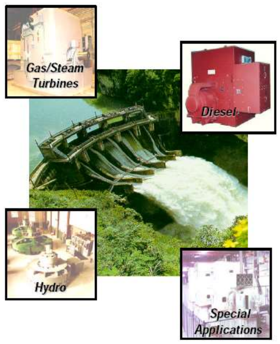
ELK 322E
POWER TRANSMISSION SYSTEMS

Chapter 3: Synchronous Machines



Synchronous Machines

- *Synchronous generators* or *alternators* are used to convert mechanical power derived from steam, gas, or hydraulic-turbine to ac electric power
- Synchronous generators are the primary source of electrical energy we consume today
- Large ac power networks rely almost exclusively on synchronous generators
- *Synchronous motors* are built in large units compare to induction motors (Induction motors are cheaper for smaller ratings) and used for constant speed industrial drives

Construction

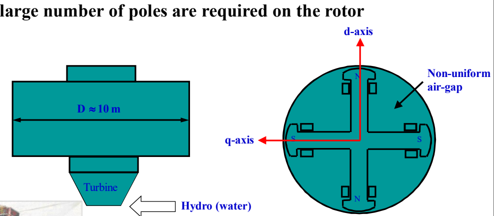
- **Basic parts of a synchronous generator:**
- **Rotor - dc excited winding**
- **Stator - 3-phase winding in which the ac emf is generated**
- **The manner in which the active parts of a synchronous machine are cooled determines its overall physical size and structure**

Various Types

- Salient-pole synchronous machine
- Cylindrical or round-rotor synchronous machine

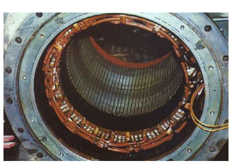
Salient-Pole Synchronous Generator

1. Most hydraulic turbines have to turn at low speeds (between 50 and 300 r/min)
2. A large number of poles are required on the rotor

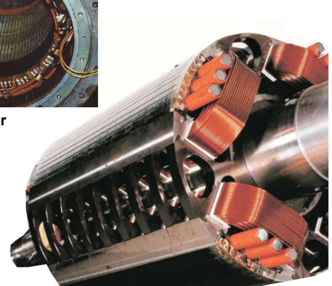


Hydrogenerator

Salient-Pole Synchronous Generator

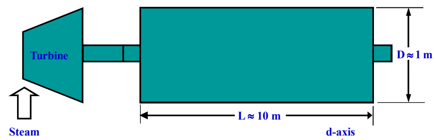


Stator

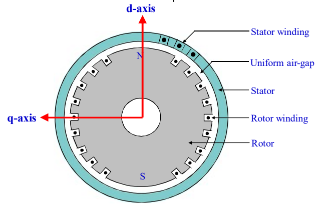


Salient-pole rotor

Cylindrical-Rotor Synchronous Generator

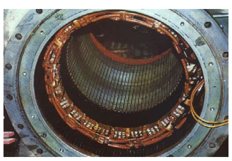


- High speed
- 3600 r/min ⇒2-pole
- 1800 r/min ⇒4-pole
- Direct-conductor cooling (using hydrogen or water as coolant)
- Rating up to 2000 MVA

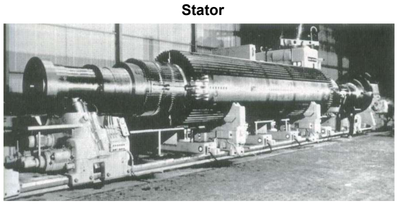


Turbogenerator

Cylindrical-Rotor Synchronous Generator



Stator



Cylindrical rotor

Operation Principle

The rotor of the generator is driven by a prime-mover

⇩

A dc current is flowing in the rotor winding which produces a rotating magnetic field within the machine

⇩

The rotating magnetic field induces a three-phase voltage in the stator winding of the generator

Electrical Frequency

Electrical frequency produced is locked or synchronized to the mechanical speed of rotation of a synchronous generator:

$$f_e = \frac{P\,n_m}{120}$$

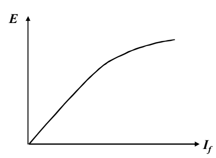
where f_e **= electrical frequency in Hz**
P **= number of poles**
n_m **= mechanical speed of the rotor, in r/min**

Generated Voltage

The generated voltage of a synchronous generator is given by

$$E = K_c \phi f_e$$

where ϕ **= flux in the machine (function of** I_f**)**
f_e **= electrical frequency**
K_c **= synchronous machine constant**



Saturation characteristic of a synchronous generator.

Voltage Regulation

A convenient way to compare the voltage behaviour of two generators is by their *voltage regulation* (*VR*). The *VR* of a synchronous generator at a given load, power factor, and at rated speed is defined as

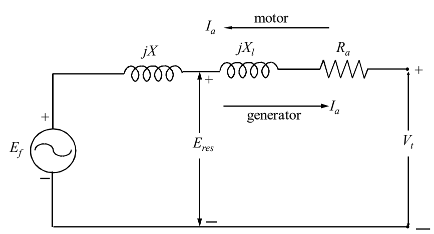
$$VR = \frac{E_{nl} - V_{fl}}{V_{fl}} \times 100\%$$

Where V_{fl} is the full-load terminal voltage, and E_{nl} (equal to E_f) is the no-load terminal voltage (internal voltage) at rated speed when the load is removed without changing the field current. For lagging power factor (*PF*), *VR* is fairly positive, for unity *PF*, *VR* is small positive and for leading *PF*, *VR* is negative.

