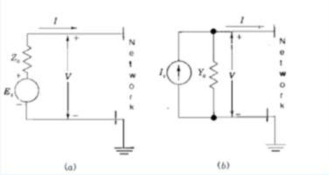
ELK322E
POWER TRANSMISSION SYSTEMS

Chapter 6.x
Bus Admittance Matrix and Load Flow Analysis

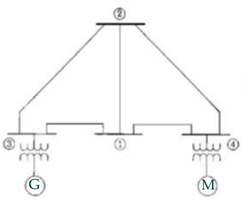
Bus Admittance Matrix

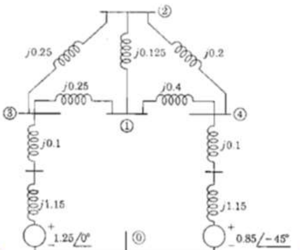
Thevenin's theorem states that any circuit composed of linear elements can be simplified to a single voltage source and a single series impedance. Norton's theorem is the same except that the voltage source and series impedance are replaced by a current source and parallel admittance. It is easy to convert a Thevenin equivalent to a Norton equivalent and vice versa, when it is required.

$$I_s = \frac{E_s}{Z_a} \quad \text{and} \quad Y_a = \frac{1}{Z_a}$$



Exampale: The single-line diagram of a small power system is shown in Fig. below. The corresponding reactance diagram, with reactances specified in per unit, is shown in the next figure. A generator with emf equal to 1.25∟0º per unit is connected through a transformer to high voltage node 3), while a motor with internal voltage equal to 0.85 ∟45º is similarly connected to node 4. Develop the nodal admittance matrix for each of the network branches and then write the nodal admittance equations of the system .

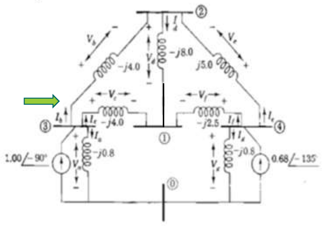




reactance diagram

Using conversion from Thevenin's equivalent circuit into Norton equivalent circuit:

Corresponding admittance diagram



the overall network nodal admittance equations

$$\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \underbrace{\begin{bmatrix} -j14.5 & j8.0 & j4.0 & j2.5 \\ j8.0 & -j17.0 & j4.0 & j5.0 \\ j4.0 & j4.0 & -j8.8 & 0.0 \\ j2.5 & j5.0 & 0.0 & -j8.3 \end{bmatrix}}_{\text{Bus admittance Matrice, } Y_{BUS}} \end{array} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1.00\angle -90^\circ \\ 0.68\angle -135^\circ \end{bmatrix}$$

An Introduction to Load Flow Studies

We should be able to analyze the performance of power systems both in normal operating conditions and under faulted conditions.

The analysis in normal steady-state operation is called a **power-flow study** (**load-flow study**) and it targets on determining **the voltages at all nodes**, **currents,** and **real and reactive power flows** in a system under a given load conditions.

Each bus in a power network can be classified to one of three types

i. **Load bus-PQ:** a bus at which the real and reactive power are specified, and for which the bus voltage will need to be calculated. All busses having no generators are load busses.

ii. Generator bus-PV bus: a bus at which the magnitude of the voltage is kept constant by adjusting the field current of a synchronous generator on the bus. We assume that the field current is adjusted to maintain a constant terminal voltage at the generator buses.

We also know that increasing the prime mover's governor set points increases the power that generator supplies to the power system. Therefore, we can control and specify the magnitude of the bus voltage and real power supplied.

Load Flow Studies

iii. Slack bus (reference): This is a special generator bus serving as the reference bus for the power system. Its voltage is assumed to be fixed in both magnitude and phase (for instance, $1\angle 0^\circ$ pu).

The real and reactive powers are uncontrolled: the bus supplies whatever real or reactive power is necessary to make the power flows in the system balance.

In practice, a voltage on a load bus may change with changing loads. **Therefore, for load busses the values of P and Q are known, while the bus voltage varies as the load changes.**

Most busses with generators will supply a fixed amount of power and the magnitude of their voltages will be maintained constant by field circuits of generators. These busses have specific values of P and $|V_i|$.

