Introduction to Digital Design

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Karnaugh Maps (K-map)

- A K-map is a collection of squares
  - Each square represents a minterm
  - The collection of squares is a graphical representation of a Boolean function
  - Adjacent squares differ in the value of one variable
  - Alternative algebraic expressions for the same function are derived by recognizing patterns of squares
- The K-map can be viewed as
  - A reorganized version of the truth table
  - A topologically-warped Venn diagram as used to visualize sets in algebra of sets

Two Variable Karnaugh Map

- Two variable: x and y
  - 4 minterms:
    - $m_0 = x'y' \rightarrow 00$
    - $m_1 = x'y \rightarrow 01$
    - $m_2 = xy' \rightarrow 10$
    - $m_3 = xy \rightarrow 11$

x \ y	0	1
0	m_0	m_1
1	m_2	m_3

Two Variable Karnaugh Map

- Two variable: x and y
  - 4 minterms:
    - $m_0 = x'y' \rightarrow 00$
    - $m_1 = x'y \rightarrow 01$
    - $m_2 = xy' \rightarrow 10$
    - $m_3 = xy \rightarrow 11$
- Example:

x	y	f
0	0	0
0	1	1
1	0	1
1	1	0

x \ y	0	1
0	m_0	m_1
1	m_2	m_3

x \ y	0	1
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K-Map Function Representation

- Example: $f(x, y) = x$

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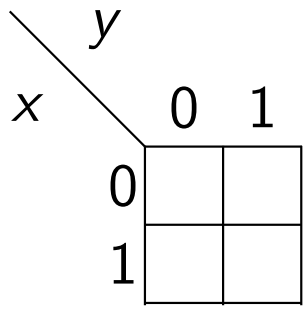

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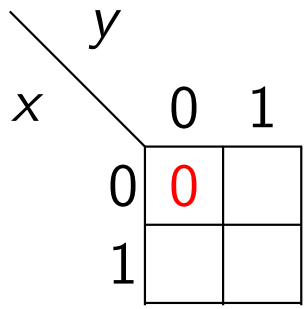

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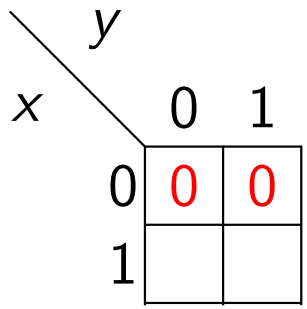

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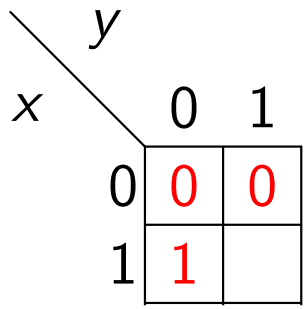

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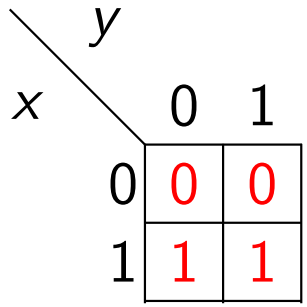

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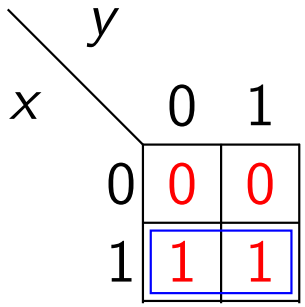

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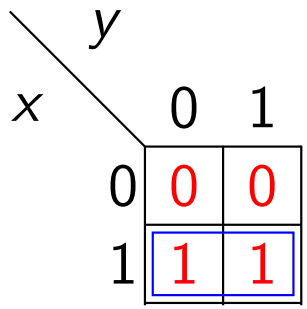

For function $f(x, y)$, the two adjacent cells containing 1's can be combined using the Minimization Theorem:

$$f(x, y) = xy\prime + xy = x$$

K-Map Function Representation

- Example: $g(x, y) = x + y$

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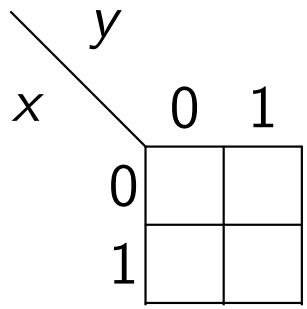

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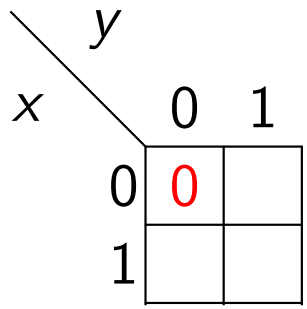

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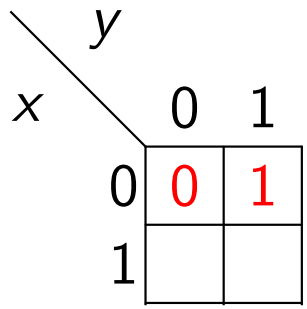

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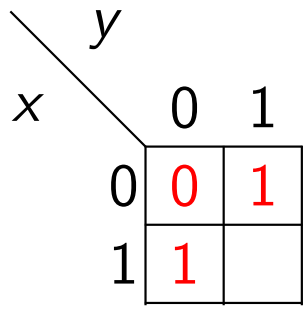

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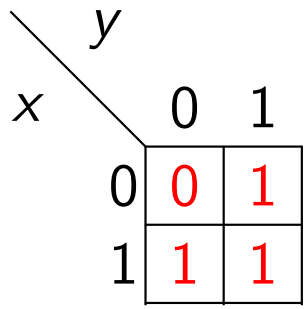

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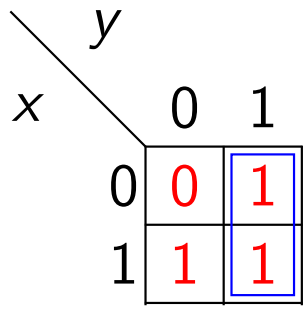

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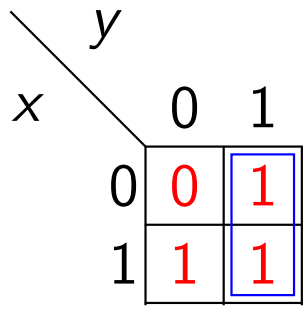

$x'y + xy = y$

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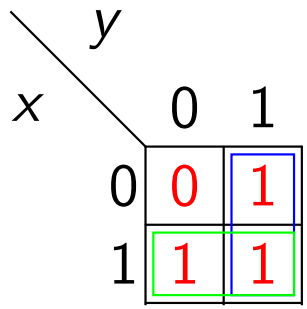

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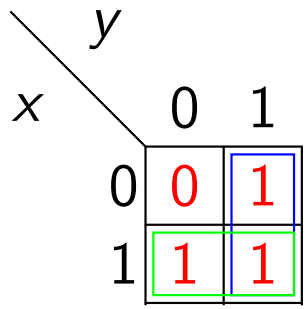

$xy' + xy = x$

$x'y + xy = y$

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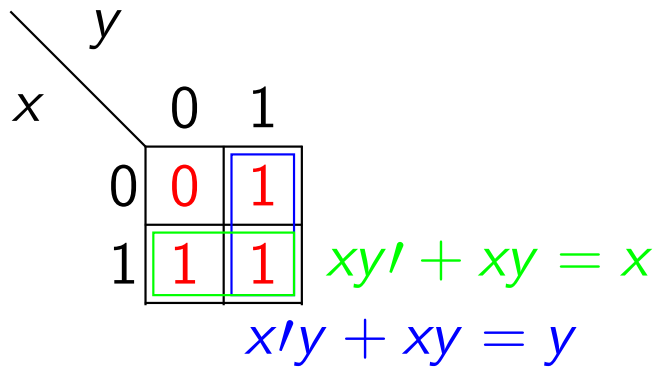


For function $g(x, y)$, the two adjacent cells containing 1's can be combined using the Minimization Theorem:

$$g(x, y) = (x'y + xy) + (xy' + xy) = x + y$$

Three Variable Karnaugh Map

Three variable: x, y, z

8 minterms:

m_0	$=$	$x'y'z'$	$\rightarrow$	000
m_1	$=$	$x'y'z$	$\rightarrow$	001
m_2	$=$	$x'yz'$	$\rightarrow$	010
m_3	$=$	$x'yz$	$\rightarrow$	011
m_4	$=$	$xy'z'$	$\rightarrow$	100
m_5	$=$	$xy'z$	$\rightarrow$	101
m_6	$=$	xyz'	$\rightarrow$	110
m_7	$=$	xyz	$\rightarrow$	111

z \ xy	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

Three Variable Karnaugh Map

Example:

x	y	z	f_1	f_2
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

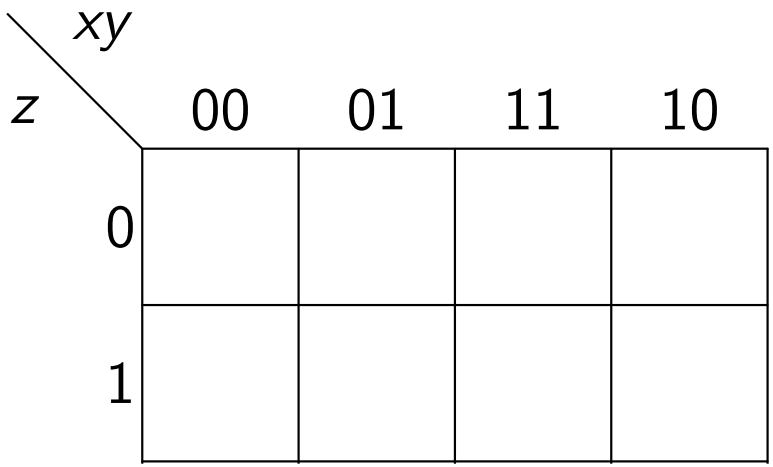
For f_1

z \ xy	00	01	11	10
0	0	0	1	0
1	0	1	1	1

For f_2

z \ xy	00	01	11	10
0	0	1	0	1
1	1	0	1	0

Three Variable Karnaugh Map



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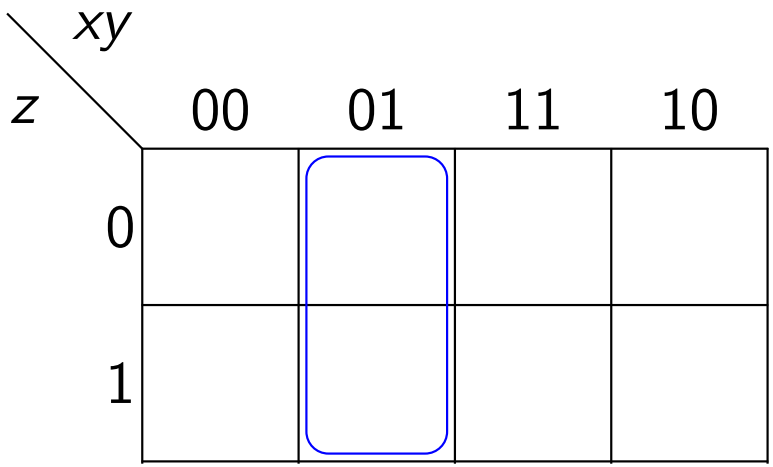

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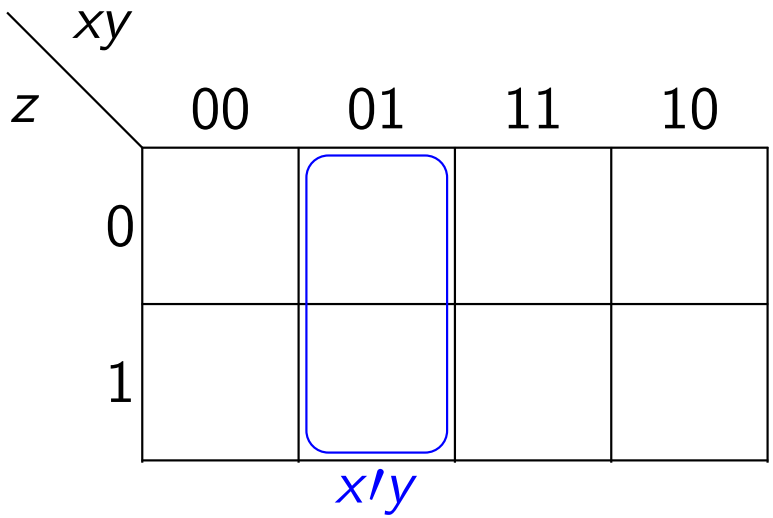