Equivalent Circuit 1

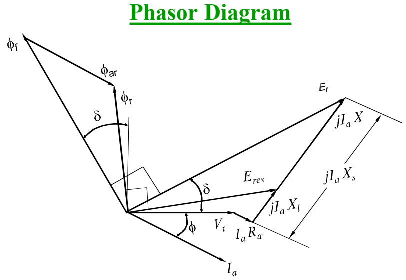
- The internal voltage E_f produced in a machine is not usually the voltage that appears at the terminals of the generator.
- The only time E_f is same as the output voltage of a phase is when there is no armature current flowing in the machine.
- There are a number of factors that cause the difference between E_f and V_t:
  - The distortion of the air-gap magnetic field by the current flowing in the stator, called the armature reaction
  - The self-inductance of the armature coils.
  - The resistance of the armature coils.
  - The effect of salient-pole rotor shapes.

Equivalent Circuit 2



Equivalent circuit of a cylindrical-rotor synchronous machine

Phasor Diagram

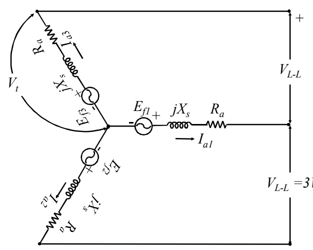


Phasor diagram of a cylindrical-rotor synchronous generator, for the case of lagging power factor

Lagging PF: $|V_t|<|E_f|$ for overexcited condition
Leading PF: $|V_t|>|E_f|$ for underexcited condition

Three-phase equivalent circuit of a cylindrical-rotor synchronous machine

The voltages and currents of the three phases are 120º apart in angle, but otherwise the three phases are identical.

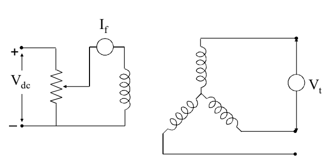


Determination of the parameters of the equivalent circuit from test data

- The equivalent circuit of a synchronous generator that has been derived contains three quantities that must be determined in order to completely describe the behaviour of a real synchronous generator:
  - The saturation characteristic: relationship between I_f and ϕ (and therefore between I_f and E_f)
  - The synchronous reactance, X_s
  - The armature resistance, R_a
- The above three quantities could be determined by performing the following three tests:
  - Open-circuit test
  - Short-circuit test
  - DC test

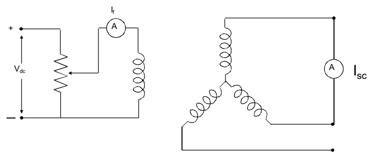
Open-circuit test

- The generator is turned at the rated speed
- The terminals are disconnected from all loads, and the field current is set to zero.
- Then the field current is gradually increased in steps, and the terminal voltage is measured at each step along the way.
- It is thus possible to obtain an open-circuit characteristic of a generator (E_f or V_t versus I_f) from this information



Short-circuit test

- Adjust the field current to zero and short-circuit the terminals of the generator through a set of ammeters.
- Record the armature current I_{sc} as the field current is increased.
- Such a plot is called short-circuit characteristic.



DC Test

- The purpose of the DC test is to determine R_a. A variable DC voltage source is connected between two stator terminals.
- The DC source is adjusted to provide approximately rated stator current, and the resistance between the two stator leads is determined from the voltmeter and ammeter readings
- then $R_{DC} = \frac{V_{DC}}{I_{DC}}$
- If the stator is Y-connected, the per phase stator resistance is

$$R_a = \frac{R_{DC}}{2}$$

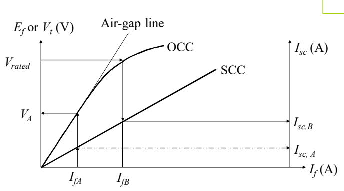
- If the stator is delta-connected, the per phase stator resistance is

$$R_a = \frac{3}{2} R_{DC}$$

Determination of X_s

- For a particular field current I_{fA}, the internal voltage E_f ($=V_A$) could be found from the occ and the short-circuit current flow $I_{sc,A}$ could be found from the scc.
- Then the synchronous reactance X_s could be obtained using

$$Z_{s,unsat} = \sqrt{R_a^2 + X_{s,unsat}^2} = \frac{V_A(= E_f)}{|I_{scA}|}$$



$$X_{s,unsat} = \sqrt{Z_{s,unsat}^2 - R_a^2}$$

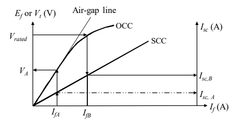
: R_a is known from the DC test.

Since $X_{s,unsat} >> R_a$,

$$X_{s,unsat} \approx \frac{E_f}{I_{scA}} = \frac{V_{t,oc}}{I_{scA}}$$

X_s under saturated condition

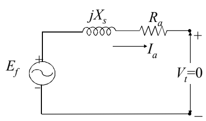
At $V = V_{rated}$,

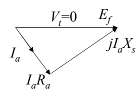


$$Z_{s,sat} = \sqrt{R_a^2 + X_{s,sat}^2} = \frac{V_{rated}(= E_f)}{|I_{scB}|}$$

$$X_{s,sat} = \sqrt{Z_{s,sat}^2 - R_a^2}$$ R_a is known from the DC test.

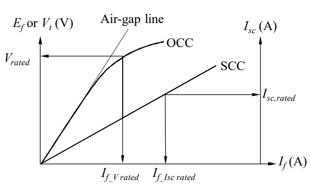
Equivalent circuit and phasor diagram under condition





Short-circuit Ratio

Another parameter used to describe synchronous generators is the short-circuit ratio (*SCR*). The SCR of a generator defined as the ratio of the *field current required for the rated voltage at open circuit* to the *field current required for the rated armature current at short circuit*. *SCR* is just the reciprocal of the per unit value of the saturated synchronous reactance calculated by



$$SCR = \frac{I_{f\_Vrated}}{I_{f\_Iscrated}} = \frac{1}{X_{s\_sat}\,[in\ p.u.]}$$

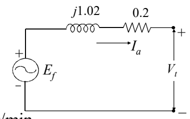
Example 1

A 200 kVA, 480-V, 60-Hz, 4-pole, Y-Connected synchronous generator with a rated field current of 5 A was tested and the following data was taken.

a) from OC test – terminal voltage = 540 V at rated field current
b) from SC test – line current = 300A at rated field current
c) from Dc test – DC voltage of 10 V applied to two terminals, a current of 25 A was measured.

