ELK331E/331

Power Electronic Circuits/Güç Elektroniği Devreleri

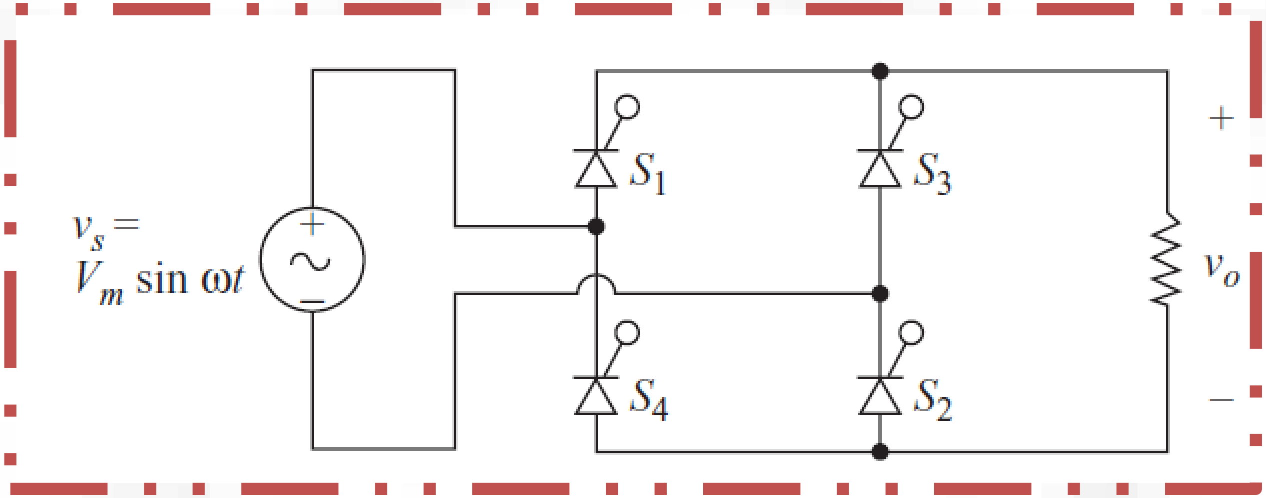
The Single Phase Full-Wave Controlled Rectifier

Tek Fazlı Tam Dalga Kontrollü Doğrultucular

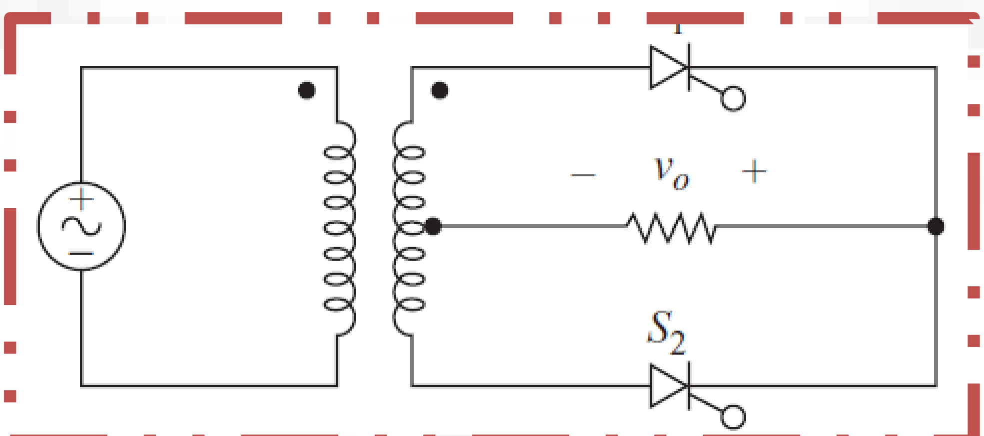
Dr. Mehmet Onur GÜLBAHÇE

The Full-Wave Controlled Rectifier

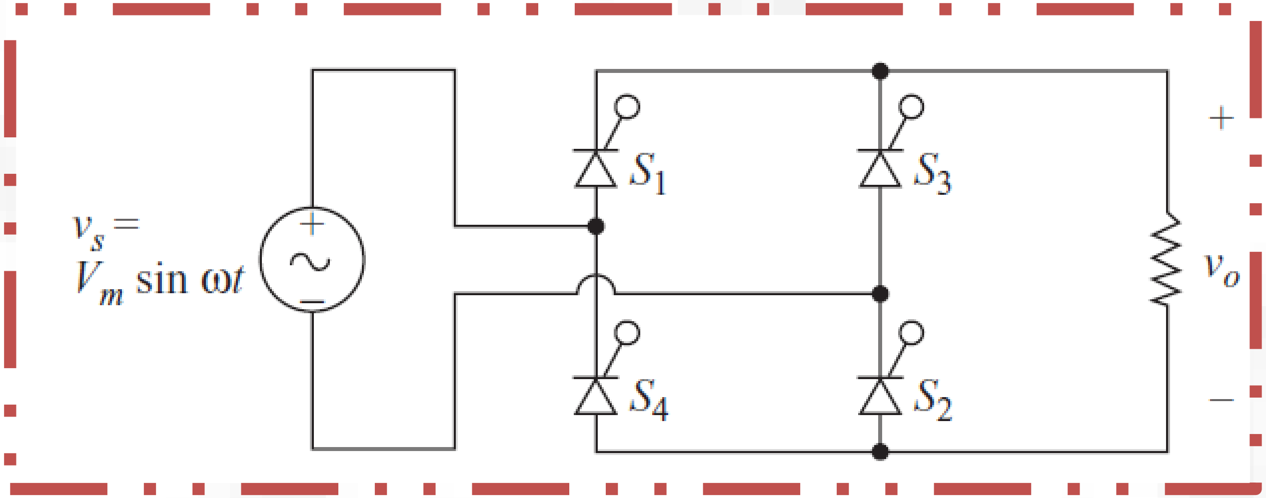
- Output is controlled by adjusting the delay angle of each SCR, resulting in an output voltage that is adjustable over a limited range.
- The controlled bridge rectifier and



