

Introduction to Digital Design

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November 3, 2023

Boolean Function Optimization

- ▶ Minimizing the literal (or gate input) cost of a (a set of) Boolean equation(s) reduces circuit cost.
- ▶ We choose literal cost.
- ▶ Boolean Algebra and graphical techniques are tools to minimize cost criteria values.
- ▶ Some important questions:
 - ▶ When do we stop trying to reduce the cost?
 - ▶ Do we know when we have a minimum cost?
- ▶ Methods
 - ▶ Quine-McCluskey
 - ▶ Karnaugh (K-) map
 - ▶ Espresso

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

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$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4'$$

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$$f(x_1, x_2, x_3, x_4) = \sum_m (1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = \textcircled{x_1'x_2'x_3'x_4} + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

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$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + \text{red circle } x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + \text{blue circle } x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

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$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + \text{circled term} + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

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$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + \textcolor{red}{x_1'x_2x_3'x_4} + \textcolor{blue}{x_1'x_2x_3x_4'}$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

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$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + \text{circled term} + x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4' - \text{circled term}$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + \textcolor{red}{x_1'x_2x_3x_4'}$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m (1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + \textcolor{red}{x_1'x_2x_3x_4'}$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m (1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + \textcolor{red}{x_1'x_2x_3x_4'}$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m (1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + \textcolor{red}{x_1'x_2x_3x_4'}$$

$$+ x_1'x_2x_3x_4 + \textcolor{blue}{x_1x_2'x_3'x_4'} + x_1x_2x_3'x_4' + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + \textcolor{red}{x_1'x_2x_3x_4'}$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + \textcolor{blue}{x_1x_2x_3'x_4'} + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_3/x_4 + x_1/x_2/x_3x_4 + x_1/x_2x_3/x_4 + \textcolor{red}{x_1/x_2x_3x_4/}$$

$$+ x_1/x_2x_3x_4 + x_1x_2/x_3/x_4/ + x_1x_2x_3/x_4/ + \textcolor{blue}{x_1x_2x_3x_4/} + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_4(x_3/ + x_3) + x_1/x_3/x_4(x_2/ + x_2)$$

$$+ x_1/x_3x_4(x_2/ + x_2) + x_1/x_2x_4(x_3/ + x_3) + x_1/x_2x_3(x_4/ + x_4)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_3/x_4 + x_1/x_2/x_3x_4 + x_1/x_2x_3/x_4 + \textcolor{red}{x_1/x_2x_3x_4/}$$

$$+ x_1/x_2x_3x_4 + x_1x_2/x_3/x_4/ + x_1x_2x_3/x_4/ + \textcolor{blue}{x_1x_2x_3x_4/} + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_4(x_3/ + x_3) + x_1/x_3/x_4(x_2/ + x_2)$$

$$+ x_1/x_3x_4(x_2/ + x_2) + x_1/x_2x_4(x_3/ + x_3) + x_1/x_2x_3(x_4/ + x_4)$$

$$+ x_2x_3x_4/(x_1/ + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_3/x_4 + x_1/x_2/x_3x_4 + x_1/x_2x_3/x_4 + \textcolor{red}{x_1/x_2x_3x_4/}$$

$$+ x_1/x_2x_3x_4 + x_1x_2/x_3/x_4/ + x_1x_2x_3/x_4/ + x_1x_2x_3x_4/ - \textcolor{blue}{x_1x_2x_3x_4/}$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_4(x_3/ + x_3) + x_1/x_3/x_4(x_2/ + x_2)$$

$$+ x_1/x_3x_4(x_2/ + x_2) + x_1/x_2x_4(x_3/ + x_3) + x_1/x_2x_3(x_4/ + x_4)$$

$$+ x_2x_3x_4/(x_1/ + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_3/x_4 + x_1/x_2/x_3x_4 + x_1/x_2x_3/x_4 + x_1/x_2x_3x_4/$$

$$+ x_1/x_2x_3x_4 + x_1x_2/x_3/x_4/ + x_1x_2x_3/x_4/ + x_1x_2x_3x_4/ + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_4(x_3/ + x_3) + x_1/x_3/x_4(x_2/ + x_2)$$

$$+ x_1/x_3x_4(x_2/ + x_2) + x_1/x_2x_4(x_3/ + x_3) + x_1/x_2x_3(x_4/ + x_4)$$

$$+ x_2x_3x_4/(x_1/ + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_3/x_4 + x_1/x_2/x_3x_4 + x_1/x_2x_3/x_4 + x_1/x_2x_3x_4/$$

$$+ x_1/x_2x_3x_4 + x_1x_2/x_3/x_4/ + x_1x_2x_3/x_4/ + x_1x_2x_3x_4/ + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_4(x_3/ + x_3) + x_1/x_3/x_4(x_2/ + x_2)$$

$$+ x_1/x_3x_4(x_2/ + x_2) + x_1/x_2x_4(x_3/ + x_3) + x_1/x_2x_3(x_4/ + x_4)$$

$$+ x_2x_3x_4/(x_1/ + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_3/x_4 + x_1/x_2/x_3x_4 + x_1/x_2x_3/x_4 + x_1/x_2x_3x_4/$$

$$+ x_1/x_2x_3x_4 + x_1x_2/x_3/x_4/ + x_1x_2x_3/x_4/ + x_1x_2x_3x_4/ + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_4(x_3/ + x_3) + x_1/x_3/x_4(x_2/ + x_2)$$

$$+ x_1/x_3x_4(x_2/ + x_2) + x_1/x_2x_4(x_3/ + x_3) + x_1/x_2x_3(x_4/ + x_4)$$

$$+ x_2x_3x_4/(x_1/ + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_3/x_4 + x_1/x_2/x_3x_4 + x_1/x_2x_3/x_4 + x_1/x_2x_3x_4/$$

$$+ x_1/x_2x_3x_4 + x_1x_2/x_3/x_4/ + x_1x_2x_3/x_4/ + x_1x_2x_3x_4/ + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_4(x_3/ + x_3) + x_1/x_3/x_4(x_2/ + x_2)$$