1. Calculate the speed of rotation in r/min
2. Calculate the generated emf and saturated equivalent circuit parameters (armature resistance and synchronous reactance)

Solution to Example 1



1.
f_e = electrical frequency = $Pn_m/120$
f_e = 60Hz
P = number of poles = 4
n_m = mechanical speed of rotation in r/min.
So, speed of rotation n_m = $120\, f_e / P$
= (120 x 60)/4 = 1800 r/min

2. In open-circuit test, $I_a = 0$ and $E_f = V_t$
E_f = 540/1.732
= 311.8 V (as the machine is Y-connected)
In short-circuit test, terminals are shorted, $V_t = 0$
$E_f = I_a Z_s$ or $Z_s = E_f/I_a$=311.8/300=1.04 ohm
From the DC test, $R_a = V_{DC}/(2I_{DC})$
= 10/(2X25) = 0.2 ohm

Synchronous reactance $Z_{s,sat} = \sqrt{R_a^2 + X_{s,sat}^2}$

$$X_{s,sat} = \sqrt{Z_{s,sat}^2 - R_a^2} = \sqrt{1.04^2 - 0.2^2} = 1.02$$

Problem 1

A 480-V, 60-Hz, Y-Connected synchronous generator, having the synchronous reactance of 1.04 ohm and negligible armature resistance, is operating alone. The terminal voltage at rated field current at open circuit condition is 480V.

1. Calculate the voltage regulation
   1. If load current is 100A at 0.8 PF lagging
   2. If load current is 100A at 0.8 PF leading
   3. If load current is 100A at unity PF
2. Calculate the real and reactive power delivered in each case.
3. State and explain whether the voltage regulation will improve or not if the load current is decreased to 50 A from 100 A at 0.8 PF lagging.

Parallel operation of synchronous generators

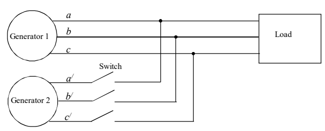
There are several major advantages to operate generators in parallel:

- Several generators can supply a bigger load than one machine by itself.
- Having many generators increases the reliability of the power system.
- It allows one or more generators to be removed for shutdown or preventive maintenance.

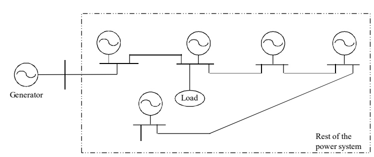
Synchronization

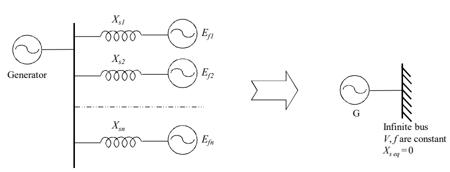
Before connecting a generator in parallel with another generator, it must be synchronized. A generator is said to be synchronized when it meets all the following conditions:

- The *rms line voltages* of the two generators must be equal.
- The two generators must have the same *phase sequence*.
- The *phase angles* of the two *a* phases must be equal.
- The *oncoming generator frequency* is equal to the running system frequency.



Synchronization





Concept of the infinite bus

When a synchronous generator is connected to a power system, the power system is often so large that nothing the operator of the generator does will have much of an effect on the power system.

An example of this situation is the connection of a single generator to the Canadian power grid. Our Canadian power grid is so large that no reasonable action on the part of one generator can cause an observable change in overall grid frequency. This idea is idealized in the concept of an infinite bus.

An infinite bus is a power system so large that its voltage and frequency do not vary regardless of how much real or reactive power is drawn from or supplied to it.

Active and reactive power-angle characteristics

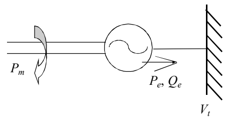
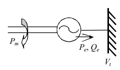


Fig. Synchronous generator connected to an infinite bus.

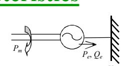
- P>0: generator operation
- P<0: motor operation
- Positive Q: delivering inductive vars for a generator action or receiving inductive vars for a motor action
- Negaive Q: delivering capacitive vars for a generator action or receiving capacitive vars for a motor action

Active and reactive power-angle characteristics



- The real and reactive power delivered by a synchronous generator or consumed by a synchronous motor can be expressed in terms of the terminal voltage V_t, generated voltage E_f, synchronous impedance Z_s, and the power angle or torque angle δ.
- Referring to Fig. 8, it is convenient to adopt a convention that makes positive real power P and positive reactive power Q delivered by an *overexcited generator*.
- The generator action corresponds to positive value of δ, while the motor action corresponds to negative value of δ.

Active and reactive power-angle characteristics



The complex power output of the generator in volt-amperes per phase is given by

$$S = P + jQ = \bar{V}_t I_a^*$$

where:
V_t = terminal voltage per phase
I_a^* = complex conjugate of the armature current per phase

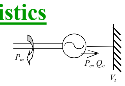
Taking the terminal voltage as reference

$$\bar{V}_t = V_t + j0$$

the excitation or the generated voltage,

$$\bar{E}_f = E_f(\cos\delta + j\sin\delta)$$

Active and reactive power-angle characteristics



and the armature current,

$$\bar{I}_a = \frac{\bar{E}_f - \bar{V}_t}{jX_s} = \frac{\left(E_f \cos\delta - V_t\right) + jE_f \sin\delta}{jX_s}$$

where X_s is the synchronous reactance per phase.

