

Introduction to Digital Design

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Boolean Function Optimization

- ▶ Minimizing the literal (or gate input) cost of a (a set of) Boolean equation(s) reduces circuit cost.
- ▶ We choose literal cost.
- ▶ Boolean Algebra and graphical techniques are tools to minimize cost criteria values.
- ▶ Some important questions:
 - ▶ When do we stop trying to reduce the cost?
 - ▶ Do we know when we have a minimum cost?
- ▶ Methods
 - ▶ Quine-McCluskey
 - ▶ Karnaugh (K-) map
 - ▶ Espresso

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

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$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2) \\ + x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4' \\ + x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2'x_3'x_4' + x_1x_2'x_3x_4' + x_1x_2'x_3x_4' + x_1x_2x_3'x_4' + x_1x_2x_3'x_4' \\ + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2) \\ + x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

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$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2) + x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

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$$+ x_1x_2'x_3'x_4' + x_1x_2'x_3x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

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$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

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$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2'x_3'x_4' + x_1x_2'x_3x_4' + x_1x_2'x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

$$+ x_2x_3x_4'(x_1' + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

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$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

$$+ x_2x_3x_4'(x_1' + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

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$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

$$+ x_2x_3x_4'(x_1' + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

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$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

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Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

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$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

$$+ x_2x_3x_4'(x_1' + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2'x_3'x_4' + x_1x_2'x_3x_4' + x_1x_2'x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

$$+ x_2x_3x_4'(x_1' + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2'x_3'x_4' + x_1x_2'x_3x_4' + x_1x_2'x_3x_4' + x_1x_2x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4'$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

$$+ x_2x_3x_4'(x_1' + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

$$+ x_2x_3x_4'(x_1' + x_1) + x_2x_3x_4(x_1' + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2'x_3'x_4' + x_1x_2'x_3x_4' + x_1x_2'x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

$$+ x_2x_3x_4'(x_1' + x_1) + x_2x_3x_4(x_1' + x_1)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2'x_3'x_4' + x_1x_2'x_3x_4' + x_1x_2'x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

$$+ x_2x_3x_4'(x_1' + x_1) + x_2x_3x_4(x_1' + x_1) + x_1x_3'x_4'(x_2' + x_2)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

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$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

$$+ x_2x_3x_4'(x_1' + x_1) + x_2x_3x_4(x_1' + x_1) + x_1x_3'x_4'(x_2' + x_2)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$\begin{aligned} f(x_1, x_2, x_3, x_4) = & x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4' \\ & + x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4' + x_1x_2x_3x_4 \end{aligned}$$

$$\begin{aligned} f(x_1, x_2, x_3, x_4) = & x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2) \\ & + x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4) \\ & + x_2x_3x_4'(x_1' + x_1) + x_2x_3x_4(x_1' + x_1) + x_1x_3'x_4'(x_2' + x_2) \end{aligned}$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

$$+ x_2x_3x_4'(x_1' + x_1) + x_2x_3x_4(x_1' + x_1) + x_1x_3'x_4'(x_2' + x_2)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

$$+ x_2x_3x_4'(x_1' + x_1) + x_2x_3x_4(x_1' + x_1) + x_1x_3'x_4'(x_2' + x_2)$$

$$+ x_1x_2x_4'(x_3' + x_3)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

$$+ x_2x_3x_4'(x_1' + x_1) + x_2x_3x_4(x_1' + x_1) + x_1x_3'x_4'(x_2' + x_2)$$

$$+ x_1x_2x_4'(x_3' + x_3)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_3'x_4 + x_1'x_2'x_3x_4 + x_1'x_2x_3'x_4 + x_1'x_2x_3x_4'$$

$$+ x_1'x_2x_3x_4 + x_1x_2'x_3'x_4' + x_1x_2x_3'x_4' + x_1x_2x_3x_4' + x_1x_2x_3x_4$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4(x_3' + x_3) + x_1'x_3'x_4(x_2' + x_2)$$

