Exercise 5: (Laws of Logic)

1.

$F \wedge (F \vee G) \equiv F$
$\Leftrightarrow (F \wedge F) \vee (F \wedge G) \equiv F$ see part 2
$\Leftrightarrow F \vee (F \wedge G) \equiv F$

So the truth table for both formulae looks like this:

F G	$F \vee G$	$F \wedge (F \vee G)$
0 0	0	0
0 1	1	0
1 0	1	1
1 1	1	1

2.

$F \wedge (G \vee H) \equiv (F \wedge G) \vee (F \wedge H)$

F G H	$G \vee H$	$F \wedge (G \vee H)$	$F \wedge G$	$F \wedge H$	$(F \wedge G) \vee (F \wedge H)$
0 0 0	0	0	0	0	0
0 0 1	1	0	0	0	0
0 1 0	1	0	0	0	0
0 1 1	1	0	0	0	0
1 0 0	0	0	0	0	0
1 0 1	1	1	0	1	1
1 1 0	1	1	1	0	1
1 1 1	1	1	1	1	1

$F \vee (G \wedge H) \equiv (F \vee G) \wedge (F \vee H)$

F G H	$G \wedge H$	$F \vee (G \wedge H)$	$F \vee G$	$F \vee H$	$(F \vee G) \wedge (F \vee H)$
0 0 0	0	0	0	0	0
0 0 1	0	0	0	1	0
0 1 0	0	0	1	0	0
0 1 1	1	1	1	1	1
1 0 0	0	1	1	1	1
1 0 1	0	1	1	1	1
1 1 0	0	1	1	1	1
1 1 1	1	1	1	1	1

3.

$\neg(F \wedge G) \equiv \neg F \vee \neg G$

F G	$F \wedge G$	$\neg F \vee \neg G$
0 0	0	1
0 1	0	1
1 0	0	1
1 1	1	0

$\neg(F \vee G) \equiv \neg F \wedge \neg G$

F G	$F \vee G$	$\neg F \wedge \neg G$
0 0	0	1
0 1	1	0
1 0	1	0
1 1	1	0

Exercise 6: (Two proofs)

Show $F_1 = (A \vee \neg(B \wedge A)) \wedge (C \vee (D \vee C)) = C \vee D = F_2$

(a) by truth table

A B C D	$\neg(B \wedge A)$	$A \vee \neg(B \wedge A) = G_1$	$D \vee C$	$C \vee (D \vee C) = G_2$	$F_1 = G_1 \wedge G_2$	$F_2 = C \vee D$
0 0 0 0	1	1	0	0	0	0
0 0 0 1	1	1	1	1	1	1
0 0 1 0	1	1	1	1	1	1
0 0 1 1	1	1	1	1	1	1
0 1 0 0	1	1	0	0	0	0
0 1 0 1	1	1	1	1	1	1
0 1 1 0	1	1	1	1	1	1
0 1 1 1	1	1	1	1	1	1
1 0 0 0	1	1	0	0	0	0
1 0 0 1	1	1	1	1	1	1
1 0 1 0	1	1	1	1	1	1
1 0 1 1	1	1	1	1	1	1
1 1 0 0	0	1	0	0	0	0
1 1 0 1	0	1	1	1	1	1
1 1 1 0	0	1	1	1	1	1
1 1 1 1	0	1	1	1	1	1

(b) by transformation

$F_1 = (A \vee \neg(B \wedge A)) \wedge (C \vee (D \vee C))$

$\overset{deMorgan}{=} (A \vee (\neg B \vee \neg A)) \wedge (C \vee (D \vee C))$

$\overset{Assoc.}{=} (A \vee \neg B \vee \neg A) \wedge (C \vee D \vee C)$

$\overset{Idemp.}{=} (A \vee \neg B \vee \neg A) \wedge (C \vee D)$

$\overset{Tautology}{=} C \vee D = F_2$

Exercise 7: (CNF and DNF)

$F = (\neg A \implies B) \vee ((A \vee \neg C) \Leftrightarrow B)$
$= (\neg\neg A \vee B) \vee (((A \wedge \neg C) \wedge B) \vee (\neg(A \wedge \neg C) \wedge \neg B))$

1.
Replace Double negations
$\Leftrightarrow (A \vee B) \vee (((A \wedge \neg C) \wedge B) \vee (\neg(A \wedge \neg C) \wedge \neg B))$

Apply De Morgan's rules
$\Leftrightarrow A \vee B \vee (A \wedge \neg C \wedge B) \vee ((\neg A \vee C) \wedge \neg B)$

2.
DNF
$\Leftrightarrow A \vee B \vee (A \wedge \neg C \wedge B) \vee (\neg A \wedge B) \vee (C \wedge \neg B)$

CNF
$\Leftrightarrow A \vee B \vee (((A \wedge \neg C \wedge B) \vee (\neg A \vee C)) \wedge (\neg B \vee (A \wedge \neg C \wedge B)))$
$\Leftrightarrow A \vee B \vee (((\underbrace{A \vee (\neg A \vee C)}_{Tautology}) \wedge (B \vee (\neg A \vee C) \wedge (\underbrace{\neg C \vee (\neg A \vee C)}_{Tautology})) \wedge ((\neg B \vee A) \wedge (\neg B \vee C) \wedge$
$(\underbrace{\neg B \vee B}_{Tautology})))$
$\Leftrightarrow A \vee B \vee ((B \vee \neg A \vee C) \wedge (\neg B \vee A) \wedge (\neg B \vee C))$
$\Leftrightarrow A \vee ((B \vee B \vee \neg A \vee C) \wedge (\underbrace{B \vee \neg B \vee A}_{Tautology}) \wedge (\underbrace{B \vee \neg B \vee C}_{Tautology}))$
$\Leftrightarrow A \vee (\underbrace{B \vee B \vee \neg A \vee C}_{Tautology}) = 1$

Exercise 8: (Switch and and or)