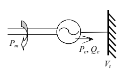
$$S = P + jQ = \bar{V}_t \bar{I}_a^* = V_t\left[\frac{\left(E_f \cos\delta - V_t\right) - jE_f \sin\delta}{-jX_s}\right]$$

$$= \frac{V_t E_f \sin\delta}{X_s} + j\frac{V_t E_f \cos\delta - V_t^2}{X_s}$$

$$\therefore \quad P = \frac{V_t E_f \sin\delta}{X_s} \quad \&$$

$$Q = \frac{V_t E_f \cos\delta - V_t^2}{X_s}$$

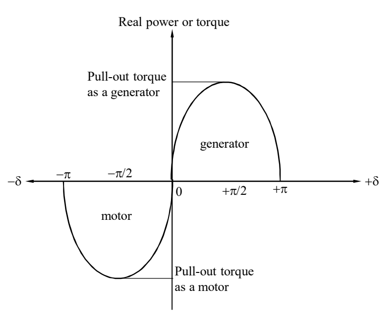
Active and reactive power-angle characteristics



$$\therefore \quad P = \frac{V_t E_f \sin\delta}{X_s} \quad \& \quad Q = \frac{V_t E_f \cos\delta - V_t^2}{X_s}$$

- The above two equations for active and reactive powers hold good for cylindrical-rotor synchronous machines for negligible resistance
- To obtain the total power for a three-phase generator, the above equations should be multiplied by 3 when the voltages are line-to-neutral
- If the line-to-line magnitudes are used for the voltages, however, these equations give the total three-phase power

Steady-state power-angle or torque-angle characteristic of a cylindrical-rotor synchronous machine (with negligible armature resistance).



Steady-state stability limit

Total three-phase power: $P = \frac{3V_t E_f}{X_s} \sin\delta$

The above equation shows that the power produced by a synchronous generator depends on the angle δ between the V_t and E_f. The maximum power that the generator can supply occurs when δ=90°.

$$P = \frac{3V_t E_f}{X_s}$$

The maximum power indicated by this equation is called *steady-state stability limit* of the generator. If we try to exceed this limit (such as by admitting more steam to the turbine), the rotor will accelerate and lose synchronism with the infinite bus. In practice, this condition is never reached because the circuit breakers trip as soon as synchronism is lost.

We have to resynchronize the generator before it can again pick up the load. Normally, real generators never even come close to the limit. Full-load torque angle of 15° to 20° are more typical of real machines.

Pull-out torque

The maximum torque or *pull-out torque* per phase that a two-pole round-rotor synchronous motor can develop is

$$T_{max} = \frac{P_{max}}{\omega_m} = \frac{P_{max}}{2\pi\left(n_s/60\right)}$$

where n_s is the synchronous speed of the motor in rpm

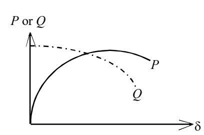


Fig. Active and reactive power as a function of the internal angle

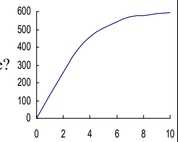
Problem 2

A 208-V, 45-kVA, 0.8-PF leading, △-connected, 60-Hz synchronous machine having 1.04 ohm synchronous reactance and negligible armature resistance is supplying a load of 12 kW at 0.8 power factor leading. Find the armature current and generated voltage and power factor if the load is increased to 20 KW. Neglect all other losses.

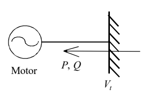
Example 5-2 (pp291)

A 480 V, 60 Hz, -connected, four pole synchronous generator has the OCC shown below. This generator has a synchronous reactance of 0.1 ohm and armature resistance of 0.015 ohm. At full load, the machine supplies 1200 A and 0.8 pf lagging. Under full-load conditions, the friction and windage losses are 40 kW, and the core losses are 30 kW. Ignore field circuit losses.

a) What is the speed of rotation of the generator?
b) How much field current must be supplied to the generator to make the terminal voltage 480 V at no load?
c) If the generator is now connected to a load and the load draws 1200 A at 0.8 pf lagging, how much field current will be required to keep the terminal voltage equal to 480 V?
d) How much power is the generator now supplying? How much power is supplied to the generator by the prime-mover? What is the machine's overall efficiency?
e) If the generator's load were suddenly disconnected from the line, what would happen to its terminal voltage?



Synchronous Motors



- A synchronous motor is the same physical machine as a generator, except that the direction of real power flow is reversed
- Synchronous motors are used to convert electric power to mechanical power
- Most synchronous motors are rated between 150 kW (200 hp) and 15 MW (20,000 hp) and turn at speed ranging from 150 to 1800 r/min. Consequently, these machines are used in heavy industry
- At the other end of the power spectrum, we find tiny single-phase synchronous motors used in control devices and electric clocks

Operation Principle

- The field current of a synchronous motor produces a steady-state magnetic field B_R
- A three-phase set of voltages is applied to the stator windings of the motor, which produces a three-phase current flow in the windings. This three-phase set of currents in the armature winding produces a uniform rotating magnetic field of B_s
- Therefore, there are two magnetic fields present in the machine, and *the rotor field will tend to line up with the stator field*, just as two bar magnets will tend to line up if placed near each other.
- Since the stator magnetic field is rotating, the rotor magnetic field (and the rotor itself) will try to catch up
- The larger the angle between the two magnetic fields (up to certain maximum), the greater the torque on the rotor of the machine

Vector Diagram

- The equivalent circuit of a synchronous motor is exactly same as the equivalent circuit of a synchronous generator, except that the reference direction of I_a is reversed.
- The basic difference between motor and generator operation in synchronous machines can be seen either in the magnetic field diagram or in the phasor diagram.
- In a generator, E_f lies ahead of V_t, and B_R lies ahead of B_{net}. In a motor, E_f lies behind V_t, and B_R lies behind B_{net}.
- In a motor the induced torque is in the direction of motion, and in a generator the induced torque is a countertorque opposing the direction of motion

Vector Diagram

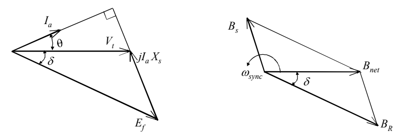


Fig. The phasor diagram (leading PF: overexcited and $|V_t|<|E_f|$) and the corresponding magnetic field diagram of a synchronous motor.

