ELK 331E/331

Power Electronic Circuits/Güç Elektroniği Devreleri

The Controlled Half-Wave Rectifier

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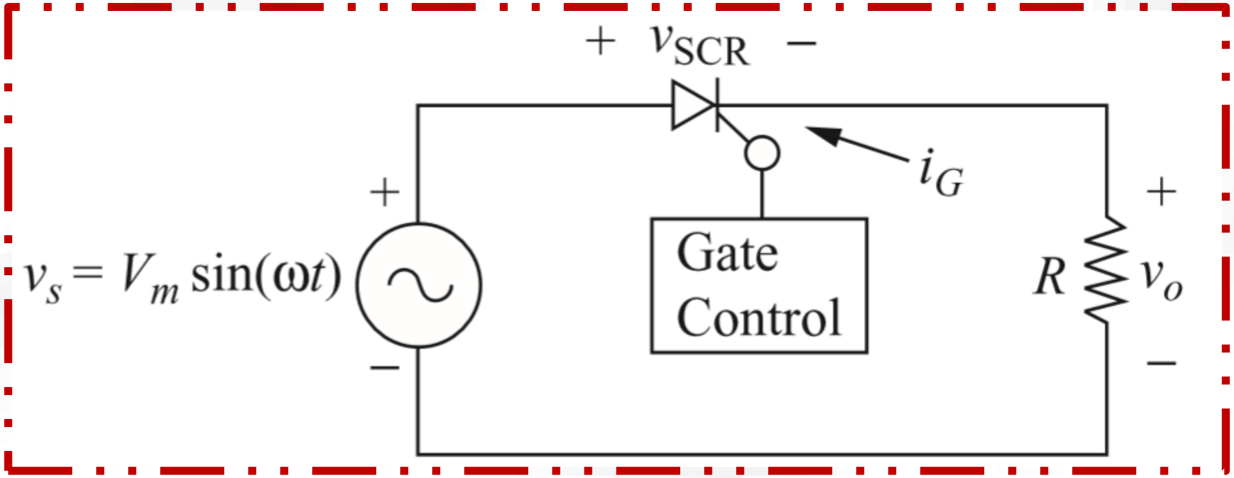
The Controlled Half-Wave Rectifier

- The uncontrolled half-wave rectifiers analysed previously
- Once the source and load parameters are established, the dc level of the output and the power transferred to the load are fixed quantities.
- Away to control the output of a half-wave rectifier is to use an SCR instead of a diode.
- A basic controlled half-wave rectifier with a resistive load.



Switching with other controlled turn-on devices such as transistors or IGBTs can be used to control the output of a converter.

The Controlled Half-Wave Rectifier

- Two conditions must be met before the SCR can conduct:

1. The SCR must be forward-biased ($V_{SCR}>0$).
2. A current must be applied to the gate of the SCR.

- Unlike the diode, the SCR will not begin to conduct as soon as the source becomes positive.

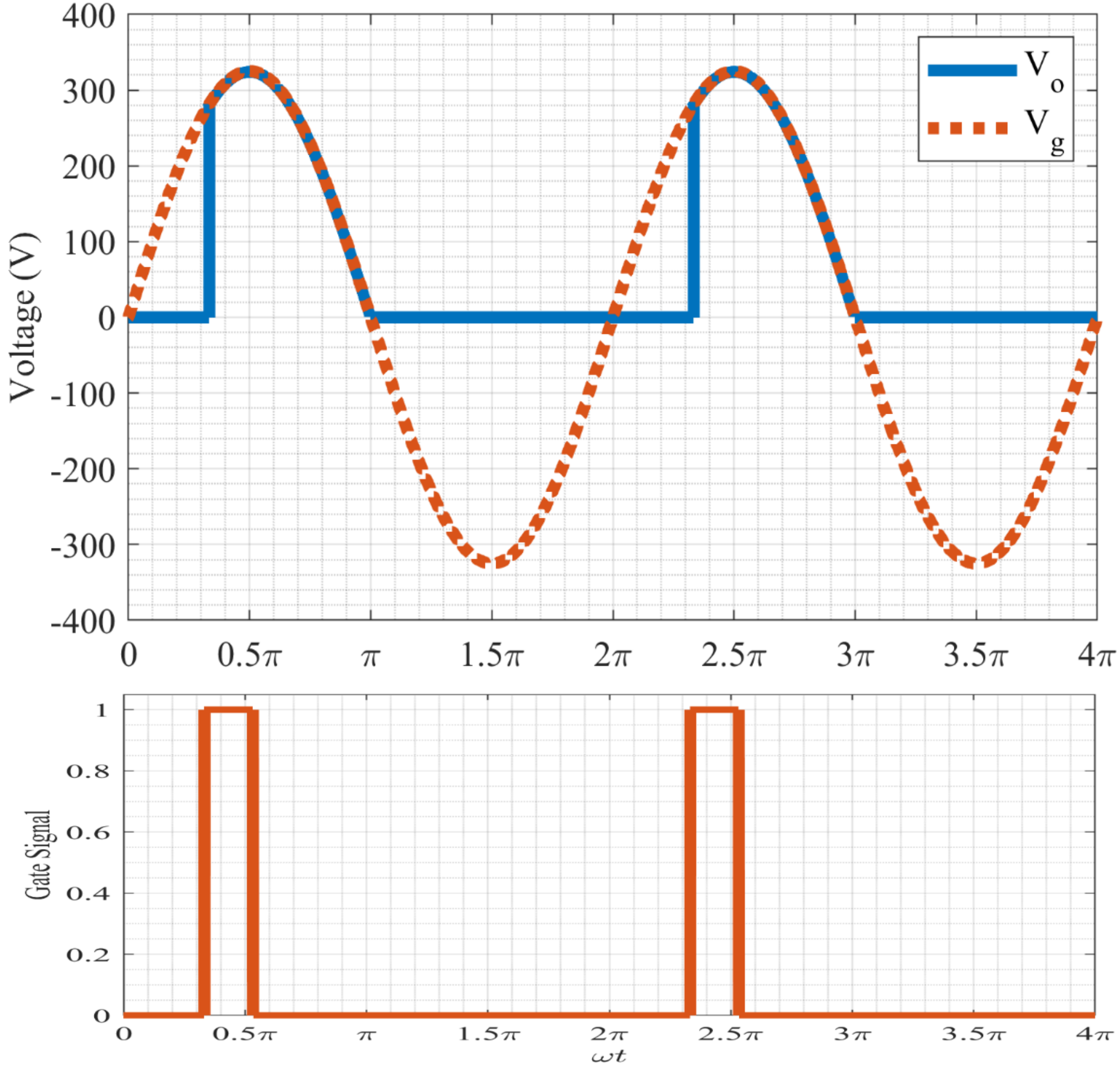
- Conduction is delayed until a gate current is applied, which is the basis for using the SCR as a means of control.

- Once the SCR is conducting, the gate current can be removed and the SCR remains on until the current goes to zero.

The Controlled Half-Wave Rectifier

Resistive Load (α=60°)

A gate signal is applied to the SCR at wt=α (α is the delay angle or firing angle)



The Controlled Half-Wave Rectifier

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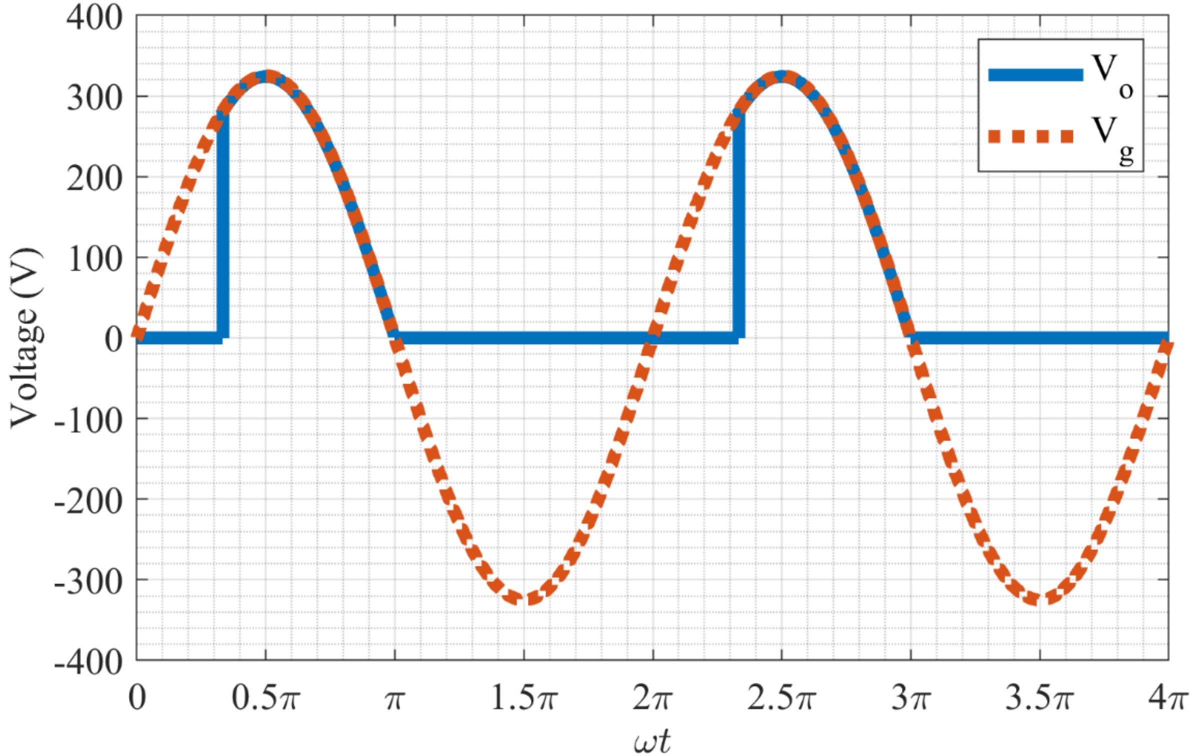
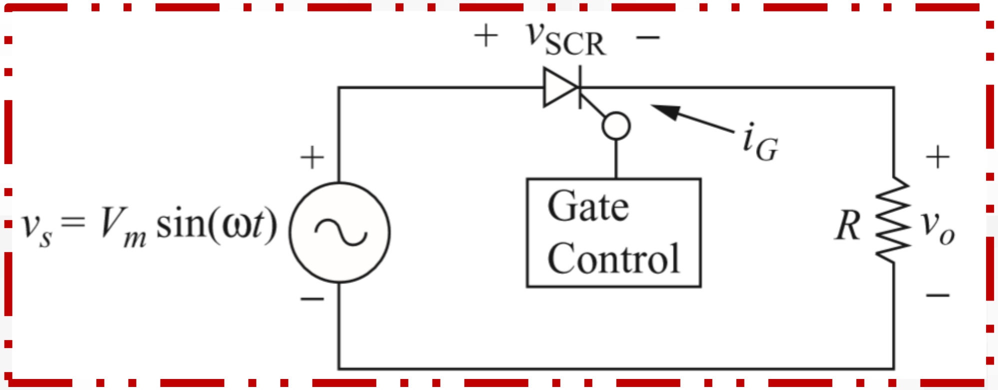


