

Homework III

1.

$$\left[\begin{array}{cccc} 1 & 3 & -5 & 1 \\ 2 & 6 & 1 & 13 \\ -1 & -3 & 5 & -1 \end{array} \right] \rightarrow \left[\begin{array}{cccc} 1 & 3 & -5 & 1 \\ 2 & 6 & 1 & 13 \\ 0 & 0 & 0 & 0 \end{array} \right]$$



$$z = 1$$

$$x + 3y = 6$$

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$$y = 1; x = 3$$

$$\left[\begin{array}{cccc} 1 & 3 & -5 & 1 \\ 0 & 0 & 11 & 11 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$y = 1; z = 0, x = -3 \rightarrow \left[\begin{array}{cccc} 1 & 3 & -5 & 0 \\ 0 & 0 & 11 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$x \text{ complete} = \left[\begin{array}{c} 3 \\ 1 \\ 1 \end{array} \right] + y \left[\begin{array}{c} -3 \\ 1 \\ 0 \end{array} \right] = \left[\begin{array}{c} 3 - 3y \\ 1 + y \\ 1 \end{array} \right]$$

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$$2. \quad A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 6 & 11 \\ 3 & 9 & 15 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 5 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Column space Row
 $C(A) = R(A) : \text{span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 11 \\ 15 \end{bmatrix} \right\}$

Null space

$$\begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad y=1, \underset{\text{pivots}}{x=0}, z=-3 \rightarrow \text{span} \left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} \right\}$$

Left Null

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad y=0, x=-3, z=1 \quad \text{span} \left\{ \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$$

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3.

$$N(A) \perp R(A^T)$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 7 & 10 \\ 4 & 9 & 14 & 17 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

pivots

$$\lambda_4 = 1 ; \quad x_3 = 1, \quad x_2 = -3, \quad x_1 = -1$$

$$S^\perp : \text{span} \left\{ \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix} \right\}$$

4.

$$A^T A \vec{x} = A^T \vec{b}$$

$$\vec{x} = (A^T A)^{-1} A^T \vec{b}$$

$$A^T A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 3 & 3 \end{bmatrix}$$

$$A^T A^{-1} = \frac{1}{15-9} \begin{bmatrix} 3 & -3 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 5/6 \end{bmatrix}$$

$$A^T \vec{b} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$$

$$\vec{x} = (A^T A)^{-1} A^T \vec{b} = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 5/16 \end{bmatrix} \begin{bmatrix} 0 \\ 6 \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$$

$$p = A \vec{x} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 5 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

$$e = b - p = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

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