ELK322E
POWER TRANSMISSION SYSTEMS

CHAPTER 1.2: BASIC CONCEPTS

PER-UNIT STEMS

1.10 Per-Unit Quantites

In an electrical power system

- $V \rightarrow$ kV, — 380 kV
- $P \rightarrow$ kW, MW — 500 MVA
- $Q \rightarrow$ kVAr, MVAr — 150 kVArs
- $S \rightarrow$ kVA, MVA
- $Z \rightarrow (\Omega)$
- $I \rightarrow$ A, kA

These quantities are often expressed as a percent or per-unit of a base or reference value specified for each. The per-unit value of ~~quantityes~~ quantity is defined as the ratio of the quantity to its based expressed as a decimal.

Advantage:
- (pu)x(pu) = (pu)
- using small ~~[illegible]~~ numbers around 1
- makes comparison very easy

PER-UNIT STEMS

$$\text{Per-Unit} = \frac{\text{The actual Value}}{\text{Base Value (Selected)}}$$

Base Values: All calculation is done using single phase values (Phase voltage, phase current).
Firstly, base voltage (kV) and base power (kVA or MVA) are chosen, the othe base quantities are then calculated

$$\text{Base current } \mathbf{I_B} = \frac{\text{Base Power (SB, MVA)}}{\text{Base Voltage (Vphase,kV)}}$$

$$\text{Base Impedance } \mathbf{Z_B} = \frac{\text{Base Voltage (Vphase,kV)}}{\text{Base current } \mathbf{I_B}} = \frac{VB^2}{\mathbf{S_B}} \text{X1000}$$

$$\text{Per-unit impedance} = \frac{\text{actual impedance}}{\mathbf{Z_B}} \text{ (pu)}$$

Example: For a power system S3 = 30 MVA= 30 000kVA, abd V_L= 120kV

Then SB =30/3 =10 MW, and VB = 120/√3 = 69.2 kV, IB= $\frac{\mathbf{S_B}}{\mathbf{V_B}} = \frac{10\ 000}{69.2}$ A

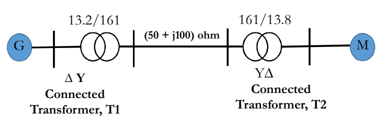
PER-UNIT STEMS

Changing the Base Valuse: Every element of power systems (generatör, transformers, motors and lines) is provided with a manufacturer data sheed in pu valuses according to their base values. Since in an electrical power system, they are connected each other, their pu values needs to be recalculated according to new base values.

$$\bar{Z}_{new} = \bar{Z}_{given}\left(\frac{V_{B given}}{V_{B new}}\right)^2\left(\frac{S_{B new}}{S_{B given}}\right)$$

Example: If the reactance of a generator X_d is given as 0.25 pu based on generator's rating of 18 kV, 500 MVA, find the X_d which is based on 20 kV, and 100 MVA

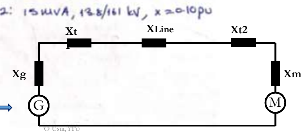
$$X_{d new} = 0.25\left(\frac{18}{20}\right)^2\left(\frac{100}{500}\right) = 0.0405 \text{ pu}$$



Base Values: SB = 20MVA, VB = 161 kV, Hence, 13.2 kV and 13.8 also become base values

Generator G: 15MVA, 13.8kV, x = 0.15 pu
Motor M: 10MVA, 13.2kV, x = 0.15 pu
Transformer T1: 25MVA, 13.2/161 kV, x = 0.10 pu
Transformer T2: 15MVA, 13.8/161 kV, x = 0.10 pu

Equievelent Circuit of the power network



21 . A

V_B = 161 kV (selected) hence 13.8 kV and 13.2 kV are also base voltages.

S_B = 20 MVA (Selected)

calculate the system parameter according to selected base values:

$$\bar{Z}_{new} = \bar{Z}_{given}\left(\frac{V_{B given}}{V_{B new}}\right)^2\left(\frac{S_{B new}}{S_{B given}}\right)$$

Generator: $\bar{X}_{new} = (0.15)\left(\frac{13.8}{13.2}\right)^2\left(\frac{20}{15}\right) = 0.219 \text{ (pu)}$

Motor $\bar{X}_{new} = (0.15)\left(\frac{13.2}{13.8}\right)\left(\frac{20}{10}\right) = 0.275 \text{ (pu)}$

T1 $\bar{X}_{new} = (0.10)\left(\frac{161}{161}\right)^2\left(\frac{20}{25}\right) = 0.08 \text{ (pu)}$

T2 $\bar{X}_{new} = (0.10)\left(\frac{161}{161}\right)^2\left(\frac{20}{14}\right) = 0.133 \text{ (pu)}$