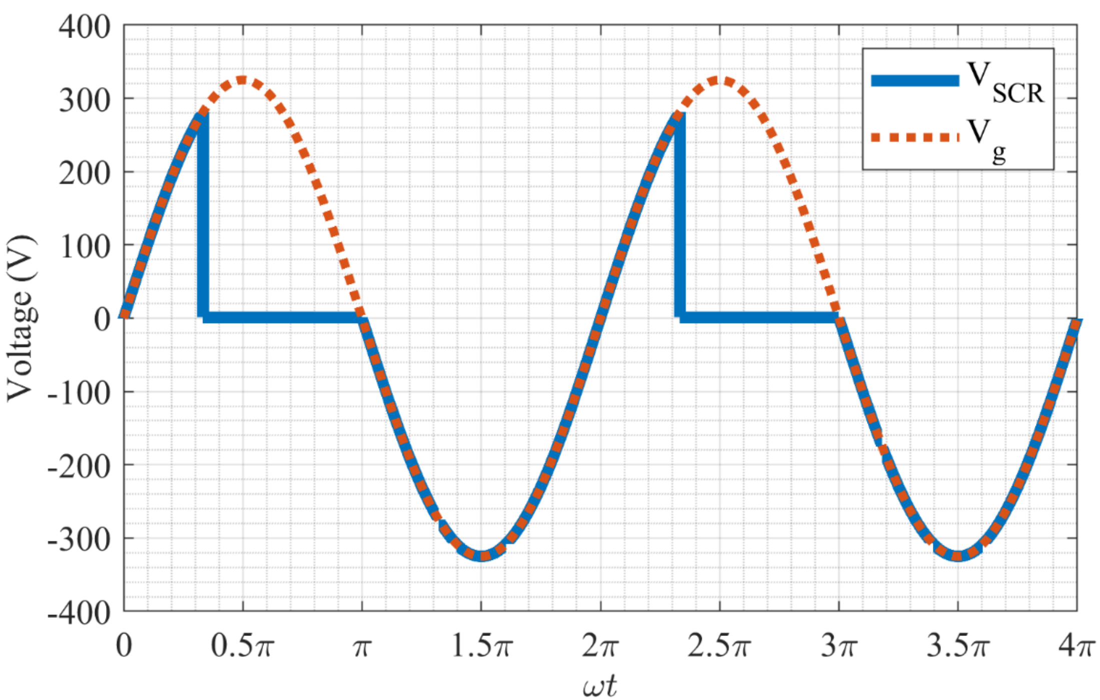
The average (dc) voltage across the load resistor in Figure is:

$$\boxed{V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi}(1 + \cos\alpha)}$$

The Controlled Half-Wave Rectifier

Resistive Load (α=60°)





The Controlled Half-Wave Rectifier

Resistive Load (α=60°)

The power absorbed by the resistor is V^2_{rms}/R, where the rms voltage across the resistor is computed from

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_o^2(\omega t)\,d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi}\int_\alpha^{\pi} [V_m \sin(\omega t)]^2\, d(\omega t)}$$

$$= \frac{V_m}{2}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}$$

The Controlled Half-Wave Rectifier

Example-1

Design a half-wave controlled rectifier circuit to produce an average voltage of 40 V across a 100-Ω load resistor from a 120 Vrms 60-Hz ac source. Determine the power absorbed by the resistance and the power factor.

$$\alpha = \cos^{-1}\left[V_o\left(\frac{2\pi}{V_m}\right) - 1\right]$$

$$= \cos^{-1}\left\{40\left[\frac{2\pi}{\sqrt{2}(120)}\right] - 1\right\} = 61.2° = 1.07 \text{ rad}$$

$$V_{\text{rms}} = \frac{\sqrt{2}(120)}{2}\sqrt{1 - \frac{1.07}{\pi} + \frac{\sin[2(1.07)]}{2\pi}} = 75.6 \text{ V}$$

Load power is

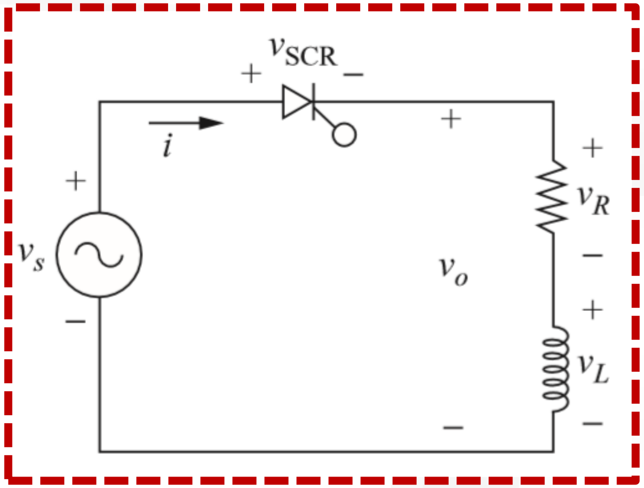
$$P_R = \frac{V_{\text{rms}}^2}{R} = \frac{(75.6)^2}{100} = 57.1 \text{ W}$$

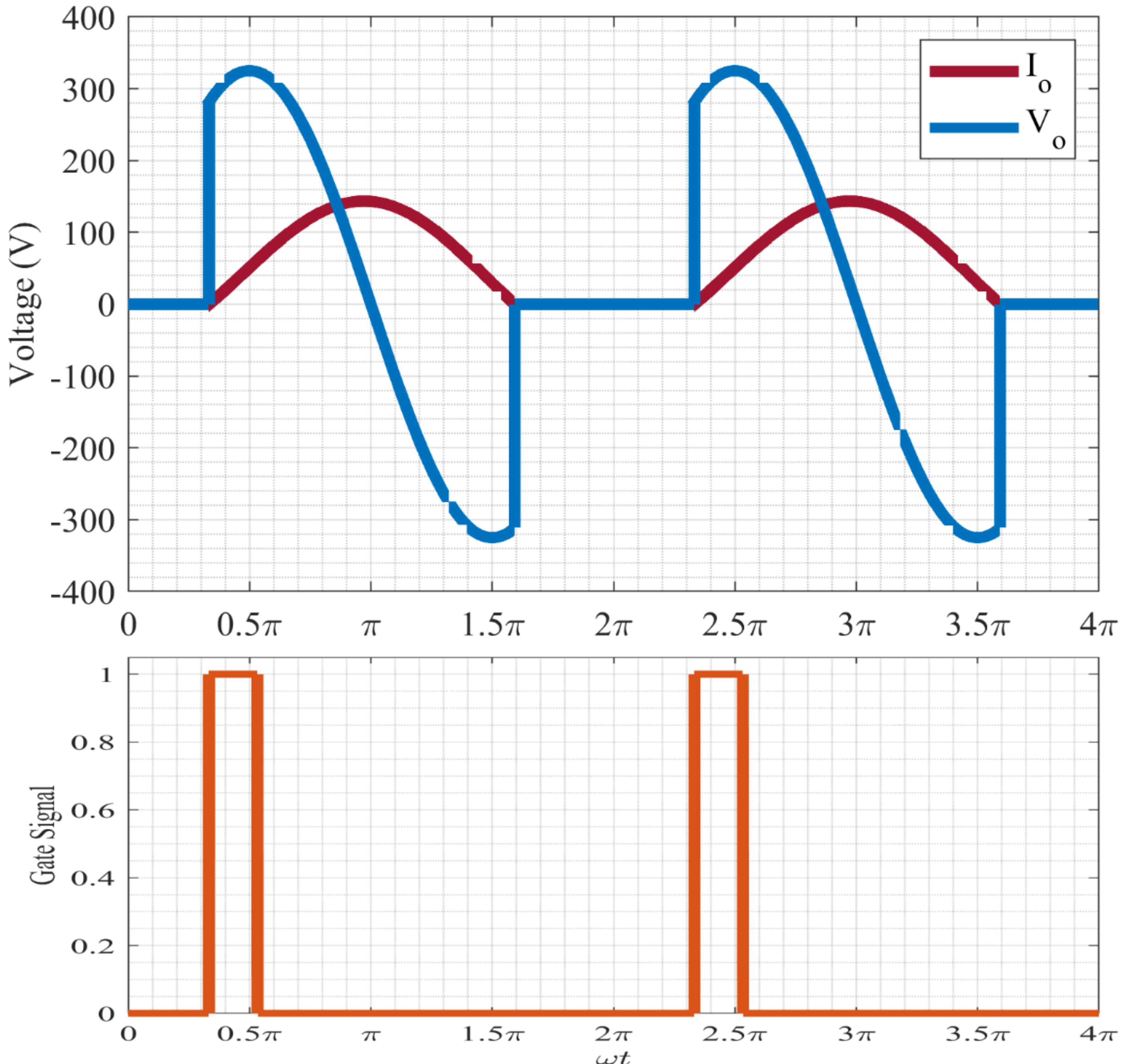
The power factor of the circuit is

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{S,\text{rms}} I_{\text{rms}}} = \frac{57.1}{(120)(75.6/100)} = 0.63$$

The Controlled Half-Wave Rectifier

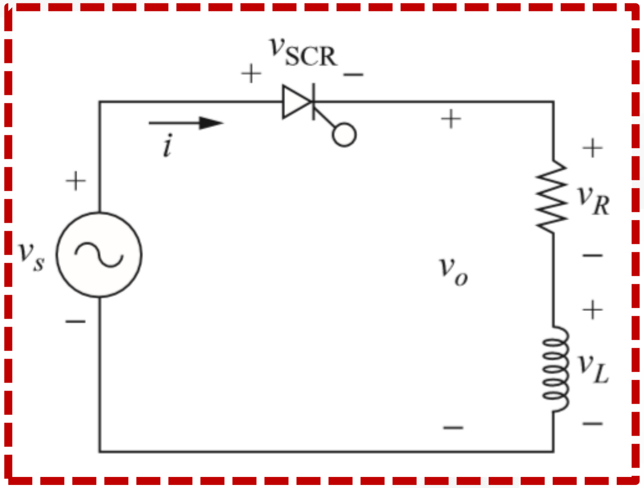
Resistive-Inductive Load (α=60°)

