

Exercise 9: (Horn formula algorithm)

First, we write the formula in the implication-form, so

$$F = (\neg A \vee \neg B \vee \neg D) \wedge \neg E \wedge (\neg C \vee A) \wedge C \wedge B \wedge (\neg G \vee D) \wedge G$$

becomes:

$$F = (A \wedge B \wedge D \Rightarrow 0) \wedge (0 \Rightarrow E) \wedge (C \Rightarrow A) \wedge (1 \Rightarrow C) \wedge (1 \Rightarrow B) \wedge (G \Rightarrow D) \wedge (1 \Rightarrow G)$$

Then we start marking all literals of the form $(1 \Rightarrow X)$:

$$F = (A \wedge B \wedge D \Rightarrow 0) \wedge (0 \Rightarrow E) \wedge (C \Rightarrow A) \wedge \underline{(1 \Rightarrow C)} \wedge \underline{(1 \Rightarrow B)} \wedge (G \Rightarrow D) \wedge \underline{(1 \Rightarrow G)}$$

Now we start with the loop part of the algorithm:

$$F = (A \wedge B \wedge D \Rightarrow 0) \wedge (0 \Rightarrow E) \wedge \underline{(C \Rightarrow A)} \wedge \underline{(1 \Rightarrow C)} \wedge \underline{(1 \Rightarrow B)} \wedge (G \Rightarrow D) \wedge \underline{(1 \Rightarrow G)}$$

$$F = (A \wedge B \wedge D \Rightarrow 0) \wedge (0 \Rightarrow E) \wedge \underline{(C \Rightarrow A)} \wedge \underline{(1 \Rightarrow C)} \wedge \underline{(1 \Rightarrow B)} \wedge \underline{(G \Rightarrow D)} \wedge \underline{(1 \Rightarrow G)}$$

In the next step we would mark the $(A \wedge B \wedge D \Rightarrow 0)$ literal, but it implies 0, so the formula is non satisfiable.

Exercise 10: (Not a Horn formula)

a.

b

Exercise 11: (\neg and \Rightarrow suffice, but \vee , \wedge and \Rightarrow dont)

a.

b

Exercise 12: (Infinitely many formulas)