EEEN451

POWER TRANSMISSION SYSTEMS

CHAPTER 1: BASIC CONCEPTS

Review of Phasors

Goal of phasor analysis is to simplify the analysis of constant frequency ac systems

v(t) = $V_{max}\cos(\omega t + \theta_v)$ → Instantaneous voltage

i(t) = $I_{max}\cos(\omega t + \theta_I)$ → Instantaneous Current

Root Mean Square (RMS) voltage of sinusoid

$$\sqrt{\frac{1}{T}\int_0^T v(t)^2 dt} = \frac{V_{\max}}{\sqrt{2}}$$

Phasor Representation

Euler's Identity: $e^{j\theta} = \cos\theta + j\sin\theta$

Phasor notation is developed by rewriting using Euler's identity

$$v(t) = \sqrt{2}|V|\cos(\omega t + \theta_V)$$

$$v(t) = \sqrt{2}|V|\mathrm{Re}\left[e^{j(\omega t + \theta_V)}\right]$$

(Note: $|V|$ is the RMS voltage)

The RMS, cosine-referenced voltage phasor is:

$$V = |V|e^{j\theta_V} = |V|\angle\theta_V$$

$$v(t) = \mathrm{Re}\sqrt{2}\,Ve^{j\omega t}e^{j\theta_V}$$

$$V = |V|\cos\theta_V + j|V|\sin\theta_V$$

$$I = |I|\cos\theta_I + j|I|\sin\theta_I$$

Advantages of Phasor Analysis

Device	Time Analysis	Phasor
Resistor	$v(t) = Ri(t)$	$V = RI$
Inductor	$v(t) = L\dfrac{di(t)}{dt}$	$V = j\omega LI$
Capacitor	$\dfrac{1}{C}\int_0^t i(t)dt + v(0)$	$V = \dfrac{1}{j\omega C}I$

$$Z = \text{Impedance} = R + jX = |Z|\angle\phi$$

$$R = \text{Resistance}$$

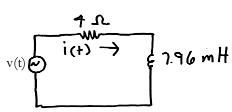
$$X = \text{Reactance}$$

$$|Z| = \sqrt{R^2 + X^2} \qquad \phi = \arctan\left(\frac{X}{R}\right)$$

(Note: Z is a complex number but not a phasor)

ELK322E Power Transmission Systems BY O Usta İTÜ 4

RL Circuit Example



$$V(t) = \sqrt{2}\,100\cos(\omega t + 30°)$$

$$f = 60\text{Hz}$$

$$R = 4\Omega \qquad X = \omega L = 3$$

$$|Z| = \sqrt{4^2 + 3^2} = 5 \quad \phi = 36.9°$$

$$I = \frac{V}{Z} = \frac{100\angle 30°}{5\angle 36.9°}$$

$$= 20\angle -6.9° \text{ Amps}$$

$$\text{i(t)} = 20\sqrt{2}\cos(\omega t - 6.9°)$$

ELK322E Power Transmission Systems BY O Usta İTÜ 5

Power in Single Phase Circuits

Power (Instantaneous Power)

$$p(t) = v(t)\, i(t)$$

$$\text{v(t)} = V_{\max}\cos(\omega t + \theta_V)$$

$$i(t) = I_{\max}\cos(\omega t + \theta_I)$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$p(t) = \frac{1}{2}V_{\max}I_{\max}[\cos(\theta_V - \theta_I) + \cos(2\omega t + \theta_V + \theta_I)]$$

Using max values

ELK322E Power Transmission Systems BY O Usta İTÜ 6

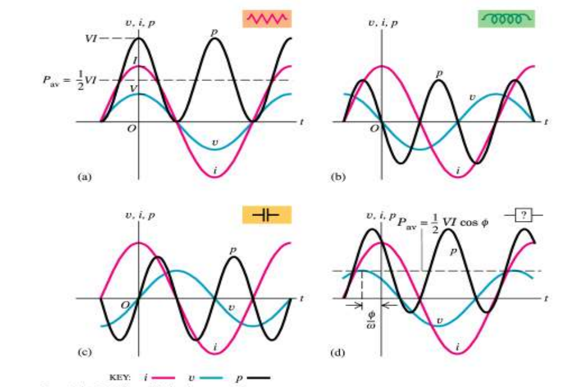
Instantaneous Power Using rms Values

$$p(t) = \underbrace{\overbrace{VICos\theta}^{\text{Aktif/real/average power}}\,(1 + Cos2\omega t)}_{\text{Instantaneous real/active power}} + \underbrace{\overbrace{VI\,Sin\theta}^{\text{Reactive Power}}\,Cos2\omega t}_{\text{Instantaneous reactive power}}$$

