Q-1) Assume an infinitely long coaxial cable shown in the figure. The inner conductor has a radius a and the outer conducting shell has a radius b. The thickness of the outer conductor can be ignored as it is very small. Between two conductors, there is a material with permeability

$$\mu(r) = \mu_0 \frac{r^2}{a^2}$$

a) Find the magnetic field intensity vector $\boldsymbol{H}$ and magnetic flux density vector $\boldsymbol{B}$ everywhere

b) Find the self-inductance per unit length of the coaxial structure.

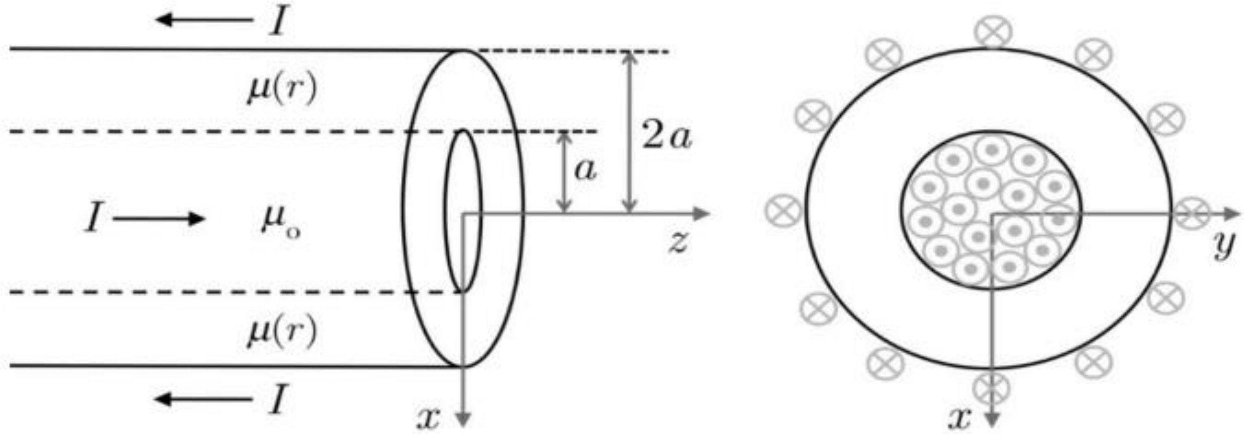
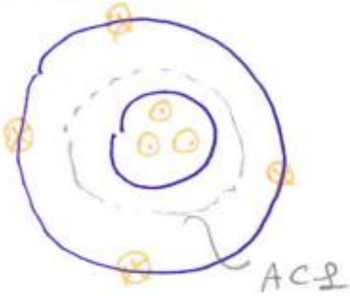


Figure 1. The geometry of Q-1.

5-1)



ⓐ $\oint \vec{H}\cdot d\vec{l} = I_{enclosed}$

for $b>r>a$ $\quad I_{enclosed} = I$

" $\quad r>b \quad$ " $\quad = 0$

for $r<a$ $\quad I_{enclosed} = \dfrac{I\pi r^2}{\pi a^2} = Ir^2/a^2$

Then, $\quad \vec{H} = H_\phi \vec{a_\phi}$

$H\,2\pi r = I_{enclosed} \Rightarrow$

$$\vec{H} = \frac{Ir}{2\pi a^2}\vec{a_\phi}, \qquad r<a$$

$$= \frac{I}{2\pi r}\vec{a_\phi}, \qquad a<r<b$$

$$= 0, \qquad r>b$$

One can find easily

$\vec{B} = \mu \vec{H}$ $\quad \mu_0$ or μ with respect to region

$$\Rightarrow B = \begin{cases} \dfrac{Ir\mu_0}{2\pi a^2}\vec{a_\phi}, & r<a \\ \dfrac{I\mu_0 r}{2\pi a^2}\vec{a_\phi}, & a<r<b \\ 0 & r>b \end{cases}$$