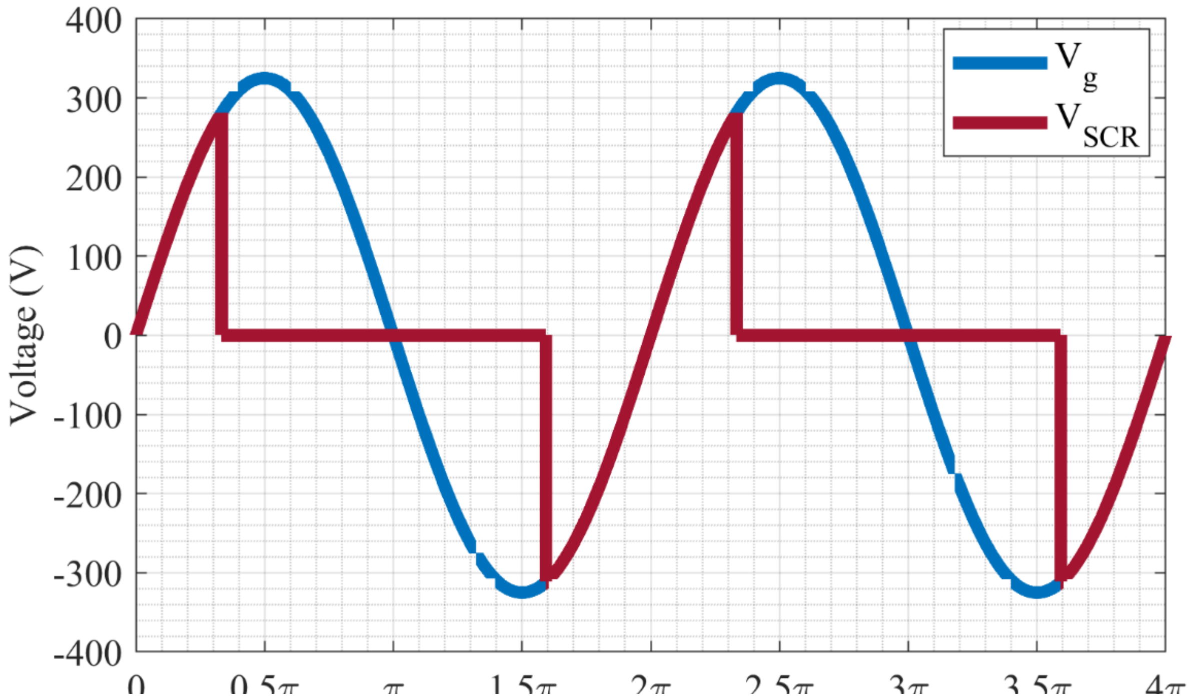


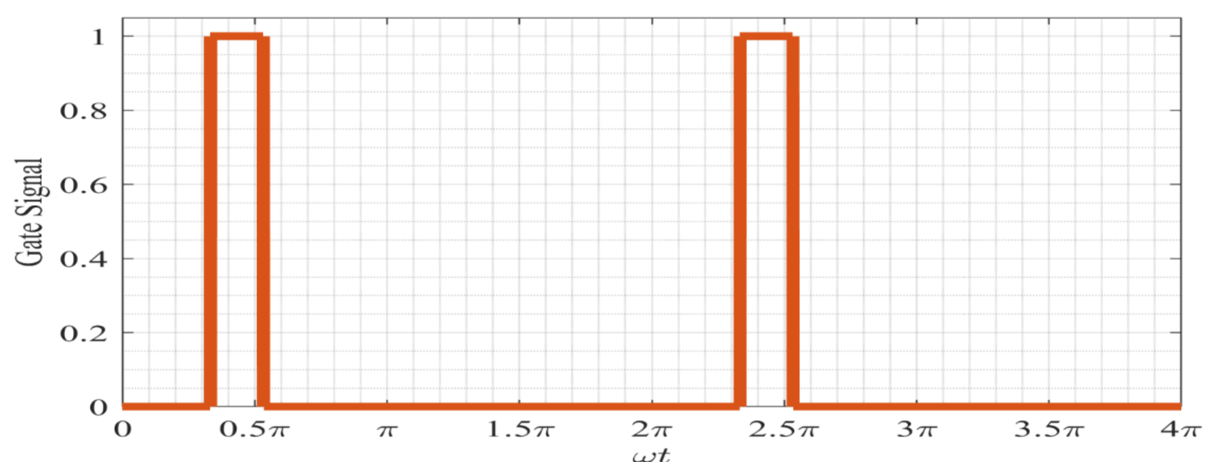


The Controlled Half-Wave Rectifier

Resistive-Inductive Load (α=60°)







The Controlled Half-Wave Rectifier

Resistive-Inductive Load (α=60°)

The analysis of this circuit is similar to that of the uncontrolled rectifier. The current is the sum of the forced and natural responses:

$$i(\omega t) = i_f(\omega t) + i_n(\omega t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-\omega t/\omega\tau}$$

The constant A is determined from the initial condition i(α)=0:

$$i(\alpha) = 0 = \frac{V_m}{Z}\sin(\alpha - \theta) + Ae^{-\alpha/\omega\tau}$$

$$A = \left[-\frac{V_m}{Z}\sin(\alpha - \theta)\right] = e^{\alpha/\omega\tau}$$

$$\boxed{i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)e^{(\alpha - \omega t)/\omega\tau}\right] & \text{for}\,\alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}}$$

The Controlled Half-Wave Rectifier

Resistive-Inductive Load (α=60°)

The extinction angle is defined as the angle at which the current returns to zero, as in the case of the uncontrolled rectifier. When wt =β:

$$i(\beta) = 0 = \frac{V_m}{Z}\left[\sin(\beta - \theta) - \sin(\alpha - \theta) e^{(\alpha-\beta)/\omega\tau}\right]$$

which must be solved numerically for β. The angle (β–α) is called the conduction angle γ. The average (dc) output voltage is:

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

The average and RMS current is computed from:

$$I_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(\omega t)\, d(\omega t) \qquad I_{rms} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} i^2(\omega t)\, d(\omega t)}$$

Power absorbed by the load is I2rmsR

The Controlled Half-Wave Rectifier

Example-2

In a controlled half wave rectifier circuit, the source is 120 V rms at 60 Hz, R=20 Ω, L=0.04 H, and the delay angle is 45°. Determine:

(a) an expression for i(wt),
(b) the average current,
(c) the power absorbed by the load, and
(d) the power factor.

(*a*) From the parameters given,

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$
$$Z = [R^2 + (\omega L)^2]^{0.5} = [20^2 + (377*0.04)^2]^{0.5} = 25.0\ \Omega$$
$$\theta = \tan^{-1}(\omega L/R) = \tan^{-1}(377*0.04)/20) = 0.646 \text{ rad}$$
$$\omega\tau = \omega L/R = 377*0.04/20 = 0.754$$
$$\alpha = 45^\circ = 0.785 \text{ rad}$$

Substituting the preceding quantities into Eq. (3-55), current is expressed as

$$i(\omega t) = 6.78 \sin(\omega t - 0.646) - 2.67e^{-\omega t/0.754} \quad \text{A} \qquad \text{for } \alpha \le \omega t \le \beta$$

The preceding equation is valid from α to β, where β is found numerically by setting the equation to zero and solving for ωt, with the result $\beta = 3.79$ rad (217°). The conduction angle is $\gamma = \beta - \alpha = 3.79 - 0.785 = 3.01$ rad $= 172°$.

The Controlled Half-Wave Rectifier

Example-2

In a controlled half wave rectifier circuit, the source is 120 V rms at 60 Hz, R=20 Ω, L=0.04 H, and the delay angle is 45°. Determine:

(a) an expression for i(wt),
(b) the average current,
(c) the power absorbed by the load, and
(d) the power factor.

(*b*) Average current is

$$I_o = \frac{1}{2\pi}\int_{0.785}^{3.79} \left[6.78 \sin(\omega t - 0.646) - 2.67 e^{-\omega t/0.754}\right] d(\omega t) = 2.19 \text{ A}$$

(*c*) The power absorbed by the load is computed from $I_{rms}^2 R$, where

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_{0.785}^{3.79} \left[6.78 \sin(\omega t - 0.646) - 2.67 e^{-\omega t/0.754}\right]^2 d(\omega t)} = 3.26 \text{ A}$$

yielding

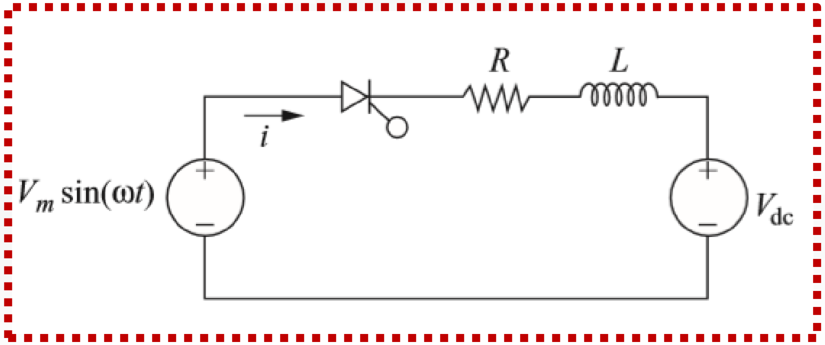
$$P = I_{rms}^2 R = (3.26)^2(20) = 213 \text{ W}$$

(*d*) The power factor is

$$\text{pf} = \frac{P}{S} = \frac{213}{(120)(3.26)} = 0.54$$

The Controlled Half-Wave Rectifier

Resistive-Inductive-Source (RLE) Load (α=60°)