$\text{Pav} = \frac{1}{T}\int p(t)dt = VICos\theta$ W, kW, MW, GW

Reactive power = $VI\ Sin\theta$, VAR, kVAR, MVAR

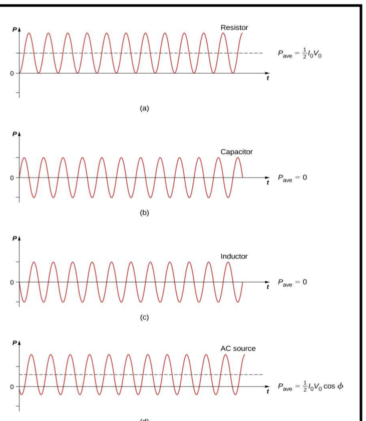
ELK322E Power Transmission Systems
BY O Usta
İTÜ
7



Copyright © Addison Wesley Longman, Inc.

ELK322E Power Transmission Systems
BY O Usta
İTÜ
8

Instantaneous Power



ELK322E Power Transmission Systems
BY O Usta
İTÜ
9

Complex Power

$$
\begin{aligned}
S &= |V||I|[\cos(\theta_V - \theta_I) + j\sin(\theta_V - \theta_I)] \\
&= P + jQ \\
&= V\ I^*
\end{aligned}
$$

P = Real Power (W, kW, MW)

Q = Reactive Power (var, kvar, Mvar)

S = Complex power (VA, kVA, MVA)

Power Factor (pf) = $\cos\phi$

If current leads voltage then pf is leading

If current lags voltage then pf is lagging

ELK322E Power Transmission Systems BY O Usta ITU 10

Relationships between real, reactive and complex power

$$P = |S|\cos\phi$$

$$Q = |S|\sin\phi = \pm|S|\sqrt{1-pf^2}$$

Example: A load draws 100 kW with a leading pf of 0.85. What are ϕ (power factor angle), Q and $|S|$?

$$\phi = -\cos^{-1}0.85 = -31.8°$$

$$|S| = \frac{100kW}{0.85} = 117.6\ \text{kVA}$$

$$Q = 117.6\sin(-31.8°) = -62.0\ \text{kVar}$$

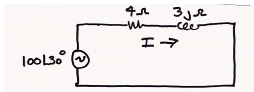
ELK322E Power Transmission Systems BY O Usta ITU 11

Conservation of Power

- At every node (bus) in the power systems
  - Sum of real power into node must equal zero
  - Sum of reactive power into node must equal zero
- This is a direct consequence of Kirchhoff's current law, which states that the total current into each node must equal zero.
  - Conservation of power follows since S = VI*

ELK322E Power Transmission Systems BY O Usta ITU 12

Conversation of Power Example



Earlier we found
$I = 20\angle\text{-}6.9°$ amps

$$
\begin{aligned}
S &= V\,I^* = 100\angle 30° \times 20\angle 6.9° = 2000\angle 36.9° \text{ VA} \\
\phi &= 36.9° \quad \text{pf} = 0.8 \text{ lagging} \\
\mathrm{S_R} &= V_R\,I^* = 4 \times 20\angle -6.9° \times 20\angle 6.9° \\
\mathrm{P_R} &= 1600W = |I|^2 R \quad (\mathrm{Q_R} = 0) \\
\mathrm{S_L} &= V_L\,I^* = 3j \times 20\angle -6.9° \times 20\angle 6.9° \\
\mathrm{Q_L} &= 1200\,\text{var} = |I|^2 X \quad (\mathrm{P_L} = 0)
\end{aligned}
$$

