

# Introduction to Digital Design

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November 5, 2023

# Karnaugh Maps (K-map)

- ▶ A K-map is a collection of squares
  - ▶ Each square represents a minterm
  - ▶ The collection of squares is a graphical representation of a Boolean function
  - ▶ Adjacent squares differ in the value of one variable
  - ▶ Alternative algebraic expressions for the same function are derived by recognizing patterns of squares
- ▶ The K-map can be viewed as
  - ▶ A reorganized version of the truth table
  - ▶ A topologically-warped Venn diagram as used to visualize sets in algebra of sets

# Two Variable Karnaugh Map

- ▶ Two variable:  $x$  and  $y$ 
  - ▶ 4 minterms:
    - ▶  $m_0 = x'y' \rightarrow 00$
    - ▶  $m_1 = x'y \rightarrow 01$
    - ▶  $m_2 = xy' \rightarrow 10$
    - ▶  $m_3 = xy \rightarrow 11$

x \ y	0	1
0	$m_0$	$m_1$
1	$m_2$	$m_3$

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- ▶ Two variable:  $x$  and  $y$ 
  - ▶ 4 minterms:
    - ▶  $m_0 = x'y' \rightarrow 00$
    - ▶  $m_1 = x'y \rightarrow 01$
    - ▶  $m_2 = xy' \rightarrow 10$
    - ▶  $m_3 = xy \rightarrow 11$

▶ **Example:**

$x$	$y$	$f$
0	0	0
0	1	1
1	0	1
1	1	0

$x \backslash y$	0	1
0	$m_0$	$m_1$
1	$m_2$	$m_3$

$x \backslash y$	0	1
0	0	1
1	1	0

# K-Map Function Representation

- ▶ Example:  $f(x, y) = x$

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- Example:  $f(x, y) = x$

		$y$	
		0	1
$x$	0		
	1		

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- Example:  $f(x, y) = x$

		$y$	
		0	1
$x$	0	0	
	1		

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- Example:  $f(x, y) = x$

		$y$	
		0	1
$x$	0	0	0
	1		



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		$y$	
		0	1
$x$	0	0	0
	1	1	

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- Example:  $f(x, y) = x$

		$y$	
		0	1
$x$	0	0	0
	1	1	1

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- Example:  $f(x, y) = x$

		$y$	
		0	1
$x$	0	0	0
	1	1	1

# K-Map Function Representation

► Example:  $f(x, y) = x$

		y	
		0	1
x	0	0	0
	1	1	1

For function  $f(x, y)$ , the two adjacent cells containing 1's can be combined using the Minimization Theorem:

$$f(x, y) = xy' + xy = x$$

# K-Map Function Representation

- ▶ **Example:**  $g(x, y) = x + y$

# K-Map Function Representation

- **Example:**  $g(x, y) = x + y$

		$y$	
		0	1
$x$	0		
	1		

# K-Map Function Representation

- Example:  $g(x, y) = x + y$

		$y$	
		0	1
$x$	0	0	
	1		

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- Example:  $g(x, y) = x + y$

		$y$	
		0	1
$x$	0	0	1
	1		



# K-Map Function Representation

- Example:  $g(x, y) = x + y$

		$y$	
		0	1
$x$	0	0	1
	1	1	

# K-Map Function Representation

- Example:  $g(x, y) = x + y$

x \ y	0	1
0	0	1
1	1	1

# K-Map Function Representation

- Example:  $g(x, y) = x + y$

		y	
		0	1
x	0	0	1
	1	1	1

# K-Map Function Representation

- Example:  $g(x, y) = x + y$

		y	
		0	1
x	0	0	1
	1	1	1

$$x'y + xy = y$$

# K-Map Function Representation

- Example:  $g(x, y) = x + y$

		y	
x	0	0	1
	1	0	1
0	0	1	1
1	1	1	1

$$x'y + xy = y$$

# K-Map Function Representation

- Example:  $g(x, y) = x + y$

		$y$	
$x$		0	1
	0	0	1
	1	1	1

$xy' + xy = x$

$x'y + xy = y$

# K-Map Function Representation

► Example:  $g(x, y) = x + y$

		y	
		0	1
x	0	0	1
	1	1	1

$xy' + xy = x$   
 $x'y + xy = y$

For function  $g(x, y)$ , the two adjacent cells containing 1's can be combined using the Minimization Theorem:

$$g(x, y) = (x'y + xy) + (xy' + xy) = x + y$$

# Three Variable Karnaugh Map

Three variable:  $x, y, z$

8 minterms:

$$m_0 = x'y'z' \rightarrow 000$$

$$m_1 = x'y'z \rightarrow 001$$

$$m_2 = x'yz' \rightarrow 010$$

$$m_3 = x'yz \rightarrow 011$$

$$m_4 = xyz' \rightarrow 100$$

$$m_5 = xyz \rightarrow 101$$

$$m_6 = xyz' \rightarrow 110$$

$$m_7 = xyz \rightarrow 111$$

		xy			
		00	01	11	10
z	0	$m_0$	$m_1$	$m_3$	$m_2$
	1	$m_4$	$m_5$	$m_7$	$m_6$



# Three Variable Karnaugh Map

Example:

x	y	z	$f_1$	$f_2$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

		xy			
		00	01	11	10
z	0	0	0	1	0
	1	0	1	1	1

For  $f_1$

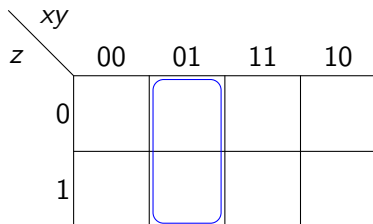
		xy			
		00	01	11	10
z	0	0	1	0	1
	1	1	0	1	0

For  $f_2$

# Three Variable Karnaugh Map

$xy$		$z$			
		00	01	11	10
$z$	0				
	1				

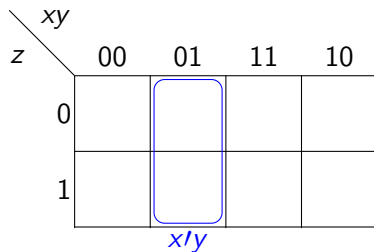
# Three Variable Karnaugh Map



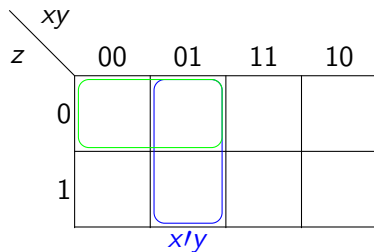
A three-variable Karnaugh map for variables x, y, and z. The map is a 2x4 grid. The vertical axis is labeled 'z' with values 0 and 1. The horizontal axis is labeled 'xy' with values 00, 01, 11, and 10. A blue rounded rectangle highlights the two cells in the '01' column, representing the expression  $xy$ .

z \ xy	00	01	11	10
0				
1				

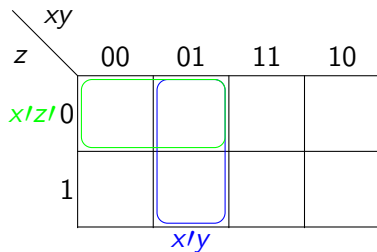
# Three Variable Karnaugh Map



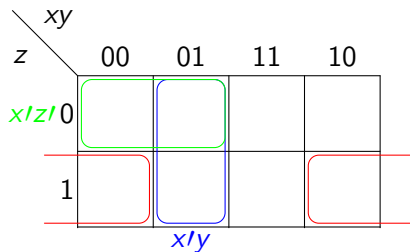
# Three Variable Karnaugh Map



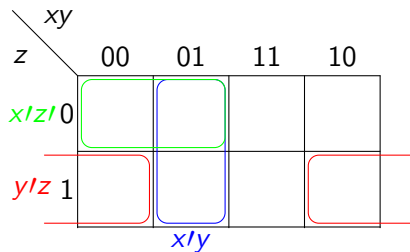
# Three Variable Karnaugh Map



# Three Variable Karnaugh Map

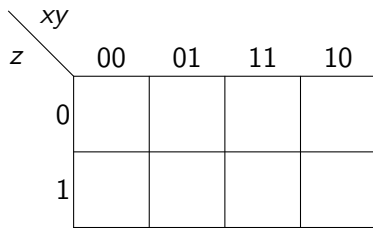
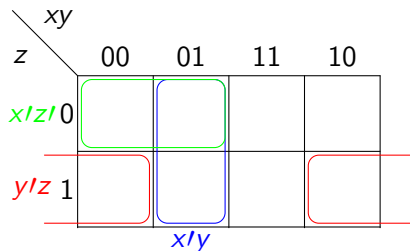