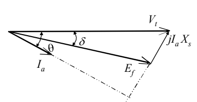


Fig. The phasor diagram of an underexcited synchronous motor (lagging PF and $|V_t|>|E_f|$).

Application of Synchronous Motors

Synchronous motors are usually used in large sizes because in small sizes they are costlier as compared with induction machines. The principal advantages of using synchronous machine are as follows:

- Power factor of synchronous machine can be controlled very easily by controlling the field current.
- It has very high operating efficiency and constant speed.
- For operating speed less than about 500 rpm and for high-power requirements (above 600KW) synchronous motor is cheaper than induction motor.

In view of these advantages, synchronous motors are preferred for driving the loads requiring high power at low speed; e.g; reciprocating pumps and compressor, crushers, rolling mills, pulp grinders etc.

Problem 5-22 (pp.343)

A 100-MVA, 12.5-kV, 0.85 power lagging, 50 Hz, two-pole, Y-connected, synchronous generator has a pu synchronous reactance of 1.1 and pu armature resistance of 0.012.

a) What are its synchronous reactance and armature resistance in ohms?
b) What is the magnitude of the internal voltage E_f at the rated conditions? What is its load angle δ at these conditions?
c) Ignoring losses in the generator, what torque must be applied to its shaft by the prime-mover at full load?

Problem 5-23 (pp.343)

A three-phase, Y-connected synchronous generator is rated 120 MVA, 13.2 kV, 0.8 power lagging, and 60 Hz. Its synchronous reactance is 0.9 ohm and its armature resistance may be ignored.

a) What is its voltage regulation at rated load?
b) What would the voltage and apparent power rating of this generator be if it were operated at 50 Hz with the same armature and field losses as it had at 60 Hz?
c) What would the voltage regulation of the generator be at 50 Hz?

Example 1: A 480V, 60Hz, 400kVA 4-pole Y-connected synchronous generator opertes at rated VA, with power factor 0.9 lagging. Rotational losses are 10kW and the armature resistance is R_A=0.02Ω.

Calculate:
1. Armature current
2. Generator efficiency
3. Prime mover torque

1.Armature current.
The line current can be found from the apparent power:
$S=\sqrt{3}V_{LL}I_L$, $I_L=400{,}000/\sqrt{3}480$, $I_L=481A$

The generator is Y-connected, $I_L=I_A$ and therefore:
$I_A=481A$,

2.Generator efficiency.
To find the efficiency it is first necessary to find the input power. Using simplified generator power flow:

$P_{in} = P_{out} + P_{rotational} + 3 I^2 R_A$

$=400*0.9= 360kW$
$P_{in} = 360kW + 10kW + 3 \times 481^2 \times 0.02 = 383.9kW$

And the efficiency is
$\eta= P_{in} / P_{out}$, then $\eta = 360/ 383.9 = 93.8\%$

Prime mover torque: The input power is mechanically given by
$P_{in} = \tau\ \omega_s$,

$\Omega_s = 4\pi f/p = 60\pi$

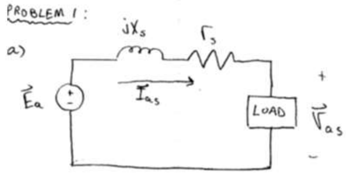
$T = P_{in} / \omega_s = 2037$ Nm

Problem 1: A 4-pole, 60Hz, Y-connected, 3-phase generator has a regulated terminal line-to-neutral voltage of $\vec{V}_{as} = 260V_{rms}\angle 0^{\circ}$, a synchronous reactance of 0.06Ω, a stator resistance of 0.003Ω, and $L_{sf} = 0.02H$. The balanced 3-phase load draws 2MW at a lagging 0.8 power factor.

a. Compute the 3-phase complex power of the load (magnitude and angle)
b. Compute the generator phase current (magnitude and angle; hint: the angle of the current should be negative for the load to have a lagging power factor)
c. Find the required excitation voltage ($\tilde{E}_a$)
d. Calculate the required field current
e. Calculate the required speed of the prime mover

PROBLEM 1:

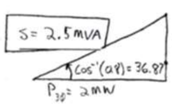
a)



$X_s = 0.06\,\Omega$

$r_s = 0.003\,\Omega$

$L_{sf} = 0.02\ H$



$Q = P_{3\phi} \tan 36.87°$
$= 1.5$ MVAR

b) Generator phase current

$S_{3\phi} = 3\vec{V}_{as}\vec{I}_{as}^{*} \Rightarrow \vec{I}_{as}^{*} = \dfrac{S_{3\phi}}{3\vec{V}_{as}} = \dfrac{2.5\times10^{6}\angle 36.87°}{3(260\angle 0°)}$

$I_{as}^{*} = 3205\angle 36.87°\ A$

$\boxed{\vec{I}_{as} = 3205\angle -36.87°\ A}$

c) Find $\vec{E}_a$

KVL:

$\vec{E}_a - \vec{I}_{as}(jX_s) - \vec{I}_{as}(r_s) - \vec{V}_{as} = 0$

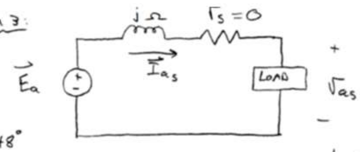
$\vec{E}_a = (3205\angle -36.87)(j0.06) + (3205\angle -36.87°)(0.003) + 260\angle 0°$

$\boxed{\vec{E}_a = 410.7\angle 21.1°\ V}$

Problem 3: A 3-phase, Y-connected, 6-pole, 60Hz generator has $X_s = 1\Omega$, negligible r_s (stator resistance), and $L_{sf} = 0.2H$. The machine operates at a power angle of 31.48° with a field current of 100A. The line-to-neutral terminal voltage is 2400Vrms.

