CSE 211: Discrete Mathematics **(Due: 17/01/21)**

Homework #4

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Moodle.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

Problem 1 (15+15=30 points)

Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

(a) Show that whether $a_n = -2^{n+1}$ is a solution of the given recurrence relation or not. Show your work step by step.
(Solution)
$a_n = -2^{n+1} \Rightarrow$a$_{n-1} = -2^n$
$a_n = 3a_{n-1} + 2^n = 3. - 2^n + 2^n = -2.2^n = -2^{n+1}$

(b) Find the solution with $a_0 = 1$.
(Solution)

$P_n = c2^n$, $P_n = 3P_{n-1} + 2^n$
$\frac{c2^n}{2^{n-1}} = \frac{3c2^{n-1}}{2^{n-1}} + \frac{2^n}{2^{n-1}}$
2c = 3c + 2
c = -2
$a_n^{(p)}$ = -2.2$_n$
$a_n = 3a_{n-1}$
r = 3
$a_n^{(p)} = \alpha\ 3^n$
a_n = -2.2^n + $\alpha\ 3^n$
a_0 = 1 = -2 + $\alpha\ 3^0$ = -2 + α
$\alpha = 3$
$a_n = -2^{n+1} + 3^{n+1}$

Problem 2 (35 points)

Solve the recurrence relation f(n) = 4f(n-1) - 4f(n-2) + n^2 for f(0) = 2 and f(1) = 5.
(Solution)

a_n = 4a_{n-1} - 4a_{n-2} + n^2
F(n) = n^2
P_n = cn^2 + dn + e, P_n = 4P_{n-1} - 4P_{n-2} + n^2
cn^2 + dn + e = 4(c$(n-1)^2$ + d(n-1) + e) -4(c$(n-2)^2$ + d(n-2) + e) + n^2
cn^2 + dn + e = n^2 + 8cn -12c +4d
c-1 = 0 $\Rightarrow c = 1$
d-8c = 0
d-8 = 0 $\Rightarrow d = 8$
e + 12c -4d = 0
e + 12 - 32 = 0 $\Rightarrow e = 20$
$a_n^{(p)}$ = n^2 + 8n + 20
a_n = 4a_{n-1} - 4a_{n-2}
r^2 = 4r - 4
r^2 - 4r + 4 = 0
$r_{1,2}$ = 2
$a_n^{(h)}$ = (α_1 + α_2n)2^n
a_n = $a_n^{(p)}$ + $a_n^{(h)}$
a_n = n^2 + 8n + 20 + (α_1 + α_2n)2^n
a_0 = 2 = 20 + α_1
α_1 = -18
a_1 = 5 = 1 + 8 + 20 + 2 (α_2 - 18)
-24 = 2 (α_2 - 18)
-12 = α_2 - 18
α_2 = 6
a_n = n^2 + 8n + 20 + (-18 + 6n)2^n

Problem 3 (20+15 = 35 points)

Consider the linear homogeneous recurrence relation a_n = 2a_{n-1} - 2a_{n-2}.
(a) Find the characteristic roots of the recurrence relation.
(Solution)

a_n = 2a_{n-1} - 2a_{n-2}
a_n = r^k
r^n = 2$r^n - 1$ - 2$r^n - 2$
r^k - 2$r^k - 1$ + 2$r^k - 2 = 0$
r^2 - 2r + 2 = 0
r_1 = 1 - i
r_2 = 1 + i

(b) Find the solution of the recurrence relation with a_0 = 1 and a_1 = 2.
(Solution)

$a_n = \alpha_1\ (1-i)^n + \alpha_2\ (1+i)^n$
$a_0 = \alpha_1 + \alpha_2 = 1$
$a_1 = \alpha_1\ (1-i) + \alpha_2\ (1+i) = 2$
$\alpha_1 = \frac{1+i}{2}$
$\alpha_2 = \frac{1-i}{2}$
$a_n = \frac{(1+i)(1-i)^n+(1-i)(1+i)^n}{2}$