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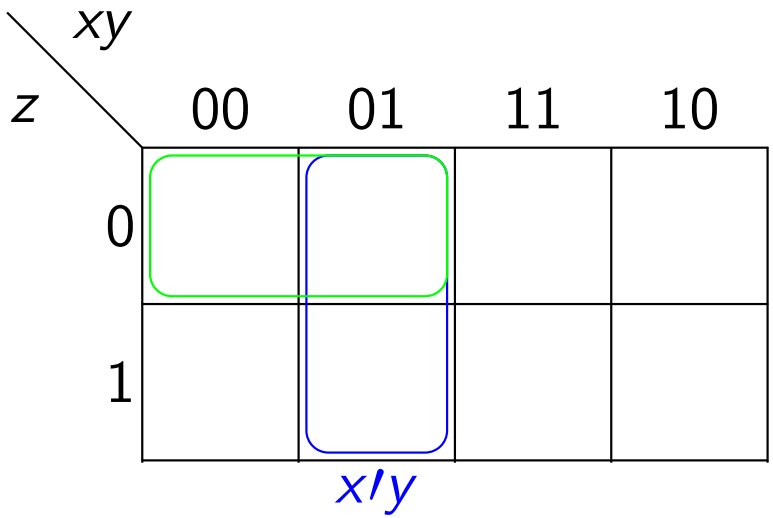

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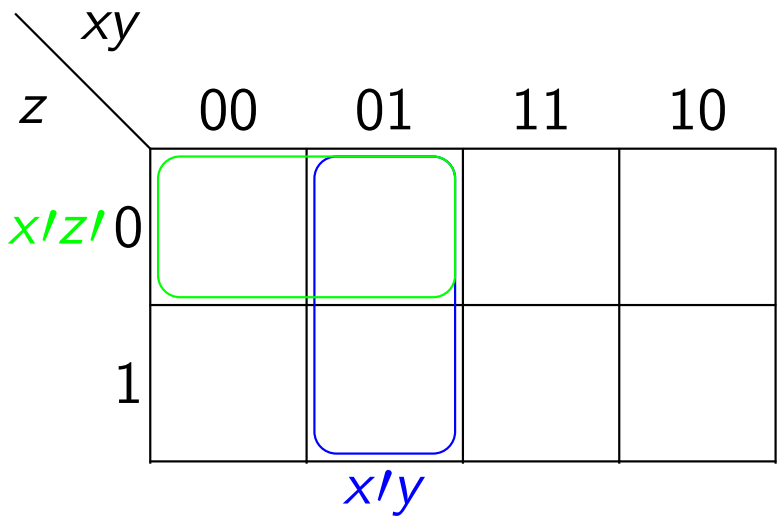

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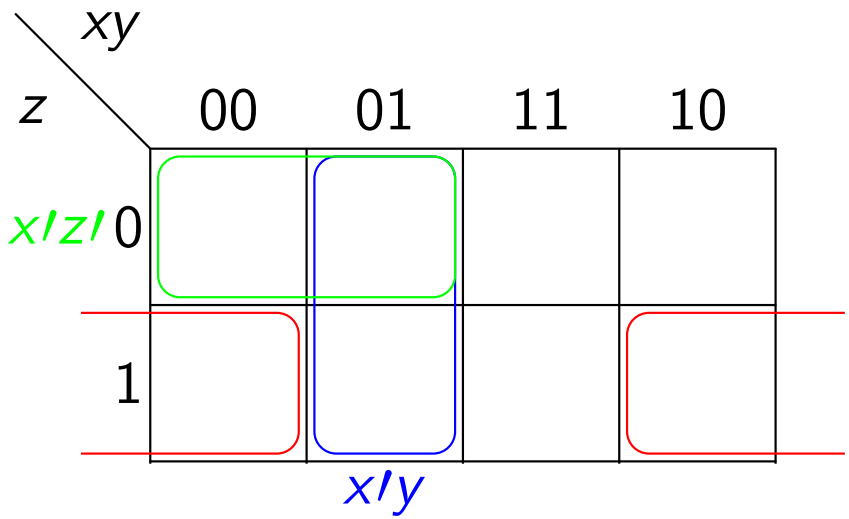

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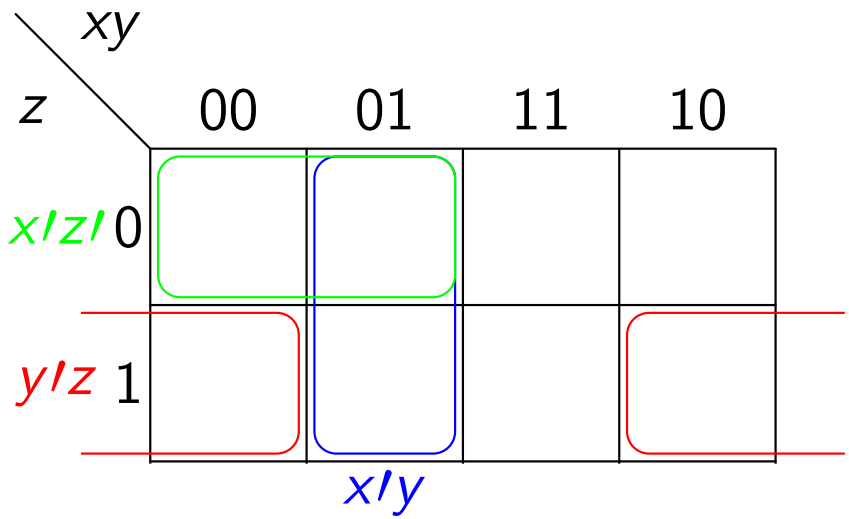

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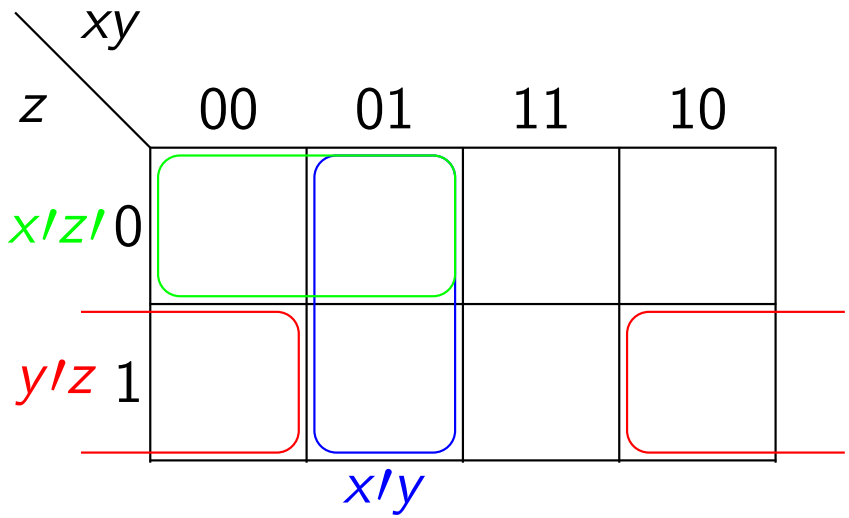

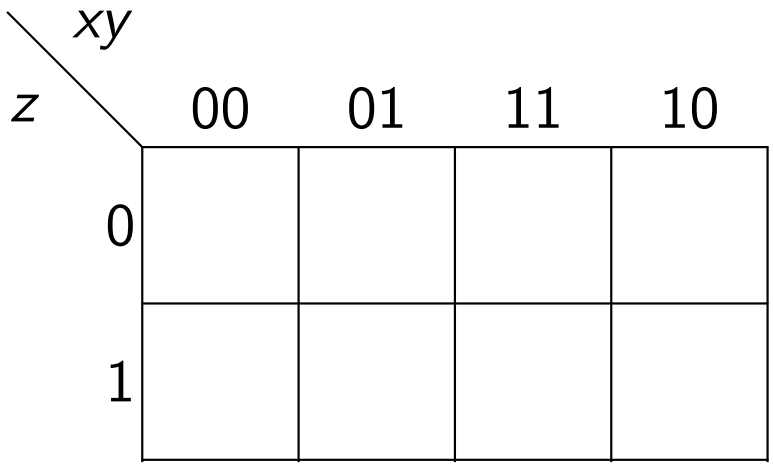

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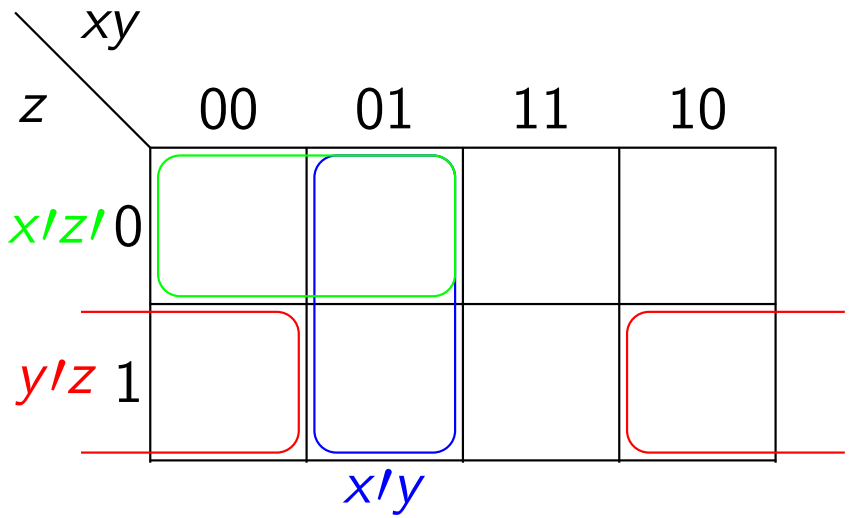

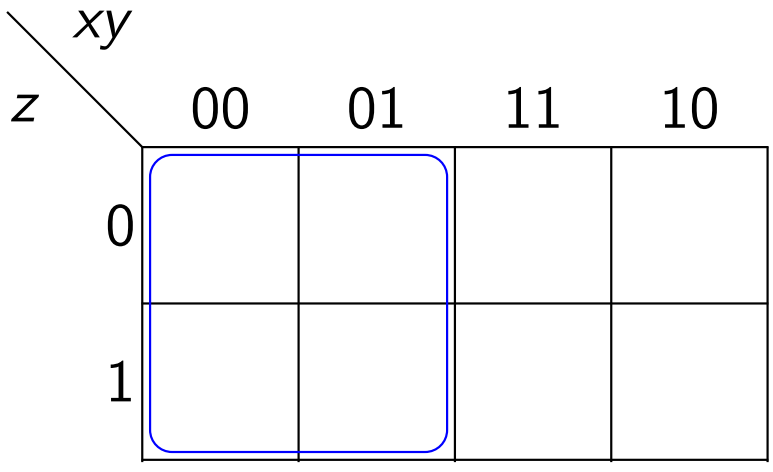

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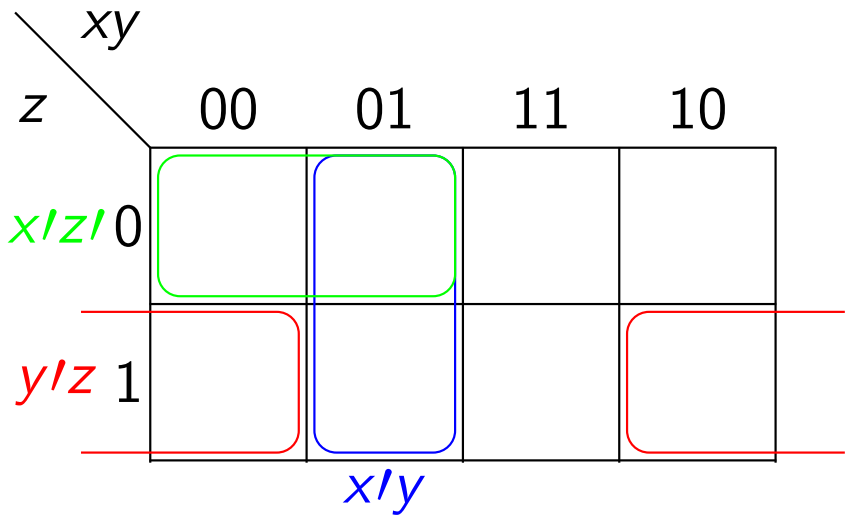