a. Find the speed of the prime mover in rad/sec
b. Determine the excitation voltage
c. Find the line current
d. Find the power consumed by the load and its power factor
e. Find the developed torque

PROBLEM 3:



$\delta = 31.48^\circ$

$I_f = 100A$

$\vec{V}_{as} = 2400 \angle 0^\circ$ V

$L_{sf} = 0.2H$

a) Find ω_m

$\omega_m = \omega_s = \frac{2}{P}\omega_e = \frac{2}{6}(2\pi 60) = 125.7 \frac{rad}{sec}$

b) Find $\vec{E}_a$

$|\vec{E}_a| = \frac{L_{sf} I_f \omega_e}{\sqrt{2}} = \frac{(0.2)(100)(2\pi 60)}{\sqrt{2}} = 5331.5$

$\boxed{\vec{E}_a = 5331.5 \angle 31.48^\circ \text{ V}}$

c) Find $\vec{I}_{as}$

$\vec{I}_{as} = \frac{\vec{E}_a - \vec{V}_{as}}{j} = \frac{5331.5 \angle 31.48^\circ - 2400 \angle 0^\circ}{j}$

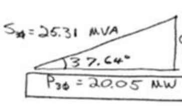
$\boxed{\vec{I}_{as} = 3515.7 \angle -37.6^\circ \text{ A}}$

d) Find $P_{3\phi}$ and p.f.

$\vec{S}_{3\phi} = 3 \vec{V}_{as} \vec{I}_{as}^{\,*} = 3(2400\angle 0^\circ)(3515.7\angle 37.6^\circ)$
$= 25.31 \angle 37.64^\circ$ MVA

Problem 3: (contd)

d) Find $P_{3\phi}$



Pf $= \cos^{-1}(37.64^\circ)$

$\boxed{= 0.792}$

e) find $\tau_{dev} = \tau_{ind}$

No losses, $r_s = 0$

$\tau_{dev} = \frac{P_{mech}}{\omega_m} = \frac{P_{3\phi}}{\omega_m} = \frac{20.05 \times 10^6}{125.7} = \boxed{159.5 \text{ kN·m}}$

or you can solve

$\tau_{dev} = \frac{3 V_{as} E_a \sin\delta}{\omega_m X_s} = \frac{3(2400)(5331.5)\sin(31.48^\circ)}{125.7}$

$\boxed{= 159.5 \text{ kN·m}}$

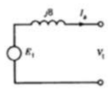
EXAMPLE

A 3ϕ, 5 kVA, 208 V, four-pole, 60 Hz, star-connected synchronous machine has negligible stator winding resistance and a synchronous reactance of 8 ohms per phase at rated terminal voltage.

Determine the excitation voltage and the power angle when the machine is delivering rated kVA at 0.8 PF lagging. Draw the phasor diagram for this condition.

Solution

The per-phase equivalent circuit for the synchronous generator



$$V_t = \frac{208}{\sqrt{3}} = 120 \text{ V/phase}$$

Stator current at rated kVA:

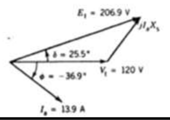
$$I_a = \frac{5000}{\sqrt{3} \times 208} = 13.9 \text{ A}$$

$\phi = -36.9^\circ$ for lagging pf of 0.8

$$E_f = V_t\angle 0^\circ + I_a jX_s$$
$$= 120\angle 0^\circ + 13.9\angle -36.9^\circ \cdot 8\angle 90^\circ$$
$$= 206.9\angle 25.5^\circ$$

Excitation voltage $E_f = 206.9$ V/phase

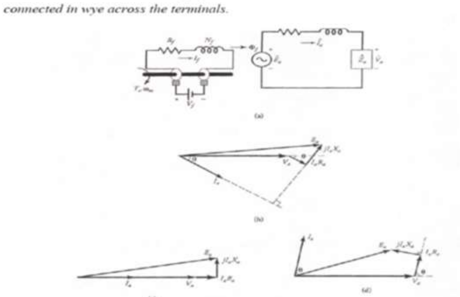
Power angle $\delta = +25.5^\circ$

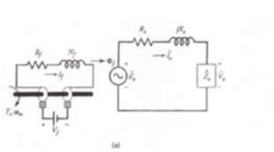


Example *A 3-phase synchronous generator produces an open-circuit line voltage of 6928 V when the do exciting current is 50 A. The ac terminals are then short-circuited, and the three line currents are found to be 800 A.*

a. Calculate the synchronous reactance per phase.

b. Calculate the terminal voltage if three 12 Ω resistors are connected in wye across the terminals.





(a)

a. The line-to-neutral induced voltage is

$$E_a = E_L / \sqrt{3} = 6928 / \sqrt{3} = 4000V$$

When the terminals are short-circuited, the only impedance limiting the current flow is that due to the synchronous reactance. Consequently,

$$X_S = E_a / I = 4000/80 = 5\Omega$$

b. The equivalent circuit per phase is shown in Fig. 1

The impedance of the circuit is:

$Z = \sqrt{R^2 + X_S^2} = \sqrt{12^2 + 5^2} = 13\Omega$

The current is :

$I = E_o / Z = 4000/13 = 308A$

The voltage across the load resistor is

$E = IR = 308 * 12 = 3696\ V$

The line voltage under load is:

$E_L = \sqrt{3}E = \sqrt{3} * 3696 = 6402\ V$

The schematic diagram of Fig. 1 helps us visualize what is happening in the actual circuit.

