1. $u \cdot v = 2 \cdot 2 + 2(-1) + (-1) \cdot 2 = 0$

$u \cdot w = 2 \cdot 3 + 2 \cdot 0 + (-1) \cdot 4 = 2$

$w \cdot v = 3 \cdot 2 + 0 \cdot (-1) + 4 \cdot 2 = 14$

$u \cdot (v - 2w) = 2 \cdot (-4) + 2(-1) + (-1)(-6) = -4$

2.

$\|u\|^2 = u \cdot u = 2 \cdot 2 + 2 \cdot 2 + (-1)(-1) = 9$

$\|u\| = 3$

$\|v\|^2 = 2 \cdot 2 + (-1) \cdot (-1) + 2 \cdot 2 = 9$

$\|v\| = 3$

$\|w\|^2 = 3 \cdot 3 + 4 \cdot 4 = 25$

$\|w\| = 5$

$|u \cdot v| \leq \|u\| \cdot \|v\|$

$0 \leq 3 \cdot 3$

$|v \cdot w| \leq \|v\| \cdot \|w\|$

$14 < 3 \cdot 5$

$14 \leq 15$

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3.

a) $\hat{u} = \frac{u}{\|u\|} = \frac{1}{3}\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 2/3 \\ -1/3 \end{bmatrix}$

b) $v = \frac{v}{\|v\|} = \frac{1}{3}\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2/3 \\ -1/3 \\ 2/3 \end{bmatrix}$

4.

a) $\cos\theta = \frac{0}{3 \cdot 3} = 0$

b) $\cos\theta = \frac{2}{3 \cdot 5} = \frac{2}{15}$

c) $\cos\theta = \frac{14}{3 \cdot 5} = \frac{14}{15}$

5. If two vectors are perpendicular, their dot products must be equal to zero.

a) $u \cdot v = 0 \rightarrow$ Perpendicular

b) $w \cdot w = 2 \neq 0$

c) $v \cdot w = 14 \neq 0$

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6.

a) $x + 2y - 4z = h_1$ $\quad P_1(2,1,2)$

$h_1 = 2 + 2 - 8 = -4$

$H_1: x + 2y - 4z = -4$

b) $2x + 5y - 9z = h_2$ $\quad P_2(1,3,3)$

$h_2 = 2 + 15 - 27 = -10$

$H_2: 2x + 5y - 9z = -10$

c) $3x - 2y + 3z = h_3$ $\quad P_3(2,2,3)$

$h_3 = 6 - 4 + 9 = 11$

$H_3: 3x - 2y + 3z = 11$

d)
$$\begin{bmatrix} 1 & 2 & -4 & -4 \\ 2 & 5 & -9 & -10 \\ 3 & -2 & 3 & 11 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -4 & -4 \\ 0 & 1 & -1 & -2 \\ 0 & -8 & 15 & 23 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 2 & -4 & -4 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 7 & 7 \end{bmatrix}$$

$z = 1$

$y = -1$

$x = 2$

$P(2,-1,1)$

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7) $\begin{bmatrix} 1 & 2 & -4 & -4 \\ 3 & -2 & 3 & 11 \\ 4 & 0 & -1 & 7 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -4 & -4 \\ 0 & -8 & 15 & 23 \\ 0 & -8 & 15 & 23 \end{bmatrix}$

$\downarrow$

Singular system

infinitely many solutions

$x + 2y - 4z = -4$

$-8y + 15z = 23$

$-8y + 15z = 23$

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