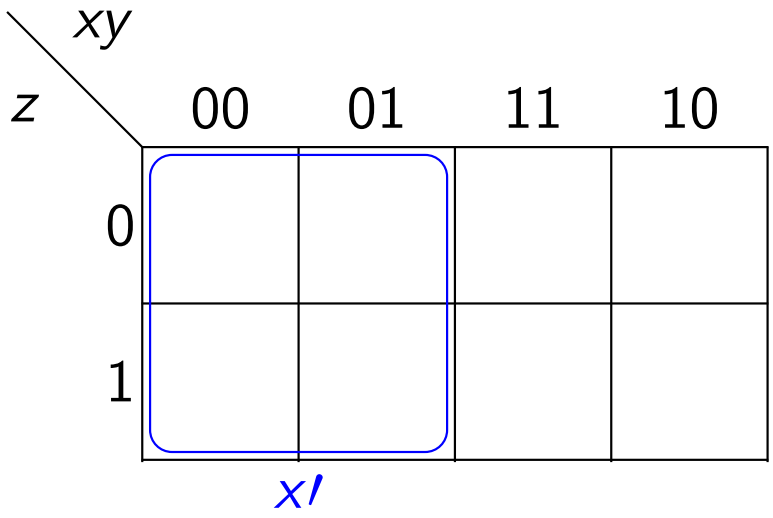

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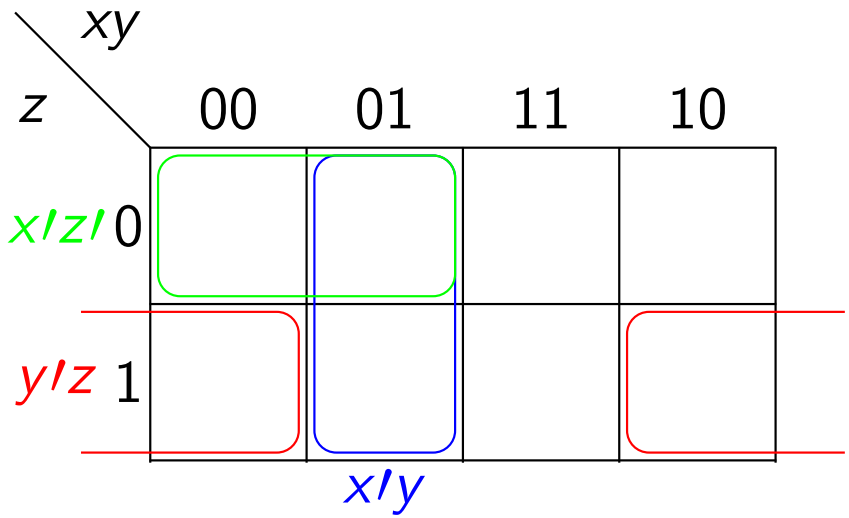

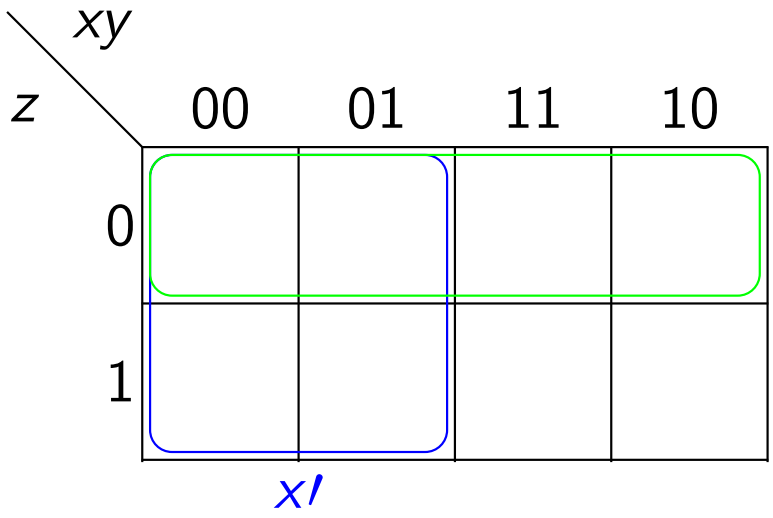

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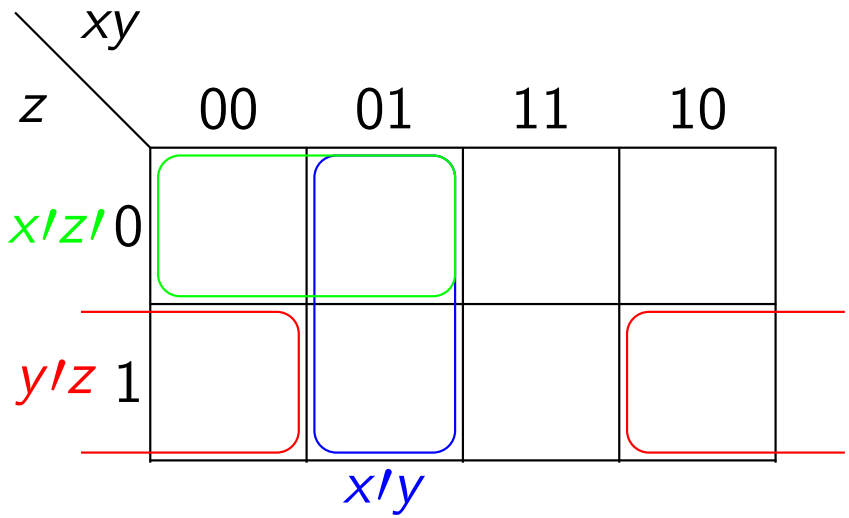

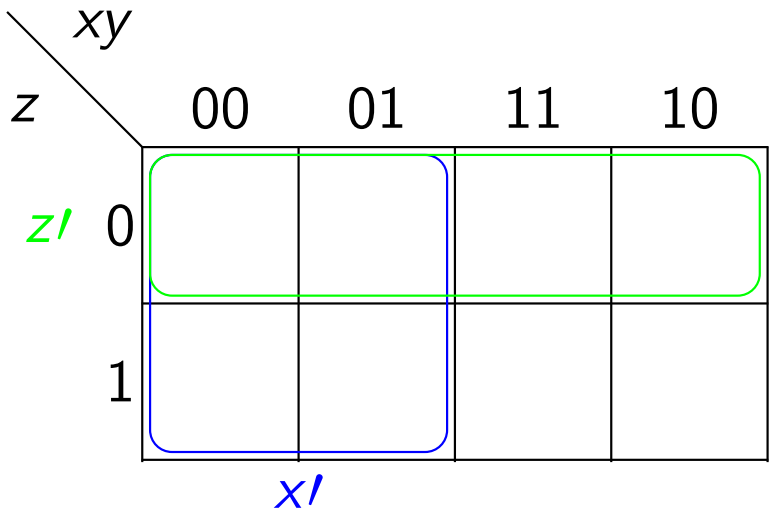

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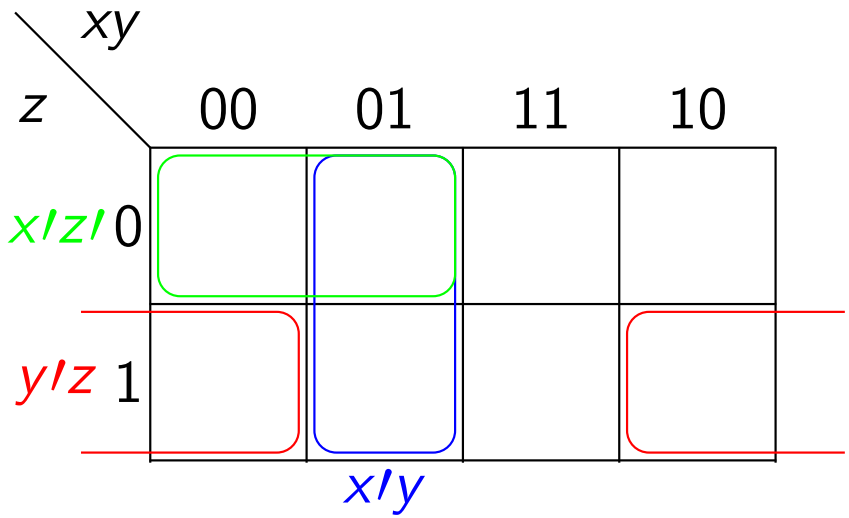