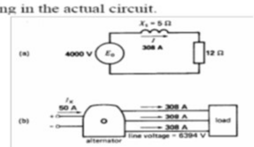
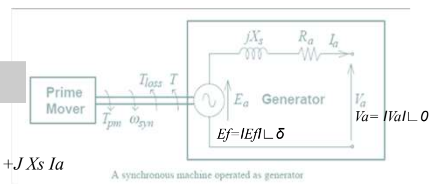


Fig. 1 See Example b. Actual line voltages and currents.

2.1.1. Production and Absorption of Reactive Power

Internal /Produced Voltage



Ef=Va +Ra Ia +J Xs Ia

Ef=IEfI∟δ and Va=IVaI∟0

Generator Power Output

$$S = P + jQ = \frac{Va * E_f}{Xs} Sin\delta + J\left(\frac{Va * E_f}{Xs} Cos\delta - \frac{Va^2}{Xs}\right)$$

Real/active Power (W,kW,MW)

Reactive Power (VAR, kVAr, MVAr)

2.1.1. Production and Absorption of Reactive Power

Synchronous Generators:

They can generate and absorb reactive power.

They produce reactive power when they are overexcited.

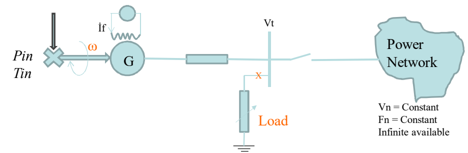
They absorb reactive power when they are under-excited.

Generated by excitation systems

$$S = P + jQ = \frac{Va * E_f}{Xs} Sin\delta + J\left(\frac{Va * E_f}{Xs} Cos\delta - \frac{Va^2}{Xs}\right)$$

Taken from the network

While the generator is operating independently from the main network,



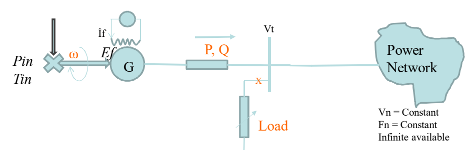
If ↗ then *Ef* ↗ then *Vt* ↗
If ↘ then *Ef* ↘ then *Vt* ↘

The terminal voltage of the generator can be controlled by changing the excitation current via an AVR

Tin, Pin ↗ then ω ↗ then *F* ↗
Tin, Pin ↘ then ω ↘ then *F* ↘

The frequency of the generated voltage can be controlled by changing the input power/torque using a speed governor.

While the generator is operating in parallel with the main network,

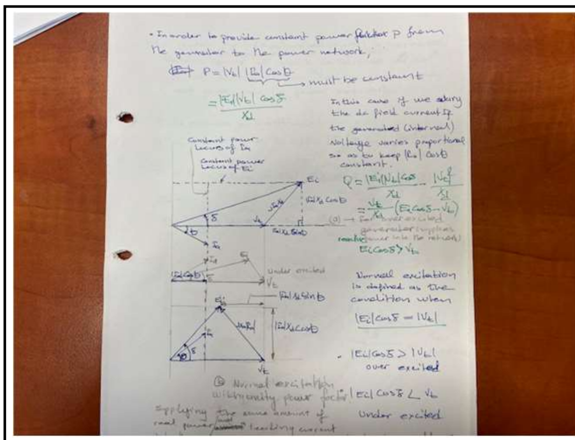


If ↗ then *Ef* ↗ then *Vt* ✕ ⟹ *Q* ↗
If ↘ then *Ef* ↘ then *Vt* ✕ ⟹ *Q* ↘

The reactive power generated can be controlled by changing the excitation current of the generator.

Tin, Pin ↗ then ω ↗ then *F* ✕ ⟹ δ ↗ *P* ↗
Tin, Pin ↘ then ω ↘ then *F* ✕ ⟹ δ ↘ *P* ↘

The active power output of the generator can be controlled by changing the input power/torque using a speed governor.



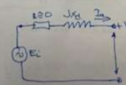
Example: The generator of above example has new synchronous reactance $X_d = 1.7241$ pu and is connected to a very large system. The terminal voltage is $1.0\angle 0°$ pu and generator supplying to the system a current of 0.8 pu at 0.9 power factor lagging. All pu values are on the machine base. Neglecting the resistance, find the magnitude and angle of the synchronous internal voltage E_i, and P and Q delivering to the infinite bus. If the real power output of the generator remains constant, but the excitation of the generator is:

a) increased by %20, find the angle of δ between E_i and the terminal voltage, Q delivered to the bus by the generator

b) decreased %20, find the angle of δ, terminal voltage and Q again.

Solution:

$E_i = |E_i| \angle \delta$



$S = 635$ MVA @ 0.9 pf lagging

$V_B = 24$ kV rated voltage

$n = 3600$ rpm

$X_d = 1.7241$ pu

$I = 15275.73$ A $= 15.276$ kA $= I_B$

$Z_B = \frac{24^2}{635} = 0.907\ \Omega$

33

$E_i = |E_i| \angle \delta° = V_t + jX_d I_a = 1.0\angle 0° + j1.7241 \times 0.8 \angle -25.842°$

$= 1.6012 + j1.2414 = 2.026 \angle 37.7862°$ (pu)

$S = P + jQ = V I^*$

$P = \frac{|V_t||E_i|}{X_d} \sin\delta$ real (average-active) power output of the generator delivered to the power system

$Q = \frac{|V_t|}{X_d}(|E_i|\cos\delta - |V_t|)$ reactive power output of the generator.