- the center-tapped transformer controlled rectifier

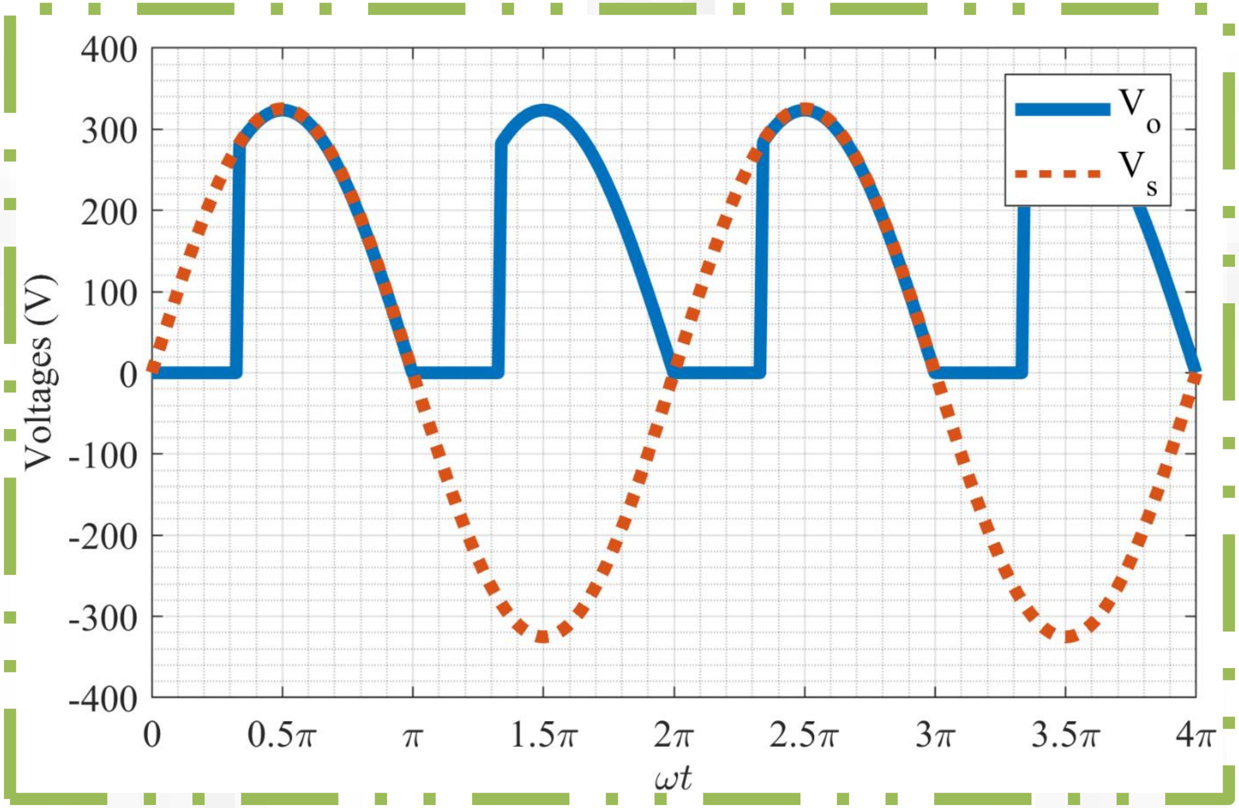


The Controlled Bridge Rectifier with R Load



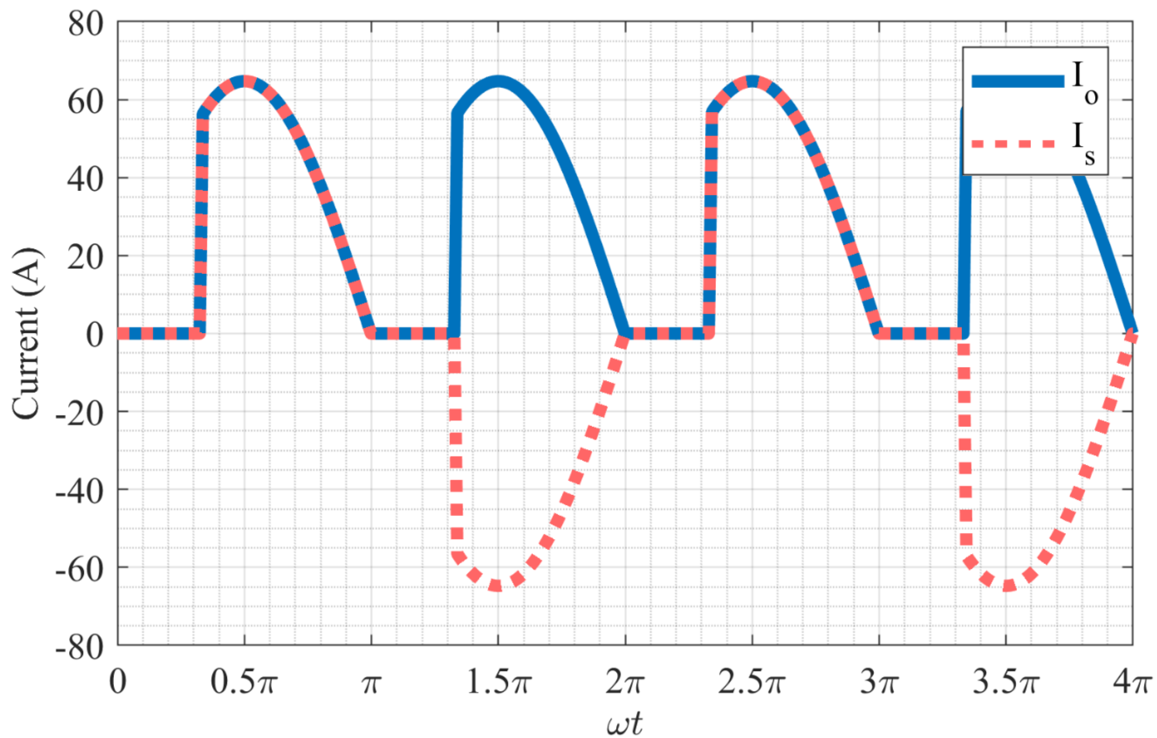
- ✓ SCRs S1 and S2 will become forward-biased when the source becomes positive but will not conduct until gate signals are applied.
- ✓ Similarly, S3 and S4 will become forward-biased when the source becomes negative but will not conduct until they receive gate signals.
- ✓ For the center-tapped transformer rectifier, S1 is forward-biased when vs is positive, and S2 is forward-biased when vs is negative, but each will not conduct until it receives a gate signal.
- ✓ The delay angle is the angle interval between the forward biasing of the SCR and the gate signal application.

The Controlled Bridge Rectifier



$$V_o = \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{\pi}(1 + \cos\alpha)$$

The Controlled Bridge Rectifier



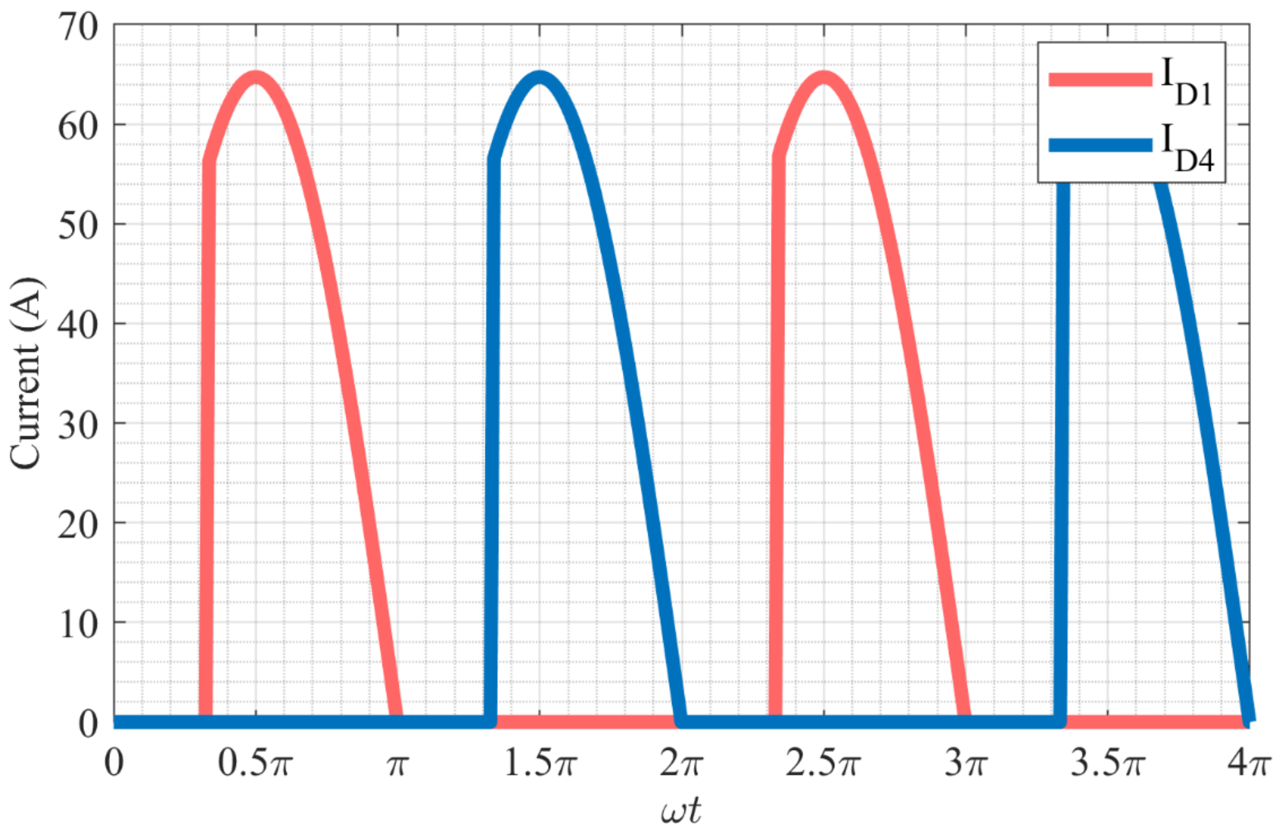
$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R}(1 + \cos\alpha)$$

$$I_{\text{rms}} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\pi}\left(\frac{V_m}{R}\sin\omega t\right)^2 d(\omega t)}$$

$$= \frac{V_m}{R}\sqrt{\frac{1}{2} - \frac{\alpha}{2\pi} + \frac{\sin(2\alpha)}{4\pi}}$$

The rms current in the source is the same as the rms current in the load.

The Controlled Bridge Rectifier



Example-1

The full-wave controlled bridge rectifier has an ac input of 120 V rms at 60 Hz and a 20-Ω load resistor. The delay angle is 40°. Determine the average current in the load, the power absorbed by the load, and the source voltamperes.

$$\boxed{V_o = \frac{V_m}{\pi}\left(1 + \cos\alpha\right) = \frac{\sqrt{2}\,(120)}{\pi}\left(1 + \cos 40^\circ\right) = 95.4\ \text{V}}$$

$$\boxed{I_o = \frac{V_o}{R} = \frac{95.4}{20} = 4.77\ \text{A}}$$

$$\boxed{I_{\text{rms}} = \frac{\sqrt{2}(120)}{20}\sqrt{\frac{1}{2} - \frac{0.698}{2\pi} + \frac{\sin[2(0.698)]}{4\pi}} = 5.80\ \text{A}}$$