# Three Variable Karnaugh Map

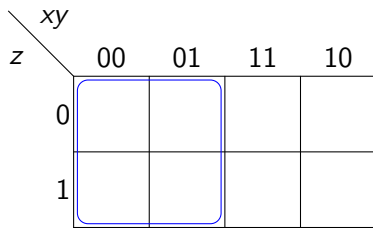
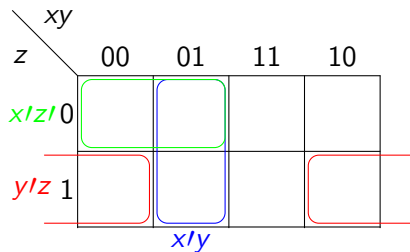




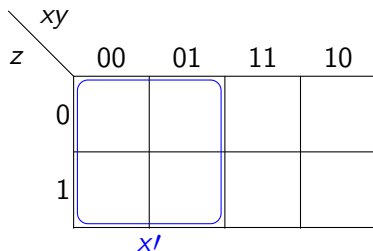
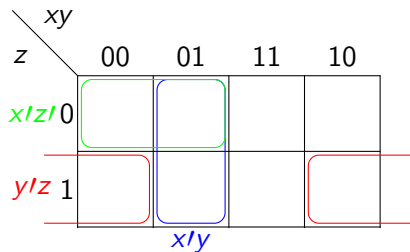
# Three Variable Karnaugh Map



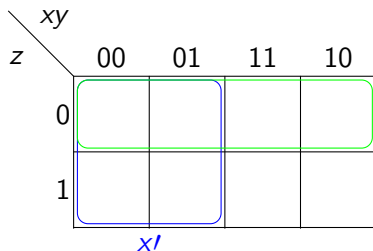
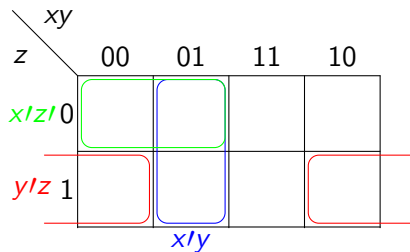
# Three Variable Karnaugh Map



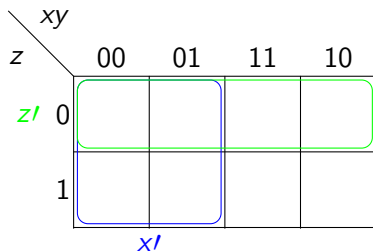
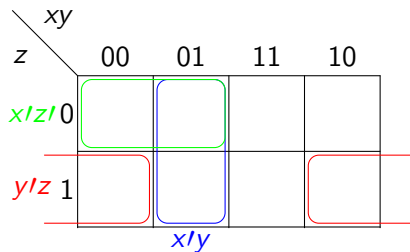
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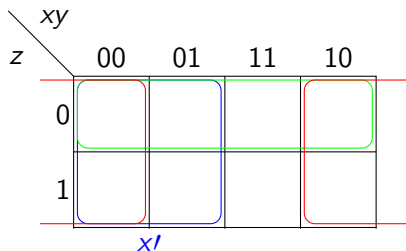
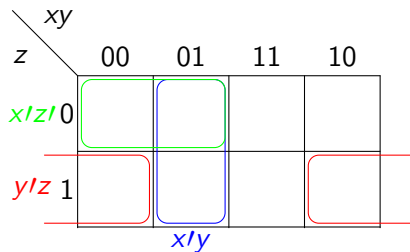
# Three Variable Karnaugh Map



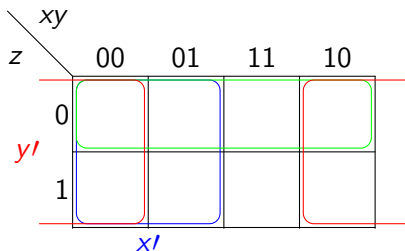
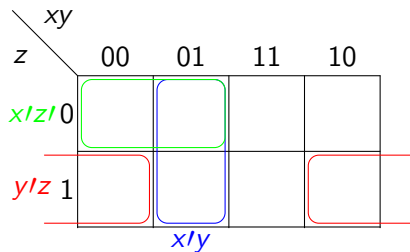
# Three Variable Karnaugh Map



