Exercise 29: (Insane Vampires)

Exercise 30: (Infinite Models)

$F = \forall x P(x, f(x)) \wedge \forall y \neg P(y, y) \wedge \forall x \forall y \forall z((P(x, y) \wedge P(y, z)) \Rightarrow P(x, z))$

a

Find a model that satisfies F
$U^F = \mathbb{Z}$
$P^F(x, y) = \{(x, y) | x < y\}$
$f(x) : x \rightarrow x + 1$

Exercise 31: (Undecidable Problem I)

a

$$A = \begin{bmatrix} 010 \\ 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 110 \end{bmatrix}, C = \begin{bmatrix} 10 \\ 01 \end{bmatrix}$$

$$B + C + A = \begin{bmatrix} 110010 \\ 110010 \end{bmatrix}$$

b

$$A = \begin{bmatrix} 010 \\ 01 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 10 \end{bmatrix}, C = \begin{bmatrix} 10 \\ 101 \end{bmatrix}$$

In order to be able to finish a sequence one needs a piece where the shorter sequence matches the end of the longer one, eg in a) piece A has the shorter sequence "0" in the bottom row which matches the end of the upper sequence "010".
The pieces given here have no such ending piece, hence no finite sequence can be found.

c

$$A = \begin{bmatrix} 1 \\ 10 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 10 \end{bmatrix}, C = \begin{bmatrix} 010 \\ 01 \end{bmatrix} D = \begin{bmatrix} 11 \\ 1 \end{bmatrix}$$

$$D + B = \begin{bmatrix} 110 \\ 110 \end{bmatrix}$$

d

$$A = \begin{bmatrix} 10 \\ 101 \end{bmatrix}, B = \begin{bmatrix} 011 \\ 11 \end{bmatrix}, C = \begin{bmatrix} 101 \\ 011 \end{bmatrix}$$

It is also not possible to find a sequence here for a similar argument as given in b). The only piece we can finish a sequence with is B. If B is the last piece, the piece before last would have to have a zero as the last symbol in the bottom row to match the zero in the first row of B. No such piece is provided.

Exercise 32: (Undecidable Problem II)

For the sake of simplification, we will enumerate both set of tiles from 1 to 4, from the left to the right.

Left set of tiles

Consider the following plane:
3 4 1 3
3 2 4 3
3 4 1 3
3 2 4 3
This plane can easily be infinitely extended in any direction.

Right set of tiles

We will show that it is not possible to lie a plane using this tileset by showing that if tiles 2, 3 or 4 are lied down they ultimately require a tile 1 to be laid and that lying of tile 1 makes any infinite plane impossible, thus violating the Wang condition.

If tile 2 is laid, tile 4 has to be laid on top of it to complete the red edge. If tile 3 or 4 are laid, tile 1 has to be placed to the right of them, to complete the green edge.

However, if tile 1 is laid, it needs to be laid to the right of it again, because it is the only tile which has a green edge to the left. Consider now the tile below tile 1: It needs to have a blue edge on top, thus it must either be tile 3 or tile 4. Both of them require tile 1 to the right of them, to complete their right green edge. This way the tile which shares the lower right corner with/ is diagonal to the original laid tile 1 has also to be tile 1. But on top of that tile (thus to the right of the original tile 1) has to be another tile 1, which means that a red upper edge would border an blue lower edge, which would violate the condition of the Wang tiles.
Illustration:
1 1 1 ...
3 1 1 1 ...