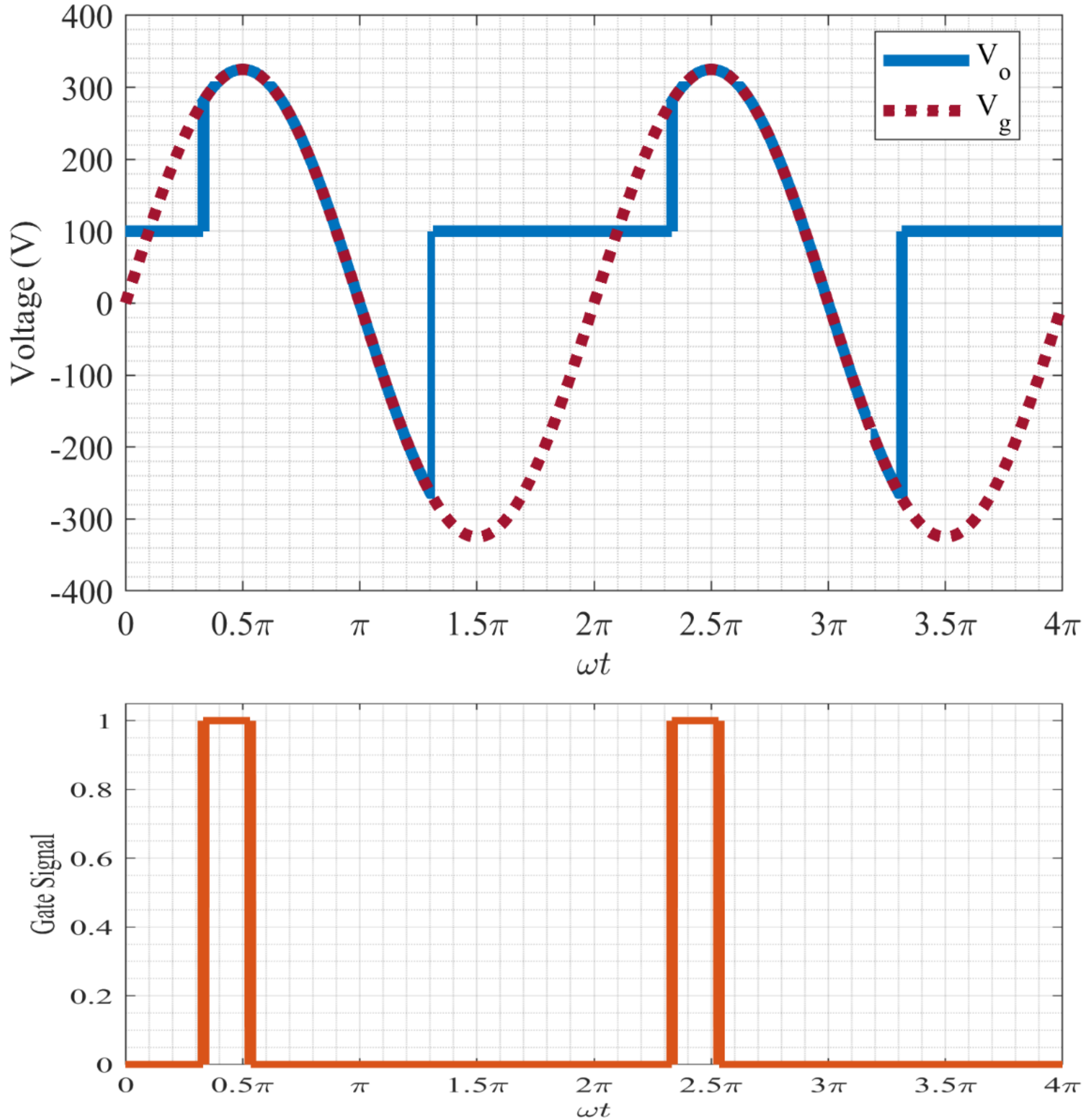


A controlled rectifier with a series resistance, inductance, and dc source is shown in Figure. The analysis of this circuit is very similar to that of the uncontrolled half-wave rectifier discussed earlier lectures. The major difference is that for the uncontrolled rectifier, conduction begins as soon as the source voltage reaches the level of the dc voltage. For the controlled rectifier, conduction begins when a gate signal is applied to the SCR, provided the SCR is forward-biased. Thus, the gate signal may be applied at any time that the ac source is larger than the dc source:

$$\alpha_{min} = \sin^{-1}\left(\frac{V_{dc}}{V_m}\right)$$

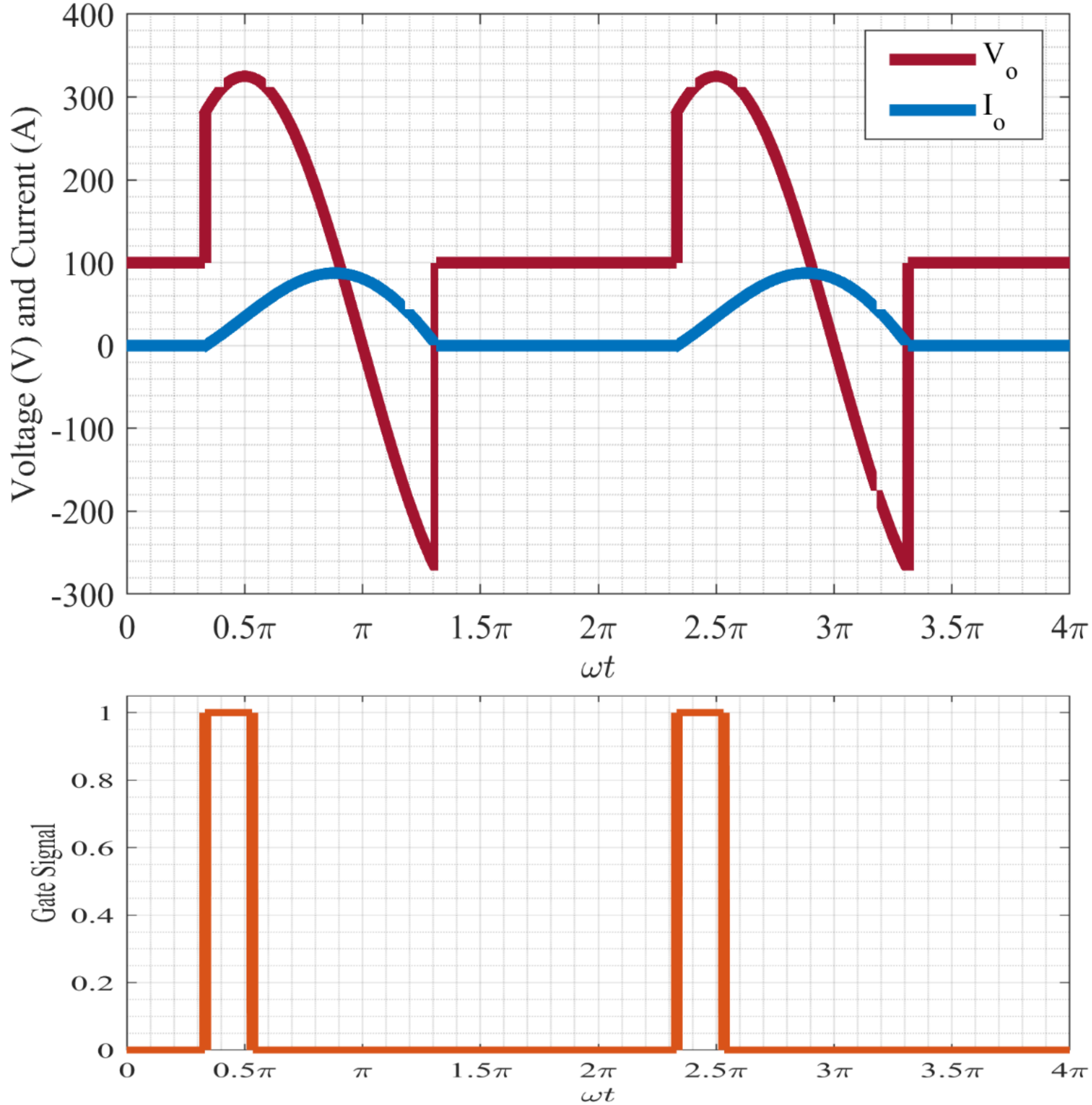
The Controlled Half-Wave Rectifier

Resistive-Inductive-Source (RLE) Load (α=60°)



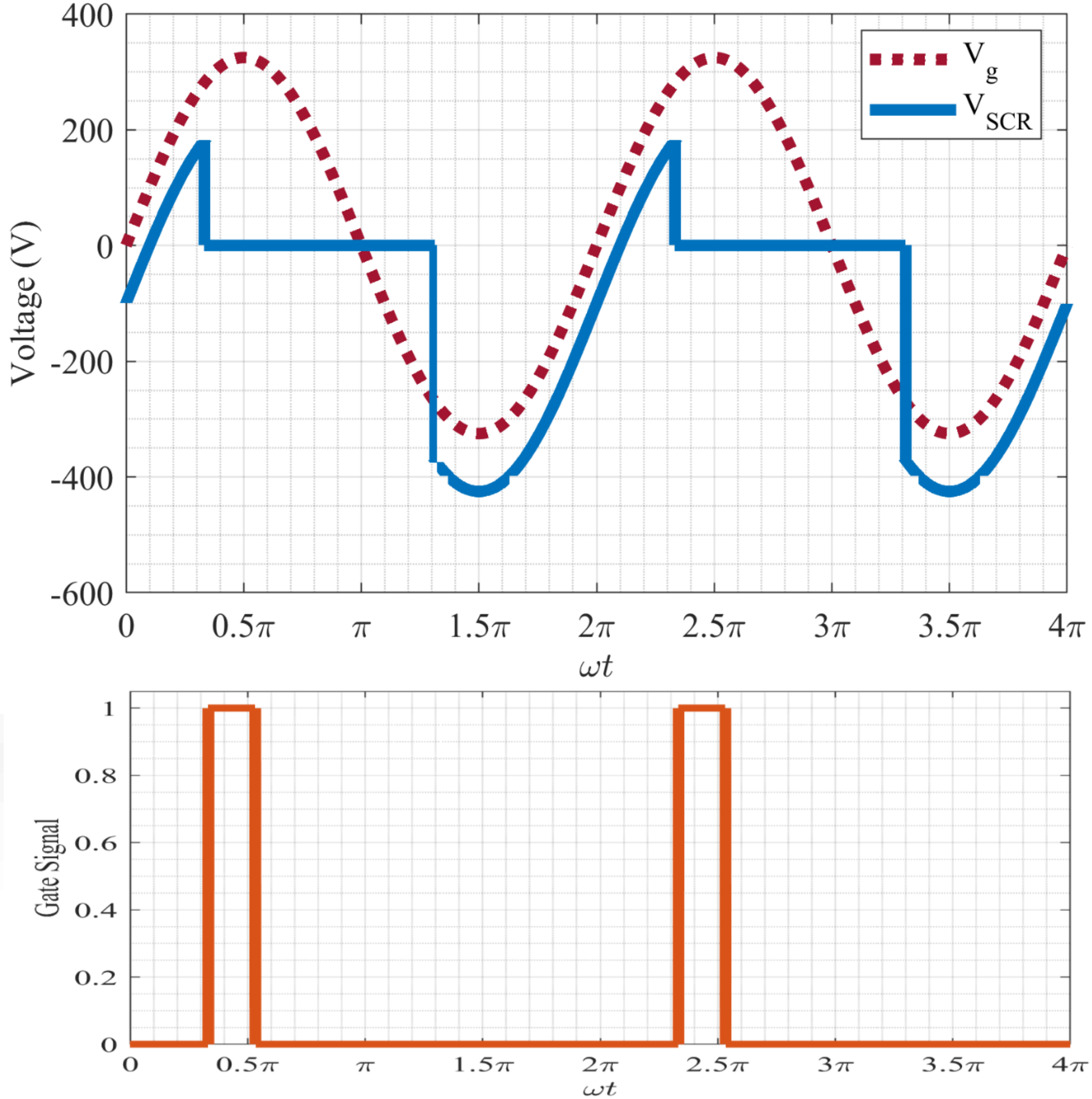
The Controlled Half-Wave Rectifier

Resistive-Inductive-Source (RLE) Load (α=60°)