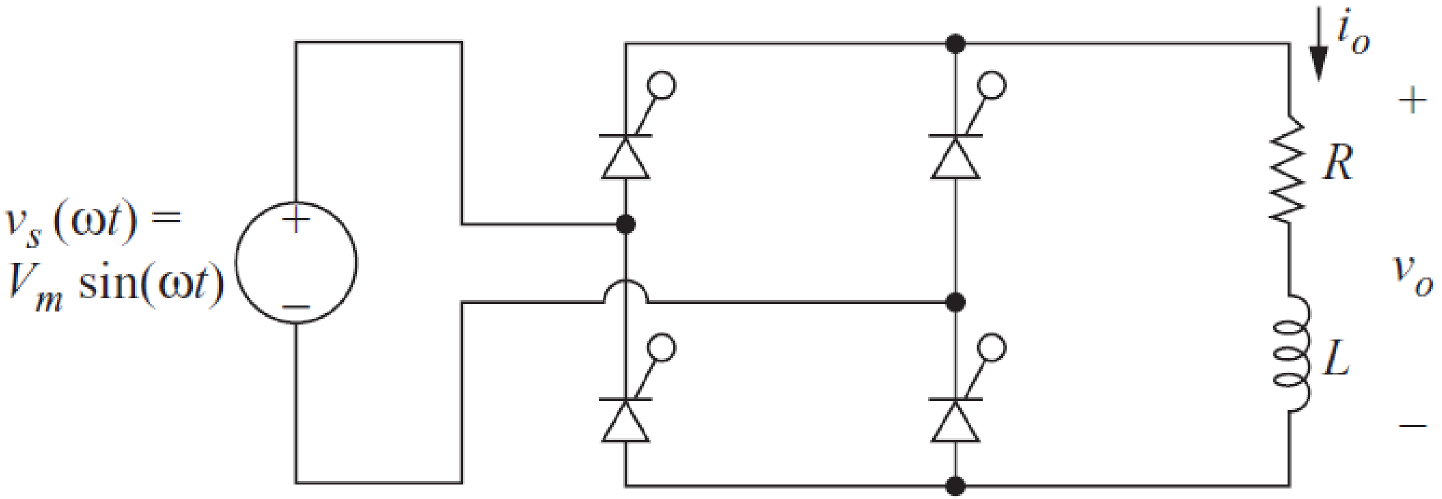
$$\boxed{P = I_{\text{rms}}^2 R = (5.80)^2\,(20) = 673\ \text{W}}$$

$$\boxed{S = V_{\text{rms}}\, I_{\text{rms}} = (120)(5.80) = 696\ \text{VA}}$$

$$\boxed{\text{pf} = \frac{P}{S} = \frac{672}{696} = 0.967}$$

The Controlled Bridge Rectifier with RL Load

Discontinuous Current



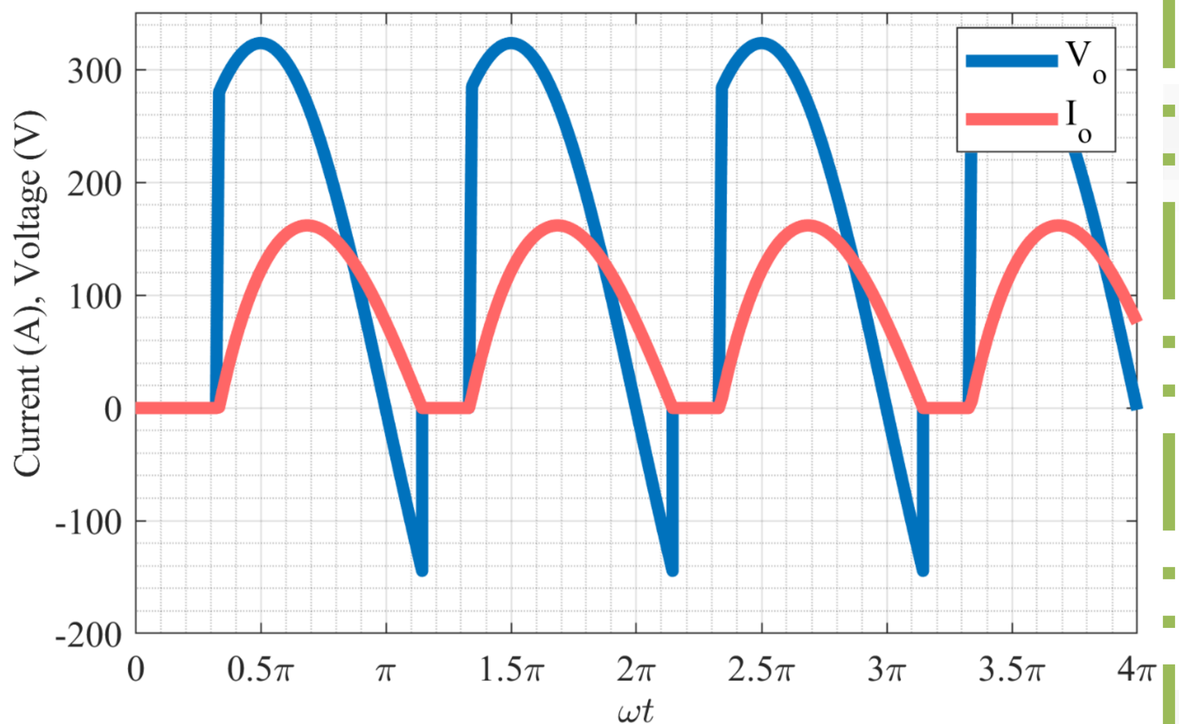
$$i(\omega t) = i_f(\omega t) + i_n(\omega t) = \frac{V_m}{Z}\sin(\omega t - \theta) + Ae^{-\omega t/\omega\tau}$$

$$i_o(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)\, e^{-(\omega t - \alpha)/\omega\tau}\right] \qquad \text{for } \alpha \le \omega t \le \beta$$

$$Z = \sqrt{R^2 + (\omega L)^2} \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \qquad \text{and} \qquad \tau = \frac{L}{R}$$

The Controlled Bridge Rectifier with RL Load

Discontinuous Current

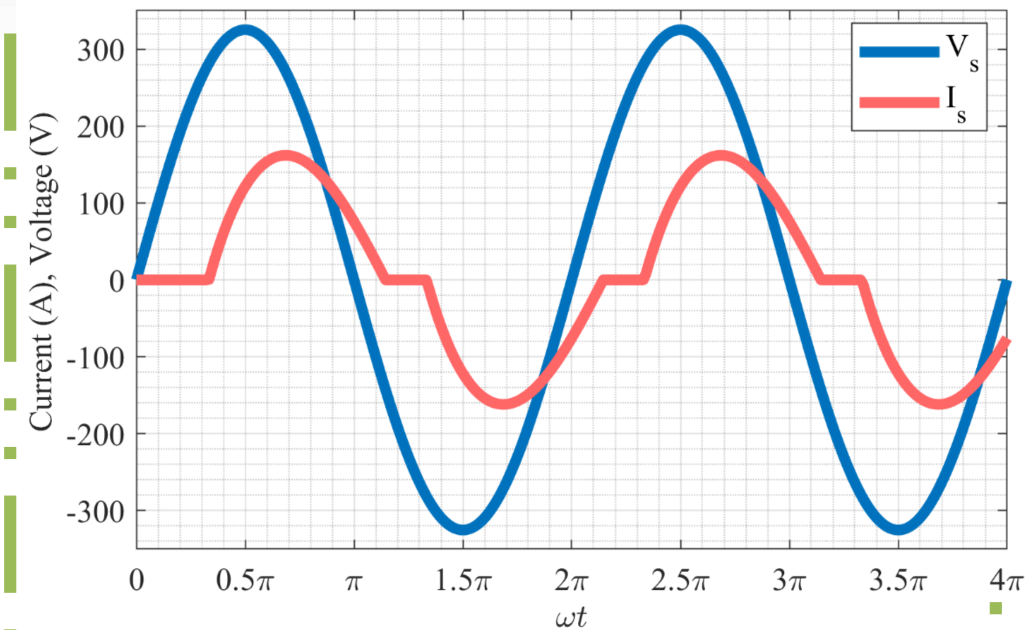


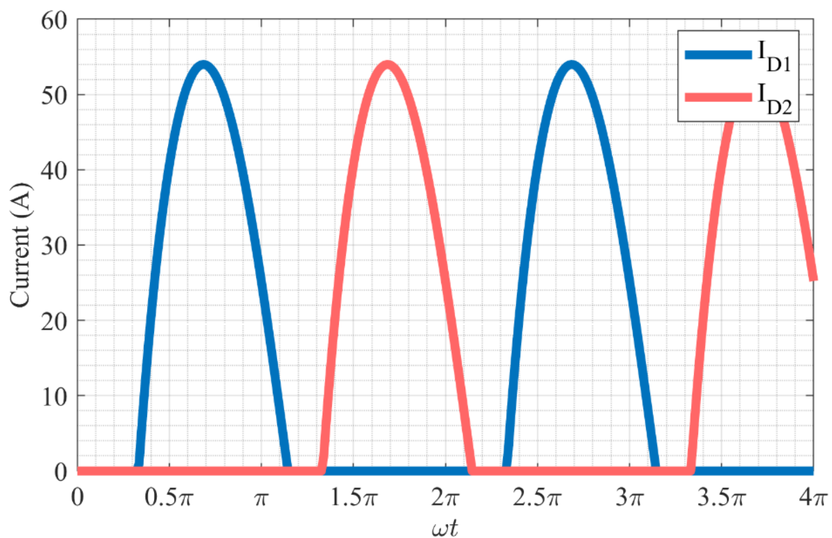
$\beta<\pi+\alpha$

The above current function becomes zero at $\omega t=\beta$. If $\beta<\pi+\alpha$ the current remains at zero until $\omega t=\pi+\alpha$ when gate signals are applied to S3 and S4 which are then forward-biased and begin to conduct. This mode of operation is called discontinuous current.

