1.a.
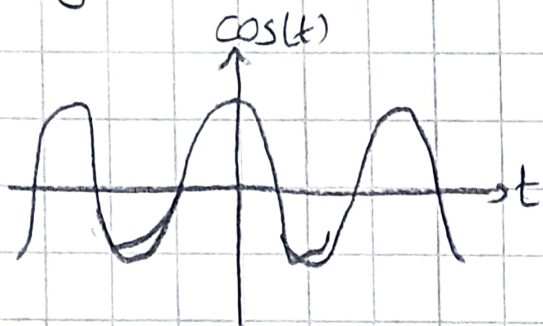
$y(t) = \cos(x(t))$ → the output depends on the
input at that specific time

$\cos(t)$

→ memoryless

→ the outputs are bounded by t
cosinus is oscillating $[-1, 1]$
→ stable

from the graph
and the Taylor
series of cos function

→ the output does not depend
on future values
→ causal

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \cdots$$

we can clearly see that
there's no linear combination

$ax_1(t) + b\,x_2(t) \to ay_1(t) + by_2(t)$

→ non-linear

$y(t) \to \cos(x(t))$
$y(t-t_0) \to \cos(x(t-t_0))$

→ output does not change
the same amount of time

→ time-variant

1.b.

$$y[n] = 2x[n]\,u[n]$$

→ the output does not depend on previous <u>memoryless</u> values

→ we cannot bound outputs based on <u>unstable</u> inputs

→ output does not depend on future <u>causal</u> values

→ $y_1[n] + y_2[n] = 2u[n]\,x_1[n] + 2u[n]\,x_2[n]$ <u>linear</u>

→ different inputs produce different $x[n]$ values. Therefore $y[n]$ is <u>time-varient</u>

Sena ERSOY
04020004,54
SENA

2.

$$x(t) = u(t+1) - u(t-1)$$

$$y(t) = u(t) + u(t-2) - 2u(t-4)$$

$$m(t) = \int_{-\infty}^{\infty} x(t) * y(t)\, dt = \int_{-\infty}^{\infty} y(t) \, x(t-\tau)\, d\tau$$

$$m(t) = \mathcal{L}^{-1} m(s)$$

$$x(s) = \frac{e^{s}}{s} - \frac{e^{-s}}{s}$$

$$y(s) = \frac{1}{s} + \frac{e^{-2s}}{s} - 2\frac{e^{-4s}}{s}$$

$$m(s) = x(s) \cdot y(s) = \left(\frac{e^{s} - e^{-s}}{s}\right) \frac{1}{s}\left(1 + e^{-2s} - 2e^{-4s}\right)$$

$$m(s) = \frac{1}{s^{2}} e^{s} - \frac{3}{s^{2}} e^{-3s} + \frac{2}{s^{2}} e^{-5s}$$

$$m(t) = (t+1)u(t+1) - 3(t-3)u(t-3) + 2(t-5)u(t-5)$$

$$m(t) = r(t+1) - 3r(t-3) + 2r(t-5)$$

3.

$$\sin\theta = \left[e^{j\theta} - e^{-j\theta}\right]\frac{1}{2j}$$

$$\cos\theta = \left[e^{j\theta} + e^{-j\theta}\right]\frac{1}{2}$$

$$x(t) = \frac{1}{2j}\left[e^{j3\pi t} - e^{-j3\pi t}\right] + \frac{1}{2}\left[e^{j\pi t} + e^{-j\pi t}\right]$$

$$c_k = \frac{-2}{2\pi}\left[\frac{1}{2k-1} - \frac{1}{2k+1}\right] = \frac{-1}{\pi}\left[\frac{2k+1-(2k-1)}{4k^2-1)}\right.$$

$$c_k = \frac{-2}{\pi(4k^2-1)} \qquad \boxed{c_0 = \frac{1}{T}\int_0^{} x(t)\,dt}$$

$$c_0 = \frac{-\cos\pi t}{\pi}\Big|_0^1 = \frac{-[\cos\pi - \cos 0]}{\pi} = \frac{2}{\pi}$$

$$x(t) = \frac{2}{\pi} + \sum_{k=1}^{\infty} c_k\, e^{j\pi k t} \quad , \quad c_k = \frac{-2}{\pi(4k^2-1)}$$

4.