The Controlled Half-Wave Rectifier

Resistive-Inductive-Source (RLE) Load (α=60°)



The Controlled Half-Wave Rectifier

Resistive-Inductive-Source (RLE) Load

$$i(\omega t)\begin{cases} \dfrac{V_m}{Z}\sin(\omega t-\theta)-\dfrac{V_{dc}}{R}+Ae^{-\omega t/\omega\tau} & \text{for } \alpha \le \omega t \le \beta \\ 0 & \text{otherwise} \end{cases}$$

$$A=\left[-\frac{V_m}{Z}\sin(\alpha-\theta)+\frac{V_{dc}}{R}\right]e^{\alpha/\omega\tau}$$

The Controlled Half-Wave Rectifier

Example-3

The controlled half-wave rectifier has an ac input of 120 V rms at 60 Hz, R=2 Ω, L=20 mH, and Vdc =100 V. The delay angle α is 45°. Determine

(a) an expression for the current,
(b) the power absorbed by the resistor, and
(c) the power absorbed by the dc source in the load.

From the parameters given,

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$

$$Z = [R^2 + (\omega L)^2]^{0.5} = [2^2 + (377*0.02)^2]^{0.5} = 7.80\ \Omega$$

$$\theta = \tan^{-1}(\omega L/R) = \tan^{-1}(377*0.02/2) = 1.312 \text{ rad}$$

$$\omega\tau = \omega L/R = 377*0.02/2 = 3.77$$

$$\alpha = 45° = 0.785 \text{ rad}$$

The Controlled Half-Wave Rectifier

Example-3

(*a*) First, use Eq. (3-60) to determine if $\alpha = 45°$ is allowable. The minimum delay angle is

$$\alpha_{\min} = \sin^{-1}\left(\frac{100}{120\sqrt{2}}\right) = 36°$$

which indicates that 45° is allowable. Equation (3-61) becomes

$$i(\omega t) = 21.8 \sin(\omega t - 1.312) - 50 + 75.0e^{-\omega t/3.77} \text{ A} \quad \text{for } 0.785 \leq \omega t \leq 3.37 \text{ rad}$$

where the extinction angle β is found numerically to be 3.37 rad from the equation $i(\beta) = 0$.

(*b*) Power absorbed by the resistor is $I_{\text{rms}}^2 R$, where I_{rms} is computed from Eq. (3-59) using the preceding expression for $i(\omega t)$.

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_{\alpha}^{\beta} i^2(\omega t)\, d(\omega t)} = 3.90 \text{ A}$$

$$P = (3.90)^2(2) = 30.4 \text{ W}$$

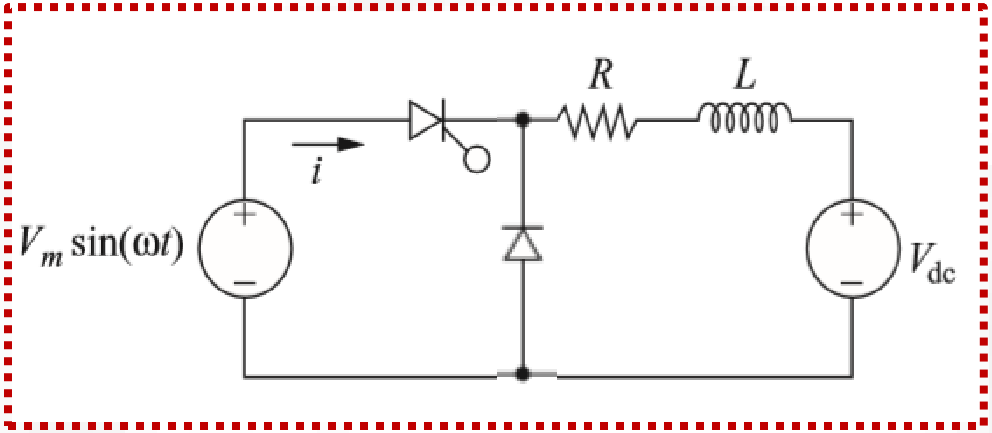
(*c*) Power absorbed by the dc source is $I_o V_{\text{dc}}$, where I_o is computed from Eq. (3-58).

$$I_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} i(\omega t)\, d(\omega t) = 2.19 \text{ A}$$

$$P_{\text{dc}} = I_o V_{\text{dc}} = (2.19)(100) = 219 \text{ W}$$

The Controlled Half-Wave Rectifier

Resistive-Inductive-Source (RLE) Load with free-wheeling diode (α=60°)



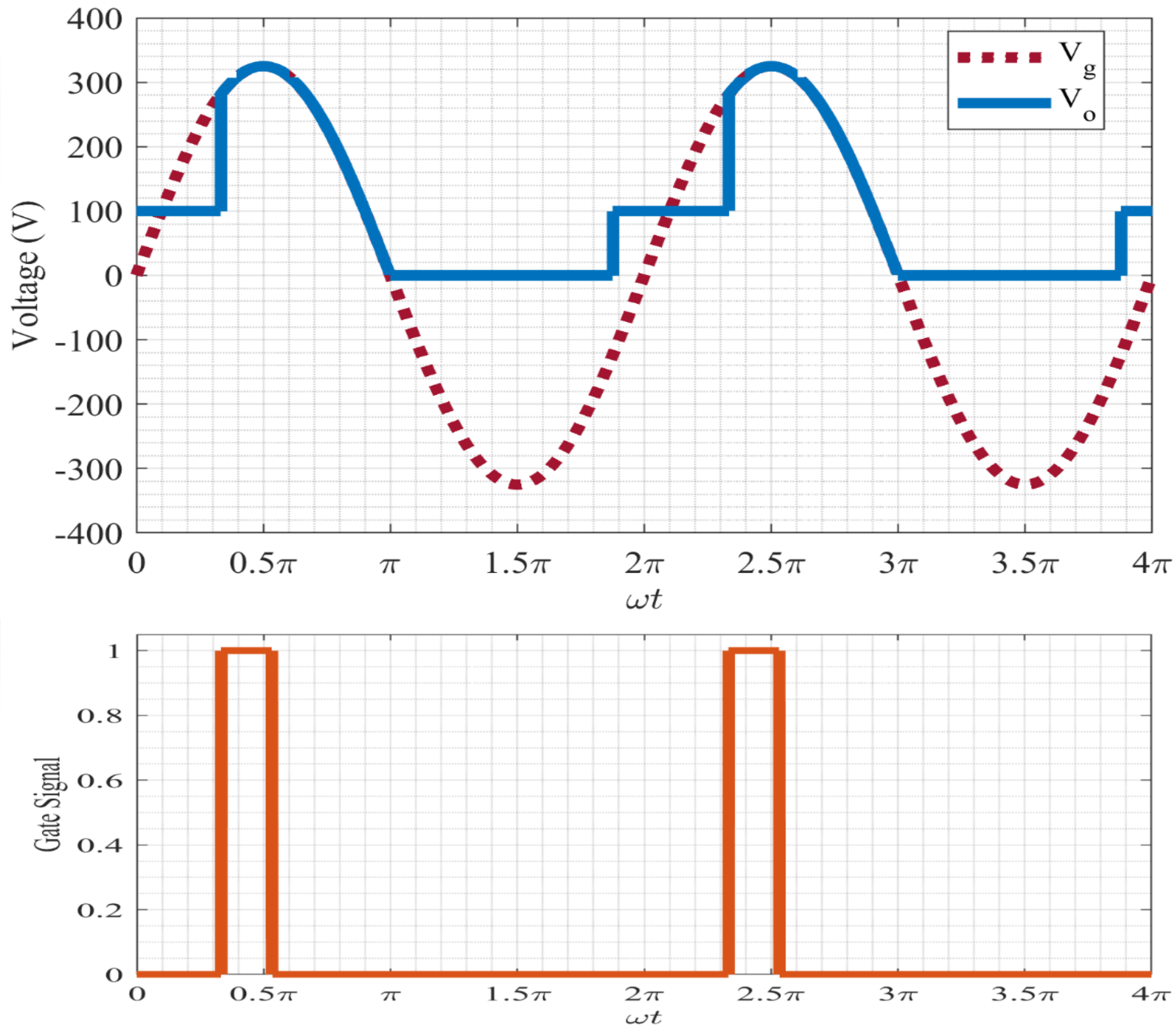
For a positive source voltage,
- SCR is on.
- FWD is off.

For a negative source voltage,
- SCR is off.
- FWD is on.

