Exercise 41: (Relations and directed graphs)

See Figure below

Exercise 42: (Rules)

See Figure below

Exercise 43: (More rules)

See Figure below

Exercise 44: (Tautologies)

a

$\Box F \Rightarrow \diamond F$

$(\forall s, (s,t) \in W : (A(F,s) = 1)) \Rightarrow (\exists s, (s,t) W : (A(F,s) = 1))$

$\neg(\forall s, (s,t) \in W : (A(F,s) = 1)) \vee (\exists s, (s,t) W : (A(F,s) = 1))$

$(\exists s, (s,t) \in W : (A(\neg F,s) = 1)) \vee (\exists (s,t) W : (A(F,s) = 1))$

$(A(\diamond \neg F, s) = 1) \vee (A(\diamond F, s) = 1)$

$\diamond \neg F \vee \diamond F$

$\overset{7}{=} \diamond(F \vee \neg F)$
Is a tauntology.

b

$F \Rightarrow \diamond F$

c

$\Box F \Leftrightarrow \neg \diamond \neg F$

$\overset{2}{=} \Box F \Leftrightarrow \neg\neg \Box F$
Is a tauntology.

d

$(\Box F \wedge \Box(F \Rightarrow G)) \Rightarrow \Box G$

$\overset{6}{=} \Box(F \wedge (F \Rightarrow G)) \Rightarrow \Box G$

$\overset{3}{=} F \wedge (F \Rightarrow G) \Rightarrow G$
Is not a tauntology.

e

$\neg(\Box(F \Rightarrow G) \wedge \diamond F \wedge \Box\neg G)$

$\overset{6}{=} \neg(\Box((F \Rightarrow G) \wedge \neg G) \wedge \diamond F)$

$\overset{2}{=} \neg(\Box((F \Rightarrow G) \wedge \neg G) \wedge \neg\Box\neg F)$