The Controlled Bridge Rectifier with RL Load

Discontinuous Current





Example-2

A controlled full-wave bridge rectifier of Fig. 4-11a has a source of 120 V rms at 60 Hz, R =10 Ω, L =20 mH, and α=60°. Determine

(a) an expression for load current,

(b) the average load current, and

(c) the power absorbed by the load.

$$V_m = \frac{120}{\sqrt{2}} = 169.7 \text{ V}$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + [(377)(0.02)]^2} = 12.5\ \Omega$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left[\frac{(377)(0.02)}{10}\right] = 0.646 \text{ rad}$$

$$\omega\tau = \frac{\omega L}{R} = \frac{(377)(0.02)}{10} = 0.754 \text{ rad}$$

$$\alpha = 60° = 1.047 \text{ rad}$$

Example-2

$$i_o(\omega t) = 13.6 \sin(\omega t - 0.646) - 21.2e^{-\omega t/0.754} \text{ A} \qquad \text{for } \alpha \le \omega t \le \beta$$

Solving $i_o(\beta) = 0$ numerically for β, $\beta = 3.78$ rad (216°). Since $\pi + \alpha = 4.19 > \beta$, the current is discontinuous, and the above expression for current is valid.

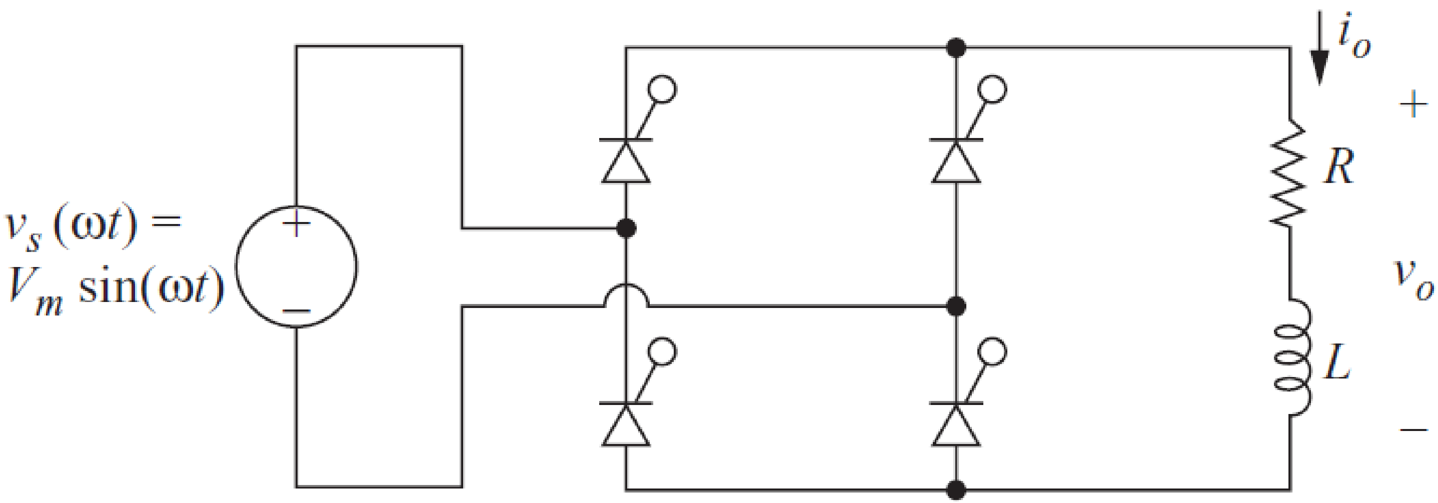
$$I_o = \frac{1}{\pi}\int_{\alpha}^{\beta} i_o(\omega t)\, d(\omega t) = 7.05 \text{ A}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{\pi}\int_{\alpha}^{\beta} i_o(\omega t)\, d(\omega t)} = 8.35 \text{ A}$$

$$P = (8.35)^2(10) = 697 \text{ W}$$

The Controlled Bridge Rectifier with RL Load

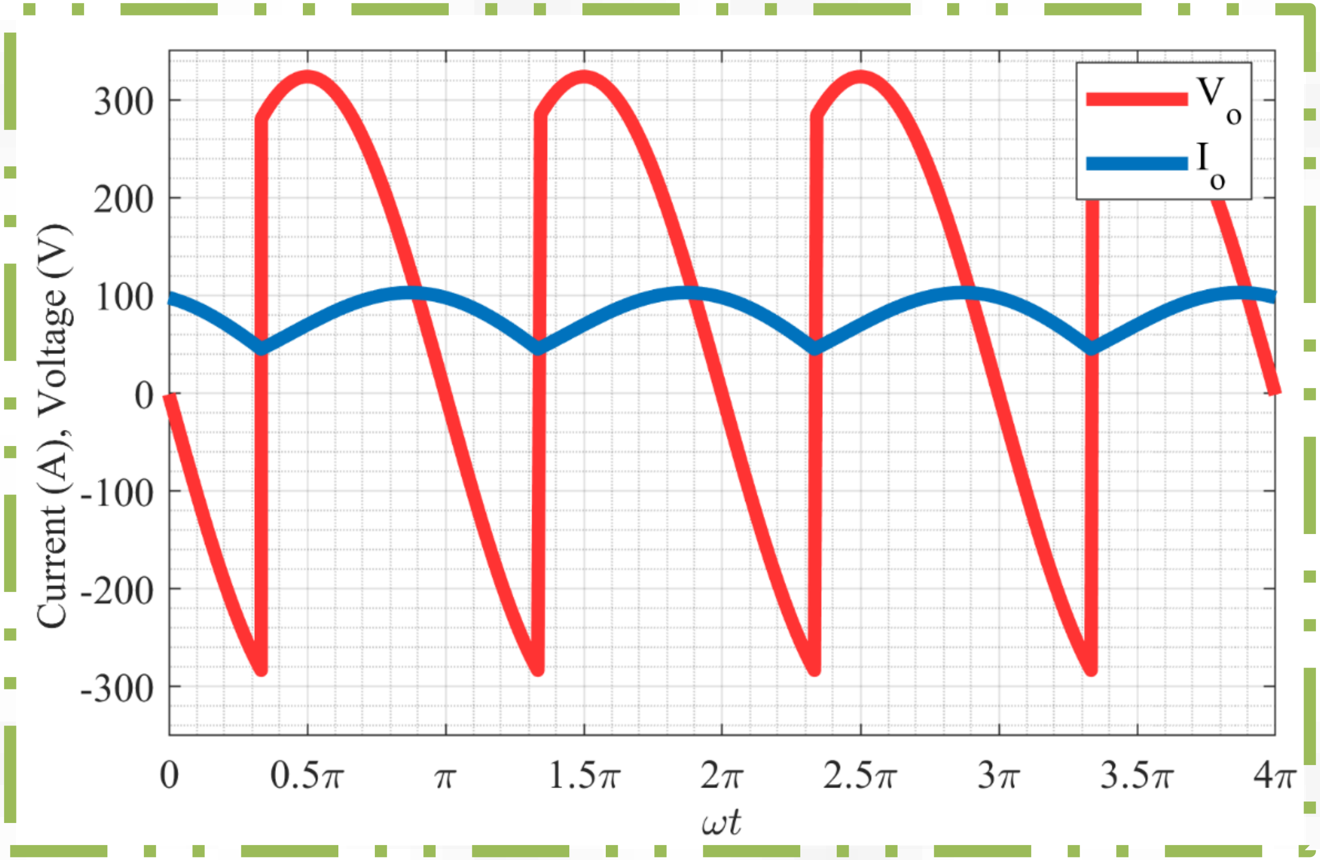
Continuous Current



- ✓ Load current is still positive at $\omega t=\pi+\alpha$ when gate signals are applied to S3 and S4 in the above analysis, S3 and S4 are turned on and S1 and S2 are forced off.
- ✓ Since the initial condition for current in the second half-cycle is not zero, the current function does not repeat. Equation is not valid in the steady state for continuous current.
- ✓ The boundary between continuous and discontinuous current occurs when β for Equation is $\pi+\alpha$. The current at $\omega t=\pi+\alpha$ must be greater than zero for continuous-current operation.

