Homework I

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Q1

If it's irrotational, the curl $F = 0$

$$\begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x+2y+az & bx-3y-z & 4x+cy+2z \end{vmatrix}$$

$$\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)\vec{a_x}+\left(\frac{\partial A_z}{\partial x}-\frac{\partial A_x}{\partial z}\right)\vec{a_y}+\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)\vec{a_z}=0$$

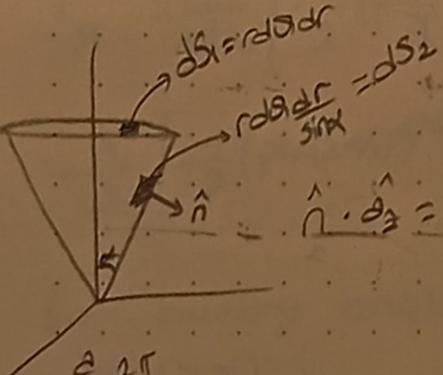
$$(c+1)\,\vec{a_x}+(4-a)\,\vec{a_y}+(b-2)\,\vec{a_z}=0$$

$c=-1 \qquad a=4 \qquad b=2$

Q2

$$\nabla\cdot A=\frac{\partial}{\partial z}(x^2+y^2)=0$$

volume integral of B over any volume is zero



$\hat{n}\cdot\hat{a_z}=-\sin\alpha$

$x^2+y^2=r^2$

$$\int_0^a\int_0^{2\pi} r^2\left(r\,d\theta\,\frac{dr}{\sin\alpha}\right)(-\sin\alpha)$$

$$S_2=-\frac{a^4}{4}\cdot 2\pi$$

$$\int_0^a\int_0^{2\pi} r^2(r\,d\theta\,dr)(\hat{a_z}\cdot\hat{a_z})$$

$$S_1=\frac{\pi\cdot a^4}{4}$$

$$\iiint \underbrace{\nabla\cdot A}_{0}\,dV=\iint A\cdot dS$$

$$=\cancel{\frac{\pi a^4}{4}}-\cancel{\frac{\pi a^4}{4}}$$ ✓ Divergence Theorem verified

$\text{curl } A = 2y\,\vec{a_x}+2x\,\vec{a_y}$ } not conservative, thus it cannot be expressed as grad of a scalar field

Q3

$$\int_0^a\int_0^{2\Phi_0} \frac{q_{s0}\, d\theta\, dr}{4\pi\varepsilon_0 a} = \int_0^{2\Phi_0} \frac{q_{\ell 0}\cdot a \cdot d\theta}{4\pi\varepsilon_0 a^2}$$ → the line charge distribution contributes the same

$$\frac{q_{s0}\,\cancel{2\Phi_0\, a}}{\cancel{4\pi\varepsilon_0 a}} = \frac{q_{\ell 0}\,\cancel{2\Phi_0}}{\cancel{4\pi\varepsilon_0 a}} \longrightarrow q_{s0}\cdot a = q_{\ell 0}$$

Q4

lateral → $dS_1 = \rho\, d\phi\, dz\, \vec{a_\rho}$ $\qquad \oint E\cdot dS = \frac{Q}{\varepsilon}$

$$\int_{-a}^{a}\int_0^{2\pi} \frac{Q_0}{\varepsilon a^2}\frac{\rho^2}{a^2}\hat{a}_\rho \hat{a}_\rho\, \rho\, d\phi\, dz = \frac{Q_0 \rho^3}{\varepsilon a^4}\int_{-a}^{a}\int_0^{2\pi} d\phi\, dz \qquad \boxed{\rho = a}$$

$$= \frac{Q_0}{\varepsilon a} 2\pi \cdot 2a = \frac{4\pi Q_0}{\varepsilon}$$

top → $dS_2 = \rho\, d\theta\, d\rho\, \vec{a_z}$ $\qquad$ bottom → $dS_3 = \rho\, d\theta\, d\rho\, (-\vec{a_z})$

$$\int_0^a\int_0^{2\pi} \frac{Q_0}{\varepsilon a^2}\underbrace{\sin\left(\frac{\pi z}{2a}\right)}_{1}\rho\, d\theta\, d\rho = 2\pi\frac{a^2}{2}\frac{Q_0}{\varepsilon a^2} = \frac{\pi Q_0}{\varepsilon} \qquad \boxed{z = a}$$

$$\int_0^a\int_0^{2\pi} \underbrace{-\frac{Q_0}{\varepsilon a^2}\underbrace{\sin\left(\frac{\pi z}{2a}\right)}_{-1}}_{+1}\rho\, d\theta\, d\rho = \frac{\pi Q_0}{\varepsilon} \qquad \boxed{z = -a}$$

$$\frac{4\pi Q_0}{\varepsilon} + \frac{\pi Q_0}{\varepsilon} + \frac{\pi Q_0}{\varepsilon} = \frac{Q}{\varepsilon}$$

$$6\pi Q_0 = Q$$

$\vec{E}$ depends only on ρ and z, so it has symmetry; Thus, we could use Gauss' Law.

