

Introduction to Digital Design

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November 5, 2023

Karnaugh Maps (K-map)

- ▶ A K-map is a collection of squares
 - ▶ Each square represents a minterm
 - ▶ The collection of squares is a graphical representation of a Boolean function
 - ▶ Adjacent squares differ in the value of one variable
 - ▶ Alternative algebraic expressions for the same function are derived by recognizing patterns of squares
- ▶ The K-map can be viewed as
 - ▶ A reorganized version of the truth table
 - ▶ A topologically-warped Venn diagram as used to visualize sets in algebra of sets

Two Variable Karnaugh Map

- ▶ Two variable: x and y
 - ▶ 4 minterms:
 - ▶ $m_0 = x'y' \rightarrow 00$
 - ▶ $m_1 = x'y \rightarrow 01$
 - ▶ $m_2 = xy' \rightarrow 10$
 - ▶ $m_3 = xy \rightarrow 11$

		y	0	1
		x	0	1
x	y	00	m_0	m_1
		01	m_2	m_3

Two Variable Karnaugh Map

- ▶ Two variable: x and y
 - ▶ 4 minterms:
 - ▶ $m_0 = x'y' \rightarrow 00$
 - ▶ $m_1 = x'y \rightarrow 01$
 - ▶ $m_2 = xy' \rightarrow 10$
 - ▶ $m_3 = xy \rightarrow 11$

▶ Example:

x	y	f
0	0	0
0	1	1
1	0	1
1	1	0

\diagdown y 0 1
x

		0	1
0		m_0	m_1
1		m_2	m_3

\diagdown y 0 1
x

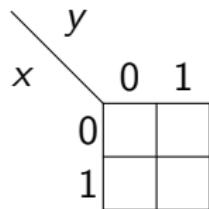
		0	1
0		0	1
1		1	0

K-Map Function Representation

- ▶ Example: $f(x, y) = x$

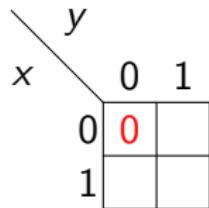
K-Map Function Representation

- ▶ Example: $f(x, y) = x$



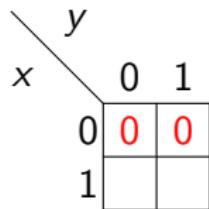
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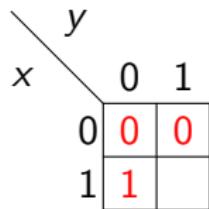
K-Map Function Representation

- ▶ Example: $f(x, y) = x$



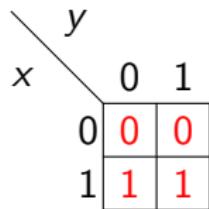
K-Map Function Representation

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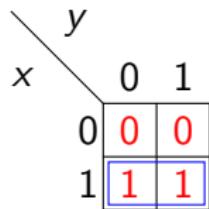


A Karnaugh map for two variables, x and y. The horizontal axis is labeled x and the vertical axis is labeled y. The axes intersect at the top-left corner of the map. The map consists of four cells arranged in a 2x2 grid. The top row is labeled 0 and the bottom row is labeled 1. The left column is labeled 0 and the right column is labeled 1. The top-left cell (0,0) contains the value 0. The top-right cell (0,1) contains the value 0. The bottom-left cell (1,0) contains the value 1. The bottom-right cell (1,1) contains the value 1. The values 0 and 1 are written in red.

	x	y
0	0	0
1	1	1

K-Map Function Representation

- ▶ Example: $f(x, y) = x$



K-Map Function Representation

- ▶ Example: $f(x, y) = x$

	y	
x		0 1
0		0 0
1		1 1

For function $f(x, y)$, the two adjacent cells containing 1's can be combined using the Minimization Theorem:

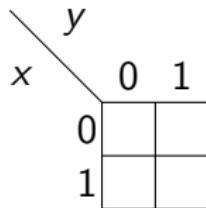
$$f(x, y) = xy' + xy = x$$

K-Map Function Representation

- ▶ Example: $g(x, y) = x + y$

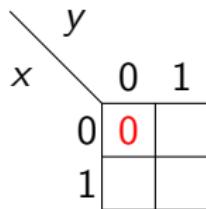
K-Map Function Representation

- ▶ Example: $g(x, y) = x + y$



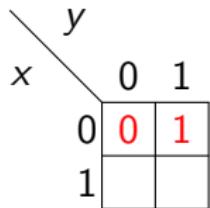
K-Map Function Representation

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K-Map Function Representation

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K-Map Function Representation

- ▶ Example: $g(x, y) = x + y$

	y
x	
0	0 1
1	1

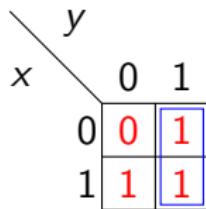
K-Map Function Representation

- ▶ Example: $g(x, y) = x + y$

	x	y
0	0	1
1	1	1

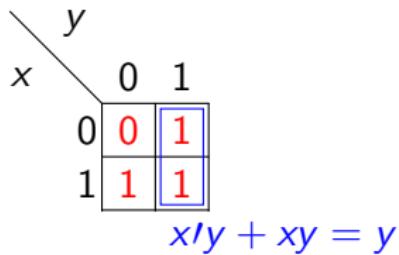
K-Map Function Representation

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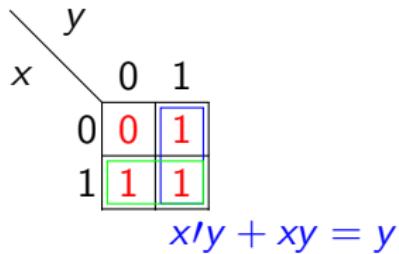
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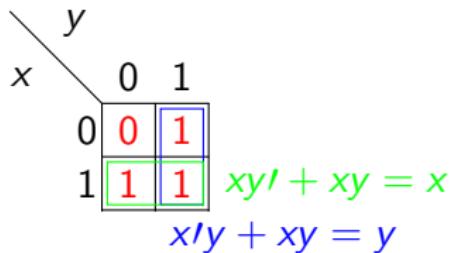
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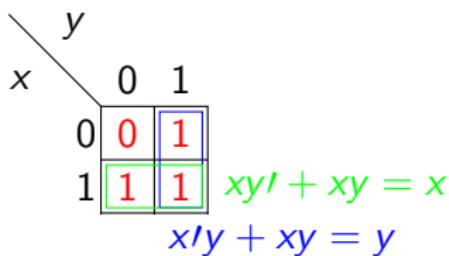
K-Map Function Representation

- ▶ Example: $g(x, y) = x + y$



K-Map Function Representation

- ▶ Example: $g(x, y) = x + y$



For function $g(x, y)$, the two adjacent cells containing 1's can be combined using the Minimization Theorem:

$$g(x, y) = (x'y + xy) + (xy' + xy) = x + y$$

Three Variable Karnaugh Map

Three variable: x, y, z

8 minterms:

m_0	$=$	$x'y'z'$	\rightarrow	000
m_1	$=$	$x'y'z$	\rightarrow	001
m_2	$=$	$x'y'z'$	\rightarrow	010
m_3	$=$	$x'y'z$	\rightarrow	011
m_4	$=$	$xy'z'$	\rightarrow	100
m_5	$=$	$xy'z$	\rightarrow	101
m_6	$=$	xyz'	\rightarrow	110
m_7	$=$	xyz	\rightarrow	111

		xy	00	01	11	10	
		z	0	m_0	m_1	m_3	m_2
		1	m_4	m_5	m_7	m_6	
0	0						
1	0						
1	1						
0	1						