$$+ x_1'x_3x_4(x_2' + x_2) + x_1'x_2x_4(x_3' + x_3) + x_1'x_2x_3(x_4' + x_4)$$

$$+ x_2x_3x_4'(x_1' + x_1) + x_2x_3x_4(x_1' + x_1) + x_1x_3'x_4'(x_2' + x_2)$$

$$+ x_1x_2x_4'(x_3' + x_3) + x_1x_2x_3(x_4' + x_4)$$

Optimization Example

$$\begin{aligned} f(x_1, x_2, x_3, x_4) = & x_1'x_2'x_4 + x_1'x_3'x_4 + x_1'x_3x_4 + x_1'x_2x_4 + x_1'x_2x_3 \\ & + x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + x_1x_2x_3 \end{aligned}$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4 + x_1'x_3'x_4 + x_1'x_3x_4 + x_1'x_2x_4 + x_1'x_2x_3 \\ + x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + x_1x_2x_3$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4 + x_1'x_3'x_4 + x_1'x_3x_4 + x_1'x_2x_4 + x_1'x_2x_3 \\ + x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + x_1x_2x_3$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_4(x_2' + x_2)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \underbrace{x_1'x_2'x_4}_{\text{red}} + \underbrace{x_1'x_3'x_4}_{\text{blue}} + \underbrace{x_1'x_3x_4}_{\text{blue}} + \underbrace{x_1'x_2x_4}_{\text{red}} + x_1'x_2x_3 \\ + x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + x_1x_2x_3$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_4(x_2' + x_2)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = \underbrace{x_1/x_2/x_4}_{\text{red}} + \underbrace{x_1/x_3/x_4}_{\text{blue}} + \underbrace{x_1/x_3x_4}_{\text{blue}} + \underbrace{x_1/x_2x_4}_{\text{red}} + x_1/x_2x_3 \\ + x_2x_3x_4' + x_2x_3x_4 + x_1x_3/x_4' + x_1x_2x_4' + x_1x_2x_3$$

$$f(x_1, x_2, x_3, x_4) = \underbrace{x_1/x_4(x_2' + x_2)}_{\text{red}} + \underbrace{x_1/x_4(x_3' + x_3)}_{\text{blue}}$$

Optimization Example

$$\begin{aligned} f(x_1, x_2, x_3, x_4) = & \underbrace{x_1/x_2/x_4}_{\text{red}} + \underbrace{x_1/x_3/x_4}_{\text{blue}} + \underbrace{x_1/x_3x_4}_{\text{blue}} + \underbrace{x_1/x_2x_4}_{\text{red}} + \underbrace{x_1/x_2x_3}_{\text{green}} \\ & + x_2x_3x_4' + x_2x_3x_4 + x_1x_3'/x_4' + x_1x_2x_4' + \underbrace{x_1x_2x_3}_{\text{green}} \end{aligned}$$
$$f(x_1, x_2, x_3, x_4) = x_1/x_4(x_2' + x_2) + x_1/x_4(x_3' + x_3)$$

Optimization Example

$$\begin{aligned} f(x_1, x_2, x_3, x_4) = & \underbrace{x_1'x_2'x_4}_{\text{red}} + \underbrace{x_1'x_3'x_4}_{\text{blue}} + \underbrace{x_1'x_3x_4}_{\text{blue}} + \underbrace{x_1'x_2x_4}_{\text{red}} + \underbrace{x_1'x_2x_3}_{\text{green}} \\ & + x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + \underbrace{x_1x_2x_3}_{\text{green}} \end{aligned}$$
$$f(x_1, x_2, x_3, x_4) = \underbrace{x_1'x_4}_{\text{red}}(\underbrace{x_2' + x_2}_{\text{red}}) + \underbrace{x_1'x_4}_{\text{blue}}(\underbrace{x_3' + x_3}_{\text{blue}}) + \underbrace{x_2x_3}_{\text{green}}(\underbrace{x_1' + x_1}_{\text{green}})$$

Optimization Example

$$\begin{aligned} f(x_1, x_2, x_3, x_4) = & \underbrace{x_1'x_2'x_4}_{\text{red}} + \underbrace{x_1'x_3'x_4}_{\text{blue}} + \underbrace{x_1'x_3x_4}_{\text{blue}} + \underbrace{x_1'x_2x_4}_{\text{red}} + \underbrace{x_1'x_2x_3}_{\text{green}} \\ & + \underbrace{x_2x_3x_4'}_{\text{yellow}} + \underbrace{x_2x_3x_4}_{\text{yellow}} + x_1x_3'x_4' + x_1x_2x_4' + \underbrace{x_1x_2x_3}_{\text{green}} \end{aligned}$$
$$f(x_1, x_2, x_3, x_4) = \underbrace{x_1'x_4}_{\text{red}}(\underbrace{x_2' + x_2}_{\text{red}}) + \underbrace{x_1'x_4}_{\text{blue}}(\underbrace{x_3' + x_3}_{\text{blue}}) + \underbrace{x_2x_3}_{\text{green}}(\underbrace{x_1' + x_1}_{\text{green}})$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4 + x_1'x_3'x_4 + x_1'x_3x_4 + x_1'x_2x_4 + x_1'x_2x_3 \\ + x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + x_1x_2x_3$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_4(x_2' + x_2) + x_1'x_4(x_3' + x_3) + x_2x_3(x_1' + x_1) \\ + x_2x_3(x_4' + x_4)$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4 + x_1'x_3'x_4 + x_1'x_3x_4 + x_1'x_2x_4 + x_1'x_2x_3 \\ + x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + x_1x_2x_3$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_4(x_2' + x_2) + x_1'x_4(x_3' + x_3) + x_2x_3(x_1' + x_1) \\ + x_2x_3(x_4' + x_4) + x_1x_3'x_4' + x_1x_2x_4'$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4 + x_1'x_3'x_4 + x_1'x_3x_4 + x_1'x_2x_4 + x_1'x_2x_3 \\ + x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + x_1x_2x_3$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_4(x_2' + x_2) + x_1'x_4(x_3' + x_3) + x_2x_3(x_1' + x_1) \\ + x_2x_3(x_4' + x_4) + x_1x_3'x_4' + x_1x_2x_4'$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_4 + x_1'x_4 + x_2x_3 + x_2x_3 + x_1x_3'x_4' + x_1x_2x_4'$$

