

**Exercise 41: (Relations and directed graphs)****a****b****c****Exercise 42: (Rules)****Exercise 43: (More rules)****a****b****c****Exercise 44: (Tautologies)****a**

$$\Box F \Rightarrow \Diamond F$$

**b**

$$F \Rightarrow \Diamond F$$

**c**

$$\Box F \Leftrightarrow \neg \Diamond \neg F$$

$$\stackrel{2}{=} \Box F \Leftrightarrow \neg \neg \Box F$$

Is a tautology.

**d**

$$(\Box F \wedge \Box(F \Rightarrow G)) \Rightarrow \Box G$$

$$\stackrel{6}{=} \Box(F \wedge (F \Rightarrow G)) \Rightarrow \Box G$$

$$\stackrel{3}{=} F \wedge (F \Rightarrow G) \Rightarrow G$$

Is not a tautology.

**e**

$$\neg(\Box(F \Rightarrow G) \wedge \Diamond F \wedge \Box\neg G)$$

$$\stackrel{6}{=} \neg(\Box((F \Rightarrow G) \wedge \neg G) \wedge \Diamond F)$$

$$\stackrel{2}{=} \neg(\Box((F \Rightarrow G) \wedge \neg G) \wedge \neg\Box\neg F)$$

$$\stackrel{6}{=} \neg(\Box((F \Rightarrow G) \wedge \neg G \wedge F))$$