kinds of Propositional Statements**
  - A **tautology** or **valid sentence** is a sentence that is true for all possible values of its propositional variables.
    Example:
  - A **contradiction** or **absurdity** is a sentence that is false for all possible values of its propositional variables.
    Example:

Propositional Logic

- **Kinds of Propositional Statements**
  - A **contingency** is a sentence that is either true or false depending on the truth values of its propositional variables.
    Example:
  - **Can you give other examples of (simple) tautologies, contradictions and contingencies?**

Propositional Logic

- **Kinds of Propositional Statements**
  - **How to Create Truth Tables**
    Step 1: Label the first n columns of the table with the n component propositional variables. Other columns are labeled by combinations of statements culminating with the entire given statement.
    Step 2: List down all possible 2^n n-tuples of truth values for all n component propositional variables.
    Step 3: Determine the truth values for the remaining columns in the table.

Propositional Logic

- ***Examples***:
  Tautology, contradiction or contingency???
  1) F: $\mathbf{\sim Q \to \sim P}$
  2) F: $\mathbf{(P \to Q) \leftrightarrow (\sim Q \to \sim P)}$
  3) F: $\mathbf{(P \wedge Q) \wedge \sim P}$
  4) F: $\mathbf{[(P \to Q) \wedge P] \wedge \sim Q}$
  5) F: $\mathbf{P \to \sim (Q \wedge R)}$

Propositional Logic

- **Properties of Propositional Statements**
  - A statement is **satisfiable** if there is some assignment of truth values to its propositional variables such that the statement is true.
    Which of tautology, contradiction and contingency is/are satisfiable?

    Example:
    Determine if the following proposition is satisfiable:
    $\mathbf{P \wedge R \to \sim P}$

Propositional Logic

- **Properties of Propositional Statements**
  - Two propositional statements are **equivalent** if they have the same truth value under every interpretation.
    - *Use truth tables.*
    - *Use laws of equivalence (discussed later).*

    Example:
    Check if the statements
    $\mathbf{P \to Q}$ and $\mathbf{(P \wedge \sim Q) \to (R \wedge \sim R)}$
    are logically equivalent.