Optimization Example

$$f(x_1, x_2, x_3, x_4) = x_1'x_2'x_4 + x_1'x_3'x_4 + x_1'x_3x_4 + x_1'x_2x_4 + x_1'x_2x_3$$

$$+ x_2x_3x_4' + x_2x_3x_4 + x_1x_3'x_4' + x_1x_2x_4' + x_1x_2x_3$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_4(x_2' + x_2) + x_1'x_4(x_3' + x_3) + x_2x_3(x_1' + x_1)$$

$$+ x_2x_3(x_4' + x_4) + x_1x_3'x_4' + x_1x_2x_4'$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_4 + x_1'x_4 + x_2x_3 + x_2x_3 + x_1x_3'x_4' + x_1x_2x_4'$$

$$f(x_1, x_2, x_3, x_4) = x_1'x_4 + x_2x_3 + x_1x_3'x_4' + x_1x_2x_4'$$

Quine-McCluskey Method

- ▶ The Boolean function is represented in SOP or POS.
- ▶ **1st Phase:** Finding the prime implicants of f .
- ▶ **2nd Phase:** Finding optimal representation of f by taking some of the prime implicants.

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4					
1	0	0	0	1					

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4					
1	0	0	0	1					
8	1	0	0	0					

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4					
1	0	0	0	1					
8	1	0	0	0					
3	0	0	1	1					

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4					
1	0	0	0	1					
8	1	0	0	0					
3	0	0	1	1					
5	0	1	0	1					

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4					
1	0	0	0	1					
8	1	0	0	0					
3	0	0	1	1					
5	0	1	0	1					
6	0	1	1	0					

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4					
1	0	0	0	1					
8	1	0	0	0					
3	0	0	1	1					
5	0	1	0	1					
6	0	1	1	0					
12	1	1	0	0					

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4					
1	0	0	0	1					
8	1	0	0	0					
3	0	0	1	1					
5	0	1	0	1					
6	0	1	1	0					
12	1	1	0	0					
7	0	1	1	1					

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4					
1	0	0	0	1					
8	1	0	0	0					
3	0	0	1	1					
5	0	1	0	1					
6	0	1	1	0					
12	1	1	0	0					
7	0	1	1	1					
14	1	1	1	0					

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4					
1	0	0	0	1					
8	1	0	0	0					
3	0	0	1	1					
5	0	1	0	1					
6	0	1	1	0					
12	1	1	0	0					
7	0	1	1	1					
14	1	1	1	0					
15	1	1	1	1					

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4			
1	0	0	0	1								
8	1	0	0	0								
3	0	0	1	1								
5	0	1	0	1								
6	0	1	1	0								
12	1	1	0	0								
7	0	1	1	1								
14	1	1	1	0								
15	1	1	1	1								

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4			
✓	1	0	0	0	1								
	8	1	0	0	0								
✓	3	0	0	1	1								
	5	0	1	0	1								
	6	0	1	1	0								
	12	1	1	0	0								
	7	0	1	1	1								
	14	1	1	1	0								
	15	1	1	1	1								

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		
✓	1	0	0	0	1		1,3	0	0	-	1	
	8	1	0	0	0							
✓	3	0	0	1	1							
	5	0	1	0	1							
	6	0	1	1	0							
	12	1	1	0	0							
	7	0	1	1	1							
	14	1	1	1	0							
	15	1	1	1	1							

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		
✓	1	0	0	0	1		1,3	0	0	-	1	
	8	1	0	0	0							
✓	3	0	0	1	1							
✓	5	0	1	0	1							
	6	0	1	1	0							
	12	1	1	0	0							
	7	0	1	1	1							
	14	1	1	1	0							
	15	1	1	1	1							

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		
✓	1	0	0	0	1		1,3	0	0	-	1	
	8	1	0	0	0		1,5	0	-	0	1	
✓	3	0	0	1	1							
✓	5	0	1	0	1							
	6	0	1	1	0							
	12	1	1	0	0							
	7	0	1	1	1							
	14	1	1	1	0							
	15	1	1	1	1							

