INFORMATION TECHNOLOGY LITERACY

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Computer Representation and Arithmetic

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Other Numbering Systems

Octal Numbering System

Hexadecimal Numbering System

Other Numbering Systems

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Octal Numbering System

- Base 8
- Based on a system with a 3-bit boundary
- Digits include 0 to 7; other digits are invalid

Hexadecimal Numbering System

- Base 16
- "Hex" meaning *six* and "dec" meaning *ten*
- Symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and A, B, C, D, E, and F

Octal Numbering System

The weighted value for each position is as follows:

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0
2 097 152	262 144	32768	4096	512	64	8	1

Convert Binary to Octal

Convert 01000101 to octal

1. Break the binary number into 3-bit sections starting from the right.

NOTE: Add 0 at the start of the sequence to make a 3-bit section

001 000 101

2. Convert the 3-bit binary number to its octal equivalent.

001 = 1

000 = 0

101= 5

Answer: 105

Convert Octal to Decimal

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Now convert 105 to decimal

1. Place each digit in their proper positions.

8^7	8^6	8^5	8^4	8^3	8^2	8^1	8^0
2 097 152	262 144	32768	4096	512	64	8	1
					1	0	5

2. Multiply the decimal equivalent by the corresponding digit of the octal number, then add this column of figures for the final solution.

(1)(5) + (8)(0) + (64)(1) = **69**

Hexadecimal Numbering System

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- Hexadecimal is more often converted to and from binary than octal.
- It’s because you can work with fewer numbers or symbols.
- The 16 symbols are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F.
- Letters A to F represent the numbers 10, 11, 12, 13, 14, and 15 respectively.

Hexadecimal Numbering System

- We can also represent the value of the hexadecimal placeholders the long way:

1 = 1s column
16×1 = 16s column
16×16 = 256s column
16×16×16 = 4096s column
16×16×16×16 = 65,536s column

- Or, we can represent the value of each placeholder column in exponential notation:

$16^0 = 1$
$16^1 = 16$
$16^2 = 256$
$16^3 = 4096$
$16^4 = 65,536$

Counting in Hexadecimal

Let's say we count in hex:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

How do we represent the number 16 and onwards?

16	1	0	24	1	8
17	1	1	25	1	9
18	1	2	26	1	A
19	1	3	27	1	B
20	1	4	28	1	C
21	1	5	29	1	D
22	1	6	30	1	E
23	1	7	31	1	F

Create another column to count with and call it the "16s" column, a "groups of 16 column".

How it works?

	16s	1s		16s	1s
16	1	0	24	1	8
17	1	1	25	1	9
18	1	2	26	1	A
19	1	3	27	1	B
20	1	4	28	1	C
21	1	5	29	1	D
22	1	6	30	1	E
23	1	7	31	1	F

Example:

To represent the number "26" in hex we take the hex symbol "A" which equals 10 and multiply it by the column value which is 1. We do the same for the column to the left multiplying 1 by its placeholder value of 16. Thus, we have 10 + 16 = 26.

Converting Binary to Hexadecimal

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Convert 1000101 to hexadecimal

1. Group the binary digits into 4-bit sections.

NOTE: Add 0 at the start of the sequence to make a 3-bit section

2. Convert each group into hexadecimal.

1000101

0100 0101
4 5

Answer: **45_{16}**

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Exercise

Convert 1011011110100011 to hexadecimal

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Converting Binary to Hexadecimal

Binary: 1011011110100011

- Group into 4-bit sections.

1011 0111 1010 0011

- Convert each group into hexadecimal.

1011 0111 1010 0011

B **7** **A** **3** → Answer

Converting Hexadecimal to Binary

You need to become VERY familiar with the patterns below:

HEX	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
BIN	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
DEC	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

There is no easy way to remember the Hex to Binary conversions for A to F. You need to learn them so you can automatically write them down without thinking. Once you have learnt the A to F conversion the process of general conversion from Hex to Binary and back becomes very simple. (So learn them!!)

Representation of Integers

Two’s Complement Representation

Representation of Integers

- Previously, we used binary numbers to represent non-negative numbers. For example, 3 bits can be used to represent 0,1,2,3,4,5,6,7.
- Binary numbers can also be used to represent negative numbers. For example, 3 bits can be used to represent –4,-3,-2,-1,0,1,2,3.

Representation of Integers

- The binary numbers (base 2) can represent the decimal number 0 to 7 (base 10):

0 0 0	represents	0
0 0 1	represents	1
0 1 0	represents	2
0 1 1	represents	3
1 0 0	represents	4
1 0 1	represents	5
1 1 0	represents	6
1 1 1	represents	7

Representation of Integers

Using two's complement representation,

1 0 0	represents	-4
1 0 1	represents	-3
1 1 0	represents	-2
1 1 1	represents	-1
0 0 0	represents	0
0 0 1	represents	1
0 1 0	represents	2
0 1 1	represents	3

REMEMBER:

If the leftmost bit is 1, then the number is negative.

Two's Complement

Given a binary number N.

The two's complement representation of –N is formed by **reversing the bits of N, and adding 1 to the result**.

Ex. 1 (base 10) = 001 (base 2),
-1 (base 10) = ??? (base 2)

Ex. 2 (base 10) = 010 (base 2),
-2 (base 10) = ??? (base 2)

Ex. 3 (base 10) = 011 (base 2),
-3 (base 10) = ??? (base 2)

For 4-bit numbers, the two's complement representation yields:

1000	-8	0000	0
1001	-7	0001	1
1010	-6	0010	2
1011	-5	0011	3
1100	-4	0100	4
1101	-3	0101	5
1110	-2	0110	6
1111	-1	0111	7

Binary Arithmetic

Addition

- Two's complement addition follows the rules of binary addition.
- When two n-bit integers are added using two's complement arithmetic, any carry at the left-hand end is **ignored**.

Example:

```
      5       0101
  +  -3        1101
---------------------
  8          10010
```

Extra bit

What happens if we add

5 + 4 = ???

or

-6 + -7 = ???

What is the problem?

Overflow

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OVERFLOW occurs when the result cannot be represented.

OVERFLOW will <u>not</u> occur if the addends have different signs.

To check if an OVERFLOW has occurred,
the SUM should have the <u>same sign</u>
as that of the ADDENDS.

Subtraction

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How does a computer subtract two numbers A - B?

Answer: Convert B to -B and add to A.

Example:

7	7	0111
- 3	+ (-3)	1101
---------	---------	--------
		<u>1</u>0100 ➔ <u>**4**</u>

Multiplication

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- Two's complement multiplication follows the binary multiplication.
- Same as the decimal method

Example:

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```
  -2  1110
x  3  0011
-------      --------
  1110
       1110
     0000
   0000
  -----------
  101010   ➔  -6
```