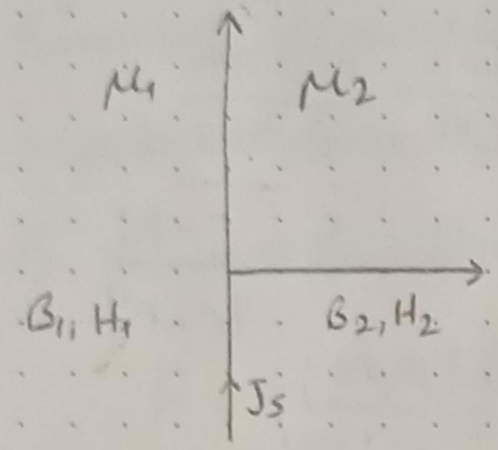
HW 2

Sena ERSOY

040200434

Sena

Q1



$\bar{B}_1 = B_{1x}\hat{a}_x + B_{1y}\hat{a}_y$
$\bar{B}_2 = B_{2x}\hat{a}_x + B_{2y}\hat{a}_y$

B_{1x}, B_{1y}, B_{2y} given. Find B_{2x}, $\vec{H}_1, \vec{H}_2$ and $\bar{J}_s$ in terms of $B_{1x}, B_{1y}, \mu_1, \mu_2$.

$\underbrace{B_{1x}}_{\text{given}} = \underbrace{B_{2x}}_{\text{found}}$) boundary condition: the normal component of B is continuous

$H_{t1} - H_{t2} = J_s$) the discontinuity of the tangential H equals surface current

$\underbrace{n \times}_{\hat{e}_z} (H_{1t} - H_{2t}) = J_s$

$\vec{H}_1 = H_{1x}\hat{a}_x + H_{1y}\hat{a}_y$
$\vec{H}_2 = H_{2x}\hat{a}_x + H_{2y}\hat{a}_y$

all given $\left(\frac{B_{1y}}{\mu_1} - \frac{B_{2y}}{\mu_2}\right) = J_s \hat{e}_z$ ✓

$\vec{H}_1 = \frac{B_{1x}}{\mu_1}\hat{a}_x + \frac{B_{1y}}{\mu_1}\hat{a}_y$ given

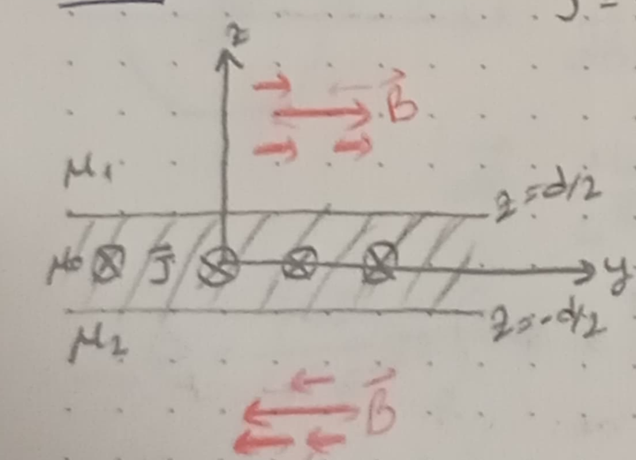
$\vec{H}_2 = \frac{B_{2x}}{\mu_2}\hat{a}_x + \frac{B_{2y}}{\mu_2}\hat{a}_y$ given
→found

Sena ERSOY
040200434

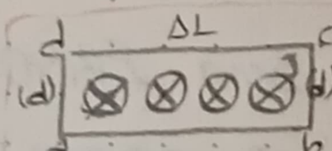
Q2

$\vec{J} = -J_0\left[\frac{|z|}{d}\right]\hat{a}_x \left[\frac{A}{m^2}\right]$

Find H and B everywhere



due to right hand rule and infinite slab B field occurs like that



$\oint H dl = I_{enc}$

$\int_a^b H dl + \underbrace{\int_b^c H dl}_{\alpha \perp dy = 0} + \int_c^d H dl + \cancel{\int_d^a H dl} = J_0 d \Delta L$