$\overset{6}{=} \neg(\Box((F \Rightarrow G) \wedge \neg G \wedge F))$

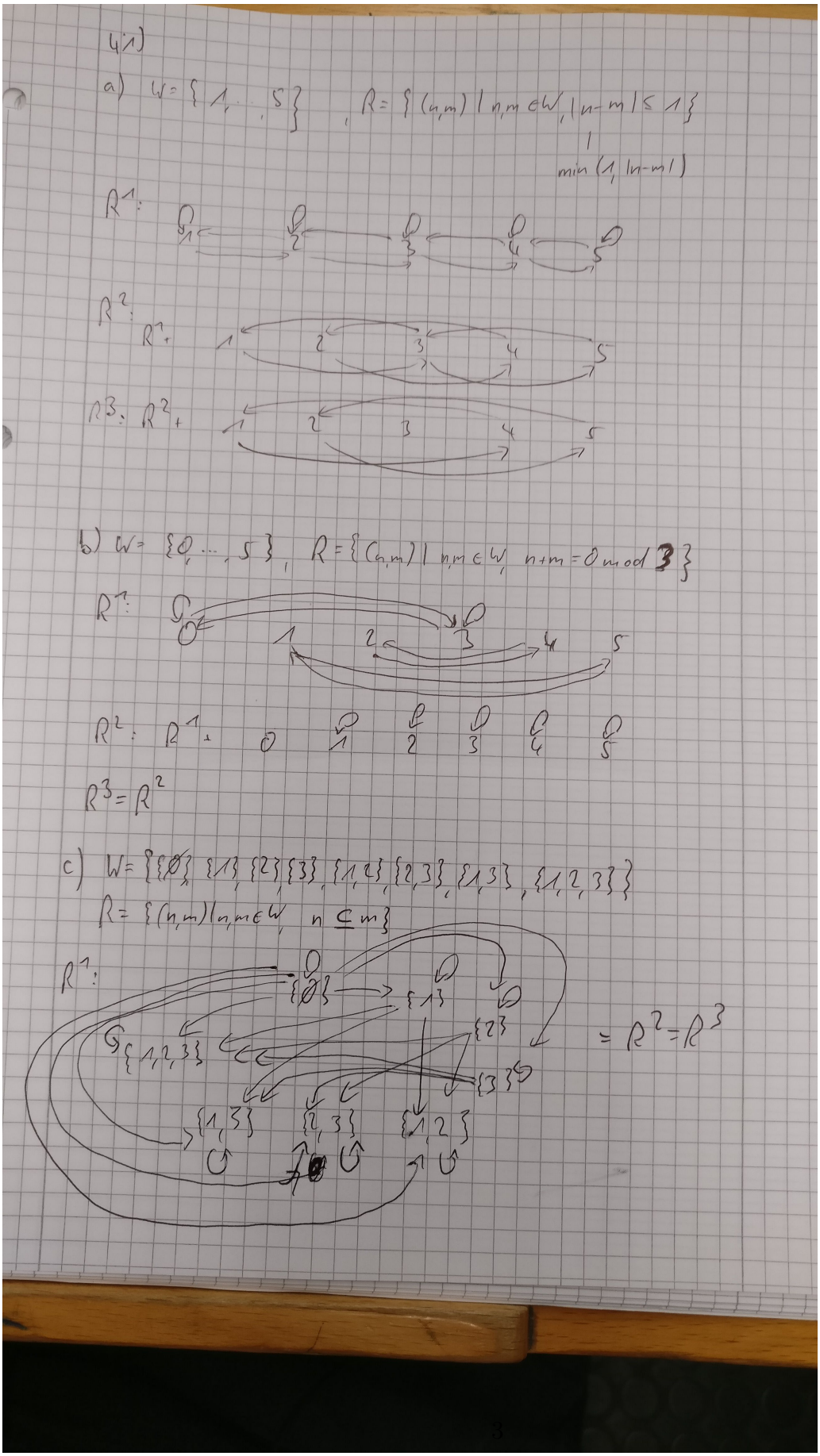

Figure 1: Solution for Exercise 41

