HW 3

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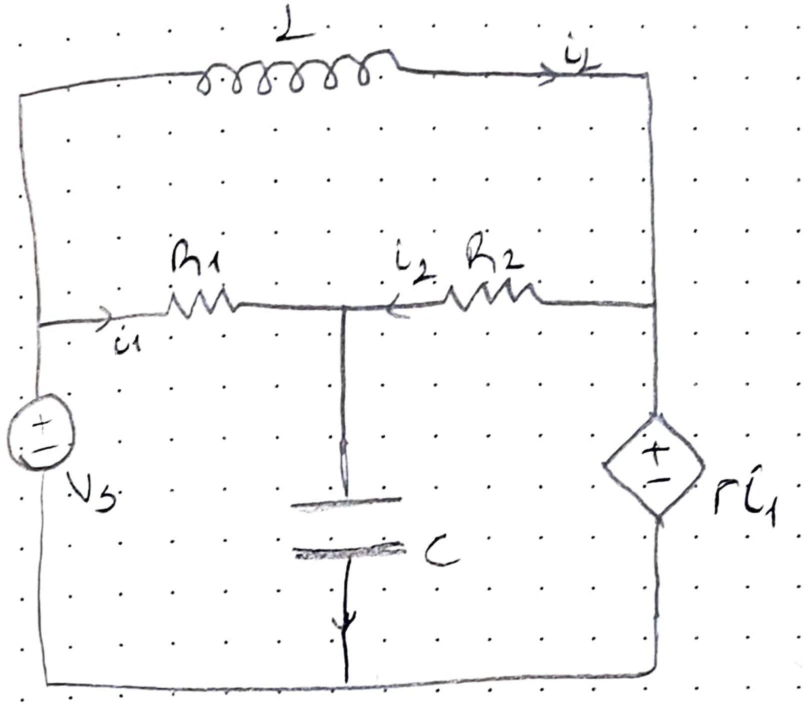
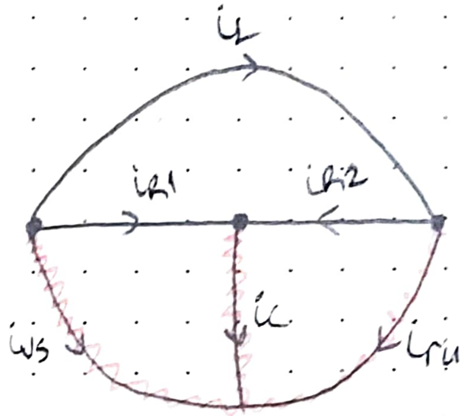




state veriables:

$[V_C , i_L]$

① $i_1 + i_2 - i_C = 0$

$$i_C = C\frac{dV_C}{dt} , \quad i_1 = \frac{V_S - V_C}{R_1} , \quad i_2 = \frac{ri_1 - V_C}{R_2}$$

$$\frac{V_S - V_C}{CR_1} + \frac{ri_1 - V_C}{CR_2} = \frac{dV_C}{dt}$$

$$\frac{1}{CR_1}V_S - \frac{R_2}{CR_1R_2}V_C + \frac{r}{CR_1R_2}V_S - \frac{r}{CR_1R_2}V_C - \frac{R_1}{CR_1R_2}V_C = \frac{dV_C}{dt}$$

$$\frac{-(R_1+R_2+r)}{CR_1R_2}V_C + \frac{R_2+r}{CR_1R_2} = \frac{dV_C}{dt}$$

② $V_L + V_{R2} - V_{R1} = 0$

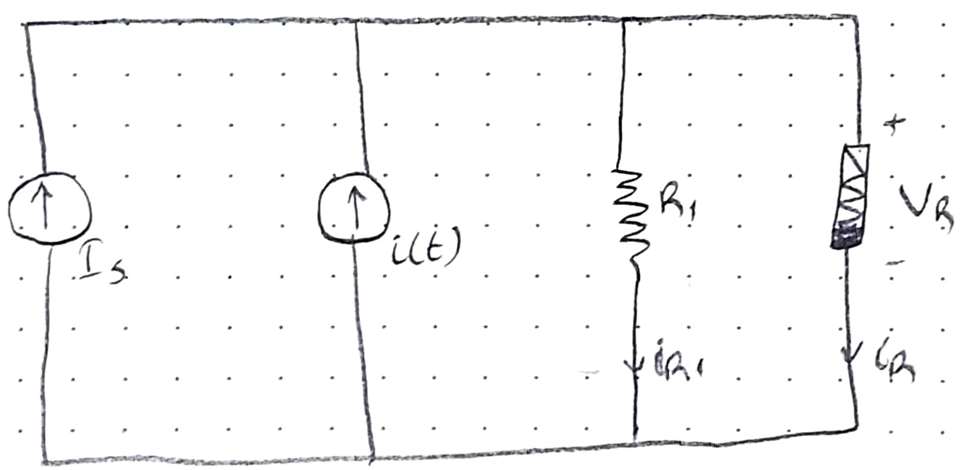
$$L\cdot\frac{di_L}{dt} = -(ri_1 - V_C) + V_S - V_C$$