$H\Delta L + H\Delta L = J_0 d \cdot \Delta L$

in the question:

$$\frac{B}{\mu_2} \cancel{\lim_{\ell \to \infty} \Delta I} + \frac{B}{\mu_1} \cancel{\lim_{\ell \to \infty} \Delta I} = -J_0 \left[\frac{|z|}{\cancel{z}} \right] \cancel{z} \cdot \cancel{\lim_{\ell \to \infty} \Delta I}$$

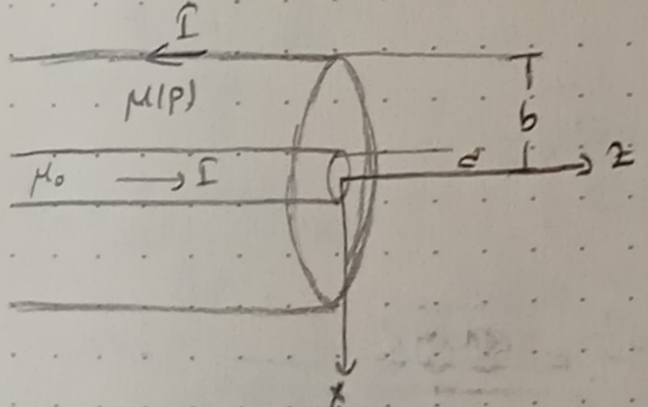
$$\frac{B}{\mu_2} + \frac{B}{\mu_1} = -J_0 |z| \rightarrow B = \frac{J_0 |z| \mu_1 \mu_2}{\mu_1 + \mu_2}$$

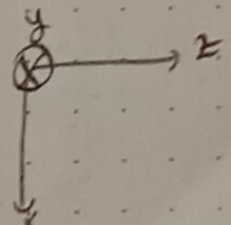
$$B = \begin{cases} J_0 \cdot z \cdot \dfrac{\mu_1 \mu_2}{\mu_1 + \mu_2} \hat{a}_y, & z > 0 \\ -J_0 \cdot z \cdot \dfrac{\mu_1 \mu_2}{\mu_1 + \mu_2} \hat{a}_y, & z < 0 \end{cases}$$

$$H = \begin{cases} J_0 \cdot z \dfrac{\mu_2}{\mu_1 + \mu_2} \hat{a}_y, & z > 0 \\ -J_0 z \dfrac{\mu_1}{\mu_1 + \mu_2} \hat{a}_y, & z < 0 \end{cases}$$

Sena ERSOY
040200434

Q3





$\vec{B} = B_\phi(\rho)\, \hat{a}_\phi$

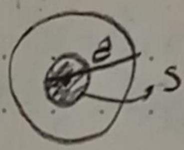
$\mu(\rho) = \mu_0 \ln \rho$

Find B, H everywhere

Find stored magnetic energy per unit ℓ

Find self inductance per unit length

for $\rho \leq a$



$d\ell = \rho \, d\phi \, \hat{a}_\phi$

$$\frac{1}{\mu_0} \oint B \cdot d\ell = I_{enc} = \int J \, ds = I \cdot \frac{\pi \rho^2}{\pi a^2} = \frac{I \rho^2}{a^2}$$

$$\frac{B \cdot 2\pi R}{\mu_0} = \frac{I \rho^2}{a^2} \qquad \boxed{B_\phi = \mu_0 \cdot \frac{I \cdot \rho}{2\pi a^2}} \qquad \boxed{H_\phi = \frac{I \rho}{2\pi a^2}}$$

for $a \leq \rho < b$

$$\frac{1}{\mu_0} \oint \frac{B}{\ln \rho} \, d\ell = I_{enc} = I \rightarrow \frac{B \, 2\pi \rho}{\mu_0 \cdot \ln \rho} = I$$

$$\boxed{B_\phi = \frac{\mu_0 \ln \rho \cdot I}{2\pi \rho}} \qquad \boxed{H_\phi = \frac{I}{2\pi \rho}}$$

for $\rho \geq b$

$$\int B \, d\ell = \mu_0 \underbrace{I_{enc}}_{0} \Rightarrow \boxed{\begin{matrix} B = 0 \\ H = 0 \end{matrix}}$$

$$u_B = \frac{1}{2} B H = \frac{1}{2} \mu(\rho) H^2 = \frac{\mu_0 \ln \rho}{2} \, \frac{I^2}{4\pi^2 \rho^2}$$

