Probability and Statistics
HW #2
Due 24.12.2023

-The students are obligated to do the homework themselves.
-The copied homework will not be considered or graded.
-Homework should be ready before 23.00 on the Ninova in electronic form.

1) There are 2 Blue, 4 Red and 6 Yellow balls in a box. Two balls are selected at random from a box containing 2 Blue, 4 Red and 6 Yellow balls. Consider random variables X and Y as, respectively, the numbers of Blue and Red balls.

    (a) Find the probabilities associated with all possible pairs of values of X and Y, obtain the joint probability distribution function $f(x,y)$. Draw a probability table corresponding to the values of X, Y.

    (b) Find the joint cumulative distribution function $F(1,1)$.

    (c) Find the marginal distributions $g(x)$ and $h(y)$ for each x, y within the range of X and Y. Re-draw the table with marginal values.

    (d) Find the conditional distribution of X given $Y = 1$.

2) Consider the PDF of a random variable X given below;

$$f(x) = \begin{cases} 2(1-x) & 0 \le x \le 1 \\ 0 & otherwise \end{cases}.$$

    (a) Verify that the given function is a valid PDF.

    (b) Calculate the standard deviation σ of X.

3) With reference to *Question-1*,

    (a) Find the expected value of $E[g(x,y)]$ where $g(x,y) = X + Y$.

    (b) Find the covariance of X and Y.

    (c) Find the conditional mean of X given $Y = 1$.

4) A professor conducted a study on the sleep habits of her students. She found that their nightly sleep duration follows a normal distribution with an average of 7 hours and a standard deviation of 1.5 hours.

(a) What is the probability of a student randomly chosen from the class getting less than 5 hours of sleep on a given night?

(b) What percentage of students get between 6 and 8 hours of sleep?

(c) If the professor wants to offer extra help sessions for students who chronically sleep less than 6 hours, what percentage of the class would be eligible?

(Hint: Convert the desired values to Z-scores and use standard normal table to find the probabilities.)

The questions will be evaluated as 1(a)- 15pts., 1(b) - 10pts., 1(c) - 10pts., (d) - 5pts.
2(a)-5pts, 2(b)-10pts, 3(a)-10pts, 3(b)-15pts, 3(c)-5pts 4(a)-5pts 4(b)-5pts 4(c)-5pts
D.T.