Power Consumption in Devices

Resistors only consume real power

$$\mathrm{P_{Resistor}} = |I_{\mathrm{Resistor}}|^2 R$$

Inductors only consume reactive power

$$\mathrm{Q_{Inductor}} = |I_{\mathrm{Inductor}}|^2 X_{\mathrm{L}}$$

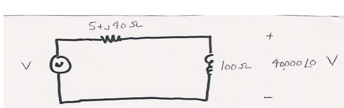
Capacitors only generate reactive power

$$\mathrm{Q_{Capacitor}} = -|I_{\mathrm{Capacitor}}|^2 X_{\mathrm{C}} \qquad X_C = \frac{1}{\omega C}$$

$$\mathrm{Q_{Capacitor}} = -\frac{|V_{\mathrm{Capacitor}}|^2}{X_{\mathrm{C}}} \quad \text{(Note-some define } \mathrm{X_C} \text{ negative)}$$

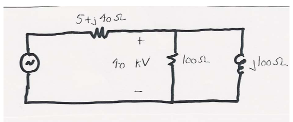
Example

First solve basic circuit



$$
\begin{aligned}
I &= \frac{40000\angle 0°\,V}{100\angle 0°\,\Omega} = 400\angle 0° \text{ Amps} \\
V &= 40000\angle 0° + (5 + j40)\,400\angle 0° \\
&= 42000 + j16000 = 44.9\angle 20.8° \text{ kV} \\
\mathrm{S} &= V I^* = 44.9\text{k}\angle 20.8° \times 400\angle 0° \\
&= 17.98\angle 20.8° \text{ MVA} \quad = 16.8 + j6.4 \text{ MVA}
\end{aligned}
$$

Example, cont'd

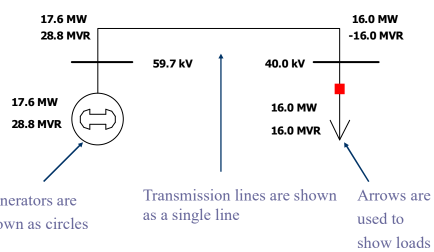


Now add additional reactive power load and resolve

$$
\begin{aligned}
Z_{Load} &= 70.7\angle 45^\circ \quad pf = 0.7 \text{ lagging} \\
I &= 564\angle -45^\circ \text{ Amps} \\
V &= 59.7\angle 13.6^\circ \text{ kV} \\
S &= 33.7\angle 58.6^\circ \text{ MVA} \quad = 17.6 + j28.8 \text{ MVA}
\end{aligned}
$$

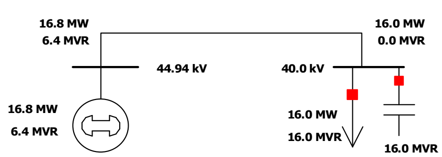
Power System Notation

Power system components are usually shown as "one-line diagrams." Previous circuit redrawn



Reactive Compensation

Key idea of reactive compensation is to supply reactive power locally. In the previous example this can be done by adding a 16 MVAR capacitor at the load



Compensated circuit is identical to first example with just real power load

- Reactive compensation decreased the line flow from 564 Amps to 400 Amps.

This has advantages

- Lines losses, which are equal to $I^2 R$ decrease
- Lower current allows utility to use small wires, or alternatively, supply more load over the same wires
- Voltage drop on the line is less

- Reactive compensation is used extensively by utilities
- Capacitors can be used to "correct" a load's power factor to an arbitrary value.

Power Factor Correction Example

Assume we have 100 kVA load with pf=0.8 lagging, and would like to correct the pf to 0.95 lagging

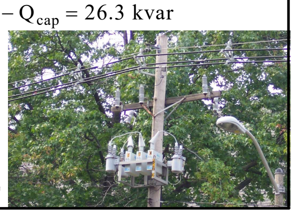
$$S = 80 + j60 \text{ kVA} \qquad \phi = \cos^{-1} 0.8 = 36.9°$$

$$\text{PF of 0.95 requires } \phi_{\text{desired}} = \cos^{-1} 0.95 = 18.2°$$

$$S_{new} = 80 + j(60 - Q_{cap})$$

$$\frac{60 - Q_{cap}}{80} = \tan 18.2° \quad \Rightarrow \quad 60 - Q_{cap} = 26.3 \text{ kvar}$$