$$+ x_1/x_3x_4(x_2/ + x_2) + x_1/x_2x_4(x_3/ + x_3) + x_1/x_2x_3(x_4/ + x_4)$$

$$+ x_2x_3x_4/(x_1/ + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_3/x_4 + x_1/x_2/x_3x_4 + x_1/x_2x_3/x_4 + x_1/x_2x_3x_4/$$

$$+ x_1/x_2x_3x_4 + x_1x_2/x_3/x_4/ + x_1x_2x_3/x_4/ + x_1x_2x_3x_4/ - x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_4(x_3/ + x_3) + x_1/x_3/x_4(x_2/ + x_2)$$

$$+ x_1/x_3x_4(x_2/ + x_2) + x_1/x_2x_4(x_3/ + x_3) + x_1/x_2x_3(x_4/ + x_4)$$

$$+ x_2x_3x_4/(x_1/ + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_3/x_4 + x_1/x_2/x_3x_4 + x_1/x_2x_3/x_4 + x_1/x_2x_3x_4/$$

$$+ x_1/x_2x_3x_4 + x_1x_2/x_3/x_4/ + x_1x_2x_3/x_4/ + x_1x_2x_3x_4/ - x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_4(x_3/ + x_3) + x_1/x_3/x_4(x_2/ + x_2)$$

$$+ x_1/x_3x_4(x_2/ + x_2) + x_1/x_2x_4(x_3/ + x_3) + x_1/x_2x_3(x_4/ + x_4)$$

$$+ x_2x_3x_4/(x_1/ + x_1) + x_2x_3x_4(x_1/ + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_3/x_4 + x_1/x_2/x_3x_4 + x_1/x_2x_3/x_4 + x_1/x_2x_3x_4/$$

$$+ x_1/x_2x_3x_4 + x_1x_2/x_3/x_4/ + x_1x_2x_3/x_4/ + x_1x_2x_3x_4/ + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_4(x_3/ + x_3) + x_1/x_3/x_4(x_2/ + x_2)$$

$$+ x_1/x_3x_4(x_2/ + x_2) + x_1/x_2x_4(x_3/ + x_3) + x_1/x_2x_3(x_4/ + x_4)$$

$$+ x_2x_3x_4/(x_1/ + x_1) + x_2x_3x_4(x_1/ + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_3/x_4 + x_1/x_2/x_3x_4 + x_1/x_2x_3/x_4 + x_1/x_2x_3x_4/$$

$$+ x_1/x_2x_3x_4 + x_1x_2/x_3/x_4/ + x_1x_2x_3/x_4/ + x_1x_2x_3x_4/ + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_4(x_3/ + x_3) + x_1/x_3/x_4(x_2/ + x_2)$$

$$+ x_1/x_3x_4(x_2/ + x_2) + x_1/x_2x_4(x_3/ + x_3) + x_1/x_2x_3(x_4/ + x_4)$$

$$+ x_2x_3x_4/(x_1/ + x_1) + x_2x_3x_4(x_1/ + x_1) + x_1x_3/x_4/(x_2/ + x_2)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_3/x_4 + x_1/x_2/x_3x_4 + x_1/x_2x_3/x_4 + x_1/x_2x_3x_4/$$

$$+ x_1/x_2x_3x_4 + x_1x_2/x_3/x_4/ + x_1x_2x_3/x_4/ + x_1x_2x_3x_4/ + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_4(x_3/ + x_3) + x_1/x_3/x_4(x_2/ + x_2)$$

$$+ x_1/x_3x_4(x_2/ + x_2) + x_1/x_2x_4(x_3/ + x_3) + x_1/x_2x_3(x_4/ + x_4)$$

$$+ x_2x_3x_4/(x_1/ + x_1) + x_2x_3x_4(x_1/ + x_1) + x_1x_3/x_4/(x_2/ + x_2)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_3/x_4 + x_1/x_2/x_3x_4 + x_1/x_2x_3/x_4 + x_1/x_2x_3x_4/$$

$$+ x_1/x_2x_3x_4 + \textcircled{x_1x_2/x_3/x_4/} + x_1x_2x_3/x_4/ + x_1x_2x_3x_4/ - \textcircled{x_1x_2x_3x_4}$$

$$f(x_1, x_2, x_3, x_4) = x_1/x_2/x_4(x_3/ + x_3) + x_1/x_3/x_4(x_2/ + x_2)$$

$$+ x_1/x_3x_4(x_2/ + x_2) + x_1/x_2x_4(x_3/ + x_3) + x_1/x_2x_3(x_4/ + x_4)$$

$$+ x_2x_3x_4/(x_1/ + x_1) + x_2x_3x_4(x_1/ + x_1) + x_1x_3/x_4/(x_2/ + x_2)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m (1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + \text{x₁x₂x₃'x₄'} + \text{x₁x₂x₃'x₄'} + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

$$+ x_2x_3x_4'(x_1' + x_1) + x_2x_3x_4(x_1' + x_1) + x_1x_3'x_4'(x_2' + x_2)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m (1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + \text{x₁x₂x₃'x₄'} + \text{x₁x₂x₃'x₄'} + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

$$+ x_2x_3x_4'(x_1' + x_1) + x_2x_3x_4(x_1' + x_1) + x_1x_3'x_4'(x_2' + x_2)$$

$$+ x_1x_2x_4'(x_3' + x_3)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m (1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + \text{circled term} + x_1x_2x_3x_4' + \text{circled term}$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

$$+ x_2x_3x_4'(x_1' + x_1) + x_2x_3x_4(x_1' + x_1) + x_1x_3'x_4'(x_2' + x_2)$$

$$+ x_1x_2x_4'(x_3' + x_3)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m (1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4'$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