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		
✓	1	0	0	0	1		1,3	0	0	-	1	
✓	8	1	0	0	0		1,5	0	-	0	1	
✓	3	0	0	1	1							
✓	5	0	1	0	1							
	6	0	1	1	0							
✓	12	1	1	0	0							
	7	0	1	1	1							
	14	1	1	1	0							
	15	1	1	1	1							

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		
✓	1	0	0	0	1		1,3	0	0	-	1	
✓	8	1	0	0	0		1,5	0	-	0	1	
✓	3	0	0	1	1		8,12	1	-	0	0	
✓	5	0	1	0	1							
	6	0	1	1	0							
✓	12	1	1	0	0							
	7	0	1	1	1							
	14	1	1	1	0							
	15	1	1	1	1							

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		
✓	1	0	0	0	1		1,3	0	0	-	1	
✓	8	1	0	0	0		1,5	0	-	0	1	
✓	3	0	0	1	1		8,12	1	-	0	0	
✓	5	0	1	0	1							
	6	0	1	1	0							
✓	12	1	1	0	0							
✓	7	0	1	1	1							
	14	1	1	1	0							
	15	1	1	1	1							

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		
✓	1	0	0	0	1		1,3	0	0	-	1	
✓	8	1	0	0	0		1,5	0	-	0	1	
✓	3	0	0	1	1		8,12	1	-	0	0	
✓	5	0	1	0	1		3,7	0	-	1	1	
	6	0	1	1	0							
✓	12	1	1	0	0							
✓	7	0	1	1	1							
	14	1	1	1	0							
	15	1	1	1	1							

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		
✓	1	0	0	0	1		1,3	0	0	-	1	
✓	8	1	0	0	0		1,5	0	-	0	1	
✓	3	0	0	1	1		8,12	1	-	0	0	
✓	5	0	1	0	1		3,7	0	-	1	1	
	6	0	1	1	0		5,7	0	1	-	1	
✓	12	1	1	0	0							
✓	7	0	1	1	1							
	14	1	1	1	0							
	15	1	1	1	1							

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		
✓	1	0	0	0	1		1,3	0	0	-	1	
✓	8	1	0	0	0		1,5	0	-	0	1	
✓	3	0	0	1	1		8,12	1	-	0	0	
✓	5	0	1	0	1		3,7	0	-	1	1	
✓	6	0	1	1	0		5,7	0	1	-	1	
✓	12	1	1	0	0							
✓	7	0	1	1	1							
	14	1	1	1	0							
	15	1	1	1	1							

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		
✓	1	0	0	0	1		1,3	0	0	-	1	
✓	8	1	0	0	0		1,5	0	-	0	1	
✓	3	0	0	1	1		8,12	1	-	0	0	
✓	5	0	1	0	1		3,7	0	-	1	1	
✓	6	0	1	1	0		5,7	0	1	-	1	
✓	12	1	1	0	0		6,7	0	1	1	-	
✓	7	0	1	1	1							
	14	1	1	1	0							
	15	1	1	1	1							

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		
✓	1	0	0	0	1		1,3	0	0	-	1	
✓	8	1	0	0	0		1,5	0	-	0	1	
✓	3	0	0	1	1		8,12	1	-	0	0	
✓	5	0	1	0	1		3,7	0	-	1	1	
✓	6	0	1	1	0		5,7	0	1	-	1	
✓	12	1	1	0	0		6,7	0	1	1	-	
✓	7	0	1	1	1							
✓	14	1	1	1	0							
	15	1	1	1	1							

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		
✓	1	0	0	0	1		1,3	0	0	-	1	
✓	8	1	0	0	0		1,5	0	-	0	1	
✓	3	0	0	1	1		8,12	1	-	0	0	
✓	5	0	1	0	1		3,7	0	-	1	1	
✓	6	0	1	1	0		5,7	0	1	-	1	
✓	12	1	1	0	0		6,7	0	1	1	-	
✓	7	0	1	1	1		6,14	-	1	1	0	
✓	14	1	1	1	0							
	15	1	1	1	1							

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		
✓	1	0	0	0	1		1,3	0	0	-	1	
✓	8	1	0	0	0		1,5	0	-	0	1	
✓	3	0	0	1	1		8,12	1	-	0	0	
✓	5	0	1	0	1		3,7	0	-	1	1	
✓	6	0	1	1	0		5,7	0	1	-	1	
✓	12	1	1	0	0		6,7	0	1	1	-	
✓	7	0	1	1	1		6,14	-	1	1	0	
✓	14	1	1	1	0		12,14	1	1	-	0	
	15	1	1	1	1							

