HW2

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1.a)

y (Red) \ x (Blue)	0	1	2
0	15/66	12/66	1/66
1	24/66	8/66	
2	6/66		

$f(0,0) = \frac{15}{66}$
$f(0,1) = \frac{24}{66}$
$f(0,2) = \frac{6}{66}$
$f(1,0) = \frac{12}{66}$
$f(2,0) = \frac{1}{66}$
$f(1,1) = \frac{8}{66}$

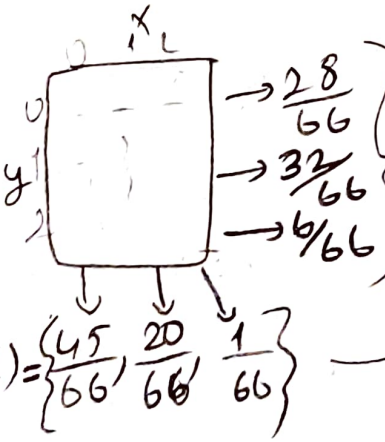
1.b) $f(1,1) \Longrightarrow P(X \leq 1, y \leq 1)$

$= f(0,0) + f(0,1) + f(1,0) + f(1,1)$

$= \frac{\binom{5}{2}}{\binom{12}{2}} + \frac{\binom{4}{1}\binom{6}{1}}{\binom{12}{2}} + \frac{\binom{2}{1}\binom{6}{1}}{\binom{12}{2}} + \frac{\binom{2}{1}\binom{4}{1}}{\binom{12}{2}}$

$= \frac{15}{66} + \frac{24}{66} + \frac{12}{66} + \frac{8}{66} = \boxed{\frac{59}{66}}$

1.c)



$\rightarrow \frac{28}{66}$
$\rightarrow \frac{32}{66}$
$\rightarrow \frac{6}{66}$
$h(y) \longrightarrow f(x,0) = \frac{28}{66}$
$\longrightarrow f(x,1) = \frac{32}{66}$
$\longrightarrow f(x,2) = \frac{6}{66}$

$h(x) = \{\frac{45}{66}, \frac{20}{66}, \frac{1}{66}\} \longrightarrow f(0,y) = \frac{45}{66}$

$f(1,y) = \frac{20}{66}$

$f(2,y) = \frac{1}{66}$

1.d) $f(x|y=1) = \frac{f(x,y)}{h(y)} = \frac{f(x,y)}{h(1)}$

$h(1) = \sum_{x=0}^{1} f(x,1) = \frac{32}{66}$

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$f(0,1) = \frac{\frac{24}{66}}{\frac{32}{66}} = \frac{3}{4}$

$f(1,1) = \frac{\frac{8}{66}}{\frac{32}{66}} = \frac{1}{4}$

$\Big\} = (1)$

2.a)

$$f(x) = \begin{cases} 2(1-x) & 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

$\int_0^1 (2-2x)\,dx = 2x - x^2 \Big|_0^1 = 2-1 = (1)$

$\int_{-\infty}^{\infty} f(x)\,dx = 1$ ✓

2.b) $Var(x) = E[x^2] - E[x]^2$

$E[x] = \int_0^1 2(1-x)\,x\,dx \longrightarrow x^2 - \frac{2x^3}{3}\Big|_0^1 = 1/3$

$E[x^2] = \int_0^1 2(1-x)x^2 dx \rightarrow \frac{2x^3}{3} - \frac{2x^4}{4}\Big|_0^1 = 1/6$

$\frac{1}{6} - \frac{1}{9} = \frac{1}{18} \rightarrow \sigma = \sqrt{Var} = \boxed{0,235}$

3.a) $E[g(x,y)] = \sum_{x=0}^{2} \sum_{y=0}^{2} (x+y) \cdot f(x,y)$

$= 0 \cdot f(0,0) + 1 \cdot f(1,0) + 2 \cdot f(2,0) + 2 \cdot f(1,1)$

$+ 1 \cdot f(0,1) + 2 f(0,2) = \frac{12}{66} + \frac{2}{66} + \frac{16}{66} + \frac{24}{66} + \frac{12}{66}$

$2 \cdot \frac{1}{66}$ $2 \cdot \frac{8}{66}$ $2 \cdot \frac{6}{66}$

$= 1$

3.b) $cov(x,y) = E(xy) - E(x)\,E(y)$

$E(X,Y) = \sum_{x=0}^{2} \sum_{y=0}^{2} xy f(x,y) = 0 \cdot \frac{15}{16} + 1 \cdot 0 \cdot \frac{12}{16} + 2 \cdot 0 \cdot \frac{1}{66}$

$+ 0 \cdot 1 \cdot \frac{24}{66} + 1 \cdot 1 \cdot \frac{8}{66} + 0 \cdot 2 \cdot \frac{6}{66}$

$= 8/66$

$E(x) = 1/3 \qquad E(y) = 2/3$

$\sigma_{xy} = \frac{4}{33} - \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{3}\left(\frac{6-11}{33}\right) = -\frac{10}{99}$

3.c) $P(x \mid y=1) = \frac{P(x=1, y=1)}{P(y=1)}$

$= \frac{\binom{2}{1}\binom{4}{1}}{\binom{12}{2}} \cdot \frac{\binom{12}{2}}{\binom{4}{1}\binom{8}{1}} = \frac{8}{66} \cdot \frac{66}{32}$

$= \frac{1}{4}$

4-a) $P(X < 5)$

$P\left(Z < \frac{5-7}{1,5}\right) = P(Z < -1,33)$

$= P(Z > 1.33) = 1 - (\Phi\, 1,33)$

$= 1 - 0,908241 = 0,091759$

4-b) $P(6 < X < 8)$

$P\left(\frac{6-7}{1.5} < Z < \frac{8-7}{1.5}\right) = P(-0,67 < Z < 0,67$

$= P(Z < 0,67) - P(Z < -0,67)$

$= \Phi\, 0,67 - (1 - P(Z < 0,67)) = 2\,\Phi\, 0,67 - 1$

$= (2 \cdot 0,74857) - 1 = 0,49714$

4-c) $P(X < 6)$

$P\left(Z < \frac{6-7}{1,5}\right) = P(Z < -0,67)$

$= P(Z > 0,67)$

$= 1 - 0,74857 = 0,25143$

% 25,143

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