Predicate Logic

- **OUTLINE**
  - **Introduction**
  - **Basic Terms and Concepts**
    - Open propositions
    - Quantifiers
  - **Syntax & Semantics**
  - Translating of English Statements
  - Inference Rules / Laws of Equivalence
  - Methods of Proof
    - Chain of Reasoning
    - Proof of Resolution

Predicate Logic

- **Propositional logic disadvantages**
  - Too coarse to describe properties of objects
  - Cannot express relations among two or more entities
  - Incapable of representing statements most often used in mathematics

  - ***Example***:
    All CMSC 56 students have passed MATH 17.
    Paul is a CMSC 56 student.
    Therefore, Paul must passed MATH 17.
    Valid or invalid argument???

Predicate Logic

- **Features of Predicate Logic**
  - Predicate Logic extends the expressiveness of propositional logic by also considering:
    - subject or individual
    - an object's meaning or characteristic

Predicate Logic

- **Basic Terms and Concepts**

DEFN. A declarative sentence is an **open proposition** if
a) it contains one or more variables
b) it is not a proposition
c) it becomes a (closed) proposition when the variables in it are replaced by certain allowable choices.

Examples:
- *He* is a singer and *she* is not pianist.
- $x \in Q$ (x is a rational number).

Predicate Logic

- **Basic Terms and Concepts**
  - *NOTATION*:
    Notice form of (open) proposition:

He **is a Computer Science major.**

Subject (constant/variable)

Meaning/Attribute (function)

CSMajor(x)

Predicate Logic

- **Basic Terms and Concepts**
  - *NOTATION*:
    variable: lower case letters
    constant: name of person/thing
    attribute: upper case letters or capitalized words

Examples:
- Francis is a teacher.
- He is a student.
- x is a negative integer.
- Joe likes Kathleen.
- If x is odd then x is not a multiple of 2.
- Poochie is not a dog
  but a python.

Predicate Logic

- **Quantifiers in Predicate Logic**
  - **Universal Quantifier**

    x "for all x"
    "for each x"
    "for every x"

    (x)(y) or **x, y** "for all x and y"

    Example:
    "All things are beautiful."
    (x)Beautiful(x) or **(x)B(x)**

Predicate Logic

- **Quantifiers in Predicate Logic**
  - **Existential Quantifier**

    x "there exists an x (such that)"
    "for some x"
    "for at least one x"

    (x)(y) or **x, y** "for some x and y"

    Example:
    "Some things are beautiful."
    (x)Beautiful(x) or **(x)B(x)**

Predicate Logic

- **Quantifiers in Predicate Logic**
  - **Universe** or **universe of discourse** is the collection of allowable choices for the variable x.
  - If x is a variable and F is a sentence, then (x) F and (x) F are sentences and F is called the **scope** (of the quantifier).

Example:
Everyone is a college student and is at least 15 years old and he is an instructor.
(x)[S(x) F(x)] I(y)

Predicate Logic

- **Syntax & Semantics of Predicate Logic**
  - **Syntax**
    - Any simple open proposition (e.g. **P(x)**) is a valid open proposition.
    - If **P** and **Q** are valid propositions, then the following are also valid (open) propositions:
      ~P, **P Q**, **P Q**, **P Q**, **P Q**,
      (x) P(x) and **(x) P(x)**
    - All valid (open) propositions generated from a finite number of the above.

Predicate Logic

- **Syntax & Semantics of Predicate Logic**
  - **Semantics**

    Examples

    Are each of the following quantified statements true or false?
    - All fish live in salt water.
      False
    - Some people cannot read.
      True
    - For all real numbers x, x – 1 < x .
      True
    - Some people have horns.
      False

Predicate Logic

- **Syntax & Semantics of Predicate Logic**
  - **Semantics**
    - (x) F is true if F is true for all x in the universe.
    - (x) F is false if F is false for at least one x in the universe.
    - (x) F if true if F is true for at least one x in the universe.
    - (x) F if false if F is false for all x in the universe.

Predicate Logic

- **Translating Quantified Statements**
  - ***Preliminary exercises in translation***

    Restate the following so that a quantifier is clearly used:
    - No UP student is not intelligent.
    - Not all people can walk.

Predicate Logic

- **Translating to Symbolic Form**

1. **All students can read.**
2. **Some children are tall.**
3. **Not all students are 15 years old.**

Predicate Logic

- **Translating to Symbolic Form**

4. **Nothing is perfect.**
5. **Some musicians can sing but cannot dance.**
6. **No chef is thin and malnourished**.
7. **Mike loves some things.**

Predicate Logic

- **Translating to Symbolic Form**

8. **Lizzy and Willy like each other.**
9. **Tina bakes cookies only.**
10. **Tina bakes all cookies.**
11. **Some girls bake cookies only.**

Predicate Logic

- **Translating to Symbolic Form**

12. **Some men fix some cars.**
13. **Some men fix all cars.**
14. **All teens drink softdrinks only.**
15. **All roses and jasmines are fragrant.**

Predicate Logic

- **Translating to Symbolic Form**

16. All children cannot fly.
17. Some people cannot talk.
18. Not all birds are flightless.
19. Manuel, a musician, is neither tone deaf nor a pianist.
20. Some integers greater than ten are even integers.
21. All old computers are slow.
22. Some movies are either too long or boring.
24. All dogs who are pets are housebroken.
25. All teens drink sugarless softdrinks only.
26. All teens do not drink fruit juices.

Predicate Logic

- **Translating to Symbolic Form**

  Some remarks:
  - Common (but not necessarily universal) patterns:
    - **(x)** followed by an *implication*
    - **(∃x)** followed by an *conjunction*
  - Order of writing a combination of and ∃ is *important*.

    Example:

    Consider the meanings of the following:
    - **(x)(∃y) [y is the mother of x]**
    - **(∃y)(x) [y is the mother of x]**