$$\frac{di_L}{dt} = -\frac{1}{L}\left(\frac{r(V_S - V_C)}{R_1} - V_C\right) + \frac{V_S - V_C}{L}$$

$$\frac{di_L}{dt} = -\frac{rV_S}{LR_1} + \frac{rV_C}{LR_1} + \cancel{\frac{V_C}{L}} + \frac{V_S}{L} - \cancel{\frac{V_C}{L}}$$

$$\frac{di_L}{dt} = \frac{R_1 - r}{LR_1}V_S + \frac{r}{LR_1}V_C$$

$$\begin{bmatrix} \dot{V}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -\frac{(R_1+R_2+r)}{CR_1R_2} & 0 \\ \frac{r}{LR_1} & 0 \end{bmatrix} \begin{bmatrix} V_C \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{R_2+r}{CR_1R_2} \\ \frac{R_1-r}{LR_1} \end{bmatrix} V_S$$



$R_1 = 1/2\ \Omega$

$I_S = 8\ A$

$i(t) = 0.1\sin(10t)$

$i_R = V_R^2 + V_R - 2$

First, DC Analysis

$I_S - i_{R1} = i_R \qquad i_{R1} = \frac{V_R}{1/2} = 2V_R$

$8 - 2V_R = i_R$

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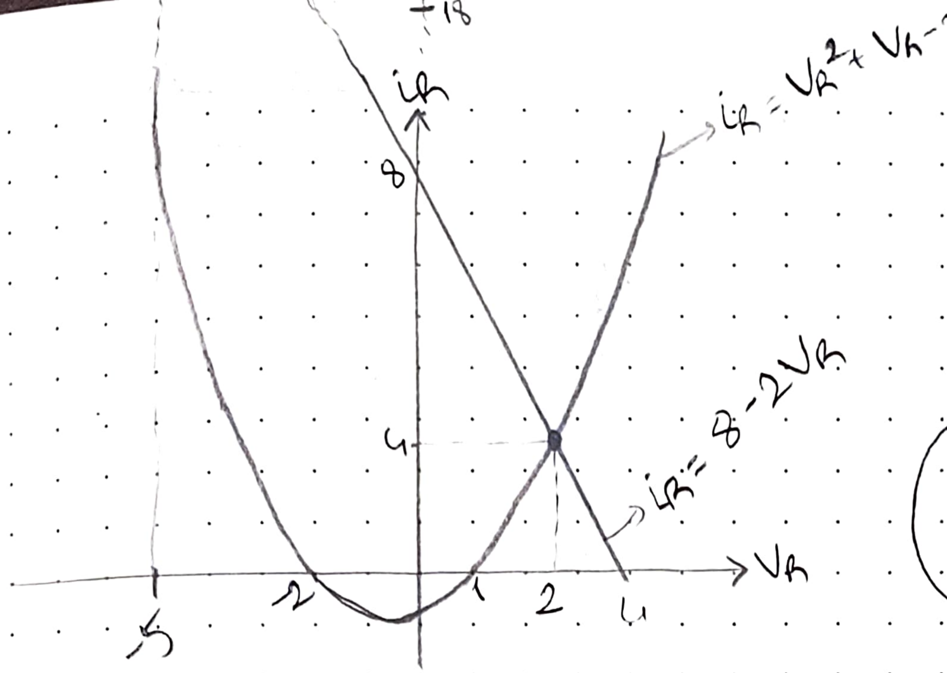


$8 - 2V_R = V_R^2 + V_R - 2$

$V_R^2 + 3V_R - 10 = 0$

$\boxed{V_R = -5,\ i_R = 18}$

$V_R = 2,\ i_R = 4$

neglected due to small signal

$$1/R_{Q1} = \frac{di_R}{dV_R}\Big|_{V_R=2} = 2V_R + 1\Big|_2 = 5,\quad R_{Q1} = \frac{1}{5}$$