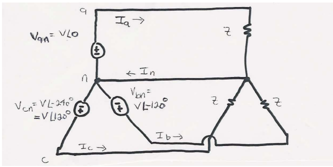
$$Q_{cap} = 33.7 \text{ kvar}$$



Balanced 3 Phase (ϕ) Systems

- A balanced 3 phase (ϕ) system has
  - three voltage sources with equal magnitude, but with an angle shift of 120°
  - equal loads on each phase
  - equal impedance on the lines connecting the generators to the loads
- Bulk power systems are almost exclusively 3ϕ
- Single phase is used primarily only in low voltage, low power settings, such as residential and some commercial

Balanced 3ϕ -- No Neutral Current



$$I_n = I_a + I_b + I_c$$

$$I_n = \frac{V}{Z}(1\angle 0° + 1\angle -120° + 1\angle 120°) = 0$$

$$S = V_{an}I_{an}^* + V_{bn}I_{bn}^* + V_{cn}I_{cn}^* \quad = 3\,V_{an}I_{an}^*$$

ELK322E Power Transmission Systems BY O Usta İTÜ 22

Advantages of 3ϕ Power

- Can transmit more power for same amount of wire (twice as much as single phase)
- Torque produced by 3ϕ machines is constrant
- Three phase machines use less material for same power rating
- Three phase machines start more easily than single phase machines

ELK322E Power Transmission Systems BY O Usta İTÜ 23

Three Phase - Wye Connection

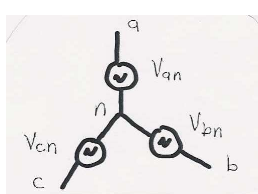
- There are two ways to connect 3ϕ systems
  - Wye (Y)
  - Delta (Δ)

Wye Connection Voltages

$$V_{an} = |V|\angle\alpha°$$

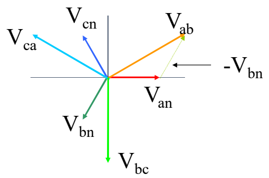
$$V_{bn} = |V|\angle\alpha° - 120°$$

$$V_{cn} = |V|\angle\alpha° + 120°$$



ELK322E Power Transmission Systems BY O Usta İTÜ 24

Wye Connection Line Voltages



(α = 0 in this case)

$$
\begin{aligned}
V_{ab} &= V_{an} - V_{bn} = |V|(1\angle\alpha - 1\angle\alpha + 120°) \\
&= \sqrt{3}\,|V|\angle\alpha + 30° \\
V_{bc} &= \sqrt{3}\,|V|\angle\alpha - 90° \\
V_{ca} &= \sqrt{3}\,|V|\angle\alpha + 150°
\end{aligned}
$$

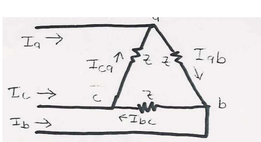
Line to line voltages are also balanced

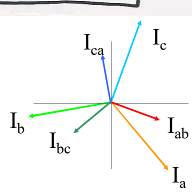
Wye Connection, cont'd

- Define voltage/current across/through device to be phase voltage/current
- Define voltage/current across/through lines to be line voltage/current

$$
\begin{aligned}
V_{Line} &= \sqrt{3}\,V_{Phase}\,1\angle 30° = \sqrt{3}\,V_{Phase}\,e^{j\pi/6} \\
I_{Line} &= I_{Phase} \\
S_{3\phi} &= 3\,V_{Phase}\,I^{*}_{Phase}
\end{aligned}
$$

Delta Connection





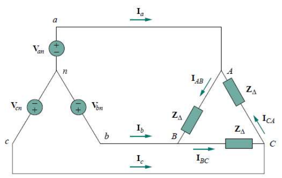
For the Delta phase voltages equal line voltages

For currents

$$
\begin{aligned}
I_a &= I_{ab} - I_{ca} \\
&= \sqrt{3}\,I_{ab}\angle -30° \\
I_b &= I_{bc} - I_{ab} \\
I_c &= I_{ca} - I_{bc} \\
S_{3\phi} &= 3\,V_{Phase}\,I^{*}_{Phase}
\end{aligned}
$$