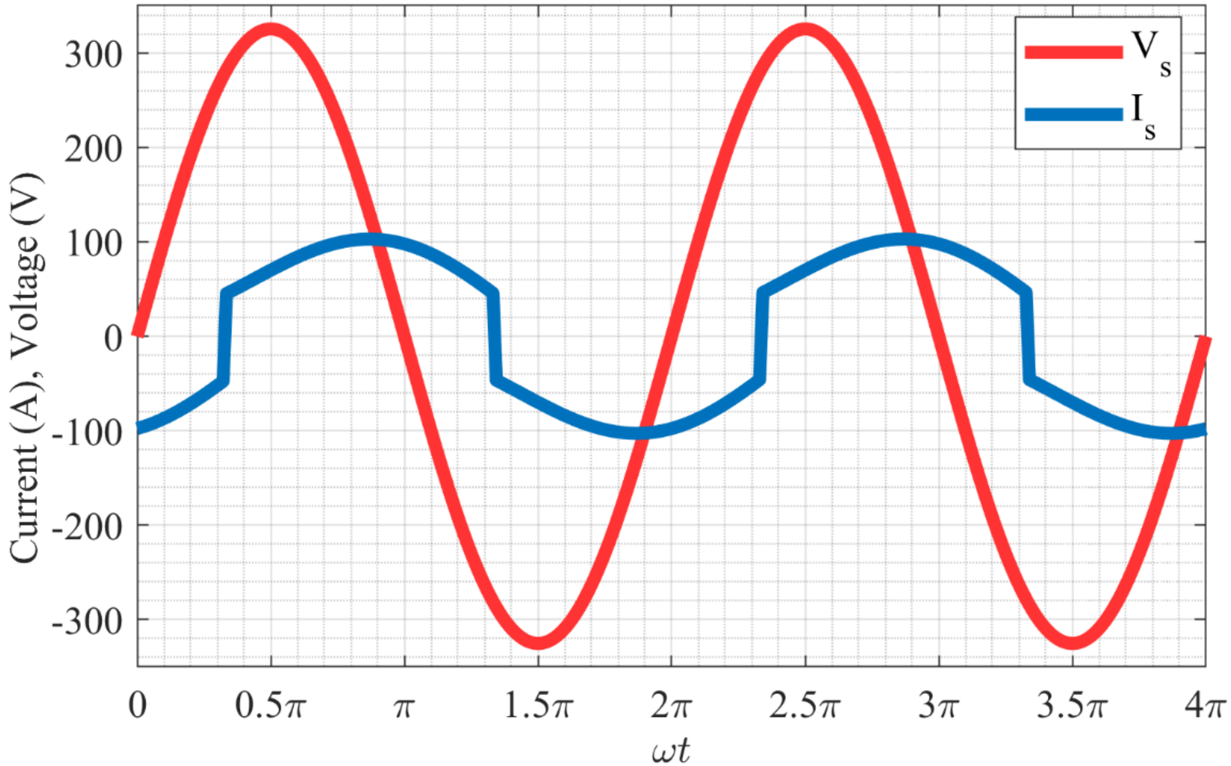
The Controlled Bridge Rectifier with RL Load

Continuous Current



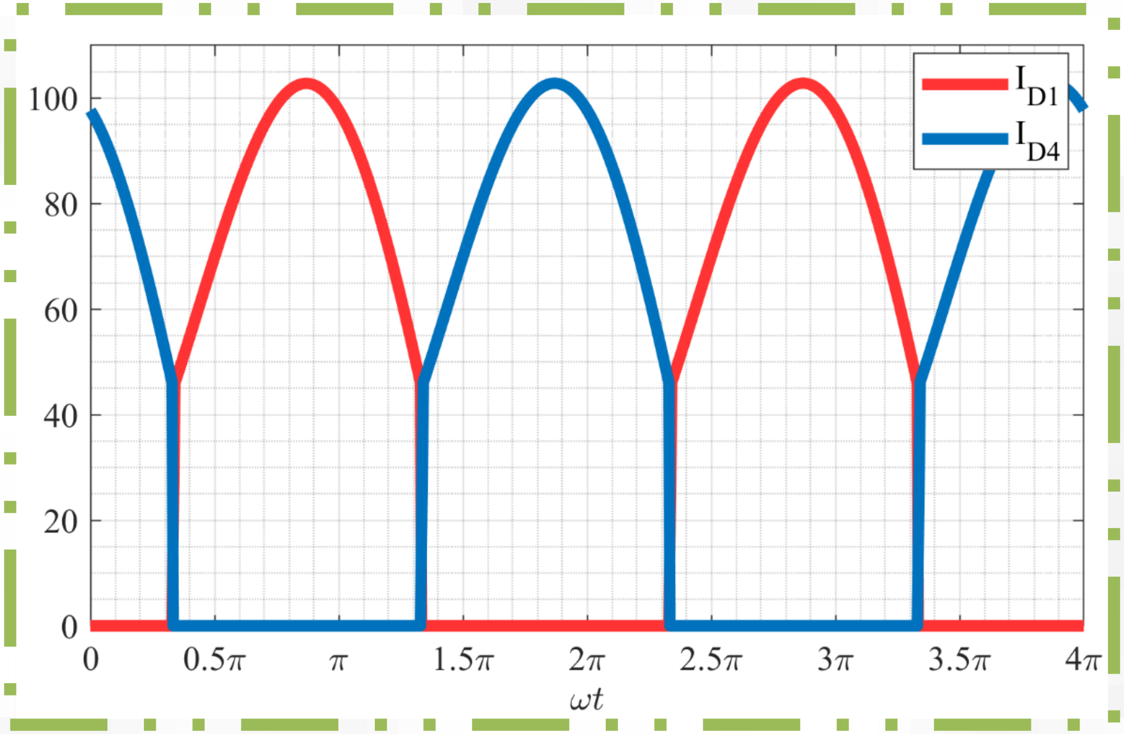
The Controlled Bridge Rectifier with RL Load

Continuous Current



The Controlled Bridge Rectifier with RL Load

Continuous Current



The Controlled Bridge Rectifier with RL Load

Continuous Current

$$i(\pi + \alpha) \geq 0$$

$$\sin(\pi + \alpha - \theta) - \sin(\pi + \alpha - \theta)\, e^{-(\pi + \alpha - \alpha)/\omega\tau} \geq 0$$

$$\sin(\pi + \alpha - \theta) = \sin(\theta - \alpha)$$

$$\sin(\theta - \alpha)\left(1 - e^{-(\pi/\omega\tau)}\right) \geq 0$$

$$\alpha \leq \theta$$

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\alpha \leq \tan^{-1}\left(\frac{\omega L}{R}\right) \qquad \text{for continuous current}$$

The Controlled Bridge Rectifier with RL Load

Continuous Current

A method for determining the output voltage and current for the Continuous current case is to use the Fourier series.

$$v_o(\omega t) = V_o + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \theta_n)$$

$$V_o = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{2V_m}{\pi}\cos\alpha$$

$$V_n = \sqrt{a_n^2 + b_n^2}$$

$$a_n = \frac{2V_m}{\pi}\left[\frac{\cos(n+1)\alpha}{n+1} - \frac{\cos(n-1)\alpha}{n-1}\right]$$

$$b_n = \frac{2V_m}{\pi}\left[\frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1}\right]$$

$$n = 2, 4, 6, \ldots$$

The Controlled Bridge Rectifier with RL Load

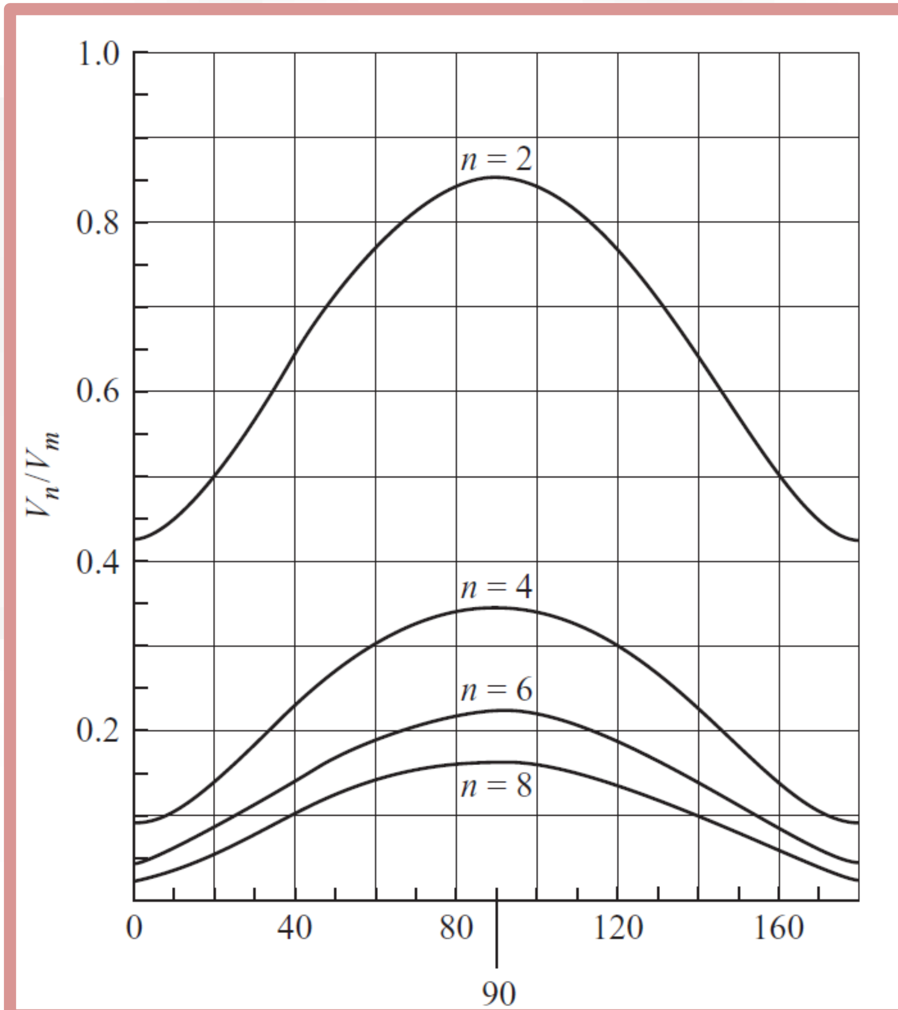
Continuous Current

$$I_{\text{rms}} = \sqrt{I_o^2 + \sum_{n=2,4,6\,\ldots}^{\infty} \left(\frac{I_n}{\sqrt{2}} \right)^2}$$

$$I_o = \frac{V_o}{R} \qquad \text{and} \qquad I_n = \frac{V_n}{Z_n} = \frac{V_n}{|R + jn\omega_0 L|}$$

The Controlled Bridge Rectifier with RL Load

Continuous Current-Output harmonic voltages as a function of delay angle for a single-phase controlled rectifier.