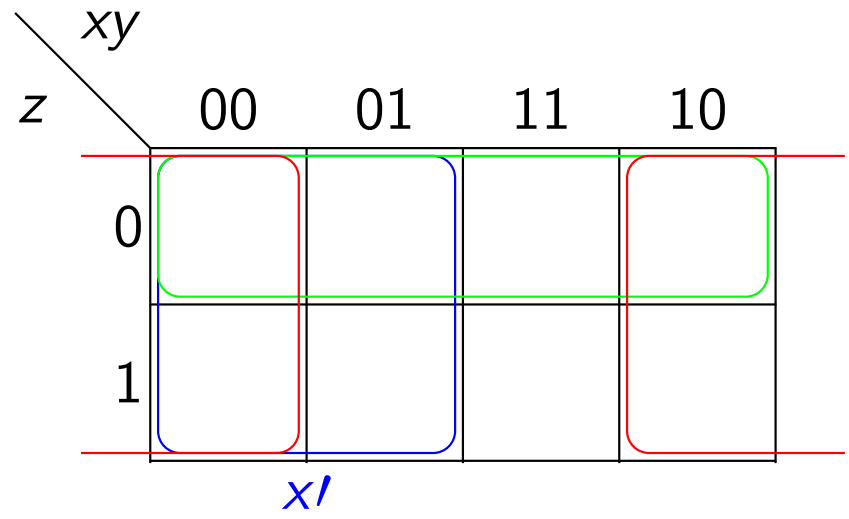

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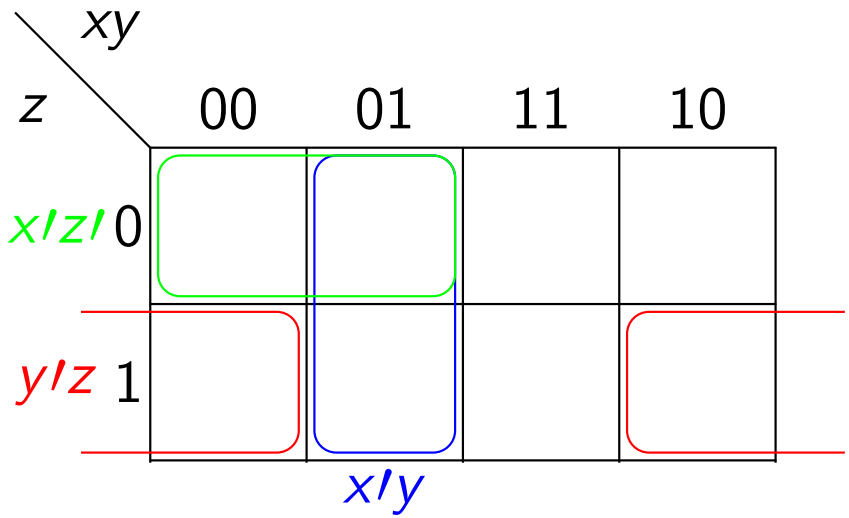
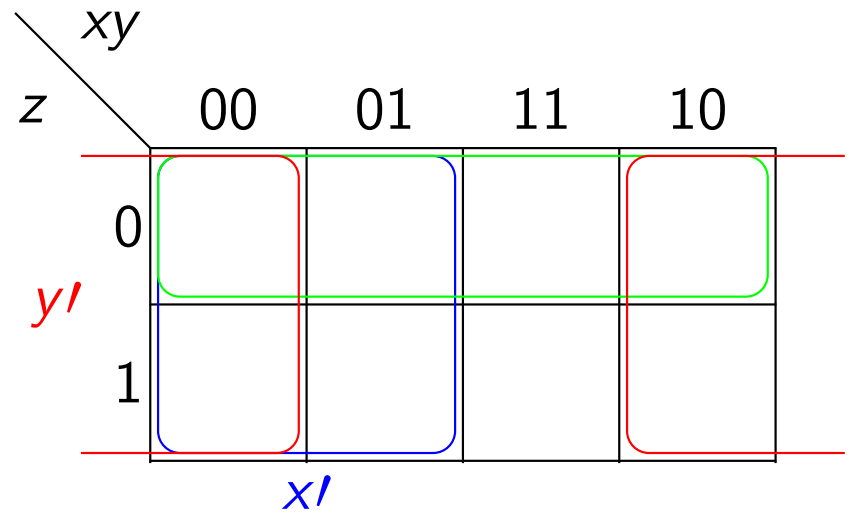
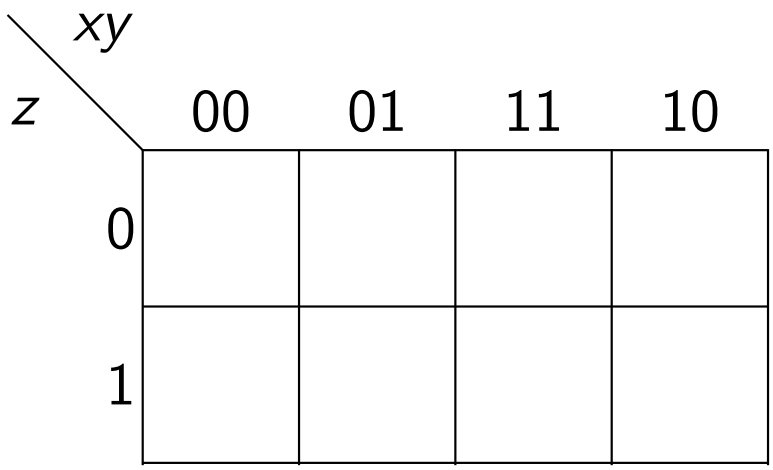




Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$



	x	y	z

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

z \ xy	00	01	11	10
0				
1	1			

	x	y	z
1	0	0	1

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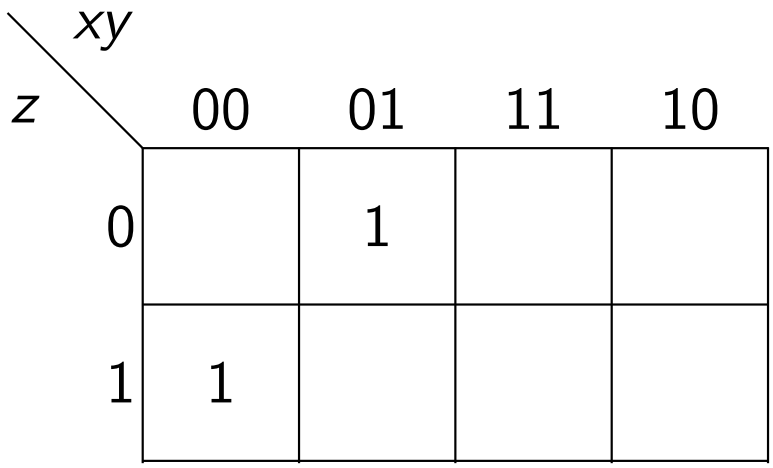

	x	y	z
1	0	0	1
2	0	1	0

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

z \ xy	00	01	11	10
0		1		
1	1			

	x	y	z
1	0	0	1
2	0	1	0

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z \ xy	00	01	11	10
0		1		
1	1	1		

	x	y	z
1	0	0	1
2	0	1	0
3	0	1	1

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

z \ xy	00	01	11	10
0		1		
1	1	1		1

	x	y	z
1	0	0	1
2	0	1	0
3	0	1	1
5	1	0	1

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z \ xy	00	01	11	10
0		1		
1	1	1		1

	x	y	z
1	0	0	1
2	0	1	0
3	0	1	1
5	1	0	1

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

z \ xy	00	01	11	10
0		1		
1	1	1	1	1

	x	y	z
1	0	0	1
2	0	1	0
3	0	1	1
5	1	0	1
7	1	1	1

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

z \ xy	00	01	11	10
0		1		
1	1	1	1	1

	x	y	z			x	y	z
1	0	0	1					
2	0	1	0					
3	0	1	1					
5	1	0	1					
7	1	1	1					

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

z \ xy	00	01	11	10
0		1		
1	1	1	1	1

		x	y	z			x	y	z
√	1	0	0	1		1,3	0	-	1
	2	0	1	0					
√	3	0	1	1					
	5	1	0	1					
	7	1	1	1					