# Three Variable Karnaugh Map



# Three Variable Karnaugh Map



Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

$xy$					
$z$		00	01	11	10
	0				
	1				

	$x$	$y$	$z$



Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

$xy$					
$z$		00	01	11	10
0					
1	1				

	$x$	$y$	$z$
1	0	0	1

Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

$xy$					
$z$		00	01	11	10
	0		1		
1	1				

	$x$	$y$	$z$
1	0	0	1
2	0	1	0

Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

$xy$					
$z$		00	01	11	10
	0		1		
1	1				

	$x$	$y$	$z$
1	0	0	1
2	0	1	0

Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

$xy$					
$z$		00	01	11	10
	0		1		
1	1	1	1		

	$x$	$y$	$z$
1	0	0	1
2	0	1	0
3	0	1	1

Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

$xy \backslash z$					
		00	01	11	10
$z$	0		1		
	1	1	1		1

	$x$	$y$	$z$
1	0	0	1
2	0	1	0
3	0	1	1
5	1	0	1

Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

$xy \backslash z$					
		00	01	11	10
$z$	0		1		
	1	1	1		1

	$x$	$y$	$z$
1	0	0	1
2	0	1	0
3	0	1	1
5	1	0	1

Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

$xy$					
$z$		00	01	11	10
	0		1		
1	1	1	1	1	1

	$x$	$y$	$z$
1	0	0	1
2	0	1	0
3	0	1	1
5	1	0	1
7	1	1	1

Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

$xy \backslash z$					
		00	01	11	10
$z$	0		1		
	1	1	1	1	1

	x	y	z				x	y	z
1	0	0	1						
2	0	1	0						
3	0	1	1						
5	1	0	1						
7	1	1	1						



Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

xy \ z		xy			
		00	01	11	10
z	0		1		
	1	1	1	1	1

	x	y	z			x	y	z
✓	1	0	0	1	1,3	0	-	1
	2	0	1	0				
✓	3	0	1	1				
	5	1	0	1				
	7	1	1	1				

Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

xy z		00	01	11	10
		0	1	1	1
0	1		1		
	1	1	1	1	1

	x	y	z			x	y	z	
✓	1	0	0	1		1,3	0	-	1
	2	0	1	0		1,5	-	0	1
✓	3	0	1	1					
✓	5	1	0	1					
	7	1	1	1					

Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

$xy \backslash z$					
		00	01	11	10
$z$	0		1		
	1	1	1	1	1

	x	y	z			x	y	z
✓ 1	0	0	1		1,3	0	-	1
✓ 2	0	1	0		1,5	-	0	1
✓ 3	0	1	1		2,3	0	1	-
✓ 5	1	0	1					
7	1	1	1					

Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

$xy \backslash z$					
		00	01	11	10
$z$	0		1		
	1	1	1	1	1

	x	y	z			x	y	z	
✓	1	0	0	1		1,3	0	-	1
✓	2	0	1	0		1,5	-	0	1
✓	3	0	1	1		2,3	0	1	-
✓	5	1	0	1		3,7	-	1	1
✓	7	1	1	1					

Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

$xy \backslash z$					
		00	01	11	10
z	0		1		
	1	1	1	1	1

	x	y	z			x	y	z	
✓	1	0	0	1		1,3	0	-	1
✓	2	0	1	0		1,5	-	0	1
✓	3	0	1	1		2,3	0	1	-
✓	5	1	0	1		3,7	-	1	1
✓	7	1	1	1		5,7	1	-	1

Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

xy \ z		xy			
		00	01	11	10
z	0		1		
	1	1	1	1	1

	x	y	z			x	y	z			x	y	z
✓	1	0	0	1		1,3	0	-	1				
✓	2	0	1	0		1,5	-	0	1				
✓	3	0	1	1		2,3	0	1	-				
✓	5	1	0	1		3,7	-	1	1				
✓	7	1	1	1		5,7	1	-	1				

Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

xy \ z		xy			
		00	01	11	10
z	0		1		
	1	1	1	1	1

	x	y	z			x	y	z			x	y	z
✓	1	0	0	1	✓	1,3	0	-	1				
✓	2	0	1	0		1,5	-	0	1				
✓	3	0	1	1		2,3	0	1	-				
✓	5	1	0	1		3,7	-	1	1				
✓	7	1	1	1		5,7	1	-	1				

Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

xy					
z		00	01	11	10
	0		1		
	1	1	1	1	1

	x	y	z			x	y	z			x	y	z
✓	1	0	0	1	✓	1,3	0	-	1	1,3,5,7	-	-	1
✓	2	0	1	0		1,5	-	0	1				
✓	3	0	1	1		2,3	0	1	-				
✓	5	1	0	1		3,7	-	1	1				
✓	7	1	1	1	✓	5,7	1	-	1				



Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

$xy \backslash z$					
		00	01	11	10
$z$	0		1		
	1	1	1	1	1

	x	y	z			x	y	z			x	y	z
✓	1	0	0	1	✓	1,3	0	-	1	1,3,5,7	-	-	1
✓	2	0	1	0	✓	1,5	-	0	1				
✓	3	0	1	1		2,3	0	1	-				
✓	5	1	0	1		3,7	-	1	1				
✓	7	1	1	1	✓	5,7	1	-	1				

Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

xy \ z		xy			
		00	01	11	10
z	0		1		
	1	1	1	1	1

	x	y	z			x	y	z			x	y	z	
✓	1	0	0	1	✓	1,3	0	-	1		1,3,5,7	-	-	1
✓	2	0	1	0	✓	1,5	-	0	1		1,5,3,7	-	-	1
✓	3	0	1	1		2,3	0	1	-					
✓	5	1	0	1	✓	3,7	-	1	1					
✓	7	1	1	1	✓	5,7	1	-	1					

Example:  $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

$xy$					
$z$		00	01	11	10
	0		1		
	1	1	1	1	1
		$x/y$			

	x	y	z		x	y	z		x	y	z		
✓	1	0	0	1	✓	1,3	0	-	1	1,3,5,7	-	-	1
✓	2	0	1	0	✓	1,5	-	0	1	1,5,3,7	-	-	1
✓	3	0	1	1	A	2,3	0	1	-				
✓	5	1	0	1	✓	3,7	-	1	1				
✓	7	1	1	1	✓	5,7	1	-	1				

Example:  $f(x, y, z) = \sum_m(1, 2, 3, 5, 7)$

$xy$ $z$		00	01	11	10
0			1		
1	1	1	1	1	1
		$x/y$			

	x	y	z		x	y	z		x	y	z			
✓	1	0	0	1	✓	1,3	0	-	1	B	1,3,5,7	-	-	1
✓	2	0	1	0	✓	1,5	-	0	1		1,5,3,7	-	-	1
✓	3	0	1	1	A	2,3	0	1	-					
✓	5	1	0	1	✓	3,7	-	1	1					
✓	7	1	1	1	✓	5,7	1	-	1					

## Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$$

$xy$		00	01	11	10
$z$	0		1		1
	1		1		1

## Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$$

$xy$		00	01	11	10
$z$	0		1		1
	1		1		1

$x/y$

## Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$$

$xy$		00	01	11	10
$z$	0		1		1
	1		1		1

$x'y$        $xy'$

## Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$$

$xy \backslash z$					
		00	01	11	10
0			1		1
1			1		1

$x'y$                        $xy'$

$$F_1(x, y, z) = x'y + xy'$$



## Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$$

$xy \backslash z$					
		00	01	11	10
0			1		1
1			1		1

$x'y$        $xy'$

$$F_2(x, y, z) = \Sigma(3, 4, 6, 7)$$

$xy \backslash z$					
		00	01	11	10
0				1	1
1			1	1	

$$F_1(x, y, z) = x'y + xy'$$

## Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$$

$xy \backslash z$		00	01	11	10
		0	1		1
	1		1		1

$x'y$        $xy'$

$$F_2(x, y, z) = \Sigma(3, 4, 6, 7)$$

$xy \backslash z$		00	01	11	10
		0		1	1
	1		1	1	

$xz'$

$$F_1(x, y, z) = x'y + xy'$$

## Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$$

$xy \backslash z$		00	01	11	10
		0	1		1
	1		1		1

$x'y$        $xy'$

$$F_2(x, y, z) = \Sigma(3, 4, 6, 7)$$

$xy \backslash z$		00	01	11	10
		0		1	1
	1		1	1	

$xz'$        $yz$

$$F_1(x, y, z) = x'y + xy'$$

## Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$$

$xy \backslash z$		00	01	11	10
		0	1		1
	1		1		1

$x'y$  (blue)       $xy'$  (green)