Q5

$$\iint D\cdot dS = Q = \iint E\,\varepsilon_0(1+\sin\theta)\,r^2\sin\theta\,d\theta\,d\phi = Q$$

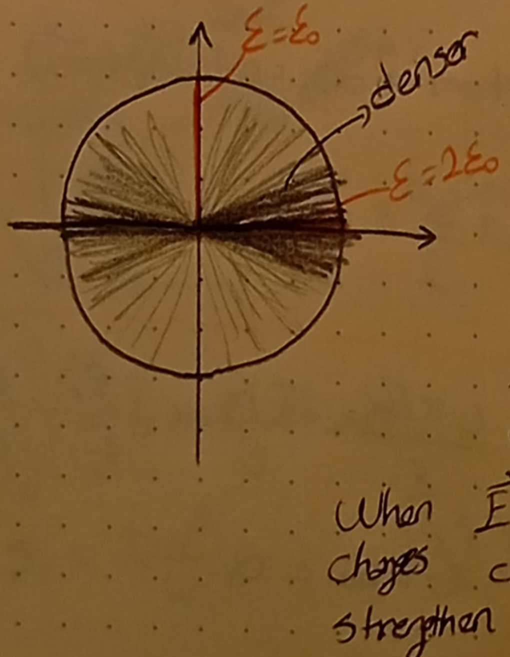
$$E\varepsilon_0 r^2\int_0^{2\pi}\int_0^{\pi}(\sin\theta+\underbrace{\sin^2\theta}_{\frac{1}{2}-\frac{\cos 2\theta}{2}})\,d\theta\,d\phi \rightarrow \int_0^{2\pi}\left[\underbrace{(-\cos\theta)\Big|_0^{\pi}}_{2}+\frac{\pi}{2}\right]d\phi$$

this term goes to zero

$$Q=(4\pi+\pi^2)E\varepsilon_0 r^2$$

$$E=\frac{Q}{(4\pi+\pi^2)\varepsilon_0 r^2} \qquad V=\int_a^b E\,dr=\frac{Q}{(4\pi+\pi^2)\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$\boxed{C=\frac{Q}{V}=\frac{(4\pi+\pi^2)\varepsilon_0}{\left(\frac{1}{a}-\frac{1}{b}\right)}} \qquad \frac{1}{2}CV^2=\frac{Q^2}{2(4\pi+\pi^2)\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$



$$C\sim\varepsilon \rightarrow \uparrow C = Q\overset{\uparrow}{V}$$

$$\vec{E}\sim\frac{1}{\varepsilon}$$

As permittivity varies with θ, the distribution of free charges on the capacitor adjusts accordingly.

When $\vec{E}$ field decreases on xy-plane, charges concentrate more in order to strengthen the $\vec{E}$ field

Q6

for ρ_z

$$dE_y = \frac{\rho_\ell}{4\pi\varepsilon_0(y^2+z^2)} \cdot \frac{y}{\sqrt{y^2+z^2}}\, dz$$

$$E_y = \frac{\rho_\ell \cdot y}{4\pi\varepsilon_0} \cdot \frac{z}{r^2(y^2+z^2)^{1/2}} = \frac{\rho_\ell \cdot y}{4\pi\varepsilon_0 y^2(y^2+1)}\, \vec{a}_y$$

$$dE_z = \frac{\rho_\ell}{4\pi\varepsilon_0(y^2+z^2)} \cdot \frac{z}{\sqrt{z^2+y^2}}\, dz \rightarrow -\frac{1}{\sqrt{z^2+y^2}}\Big|_0^1$$

$$E_z = \frac{\rho_\ell}{4\pi\varepsilon_0}\left(-\frac{1}{\sqrt{1+y^2}} + \frac{1}{y}\right)$$

for ρ_y

$$dE = \frac{-\rho_\ell\, dy}{\underbrace{(r-y)}_{u}{}^2}$$

$du = -dy$

$$\int -\frac{du}{u^2} = \frac{1}{r-y}\Big|_0^1 = \frac{1}{r-1} - \frac{1}{r} = \frac{1}{r(r-1)}$$

$$\vec{E} = -\frac{\rho_\ell}{4\pi\varepsilon_0 r(r-1)}\, \vec{a}_y$$

$$\boxed{\vec{E} = \left(\frac{\rho_\ell r}{4\pi\varepsilon_0 r^2\sqrt{r^2+1}} - \frac{\rho_\ell}{4\pi\varepsilon_0 r(r-1)}\right)\vec{a}_y - \frac{\rho_\ell}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{1+y^2}} - \frac{1}{y}\right) a_z}$$

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