Let us find $\quad w_m = \frac{1}{2}\int_{v} \vec{B}\cdot\vec{H}\,dv$

$$= w_{m,in} = \frac{1}{2}\int_0^1\int_0^{2\pi}\int_0^a \frac{\mu_0 I^2 r^2}{(2\pi a^2)^2}\,r\,d\phi\,dz$$

$$= \frac{\mu_0 I^2 2\pi}{2(2\pi a^2)}\cdot\frac{r^4}{4}\Big|_0^a \Rightarrow \frac{\mu_0 I^2 \pi a^4}{16\pi^2 a^4} = \frac{\mu_0 I^2}{16\pi}$$

By employing $\quad w_m = \frac{1}{2}L_{in} I^2 \Rightarrow \boxed{L_{in} = \frac{\mu_0}{8\pi}}$

for $a<r<b \rightarrow W_{m,out} = \frac{1}{2}\int_0^1\int_0^{2\pi}\int_a^b \frac{I^2\mu_0}{4\pi^2 a^2}\, r\,dr\,d\phi\,dz$

$$= \frac{I^2\mu_0}{8\pi^2 a^2}\, \frac{2\pi r^2}{2}\Big|_a^b = \frac{\mu_0 I^2 (b^2-a^2)}{8\pi a^2}$$

$\rightarrow W_{m,out} = \frac{1}{2} L_{out} I^2 \rightarrow L_{out} = \frac{\mu_0}{4\pi a^2}(b^2-a^2)$

for $r>b$, $W_m = 0$

$L = L_{in} + L_{out} = \frac{\mu_0}{8\pi} + \frac{\mu_0}{4\pi a^2}(b^2-a^2)$ [H]

Q-2) Try to obtain the mutual inductance between the toroidal coil and the infinite line current placed on the toroid shown below:

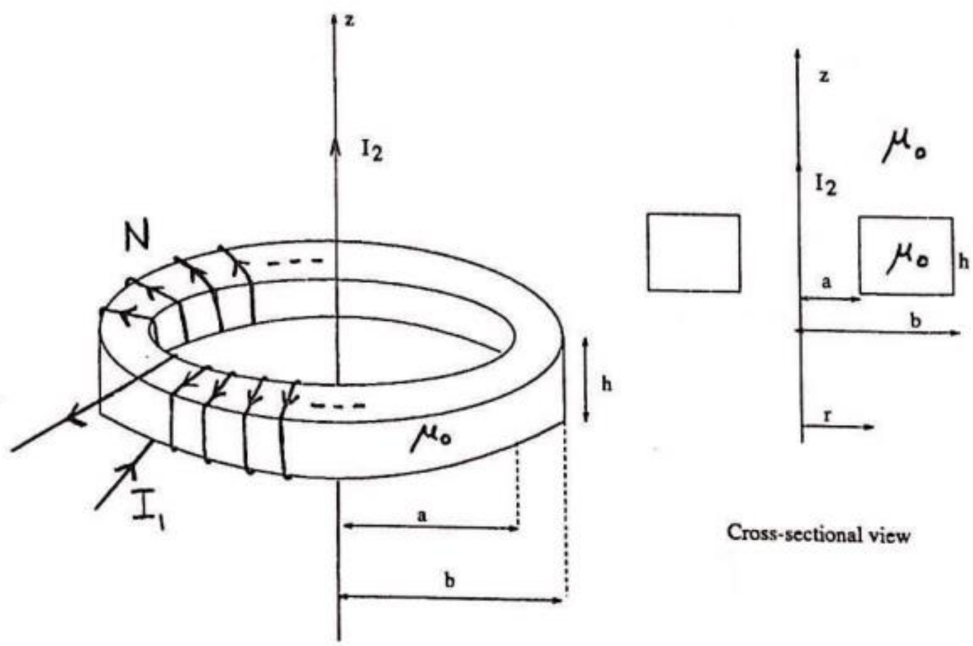


Figure 2. The geometry of Q-2.