$P = \frac{1 \times 2.0261}{1.7241} \sin 37.7862° = 0.72$ (pu)

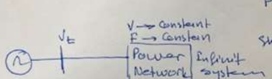
$Q = \frac{1.0}{1.7241}(1.6012 - 1.0) = 0.3487$ (pu)

$Q = \frac{1.0}{1.7241}(1.6012 - 1.0) = 0.3487$ (pu)

ⓐ Increasing excitation by 20% with P and V_t constant

$P = 0.72 = \frac{1.0 \times 1.2 \times 2.0261}{1.7241} \sin\delta$

$\sin\delta = 0.5105 \Rightarrow \delta = 30.702$



In this case the new value of Q supplied by the generator

$Q = \frac{1.0}{1.724}[1.2 \times 2.026 \cos(30.702) - 1.0] = 0.6325$ pu

(34)

(b) with excitation decreased 20%, we obtain

$$0.72 = \frac{1.0 \times 0.8 \times 2.026}{1.724} \sin\delta \Rightarrow \delta = 49.9827^\circ$$

then

$$Q = \frac{1.0}{1.724}\left[0.8 \times 2.0261 \cos(49.9827) - 1.0\right] = 0.0245 \text{ pu}$$

... equation has nothing to do with transferring the ohmic value of impedance from one side of the transformer to another. The great value of the equation is in changing the per-unit impedance given on a particular base to a new base.

Rather than using Eq. 2.52, however, the change in base may also be accomplished by converting the per-unit value on the given base to ohms and dividing by the new base impedance.

Example 2.5 The reactance of a generator designated X'' is given as 0.25 per unit based on the generator's nameplate rating of 18 kV, 500 MVA. The base for calculations is 20 kV, 100 MVA. Find X'' on the new base.

SOLUTION By Eq. (2.52)

$$X'' = 0.25\left(\frac{18}{20}\right)^2\left(\frac{100}{500}\right) = 0.0405 \text{ per unit}$$

or by converting the given value to ohms and dividing by the new base impedance

$$X'' = \frac{0.25(18^2/500)}{20^2/100} = 0.0405 \text{ per unit}$$

Resistance and reactance of a device in percent or per unit are usually available from the manufacturer. The base is understood to be the rated kilovoltamperes and kilovolts of the device. Tables A.4 and A.5 in the Appendix list some representative values of reactance for generators and transformers. We shall discuss per-unit quantities further in Chap. 6 in connection with our study of transformers.

PROBLEMS

2.1 If $v = 141.4 \sin(\omega t + 30^\circ)$ V and $i = 11.31 \cos(\omega t - 30^\circ)$ A, find for each (a) the maximum value, (b) the rms value, and (c) the phasor expression in polar and rectangular form if voltage is the reference. Is the circuit inductive or capacitive?

2.2 If the circuit of Prob. 2.1 consists of a purely resistive and a purely reactive element, find R and X, (a) if the elements are in series and (b) if the elements are in parallel.

2.3 In a single-phase circuit $V_a = 120\angle 45^\circ$ V and $V_b = 100\angle -15^\circ$ V with respect to a reference node o. Find V_{ba} in polar form.

2.4 A single-phase ac voltage of 240 V is applied to a series circuit whose impedance is [illegible] Ω. Find R, X, P, Q, and the power factor of the circuit.

2.6 A single-phase inductive load draws 10 MW at 0.6 power factor lagging. Draw the power triangle and determine the reactive power of a capacitor to be connected in parallel with the load to raise the power factor to 0.85.

2.7 A single-phase induction motor is operating at a very light load during a large part of every day and draws 10 A from the supply. A device is proposed to "increase the efficiency" of the motor. During a demonstration the device is placed in parallel with the unloaded motor and the current drawn from the supply drops to 8 A. When two of the devices are placed in parallel the current drops to 6 A. What simple device will cause this drop in current? Discuss the advantages of the device. Is the efficiency of the motor increased by the device? (Recall that an induction motor draws lagging current.)

2.8 If the impedance between machines 1 and 2 of Example 2.1 is $Z = 0 - j5$ Ω determine (a) whether each machine is generating or consuming power, (b) whether each machine is receiving or supplying positive reactive power and the amount, and (c) the value of P and Q absorbed by the impedance.

2.9 Repeat Prob. 2.8 if $Z = 5 + j0$ Ω.

2.10 A voltage source $E_{an} = -120\angle 210^\circ$ V and the current through the source is given by $I_{na} = 10\angle 60^\circ$ A. Find the values of P and Q and state whether the source is delivering or receiving each.

2.11 Solve Example 2.1 if $E_1 = 100\angle 0^\circ$ V and $E_2 = 120\angle 30^\circ$ V. Compare the results with Example 2.1 and form some conclusions about the effect of variation of the magnitude of E_1 in this circuit.

2.12 Three identical impedances of $10\angle -15^\circ$ Ω are Y-connected to balanced three-phase line voltages of 208 V. Specify all the line and phase voltages and the currents as phasors in polar form with V_{ca} as reference for a phase sequence of abc.

2.13 In a balanced three-phase system the Y-connected impedances are $10\angle 30^\circ$ Ω. If $V_{bc} = 416\angle 90^\circ$ V, specify I_{cn} in polar form.

2.14 [illegible]

2.15 Determine the current drawn from a three-phase 440-V line by a three-phase 15-hp motor operating at full load, 90% efficiency, and 80% power factor lagging. Find the values of P and Q drawn from the line.

2.16 If the impedance of each of the three lines connecting the motor of Prob. 2.15 to a bus is $0.3 + j1.0$ Ω, find the line-to-line voltage at the bus which supplies 440 V at the motor.

2.17 A balanced-Δ load consisting of pure resistances of 15 Ω per phase is in parallel with a balanced-Y load having phase impedances of $8 + j6$ Ω. Identical impedances of $2 + j5$ Ω are in each of the three lines connecting the combined loads to a 110-V three-phase supply. Find the current drawn from the supply and the line voltage at the combined loads.

2.18 A three-phase load draws 250 kW at a power factor of 0.707 lagging from a 440-V line. In parallel with this load is a three-phase capacitor bank which draws 60 kVA. Find the total current and resultant power factor.

2.19 A three-phase motor draws 20 kVA at 0.707 power factor lagging from a 220-V source. Determine the kilovoltampere rating of capacitors to make the combined power factor 0.90 lagging, [illegible]