Example-3

A controlled full-wave bridge rectifier of has a source of 120 V rms at 60 Hz, an RL load where R = 10 Ω and L =100 mH. The delay angle α= 60°.

(a) Verify that the load current is continuous.

(b) Determine the dc (average) component of the current.

(c) Determine the power absorbed by the load.

$$\tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left[\frac{(377)(0.1)}{10}\right] = 75^\circ$$

$$\alpha = 60^\circ < 75^\circ \qquad \therefore \text{ continuous current}$$

$$V_0 = \frac{2V_m}{\pi}\cos\alpha = \frac{2\sqrt{2}(120)}{\pi}\cos(60^\circ) = 54.0 \text{ V}$$

Example-3

$$a_n = \frac{2V_m}{\pi}\left[\frac{\cos(n+1)\alpha}{n+1} - \frac{\cos(n-1)\alpha}{n-1}\right]$$

$$b_n = \frac{2V_m}{\pi}\left[\frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1}\right]$$

$$n = 2, 4, 6, \ldots$$

$$V_n = \sqrt{a_n^2 + b_n^2}$$

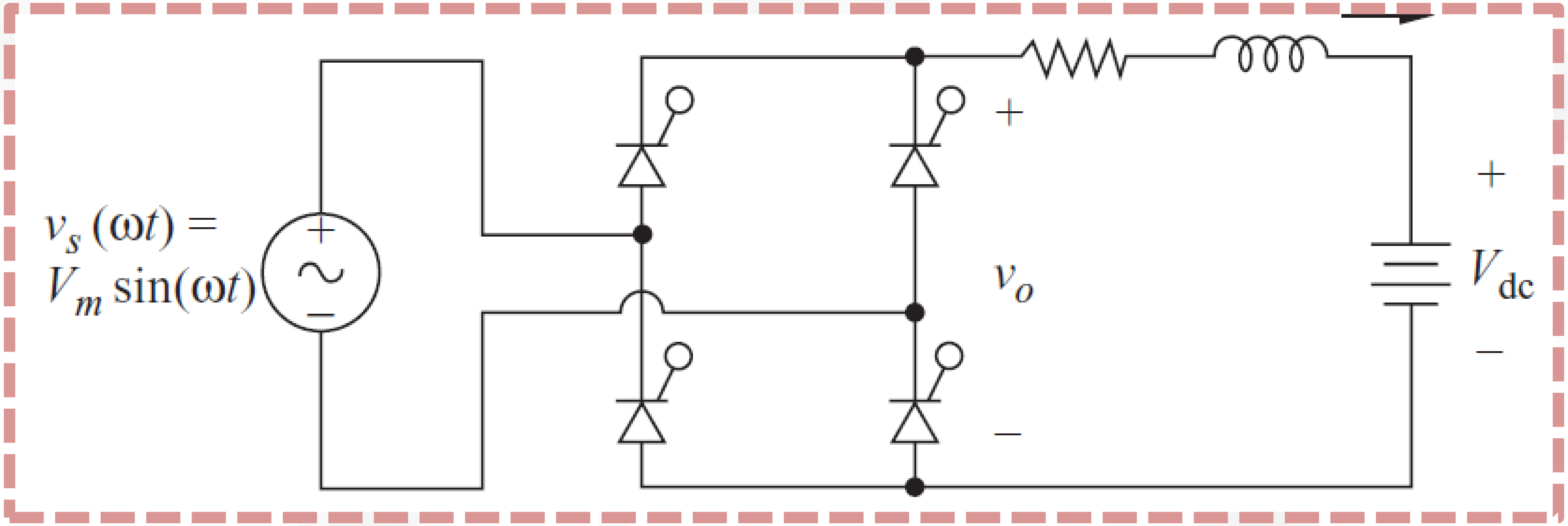
$$I_n = \frac{V_n}{Z_n} = \frac{V_n}{|R + jn\omega_0 L|}$$

n	a_n	b_n	V_n	Z_n	I_n
0 (dc)	—	—	54.0	10	5.40
2	−90	−93.5	129.8	76.0	1.71
4	46.8	−18.7	50.4	151.1	0.33
6	−3.19	32.0	32.2	226.4	0.14

$$I_{\text{rms}} = \sqrt{(5.40)^2 + \left(\frac{1.71}{\sqrt{2}}\right)^2 + \left(\frac{0.33}{\sqrt{2}}\right)^2 + \left(\frac{0.14}{\sqrt{2}}\right)^2 + \cdots} \approx 5.54 \text{ A}$$

$$P = (5.54)^2(10) = 307 \text{ W}$$

The Controlled Bridge Rectifier with RLE Load

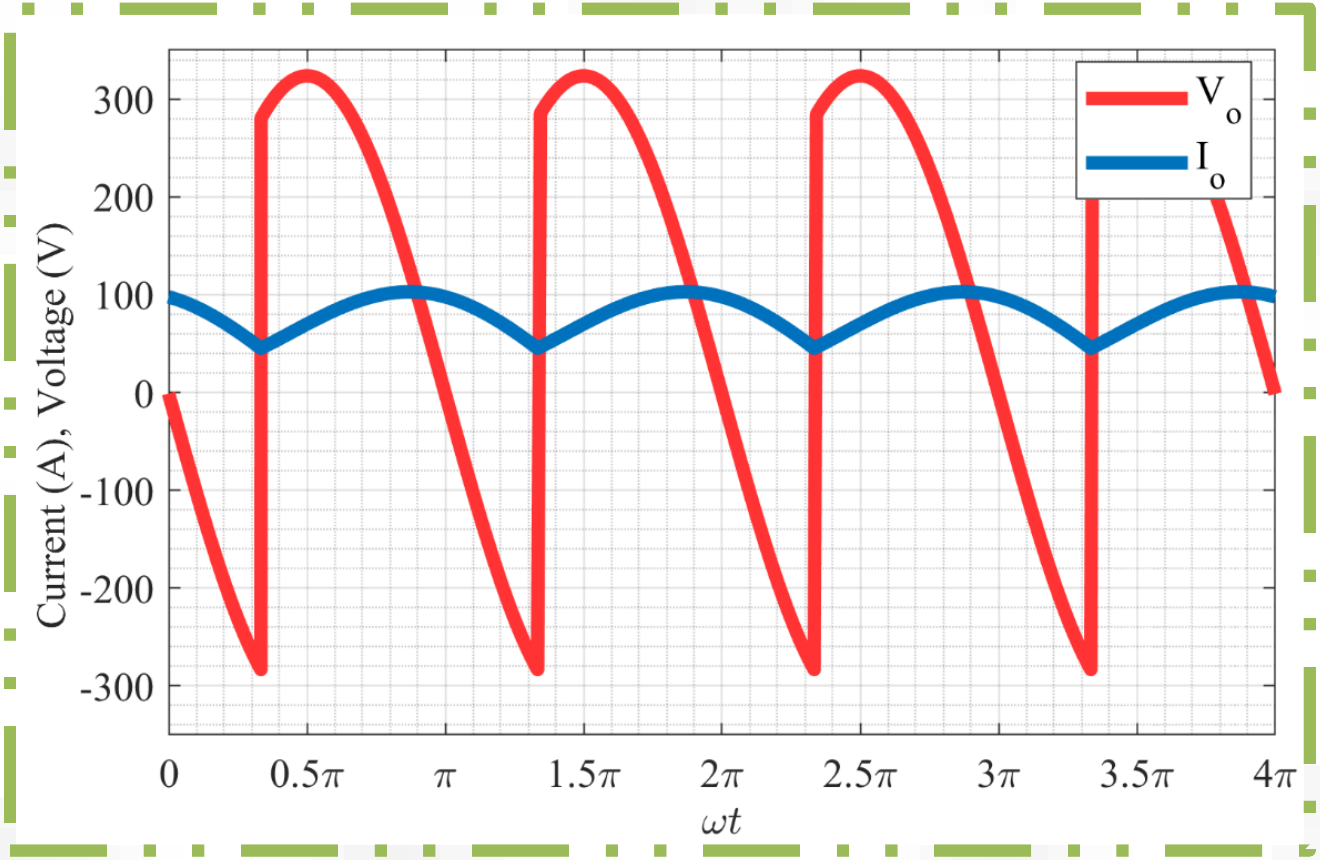


For the controlled rectifier, the SCRs may be turned on at any time that they are forward biased, which is at an angle