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		
✓	1	0	0	0	1		1,3	0	0	-	1	
✓	8	1	0	0	0		1,5	0	-	0	1	
✓	3	0	0	1	1		8,12	1	-	0	0	
✓	5	0	1	0	1		3,7	0	-	1	1	
✓	6	0	1	1	0		5,7	0	1	-	1	
✓	12	1	1	0	0		6,7	0	1	1	-	
✓	7	0	1	1	1		6,14	-	1	1	0	
✓	14	1	1	1	0		12,14	1	1	-	0	
✓	15	1	1	1	1		7,15					

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		
✓	1	0	0	0	1		1,3	0	0	-	1	
✓	8	1	0	0	0		1,5	0	-	0	1	
✓	3	0	0	1	1		8,12	1	-	0	0	
✓	5	0	1	0	1		3,7	0	-	1	1	
✓	6	0	1	1	0		5,7	0	1	-	1	
✓	12	1	1	0	0		6,7	0	1	1	-	
✓	7	0	1	1	1		6,14	-	1	1	0	
✓	14	1	1	1	0		12,14	1	1	-	0	
✓	15	1	1	1	1		7,15	-	1	1	1	

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		
✓	1	0	0	0	1		1,3	0	0	-	1	
✓	8	1	0	0	0		1,5	0	-	0	1	
✓	3	0	0	1	1		8,12	1	-	0	0	
✓	5	0	1	0	1		3,7	0	-	1	1	
✓	6	0	1	1	0		5,7	0	1	-	1	
✓	12	1	1	0	0		6,7	0	1	1	-	
✓	7	0	1	1	1		6,14	-	1	1	0	
✓	14	1	1	1	0		12,14	1	1	-	0	
✓	15	1	1	1	1		7,15	-	1	1	1	
							14,15	1	1	1	-	

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
✓	1	0	0	0	1		1,3	0	0	-	1				
✓	8	1	0	0	0		1,5	0	-	0	1				
✓	3	0	0	1	1		8,12	1	-	0	0				
✓	5	0	1	0	1		3,7	0	-	1	1				
✓	6	0	1	1	0		5,7	0	1	-	1				
✓	12	1	1	0	0		6,7	0	1	1	-				
✓	7	0	1	1	1		6,14	-	1	1	0				
✓	14	1	1	1	0		12,14	1	1	-	0				
✓	15	1	1	1	1		7,15	-	1	1	1				
							14,15	1	1	1	-				