The Controlled Half-Wave Rectifier

Resistive-Inductive-Source (RLE) Load with free-wheeling diode (α=60°)

1. Discontinuous Load Current



The Controlled Half-Wave Rectifier

Resistive-Inductive-Source (RLE) Load with free-wheeling diode (α=60°)

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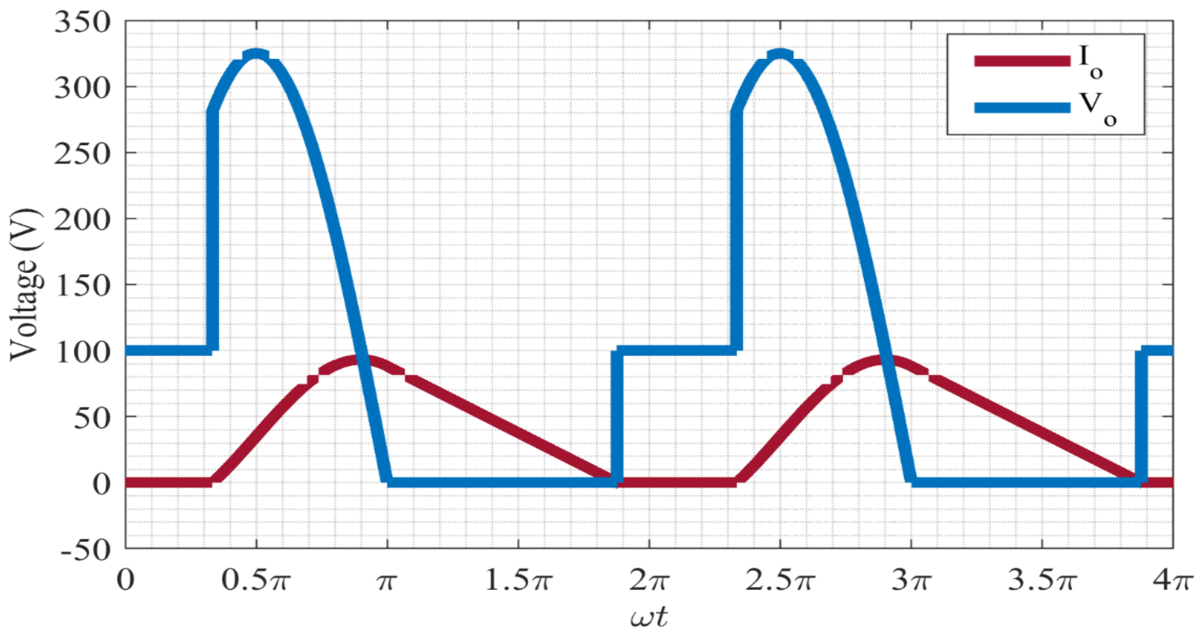

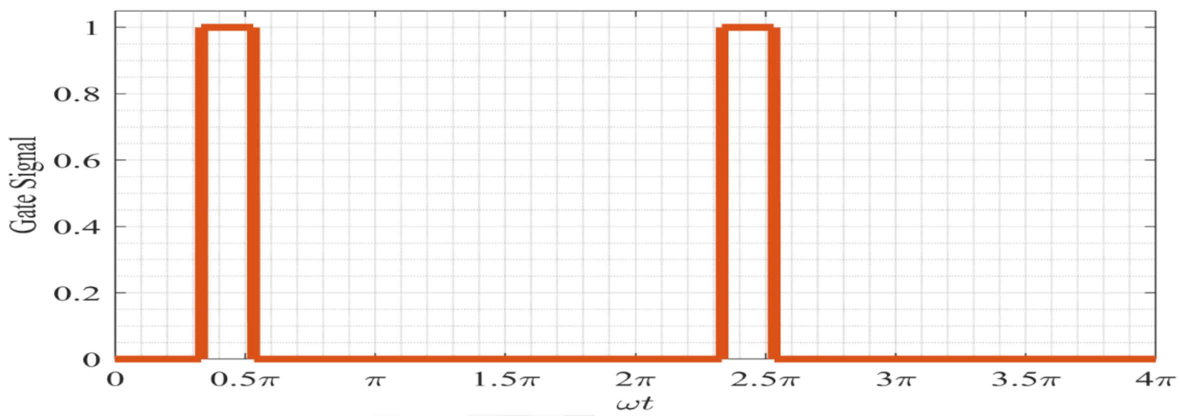

The Controlled Half-Wave Rectifier

Resistive-Inductive-Source (RLE) Load with free-wheeling diode (α=60°)

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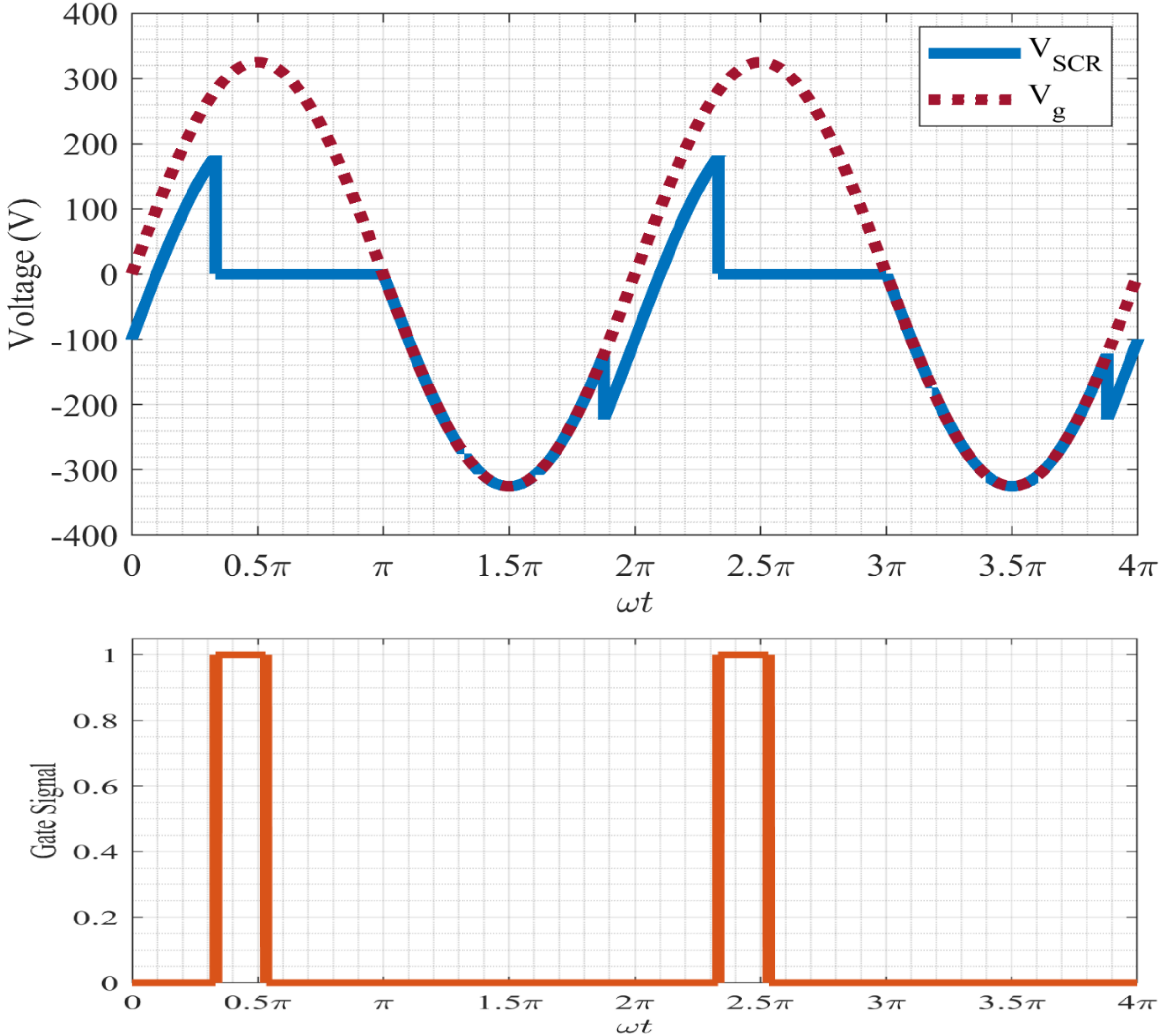
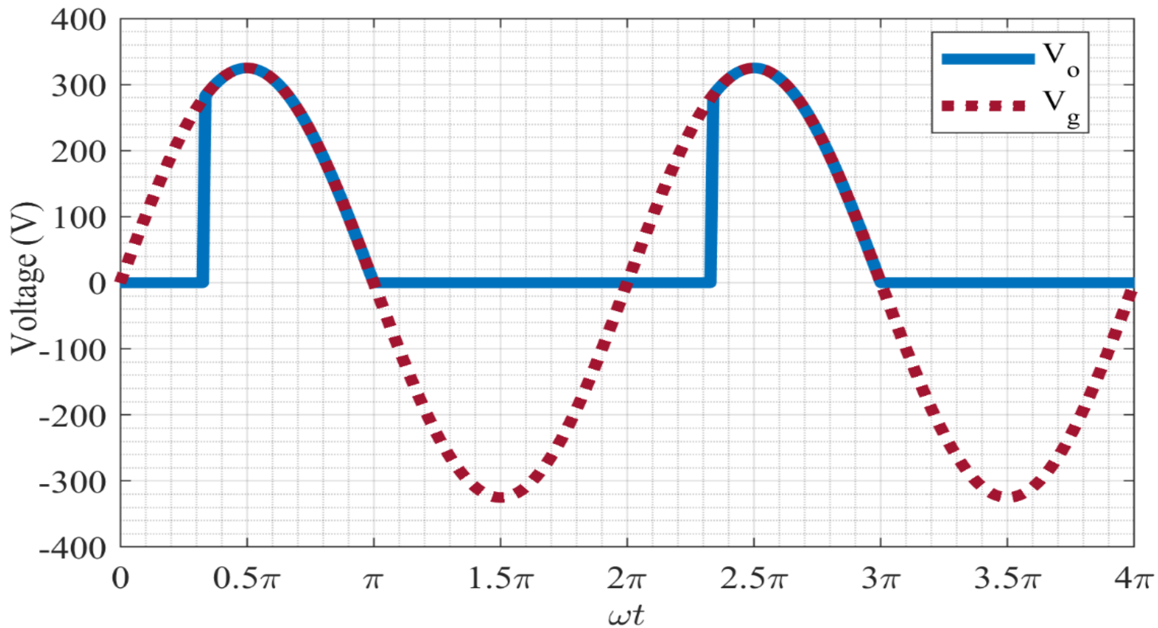
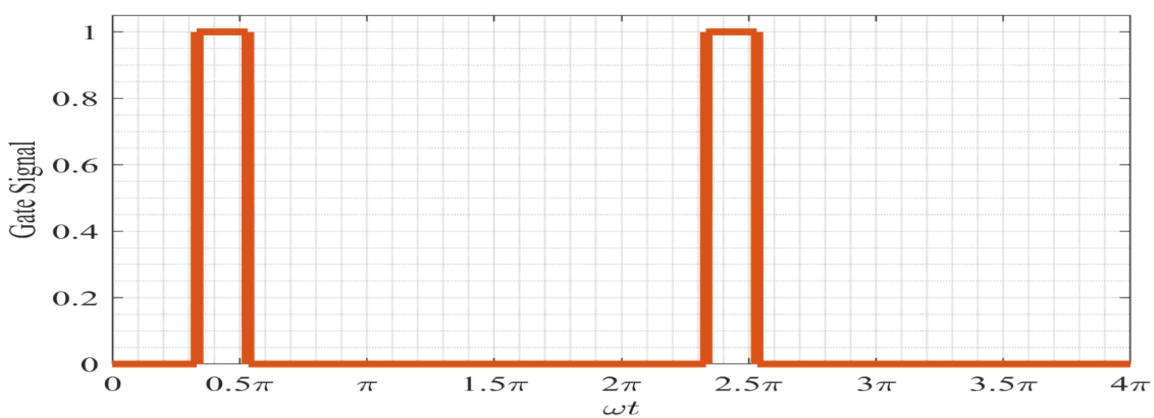


