

Exercise 41: (Relations and directed graphs)

See Figure below

Exercise 42: (Rules)

See Figure below

Exercise 43: (More rules)

See Figure below

Exercise 44: (Tautologies)

a

$$\Box F \Rightarrow \Diamond F$$

$$(\forall s, (s, t) \in W : (A(F, s) = 1)) \Rightarrow (\exists s, (s, t) \in W : (A(F, s) = 1))$$

$$\neg(\forall s, (s, t) \in W : (A(F, s) = 1)) \vee (\exists s, (s, t) \in W : (A(F, s) = 1))$$

$$(\exists s, (s, t) \in W : (A(\neg F, s) = 1)) \vee (\exists s, (s, t) \in W : (A(F, s) = 1))$$

$$(A(\Diamond \neg F, s) = 1) \vee (A(\Diamond F, s) = 1)$$

$$\Diamond \neg F \vee \Diamond F$$

$$\stackrel{7}{=} \Diamond(F \vee \neg F)$$

Is a tautology.

b

$$F \Rightarrow \Diamond F$$

c

$$\Box F \Leftrightarrow \neg \Diamond \neg F$$

$$\stackrel{2}{=} \Box F \Leftrightarrow \neg \neg \Box F$$

Is a tautology.

d

$$(\Box F \wedge \Box(F \Rightarrow G)) \Rightarrow \Box G$$

$$\stackrel{6}{=} \Box(F \wedge (F \Rightarrow G)) \Rightarrow \Box G$$

$$\stackrel{3}{=} F \wedge (F \Rightarrow G) \Rightarrow G$$

Is not a tautology.

e

$$\neg(\Box(F \Rightarrow G) \wedge \Diamond F \wedge \Box \neg G)$$

$$\stackrel{6}{=} \neg(\Box((F \Rightarrow G) \wedge \neg G) \wedge \Diamond F)$$

$$\stackrel{2}{=} \neg(\Box((F \Rightarrow G) \wedge \neg G) \wedge \neg \Box \neg F)$$