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1	✓	1,3	0	0	-	1						
✓	8	1	0	0	0		1,5	0	-	0	1						
✓	3	0	0	1	1		8,12	1	-	0	0						
✓	5	0	1	0	1		3,7	0	-	1	1						
✓	6	0	1	1	0	✓	5,7	0	1	-	1						
✓	12	1	1	0	0		6,7	0	1	1	-						
✓	7	0	1	1	1		6,14	-	1	1	0						
✓	14	1	1	1	0		12,14	1	1	-	0						
✓	15	1	1	1	1		7,15	-	1	1	1						
							14,15	1	1	1	-						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1	✓	1,3	0	0	-	1		1,3,5,7	0	-	-	1
✓	8	1	0	0	0		1,5	0	-	0	1						
✓	3	0	0	1	1		8,12	1	-	0	0						
✓	5	0	1	0	1		3,7	0	-	1	1						
✓	6	0	1	1	0	✓	5,7	0	1	-	1						
✓	12	1	1	0	0		6,7	0	1	1	-						
✓	7	0	1	1	1		6,14	-	1	1	0						
✓	14	1	1	1	0		12,14	1	1	-	0						
✓	15	1	1	1	1		7,15	-	1	1	1						
							14,15	1	1	1	-						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1	✓	1,3	0	0	-	1		1,3,5,7	0	-	-	1
✓	8	1	0	0	0	✓	1,5	0	-	0	1						
✓	3	0	0	1	1		8,12	1	-	0	0						
✓	5	0	1	0	1	✓	3,7	0	-	1	1						
✓	6	0	1	1	0	✓	5,7	0	1	-	1						
✓	12	1	1	0	0		6,7	0	1	1	-						
✓	7	0	1	1	1		6,14	-	1	1	0						
✓	14	1	1	1	0		12,14	1	1	-	0						
✓	15	1	1	1	1		7,15	-	1	1	1						
							14,15	1	1	1	-						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1	✓	1,3	0	0	-	1		1,3,5,7	0	-	-	1
✓	8	1	0	0	0	✓	1,5	0	-	0	1		1,5,3,7	0	-	-	1
✓	3	0	0	1	1		8,12	1	-	0	0						
✓	5	0	1	0	1	✓	3,7	0	-	1	1						
✓	6	0	1	1	0	✓	5,7	0	1	-	1						
✓	12	1	1	0	0		6,7	0	1	1	-						
✓	7	0	1	1	1		6,14	-	1	1	0						
✓	14	1	1	1	0		12,14	1	1	-	0						
✓	15	1	1	1	1		7,15	-	1	1	1						
							14,15	1	1	1	-						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1	✓	1,3	0	0	-	1		1,3,5,7	0	-	-	1
✓	8	1	0	0	0	✓	1,5	0	-	0	1		1,5,3,7	0	-	-	1
✓	3	0	0	1	1		8,12	1	-	0	0						
✓	5	0	1	0	1	✓	3,7	0	-	1	1						
✓	6	0	1	1	0	✓	5,7	0	1	-	1						
✓	12	1	1	0	0	✓	6,7	0	1	1	-						
✓	7	0	1	1	1		6,14	-	1	1	0						
✓	14	1	1	1	0		12,14	1	1	-	0						
✓	15	1	1	1	1		7,15	-	1	1	1						
						✓	14,15	1	1	1	-						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \sum_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1	✓	1,3	0	0	-	1		1,3,5,7	0	-	-	1
✓	8	1	0	0	0	✓	1,5	0	-	0	1		1,5,3,7	0	-	-	1
✓	3	0	0	1	1		8,12	1	-	0	0		6,7,14,15	-	1	1	-
✓	5	0	1	0	1	✓	3,7	0	-	1	1						
✓	6	0	1	1	0	✓	5,7	0	1	-	1						
✓	12	1	1	0	0	✓	6,7	0	1	1	-						
✓	7	0	1	1	1		6,14	-	1	1	0						
✓	14	1	1	1	0		12,14	1	1	-	0						
✓	15	1	1	1	1		7,15	-	1	1	1						
						✓	14,15	1	1	1	-						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1	✓	1,3	0	0	-	1		1,3,5,7	0	-	-	1
✓	8	1	0	0	0	✓	1,5	0	-	0	1		1,5,3,7	0	-	-	1
✓	3	0	0	1	1		8,12	1	-	0	0		6,7,14,15	-	1	1	-
✓	5	0	1	0	1	✓	3,7	0	-	1	1						
✓	6	0	1	1	0	✓	5,7	0	1	-	1						
✓	12	1	1	0	0	✓	6,7	0	1	1	-						
✓	7	0	1	1	1	✓	6,14	-	1	1	0						
✓	14	1	1	1	0		12,14	1	1	-	0						
✓	15	1	1	1	1	✓	7,15	-	1	1	1						
						✓	14,15	1	1	1	-						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1	✓	1,3	0	0	-	1		1,3,5,7	0	-	-	1
✓	8	1	0	0	0	✓	1,5	0	-	0	1		1,5,3,7	0	-	-	1
✓	3	0	0	1	1		8,12	1	-	0	0		6,7,14,15	-	1	1	-
✓	5	0	1	0	1	✓	3,7	0	-	1	1		6,14,7,15	-	1	1	-
✓	6	0	1	1	0	✓	5,7	0	1	-	1						
✓	12	1	1	0	0	✓	6,7	0	1	1	-						
✓	7	0	1	1	1	✓	6,14	-	1	1	0						
✓	14	1	1	1	0		12,14	1	1	-	0						
✓	15	1	1	1	1	✓	7,15	-	1	1	1						
						✓	14,15	1	1	1	-						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x ₁	x ₂	x ₃	x ₄			x ₁	x ₂	x ₃	x ₄			x ₁	x ₂	x ₃	x ₄
✓	1	0	0	0	1	✓	1,3	0	0	-	1	C	1,3,5,7	0	-	-	1
✓	8	1	0	0	0	✓	1,5	0	-	0	1		1,5,3,7	0	-	-	1
✓	3	0	0	1	1	A	8,12	1	-	0	0	D	6,7,14,15	-	1	1	-
✓	5	0	1	0	1	✓	3,7	0	-	1	1		6,14,7,15	-	1	1	-
✓	6	0	1	1	0	✓	5,7	0	1	-	1						
✓	12	1	1	0	0	✓	6,7	0	1	1	-						
✓	7	0	1	1	1	✓	6,14	-	1	1	0						
✓	14	1	1	1	0	B	12,14	1	1	-	0						
✓	15	1	1	1	1	✓	7,15	-	1	1	1						
						✓	14,15	1	1	1	-						

Optimization of SOP Representation By Quine-McCluskey Method – 1st Phase: Finding the prime implicants