The Controlled Half-Wave Rectifier

Resistive-Inductive-Source (RLE) Load with free-wheeling diode (α=60°)

2. Continuous Load Current

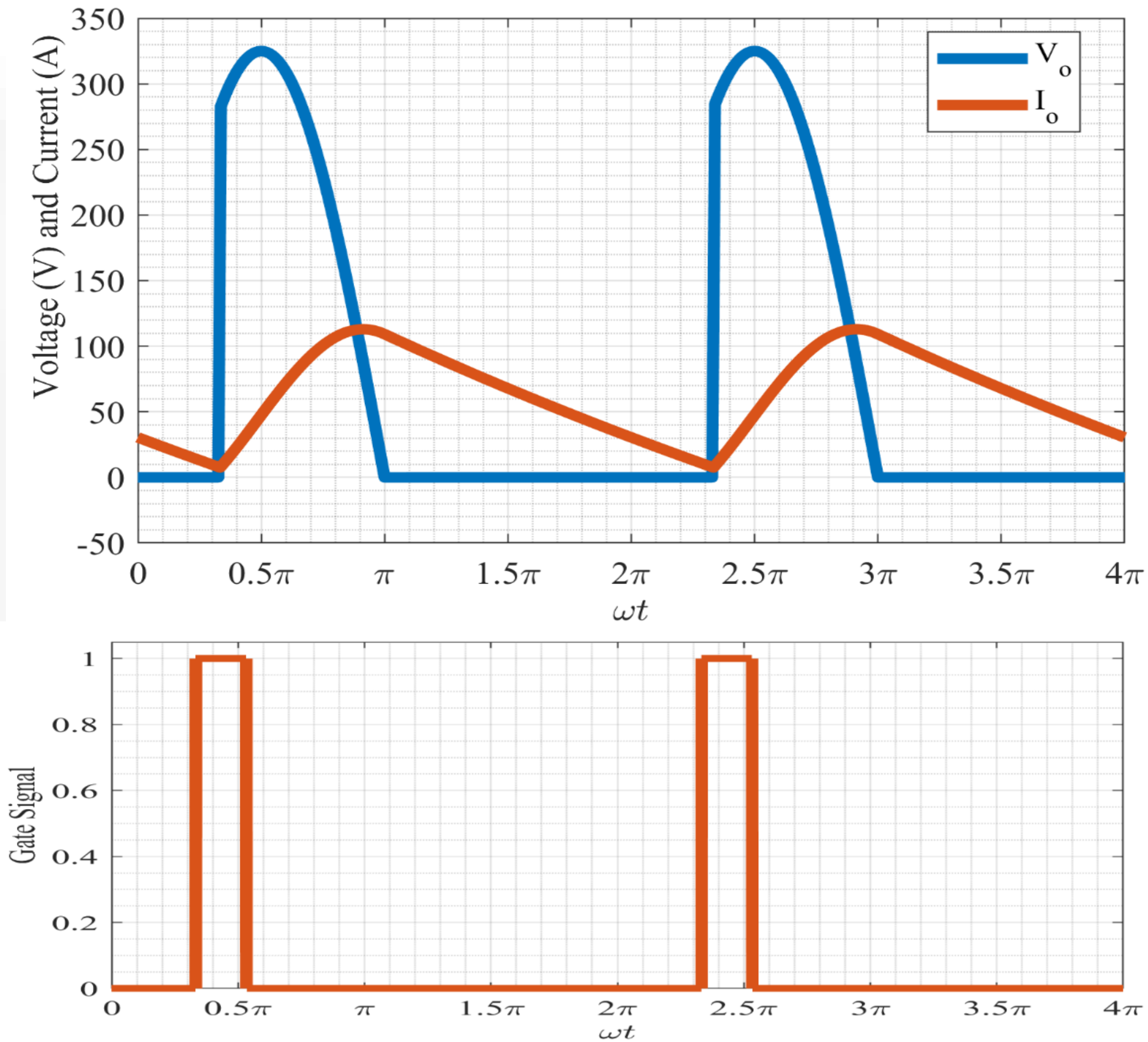




The Controlled Half-Wave Rectifier

Resistive-Inductive-Source (RLE) Load with free-wheeling diode (α=60°)

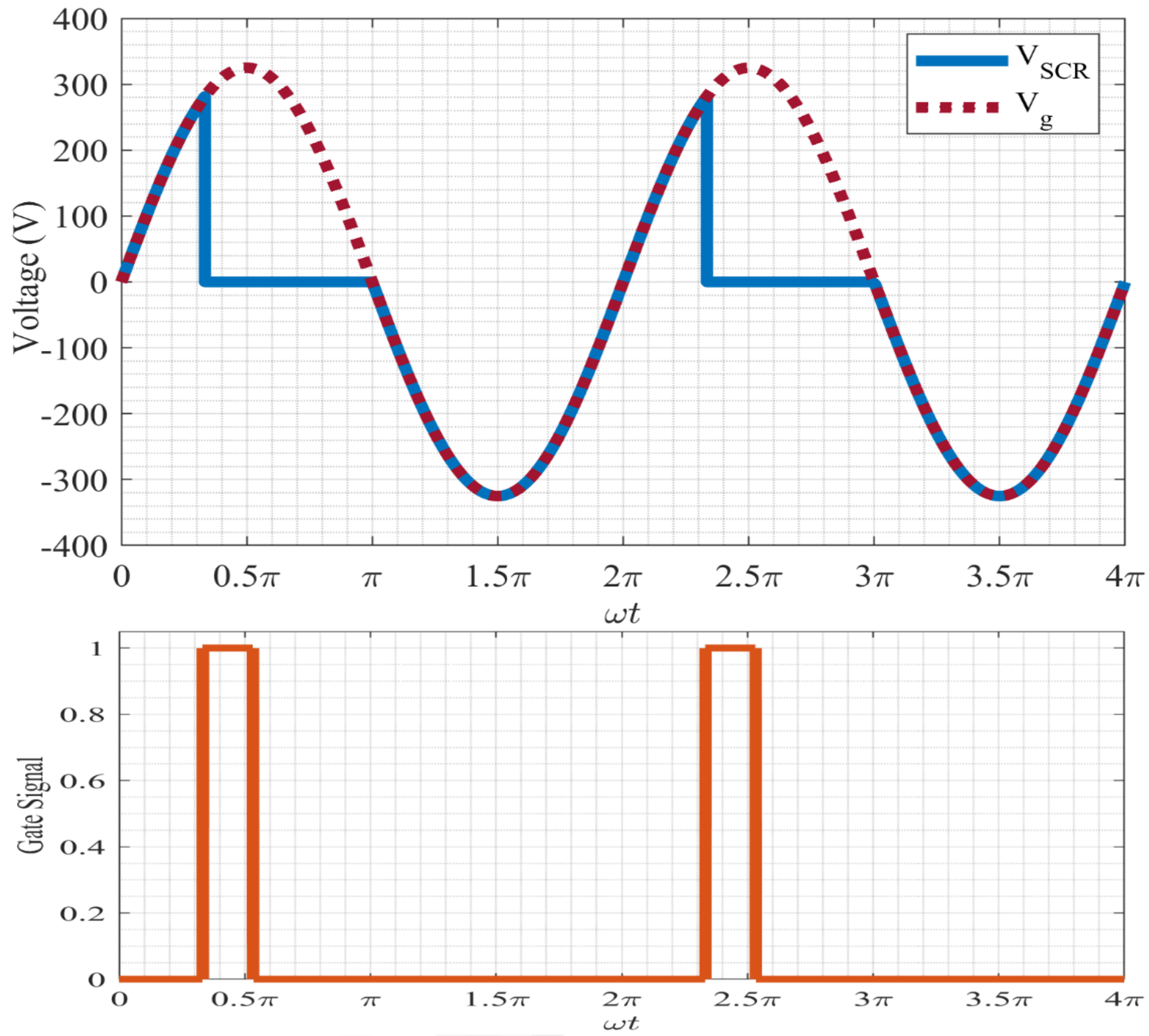
2. Continuous Load Current



The Controlled Half-Wave Rectifier

Resistive-Inductive-Source (RLE) Load with free-wheeling diode (α=60°)

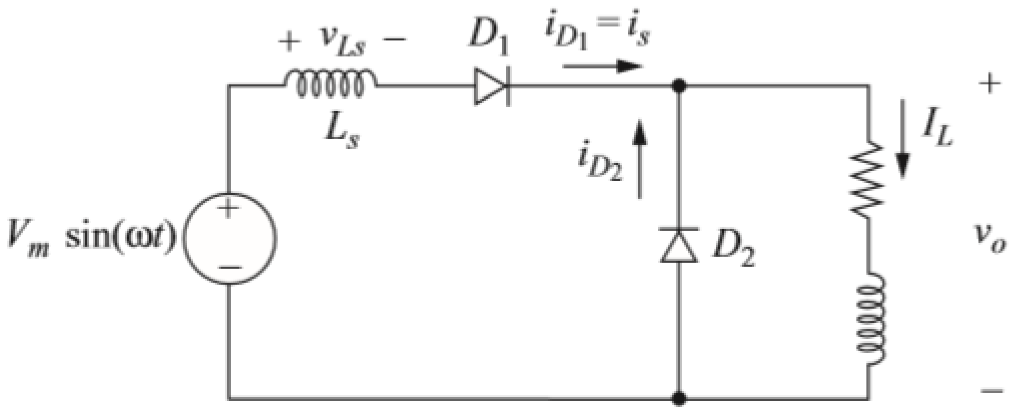
2. Continuous Load Current



COMMUTATION

The Effect of Source Inductance

- The preceding discussion on half-wave rectifiers assumed an ideal source. In practical circuits, the source has an equivalent impedance which is predominantly inductive reactance.
- For the single-diode half-wave rectifiers, the nonideal circuit is analyzed by including the source inductance with the load elements.
- However, the source inductance causes a fundamental change in circuit behavior for circuits like the half-wave rectifier with a freewheeling diode.