Three Phase Example

Assume a Δ-connected load is supplied from a 3ϕ 13.8 kV (L-L) source with Z = 100∠20°Ω



$$V_{ab} = 13.8\angle 0^\circ\ kV$$
$$V_{bc} = 13.8\angle -120^\circ\ kV$$
$$V_{ca} = 13.8\angle 120^\circ\ kV$$

$$I_{ab} = \frac{13.8\angle 0^\circ\ kV}{100\angle 20^\circ\ \Omega} = 138\angle -20^\circ\ amps$$

$$I_{bc} = 138\angle -140^\circ\ amps \qquad I_{ca} = 138\angle 100^\circ\ amps$$

Three Phase Example, cont'd

$$\begin{aligned}
I_a &= I_{ab} - I_{ca} = 138\angle -20^\circ - 138\angle 100^\circ \\
&= 239\angle -50^\circ \text{ amps} \\
I_b &= 239\angle -170^\circ \text{ amps} \quad I_c = 239\angle 70^\circ \text{ amps}
\end{aligned}$$

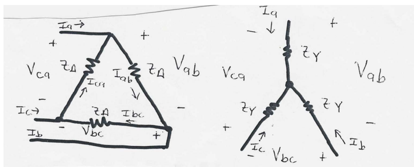
$$\begin{aligned}
S &= 3\times V_{ab} I_{ab}^{*} = 3\times 13.8\angle 0^\circ \text{kV} \times 138\angle 20^\circ \text{ amps} \\
&= 5.7\angle 20^\circ \text{ MVA} \\
&= 5.37 + j1.95 \text{ MVA} \\
\text{pf} &= \cos 20^\circ = 0.94 \text{ lagging}
\end{aligned}$$

Delta-Wye Transformation

To simplify analysis of balanced 3ϕ systems:

1) Δ-connected loads can be replaced by Y-connected loads with $Z_Y = \frac{1}{3} Z_\Delta$
2) Δ-connected sources can be replaced by Y-connected sources with $V_{phase} = \frac{V_{Line}}{\sqrt{3}\angle 30^\circ}$

Delta-Wye Transformation Proof



From the Δ side we get

$$I_a = \frac{V_{ab}}{Z_\Delta} - \frac{V_{ca}}{Z_\Delta} = \frac{V_{ab} - V_{ca}}{Z_\Delta}$$

Hence $Z_\Delta = \dfrac{V_{ab} - V_{ca}}{I_a}$

ELK322E Power Transmission Systems BY O Usta İTÜ 31

Delta-Wye Transformation, cont'd

From the Y side we get

$$V_{ab} = Z_Y(I_a - I_b) \qquad V_{ca} = Z_Y(I_c - I_a)$$

$$V_{ab} - V_{ca} = Z_Y(2I_a - I_b - I_c)$$

Since $\mathrm{I_a} + I_b + I_c = 0 \Rightarrow I_a = -I_b - I_c$

Hence $V_{ab} - V_{ca} = 3\,Z_Y I_a$

$$3\,Z_Y = \frac{V_{ab} - V_{ca}}{I_a} = Z_\Delta$$

Therefore $Z_Y = \frac{1}{3} Z_\Delta$

ELK322E Power Transmission Systems BY O Usta İTÜ 32

APPLICATIONS

Example 1-1: A DC circuit, as shown in Figure 1-2, has a DC voltage of 12 volts and a resistor of 2 Ω. What are the DC current in the circuit and the power consumed by the resistor?

$$I_{DC} = \frac{12}{2} = 6 \ \ (A)$$

$$P_{DC} = 12 \times 6 = 72 \ \ (W)$$

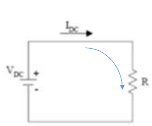
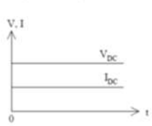


Fig. 1-2. A simple DC circuit.



Voltage and current waveforms of the simple DC circuit.

ELK322E Power Transmission Systems BY O Usta İTÜ 33

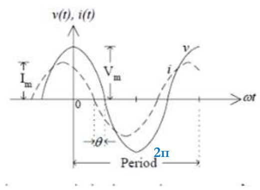
Since in power systems the sinusoidal voltages are generated, and consequently, most likely sinusoidal currents are flowed in the generation, transmission and distribution systems, sinusoidal quantities are assumed throughout this material, unless otherwise specified.