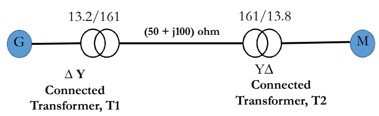
Zline (pu) $= \dfrac{Z_{\Omega}}{Z_B} = \dfrac{S_{B3}}{V^2_{n\text{-}pp}} Z_{\Omega} = \dfrac{2.0 \cdot 10^6}{(161 \cdot 10^3)^2}(50 + j100) =$

$= (0.0386 + j0.077)$

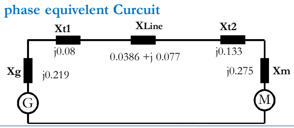
$Z_T = j0.219 + j0.08 + j0.133 + j0.275 + 0.0386 + j0.077$

$= 0.0386 + j0.784$

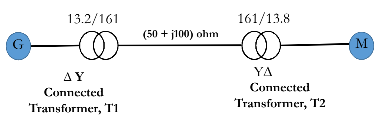
Power Network



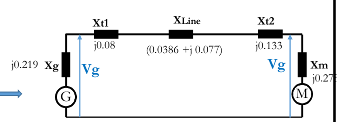
The single phase equivelent Curcuit



Power Network



The single phase equivelent circuit

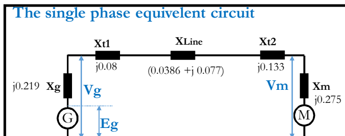


Example 1:

(a) If the motor in above system is a synchronous motor, and has a rated power of 10 MVA @ 0.9 leading power factor (comsumes real power and producing reactive power), and 1.1 pu terminal voltage? Calculate: the genrator terminal voltage and internal voltage in pu and kV.

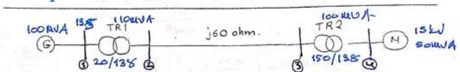
(b) If the motor operating with a power factoctor of 0.8 lagging, what will be the the generator terminal and internal voltage?

The single phase equivelent circuit



Example 2

2)



a) Find the per unit impedances of the system, and draw the impedance diagram. Use 150MVA and 138kV as the base power and the base voltage in the transmission line, respectively. The ratings of the generator, motor and transformers are:

G : 100MVA, 13.5kV, x = 0.2 pu.
T1 : 110MVA, 138/20kV, x = 0.15 pu.
T2 : 100MVA, 150/13.8kV, x = 0.1 pu.
M : 50MVA, 15kV, x = 0.3 pu.

b) The motor is drawing 45MVA, 0.8 PF lagging at a line-to-line terminal voltage of 13.5 kV. Determine the terminal voltage and the internal emf of the generator in pu and kV.

$S_{BASE} = 150MVA$

$V_{B1} = 138kV$

At the generator's side 138 / 138, 20 / ? $V_{B2} = 20kV$

At the motor's side 150 / 138, 13.8 / ? 12.7 $V_{B3} = 12.7kV$

Solution 2a

$$Z_{pu}^{new} = Z_{pu}^{old} \times \left(\frac{V^{old}}{V^{new}}\right)^2 \left(\frac{S^{new}}{S^{old}}\right)$$

$$Z_{pu} = \frac{Z_{actual}}{Z_{base}} = \frac{Z_{actual}}{\frac{V^2_{base}}{S_{base}}} = Z_{actual} \times \frac{S_{base}}{V^2_{base}}$$

G

$$X_{new} = 0.2 \times \left(\frac{13.5}{20}\right)^2 \times \left(\frac{150}{100}\right) = 0.137\ pu.$$

TR1

$$X_{new} = 0.15 \times \left(\frac{20}{20}\right) \times \left(\frac{150}{110}\right) = 0.204\ pu$$

TR2

$$X_{new} = 0.1 \times \left(\frac{150}{138}\right)^2 \times \left(\frac{150}{100}\right) = 0.177\ pu.$$

M

$$X_{new} = 0.3 \times \left(\frac{15}{12.7}\right)^2 \times \left(\frac{150}{50}\right) = 1.255\ pu.$$