Three Variable Karnaugh Map

Example:

x	y	z	f_1	f_2
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

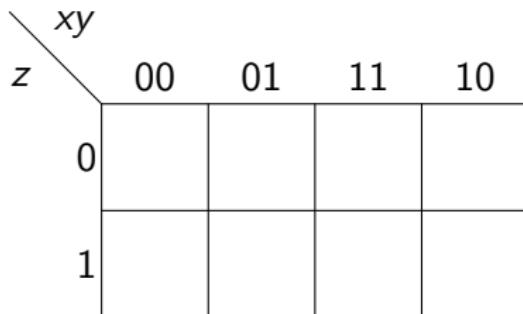
For f_1

xy	00	01	11	10
z	0	0	1	0
z	1	0	1	1

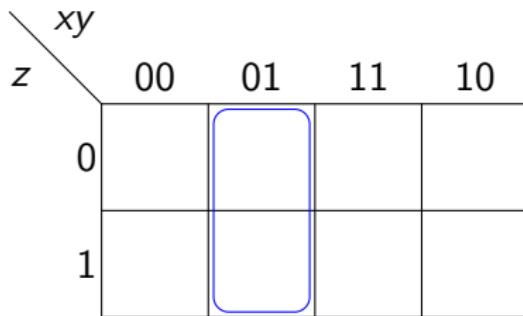
For f_2

xy	00	01	11	10
z	0	0	1	0
z	1	1	0	0

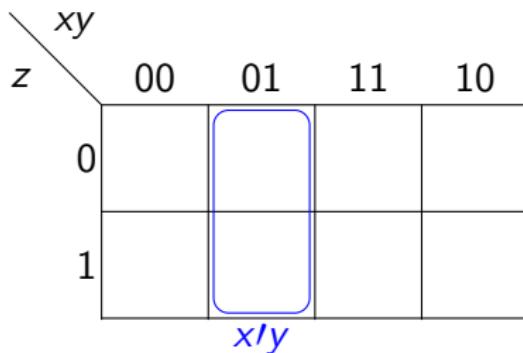
Three Variable Karnaugh Map



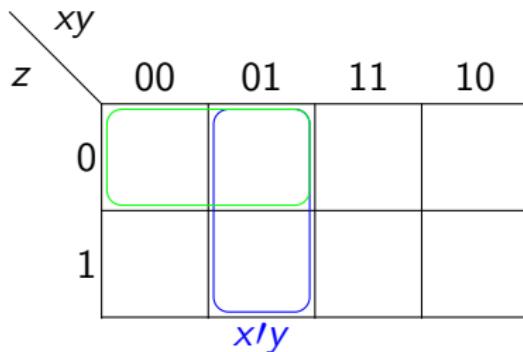
Three Variable Karnaugh Map



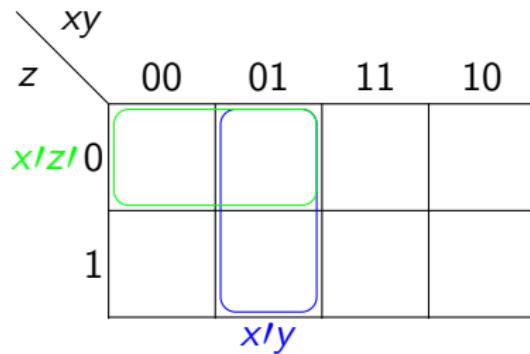
Three Variable Karnaugh Map



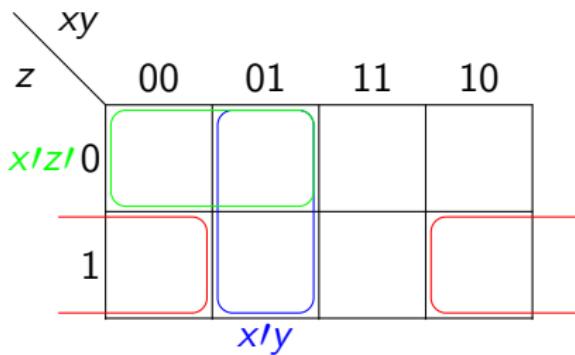
Three Variable Karnaugh Map



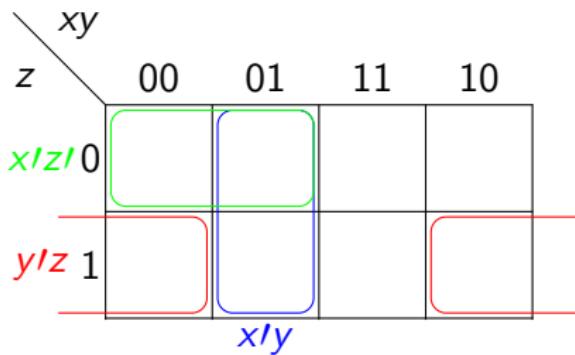
Three Variable Karnaugh Map



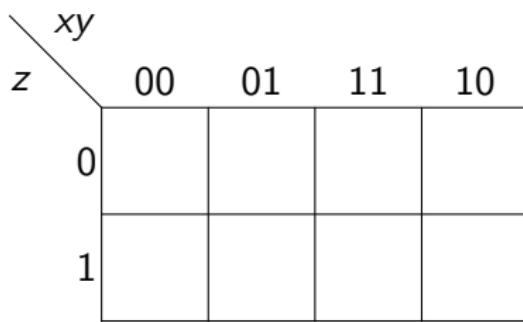
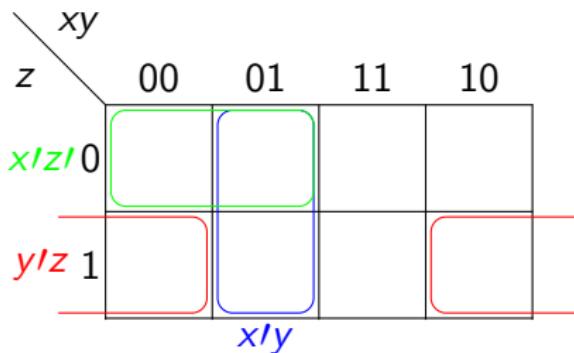
Three Variable Karnaugh Map