$$U_B = \iiint u_B \, dV = \frac{1}{2} L I^2$$

Sena ERSOY
040200434

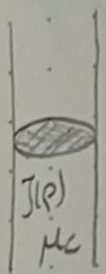
$$u_B = \frac{\mu_0 \ln\rho\, I^2}{8\pi^2 \rho^2} \longrightarrow \int_0^1\int_0^{2\pi}\int_a^b \frac{\ln\rho}{\rho^2}\, d\rho\, d\phi\, dz = \left(\frac{1+\ln a}{a} - \frac{1+\ln b}{b}\right)2\pi$$

$$U_B = \frac{\mu_0 I^2}{4\pi}\left(\frac{1+\ln a}{a} - \frac{1+\ln b}{b}\right) = \frac{1}{2} L I^2$$

$$L = \frac{\mu_0}{2\pi}\left(\frac{1+\ln a}{a} - \frac{1+\ln b}{b}\right)$$

Sena ERSOY
040200434

Q4



$$J = J_0 \rho\, \hat{a}_z \ A/m^2$$

$$\int H \cdot dl = I_{enc} \longrightarrow = \int J \cdot dS = J_0 \int \rho^2 d\phi\, d\rho$$

$$H \cdot 2\pi\rho = J_0 \frac{2\pi \rho^3}{3} \qquad \text{for } \rho \leq a$$

$$\boxed{\vec{H}_\phi = J_0 \frac{\rho^2}{3}\hat{a}_\phi \qquad B_\phi = J_0 \frac{\mu_c \rho^2}{3}\hat{a}_\phi}$$

$$\text{for } \rho \geq a \longrightarrow J_0 \int_0^a \rho^2 d\phi\, d\rho = I_{enc}$$

$$\boxed{H_\phi = J_0 \frac{a^3}{3\rho}\hat{a}_\phi \qquad B_\phi = \frac{J_0 \mu_c a^3}{3\rho}\hat{a}_\phi}$$

$$u_B = \frac{1}{2} BH$$

$$U_{B,in} = \frac{J_0^2 \mu_c}{9}\int_0^1\int_0^{2\pi}\int_0^a \rho^4\, d\rho\, d\phi\, dz$$

$$U_{B,in} = \frac{2\pi a^5 J_0^2 \mu_c}{45} = \frac{1}{2} L_{in}\Big(\underbrace{J_0 \frac{\pi a^3}{3}}_{I}\Big)^2$$

$$L_{in} = \frac{4\mu_c}{5\pi a} \qquad U_{B,out} = \iiint \frac{\mu_0}{2}\left(J_0 \frac{a^3}{3\rho}\right)^2 = \frac{1}{2} L_{ext} \frac{J_0^2 \pi^2 a^6}{9}$$

$$\frac{\mu_0 J_0^2 a^6}{18} \ln\frac{\rho}{a} \cdot 2\pi = \frac{1}{2} L_{ext} \frac{J_0^2 \pi^2 a^6}{9} \rightarrow L_{ext} = \frac{2\mu_0 \ln\frac{\rho}{a}}{\pi}$$

$$L = L_{in} + L_{ext} = \frac{4\mu_c}{5\pi a} + \frac{2\mu_0 \ln\frac{\rho}{a}}{\pi}$$

Mutual Inductance

$$B_\phi = \frac{J_0 \mu_0 a^3}{3\rho} \text{ for } \rho \geq a$$

$$\Phi = \int B_\phi \hat{a}_\phi \, dS_{a\phi} = \frac{J_0 \mu_0 a^3}{3} \int_0^H \int_\rho^{\rho+L} \frac{1}{\rho} d\rho \, dz$$

$$L = \frac{\Phi}{I} = \frac{J_0 \mu_0 \cancel{a^3}}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{J_0} 2\pi \cancel{a^3}} H \cdot \ln\frac{\rho+L}{\rho} = \frac{H\mu_0}{2\pi} \ln\frac{\rho+L}{\rho}$$

$$I \rightarrow \frac{J_0 2\pi a^3}{3}$$

Sera ERSOY
040200434
Sera