Example: $f(x_1, x_2, x_3, x_4) = \Sigma_m(1, 3, 5, 6, 7, 8, 12, 14, 15)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1	✓	1,3	0	0	-	1	C	1,3,5,7	0	-	-	1
✓	8	1	0	0	0	✓	1,5	0	-	0	1		1,5,3,7	0	-	-	1
✓	3	0	0	1	1	A	8,12	1	-	0	0	D	6,7,14,15	-	1	1	-
✓	5	0	1	0	1	✓	3,7	0	-	1	1		6,14,7,15	-	1	1	-
✓	6	0	1	1	0	✓	5,7	0	1	-	1						
✓	12	1	1	0	0	✓	6,7	0	1	1	-						
✓	7	0	1	1	1	✓	6,14	-	1	1	0						
✓	14	1	1	1	0	B	12,14	1	1	-	0						
✓	15	1	1	1	1	✓	7,15	-	1	1	1						
						✓	14,15	1	1	1	-						

Sum of Prime Implicants:

$$f(x_1, x_2, x_3, x_4) = x_1 x_3' x_4' + x_1 x_2 x_4' + x_1' x_4 + x_2 x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

A 8,12
B 12,14
C 1,3,5,7
D 6,7,14,15

	1	3	5	6	7	8	12	14	15	
A										
B										
C										
D										

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1 x_3' x_4' + x_1' x_4 + x_2 x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

A 8,12
B 12,14
C 1,3,5,7
D 6,7,14,15

	1	3	5	6	7	8	12	14	15	
A						X	X			
B										
C										
D										

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1 x_3' x_4' + x_1' x_4 + x_2 x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

A 8,12
B 12,14
C 1,3,5,7
D 6,7,14,15

	1	3	5	6	7	8	12	14	15
A						X	X		
B							X	X	
C									
D									

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1 x_3' x_4' + x_1' x_4 + x_2 x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

A 8,12
B 12,14
C 1,3,5,7
D 6,7,14,15

	1	3	5	6	7	8	12	14	15
A						X	X		
B							X	X	
C	X	X	X		X				
D									

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1 x_3' x_4' + x_1' x_4 + x_2 x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

A 8,12
B 12,14
C 1,3,5,7
D 6,7,14,15

	1	3	5	6	7	8	12	14	15	
A						X	X			
B							X	X		
C	X	X	X		X					
D				X	X			X	X	

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1 x_3' x_4' + x_1' x_4 + x_2 x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
B 12,14
C 1,3,5,7
D 6,7,14,15

	1	3	5	6	7	8	12	14	15	
A						X	X			EPI
B							X	X		
C	X	X	X		X					
D				X	X			X	X	

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1 x_3' x_4' + x_1' x_4 + x_2 x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

	1	3	5	6	7	8	12	14	15	
A						X	X			EPI
B							X	X		
C	X	X	X		X					
D				X	X			X	X	

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1 x_3' x_4' + x_1' x_4 + x_2 x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
B 12,14
C 1,3,5,7
D 6,7,14,15

	1	3	5	6	7	8	12	14	15	
A						X	X			EPI
B							X	X		
C	X	X	X		X					EPI
D				X	X			X	X	

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1 x_3' x_4' + x_1' x_4 + x_2 x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

	1	3	5	6	7	8	12	14	15	
A						X	X			EPI
B							X	X		
C	X	X	X		X					EPI
D				X	X			X	X	

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1 x_3' x_4' + x_1' x_4 + x_2 x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

	1	3	5	6	7	8	12	14	15	
A						X	X			EPI
B							X	X		
C	X	X	X		X					EPI
D				X	X			X	X	EPI

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1 x_3' x_4' + x_1' x_4 + x_2 x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
B 12,14
C 1,3,5,7
D 6,7,14,15

	1	3	5	6	7	8	12	14	15	
A						X	X			EPI
B							X	X		
C	X	X	X		X					EPI
D				X	X			X	X	EPI

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Coverage Table Method

- A 8,12
B 12,14
C 1,3,5,7
D 6,7,14,15

	1	3	5	6	7	8	12	14	15	
A						X	X			EPI
B							X	X		
C	X	X	X		X					EPI
D				X	X			X	X	EPI

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1 x_3' x_4' + x_1' x_4 + x_2 x_3$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Patrick Method

- A 8,12
- B 12,14
- C 1,3,5,7
- D 6,7,14,15

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Patrick Method

A 8,12
B 12,14
C 1,3,5,7
D 6,7,14,15

$$P = C \ C \ C \ D \ (C+D) \ A \ (A+B) \ (B+D) \ D$$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Patrick Method

A 8,12
B 12,14
C 1,3,5,7
D 6,7,14,15

$P = C \ C \ C \ D \ (C+D) \ A \ (A+B) \ (B+D) \ D$
 $C=1, D=1, A=1$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Patrick Method

A 8,12
B 12,14
C 1,3,5,7
D 6,7,14,15

$P = C \ C \ C \ D \ (C+D) \ A \ (A+B) \ (B+D) \ D$
 $C=1, D=1, A=1$
 $P=1$

Optimization of SOP Representation By Quine-McCluskey Method – 2nd Phase: Finding optimal representation