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \qquad\qquad \omega_0 = 2\pi$$

$$c_1 = j \qquad c_{-1} = -j \qquad c_3 = 1 \qquad c_{-3} = -1$$

$$x(t) = j e^{j2\pi t} - j e^{-j2\pi t} + e^{j3\times 2\pi t} + e^{-j3\times 2\pi t}$$

$$x(t) = \frac{-2\left(e^{j2\pi t} - e^{-j2\pi t}\right)}{2j} + \frac{2\left(e^{j6\pi t} + e^{-j6\pi t}\right)}{2}$$

$$\boxed{x(t) = 2\cos 6\pi t - 2\sin 2\pi t}$$

5.

$$x(t) = \begin{cases} 1 & : \; |t| < a \\ 0 & : \; |t| > a \end{cases}$$

$$x(t) = 1 \qquad -\infty \leq t < a$$

$$x(t) = 0 \qquad a \leq t < \infty$$

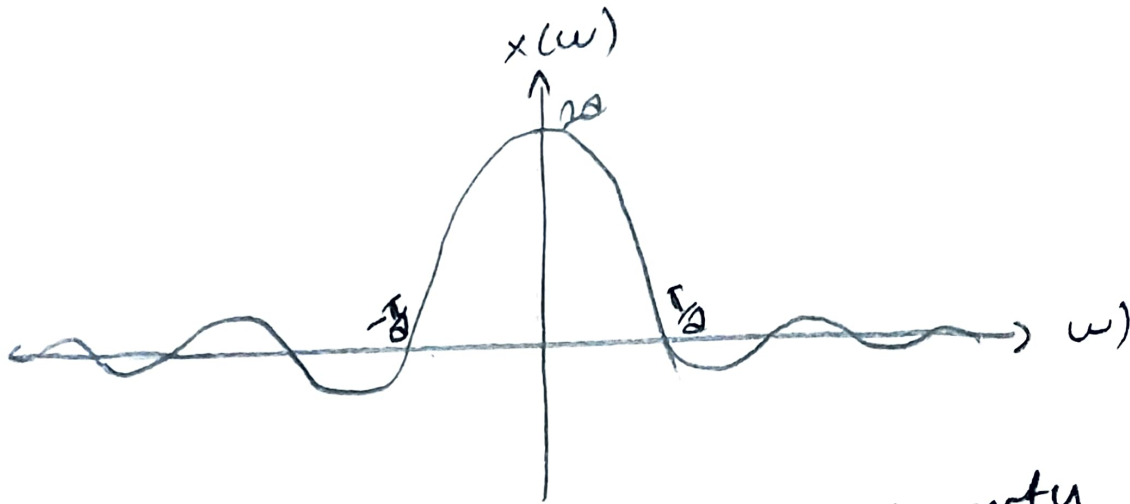$$x(\omega) = \int_{-\infty}^{\infty} x(t) \, e^{-j\omega t} \, dt$$

$x(t)$ between $-a$ and $+a$

$$x(\omega) = \int_{-a}^{a} e^{-j\omega t} \, dt = \left[\frac{e^{-j\omega t}}{j\omega} \right] \Big|_{-a}^{a}$$

$$= \left[\frac{e^{j\textcircled{a\omega}} - e^{-j\textcircled{a\omega}}}{-j\omega} \right] \qquad a\omega = x \text{ let's say}$$

$$= \underbrace{\qquad\qquad\qquad}$$

$$= \frac{2 \sin(a\omega)}{\omega} = 2a \boxed{\frac{\sin(a\omega)}{a\omega}} \qquad \begin{array}{l} \text{sinc}(x) \\ \hookrightarrow x = a\omega \end{array}$$

$X(\omega)$

$2a$

$-\frac{\pi}{a}$ $\frac{\pi}{a}$

$\omega)$

x

$x(t-a)$

1

0 a $2a$ t

shifting property

FT of $x(t-t_0) = e^{-j\omega t_0} X(\omega)$

$t_0 = a$

FT of $x(t-a) = e^{-j\omega a} \, 2a \, \dfrac{\sin a\omega}{a\omega}$

6.

$$X(j\omega) = e^{-2|\omega|} = \begin{cases} e^{-2\omega} & \omega \geq 0 \\ e^{2\omega} & \omega < 0 \end{cases}$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} \, d\omega$$

$$= \frac{1}{2\pi} \left[\int_{-\infty}^{0} e^{2\omega} \cdot e^{j\omega t} \, d\omega + \int_{0}^{\infty} e^{-2\omega} \cdot e^{j\omega t} \, d\omega \right]$$

$$= \frac{1}{2\pi} \left[\frac{e^{(2+jt)\omega}}{2+jt} \Big|_{-\infty}^{0} + \frac{e^{-(2-jt)\omega}}{-(2-jt)} \Big|_{0}^{\infty} \right]$$

$$= \frac{1}{2\pi} \left[\frac{e^{0} - e^{-\infty}}{2+jt} + \frac{e^{-\infty} - e^{0}}{-(2-jt)} \right]$$

$$= \frac{1}{2\pi} \left[\frac{1}{\underset{(2-jt)}{2+jt}} + \frac{1}{\underset{(2+jt)}{2-jt}} \right] = \frac{1}{2\pi} \left[\frac{4}{4+t^2} \right]$$

$$x(t) = \frac{2}{\pi(4+t^2)}$$