$$+ x_2x_3x_4'(x_1' + x_1) + x_2x_3x_4(x_1' + x_1) + x_1x_3'x_4'(x_2' + x_2)$$

$$+ x_1x_2x_4'(x_3' + x_3) + x_1x_2x_3(x_4' + x_4)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4 + x_1'x_3'x_4 + x_1'x_3x_4 + x_1'x_2x_4 + x_1'x_2x_3 \\ + x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + x_1x_2x_3$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \cancel{x_1'x_2'x_4} + x_1'x_3'x_4 + x_1'x_3x_4 + \cancel{x_1'x_2x_4} + x_1'x_2x_3 \\ + x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + x_1x_2x_3$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \cancel{x_1'x_2'x_4} + x_1'x_3'x_4 + x_1'x_3x_4 + \cancel{x_1'x_2x_4} + x_1'x_2x_3$$
$$+ x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + x_1x_2x_3$$
$$f(x_1, x_2, x_3, x_4) = x_1'x_4(x_2' + x_2)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \cancel{x_1'x_2'x_4} + \cancel{x_1'x_3'x_4} + \cancel{x_1'x_3x_4} + \cancel{x_1'x_2x_4} + x_1'x_2x_3$$
$$+ x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + x_1x_2x_3$$
$$f(x_1, x_2, x_3, x_4) = x_1'x_4(x_2' + x_2)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \textcolor{red}{x_1'x_2'x_4} + \textcolor{blue}{x_1'x_3'x_4} + \textcolor{blue}{x_1'x_3x_4} + \textcolor{red}{x_1'x_2x_4} + x_1'x_2x_3 \\ + x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + x_1x_2x_3$$

$$f(x_1, x_2, x_3, x_4) = \textcolor{red}{x_1'x_4(x_2' + x_2)} + \textcolor{blue}{x_1'x_4(x_3' + x_3)}$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4 + x_1'x_3'x_4 + x_1'x_3x_4 + x_1'x_2x_4 + x_1'x_2x_3 \\ + x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + x_1x_2x_3$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_4(x_2' + x_2) + x_1'x_4(x_3' + x_3)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4 + x_1'x_3'x_4 + x_1'x_3x_4 + x_1'x_2x_4 + x_1'x_2x_3 \\ + x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + x_1x_2x_3$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_4(x_2' + x_2) + x_1'x_4(x_3' + x_3) + x_2x_3(x_1' + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4 + x_1'x_3'x_4 + x_1'x_3x_4 + x_1'x_2x_4 + x_1'x_2x_3 \\ + x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + x_1x_2x_3$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_4(x_2' + x_2) + x_1'x_4(x_3' + x_3) + x_2x_3(x_1' + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \cancel{x_1'x_2'x_4} + \cancel{x_1'x_3'x_4} + \cancel{x_1'x_3x_4} + \cancel{x_1'x_2x_4} + \cancel{x_1'x_2x_3}$$
$$+ \cancel{x_2x_3x_4'} + \cancel{x_2x_3x_4} + x_1x_3'x_4' + x_1x_2x_4' + \cancel{x_1x_2x_3}$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_4(x_2' + x_2) + x_1'x_4(x_3' + x_3) + x_2x_3(x_1' + x_1)$$
$$+ x_2x_3(x_4' + x_4)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \cancel{x_1'x_2'x_4} + \cancel{x_1'x_3'x_4} + \cancel{x_1'x_3x_4} + \cancel{x_1'x_2x_4} + \cancel{x_1'x_2x_3}$$
$$+ \cancel{x_2x_3x_4'} + \cancel{x_2x_3x_4} + x_1x_3'x_4' + x_1x_2x_4' + \cancel{x_1x_2x_3}$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_4(x_2' + x_2) + x_1'x_4(x_3' + x_3) + x_2x_3(x_1' + x_1)$$
$$+ x_2x_3(x_4' + x_4) + x_1x_3'x_4' + x_1x_2x_4'$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4 + x_1'x_3'x_4 + x_1'x_3x_4 + x_1'x_2x_4 + x_1'x_2x_3 \\ + x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + x_1x_2x_3$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_4(x_2' + x_2) + x_1'x_4(x_3' + x_3) + x_2x_3(x_1' + x_1) \\ + x_2x_3(x_4' + x_4) + x_1x_3'x_4' + x_1x_2x_4'$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_4 + x_1'x_4 + x_2x_3 + x_2x_3 + x_1x_3'x_4' + x_1x_2x_4'$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4 + x_1'x_3'x_4 + x_1'x_3x_4 + x_1'x_2x_4 + x_1'x_2x_3 \\ + x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + x_1x_2x_3$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_4(x_2' + x_2) + x_1'x_4(x_3' + x_3) + x_2x_3(x_1' + x_1) \\ + x_2x_3(x_4' + x_4) + x_1x_3'x_4' + x_1x_2x_4'$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_4 + x_1'x_4 + x_2x_3 + x_2x_3 + x_1x_3'x_4' + x_1x_2x_4'$$