In general, a set of typical steady-state voltage and current waveforms of an AC circuit can be drawn as shown in following Figure, and their mathematical expressions can be written as follows:

$$v(t) = \mathrm{V_m}\cos(\omega t),$$

and

$$i(t) = \mathrm{I_m}\cos(\omega t + \theta),$$



There is an important quantity called "root mean square" value, or rms, and is defined as

$$\mathrm{V_{rms}} = \sqrt{\frac{1}{\mathrm{T}}\int_0^{\mathrm{T}} v^2(t)dt}\,.$$

For a sinusoidal voltage, its rms value equals

$$\mathrm{V_{rms}} = \sqrt{\frac{1}{\mathrm{T}}\int_0^{\mathrm{T}} [\mathrm{V_m}\cos(\omega t)]^2 dt} = \sqrt{\frac{\mathrm{V_m^2}}{\omega \mathrm{T}}\int_0^{2\pi} \frac{1+\cos 2\omega t}{2} d(\omega t)} = \sqrt{\frac{\mathrm{V_m^2}}{2\pi}\left[\frac{\omega t}{2} + \frac{\sin 2\omega t}{4}\right]_0^{2\pi}} = \frac{\mathrm{V_m}}{\sqrt{2}}\,.$$

Example 1-2: What are the phasor representations of the following instantaneous quantities?

V(t) =170 cos(ωt) volts, and i(t) = 85 cos(ωt+30°)

Solution:

$$\overline{\mathrm{V}} = \frac{170}{\sqrt{2}}\angle 0° = 120\angle 0° \text{ volts}$$

$$\overline{\mathrm{I}} = \frac{85}{\sqrt{2}}\angle 30° = 60.1\angle 30° \text{ amps}$$

Unlike in DC circuits, the loads in AC circuits,can be expressed as its impedance, consisting of resistance R and reactance X, as follows

$$\overline{\mathrm{Z}} = \frac{\overline{\mathrm{V}}}{\overline{\mathrm{I}}} = \mathrm{R} + (j\omega \mathrm{L} + \frac{1}{j\omega \mathrm{C}}) = \mathrm{R} + j(\mathrm{X_L} - \mathrm{X_C}) = \mathrm{R} + j\mathrm{X} = \mathrm{Z}\angle\theta\ \Omega,$$

where

$$\mathrm{Z} = \sqrt{\mathrm{R}^2 + \mathrm{X}^2}\,,$$

and

$$\theta = \tan^{-1}(\frac{\mathrm{X}}{\mathrm{R}})\,.$$

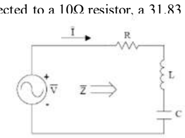
Example 1-3: A 60 Hz 120 volts AC voltage source is connected to a 10Ω resistor, a 31.83 mH inductor and 1326.26μF capacitor, as shown in Figure.

Find

(1) The total impedance Z.

(2) The current I in polar form.

(3) The voltage and current in instantaneous forms.



Solution:

(1) Since the frequency is 60 Hz, the inductive and capacitive reactances can be obtained as

$$\mathrm{X_L} = \omega \mathrm{L} = 377 \times 31.83 \times 10^{-3} = 12\Omega$$

$$\mathrm{X_C} = \frac{1}{\omega \mathrm{C}} = \frac{1}{377 \times 1326.26 \times 10^{-6}} = 2\Omega$$

The total impedance seen by the voltage source

$$\overline{\mathrm{Z}} = \mathrm{R} + j(\mathrm{X_L} - \mathrm{X_C}) = 10 + j(12-2) = 10 + j10 = \left(\sqrt{10^2 + 10^2}\right)\angle \tan^{-1}\left(\frac{10}{10}\right) = 10\sqrt{2}\angle 45°\,\Omega$$

(2) To calculate the current, the angle of the voltage is set to be the reference, namely, 0°. Then,

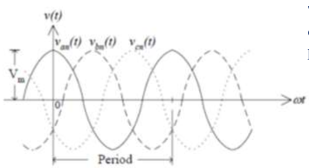
$$\overline{\mathrm{I}} = \frac{\overline{\mathrm{V}}}{\overline{\mathrm{Z}}} = \frac{120\angle 0°}{10\sqrt{2}\angle 45°} = 8.485\angle -45° \text{ amps}$$

(3) To convert the phasors to the instantaneous forms

$$v(t) = 120\sqrt{2}\cos(377t) = 170\cos(377t) \text{ volts}$$

$$i(t) = 8.485\sqrt{2}\cos(377t - 45°) = 12\cos(377t - 45°) \text{ amps}$$

In general, the phase voltages of a balanced three-phase voltage source with "positive" sequence can be expressed as



Waveforms of phase voltages of balanced three-phase systems.