$$\boxed{\alpha \geq \sin^{-1}\left(\frac{V_{dc}}{V_m}\right)}$$

The Controlled Bridge Rectifier with RLE Load

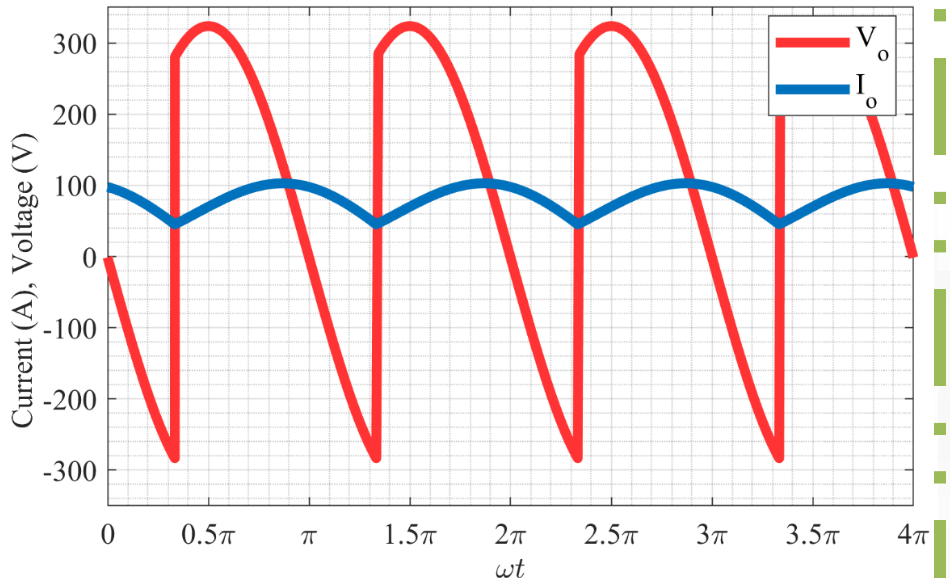
Continuous Current



For the continuous-current case, the bridge output voltage is the same as in Figure. The average bridge output voltage is

$$\boxed{V_o = \frac{2V_m}{\pi} \cos \alpha}$$

The Controlled Bridge Rectifier with RLE Load



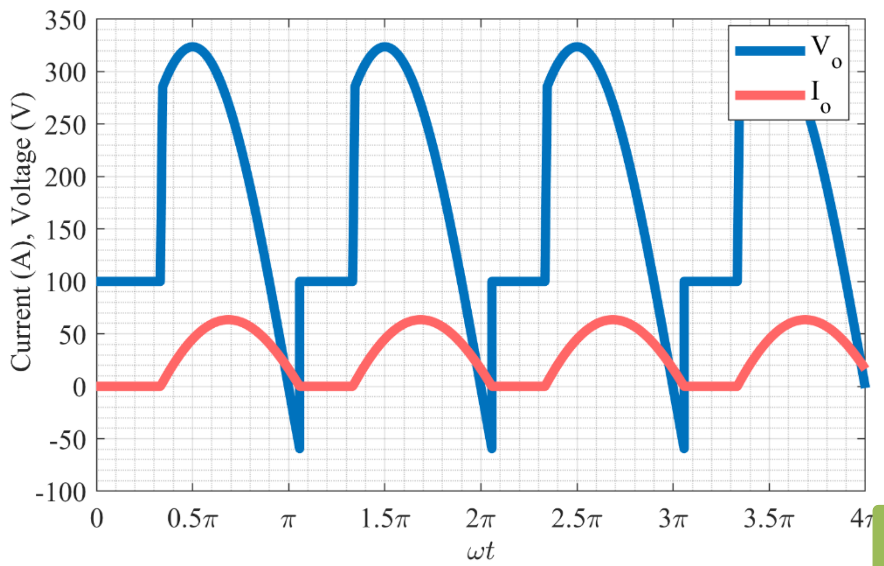
The average load current is

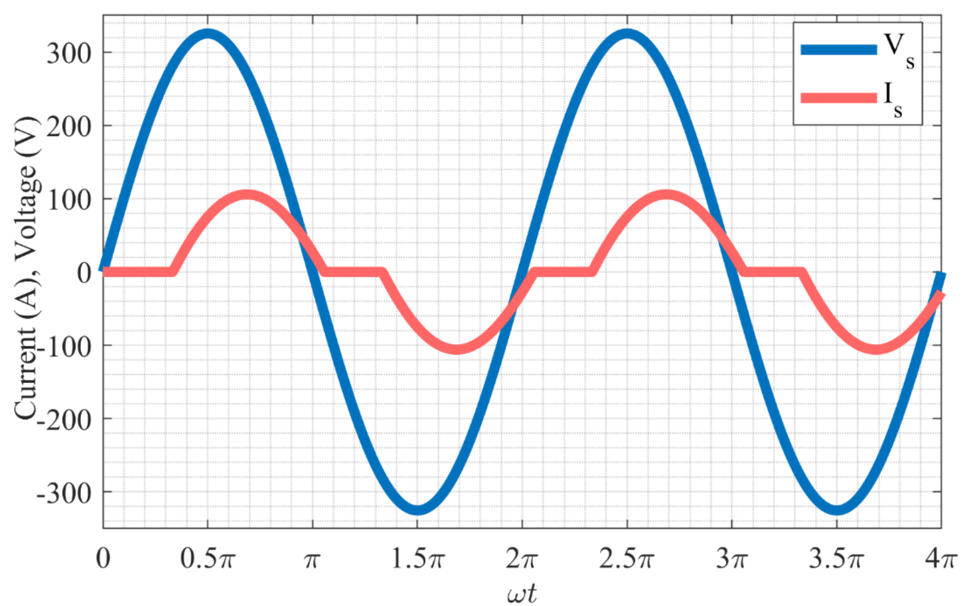
$$I_o = \frac{V_o - V_{dc}}{R}$$

$$P_{dc} = I_o V_{dc}$$

The Controlled Bridge Rectifier with RLE Load

Discontinuous Current





Example-4

The controlled rectifier has an ac source of 240 V rms at 60 Hz, Vdc=100 V, R =5 Ω, and an inductor large enough to cause continuous current.

(a) Determine the delay angle such that the power absorbed by the dc source is 1000 W.

(b) Determine the value of inductance that will limit the peak-to-peak load current variation to 2 A.

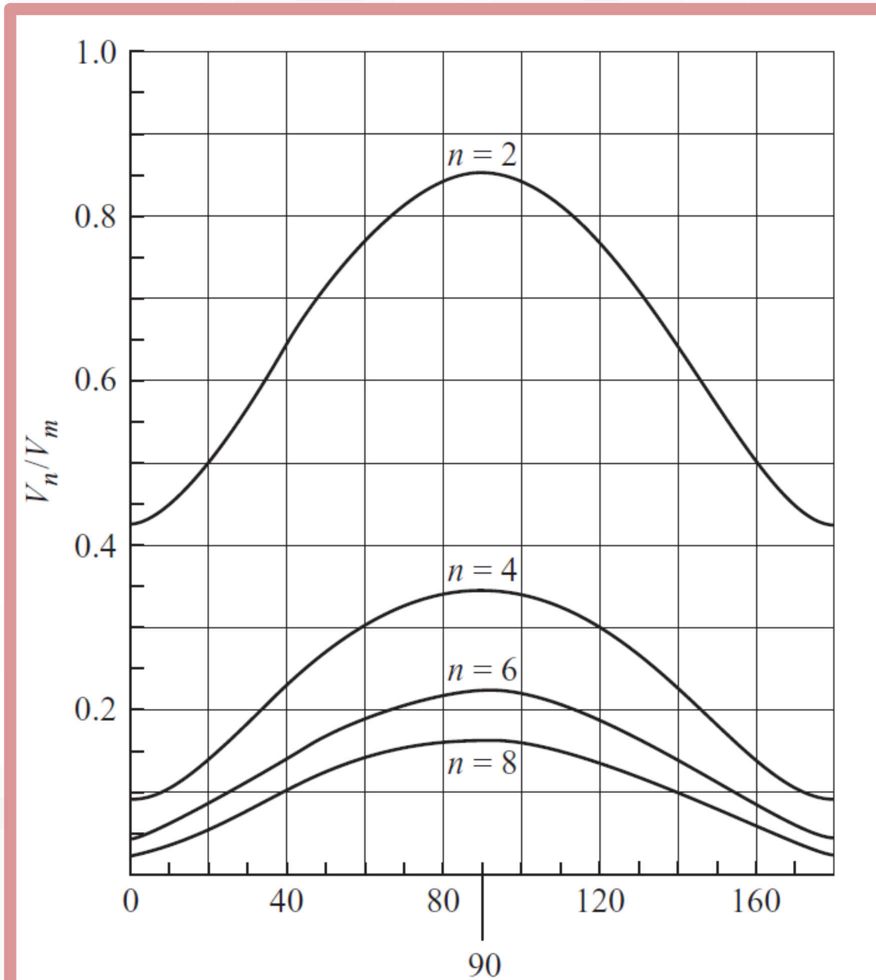
$$V_o = V_{dc} + I_o R = 100 + (10)(5) = 150 \text{ V}$$

$$\alpha = \cos^{-1}\left(\frac{V_o \pi}{2V_m}\right) = \cos^{-1}\left[\frac{(150)(\pi)}{2\sqrt{2}(240)}\right] = 46°$$

$$I_n = \frac{V_n}{Z_n}$$

$$Z_n = |R + jn\omega_0 L|$$

Since the decreasing amplitude of the voltage terms and the increasing magnitude of the impedance both contribute to diminishing ac currents as n increases, the peak-to-peak current variation will be estimated from the first ac term. For $n = 2$, V_n/V_m is estimated from Fig. 4-12 as 0.68 for $\alpha = 46°$, making $V_2 = 0.68V_m = 0.68\,(240\sqrt{2}) = 230$ V. The peak-to-peak variation of 2 A corresponds to a 1-A zero-to-peak amplitude. The required load impedance for $n = 2$ is then



$$Z_2 = \frac{V_2}{I_2} = \frac{230\text{ V}}{1\text{ A}} = 230\ \Omega$$

$$L \approx \frac{Z_2}{2\omega} = \frac{230}{2(377)} = 0.31\text{ H}$$

Controlled Single-Phase Converter Operating as an Inverter

- The above discussion focused on circuits operating as rectifiers, which means that the power flow is from the ac source to the load.