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

z \ xy	00	01	11	10
0		1		
1	1	1	1	1

		x	y	z
√	1	0	0	1
	2	0	1	0
√	3	0	1	1
√	5	1	0	1
	7	1	1	1

	x	y	z
1,3	0	-	1
1,5	-	0	1

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

z \ xy	00	01	11	10
0		1		
1	1	1	1	1

		x	y	z
√	1	0	0	1
√	2	0	1	0
√	3	0	1	1
√	5	1	0	1
	7	1	1	1

	x	y	z
1,3	0	-	1
1,5	-	0	1
2,3	0	1	-

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

z \ xy	00	01	11	10
0		1		
1	1	1	1	1

		x	y	z
√	1	0	0	1
√	2	0	1	0
√	3	0	1	1
√	5	1	0	1
√	7	1	1	1

	x	y	z
1,3	0	-	1
1,5	-	0	1
2,3	0	1	-
3,7	-	1	1

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

z \ xy	00	01	11	10
0		1		
1	1	1	1	1

		x	y	z
√	1	0	0	1
√	2	0	1	0
√	3	0	1	1
√	5	1	0	1
√	7	1	1	1

	x	y	z
1,3	0	-	1
1,5	-	0	1
2,3	0	1	-
3,7	-	1	1
5,7	1	-	1

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

z \ xy	00	01	11	10
0		1		
1	1	1	1	1

		x	y	z
√	1	0	0	1
√	2	0	1	0
√	3	0	1	1
√	5	1	0	1
√	7	1	1	1

	x	y	z
1,3	0	-	1
1,5	-	0	1
2,3	0	1	-
3,7	-	1	1
5,7	1	-	1

x	y	z

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

z \ xy	00	01	11	10
0		1		
1	1	1	1	1

		x	y	z
√	1	0	0	1
√	2	0	1	0
√	3	0	1	1
√	5	1	0	1
√	7	1	1	1

		x	y	z
√	1,3	0	-	1
	1,5	-	0	1
	2,3	0	1	-
	3,7	-	1	1
	5,7	1	-	1

x	y	z

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

z \ xy	00	01	11	10
0		1		
1	1	1	1	1

		x	y	z
√	1	0	0	1
√	2	0	1	0
√	3	0	1	1
√	5	1	0	1
√	7	1	1	1

		x	y	z
√	1,3	0	-	1
	1,5	-	0	1
	2,3	0	1	-
	3,7	-	1	1
√	5,7	1	-	1

	x	y	z
1,3,5,7	-	-	1

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

z \ xy	00	01	11	10
0		1		
1	1	1	1	1

		x	y	z
√	1	0	0	1
√	2	0	1	0
√	3	0	1	1
√	5	1	0	1
√	7	1	1	1

		x	y	z
√	1,3	0	-	1
√	1,5	-	0	1
	2,3	0	1	-
	3,7	-	1	1
√	5,7	1	-	1

	x	y	z
1,3,5,7	-	-	1

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

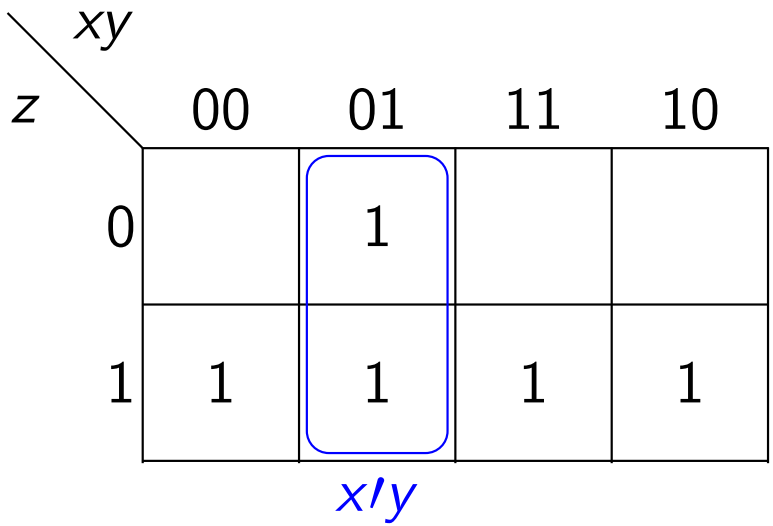
z \ xy	00	01	11	10
0		1		
1	1	1	1	1

		x	y	z
√	1	0	0	1
√	2	0	1	0
√	3	0	1	1
√	5	1	0	1
√	7	1	1	1

		x	y	z
√	1,3	0	-	1
√	1,5	-	0	1
	2,3	0	1	-
√	3,7	-	1	1
√	5,7	1	-	1

	x	y	z
1,3,5,7	-	-	1
1,5,3,7	-	-	1

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

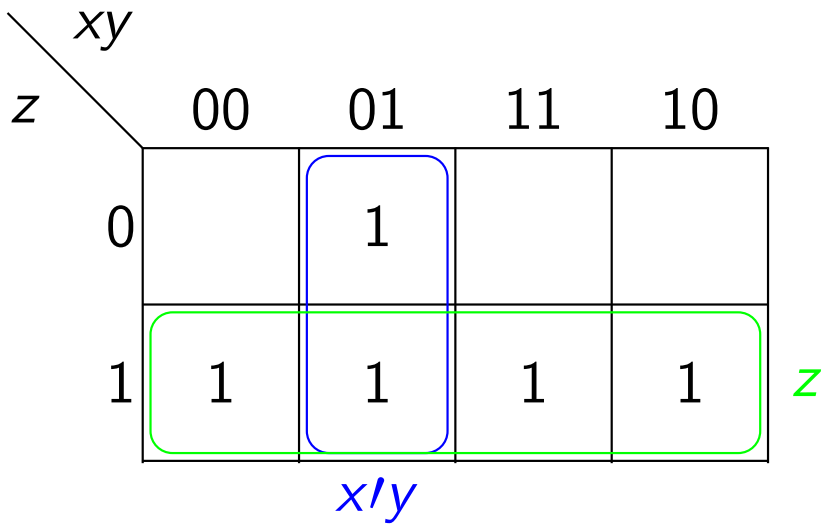


	x	y	z
√ 1	0	0	1
√ 2	0	1	0
√ 3	0	1	1
√ 5	1	0	1
√ 7	1	1	1

		x	y	z
√	1,3	0	-	1
√	1,5	-	0	1
A	2,3	0	1	-
√	3,7	-	1	1
√	5,7	1	-	1

	x	y	z
1,3,5,7	-	-	1
1,5,3,7	-	-	1

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$



	x	y	z
√ 1	0	0	1
√ 2	0	1	0
√ 3	0	1	1
√ 5	1	0	1
√ 7	1	1	1

		x	y	z
√	1,3	0	-	1
√	1,5	-	0	1
A	2,3	0	1	-
√	3,7	-	1	1
√	5,7	1	-	1

		x	y	z
B	1,3,5,7	-	-	1
	1,5,3,7	-	-	1

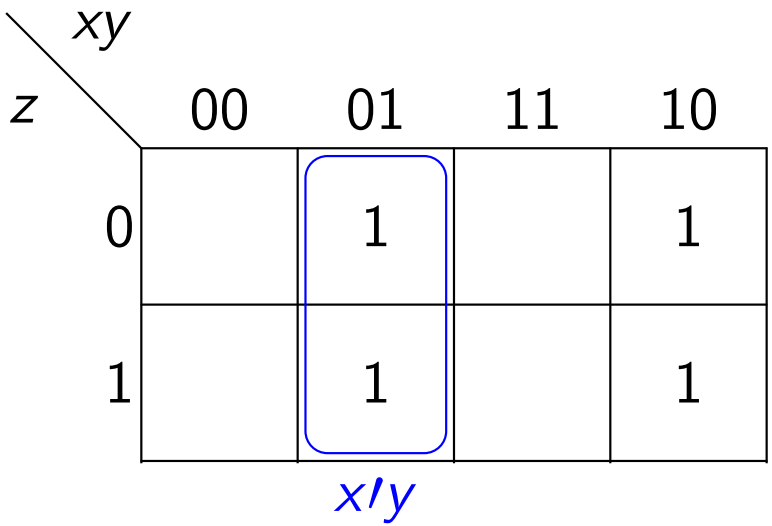
Example: Three Variable Karnaugh Map

$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$

z \ xy	00	01	11	10
0		1		1
1		1		1

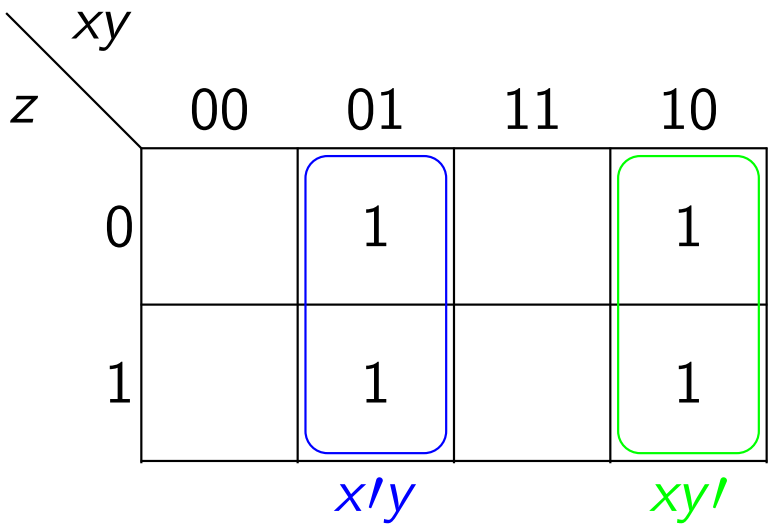
Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$$



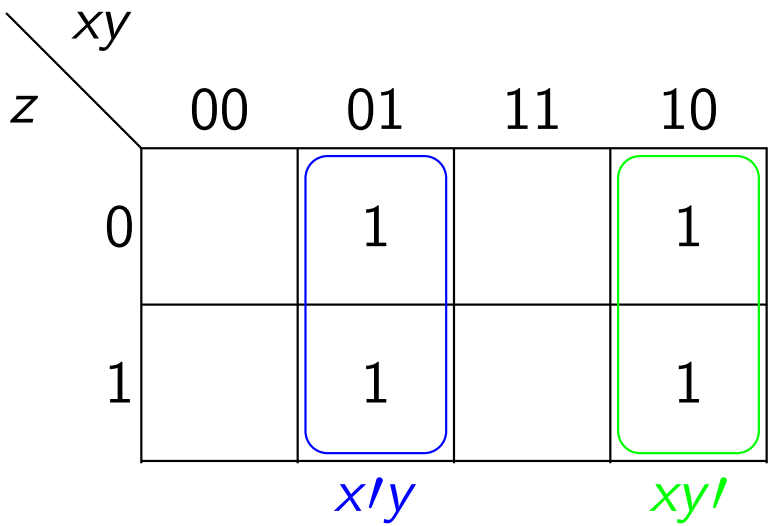
Example: Three Variable Karnaugh Map

$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$



Example: Three Variable Karnaugh Map

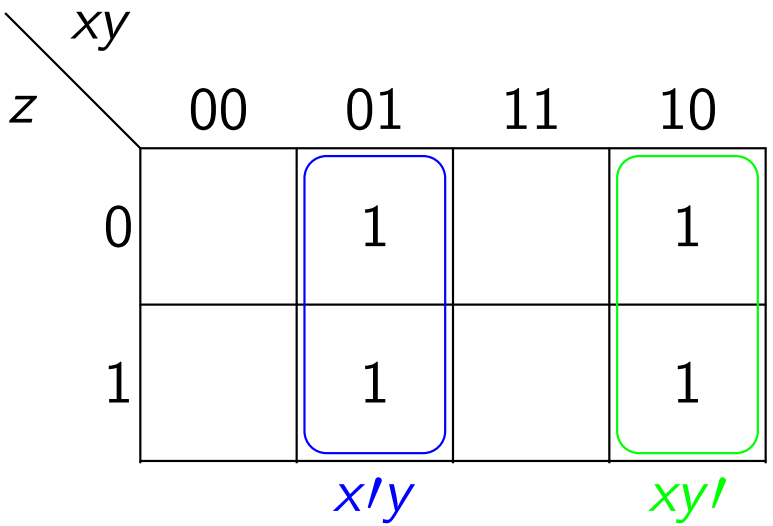
$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$