Their phasor representations are obtained as discussed for single-phase systems

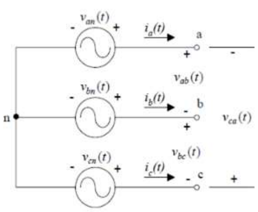
$$\overline{V}_{an} = V\angle 0^\circ,$$
$$\overline{V}_{bn} = \overline{V}_{an}\angle -120^\circ = V\angle -120^\circ,$$
$$\overline{V}_{cn} = \overline{V}_{an}\angle 120^\circ = V\angle 120^\circ.$$

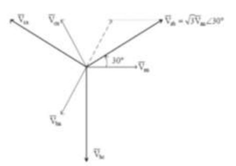
$$v_{an}(t) = \sqrt{2}V\cos(\omega t),$$
$$v_{bn}(t) = v_{an}(t - \frac{T}{3}) = \sqrt{2}V\cos(\omega t - 120^\circ),$$
$$v_{cn}(t) = v_{an}(t - \frac{2T}{3}) = \sqrt{2}V\cos(\omega t - 240^\circ) = \sqrt{2}V\cos(\omega t + 120^\circ).$$

ELK322E Power Transmission Systems BY O Usta ITU 37

Phasor values of Line voltages (phase to phase)

$$\overline{V}_{ab} = \overline{V}_{an} - \overline{V}_{bn} = V\angle 0^\circ - V\angle -120^\circ = V[1-(-0.5-j0.866)] = \sqrt{3}V\angle 30^\circ = \sqrt{3}\overline{V}_{an}\angle 30^\circ.$$
$$\overline{V}_{bc} = \sqrt{3}V\angle -90^\circ = \sqrt{3}\overline{V}_{bn}\angle 30^\circ,$$
$$\overline{V}_{ca} = \sqrt{3}V\angle 150^\circ = \sqrt{3}\overline{V}_{cn}\angle 30^\circ.$$

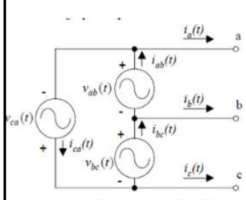


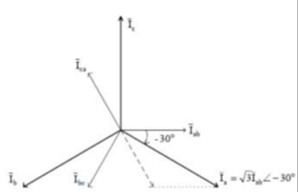


The relationships of phase voltages and line-to-line voltages in a three-phase system with wye configuration.

ELK322E Power Transmission Systems BY O Usta ITU 38

The other configuration of the three-phase voltage source is to connect them in a delta configuration, as shown in Figure





Three-phase voltage source with delta configuration.

$$\overline{I}_a = \overline{I}_{ab} - \overline{I}_{ca} = I\angle 0^\circ - I\angle 120^\circ = I[1-(-0.5+j0.866)] = \sqrt{3}I\angle -30^\circ = \sqrt{3}\overline{I}_{ab}\angle -30^\circ.$$

Similarly, at nodes b and c, the other two line currents can be obtained as follows

$$\overline{I}_b = \sqrt{3}I\angle -150^\circ = \sqrt{3}\overline{I}_{bc}\angle -30^\circ,$$
$$\overline{I}_c = \sqrt{3}I\angle 90^\circ = \sqrt{3}\overline{I}_{ca}\angle -30^\circ.$$

ELK322E Power Transmission Systems BY O Usta ITU 39

Example 1-4: A balanced three-phase load of 50 kVA, pf = 0.85 lagging is supplied from a balanced three-phase wye connected voltage source of positive sequence such that V^L = 4157 volts.
Calculate:
Phase voltage, Line & Phase currents, and 3 phase powers

