Propositional Logic

- **OUTLINE**
  - Propositions and Logical Connectives
  - Syntax and Semantics of Propositional Logic
  - **Rules of Inference**
  - **Laws of Equivalence**
  - Methods of Proof

Rules of Inference/Logic

- **Is reasoning correct or incorrect?**
- **Is conclusion drawn or inferred from facts valid?**
- An argument is a sequence of propositions or sentences written as:

$$\begin{array}{l} F_1 \\ F_2 \\ : \\ F_n \\ \hline \therefore Q \end{array} \qquad \text{OR:} \quad (F_1 \wedge F_2 \wedge \ldots \wedge F_n) \rightarrow Q$$

- The argument is **valid** provided that: if *all* the premises are true then Q must also be true.

Rules of Inference/Logic

- **Rules of Inference**
  - Common (simple) forms of *valid arguments*.
  - Specify what conclusions can be validly drawn/inferred from a set of premises *that are assumed to be true*.
  - *Example*:
    Simplification: $(P \wedge Q) \rightarrow P$
    if $P \wedge Q$ is true (as a whole),
    then P must be also true.

Rules of Inference/Logic

- **Rules of Inference** (cont'd)
  - *Example*:
    Modus Ponens: $[(P \rightarrow Q) \wedge P] \rightarrow Q$
    - *if* $P \rightarrow Q$ *is true (as a whole)*
      and P *is known to be true*
      then Q *also has to be true.*
    - ____
      ____
      ____
  - *Verify*:
    - What are the possible values of P and Q if $P \rightarrow Q$ is true?
    - If P is now known to be true, what are the possible values of Q?

Rules of Inference/Logic

- **Rules of Inference** (cont'd)
  - *Example*:
    Disjunctive Syllogism: $[(P \vee Q) \wedge \sim P] \rightarrow Q$
    - *if* $P \vee Q$ *is true (as a whole)*
      and $\sim P$ *is known to be true*
      (therefore P is false)
      then Q *has to be true.*
    - Either my dog ran away or got hit by a car. My dog did not get hit by a car. Therefore, my dog ran away.

Rules of Inference/Logic

- **Rules of Inference** (cont'd)
  - *More examples*:
    Modus Tollens:
    If I am elected president then I will reduce taxes. I did not reduce taxes. Therefore, I was not elected president.
    Hypothetical Syllogism:
    If 3 is a positive integer, then 3 > 0. If 3 > 0, then 3 is greater than any negative integer. Therefore, if 3 is a positive integer, then 3 is greater than any negative integer.

Rules of Inference/Logic

- **Rules of Inference** (cont'd)
  - *More examples*:

    Constructive Dilemma:

    If you say what is just, then men will hate you. And if you say what is unjust, the gods will hate you. But you must say either one or the other. Therefore, you will be hated by men or by the gods.

    If Mr. X goes out with Miss A, then Miss B will hate him. If Mr. X goes out with Miss B, then Miss A will hate him. But Mr. X must go out with either one. Therefore, either Miss A or Miss B will hate Mr. X.

Rules of Inference/Logic

- ***Examples***:

  Are the following simple arguments valid? If so, state which rule of inference supports the argument.

  - $\sim B$
$\underline{\sim A \vee B}$
$\therefore\ \sim A$
  - $\sim S \rightarrow Q$
$\underline{(P \wedge R) \rightarrow \sim S}$
$\therefore\ (P \wedge R) \rightarrow Q$
  - $\underline{(\sim D \wedge C) \rightarrow A}$
$\therefore\ \sim D \rightarrow A$
  - $\sim(S \vee R)$
$\underline{Q \rightarrow P}$
$\therefore\ \sim(S \vee R) \wedge (Q \rightarrow P)$
  - $\underline{(A \leftrightarrow B)}$
$\therefore\ D \wedge (A \leftrightarrow B)$
  - $\sim(S \leftrightarrow R)$
$\underline{Q}$
$\therefore\ \sim[S \leftrightarrow (R \wedge Q)]$

Rules of Inference/Logic

- ***Examples***:

  Are the following simple arguments valid? If so, state which rule of inference supports the argument.

  - *Vic forgot to send me a text message or he hasn't arrived yet. Vic already has arrived. Therefore, Vic forgot to send me a text message.*
  - *If I am invited to the party I will buy myself a new dress. I was not invited to the party. Therefore, I will not buy myself a new dress.*
  - *I will not take another GE course if the petition for CMSC 11 is approved. If I am underloaded this sem, I have to enroll in the summer. Either the petition for CMSC 11 is approved or I am underloaded this sem. Therefore, I will not take another GE course or I have to enroll in the summer.*

Rules of Inference/Logic

- **Laws of Equivalence**
  - Specify what propositional statements are *equivalent* to each other.
  - Specify what statements can be used as *substitute/replacement* for another (when used in proofs).
  - *Example*:

    De Morgan's Law:

    $\sim(P \vee Q) \leftrightarrow (\sim P \wedge \sim Q)$

    It is not true that it is hot or raining.

    $\leftrightarrow$ *It is not hot and it is not raining*.

Rules of Inference/Logic

- ***Examples***:

  According to the laws of equivalence, which of the following statements are valid?

  - $\sim(C \vee \sim D)$
$\leftrightarrow (\sim C \wedge \sim\sim D)$
  - $[Q \rightarrow \sim(P \vee R)]$
$\leftrightarrow [\sim\sim(P \vee R) \rightarrow \sim Q]$
  - $[A \wedge \sim(C \vee D)]$
$\leftrightarrow [(A \wedge \sim C) \vee (A \wedge \sim D)]$
  - *It is not raining or I will bring my umbrella.*
$\leftrightarrow$ *I will my umbrella if it is raining.*
  - *It is not true that there are no classes on Friday.*
$\leftrightarrow$ *There are classes on Friday.*