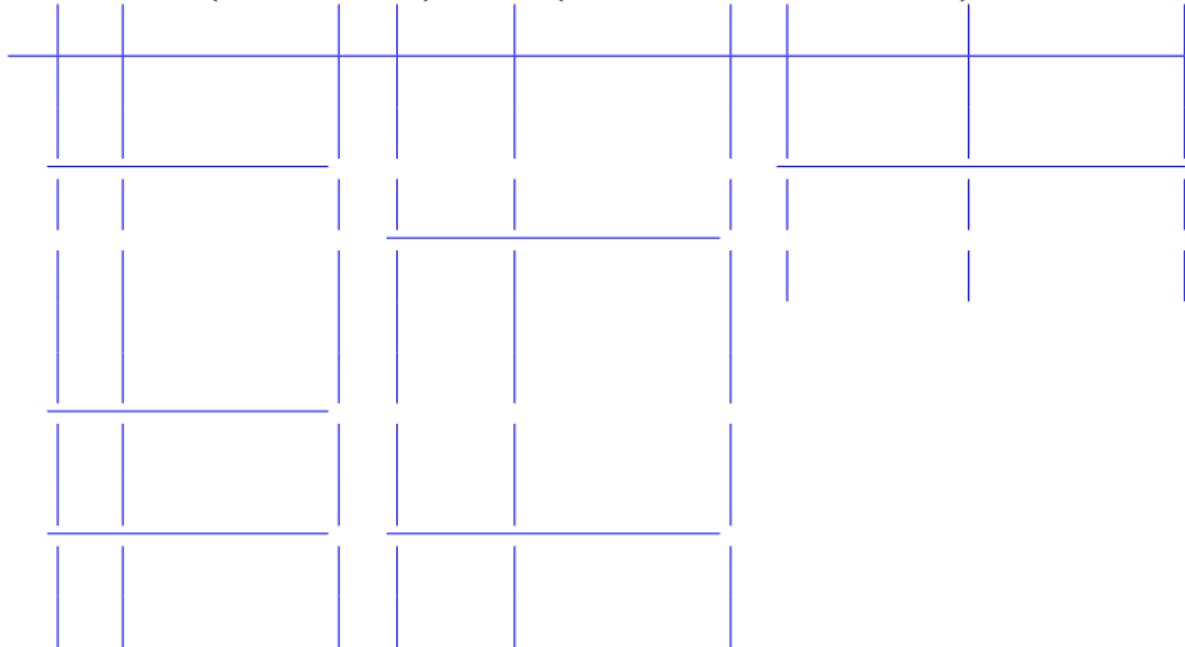
$$f(x_1, x_2, x_3, x_4) = x_1'x_4 + x_2x_3 + x_1x_3'x_4' + x_1x_2x_4'$$

Quine-McCluskey Method

- ▶ The Boolean function is represented in SOP or POS.
- ▶ **1st Phase:** Finding the prime implicants of f .
- ▶ **2nd Phase:** Finding optimal representation of f by taking some of the prime implicants.

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$



Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4								
1	0	0	0	1								

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4								
1	0	0	0	1								
8	1	0	0	0								

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4								
1	0	0	0	1								
8	1	0	0	0								
3	0	0	1	1								

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4								
1	0	0	0	1								
8	1	0	0	0								
3	0	0	1	1								
5	0	1	0	1								

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4								
1	0	0	0	1								
8	1	0	0	0								
3	0	0	1	1								
5	0	1	0	1								
6	0	1	1	0								

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4								
1	0	0	0	1								
8	1	0	0	0								
3	0	0	1	1								
5	0	1	0	1								
6	0	1	1	0								
12	1	1	0	0								

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4					
1	0	0	0	1					
8	1	0	0	0					
3	0	0	1	1					
5	0	1	0	1					
6	0	1	1	0					
12	1	1	0	0					
7	0	1	1	1					

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4						
1	0	0	0	1						
8	1	0	0	0						
3	0	0	1	1						
5	0	1	0	1						
6	0	1	1	0						
12	1	1	0	0						
7	0	1	1	1						
14	1	1	1	0						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4					
1	0	0	0	1					
8	1	0	0	0					
3	0	0	1	1					
5	0	1	0	1					
6	0	1	1	0					
12	1	1	0	0					
7	0	1	1	1					
14	1	1	1	0					
15	1	1	1	1					

