

Exercise 13: (Resolvents)

Transform F into CNF:

$$F = (A \vee \neg(B \vee \neg C)) \wedge (B \vee C) \wedge (A \Rightarrow C) \wedge (C \Rightarrow B) \wedge \neg C$$

$$\Leftrightarrow F = (A \vee \neg B \vee C) \wedge (B \vee C) \wedge (\neg A \vee C) \wedge (\neg C \vee B) \wedge \neg C$$

Resolution:

$$Res^1(F) = \{\{A, \neg B, C\}, \{B, C\}, \{\neg A, C\}, \{\neg C, B\}, \{\neg C\}\}$$

$$Res^2(F) = Res^1(F) \cup \{\{A, C\}, \{\neg B, C\}, A, \{A, \neg B\}, \{B\}, \{\neg A\}\}$$

$$Res^3(F) = Res^2(F) \cup \{\{C\}, \{\square\}\}$$

$Res^3(F)$ contains \square and by Theorem 1.9 we now can conclude that F is unsatisfiable.

Exercise 14: (Efficiency of Resolution)

Exercise 15: (The buying public)

Events:

C = "buy car", H = "make holiday", M = "buy moped to conciliate the spoilt son who is slightly mentally unstable", I = "receive incentive"

The washing machine is bought in any case, hence it can be omitted.

Model M:

$$M = (H \Rightarrow \neg C) \wedge (\neg H \Rightarrow M) \wedge (\neg C \Rightarrow M) \wedge (\neg M \Rightarrow C) \wedge (I \Rightarrow (C \wedge M))$$

$$\Leftrightarrow M = (\neg H \vee \neg C) \wedge (H \vee M) \wedge (C \vee M) \wedge (M \vee C) \wedge (\neg I \vee C) \wedge (\neg I \vee M)$$

Resolution:

$$Res^1(M) = \{\{\neg H, C\}, \{H, M\}, \{C, M\}, \{\neg I, C\}, \{\neg I, M\}\}$$

$$Res^2(M) = Res^1(M) \cup \{\{\neg C, M\}, \{\neg H, M\}, \{\neg H, I\}\}$$

$$Res^3(M) = Res^2(M) \cup \{\{M\}, \{M, I\}\}$$

\Downarrow

The moped will be bought in any case, the event does not depend on Mrs Smith receiving the incentive or not.

Exercise 16: (Dr Who pushing buttons)