The controls on the swing generator will be set up to maintain a constant voltage and frequency, allowing P and Q to increase or decrease as loads change.

Load Flow Studies

As a result, before the load flow analysis

Buses	Known Variables	Unknown Variables
Generator	Real Power Voltage Magnitude	Reactive Power Voltage Angle
Load	Real Power Reactive Power	Voltage Magnitude Voltage Angle

At the end of load flow studies, for each buses P, Q, V and δ would be known. Using these variables, real & reactive power flow and line loses will be found out for each power line for an operation conditions.

The simplest way to perform power-flow calculations is by iteration:

1. Create a bus admittance matrix $\mathbf{Y}_{bus}$ for the network given,

2. Make an initial estimate for the voltages at each bus,

3. Update the voltage estimate for each bus, based on the estimates for the voltages and power flows at every other bus and the values of the bus admittance matrix:
Since the voltage at a given bus depends on the voltages at all of the other busses in the system, the updated voltage will not be correct. However, it will usually be closer to the answer than the original guess.

4. Repeat this process to make the voltages at each bus approaching the correct answers closer and closer…

Load Flow Studies

Example: a simple power system has 4 busses, 5 transmission lines, 1 generator, and 3 loads. Series per-unit impedances are:

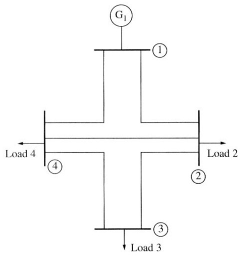


Table of Busses:

Bus 1	Slack bus
Bus 2	Load bus
Bus 3	Load bus
Bus 4	Load bus

Load Flow Studies

Line s	Bus to bus	Series Z (pu)	Equivalent Y (pu)
1	1-2	0.1+j0.4	0.5882-j2.3529
2	2-3	0.1+j0.5	0.3846-j1.9231
3	2-4	0.1+j0.4	0.5882-j2.3529
4	3-4	0.5+j0.2	1.1765-j4.7059
5	4-1	0.5+j0.2	1.1765-j4.7059

Load Flow Studies

The shunt admittances of the transmission lines are ignored. In this case, the Y_{ii} terms of the bus admittance matrix can be constructed by summing the admittances of all transmission lines connected to each bus, and the Y_{ij} $(i \neq j)$ terms are just the negative of the line admittances connected between busses *i* and *j*.

Therefore, for instance, the term Y_{11} will be the sum of the admittances of all transmission lines connected to bus 1, which are the lines 1 and 5, so Y_{11} = 1.7647 – j7.0588 pu.

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Load Flow Studies

If the shunt admittances of the transmission lines are not ignored, the self admittance Y_{ii} at each bus would also include half of the shunt admittance of each transmission line connected to the bus.

The term Y_{12} will be the negative of all the admittances stretching between bus 1 and bus 2, which will be the negative of the admittance of transmission line 1, therefore Y_{12} = -0.5882 + j2.3529.

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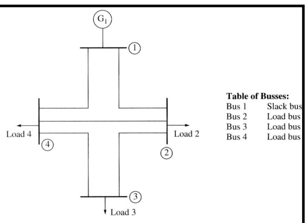
Load Flow Studies

The complete bus admittance matrix of the network can be obtained by repeating these calculations for every term in the matrix.



$$Y_{bus} = \begin{bmatrix} 1.7647-j7.0588 & -0.5882+j2.3529 & 0 & -1.1765+j4.7059 \\ -0.5882+j2.3529 & 1.5611-j6.6290 & -0.3846+j1.9231 & -0.5882+j2.3529 \\ 0 & -0.3846+j1.9231 & 1.5611-j6.6290 & -1.1765+j4.7059 \\ -1.1765+j4.7059 & -0.5882+j2.3529 & -1.1765+j4.7059 & 2.9412-j11.7647 \end{bmatrix}$$

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Load Flow Equations

The basic equation for power-flow analysis is derived from the nodal analysis equations for the power system:

$$Y_{bus}V = I$$

For the four-bus power system shown above becomes

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

where Y_{ij} are the elements of the bus admittance matrix, V_i are the bus voltages, and I_i are the currents injected at each node. For bus 2 in this system, this equation reduces to