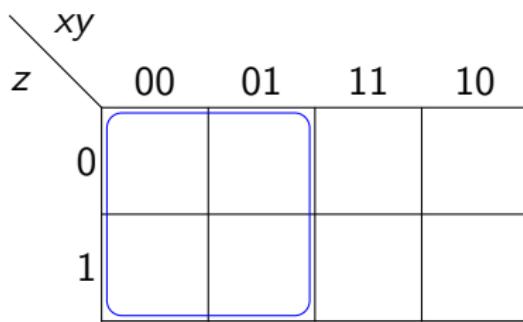
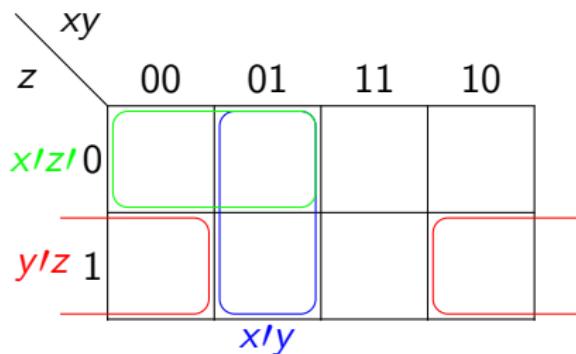
Three Variable Karnaugh Map



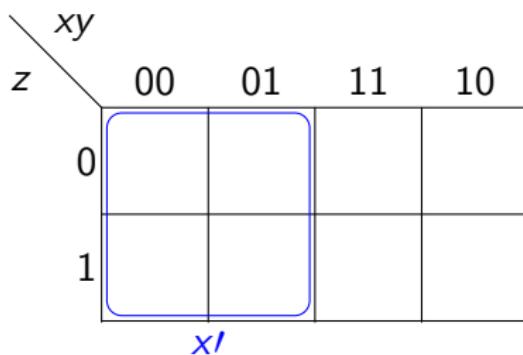
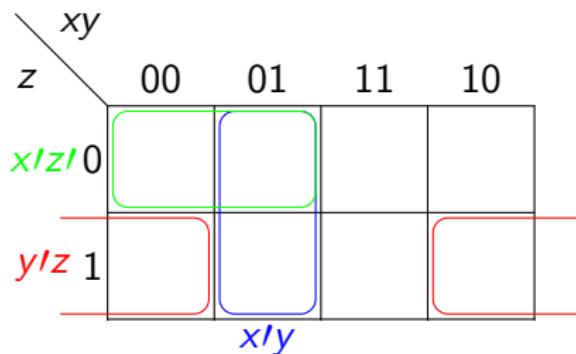
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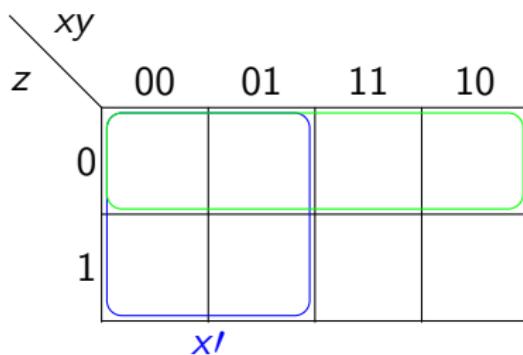
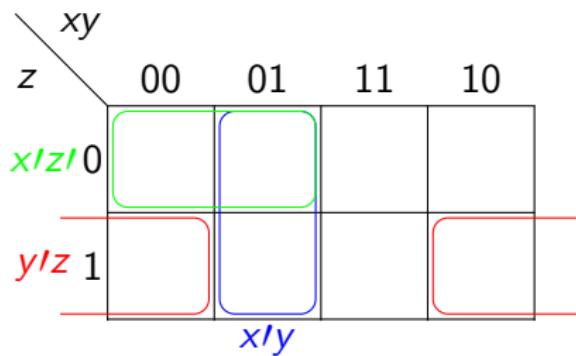
Three Variable Karnaugh Map



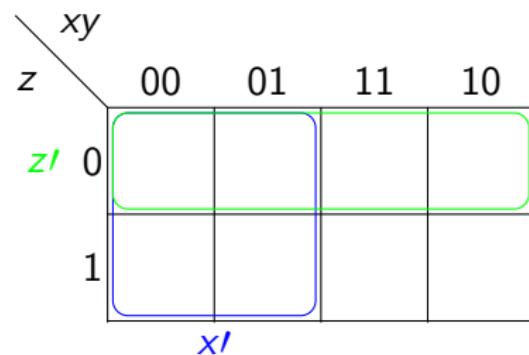
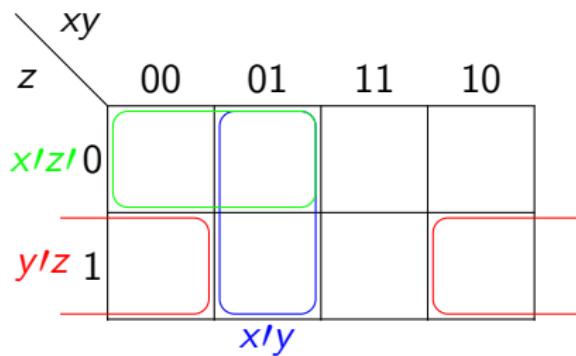
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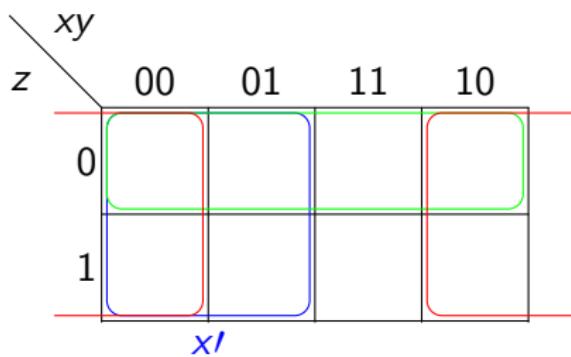
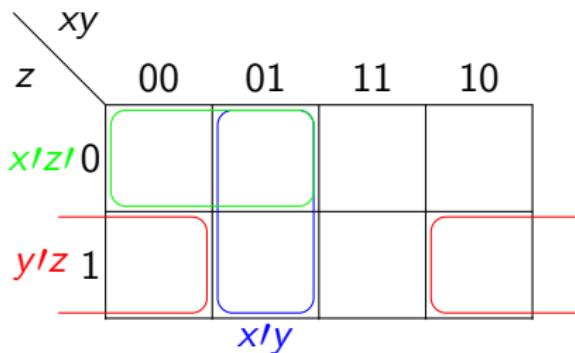
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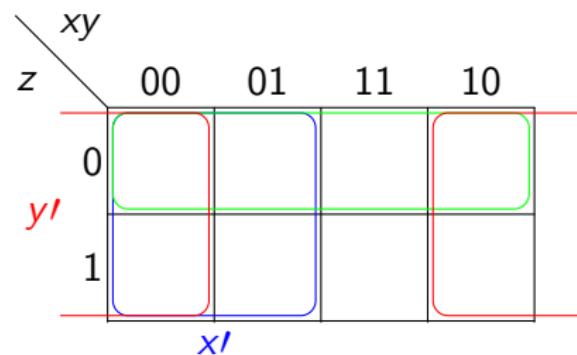
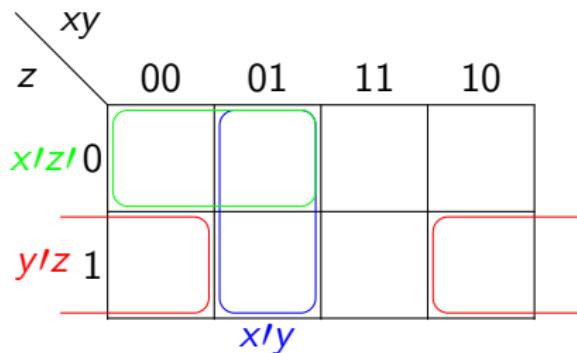
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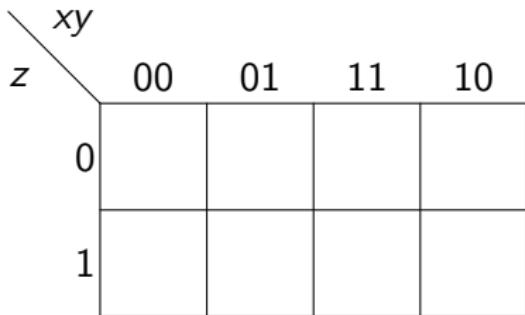
Three Variable Karnaugh Map



