Exercise 17: (Models?)

a

Choose for example $x = 3$, $y = 5$, $z = 4$
$\Rightarrow x < z$, $z < y$, $x < z$, $\neg(z < x)$
This holds true, so this is a model for F.

b

$P = \{(n, n+1) | n \in \mathbb{N}\}$
$(x, y) = (x, x+1)$,
$(z, y) = (z, x+1) \Rightarrow z = x$,
$(x, z) = (x, x+1) \Rightarrow (z = x)$
$(z = x) \wedge (z = x+1)$ unsatisfiable
So this is no model of F.

c

$x = y'$, $z = y'$, $x = z'$, $\neg(z = x')$
Derivatives are surjectiv, thus $x = z$. Furthermore either
$x = z' \wedge x' = z$ or
$\neg(x' = z) \wedge \neg(x = z')$.
In conclusion, this cannot be a model for F.

d

Choose for example $y = \{1, 2, 3\}$, $z = \{1, 2\}$, $x = \{1\}$
$\Rightarrow x \subseteq y$, $z \subseteq y$, $x \subseteq z$, $\neg(z \subseteq x)$
This holds true, so this is a model for F.

Exercise 18: (Models and non-models)

$F = \forall x \exists y P(x, y, f(z))$

Terms

$x, y, z, f(z)$

Partial Formulas

$F, \exists y P(x, y, f(z)), \forall x P(x, y, f(z)), P(x, y, f(z))$

Matrix

$P(x, y, f(z))$

Structures that are not model for F:

$U_A = \mathbb{N}$, $P^A = \{(x, y, z | x, y, z \in \mathbb{N}, y < x)\}$ $(x = 1)$
$U_A = \mathbb{R}$, $P^A = \{(x, y, z | x, y, z \in \mathbb{R}, \pi x^2 = y^2)\}$ (radius of circle/sidelength of square which have the same area)

Structures that are model for F:

$U_A = \mathbb{Z}$, $P^A = \{(x, y, z | x, y, z \in \mathbb{Z}, y < x)\}$
$U_A = \mathbb{R}$, $P^A = \{(x, y, z | x, y, z \in \mathbb{R}, y^2 < x^2, y \neq x)\}$

Exercise 19: (Reflexive, symmetric, transitive)

Exercise 20: (Small universes)