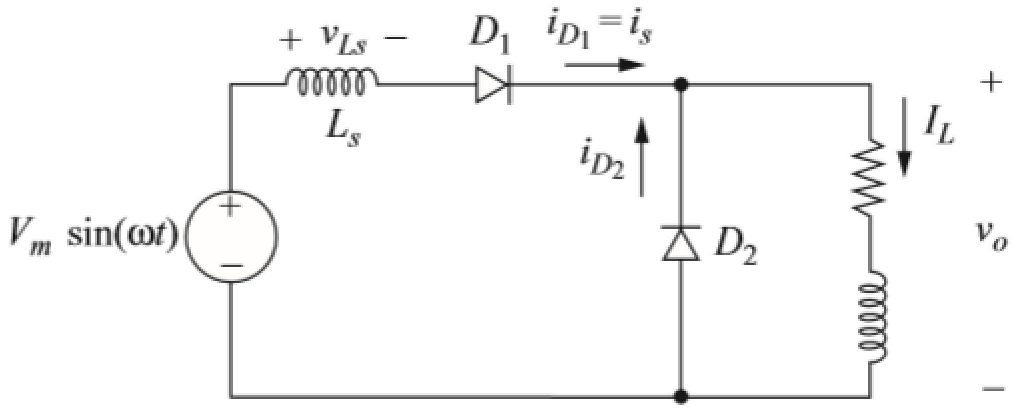


COMMUTATION

The Effect of Source Inductance

A half-wave rectifier with a freewheeling diode and source inductance L_s is shown in Fig. 3-18a. Assume that the load inductance is very large, making the load current constant. At $t = 0^-$, the load current is I_L, D_1 is off, and D_2 is on. As the source voltage becomes positive, D_1 turns on, but the source current does not instantly equal the load current because of L_s. Consequently, D_2 must remain on while the current in L_s and D_1 increases to that of the load. The interval when both D_1 and D_2 are on is called the commutation time or commutation angle. *Commutation is the process of turning off an electronic switch, which usually involves transferring the load current from one switch to another.*[2]

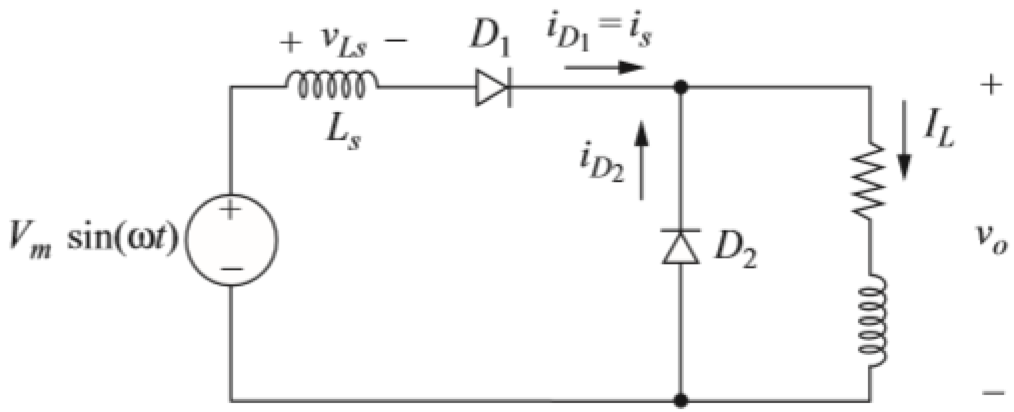
When both D_1 and D_2 are on, the voltage across L_s is



$$v_{Ls} = V_m \sin(\omega t)$$

COMMUTATION

The Effect of Source Inductance



$$v_{Ls} = V_m \sin(\omega t)$$

and current in L_s and the source is

$$i_s = \frac{1}{\omega L_s}\int_0^{\omega t} v_{Ls}\, d(\omega t) + i_s(0) = \frac{1}{\omega L_s}\int_0^{\omega t} V_m \sin(\omega t)\, d(\omega t) + 0$$

$$i_s = \frac{V_m}{\omega L_s}(1 - \cos \omega t)$$

COMMUTATION

The Effect of Source Inductance

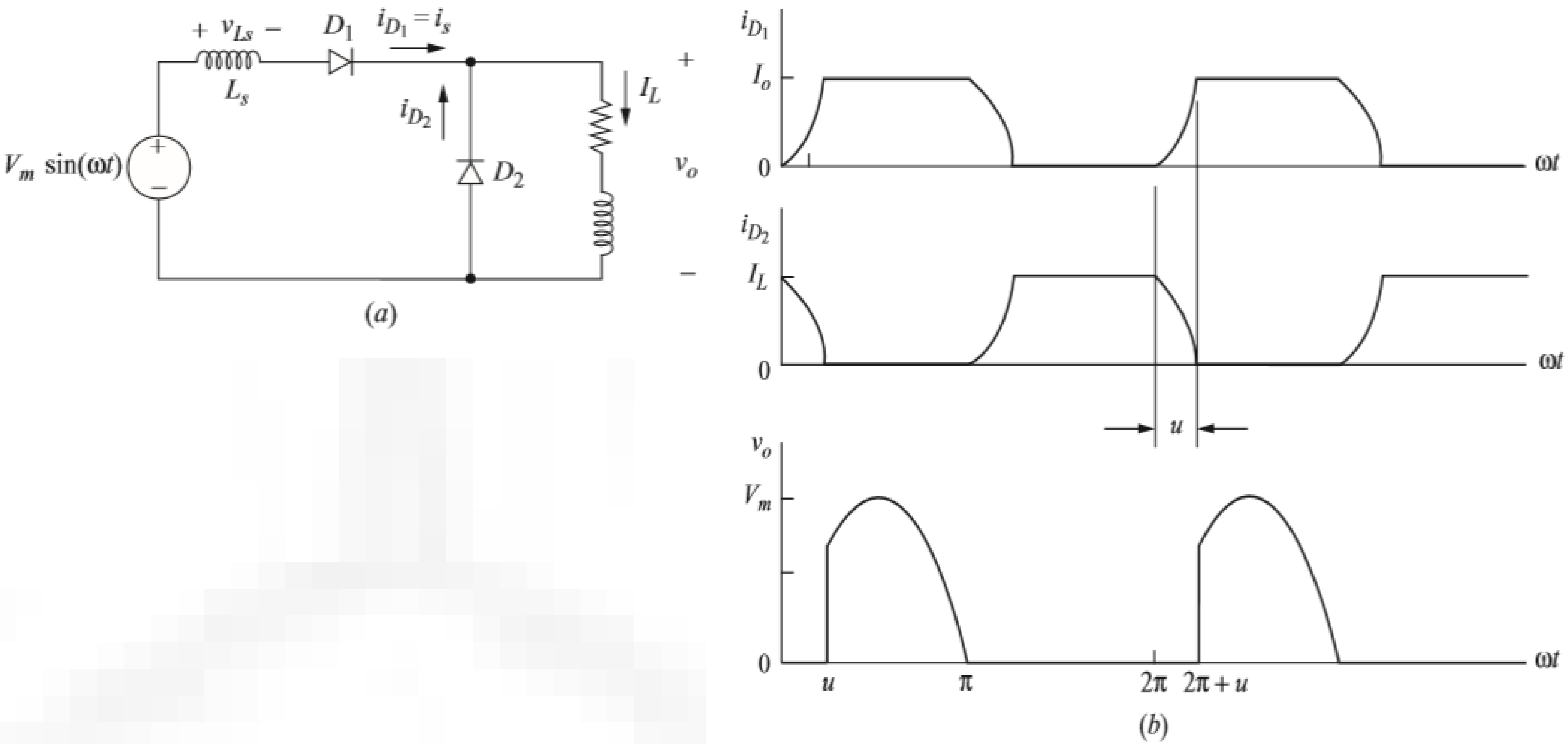


Figure 3-18 (*a*) Half-wave rectifier with freewheeling diode and source inductance; (*b*) Diode currents and load voltage showing the effects of Commutation.

COMMUTATION

The Effect of Source Inductance

Current in D_2 is

$$i_{D_2} = I_L - i_s = I_L - \frac{V_m}{\omega L_s}(1 - \cos \omega t)$$

The current in D_2 starts at I_L and decreases to zero. Letting the angle at which the current reaches zero be $\omega t = u$,

$$i_{D_2}(u) = I_L - \frac{V_m}{\omega L_s}(1 - \cos u) = 0$$

Solving for u,

$$\boxed{u = \cos^{-1}\left(1 - \frac{I_L \omega L_s}{V_m}\right) = \cos^{-1}\left(1 - \frac{I_L X_s}{V_m}\right)} \qquad (3\text{-}64)$$

COMMUTATION

The Effect of Source Inductance

where $X_s = \omega L_s$ is the reactance of the source. Figure 3-18*b* shows the effect of the source reactance on the diode currents. The commutation from D_1 to D_2 is analyzed similarly, yielding an identical result for the commutation angle u.

The commutation angle affects the voltage across the load. Since the voltage across the load is zero when D_2 is conducting, the load voltage remains at zero through the commutation angle, as shown in Fig. 3-17*b*. Recall that the load voltage is a half-wave rectified sinusoid when the source is ideal.

Average load voltage is

$$V_o = \frac{1}{2\pi} \int_u^{\pi} V_m \sin(\omega t)\, d(\omega t)$$

$$= \frac{V_m}{2\pi}\left[-\cos(\omega t)\right]\Big|_u^{\pi} = \frac{V_m}{2\pi}(1 + \cos u)$$

Using u from Eq. (3-64),

$$\boxed{V_o = \frac{V_m}{\pi}\left(1 - \frac{I_L X_s}{2V_m}\right)} \tag{3-65}$$

Recall that the average of a half-wave rectified sine wave is V_m/π. Source reactance thus reduces average load voltage.