S-2) Mutual Inductance

In this calculation, we need to consider the effect of current line I_2 to toroid. B-field due to I_2 is

$$B_2 = \frac{\mu_0 I_2}{2\pi r} a_\phi \quad (ACL)$$

→ This field induces flux at cross-section of toroid. This flux can be found as:

$$\Phi_{21} = \int B_2 \, dS_1 = \int_0^h \int_a^b \frac{\mu_0 I_2}{2\pi r} dr\, dz = \frac{\mu_0 I_2}{2\pi} h \ln(b/a)$$

The flux linking the second loop $\Lambda_{12} = N \Phi_{21}$,

$$L_{12} = \frac{\Lambda_{12}}{I_2} \Rightarrow \frac{\mu_0 I_2 N h \ln(b/a)}{2\pi I_2} = \frac{\mu_0 h N}{2\pi} \ln(b/a) \; [H]$$

Q-3) A uniform volume current density $\boldsymbol{J} = J_0 \hat{a}_x$ is distributed in a slab of thickness d (permeability, $\mu = \mu_0$) as provided below. This slab has infinite extend in x and y directions. Using Ampere's Circuital Law, evaluate the magnetic field $\boldsymbol{B}$ everywhere, inside and outside the slab.

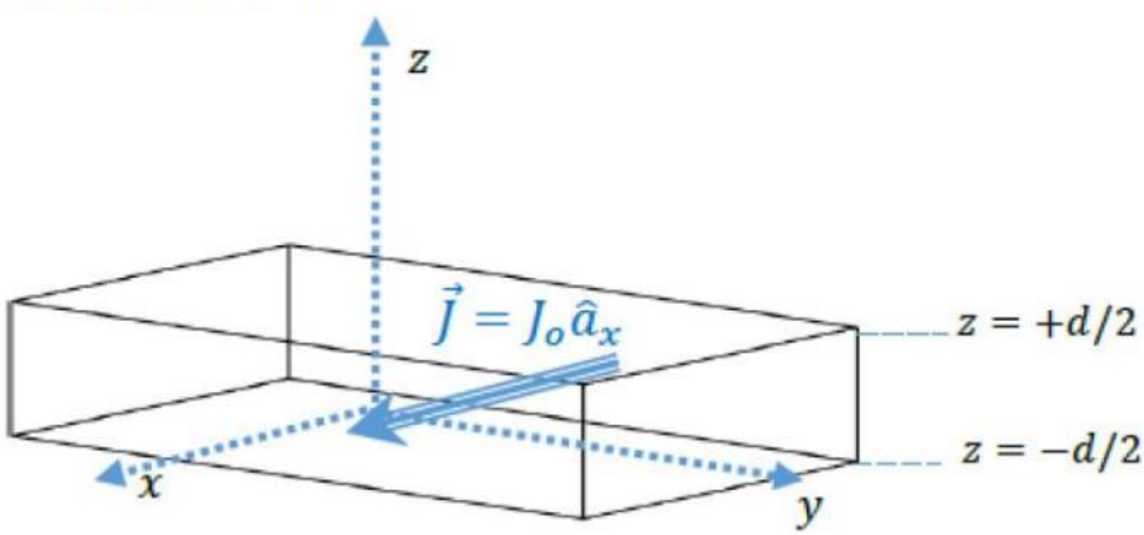
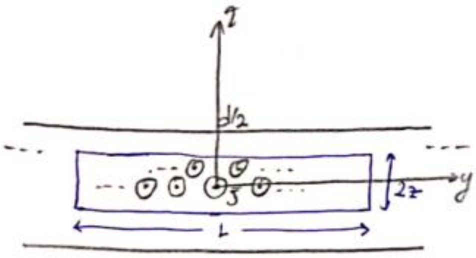


Figure 3. The geometry of Q-3.



Blue rectangle symbolises the amperé loop.

$\oint_c B.dl = \mu_0 I$ → By inspecting and using right hand rule direction of B can be understand

↳ $\bar{B}$ is $\begin{cases} \text{at} \quad -\hat{a}_y \text{ if we operate at } z>0 \\ \text{at} \quad \hat{a}_y \text{ if we operate at } z<0 \end{cases}$

Inside slab

$$\left.\begin{aligned} &\oint \bar{B}.d\bar{l} = \mu_0.I_{enc} \\ &I_{enc} = J_0 . 2z.L \end{aligned}\right\} \begin{aligned} &B.2L = \mu_0 J_0.2z.L \\ &\bar{B} = \mu_0 J_0.z\,(\pm\hat{a}_z) \end{aligned} \Rightarrow \bar{B} = \begin{cases} -\mu_0 J_0 z\,\hat{a}_y & \text{at} \quad 0<z<d/2 \\ \mu_0 J_0 z\,\hat{a}_y & \text{at} \quad -d/2<z<0 \end{cases}$$