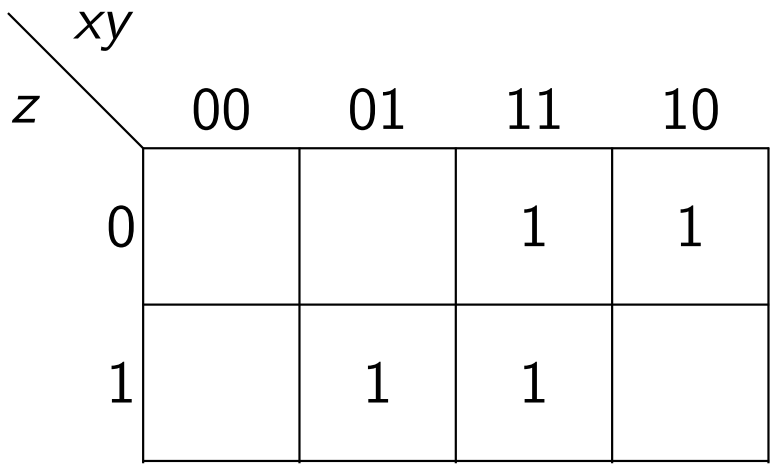
$F_1(x, y, z) = x'y + xy'$

Example: Three Variable Karnaugh Map

$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$



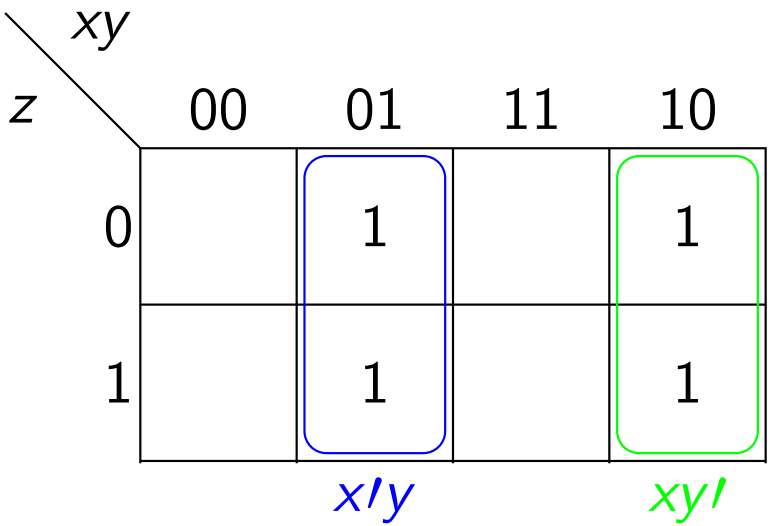
$F_2(x, y, z) = \Sigma(3, 4, 6, 7)$



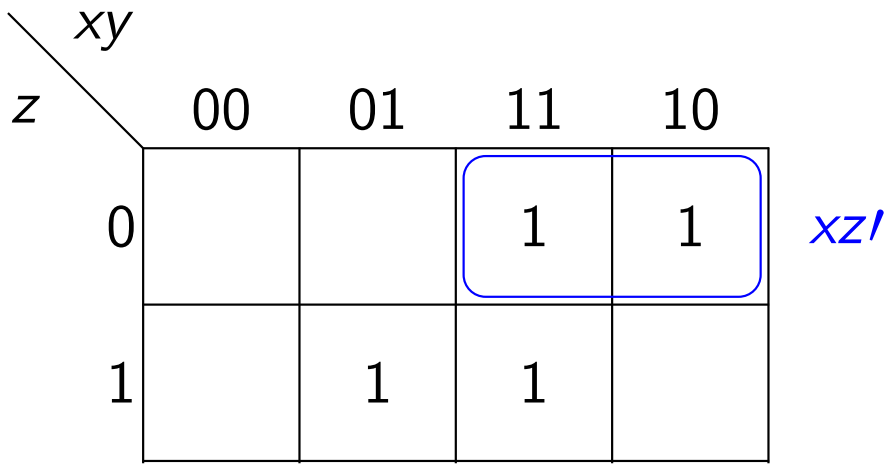
$F_1(x, y, z) = x'y + xy'$

Example: Three Variable Karnaugh Map

$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$



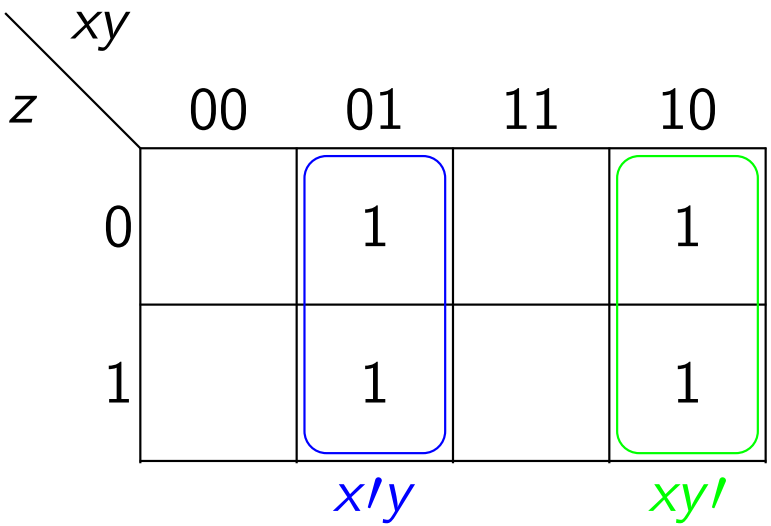
$F_2(x, y, z) = \Sigma(3, 4, 6, 7)$



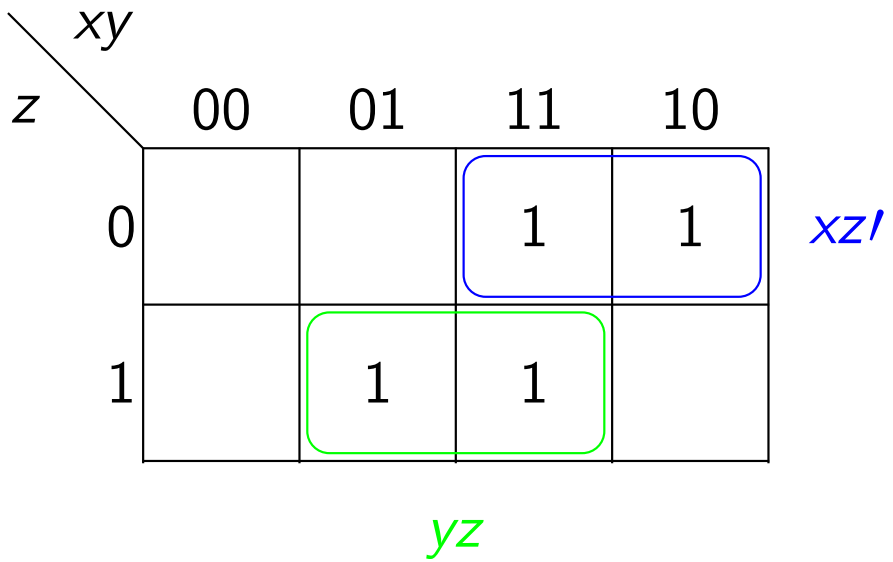
$F_1(x, y, z) = x'y + xy'$

Example: Three Variable Karnaugh Map

$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$



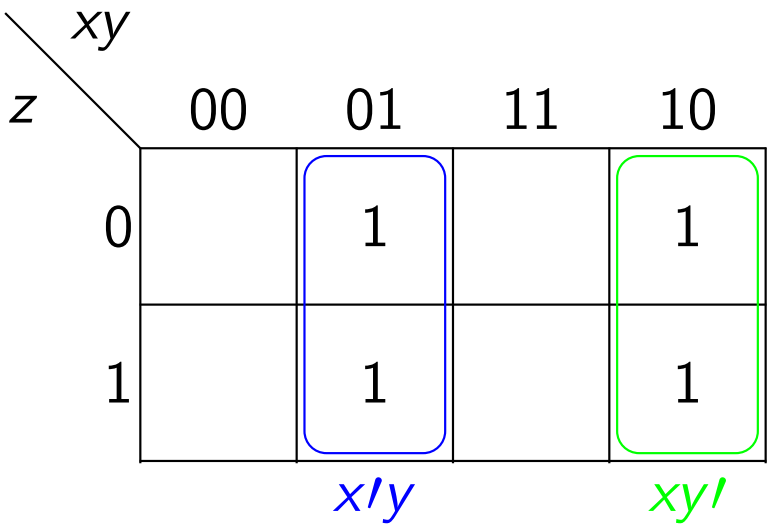
$F_2(x, y, z) = \Sigma(3, 4, 6, 7)$



$F_1(x, y, z) = x'y + xy'$

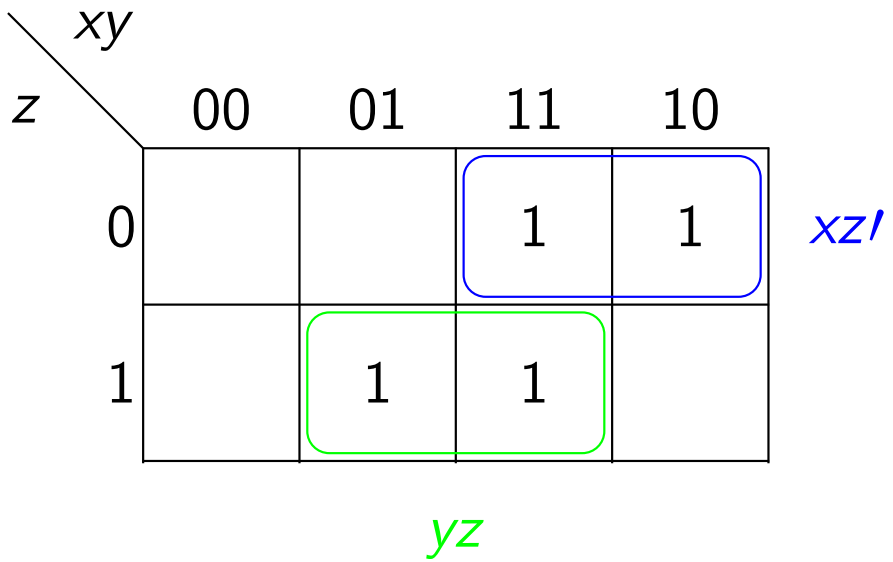
Example: Three Variable Karnaugh Map

$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$



$F_1(x, y, z) = x'y + xy'$

$F_2(x, y, z) = \Sigma(3, 4, 6, 7)$



$F_2(x, y, z) = xz' + yz$

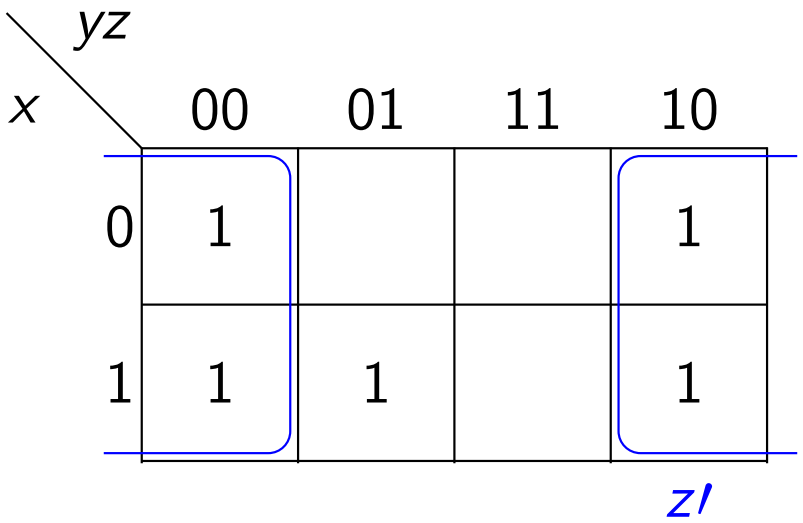
Example: Three Variable Karnaugh Map

$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$

x \ yz	00	01	11	10
0	1			1
1	1	1		1

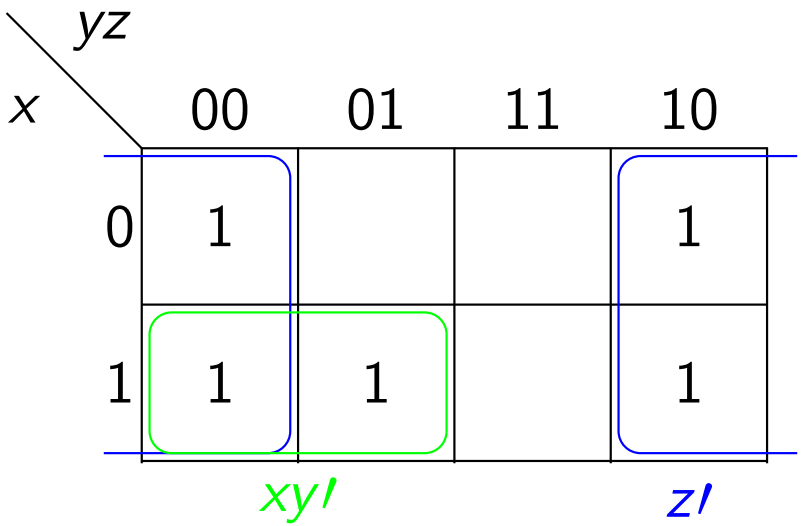
Example: Three Variable Karnaugh Map

$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$



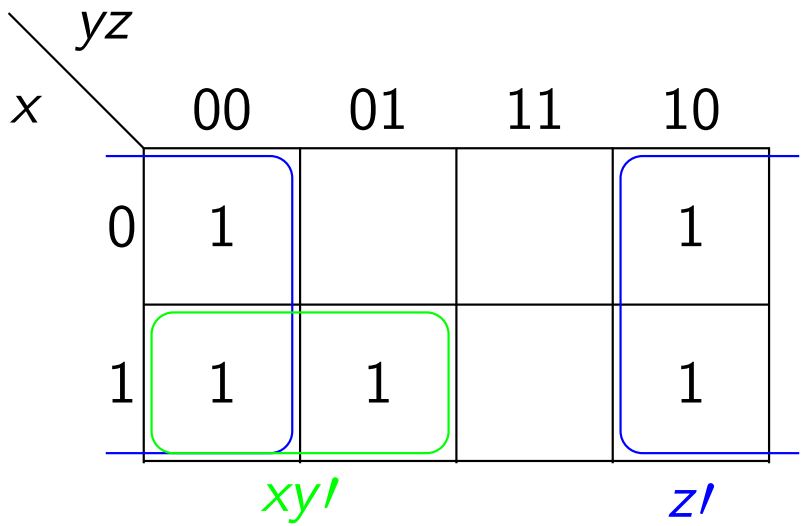
Example: Three Variable Karnaugh Map

$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$



Example: Three Variable Karnaugh Map

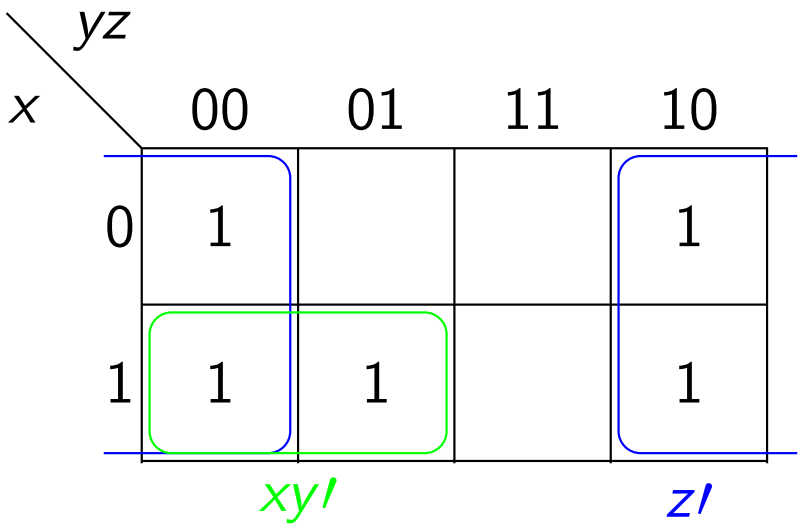
$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$

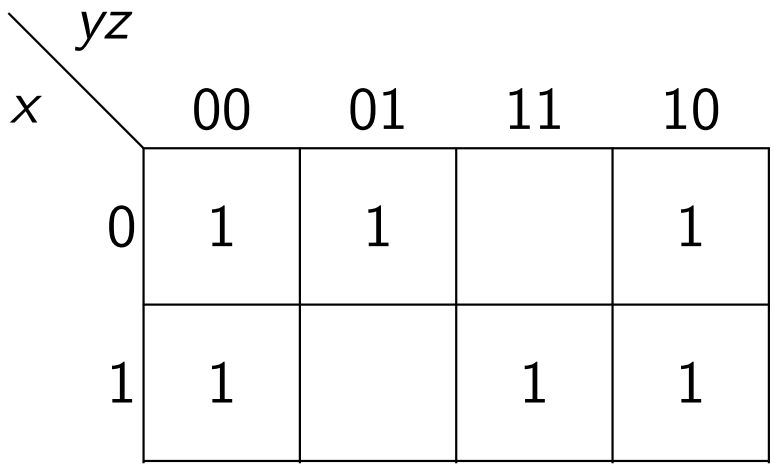


$F_1(x, y, z) = z' + xy'$

Example: Three Variable Karnaugh Map

$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$ $F_2(x, y, z) = \Sigma(0, 1, 2, 4, 6, 7)$