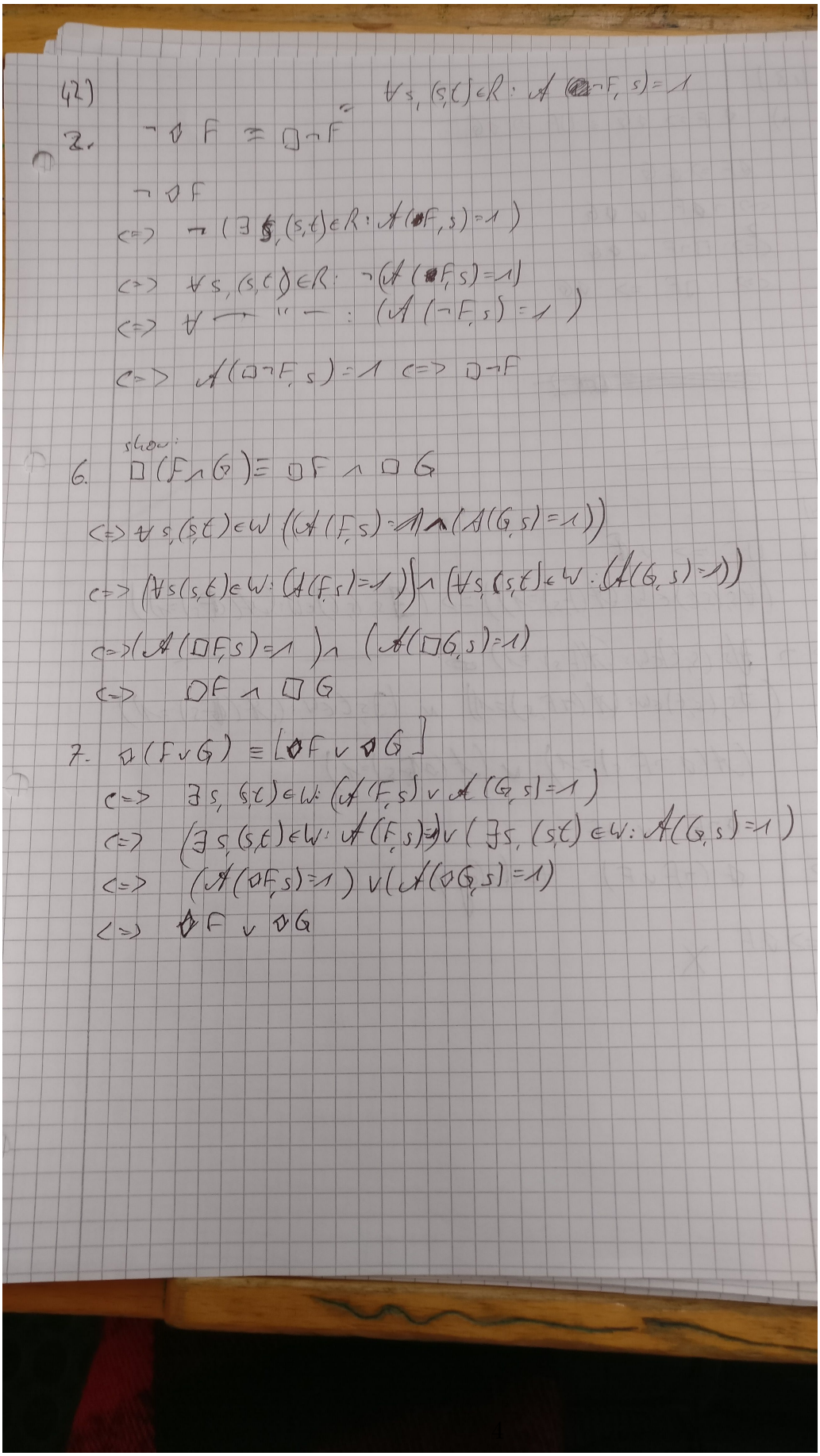

Figure 2: Solution for Exercise 42

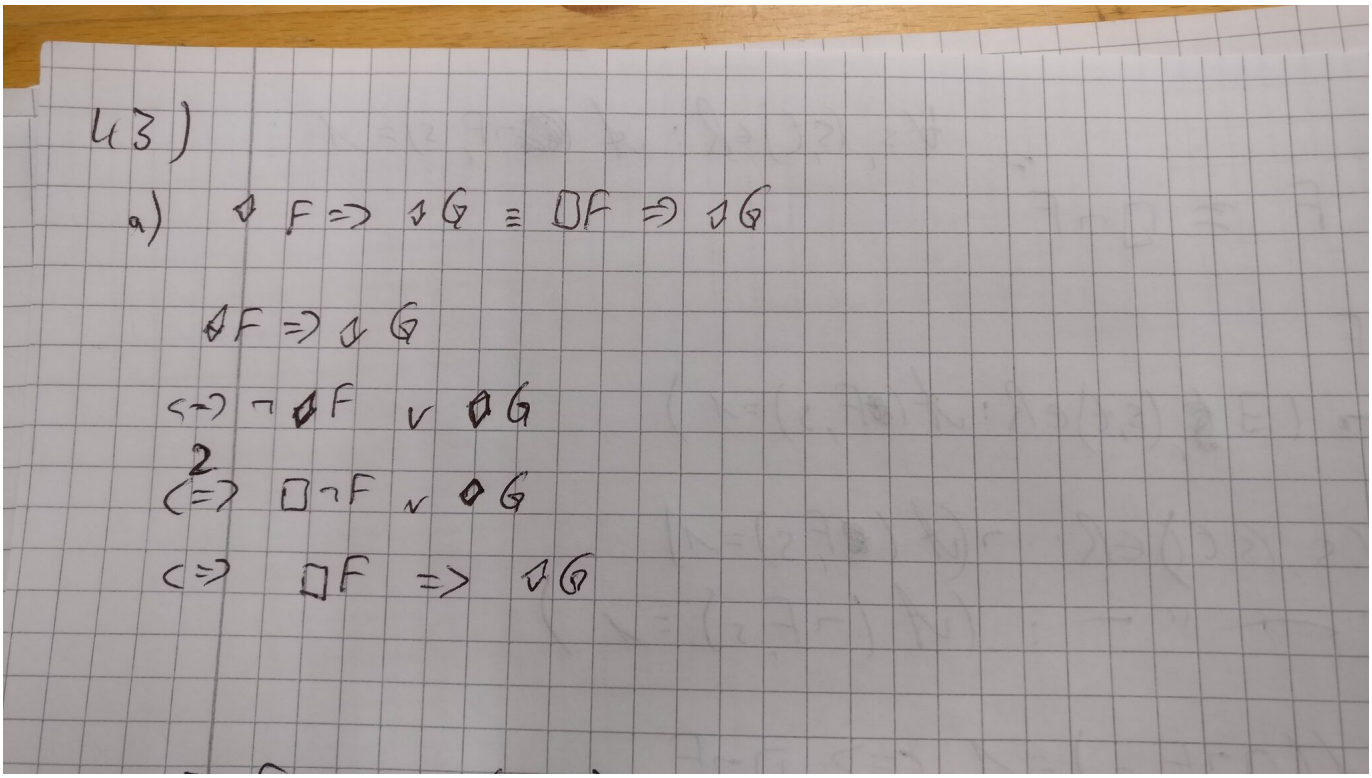

Figure 3: Solution for Exercise 43