$$F_1(x, y, z) = x'y + xy'$$

$$F_2(x, y, z) = \Sigma(3, 4, 6, 7)$$

$xy \backslash z$		00	01	11	10
		0		1	1
	1		1	1	

$xz'$  (blue)       $yz$  (green)

$$F_2(x, y, z) = xz' + yz$$

## Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

$x \backslash yz$					
		00	01	11	10
0		1			1
1		1	1		1

## Example: Three Variable Karnaugh Map

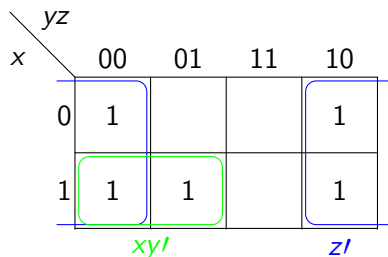
$$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

$yz$		00	01	11	10
$x$	0	1			1
	1	1	1		1

$z'$

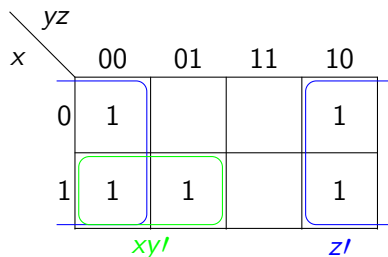
## Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$



## Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$



$$F_1(x, y, z) = z' + xy'$$



## Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

$$F_2(x, y, z) = \Sigma(0, 1, 2, 4, 6, 7)$$

$yz$		00	01	11	10
$x$		1			1
0		1			1
1		1	1		1

$xy'$   $z'$

$yz$		00	01	11	10
$x$					
0		1	1		1
1		1		1	1

$$F_1(x, y, z) = z' + xy'$$

## Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

$$F_2(x, y, z) = \Sigma(0, 1, 2, 4, 6, 7)$$

$x \backslash yz$					
		00	01	11	10
0	1				1
1	1	1			1

$xy'$  (green box)       $z'$  (blue box)

$x \backslash yz$					
		00	01	11	10
0	1	1			1
1	1			1	1

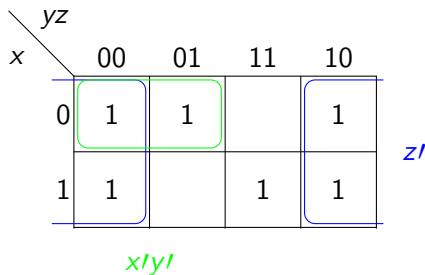
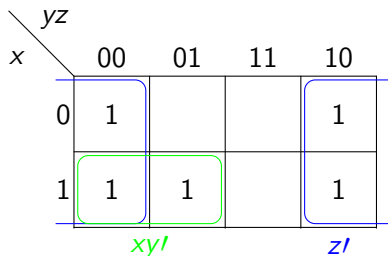
$z'$  (blue box)

$$F_1(x, y, z) = z' + xy'$$

## Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

$$F_2(x, y, z) = \Sigma(0, 1, 2, 4, 6, 7)$$

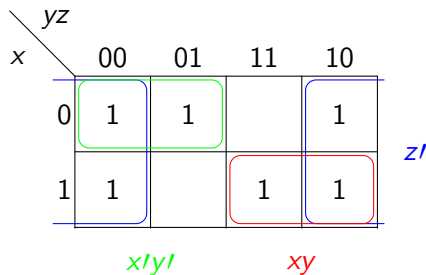
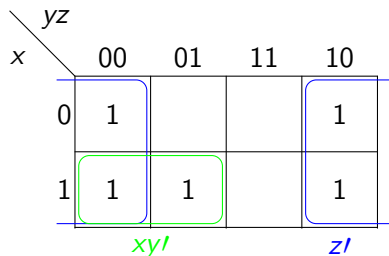


$$F_1(x, y, z) = z' + xy'$$

## Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

$$F_2(x, y, z) = \Sigma(0, 1, 2, 4, 6, 7)$$

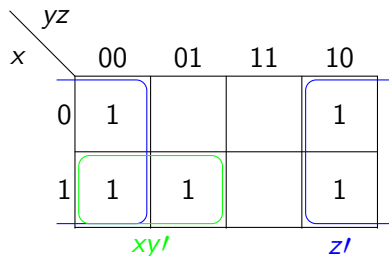


$$F_1(x, y, z) = z' + xy'$$

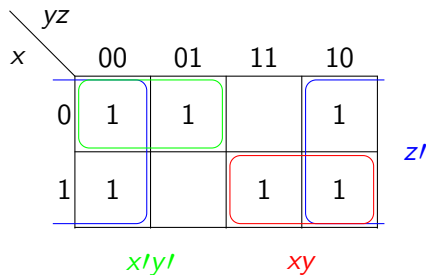
## Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