Three Variable Karnaugh Map



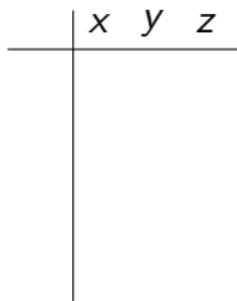
Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$



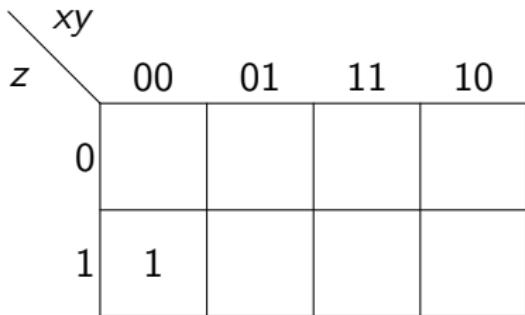
A Karnaugh map for three variables x , y , and z . The vertical axis is labeled z with values 0 and 1. The horizontal axis is labeled xy with values 00, 01, 11, and 10. The map consists of four columns and two rows of cells. The cells are labeled as follows:

$xy\backslash z$	0	1	0	1
0	0	1	0	1
1	0	1	0	1

The cells where the function value is 1 are shaded: (01, 0), (11, 0), (10, 0), and (11, 1).

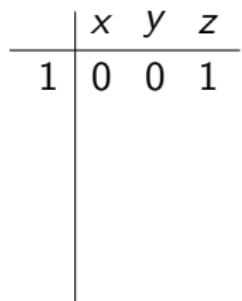


Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$



A Karnaugh map for three variables x , y , and z . The vertical axis is labeled z with values 0 and 1. The horizontal axis is labeled xy with values 00, 01, 11, and 10. The map shows minterms 1, 2, 3, 5, and 7. The cell at $(x=0, y=0, z=1)$ contains a 1, while all other cells are empty.

		00	01	11	10
		0			
z	0				
1	1				



A truth table for three variables x , y , and z . The first row is a header with columns x , y , z . The second row contains the minterm 1, with values 0, 0, 1 respectively.

	x	y	z
1	0	0	1

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

		xy		
		z	00	01
0				
1	0	1		
	1	1		

	x	y	z
1	0	0	1
2	0	1	0

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

		xy		
		z	00	01
0				
1	0	1		
	1	1		

	x	y	z
1	0	0	1
2	0	1	0

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

		xy				
		z	00	01	11	10
0				1		
1	0	1	1			
	1	1				

	x	y	z
1	0	0	1
2	0	1	0
3	0	1	1

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

		xy				
		z	00	01	11	10
0				1		
1	0		1			
	1		1			1

	x	y	z
1	0	0	1
2	0	1	0
3	0	1	1
5	1	0	1

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

		xy				
		z	00	01	11	10
0				1		
1	0		1			
	1		1			1

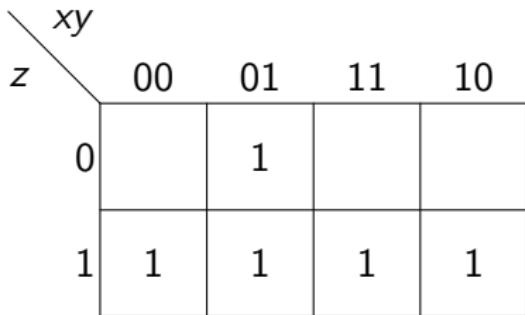
	x	y	z
1	0	0	1
2	0	1	0
3	0	1	1
5	1	0	1

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

		xy				
		z	00	01	11	10
0				1		
1			1	1	1	1

	x	y	z
1	0	0	1
2	0	1	0
3	0	1	1
5	1	0	1
7	1	1	1

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$



		xy				
		z	00	01	11	10
0				1		
1		1	1	1	1	1

	x	y	z		x	y	z
1	0	0	1				
2	0	1	0				
3	0	1	1				
5	1	0	1				
7	1	1	1				

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

		xy				
		z	00	01	11	10
0				1		
1			1	1	1	1

	x	y	z		x	y	z
✓ 1	0	0	1		1,3	0	- 1
2	0	1	0				
✓ 3	0	1	1				
5	1	0	1				
7	1	1	1				

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

		xy				
		z	00	01	11	10
0				1		
1			1	1	1	1

	x	y	z		x	y	z
✓ 1	0	0	1		1,3	0	- 1
2	0	1	0		1,5	-	0 1
✓ 3	0	1	1				
✓ 5	1	0	1				
7	1	1	1				

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

		xy				
		z	00	01	11	10
0				1		
1			1	1	1	1

	x	y	z		x	y	z
✓ 1	0	0	1		1,3	0	- 1
✓ 2	0	1	0		1,5	-	0 1
✓ 3	0	1	1		2,3	0	1 -
✓ 5	1	0	1				
7	1	1	1				

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

		xy				
		z	00	01	11	10
0				1		
1			1	1	1	1

	x	y	z		x	y	z
✓ 1	0	0	1		1,3	0	- 1
✓ 2	0	1	0		1,5	-	0 1
✓ 3	0	1	1		2,3	0	1 -
✓ 5	1	0	1		3,7	-	1 1
✓ 7	1	1	1				

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

		xy				
		z	00	01	11	10
0				1		
1			1	1	1	1

	x	y	z		x	y	z
✓ 1	0	0	1		1,3	0	- 1
✓ 2	0	1	0		1,5	-	0 1
✓ 3	0	1	1		2,3	0	1 -
✓ 5	1	0	1		3,7	-	1 1
✓ 7	1	1	1		5,7	1	- 1

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

		xy	z			
		00	01	11	10	
0			1			
1	0	1	1	1	1	
1	1					

	x	y	z		x	y	z		x	y	z
✓ 1	0	0	1		1,3	0	-	1			
✓ 2	0	1	0		1,5	-	0	1			
✓ 3	0	1	1		2,3	0	1	-			
✓ 5	1	0	1		3,7	-	1	1			
✓ 7	1	1	1		5,7	1	-	1			

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

		xy	z			
		00	01	11	10	
		0		1		
0						
1	1	1	1	1	1	1

	x	y	z		x	y	z		x	y	z
✓ 1	0	0	1	✓	1,3	0	-	1			
✓ 2	0	1	0		1,5	-	0	1			
✓ 3	0	1	1		2,3	0	1	-			
✓ 5	1	0	1		3,7	-	1	1			
✓ 7	1	1	1		5,7	1	-	1			