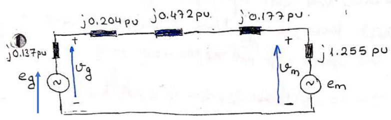
Line

$$X_{line(pu)} = 60 \times \frac{150}{138^2} = 0.472\ pu$$

Solution 2

Line

$$\bar{X}_{line\,(pu)} = 60 \times \frac{150}{138^2} = 0.472 \text{ pu}$$



v_g : terminal voltage of generator

v_m : terminal voltage of the motor

Solution 2b

$$S_M = 45 \angle \cos^{-1} 0.8 = \boxed{45 \angle 36.87 \text{ MVA}}$$

$$\bar{S}_m = \frac{45 \angle 36.87}{150} = 0.3 \angle 36.87 \text{ pu.}$$

$$|\bar{v}_m| = \frac{13.5\text{ kV}}{12.7\text{ kV}} = 1.063 \text{ pu.} \qquad \bar{v}_m = 1.063 \angle 0^\circ \text{ pu ref}$$

$$i = \frac{S_m^*}{v_m^*} = \frac{0.3 \angle -36.87}{1.063 \angle 0^\circ} = 0.282 \angle -36.87 \text{ pu}$$

Solution 2

Generator terminal voltage

$$\bar{v}_g = 1.063 \angle 0^\circ + j(0.204 + 0.177 + 0.472) \cdot 0.282 \angle -36.87$$

$$= 1.063 + 0.853 \angle 90 \times 0.282 \angle -36.87 = 1.063 + 0.240 \angle 53.13$$

$$= 1.063 + 0.144 + j0.192 = 1.207 + j0.192 = 1.222 \angle 9.04 \text{ pu.}$$

Vg = 1.222x 20 = 24.44 kV

Generator internal (produced) voltage

$$\bar{e}_g = 1.063 \angle 0^\circ + j(0.204 + 0.177 + 0.472 + 0.137) \cdot 0.282 \angle -36.87$$

$$= 1.063 + 0.99 \angle 90 \times 0.282 \angle -36.87 = 1.063 + 0.279 \angle 53.13$$

$$= 1.063 + 0.167 + j0.223 = 1.23 + j0.223 = 1.25 \angle 10.23 \text{ pu.}$$

The generator line-to-line internal emf is

$$|E_g| = 1.25 \times 20 = 25 \text{ kV} \qquad |V_g| = 24.4 \text{ kV}$$

Solution 2b

The motor is drawing 45 MVA @ 0.8 pf leading at a terminal voltage of 13.5 kV (Line to Line), Determine the terminal voltage and the internal voltage of the generator in pu and kV.

S = 45 MVA@ 0.8 leading power factor means **machine comsumes active power and produces reactive power**. Then:

$$S_M = 45 \text{ MVA @ 0.8 pf leading} = \boxed{45 \angle -36.87 \text{ MVA} \quad \angle 143.13}$$

$$\bar{S}_M = \frac{45}{150} \angle -36.87 = 0.3 \angle -36.87$$

$$\bar{V}_M = 1.063 \angle 0^\circ \text{ pu reference.}$$

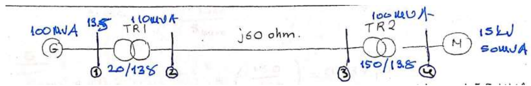
$$\bar{S}_M = V_M I_M^* \rightarrow I_M^* = \frac{\bar{S}_M^*}{V_M^*} = \frac{0.3 \angle 36.87}{1.06 \angle 0^\circ} = 0.28 \angle 36.87 \text{ pu}$$

$$Eg = (j0.137 + j0.21 + j0.47 + j0.177) \times 0.28 \angle 36.87 + 1.063$$

$$= j0.99 \times 0.28 \angle 36.87 + 1.06 = 0.278 \angle 126.87 + 1.063$$

$$Eg = -0.167 + j0.224 + 1.063 = 0.896 + j0.22 = 0.85 \angle 13.85$$

Eg =0.85 pu =0.85x20 =17kV



Motor has 45 MVA rated power @ 0.8 lagging power (**Comsumes active power and absorbs reactive power)**

Motor terminal Voltage Vm= 1.063x12.7 = **13.5 kV**
Motor Current Im = 0.282x11. 8(150 x10e6/12.7 x10e3) =**3.3∠-36.87kA**
Motor Power Sm = 50MVA @0.8 pf lagging
Terminal Voltage Vt = Vg = **24.4 kV**
Internal Voltage Eg = **25kV,**

Motor has 45 MVA rated power @ 0.8 leading power (Comsumes active power and absorbs reactive power)

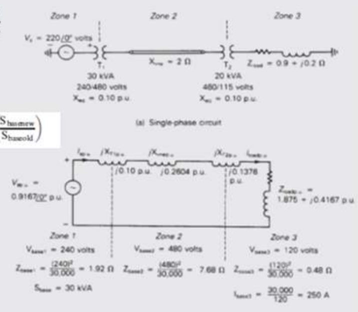
Motor terminal Voltage Vm= 1.063x12.7 = **13.5 kV**
Motor Current Im = 0.282x11. 8(150 x10e6/12.7 x10e3) =**3.3∠36.87kA**
Motor Power Sm = 50MVA @0.8 pf leading
Internal Voltage Eg = **17kV,**