$$\stackrel{6}{=} \neg(\Box((F \Rightarrow G) \wedge \neg G \wedge F))$$

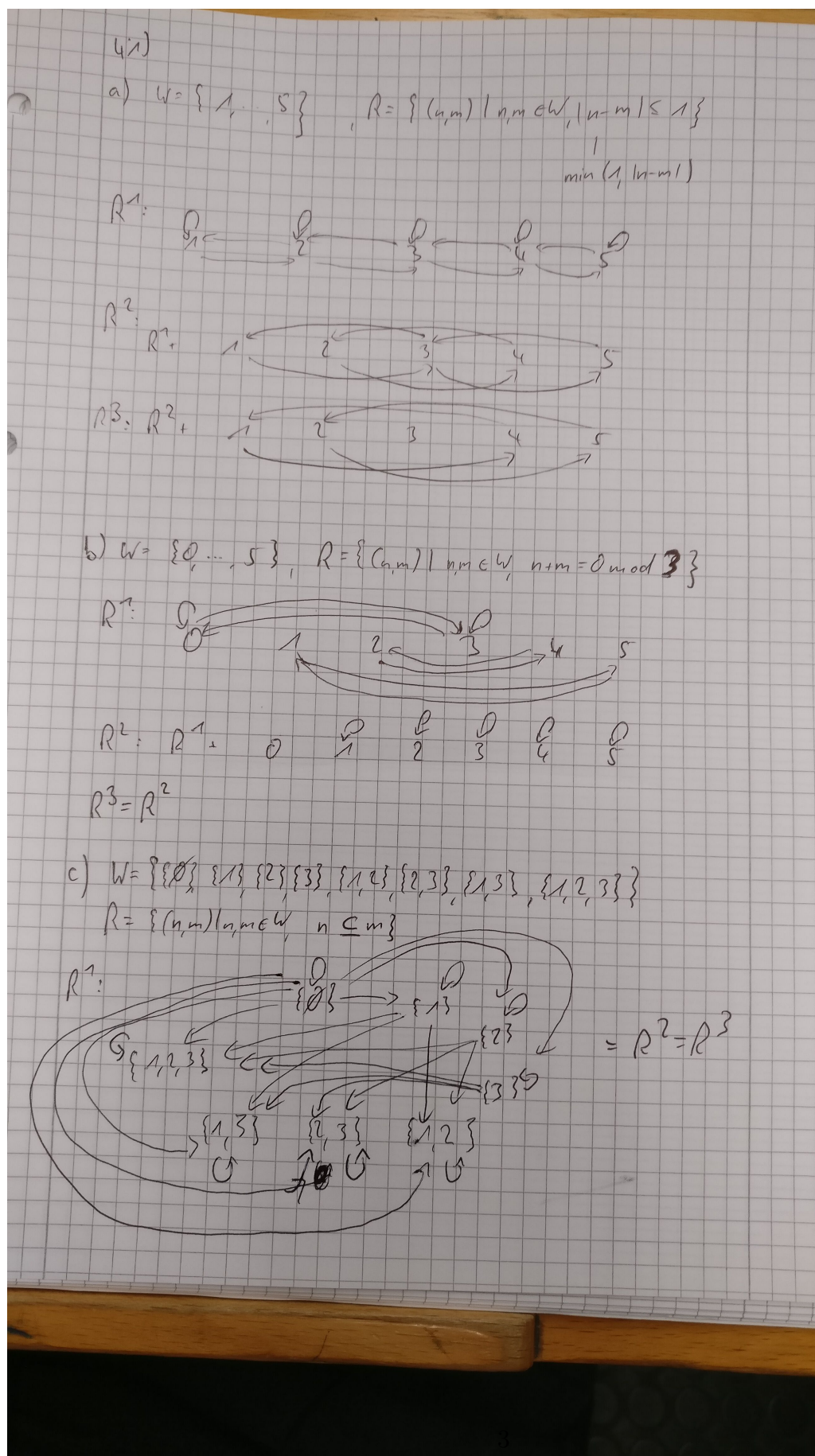


Figure 1: Solution for Exercise 41

42)

$$\neg \Diamond F \equiv \Box \neg F \quad \forall s, (s, t) \in R: \neg (A(\neg F, s) = 1)$$

\Rightarrow

$$\neg \Diamond F$$

$$\Leftrightarrow \neg (\exists s, (s, t) \in R: A(\neg F, s) = 1)$$

$$\Leftrightarrow \forall s, (s, t) \in R: \neg (A(\neg F, s) = 1)$$

$$\Leftrightarrow \forall s \rightarrow \neg : (A(\neg F, s) = 1)$$

$$\Leftrightarrow A(\Box \neg F, s) = 1 \Leftrightarrow \Box \neg F$$

\Rightarrow

$$6. \quad \Box (F \wedge G) \equiv \Box F \wedge \Box G$$

$$\Leftrightarrow \forall s, (s, t) \in W: (A(F, s) = 1 \wedge A(G, s) = 1)$$

$$\Leftrightarrow (\forall s, (s, t) \in W: A(F, s) = 1) \wedge (\forall s, (s, t) \in W: A(G, s) = 1)$$

$$\Leftrightarrow (A(\Box F, s) = 1) \wedge (A(\Box G, s) = 1)$$

$$\Leftrightarrow \Box F \wedge \Box G$$

$$7. \quad \Diamond (F \vee G) \equiv \Diamond F \vee \Diamond G$$

$$\Leftrightarrow \exists s, (s, t) \in W: (A(F, s) = 1 \vee A(G, s) = 1)$$

$$\Leftrightarrow (\exists s, (s, t) \in W: A(F, s) = 1) \vee (\exists s, (s, t) \in W: A(G, s) = 1)$$

$$\Leftrightarrow (A(\Diamond F, s) = 1) \vee (A(\Diamond G, s) = 1)$$

$$\Leftrightarrow \Diamond F \vee \Diamond G$$

Figure 2: Solution for Exercise 42

43)

a) $\Diamond F \Rightarrow \Diamond G \equiv \Box F \Rightarrow \Diamond G$

$\Diamond F \Rightarrow \Diamond G$

$\Leftrightarrow \neg \Box F \vee \Diamond G$

$\stackrel{2}{\Leftrightarrow} \Box \neg F \vee \Diamond G$

$\Leftrightarrow \Box F \Rightarrow \Diamond G$

Figure 3: Solution for Exercise 43