Exercise 9: (Horn formula algorithm)

First, we write the formula in the implication-form, so
$F = (\neg A \vee \neg B \vee \neg D) \wedge \neg E \wedge (\neg C \vee A) \wedge C \wedge B \wedge (\neg G \vee D) \wedge G$
becomes:
$F = (A \wedge B \wedge D \Rightarrow 0) \wedge (0 \Rightarrow E) \wedge (C \Rightarrow A) \wedge (1 \Rightarrow C) \wedge (1 \Rightarrow B) \wedge (G \Rightarrow D) \wedge (1 \Rightarrow G)$

Then we start marking all literals of the form $(1 \Rightarrow X)$:
$F = (A \wedge B \wedge D \Rightarrow 0) \wedge (0 \Rightarrow E) \wedge (C \Rightarrow A) \wedge \underline{(1 \Rightarrow C)} \wedge \underline{(1 \Rightarrow B)} \wedge (G \Rightarrow D) \wedge \underline{(1 \Rightarrow G)}$

Now we start with the loop part of the algorithm:
$F = (A \wedge B \wedge D \Rightarrow 0) \wedge (0 \Rightarrow E) \wedge \underline{(C \Rightarrow A)} \wedge \underline{(1 \Rightarrow C)} \wedge \underline{(1 \Rightarrow B)} \wedge (G \Rightarrow D) \wedge \underline{(1 \Rightarrow G)}$

$F = (A \wedge B \wedge D \Rightarrow 0) \wedge (0 \Rightarrow E) \wedge \underline{(C \Rightarrow A)} \wedge \underline{(1 \Rightarrow C)} \wedge \underline{(1 \Rightarrow B)} \wedge \underline{(G \Rightarrow D)} \wedge \underline{(1 \Rightarrow G)}$

In the next step we would mark the $(A \wedge B \wedge D \Rightarrow 0)$ literal, but it implies 0, so the formula is non satisfiable.

Exercise 10: (Not a Horn formula)

a.

$F = (A \vee B \vee \neg F) \wedge (\neg C)$
is in CNF and has more than one not negated atomic formula in it's disjunctive clause.

b.

Horn formulas (in CNF) in which each disjunctive clause contains at least one $\neg$ have only literals of the form $(X_1 \wedge X_2 \wedge ... \wedge X_n \Rightarrow 0)$ and $(X_1 \wedge X_2 \wedge ... \wedge X_n \Rightarrow X_{n+1})$, but none of the form $(1 \Rightarrow X)$, because the presence of at least one $\neg$ would turn them to the second form.
When now using the algorithm 1.3 to determine the satisfiability of those formulas, in the first step no atomic formula of the form $(1 \Rightarrow X)$ is present and such will not be marked. This results to the loop in the second step not executing at all. So step three is executed and the algorithm returns the formula as satisfiable.

Exercise 11: ($\neg$ and $\Rightarrow$ suffice, but $\vee$, $\wedge$ and $\Rightarrow$ dont)

a.

Every multi clause Formula F can be treated like an atomic formula and solved separately. Thus we can construct 4 distinct cases how two Formulas A, B can appear:

Formula	Conversion to representation using only $\neg, \Rightarrow$
$A \lor B$	$\neg A \Rightarrow B$
$A \lor \neg B$	$\neg A \Rightarrow \neg B$
$A \land B$	$\neg\neg(A \land B) = \neg(\neg A \lor \neg B) = \neg(A \Rightarrow \neg B)$
$A \land \neg B$	$\neg\neg(A \land \neg B) = \neg(\neg A \lor B) = \neg(A \Rightarrow B)$

b.

Proof by giving such a formula:
$F = \neg A \land \neg B$

Exercise 12: (Infinitely many formulas)

Choose the valuation of formulas with an even index opposite to formulas with an uneven index and the set of formulas is satisfied. Eg. start with $A_1 = True$ and $A_2 = False$ or vice versa.