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

		xy	z			
		00	01	11	10	
0			1			
1	0	1	1	1	1	

x	y	z		x	y	z		x	y	z
✓ 1	0	0	1	✓ 1,3	0	-	1	1,3,5,7	-	-
✓ 2	0	1	0	1,5	-	0	1			
✓ 3	0	1	1	2,3	0	1	-			
✓ 5	1	0	1	3,7	-	1	1			
✓ 7	1	1	1	✓ 5,7	1	-	1			

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

		xy				
		z	00	01	11	10
0				1		
1			1	1	1	1

x	y	z		x	y	z		x	y	z	
✓ 1	0	0	1	✓ 1,3	0	-	1	1,3,5,7	-	-	1
✓ 2	0	1	0	✓ 1,5	-	0	1				
✓ 3	0	1	1	2,3	0	1	-				
✓ 5	1	0	1	3,7	-	1	1				
✓ 7	1	1	1	✓ 5,7	1	-	1				

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

		xy	z			
		00	01	11	10	
0			1			
1	0	1	1	1	1	

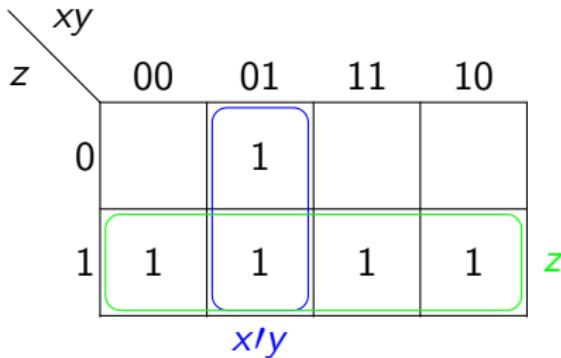
x	y	z		x	y	z		x	y	z	
✓ 1	0	0	1	✓ 1,3	0	-	1	1,3,5,7	-	-	1
✓ 2	0	1	0	✓ 1,5	-	0	1	1,5,3,7	-	-	1
✓ 3	0	1	1	2,3	0	1	-				
✓ 5	1	0	1	✓ 3,7	-	1	1				
✓ 7	1	1	1	✓ 5,7	1	-	1				

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$

		xy		
		z	00	01
0				1
1		1	1	1
		x/y		

	x	y	z		x	y	z		x	y	z
✓ 1	0	0	1	✓ 1,3	0	-	1	1,3,5,7	-	-	1
✓ 2	0	1	0	✓ 1,5	-	0	1	1,5,3,7	-	-	1
✓ 3	0	1	1	A 2,3	0	1	-				
✓ 5	1	0	1	✓ 3,7	-	1	1				
✓ 7	1	1	1	✓ 5,7	1	-	1				

Example: $f(x, y, z) = \Sigma_m(1, 2, 3, 5, 7)$



	x	y	z		x	y	z		x	y	z	
✓ 1	0	0	1	✓ 1,3	0	-	1	B	1,3,5,7	-	-	1
✓ 2	0	1	0	✓ 1,5	-	0	1		1,5,3,7	-	-	1
✓ 3	0	1	1	A 2,3	0	1	-					
✓ 5	1	0	1	✓ 3,7	-	1	1					
✓ 7	1	1	1	✓ 5,7	1	-	1					

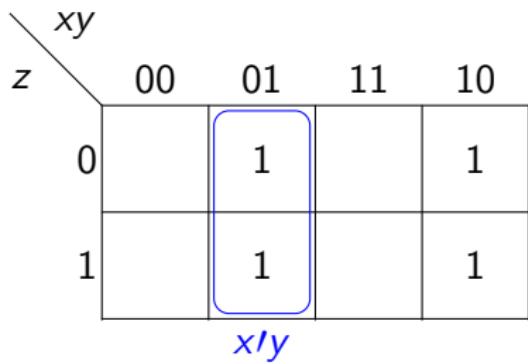
Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$$

		xy				
		z	00	01	11	10
0				1		1
1			1			1

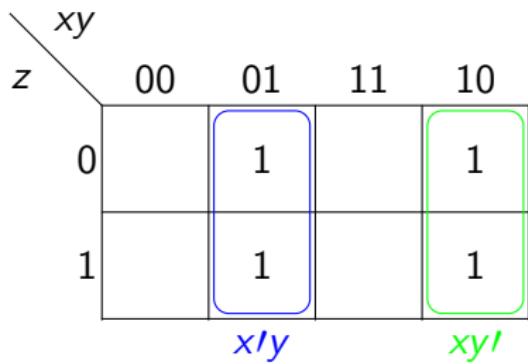
Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$$



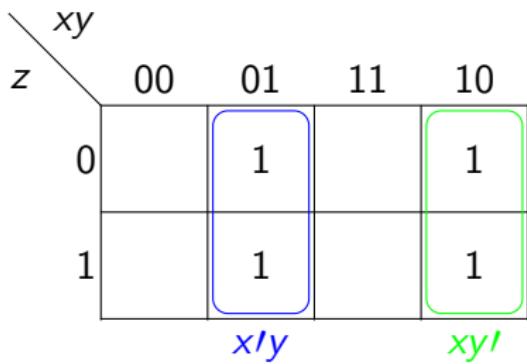
Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$$



Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$$

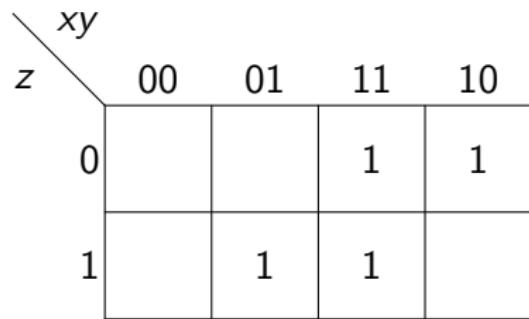
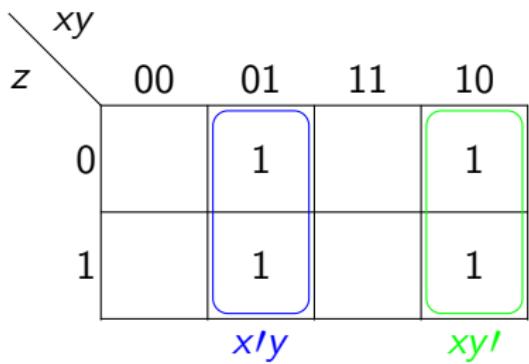


