Exercise 37: (Violate the Peano axioms!)

Standard model: $U_A = \mathbb{N}_0$
$S(x) = x + 1$
$f(x, y) = x + y$
$g(x, y) = x * y$
Change model such that...

   a)
Satisfy all except $\forall x 0 \neq S(x)$:
$U_A = \mathbb{N}_0 \cup \{-1\}$

   b)
Satisfy all except $\forall x \forall y S(x) = S(y) \Rightarrow x = y$:
$U_A = \mathbb{Z} \setminus \{-1, 1\}$
$S(n) = |n| + 1$

   c)
Satisfy all except $\forall x \forall y \;\; x * S(y) = f(g(x, y), x)$:
$U_A = \mathbb{Z} \setminus \{-1, 1\}$
$f(x, y) = x + g(-1, y)$

Exercise 38: (Consequences of Peano axioms)

Exercise 39: (Football experts)

We construct a the structure $A = \{W, R, \alpha\}$ where
$W = \{1, 2, ..., 34\}$ (days of play in Bundesliga this season. Current day of play is 11)
R the relation $<$ from ex. 40, here being interpretable as a timeline relation as $n < m =$ "n happened before m"

   $\alpha : \{M, B, D\} \times W \rightarrow \{0, 1\}$
Results from matches on 22nd July 2017: $\alpha(M, 1) = 0$, $\alpha(D, 1) = 0$, $\alpha(B, 1) = 0$

$$\alpha(B, n) = \begin{cases} 1 & , n \geq 12 \\ 1 & , \text{if Bremen won}, n < 12 \\ 0 & , \text{otherwise(lost)} \end{cases}$$

$$\alpha(M, n) = \begin{cases} 1 & , \text{Munich won on day n} \\ 0 & , \text{otherwise(lost)} \end{cases}$$

$$\alpha(d, n) = \begin{cases} 1 & , n > 12 \\ 1 & , \text{if Dortmund won}, n < 12 \\ 0 & , \text{otherwise(lost)} \end{cases}$$

Atomic Formulas: M = Munich wins, B = Bremen wins, D = Dortmund wins

a) B
b) $\neg D$
c) $F = \diamond\neg M \Rightarrow \neg B$
d) $\diamond M \Rightarrow D$
e) $M \vee B \vee D$

At least the last statement is wrong. On July 22nd 2017, Bayern lost against Mailand, Dortmund against Bochum and Bremen against St. Pauli. This violates statement e.

Exercise 40: (Frames)