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
1	0	0	0	1						
8	1	0	0	0						
3	0	0	1	1						
5	0	1	0	1						
6	0	1	1	0						
12	1	1	0	0						
7	0	1	1	1						
14	1	1	1	0						
15	1	1	1	1						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓ 1	0	0	0	1						
✓ 8	1	0	0	0						
✓ 3	0	0	1	1						
5	0	1	0	1						
6	0	1	1	0						
12	1	1	0	0						
7	0	1	1	1						
14	1	1	1	0						
15	1	1	1	1						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓ 1	0	0	0	1		1,3	0	0	-	1
✓ 8	1	0	0	0						
✓ 3	0	0	1	1						
5	0	1	0	1						
6	0	1	1	0						
12	1	1	0	0						
7	0	1	1	1						
14	1	1	1	0						
15	1	1	1	1						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓ 1	0	0	0	1		1,3	0	0	-	1
✓ 8	1	0	0	0						
✓ 3	0	0	1	1						
✓ 5	0	1	0	1						
✓ 6	0	1	1	0						
✓ 12	1	1	0	0						
✓ 7	0	1	1	1						
✓ 14	1	1	1	0						
✓ 15	1	1	1	1						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓ 1	0	0	0	1		1,3	0	0	-	1
✓ 8	1	0	0	0		1,5	0	-	0	1
✓ 3	0	0	1	1						
✓ 5	0	1	0	1						
6	0	1	1	0						
12	1	1	0	0						
7	0	1	1	1						
14	1	1	1	0						
15	1	1	1	1						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓ 1	0	0	0	1		1,3	0	0	-	1
✓ 8	1	0	0	0		1,5	0	-	0	1
✓ 3	0	0	1	1						
✓ 5	0	1	0	1						
6	0	1	1	0						
✓ 12	1	1	0	0						
7	0	1	1	1						
14	1	1	1	0						
15	1	1	1	1						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓ 1	0	0	0	1		1,3	0	0	-	1
✓ 8	1	0	0	0		1,5	0	-	0	1
✓ 3	0	0	1	1		8,12	1	-	0	0
✓ 5	0	1	0	1						
6	0	1	1	0						
✓ 12	1	1	0	0						
7	0	1	1	1						
14	1	1	1	0						
15	1	1	1	1						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓ 1	0	0	0	1		1,3	0	0	-	1
✓ 8	1	0	0	0		1,5	0	-	0	1
✓ 3	0	0	1	1		8,12	1	-	0	0
✓ 5	0	1	0	1						
6	0	1	1	0						
✓ 12	1	1	0	0						
✓ 7	0	1	1	1						
14	1	1	1	0						
15	1	1	1	1						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓ 1	0	0	0	1		1,3	0	0	-	1
✓ 8	1	0	0	0		1,5	0	-	0	1
✓ 3	0	0	1	1		8,12	1	-	0	0
✓ 5	0	1	0	1		3,7	0	-	1	1
6	0	1	1	0						
✓ 12	1	1	0	0						
✓ 7	0	1	1	1						
14	1	1	1	0						
15	1	1	1	1						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓ 1	0	0	0	1		1,3	0	0	-	1
✓ 8	1	0	0	0		1,5	0	-	0	1
✓ 3	0	0	1	1		8,12	1	-	0	0
✓ 5	0	1	0	1		3,7	0	-	1	1
6	0	1	1	0		5,7	0	1	-	1
✓ 12	1	1	0	0						
✓ 7	0	1	1	1						
14	1	1	1	0						
15	1	1	1	1						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓ 1	0	0	0	1		1,3	0	0	-	1
✓ 8	1	0	0	0		1,5	0	-	0	1
✓ 3	0	0	1	1		8,12	1	-	0	0
✓ 5	0	1	0	1		3,7	0	-	1	1
✓ 6	0	1	1	0		5,7	0	1	-	1
✓ 12	1	1	0	0						
✓ 7	0	1	1	1						
14	1	1	1	0						
15	1	1	1	1						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓ 1	0	0	0	1		1,3	0	0	-	1
✓ 8	1	0	0	0		1,5	0	-	0	1
✓ 3	0	0	1	1		8,12	1	-	0	0
✓ 5	0	1	0	1		3,7	0	-	1	1
✓ 6	0	1	1	0		5,7	0	1	-	1
✓ 12	1	1	0	0		6,7	0	1	1	-
✓ 7	0	1	1	1						
14	1	1	1	0						
15	1	1	1	1						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓ 1	0	0	0	1		1,3	0	0	- 1	
✓ 8	1	0	0	0		1,5	0	-	0 1	
✓ 3	0	0	1	1		8,12	1	-	0 0	
✓ 5	0	1	0	1		3,7	0	-	1 1	
✓ 6	0	1	1	0		5,7	0	1	- 1	
✓ 12	1	1	0	0		6,7	0	1	1 -	
✓ 7	0	1	1	1						
✓ 14	1	1	1	0						
15	1	1	1	1						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓ 1	0	0	0	1		1,3	0	0	- 1	
✓ 8	1	0	0	0		1,5	0	-	0 1	
✓ 3	0	0	1	1		8,12	1	-	0 0	
✓ 5	0	1	0	1		3,7	0	-	1 1	
✓ 6	0	1	1	0		5,7	0	1	- 1	
✓ 12	1	1	0	0		6,7	0	1	1	-
✓ 7	0	1	1	1		6,14	-	1	1	0
✓ 14	1	1	1	0						
15	1	1	1	1						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓ 1	0	0	0	1		1,3	0	0	- 1	
✓ 8	1	0	0	0		1,5	0	-	0 1	
✓ 3	0	0	1	1		8,12	1	-	0 0	
✓ 5	0	1	0	1		3,7	0	-	1 1	
✓ 6	0	1	1	0		5,7	0	1	- 1	
✓ 12	1	1	0	0		6,7	0	1	1	-
✓ 7	0	1	1	1		6,14	-	1	1	0
✓ 14	1	1	1	0		12,14	1	1	-	0
15	1	1	1	1						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓ 1	0	0	0	1		1,3	0	0	- 1	
✓ 8	1	0	0	0		1,5	0	-	0 1	
✓ 3	0	0	1	1		8,12	1	-	0 0	
✓ 5	0	1	0	1		3,7	0	-	1 1	
✓ 6	0	1	1	0		5,7	0	1	- 1	
✓ 12	1	1	0	0		6,7	0	1	1	-
✓ 7	0	1	1	1		6,14	-	1	1	0
✓ 14	1	1	1	0		12,14	1	1	-	0
✓ 15	1	1	1	1		7,15				

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓ 1	0	0	0	1		1,3	0	0	- 1	
✓ 8	1	0	0	0		1,5	0	-	0 1	
✓ 3	0	0	1	1		8,12	1	-	0 0	
✓ 5	0	1	0	1		3,7	0	-	1 1	
✓ 6	0	1	1	0		5,7	0	1	- 1	
✓ 12	1	1	0	0		6,7	0	1	1	-
✓ 7	0	1	1	1		6,14	-	1	1	0
✓ 14	1	1	1	0		12,14	1	1	-	0
✓ 15	1	1	1	1		7,15	-	1	1	1

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓ 1	0	0	0	1		1,3	0	0	- 1	
✓ 8	1	0	0	0		1,5	0	-	0 1	
✓ 3	0	0	1	1		8,12	1	-	0 0	
✓ 5	0	1	0	1		3,7	0	-	1 1	
✓ 6	0	1	1	0		5,7	0	1	- 1	
✓ 12	1	1	0	0		6,7	0	1	1	-
✓ 7	0	1	1	1		6,14	-	1	1	0
✓ 14	1	1	1	0		12,14	1	1	-	0
✓ 15	1	1	1	1		7,15	-	1	1	1
						14,15	1	1	1	-

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
✓ 1	0	0	0	1		1,3	0	0	- 1					
✓ 8	1	0	0	0		1,5	0	-	0 1					
✓ 3	0	0	1	1		8,12	1	-	0 0					
✓ 5	0	1	0	1		3,7	0	-	1 1					
✓ 6	0	1	1	0		5,7	0	1	- 1					
✓ 12	1	1	0	0		6,7	0	1	1 -					
✓ 7	0	1	1	1		6,14	-	1	1 0					
✓ 14	1	1	1	0		12,14	1	1	- 0					
✓ 15	1	1	1	1		7,15	-	1	1 1					
						14,15	1	1	1 -					

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
✓	1	0	0	1	✓	1,3	0	0	-1					
✓	8	1	0	0	0	1,5	0	-	0	1				
✓	3	0	0	1	1	8,12	1	-	0	0				
✓	5	0	1	0	1	3,7	0	-	1	1				
✓	6	0	1	1	0	5,7	0	1	-	1				
✓	12	1	1	0	0	6,7	0	1	1	-				
✓	7	0	1	1	1	6,14	-	1	1	0				
✓	14	1	1	1	0	12,14	1	1	-	0				
✓	15	1	1	1	1	7,15	-	1	1	1				
						14,15	1	1	1	-				