Outside slab

$$\left.\begin{aligned} &\oint \bar{B}.d\bar{l} = \mu_0 I_{enc} \\ &I_{enc} = J_0.d.L \end{aligned}\right\} \begin{aligned} &B.2L = \mu_0 J_0 d.L \\ &\bar{B} = \frac{\mu_0 J_0 d}{2}\,(\pm\hat{a}_y) \end{aligned} \Rightarrow \bar{B} = \begin{cases} -\frac{\mu_0 J_0 d}{2}\,\hat{a}_y & \text{at} \quad d/2<z \\ \frac{\mu_0 J_0 d}{2}\,\hat{a}_y & \text{at} \quad z<-d/2 \end{cases}$$

Q-4) There exist two conductor lines of infinite length (along z) provided in the following figure. Conductor 1 is a thin straight wire on the z-axis whereas Conductor 2 is **half** of a thin circular cylinder of radius a along the z-axis. If a uniform current of intensity I flows through the two conductors in reverse directions, determine the force per unit length on the conductors.

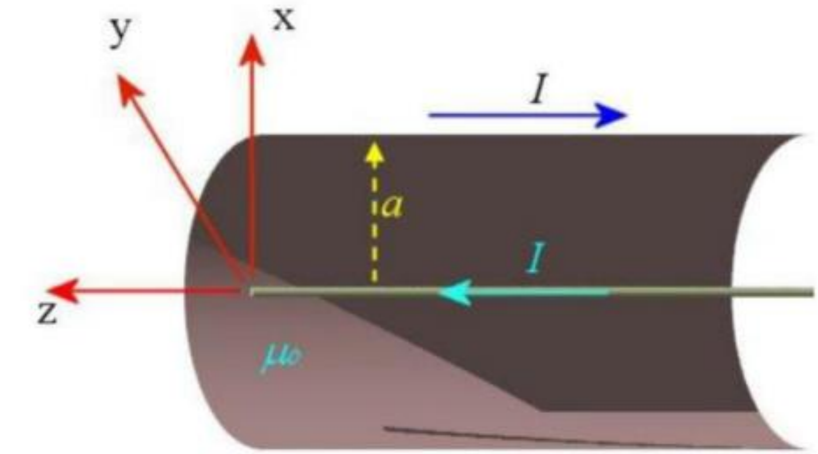


Figure 4. The geometry of Q-4.

H field due to inner conductor on the second conductor

Cross- section

$$H_\phi = \frac{I}{2\pi a}\vec{a_\phi} \rightarrow \vec{B} = \frac{I\mu_0}{2\pi a}\vec{a_\phi}$$

Current density on the outer conductor :

$$\vec{J} = -\vec{a_z}\frac{I}{\pi a} \Rightarrow dF_{12} = \vec{J}\times\vec{B}\,dl \Rightarrow \quad {}_{external}$$

$$= \vec{J}\,a\,d\phi \times \vec{B} \Rightarrow$$

$$= \vec{J}\,(a\,d\phi)\times\vec{B}_{external} \qquad (I_1 = I_2 = I)$$

J

sum all of them

$$= -\vec{a_z}\frac{I_2}{\pi a} \times \frac{I_1\mu_0}{2\pi a}\vec{a_\phi}\,a\,d\phi = \frac{I_1 I_2 \mu_0}{2\pi^2 a}\,d\phi\,\vec{a_\rho}$$

$$I_1 = I_2 = I \Rightarrow \quad \rightarrow dF = \frac{I^2}{2\pi^2 a}\mu_0\,d\phi\,\vec{a_\rho} \qquad (\vec{a_\rho} = a_x\cos\phi + a_y\sin\phi)$$

$$F = \int dF = \int_0^\pi (\vec{a_x}\cos\phi + \vec{a_y}\sin\phi)\frac{I^2\mu_0}{2\pi^2 a}\,d\phi = \frac{I^2\mu_0}{a\pi^2}\vec{a_y}$$