Homework 5

1. $\begin{bmatrix} 5 & 1 \\ 3 & 7 \end{bmatrix} = A \qquad |A - \lambda I| = 0$

$$\downarrow$$

$$\det \begin{bmatrix} 5-\lambda & 1 \\ 3 & 7-\lambda \end{bmatrix} = 0$$

$$35 - 5\lambda - 7\lambda + \lambda^2 - 3 = 0$$

$$\lambda^2 - 12\lambda + 32 = 0$$

eigenvalues $\lambda_1 = 8 \rightarrow \begin{bmatrix} -3 & 1 \\ 3 & -1 \end{bmatrix} \quad v_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$\lambda_2 = 4 \rightarrow \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} \quad v_1 = -v_2$

$$= \text{span} \left\{ \underbrace{\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}}_{\text{eigenvectors}} \right\}$$

$$A(\vec{x})_B = \begin{bmatrix} 8 & 0 \\ 0 & 4 \end{bmatrix} (\vec{x})_B$$

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2.

$A = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$ $\quad \det \begin{bmatrix} 7-\lambda & 3 \\ 3 & -1-\lambda \end{bmatrix} = 0$

$-7 - 7\lambda + \lambda + \lambda^2 - 9 = 0$

$\lambda^2 - 6\lambda - 16 = 0$ $\quad \begin{bmatrix} -1 & 3 \\ 3 & -9 \end{bmatrix} \to v_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

Diagonal Matrice $\lambda_1 = 8$

$\lambda_2 = -2 \longrightarrow \begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix} \to v_1 \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

$\begin{bmatrix} 8 & 0 \\ 0 & -2 \end{bmatrix}$

$X = \begin{bmatrix} \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & \frac{-3}{\sqrt{10}} \end{bmatrix}$ $\quad X^{-1} = \frac{1}{-\frac{9}{10} - \frac{1}{10}} \begin{bmatrix} \frac{-3}{\sqrt{10}} & -\frac{1}{\sqrt{10}} \\ -\frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \end{bmatrix}$

$\sqrt{10}$ to make it orthogonal

$X^{-1} = \begin{bmatrix} \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & -\frac{3}{\sqrt{10}} \end{bmatrix}$

$$\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & \frac{-3}{\sqrt{10}} \end{bmatrix} \begin{bmatrix} 8 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} & -\frac{3}{\sqrt{10}} \end{bmatrix}$$

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3. $A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$ Symetric ✓ $n \times n$ ✓

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad x^T = \begin{bmatrix} x_1 & x_2 \end{bmatrix}$

$$\begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \overset{?}{>} 0$$

$$\begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} x_1 + 2x_2 \\ 2x_1 + 5x_2 \end{bmatrix} = x_1(x_1 + 2x_2) + x_2(2x_1 + 5x_2)$$

$$= x_1^2 + 2x_1x_2 + 2x_1x_2 + 5x_2^2$$

$$= (x_1 + 2x_2)^2 + x_2^2 > 0$$

A is positive definite

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