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		
✓	1	0	0	0	1	✓	1,3	0	0	-	1	1,3,5,7	0	-	-	1
✓	8	1	0	0	0		1,5	0	-	0	1					
✓	3	0	0	1	1		8,12	1	-	0	0					
✓	5	0	1	0	1		3,7	0	-	1	1					
✓	6	0	1	1	0	✓	5,7	0	1	-	1					
✓	12	1	1	0	0		6,7	0	1	1	-					
✓	7	0	1	1	1		6,14	-	1	1	0					
✓	14	1	1	1	0		12,14	1	1	-	0					
✓	15	1	1	1	1		7,15	-	1	1	1					
							14,15	1	1	1	-					

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4			
✓	1	0	0	0	1	✓	1,3	0	0	-	1		1,3,5,7	0	-	-	1
✓	8	1	0	0	0	✓	1,5	0	-	0	1						
✓	3	0	0	1	1		8,12	1	-	0	0						
✓	5	0	1	0	1	✓	3,7	0	-	1	1						
✓	6	0	1	1	0	✓	5,7	0	1	-	1						
✓	12	1	1	0	0		6,7	0	1	1	-						
✓	7	0	1	1	1		6,14	-	1	1	0						
✓	14	1	1	1	0		12,14	1	1	-	0						
✓	15	1	1	1	1		7,15	-	1	1	1						
							14,15	1	1	1	-						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
✓ 1	0	0	0	1	✓	1,3	0	0	- 1		1,3,5,7	0	-	- 1
✓ 8	1	0	0	0	✓	1,5	0	-	0 1		1,5,3,7	0	-	- 1
✓ 3	0	0	1	1		8,12	1	-	0 0					
✓ 5	0	1	0	1	✓	3,7	0	-	1 1					
✓ 6	0	1	1	0	✓	5,7	0	1	- 1					
✓ 12	1	1	0	0		6,7	0	1	1	-				
✓ 7	0	1	1	1		6,14	-	1	1	0				
✓ 14	1	1	1	0		12,14	1	1	-	0				
✓ 15	1	1	1	1		7,15	-	1	1	1				
						14,15	1	1	1	-				

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		
✓	1	0	0	0	1	✓	1,3	0	0	-	1	1,3,5,7	0	-	-	1
✓	8	1	0	0	0	✓	1,5	0	-	0	1	1,5,3,7	0	-	-	1
✓	3	0	0	1	1		8,12	1	-	0	0					
✓	5	0	1	0	1	✓	3,7	0	-	1	1					
✓	6	0	1	1	0	✓	5,7	0	1	-	1					
✓	12	1	1	0	0	✓	6,7	0	1	1	-					
✓	7	0	1	1	1		6,14	-	1	1	0					
✓	14	1	1	1	0		12,14	1	1	-	0					
✓	15	1	1	1	1		7,15	-	1	1	1					
						✓	14,15	1	1	1	-					

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓	1	0	0	1	✓	1,3	0	0	-1		1,3,5,7	0	-	-1	
✓	8	1	0	0	0	✓	1,5	0	-	01		1,5,3,7	0	-	-1
✓	3	0	0	1	1		8,12	1	-	00		6,7,14,15	-	1	1
✓	5	0	1	0	1	✓	3,7	0	-	11					
✓	6	0	1	1	0	✓	5,7	0	1	-1					
✓	12	1	1	0	0	✓	6,7	0	1	1	-				
✓	7	0	1	1	1		6,14	-	1	10					
✓	14	1	1	1	0		12,14	1	1	-0					
✓	15	1	1	1	1		7,15	-	1	11	1				
						✓	14,15	1	1	1	-				

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
✓	1	0	0	1	✓	1,3	0	0	-1		1,3,5,7	0	-	-1
✓	8	1	0	0	0	✓	1,5	0	-	01	1,5,3,7	0	-	-1
✓	3	0	0	1	1		8,12	1	-	00	6,7,14,15	-	1	1
✓	5	0	1	0	1	✓	3,7	0	-	11				
✓	6	0	1	1	0	✓	5,7	0	1	-1				
✓	12	1	1	0	0	✓	6,7	0	1	1	-			
✓	7	0	1	1	1	✓	6,14	-	1	10				
✓	14	1	1	1	0		12,14	1	1	-0				
✓	15	1	1	1	1	✓	7,15	-	1	11	1			
						✓	14,15	1	1	1-				

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
✓	1	0	0	1	✓	1,3	0	0	-1		1,3,5,7	0	-	-1
✓	8	1	0	0	0	✓	1,5	0	-	01	1,5,3,7	0	-	-1
✓	3	0	0	1	1		8,12	1	-	00	6,7,14,15	-	1	1
✓	5	0	1	0	1	✓	3,7	0	-	11	6,14,7,15	-	1	1
✓	6	0	1	1	0	✓	5,7	0	1	-1				
✓	12	1	1	0	0	✓	6,7	0	1	1	-			
✓	7	0	1	1	1	✓	6,14	-	1	10				
✓	14	1	1	1	0		12,14	1	1	-0				
✓	15	1	1	1	1	✓	7,15	-	1	11	1			
						✓	14,15	1	1	1-				

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
✓ 1	0	0	0	1	✓	1,3	0	0	- 1	C	1,3,5,7	0	-	- 1
✓ 8	1	0	0	0	✓	1,5	0	-	0 1		1,5,3,7	0	-	- 1
✓ 3	0	0	1	1	A	8,12	1	-	0 0	D	6,7,14,15	-	1	1 -
✓ 5	0	1	0	1	✓	3,7	0	-	1 1		6,14,7,15	-	1	1 -
✓ 6	0	1	1	0	✓	5,7	0	1	- 1					
✓ 12	1	1	0	0	✓	6,7	0	1	1 -					
✓ 7	0	1	1	1	✓	6,14	-	1	1 0					
✓ 14	1	1	1	0	B	12,14	1	1	- 0					
✓ 15	1	1	1	1	✓	7,15	-	1	1 1					
					✓	14,15	1	1	1 -					