Load Flow Equations

$$Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4 = I_2$$

However, real loads are specified in terms of real and reactive powers, not as currents. The relationship between per-unit real and reactive power supplied to the system at a bus and the per-unit current injected into the system at that bus is:

$$S = VI^* = P + jQ$$

where *V* is the per-unit voltage at the bus; *I** - complex conjugate of the per-unit current injected at the bus; P and Q are per-unit real and reactive powers. Therefore, for instance, the current injected at bus 2 can be found as

$$V_2 I_2^* = P_2 + jQ_2 \;\Rightarrow\; I_2^* = \frac{P_2 + jQ_2}{V_2} \;\Rightarrow\; I_2 = \frac{P_2 + jQ_2}{V_2^*}$$

Load Flow Equations

Substituting into we obtain

$$Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4 = \frac{P_2 + jQ_2}{V_2^*}$$

Solving the last equation for V_2, yields

$$V_2 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - (Y_{21}V_1 + Y_{23}V_3 + Y_{24}V_4)\right]$$

Similar equations can be created for each load bus (bus 3 and bus 4) in the power system

Load Flow Equations

Above equation gives updated estimate for V_2 based on the specified values of real and reactive powers and the current estimates of all the bus voltages in the system.

Note that the updated estimate for V_2 will not be the same as the original estimate of V_2^* used in the equation to derive it. We can repeatedly update the estimate wile substituting current estimate for V_2 back to the equation.

The values of V_2 will converge; however, this would not be the correct bus voltage since voltages at the other nodes are also needed to be updated. Therefore, all voltages need to be updated during each iteration!

The iterations are repeated until voltage values no longer change much between iterations.

Load Flow Analysis: Gauss-Siedel iterative method

The basic procedure is:

1. **Calculate the bus admittance matrix Y_{bus} including the admittances of all transmission lines, transformers, etc., between busses but exclude the admittances of the loads or generators themselves.**
2. **Select a slack bus: one of the busses in the power system, whose voltage will arbitrarily be assumed as $1.0\angle 0^\circ$.**
3. **Select initial estimates for all bus voltages: usually, the voltage at every load bus assumed as $1.0\angle 0^\circ$ (flat start) lead to good convergence.**
4. **Write voltage equations for every other bus in the system. The generic form is**

Load Flow Equations

$$V_i = \frac{1}{Y_{ii}}\left(\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k\neq i}}^{N} Y_{ik}V_k\right)$$

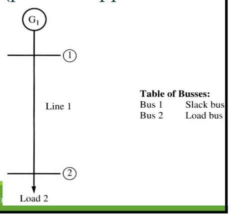
Calculate an updated estimate of the voltage at each load bus in succession using above equation except for the slack bus.

6. Compare the differences between the old and new voltage estimates: if the differences are less than some specified tolerance for all busses, stop. Otherwise, repeat step 5.
7. Confirm that the resulting solution is reasonable: a valid solution typically has bus voltages, whose phases range in less than 45˚.

Example (Load flow analysis): in a 2-bus power system, a generator attached to bus 1 and loads attached to bus 2.

The series impedance of a single transmission line connecting them is 0.1+j0.5 pu. The shunt admittance of the line may be neglected.
Assume that bus 1 is the slack bus and that it has a voltage $V_1 = 1.0\angle 0^\circ$ pu.
Real and reactive powers supplied to the loads from the system at bus 2 are P_2 = 0.3 pu, Q_2 = 0.2 pu (powers supplied to the system at each busses is negative of the above values).

Determine voltages at each bus for the specified load Conditions.



1. We start from calculating the bus admittance matrix Y_{bus}. The Y_{ii} terms can be constructed by summing the admittances of all transmission lines connected to each bus, and the Y_{ij} terms are the negative of the admittances of the line stretching between busses *i* and *j*. For instance, the term Y_{11} is the sum of the admittances of all transmission lines connected to bus 1 (a single line in our case). The series admittance of line 1 is

$$Y_{line1} = \frac{1}{Z_{line1}} = \frac{1}{0.1 + j0.5} = 0.3846 - j1.9231 = Y_{11}$$

Applying similar calculations to other terms, we complete the admittance matrix as

$$Y_{bus} = \begin{bmatrix} 0.3846 - j1.9231 & -0.3846 + j1.9231 \\ -0.3846 + j1.9231 & 0.3846 - j1.9231 \end{bmatrix}$$

2. Next, we select bus 1 as the slack bus since it is the only bus in the system connected to the generator. The voltage at bus 1 will be assumed $1.0\angle 0^\circ$.