$$F_1(x, y, z) = x'y + xy'$$

Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$$

$$F_2(x, y, z) = \Sigma(3, 4, 6, 7)$$

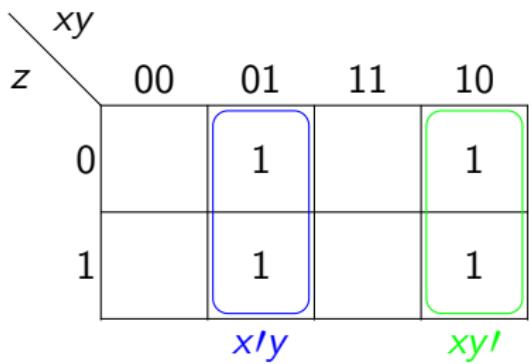


$$F_1(x, y, z) = x'y + xy'$$

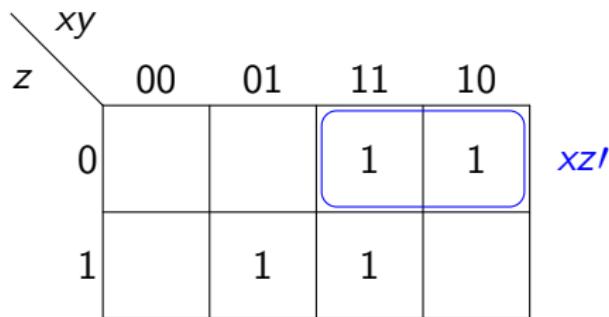
Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$$

$$F_2(x, y, z) = \Sigma(3, 4, 6, 7)$$



$$F_1(x, y, z) = x'y + xy'$$

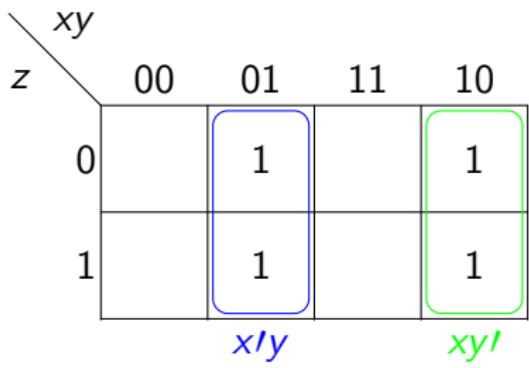


xz'

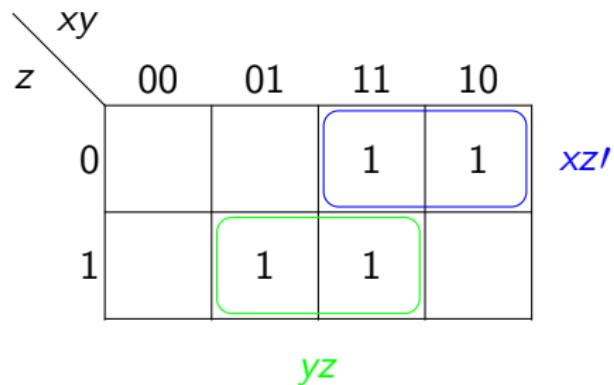
Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$$

$$F_2(x, y, z) = \Sigma(3, 4, 6, 7)$$



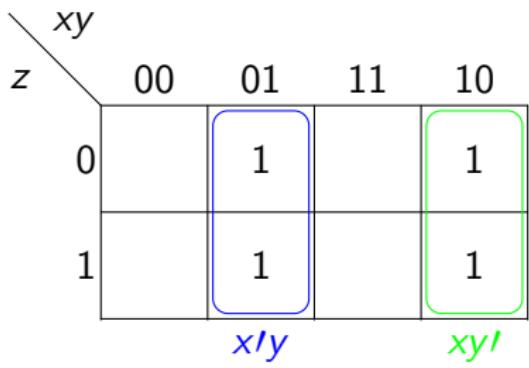
$$F_1(x, y, z) = x'y + xy'$$



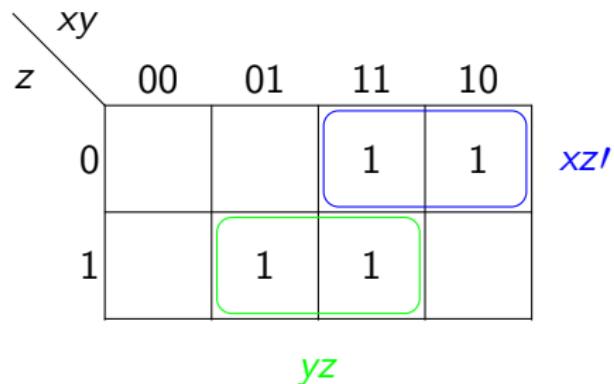
xz'

Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(2, 3, 4, 5)$$



$$F_2(x, y, z) = \Sigma(3, 4, 6, 7)$$



$$F_1(x, y, z) = x'y + xy'$$

$$F_2(x, y, z) = xz' + yz$$

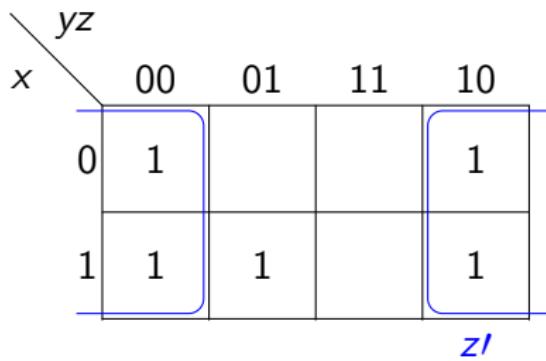
Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

		yz	00	01	11	10
		x	0			
		0	1			1
		1	1	1		1

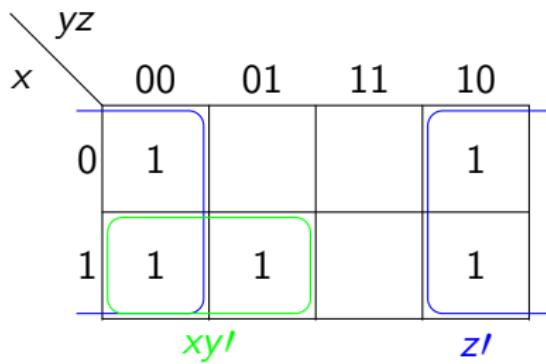
Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$



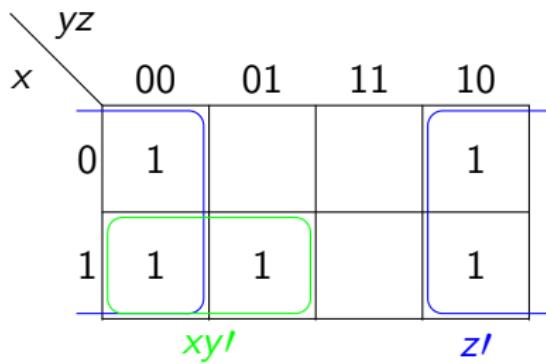
Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$



Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

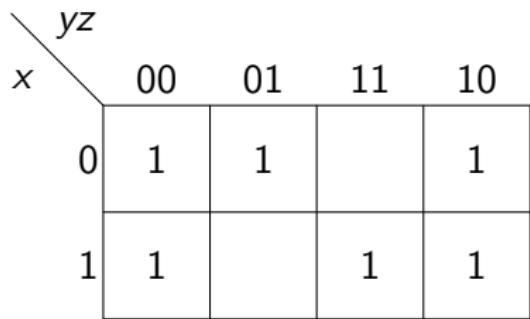
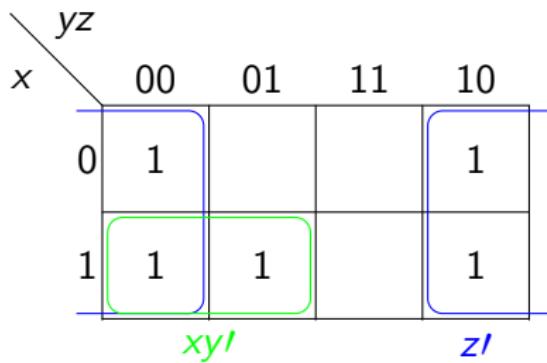


$$F_1(x, y, z) = z' + xy'$$

Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

$$F_2(x, y, z) = \Sigma(0, 1, 2, 4, 6, 7)$$

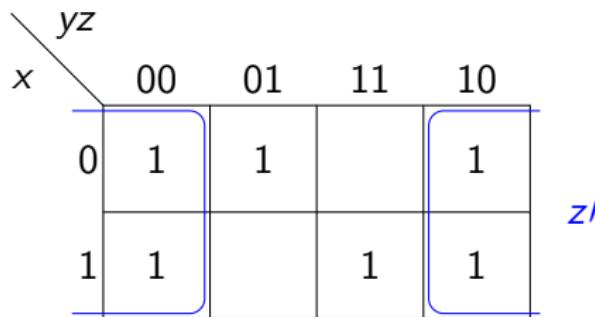
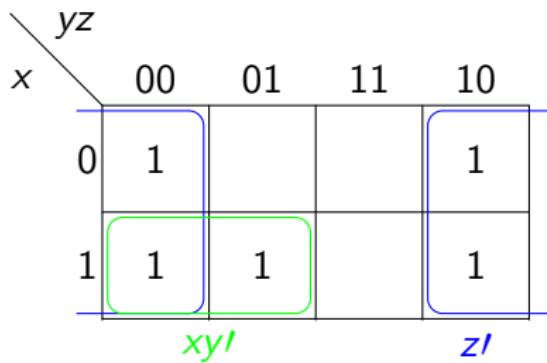


$$F_1(x, y, z) = z' + xy'$$

Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

$$F_2(x, y, z) = \Sigma(0, 1, 2, 4, 6, 7)$$

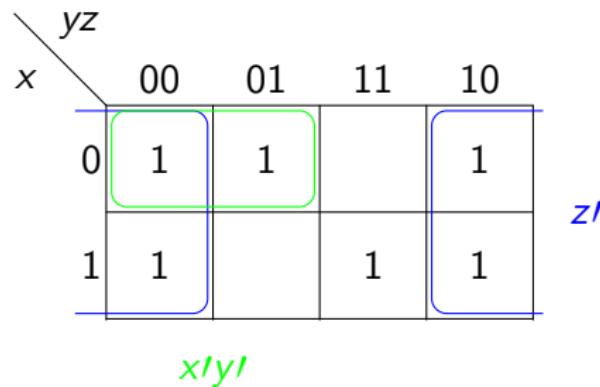
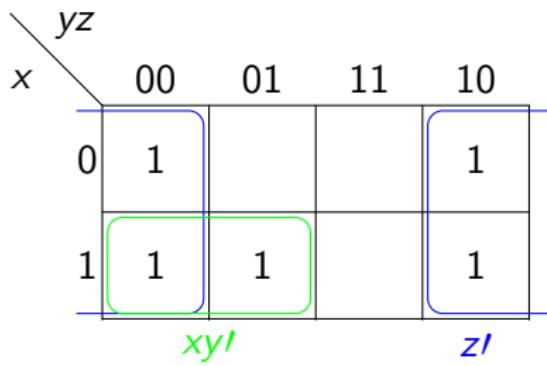


$$F_1(x, y, z) = z' + xy'$$

Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

$$F_2(x, y, z) = \Sigma(0, 1, 2, 4, 6, 7)$$

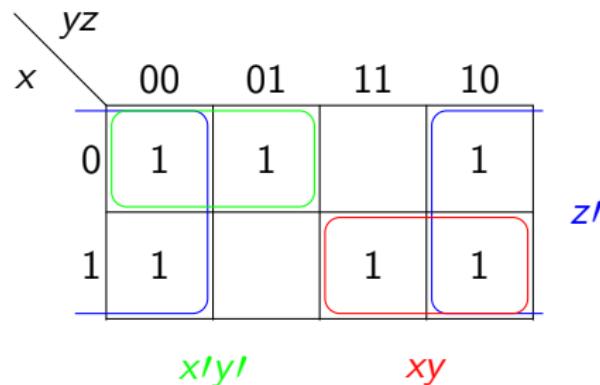
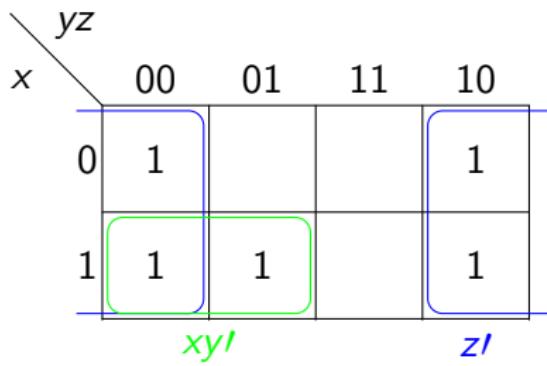


$$F_1(x, y, z) = z' + xy'$$

Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

$$F_2(x, y, z) = \Sigma(0, 1, 2, 4, 6, 7)$$

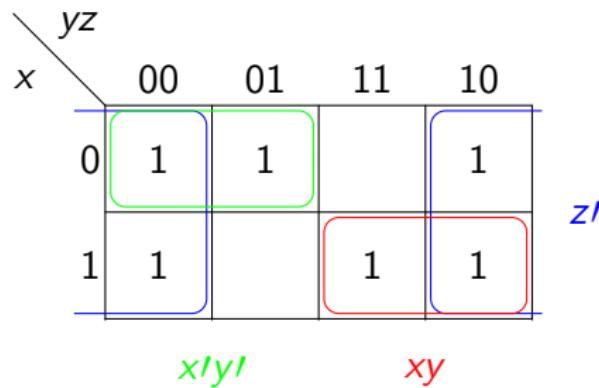
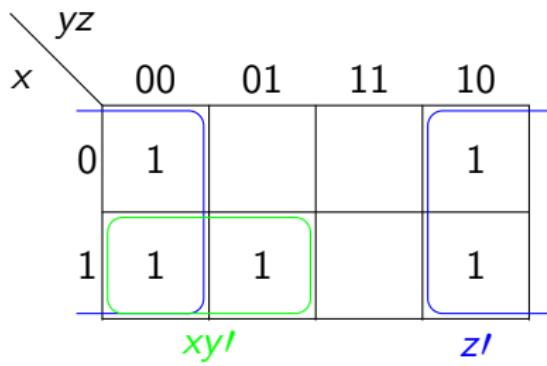