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
✓ 1	0	0	0	1	✓	1,3	0	0	- 1	C	1,3,5,7	0	-	- 1
✓ 8	1	0	0	0	✓	1,5	0	-	0 1		1,5,3,7	0	-	- 1
✓ 3	0	0	1	1	A	8,12	1	-	0 0	D	6,7,14,15	-	1	1 -
✓ 5	0	1	0	1	✓	3,7	0	-	1 1		6,14,7,15	-	1	1 -
✓ 6	0	1	1	0	✓	5,7	0	1	- 1					
✓ 12	1	1	0	0	✓	6,7	0	1	1 -					
✓ 7	0	1	1	1	✓	6,14	-	1	1 0					
✓ 14	1	1	1	0	B	12,14	1	1	- 0					
✓ 15	1	1	1	1	✓	7,15	-	1	1 1					
					✓	14,15	1	1	1 -					

Sum of Prime Implicants:

$$f(x_1, x_2, x_3, x_4) = x_1x_3'x_4' + x_1x_2x_4' + x_1'x_4 + x_2x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

	1	3	5	6	7	8	12	14	15
A									
B									
C									
D									

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1x_3'x_4' + x_1'x_4 + x_2x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

	1	3	5	6	7	8	12	14	15
A						X	X		
B									
C									
D									

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1x_3'x_4' + x_1'x_4 + x_2x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

	1	3	5	6	7	8	12	14	15
A						X	X		
B							X	X	
C									
D									

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1x_3'x_4' + x_1'x_4 + x_2x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

	1	3	5	6	7	8	12	14	15
A						X	X		
B							X	X	
C	X	X	X		X				
D									

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1x_3'x_4' + x_1'x_4 + x_2x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

	1	3	5	6	7	8	12	14	15
A						X	X		
B							X	X	
C	X	X	X		X				
D			X	X			X	X	

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1x_3'x_4' + x_1'x_4 + x_2x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

	1	3	5	6	7	8	12	14	15	
A						X	X			EPI
B							X	X		
C	X	X	X		X					
D				X	X			X	X	

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1x_3'x_4' + x_1'x_4 + x_2x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

	1	3	5	6	7	8	12	14	15	
A						X	X			EPI
B							X	X		
C	X	X	X		X					
D				X	X			X	X	

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1x_3'x_4' + x_1'x_4 + x_2x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

	1	3	5	6	7	8	12	14	15	
A						X	X			EPI
B							X	X		
C	X	X	X		X					EPI
D				X	X			X	X	

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1x_3'x_4' + x_1'x_4 + x_2x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

	1	3	5	6	7	8	12	14	15	
A						X	X			EPI
B							X	X		
C	X	X	X		X					EPI
D				X	X			X	X	

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1x_3'x_4' + x_1'x_4 + x_2x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

	1	3	5	6	7	8	12	14	15	
A						X	X			EPI
B							X	X		
C	X	X	X			X				EPI
D				X	X			X	X	EPI

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1x_3'x_4' + x_1'x_4 + x_2x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

	1	3	5	6	7	8	12	14	15	
A						X	X			EPI
B							X	X		
C	X	X	X		X					EPI
D				X	X			X	X	EPI

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

	1	3	5	6	7	8	12	14	15	
A						○X	X			EPI
B							X	X		
C	○X	X	X		X					EPI
D				○X	X			X	X	EPI

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1x_3'x_4' + x_1'x_4 + x_2x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Patrick Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Patrick Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

$$P = C \bar{C} C D (C+D) A (A+B) (B+D) \bar{D}$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Patrick Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

$$P = C \ C \ C \ D \ (C+D) \ A \ (A+B) \ (B+D) \ D$$
$$C=1, \ D=1, \ A=1$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Patrick Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

$$\begin{aligned}P &= C \ C \ C \ D \ (C+D) \ A \ (A+B) \ (B+D) \ D \\C &= 1, \ D = 1, \ A = 1 \\P &= 1\end{aligned}$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Patrick Method

A 8,12

B 12,14

C 1,3,5,7

D 6,7,14,15

P=C C C D (C+D) A (A+B) (B+D) D

C=1, D=1, A=1

P=1

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1x_3/x_4' + x_1/x_4 + x_2x_3$$

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4			

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4			
1	0	0	0	1			

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4			
1	0	0	0	1			
4	0	1	0	0			

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4			
1	0	0	0	1			
4	0	1	0	0			
8	1	0	0	0			

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4			
1	0	0	0	1			
4	0	1	0	0			
8	1	0	0	0			
5	0	1	0	1			

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4			
1	0	0	0	1			
4	0	1	0	0			
8	1	0	0	0			
5	0	1	0	1			
12	1	1	0	0			

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4			
1	0	0	0	1			
4	0	1	0	0			
8	1	0	0	0			
5	0	1	0	1			
12	1	1	0	0			
11	1	0	1	1			

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4			
1	0	0	0	1			
4	0	1	0	0			
8	1	0	0	0			
5	0	1	0	1			
12	1	1	0	0			
11	1	0	1	1			
14	1	1	1	0			

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
1	0	0	0	1					
4	0	1	0	0					
8	1	0	0	0					
5	0	1	0	1					
12	1	1	0	0					
11	1	0	1	1					
14	1	1	1	0					

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
✓	1	0	0	1					
	4	0	1	0	0				
	8	1	0	0	0				
✓	5	0	1	0	1				
	12	1	1	0	0				
	11	1	0	1	1				
	14	1	1	1	0				

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓	1	0	0	1		1,5	0	-	0	1
	4	0	1	0	0					
	8	1	0	0	0					
✓	5	0	1	0	1					
	12	1	1	0	0					
	11	1	0	1	1					
	14	1	1	1	0					

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓	1	0	0	1		1,5	0	-	0	1
✓	4	0	1	0	0					
	8	1	0	0	0					
✓	5	0	1	0	1					
	12	1	1	0	0					
	11	1	0	1	1					
	14	1	1	1	0					