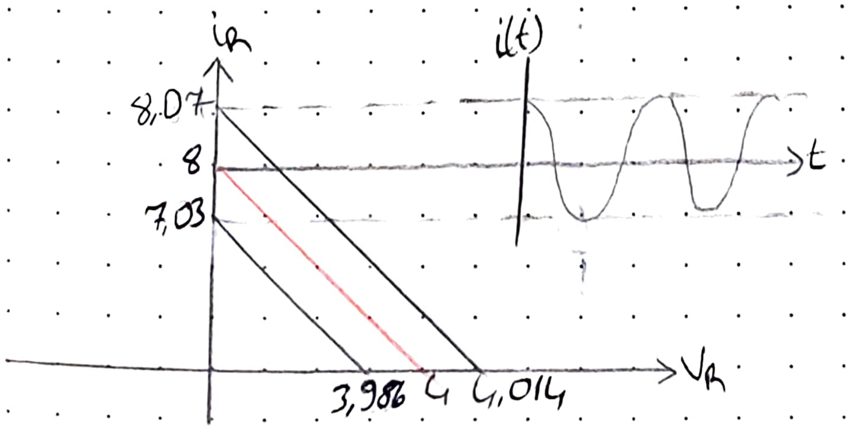
$$\boxed{1/R_{Q2} = \frac{di_R}{dV_R}\Big|_{V_R=-5} = 2V_R + 1\Big|_{-5} = -9,\quad R_{Q2} = -\frac{1}{9}}$$

Second, small signal analysis

$$\tilde{v}(t) = 0.1\sin(10t) \cdot R_Q // \frac{1}{2} = \frac{0.1}{7}\sin(10t)$$

$$\tilde{i}(t) = 0.1\sin(10t) - \frac{\tilde{v}(t)}{1/2} = \frac{0.5}{7}\sin(10t)$$

$V_R = V_Q + \tilde{v}(t)$ $i_R = i_Q + \tilde{i}(t)$



$V_R = 2 + \frac{0.1}{7}\sin(10t)$

$i_R = 4 + \frac{0.5}{7}\sin(10t)$

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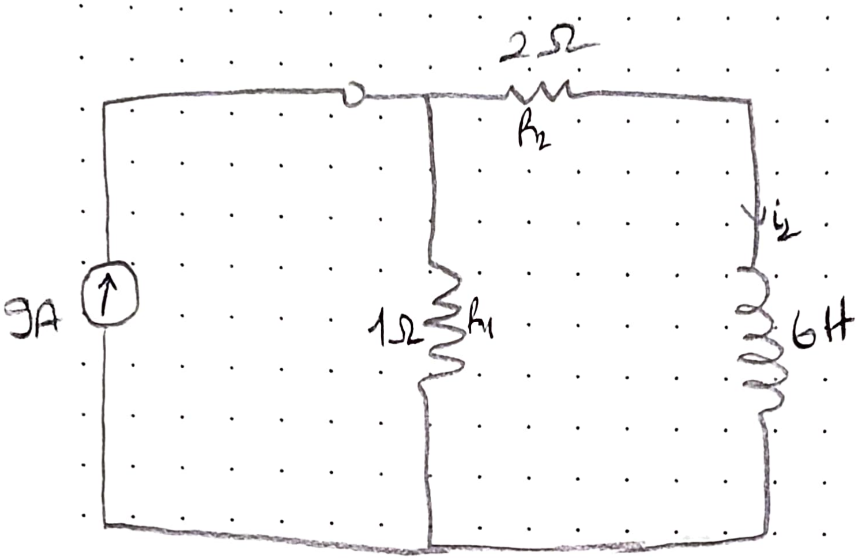

$V_L = 0$ for $t < 0$

$2 \cdot i_0 = 1 \cdot i_{R1}$, $i_0 + i_{R1} = 9A$

$1 // 2 = \frac{2}{3}\,\Omega = R_{eq}$

$I_0 = 3\,[A]$

$\tau = \frac{6}{3} = 2\,[s]$

$i(t) = 3e^{-t/2}$ [A]

$V_L = L \cdot \frac{di(t)}{dt} = -6 \cdot \frac{3}{2} e^{-t/2} = -9e^{-t/2}$