$$F_1(x, y, z) = z' + xy'$$

Example: Three Variable Karnaugh Map

$$F_1(x, y, z) = \Sigma(0, 2, 4, 5, 6)$$

$$F_2(x, y, z) = \Sigma(0, 1, 2, 4, 6, 7)$$



$$F_1(x, y, z) = z' + xy'$$

$$F_2(x, y, z) = z' + xy' + xy$$

Four Variable Karnaugh Map

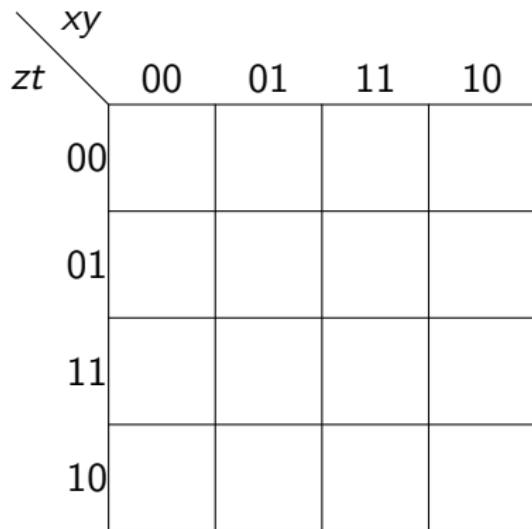
Four variable: x, y, z, t

16 minterms:

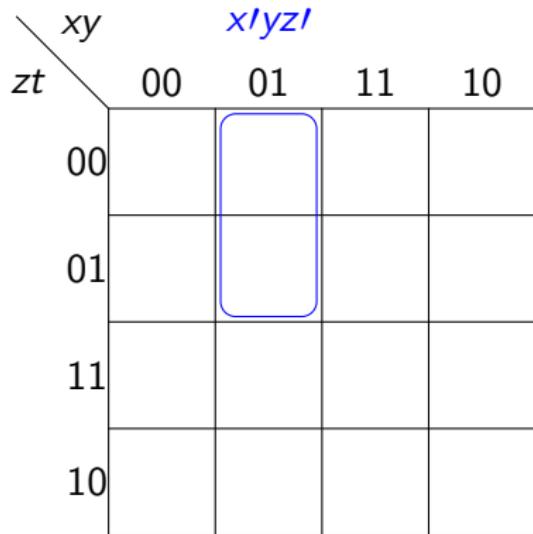
m_0	$= x'y'z't'$	$\rightarrow 0000$
m_1	$= x'y'z't$	$\rightarrow 0001$
m_2	$= x'y'zt'$	$\rightarrow 0010$
m_3	$= x'y'zt$	$\rightarrow 0011$
m_4	$= x'yz't'$	$\rightarrow 0100$
m_5	$= x'yz't$	$\rightarrow 0101$
m_6	$= x'yzt'$	$\rightarrow 0110$
m_7	$= x'yzt$	$\rightarrow 0111$
m_8	$= xy'z't'$	$\rightarrow 1000$
m_9	$= xy'z't$	$\rightarrow 1001$
m_{10}	$= xy'zt'$	$\rightarrow 1010$
m_{11}	$= xy'zt$	$\rightarrow 1011$
m_{12}	$= xyz't'$	$\rightarrow 1100$
m_{13}	$= xyz't$	$\rightarrow 1101$
m_{14}	$= xyzt'$	$\rightarrow 1110$
m_{15}	$= xyzt$	$\rightarrow 1111$

		xy	00	01	11	10
zt			m_0	m_1	m_3	m_2
	00		m_4	m_5	m_7	m_6
	01		m_{12}	m_{13}	m_{15}	m_{14}
	11		m_8	m_9	m_{11}	m_{10}
	10					

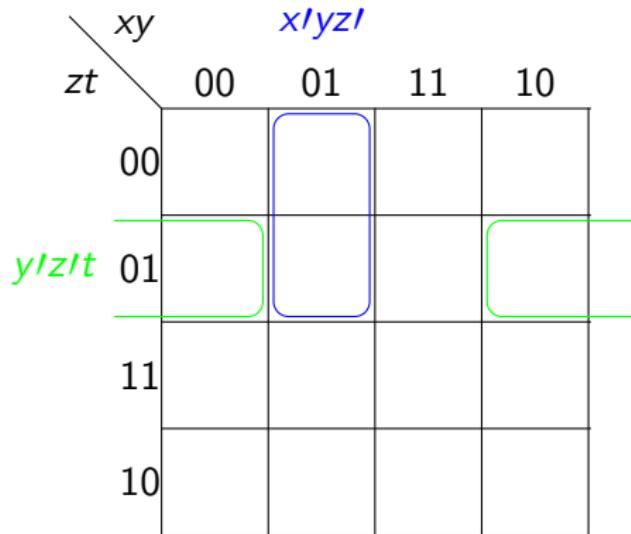
Four Variable Karnaugh Map



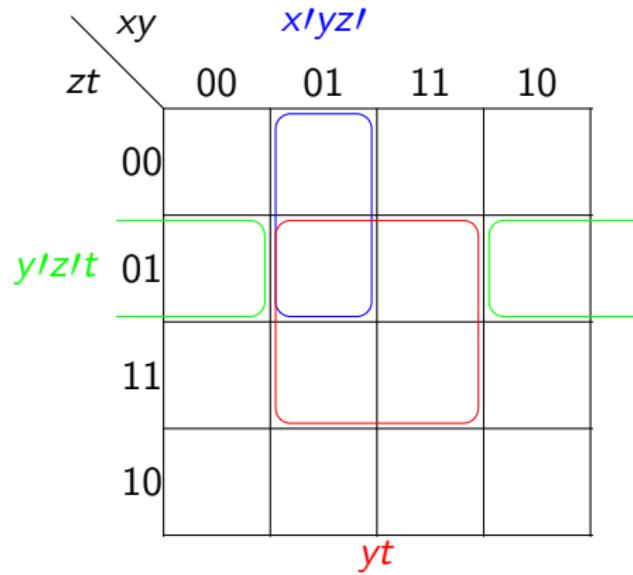
Four Variable Karnaugh Map



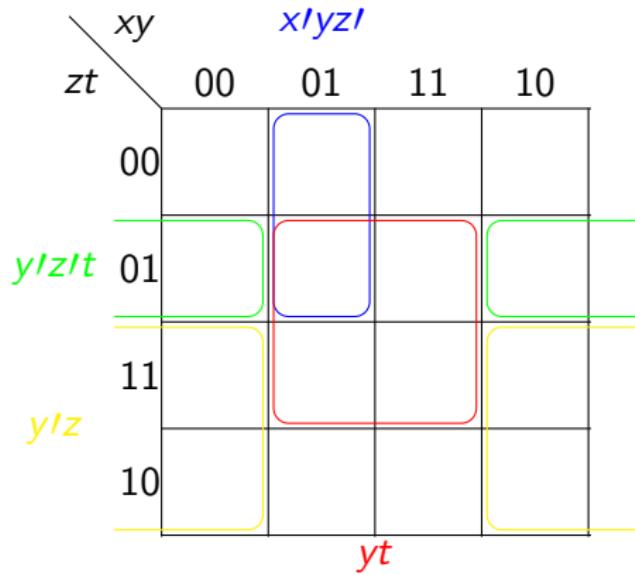
Four Variable Karnaugh Map



Four Variable Karnaugh Map



Four Variable Karnaugh Map



Four Variable Karnaugh Map