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
✓ 1	0	0	0	1		1,5	0	-	0
✓ 4	0	1	0	0		4,5	0	1	0
8	1	0	0	0					-
✓ 5	0	1	0	1					
12	1	1	0	0					
11	1	0	1	1					
14	1	1	1	0					

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
✓ 1	0	0	0	1		1,5	0	-	0
✓ 4	0	1	0	0		4,5	0	1	0
8	1	0	0	0					-
✓ 5	0	1	0	1					
✓ 12	1	1	0	0					
11	1	0	1	1					
14	1	1	1	0					

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4	
✓ 1	0	0	0	1		1,5	0	-	0	1
✓ 4	0	1	0	0		4,5	0	1	0	-
✓ 8	1	0	0	0		4,12	-	1	0	0
✓ 5	0	1	0	1						
✓ 12	1	1	0	0						
11	1	0	1	1						
14	1	1	1	0						

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4	
✓	1	0	0	0	1			1,5	0	-	0	1
✓	4	0	1	0	0			4,5	0	1	0	-
✓	8	1	0	0	0			4,12	-	1	0	0
✓	5	0	1	0	1							
✓	12	1	1	0	0							
	11	1	0	1	1							
	14	1	1	1	0							

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
✓	1	0	0	1		1,5	0	-	0
✓	4	0	1	0	0	4,5	0	1	0
✓	8	1	0	0	0	4,12	-	1	0
✓	5	0	1	0	1	8,12	1	-	0
✓	12	1	1	0	0				
	11	1	0	1	1				
	14	1	1	1	0				

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4	
✓	1	0	0	0	1			1,5	0	-	0	1
✓	4	0	1	0	0			4,5	0	1	0	-
✓	8	1	0	0	0			4,12	-	1	0	0
✓	5	0	1	0	1			8,12	1	-	0	0
✓	12	1	1	0	0							
	11	1	0	1	1							
✓	14	1	1	1	0							

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4	
✓	1	0	0	0	1			1,5	0	-	0	1
✓	4	0	1	0	0			4,5	0	1	0	-
✓	8	1	0	0	0			4,12	-	1	0	0
✓	5	0	1	0	1			8,12	1	-	0	0
✓	12	1	1	0	0			12,14	1	1	-	0
	11	1	0	1	1							
✓	14	1	1	1	0							

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4	
✓	1	0	0	0	1	B	1,5	0	-	0	1	
✓	4	0	1	0	0	C	4,5	0	1	0	-	
✓	8	1	0	0	0	D	4,12	-	1	0	0	
✓	5	0	1	0	1	E	8,12	1	-	0	0	
✓	12	1	1	0	0	F	12,14	1	1	-	0	
A	11	1	0	1	1							
✓	14	1	1	1	0							

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
✓	1	0	0	1	B	1,5	0	-	0
✓	4	0	1	0	C	4,5	0	1	0
✓	8	1	0	0	D	4,12	-	1	0
✓	5	0	1	0	E	8,12	1	-	0
✓	12	1	1	0	F	12,14	1	1	-
A	11	1	0	1					
✓	14	1	1	1					

Product of Prime Implicants:

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')$$

$$(x_1 + x_3 + x_4')(x_1 + x_2' + x_3)(x_2' + x_3 + x_4)$$

$$(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

Optimization of POS Representation By Quine-McCluskey Method

- A 11
- B 1,5
- C 4,5
- D 4,12
- E 8,12
- F 12,14

Optimization of POS Representation By Quine-McCluskey Method

	1	4	5	8	11	12	14
A							
B							
C							
D							
E							
F							

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_2' + x_3)(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

or

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_3 + x_4')(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

Optimization of POS Representation By Quine-McCluskey Method

	1	4	5	8	11	12	14
A					X		
B	X		X				
C		X	X				
D	X					X	
E				X	X		
F					X	X	

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_2' + x_3)(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

or

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_3 + x_4')(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

Optimization of POS Representation By Quine-McCluskey Method

	1	4	5	8	11	12	14	
A					X			EPI
B	X		X					
C		X	X					
D	X					X		
E				X		X		
F					X	X		

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_2' + x_3)(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

or

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_3 + x_4')(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

Optimization of POS Representation By Quine-McCluskey Method

- A 11
- B 1,5
- C 4,5
- D 4,12
- E 8,12
- F 12,14

	1	4	5	8	11	12	14	
A					X			EPI
B	X			X				EPI
C		X	X					
D		X				X		
E				X	X			
F					X	X		

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_2' + x_3)(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

or

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_3 + x_4')(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

Optimization of POS Representation By Quine-McCluskey Method

A 11
B 1,5
C 4,5
D 4,12
E 8,12
F 12,14

	1	4	5	8	11	12	14	
A					X			EPI
B	X			X			EPI	
C		X	X					
D		X				X		
E				X	X		EPI	
F					X	X		

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_2' + x_3)(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

or

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_3 + x_4')(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

Optimization of POS Representation By Quine-McCluskey Method

A 11
B 1,5
C 4,5
D 4,12
E 8,12
F 12,14

	1	4	5	8	11	12	14	
A					X			EPI
B	X			X				EPI
C		X	X					
D		X				X		
E			X		X			EPI
F				X	X	X		

Optimization of POS Representation By Quine-McCluskey Method

A 11
B 1,5
C 4,5
D 4,12
E 8,12
F 12,14

	1	4	5	8	11	12	14	
A					X			EPI
B	X			X			EPI	
C		X	X					
D		X				X		
E				X	X		EPI	
F					X	X	EPI	

Optimal Representation:

Optimization of POS Representation By Quine-McCluskey Method

- A 11
- B 1,5
- C 4,5
- D 4,12
- E 8,12
- F 12,14

	1	4	5	8	11	12	14	
A					X			EPI
B	X			X				EPI
C		X	X					
D		X				X		
E			X		X			EPI
F				X	X	X		EPI

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_2' + x_3)(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

or

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_3 + x_4')(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$