$$F_2(x, y, z) = \Sigma(0, 1, 2, 4, 6, 7)$$



$$F_1(x, y, z) = z' + xy'$$



$$F_2(x, y, z) = z' + x'y' + xy$$

# Four Variable Karnaugh Map

Four variable:  $x, y, z, t$

16 minterms:

$$m_0 = x/y/z/t/ \rightarrow 0000$$

$$m_1 = x/y/z/t \rightarrow 0001$$

$$m_2 = x/y/z/t/ \rightarrow 0010$$

$$m_3 = x/y/z/t \rightarrow 0011$$

$$m_4 = x/y/z/t/ \rightarrow 0100$$

$$m_5 = x/y/z/t \rightarrow 0101$$

$$m_6 = x/y/z/t/ \rightarrow 0110$$

$$m_7 = x/y/z/t \rightarrow 0111$$

$$m_8 = xy/z/t/ \rightarrow 1000$$

$$m_9 = xy/z/t \rightarrow 1001$$

$$m_{10} = xy/z/t/ \rightarrow 1010$$

$$m_{11} = xy/z/t \rightarrow 1011$$

$$m_{12} = xyz/t/ \rightarrow 1100$$

$$m_{13} = xyz/t \rightarrow 1101$$

$$m_{14} = xyzt/ \rightarrow 1110$$

$$m_{15} = xyzt \rightarrow 1111$$

		xy			
		00	01	11	10
zt	00	$m_0$	$m_1$	$m_3$	$m_2$
	01	$m_4$	$m_5$	$m_7$	$m_6$
	11	$m_{12}$	$m_{13}$	$m_{15}$	$m_{14}$
	10	$m_8$	$m_9$	$m_{11}$	$m_{10}$

# Four Variable Karnaugh Map

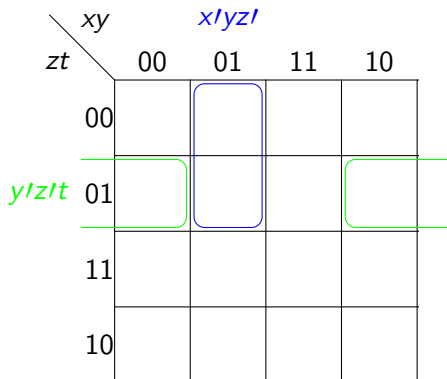
$xy$					
$zt$		00	01	11	10
	00				
	01				
	11				
	10				

# Four Variable Karnaugh Map

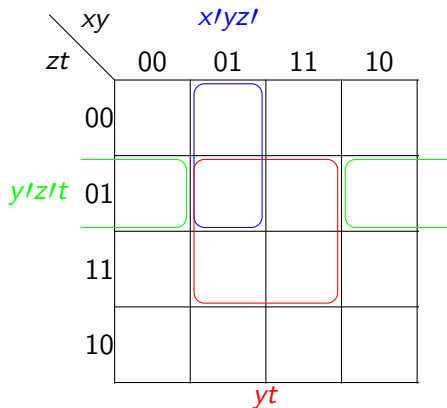
$xy$		$x/yz'$			
$zt$		00	01	11	10
00					
01					
11					
10					



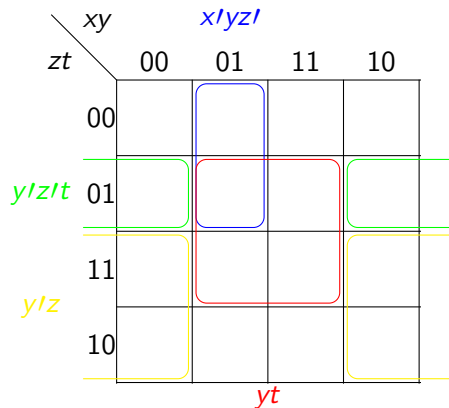
# Four Variable Karnaugh Map



# Four Variable Karnaugh Map



# Four Variable Karnaugh Map



# Four Variable Karnaugh Map