- It is also possible for power to flow from the load to the ac source, which classifies the circuit as an inverter.

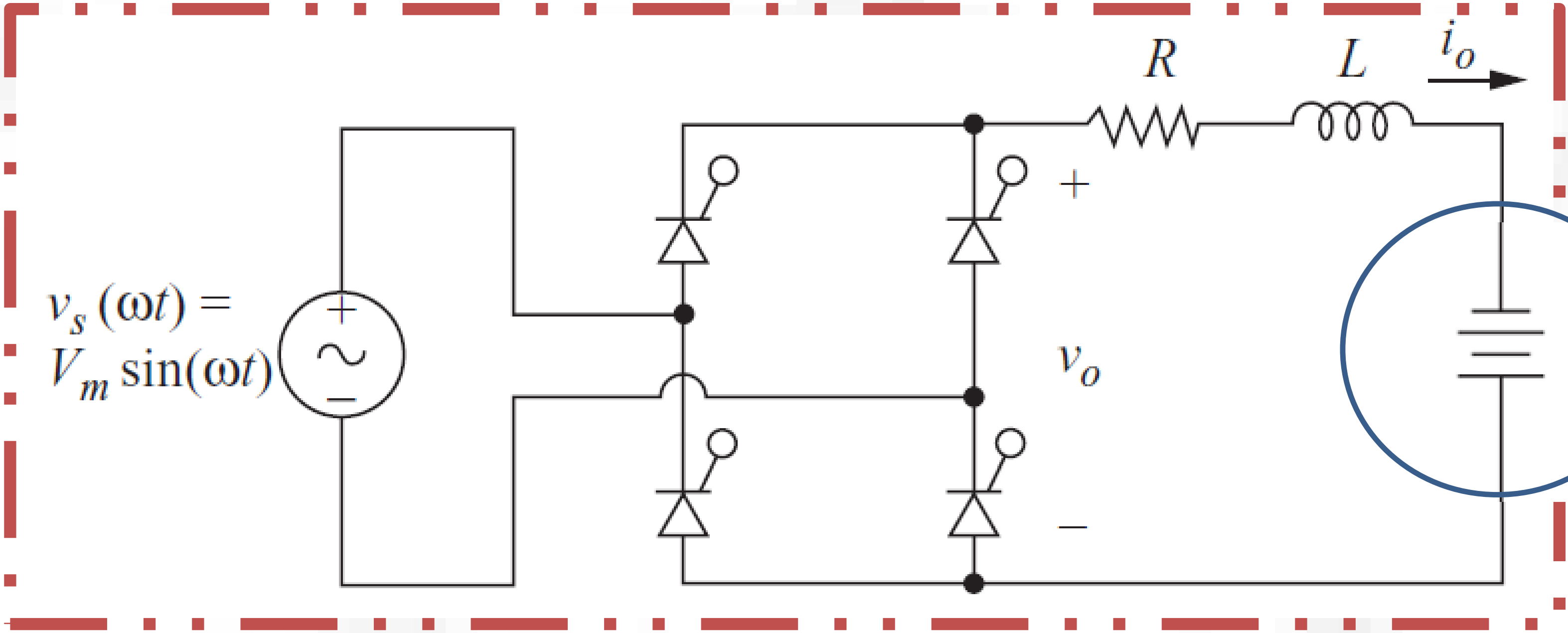
- For inverter operation of the converter, power is supplied by the dc source, and power is absorbed by the bridge and is transferred to the ac system.

- The load current must be in the direction shown because of the SCRs in the bridge. For power to be supplied by the dc source, Vdc must be negative.

- For power to be absorbed by the bridge and transferred to the ac system, the bridge output voltage Vo must also be negative. A delay angle larger than 90° will result in a negative output voltage.

$$\alpha \geq \sin^{-1}\left(\frac{V_{dc}}{V_m}\right) \qquad V_o = \frac{2V_m}{\pi}\cos\alpha$$

Controlled Single-Phase Converter Operating as an Inverter

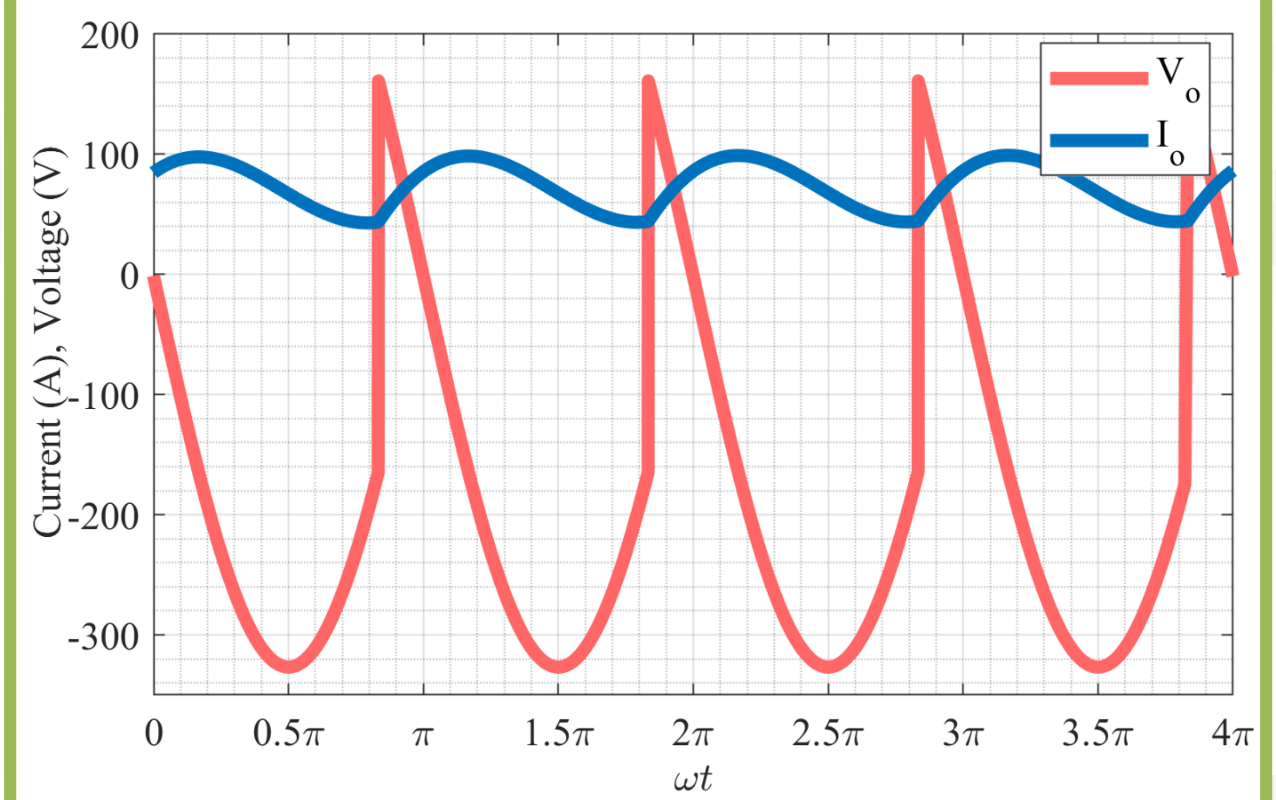


$$V_o = \frac{2V_m}{\pi}\cos\alpha$$

$$0 < \alpha < 90° \rightarrow V_o > 0 \qquad \text{rectifier operation}$$

$$90° < \alpha < 180° \rightarrow V_o < 0 \qquad \text{inverter operation}$$

Controlled Single-Phase Converter Operating as an Inverter



$$P_{\text{bridge}} = P_{\text{ac}} = -I_o V_o$$

Example-5

The voltage generated by an array of solar cells is 110 V. The solar cells are capable of producing 1000 W. The ac source is 120 V rms, R = 0.5 Ω, and L is large enough to cause the load current to be essentially dc. Determine the delay angle α such that 1000 W is supplied by the solar cell array. Determine the power transferred to the ac system and the losses in the resistance. Assume ideal SCRs.

$$I_o = \frac{P_{dc}}{V_{dc}} = \frac{1000}{110} = 9.09 \text{ A}$$

$$V_o = I_o R + V_{dc} = (9.09)(0.5) + (-110) = -105.5 \text{ V}$$

$$\alpha = \cos^{-1}\left(\frac{V_o \pi}{2V_m}\right) = \cos^{-1}\left[\frac{-105.5\pi}{2\sqrt{2}(120)}\right] = 165.5^\circ$$

$$P_{ac} = -V_o I_o = (-9.09)(-105.5) = 959 \text{ W}$$

$$P_R = I_{rms}^2 R \approx I_o^2 R = (9.09)^2(0.5) = 41 \text{ W}$$