3. We select initial estimates for all bus voltages. Making a flat start, the initial voltage estimates at every bus are $1.0\angle 0^\circ$.

4. Next, we write voltage equations for every other bus in the system. For bus 2:

$$V_2 = \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2}{V_{2,old}^*} - Y_{21} V_1 \right]$$

Since the real and reactive powers supplied to the system at bus 2 are P_2 = -0.3 pu and Q_2 = -0.2 pu and since *Ys* and V_1 are known, we may reduce the last equation:

$$V_2 = \frac{1}{0.3846 - j1.9231}\left[\frac{-0.3 - j0.2}{V_{2,old}^*} - \left((-0.3846 + j1.9231)V_1\right)\right]$$
$$= \frac{1}{1.9612\angle -78.8°}\left[\frac{0.3603\angle -146.3°}{V_{2,old}^*} - (1.9612\angle 101.3°)(1\angle 0°)\right]$$

5. Next, we calculate an updated estimate of the voltages at each load bus in succession. In this problem we only need to calculate updated voltages for bus 2, since the voltage at the slack bus (bus 1) is assumed constant. We repeat this calculation until the voltage converges to a constant value.

The initial estimate for the voltage is $V_{2,0} = 1\angle 0°$. The next estimate for the voltage is

$$V_{2,1} = \frac{1}{1.9612\angle -78.8°}\left[\frac{0.3603\angle -146.3°}{V_{2,old}^*} - (1.9612\angle 101.3°)(1\angle 0°)\right]$$
$$= \frac{1}{1.9612\angle -78.8°}\left[\frac{0.3603\angle -146.3°}{1\angle 0°} - (1.9612\angle 101.3°)\right]$$
$$= 0.8797\angle -8.499°$$

This new estimate for V_2 substituted back to the equation will produce the second estimate:

$$V_{2,2} = \frac{1}{1.9612\angle -78.8°}\left[\frac{0.3603\angle -146.3°}{0.8797\angle -8.499°} - (1.9612\angle 101.3°)(1\angle 0°)\right]$$
$$= 0.8412\angle -8.499°$$

The third iteration will be

$$V_{2,3} = \frac{1}{1.9612\angle -78.8°}\left[\frac{0.3603\angle -146.3°}{0.8412\angle -8.499°} - (1.9612\angle 101.3°)(1\angle 0°)\right]$$
$$= 0.8345\angle -8.962°$$

The fourth iteration will be

$$V_{2,4} = \frac{1}{1.9612\angle -78.8°}\left[\frac{0.3603\angle -146.3°}{0.8345\angle -8.962°} - (1.9612\angle 101.3°)(1\angle 0°)\right]$$
$$= 0.8320\angle -8.962°$$

The fifth iteration will be

$$V_{2,5} = \frac{1}{1.9612\angle -78.8°}\left[\frac{0.3603\angle -146.3°}{0.8320\angle -8.962°} - (1.9612\angle 101.3°)(1\angle 0°)\right]$$
$$= 0.8315\angle -8.994°$$

6. We observe that the magnitude of the voltage is barely changing and may conclude that this value is close to the correct answer and, therefore, stop the iterations.

This power system converged to the answer in five iterations. The voltages at each bus in the power system are:

$$V_1 = 1.0\angle 0°$$
$$V_2 = 0.8315\angle -8.994°$$

Finally, we need to confirm that the resulting solution is reasonable. The results seem reasonable since the phase angles of the voltages in the system differ by only 10˚. The current flowing from bus 1 to bus 2 is

$$I_1 = \frac{V_1 - V_2}{Z_{line1}} = \frac{1\angle 0° - 0.8315\angle -8.994°}{0.1 + j0.5} = 0.4333\angle -42.65°$$

The power supplied by the transmission line to bus 2 is

$$S = VI^* = (0.8315\angle -8.994°)(0.4333\angle -42.65°)^* = 0.2999 + j0.1997$$

This is the amount of power consumed by the loads; therefore, this solution appears to be correct.

