$\underline{Q_5}$

$$p(x)y'' + q(x)y' + r(x)y = 0$$

$$\underbrace{\lim_{x\to 0} x\frac{q(x)}{p(x)}}_{2} \qquad \underbrace{\lim_{x\to 0} x^2\frac{2r(x)}{p(x)}}_{-2}$$

$$p(x) = x^2 \qquad p(0) = 0$$

$$y = \sum_{n=0}^{\infty} a_n x^{n+r}$$

$$y' = \left(\sum_{n=0}^{\infty} a_n x^{n+r}\right)' = \left(\sum a_n (n+r) x^{n+r-1}\right)'$$

$$a_n(n+r)\,x^{n+r-1}$$

$$y' = \sum_{0}^{\infty} a_n(n+r)\,x^{n+r-1}$$

$$y'' = \sum_{0}^{\infty} a_n (n+r)(n+r-1)\,x^{n+r-2}$$

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$$x^2 \sum_0^\infty a_n(n+r)(n+r-1)x^{n+r-2} + (2x+3x^2) \cdot$$

$$\sum_0^\infty a_n(n+r)x^{n+r-1} - 2\sum_0^\infty a_n x^{n+r} = 0$$

$$= \sum_0^\infty a_n(2nr+r^2-r+n^2-n)x^{n+r} + \sum_0^\infty 3a_n(n+r) \cdot$$

$$x^{n+r+1} + \sum_0^\infty 2a_n(n+r)x^{n+r} - \sum_0^\infty 2a_n x^{n+r}$$

$$= (a_0r^2 + a_0r - 2a_0)x^r + \sum_{n=1}^\infty (a_nr^2 + a_nr + 3a_{n-1}r$$

$$+ 2a_nnr - 2a_n - 3a_{n-1} + a_nn^2 + a_nn + 3a_{n-1}n)x^{n+r}$$

$$= 0 \rightarrow r_1 = -2 \quad r_2 = 1$$

$$(a_0 + a_0 - 2a_0)x + \sum_1^\infty (a_n + a_n + 3a_{n-1} + 2a_nn$$

$$-2a_n - 3a_{n-1} + a_nn^2 + a_nn + 3a_{n-1}n)x^{n+1} = 0$$

$$\sum_1^\infty (n^2a_n + 3na_n + 3na_{n-1})x^{n+1}$$

$$(n^2a_n + 3na_n + 3na_{n-1}) = 0 \qquad a_n = \frac{-3a_{n-1}}{n+3}$$

$$a_1 = -\frac{3a_0}{4} \qquad a_2 = -\frac{3a_1}{5} = \frac{9a_0}{20}$$

$$a_3 = \frac{3a_2}{6} = \frac{3 \cdot 9}{6 \cdot 20} a_0 \,---$$

$$y_1 = a_0 x \left(1 - \frac{3x}{4} + \frac{9x^2}{20} + ---\right)$$

$$y_1 = c_1 x \left(1 - \frac{3x}{4} + \frac{9x^2}{20} ---\right)$$

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