Solution:

$$(1)\ V_\phi = \frac{V_L}{\sqrt{3}} = \frac{4157}{\sqrt{3}} = 2400 \text{ volts} \qquad I_\phi = I_L = \frac{S_{3\phi}}{\sqrt{3}V_L} = \frac{50\times10^3}{\sqrt{3}\times4157} = 6.94 \text{ amps}$$

$$(2)\ \overline{S}_{3\phi} = 50\angle\cos^{-1}(0.85) = 50\angle31.8^\circ \text{ kVA}$$

$$S_{3\phi} = 50 \text{ kVA}$$

$$P_{3\phi} = 50\times0.85 = 42.5 \text{ kW}$$

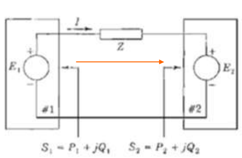
$$Q_{3\phi} = 50\sin[\cos^{-1}(0.85)] = 50\sin31.8^\circ = 26.34 \text{ kvar}$$

DIRECTION OF POWER FLOW

The relation among I, Q, and bus voltage, or generated voltage Ef, with respect t the sign of P and Q is important when the flow of power in a system is considered. The question involves the direction of flow of power, that is, whether power is being generated or absorbed when a voltage and a current are specified.

Example 1.5. Two ideal vol tage sourccs designated as mach ines 1 and 2 are connected, as shown below. If E1 = 100 ∟0° V, E2= 100 ∟30° V, and Z = (0 + j5) ohm.
Determine
(a) whether each machine is generating or consuming real power and the amount,
(b) whether each machine is receiving or supplying reactive power and the amount,
(c) the P and Q absorbed by the impedance.

Assume tha Mach 1 generator and mach 2 motor:
If the result +, assumption is correct
If the result - , asumption is incorrect



Solution

$$I = \frac{E_1 - E_2}{Z} = \frac{100 + j0 - (86.6 + j50)}{j5}$$

$$= \frac{13.4 - j50}{j5} = -10 - j2.68 = 10.35\angle 195^\circ \text{ A}$$

S1=E1I*= 100 (-10+j2.68)= (-1000+j268) VA
Mach 1 is motor and consuming active power, producing reactive power

S2=E2I*= (86.6+j50) (-10+j2.68)= (-1000-j268) VA
Mach 2 is generator and generating active and reactive power

The reactive power absorb by the line (r=0, and X=5 ohm)

$XI^2 = 5\text{x}10.35^2 = 536 = 268+268$ Produced Absorbed

Example 1.6: The terminal voltage of a Y-connected load consisting of 3 equal impedances of 20∟30°ohm is 4.4 kV. The impedance in each of the 3 lines connecting the load to a bus bar at a sub-station is ZL 1.4 ∟75°ohm. Find out the line-to-line voltage at the substation.

Solution: Using single phase equavelent circuit:

Solution. The magnitude of the voltage to neutral at the load is $4400/\sqrt{3} = 2540$ V. If V_{an}, the voltage across the load, is chosen as reference,

$$V_{an} = 2540\angle 0^\circ \text{ V} \quad \text{and} \quad I_{an} = \frac{2540\angle 0^\circ}{20\angle 30^\circ} = 127.0\angle -30^\circ \text{ A}$$

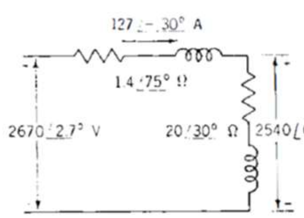
The line-to-neutral voltage at the substation is

$$\begin{aligned} V_{an} + I_{an}Z_L &= 2540\angle 0^\circ + 127\angle -30^\circ \times 1.4\angle 75^\circ \\ &= 2540\angle 0^\circ + 177.8\angle 45^\circ \\ &= 2666 + j125.7 = 2670\angle 2.70^\circ \text{ V} \end{aligned}$$

43

and the magnitude of the voltage at the substation bus is

$$\sqrt{3} \times 2.67 = 4.62 \text{ kV}$$



ELK322E Power Transmission Systems
BY O Usta
İTÜ
44

THANKS

ELK322E Power Transmission Systems
BY O Usta
İTÜ
45