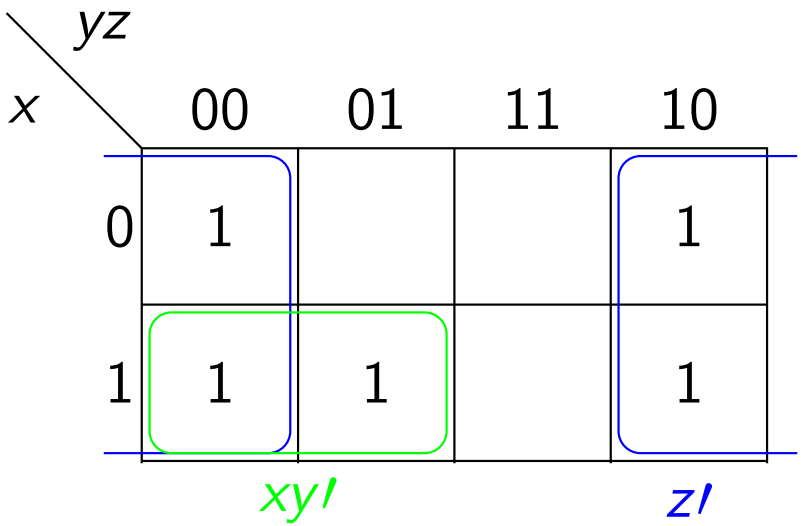


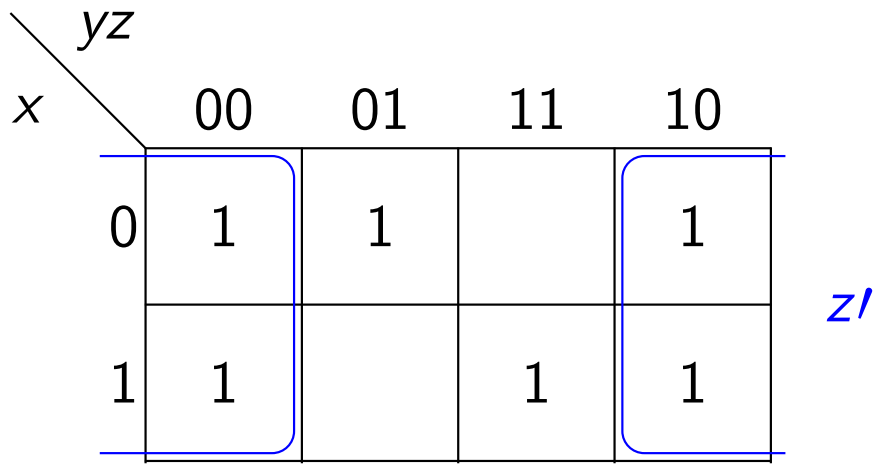


$F_1(x, y, z) = z' + xy'$

Example: Three Variable Karnaugh Map

$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$ $F_2(x, y, z) = \Sigma(0, 1, 2, 4, 6, 7)$

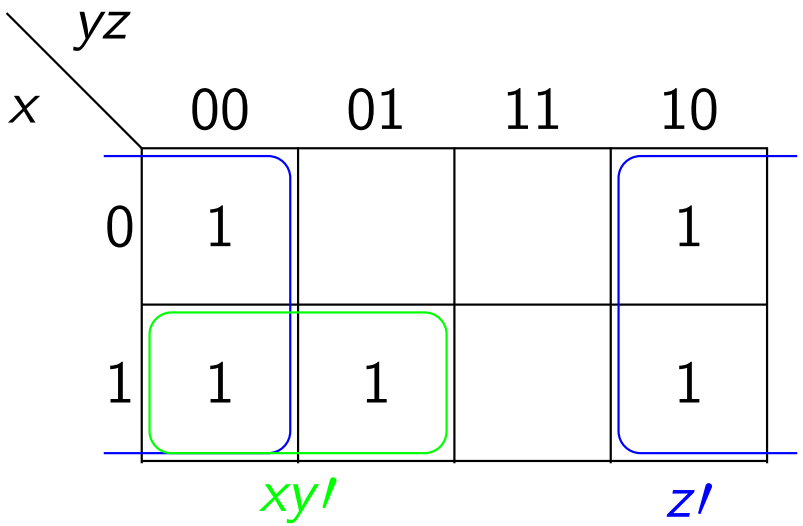


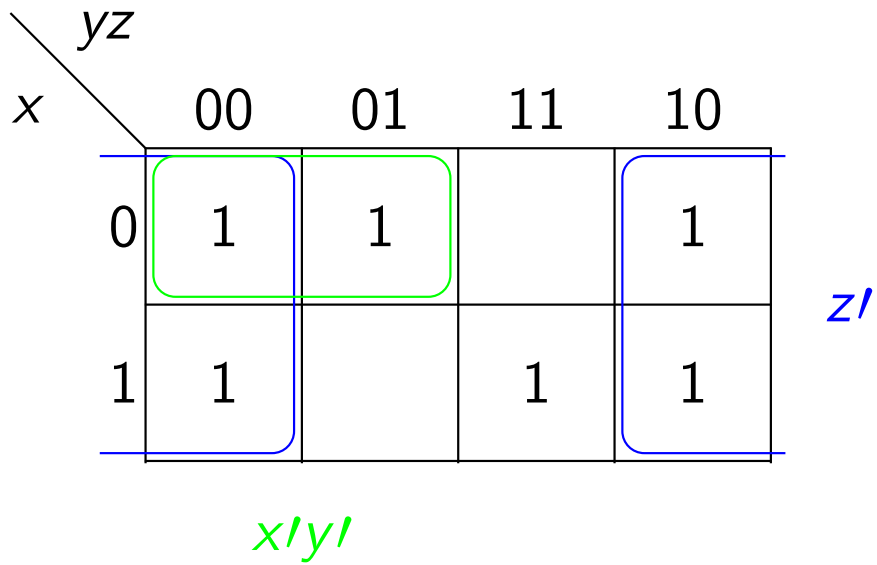


$F_1(x, y, z) = z' + xy'$

Example: Three Variable Karnaugh Map

$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$ $F_2(x, y, z) = \Sigma(0, 1, 2, 4, 6, 7)$

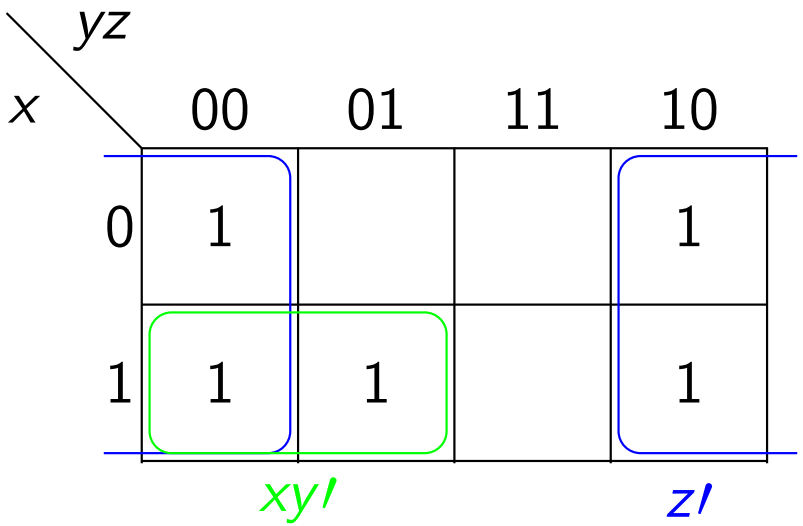


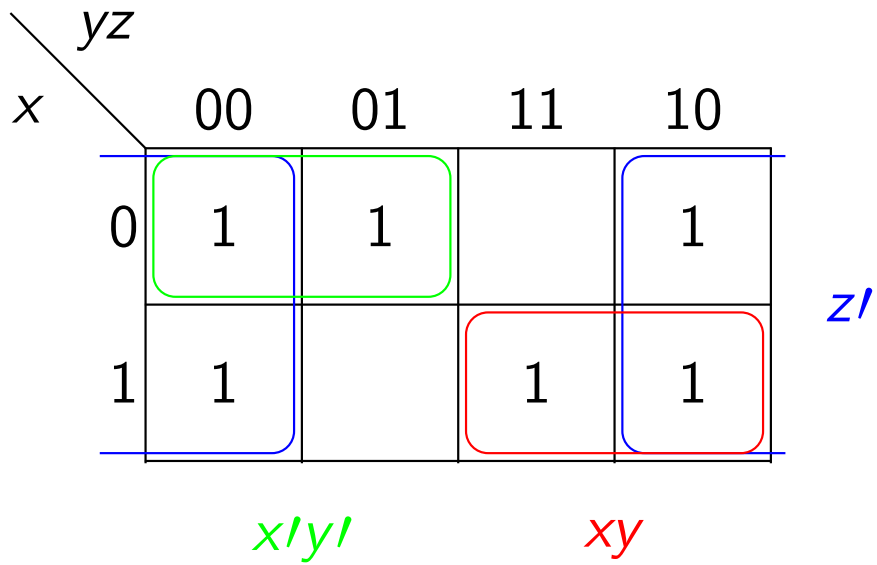


$F_1(x, y, z) = z' + xy'$

Example: Three Variable Karnaugh Map

$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$ $F_2(x, y, z) = \Sigma(0, 1, 2, 4, 6, 7)$

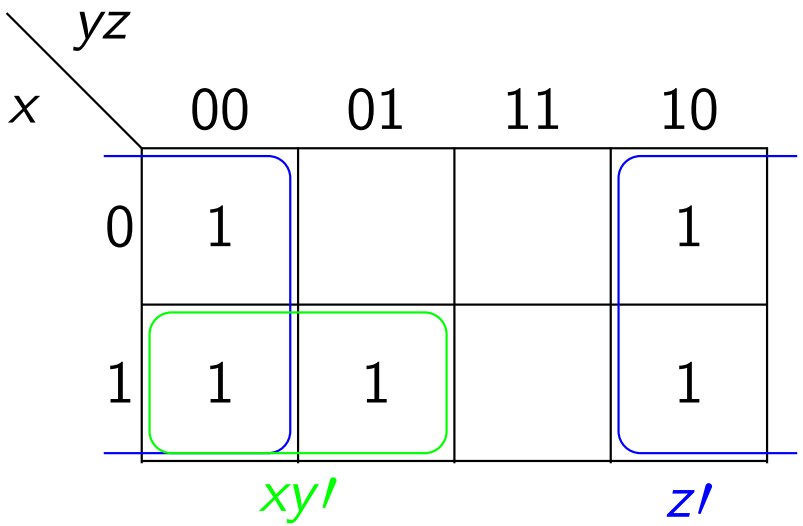


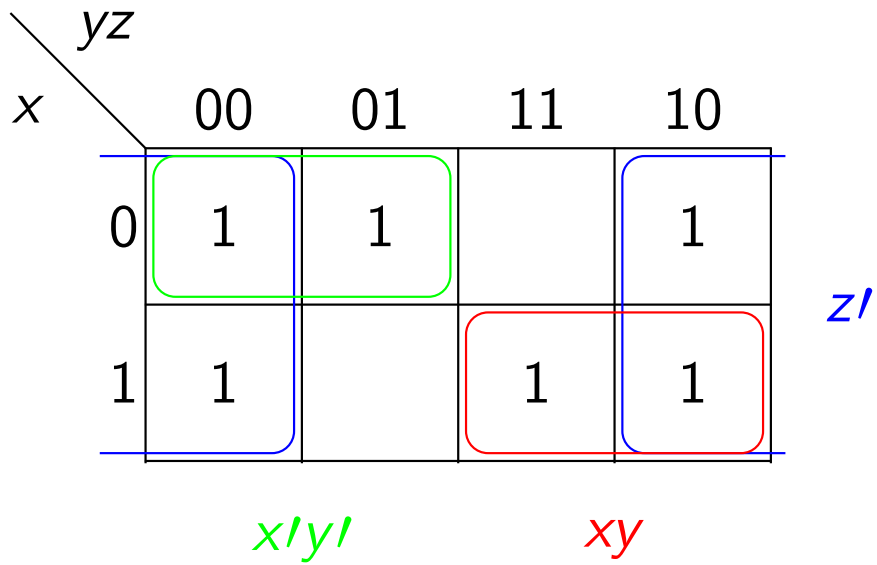


$F_1(x, y, z) = z' + xy'$

Example: Three Variable Karnaugh Map

$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$ $\quad$ $F_2(x, y, z) = \Sigma(0, 1, 2, 4, 6, 7)$





$F_1(x, y, z) = z' + xy'$ $\quad$ $F_2(x, y, z) = z' + x'y' + xy$

Four Variable Karnaugh Map

Four variable: x, y, z, t

16 minterms:

$m_0 = x'y'z't' \rightarrow 0000$

$m_1 = x'y'z't \rightarrow 0001$

$m_2 = x'y'zt' \rightarrow 0010$

$m_3 = x'y'zt \rightarrow 0011$

$m_4 = x'yz't' \rightarrow 0100$

$m_5 = x'yz't \rightarrow 0101$

$m_6 = x'yzt' \rightarrow 0110$

$m_7 = x'yzt \rightarrow 0111$

$m_8 = xy'z't' \rightarrow 1000$

$m_9 = xy'z't \rightarrow 1001$

$m_{10} = xy'zt' \rightarrow 1010$

$m_{11} = xy'zt \rightarrow 1011$

$m_{12} = xyz't' \rightarrow 1100$

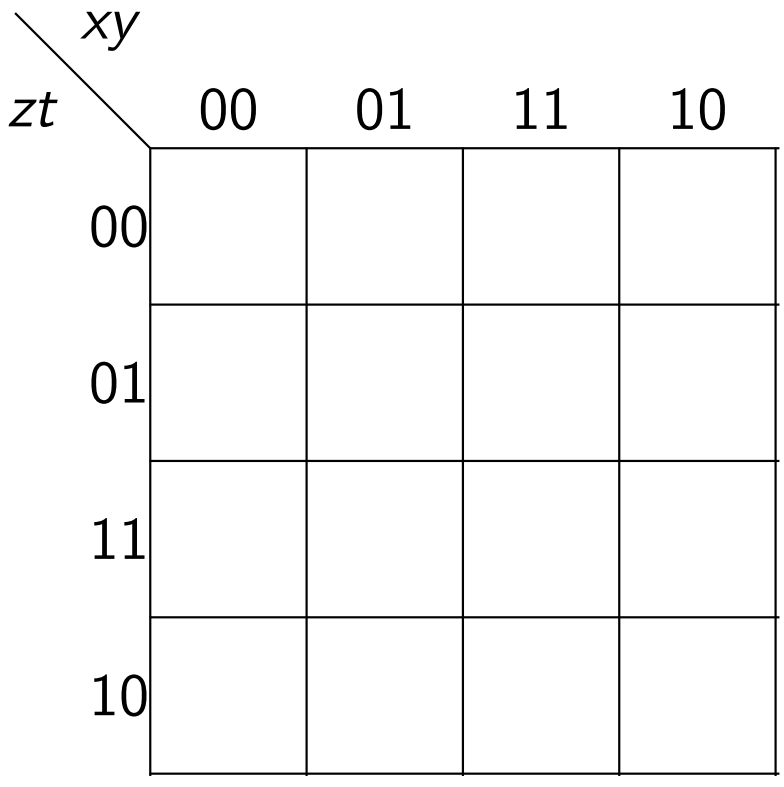
$m_{13} = xyz't \rightarrow 1101$

$m_{14} = xyzt' \rightarrow 1110$

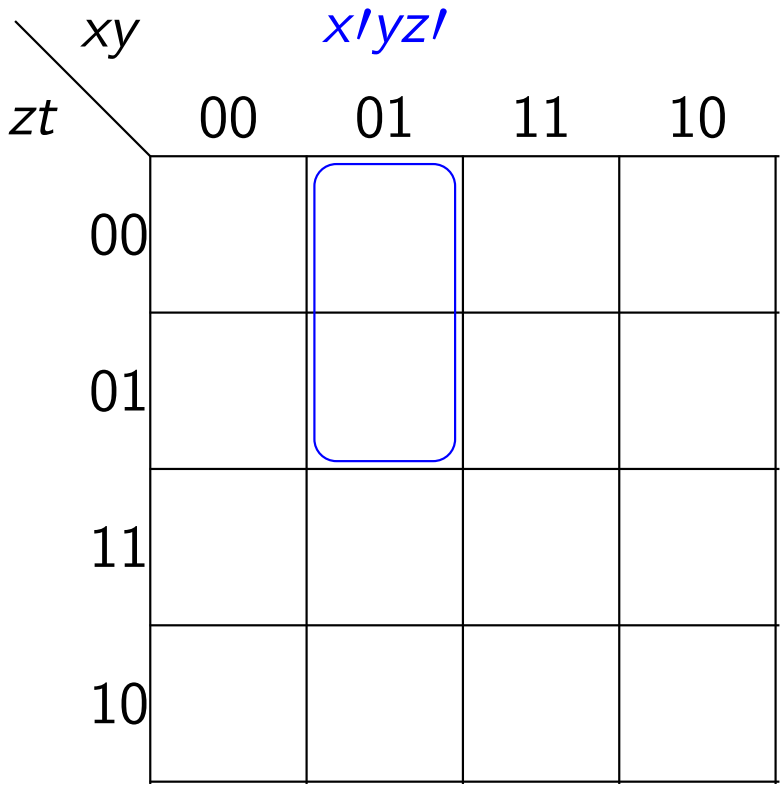
$m_{15} = xyzt \rightarrow 1111$

zt \ xy	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

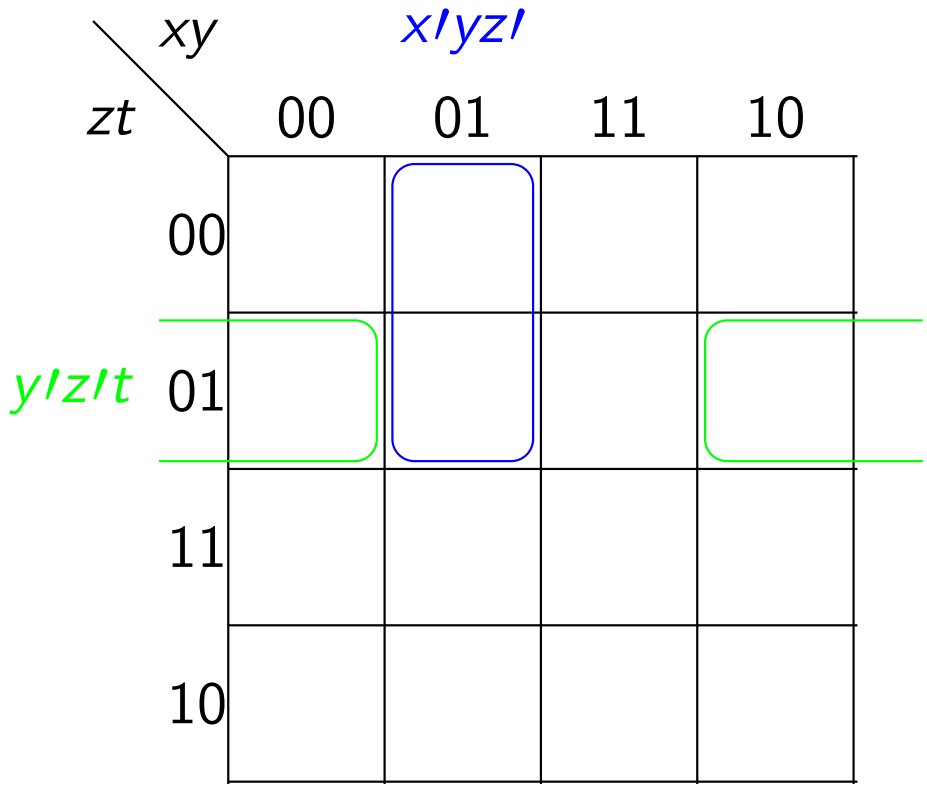
Four Variable Karnaugh Map



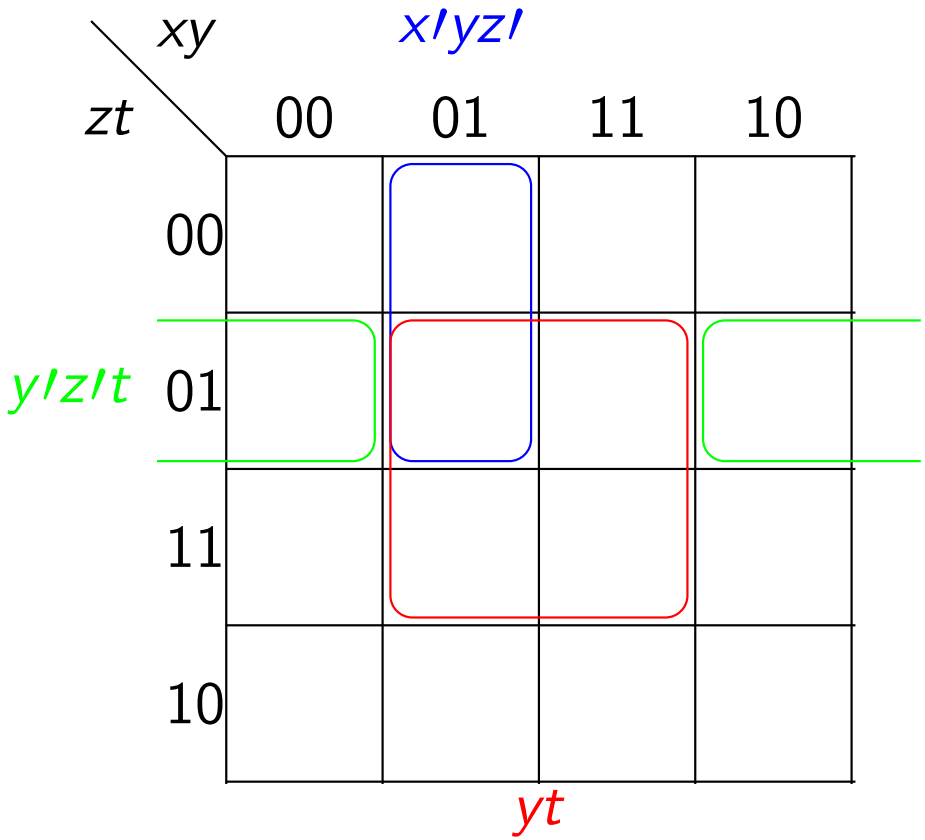
Four Variable Karnaugh Map



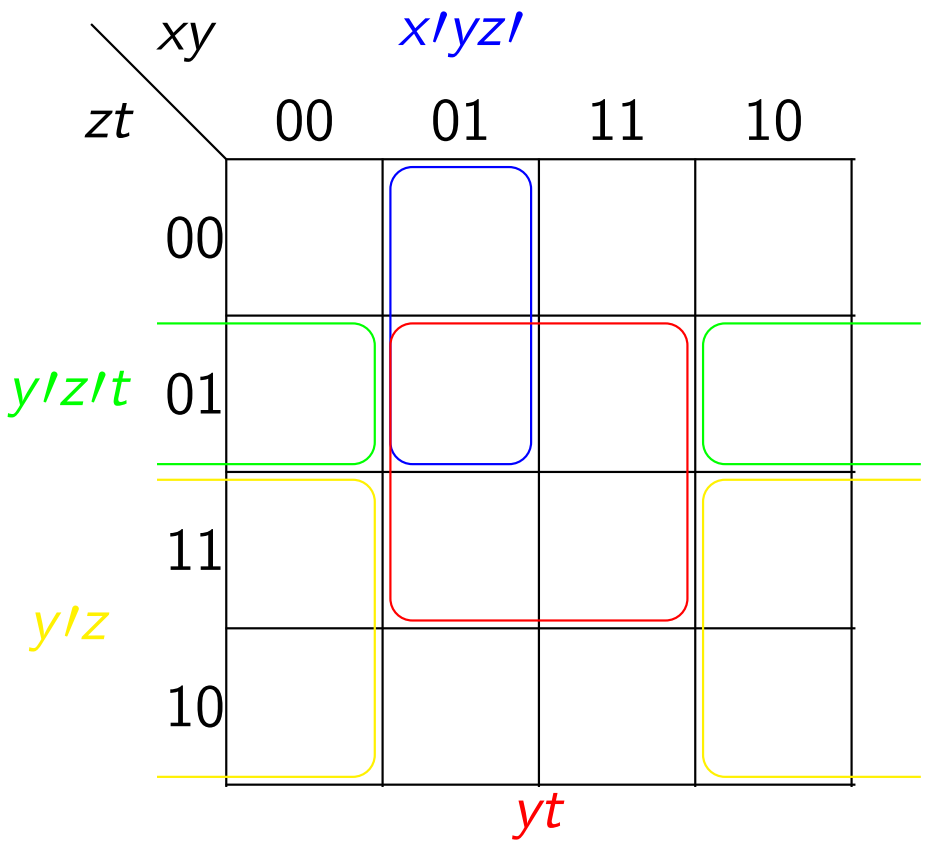
Four Variable Karnaugh Map



Four Variable Karnaugh Map



Four Variable Karnaugh Map



Four Variable Karnaugh Map