Note that this example must be interpreted as follows: if the real and reactive power supplied by bus 2 is 0.3 + j0.2 pu and if the voltage on the slack bus is 1∠0˚ pu, then the voltage at bus 2 will be V_2 = 0.8315∠-8.994˚.

This voltage is correct only for the assumed conditions; another amount of power supplied by bus 2 will result in a different voltage V_2.

Therefore, we usually postulate some reasonable combination of powers supplied to loads, and determine the resulting voltages at all the busses in the power system. Once the voltages are known, currents through each line can be calculated.

The relationship between voltage and current at a load bus as given by

$$V_i = \frac{1}{Y_{ii}}\left(\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k\neq i}}^{N} Y_{ik}V_k\right)$$

is fundamentally nonlinear! Therefore, solution greatly depends on the initial conditions.

At a generator bus, the real power P_i and the magnitude of the bus voltage $|V_i|$ are specified. Since the reactive power for that bus is usually unknown, we need to estimate it before applying above equation to get updated voltage estimates. The values of reactive power at the generator bus can be estimated by solving above equation for Q_i:

$$V_i = \frac{1}{Y_{ii}}\left(\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k\neq i}}^{N} Y_{ik}V_k\right) \Leftrightarrow P_i - jQ_i = V_i^*\left(Y_{ii}V_i - \sum_{\substack{k=1 \\ k\neq i}}^{N} Y_{ik}V_k\right)$$

Bringing the case $k = I$ into summation, we obtain

$$P_i - jQ_i = V_i^* \sum_{k=1}^{N} Y_{ik} V_k \Rightarrow Q_i = -\mathrm{Im}\left\{ V_i^* \sum_{k=1}^{N} Y_{ik} V_k \right\}$$

Once the reactive power at the bus is estimated, we can update the bus voltage at a generator bus using P_i and Q_i as we would at a load bus.

However, the magnitude of the generator bus voltage is also forced to remain constant. Therefore, we must multiply the new voltage estimate by the ratio of magnitudes of old to new estimates.

Therefore, the steps required to update the voltage at a generator bus are:

1. Estimate the reactive power Q_i according to

$$P_i - jQ_i = V_i^* \sum_{k=1}^{N} Y_{ik} V_k \Rightarrow Q_i = -\mathrm{Im}\left\{ V_i^* \sum_{k=1}^{N} Y_{ik} V_k \right\}$$

2. Update the estimated voltage at the bus according to

as if the bus was a load bus; $$V_i = \frac{1}{Y_{ii}} \left(\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{N} Y_{ik} V_k \right)$$

3. Force the magnitude of the estimated voltage to be constant by multiplying the new voltage estimate by the ratio of the magnitude of the original estimate to the magnitude of the new estimate. This has the effect of updating the voltage phase estimate without changing the voltage amplitude.

Example: a 4-bus power system with 5 transmission lines, 2 generators, and 2 loads. Since the system has generators connected to 2 busses, it will have one slack bus, one generator bus, and two load busses.

Assume that bus 1 is the slack bus and that it has a voltage V_1 = 1.0∠0° pu. Bus 3 is a generator bus. The generator is supplying a real power P_3 = 0.3 pu to the system with a voltage magnitude 1 pu.

The per-unit real and reactive power loads at busses 2 and 4 are P_2 =0.3 pu, Q_2 = 0.2 pu, P_4 = 0.2 pu, Q_4 = 0.15 pu (powers supplied to the system at each busses are negative of the above values)..

Determine voltages at each bus for the specified load conditions.

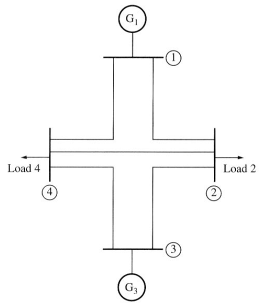


Table of Buses:

Bus 1	Slack bus
Bus 2	Load bus
Bus 3	Generator bus
Bus 4	Load bus

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