EXAMPLE 3.4 Per-unit circuit: three-zone single-phase network

Three zones of a single-phase circuit are identified in Figure 3.10(a). The zones are connected by transformers T_1 and T_2, whose ratings are also shown. Using base values of 30 kVA and 240 volts in zone 1, draw the per-unit circuit and

$$Z_{p.u.new} = Z_{p.u.old}\left(\frac{V_{baseold}}{V_{basenew}}\right)^2\left(\frac{S_{basenew}}{S_{baseold}}\right)$$

FIGURE 3.10
Circuits for Example 3.4



determine the per-unit impedances and the per-unit source voltage. Then calculate the load current both in per-unit and in amperes. Transformer winding resistances and shunt admittance branches are neglected.

SOLUTION First the base values in each zone are determined. S_{base} = 30 kVA is the same for the entire network. Also, V_{base1} = 240 volts, as specified for zone 1. When moving across a transformer, the voltage base is changed in proportion to the transformer voltage ratings. Thus,

$$V_{base2} = \left(\frac{480}{240}\right)(240) = 480 \quad \text{volts}$$

and

$$V_{base3} = \left(\frac{115}{460}\right)(480) = 120 \quad \text{volts}$$

The base impedances in zones 2 and 3 are

$$Z_{base2} = \frac{V_{base2}^2}{S_{base}} = \frac{480^2}{30,000} = 7.68 \quad \Omega$$

and

$$Z_{base3} = \frac{V_{base3}^2}{S_{base}} = \frac{120^2}{30,000} = 0.48 \quad \Omega$$

and the base current in zone 3 is

$$I_{base3} = \frac{S_{base}}{V_{base3}} = \frac{30,000}{120} = 250 \quad A$$

Next, the per-unit circuit impedances are calculated using the system base values. Since $S_{base} = 30$ kVA is the same as the kVA rating of transformer T_1, and $V_{base1} = 240$ volts is the same as the voltage rating of the zone 1 side of transformer T_1, the per-unit leakage reactance of T_1 is the same as its nameplate value, $X_{T1p.u.} = 0.1$ per unit. However, the per-unit leakage reactance of transformer T_2 must be converted from its nameplate rating to the system base. Using (3.3.11) and $V_{base2} = 480$ volts,

$$X_{T2p.u.} = (0.10)\left(\frac{460}{480}\right)^2\left(\frac{30{,}000}{20{,}000}\right) = 0.1378 \quad \text{per unit}$$

Alternatively, using $V_{base3} = 120$ volts,

$$X_{T2p.u.} = (0.10)\left(\frac{115}{120}\right)^2\left(\frac{30{,}000}{20{,}000}\right) = 0.1378 \quad \text{per unit}$$

which gives the same result. The line, which is located in zone 2, has a per-unit reactance

$$X_{linep.u.} = \frac{X_{line}}{Z_{base2}} = \frac{2}{7.68} = 0.2604 \quad \text{per unit}$$

and the load, which is located in zone 3, has a per-unit impedance

$$Z_{loadp.u.} = \frac{Z_{load}}{Z_{base3}} = \frac{0.9 + j0.2}{0.48} = 1.875 + j0.4167 \quad \text{per unit}$$

The per-unit circuit is shown in Figure 3.10(b), where the base values for each zone, per-unit impedances, and the per-unit source voltage are shown. The per-unit load current is then easily calculated from Figure 3.10(b) as follows:

$$\begin{aligned} I_{loadp.u.} = I_{sp.u.} &= \frac{V_{sp.u.}}{j(X_{T1p.u.} + X_{linep.u.} + X_{T2p.u.}) + Z_{loadp.u.}} \\ &= \frac{0.9167\underline{/0^\circ}}{j(0.10 + 0.2604 + 0.1378) + (1.875 + j0.4167)} \\ &= \frac{0.9167\underline{/0^\circ}}{1.875 + j0.9149} = \frac{0.9167\underline{/0^\circ}}{2.086\underline{/26.01^\circ}} \\ &= 0.4395\underline{/-26.01^\circ} \quad \text{per unit} \end{aligned}$$

The actual load current is

$$I_{load} = (I_{loadp.u.})I_{base3} = (0.4395\underline{/-26.01^\circ})(250) = 109.9\underline{/-26.01^\circ} \quad \text{A}$$

Note that the per-unit equivalent circuit of Figure 3.10(b) is relatively easy to analyze, since ideal transformer windings have been eliminated by proper selection of base values. ■

THANKS