Patrick Method

A 8,12

B 12,14

C 1,3,5,7

D 6,7,14,15

$P=C$ C C D $(C+D)$ A $(A+B)$ $(B+D)$ D

$C=1$, $D=1$, $A=1$

$P=1$

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = x_1 x_3' x_4' + x_1' x_4 + x_2 x_3$$

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4			
1	0	0	0	1			

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4			
1	0	0	0	1			
4	0	1	0	0			

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4		
1	0	0	0	1		
4	0	1	0	0		
8	1	0	0	0		

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4		
1	0	0	0	1		
4	0	1	0	0		
8	1	0	0	0		
5	0	1	0	1		

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4		
1	0	0	0	1		
4	0	1	0	0		
8	1	0	0	0		
5	0	1	0	1		
12	1	1	0	0		

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4		
1	0	0	0	1		
4	0	1	0	0		
8	1	0	0	0		
5	0	1	0	1		
12	1	1	0	0		
11	1	0	1	1		

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4		
1	0	0	0	1		
4	0	1	0	0		
8	1	0	0	0		
5	0	1	0	1		
12	1	1	0	0		
11	1	0	1	1		
14	1	1	1	0		

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

	x_1	x_2	x_3	x_4		x_1	x_2	x_3	x_4
1	0	0	0	1					
4	0	1	0	0					
8	1	0	0	0					
5	0	1	0	1					
12	1	1	0	0					
11	1	0	1	1					
14	1	1	1	0					

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1						
	4	0	1	0	0						
	8	1	0	0	0						
✓	5	0	1	0	1						
	12	1	1	0	0						
	11	1	0	1	1						
	14	1	1	1	0						

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1		1,5	0	-	0	1
	4	0	1	0	0						
	8	1	0	0	0						
✓	5	0	1	0	1						
	12	1	1	0	0						
	11	1	0	1	1						
	14	1	1	1	0						

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1		1,5	0	-	0	1
✓	4	0	1	0	0						
	8	1	0	0	0						
✓	5	0	1	0	1						
	12	1	1	0	0						
	11	1	0	1	1						
	14	1	1	1	0						

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1		1,5	0	-	0	1
✓	4	0	1	0	0		4,5	0	1	0	-
	8	1	0	0	0						
✓	5	0	1	0	1						
	12	1	1	0	0						
	11	1	0	1	1						
	14	1	1	1	0						

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1		1,5	0	-	0	1
✓	4	0	1	0	0		4,5	0	1	0	-
	8	1	0	0	0						
✓	5	0	1	0	1						
✓	12	1	1	0	0						
	11	1	0	1	1						
	14	1	1	1	0						

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1		1,5	0	-	0	1
✓	4	0	1	0	0		4,5	0	1	0	-
	8	1	0	0	0		4,12	-	1	0	0
✓	5	0	1	0	1						
✓	12	1	1	0	0						
	11	1	0	1	1						
	14	1	1	1	0						

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1		1,5	0	-	0	1
✓	4	0	1	0	0		4,5	0	1	0	-
✓	8	1	0	0	0		4,12	-	1	0	0
✓	5	0	1	0	1						
✓	12	1	1	0	0						
	11	1	0	1	1						
	14	1	1	1	0						

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1		1,5	0	-	0	1
✓	4	0	1	0	0		4,5	0	1	0	-
✓	8	1	0	0	0		4,12	-	1	0	0
✓	5	0	1	0	1		8,12	1	-	0	0
✓	12	1	1	0	0						
	11	1	0	1	1						
	14	1	1	1	0						

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1		1,5	0	-	0	1
✓	4	0	1	0	0		4,5	0	1	0	-
✓	8	1	0	0	0		4,12	-	1	0	0
✓	5	0	1	0	1		8,12	1	-	0	0
✓	12	1	1	0	0						
	11	1	0	1	1						
✓	14	1	1	1	0						

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1		1,5	0	-	0	1
✓	4	0	1	0	0		4,5	0	1	0	-
✓	8	1	0	0	0		4,12	-	1	0	0
✓	5	0	1	0	1		8,12	1	-	0	0
✓	12	1	1	0	0		12,14	1	1	-	0
	11	1	0	1	1						
✓	14	1	1	1	0						

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1	B	1,5	0	-	0	1
✓	4	0	1	0	0	C	4,5	0	1	0	-
✓	8	1	0	0	0	D	4,12	-	1	0	0
✓	5	0	1	0	1	E	8,12	1	-	0	0
✓	12	1	1	0	0	F	12,14	1	1	-	0
A	11	1	0	1	1						
✓	14	1	1	1	0						

Optimization of POS Representation By Quine-McCluskey Method

Example: $f(x_1, x_2, x_3, x_4) = \prod_M(1, 4, 5, 8, 11, 12, 14)$

		x_1	x_2	x_3	x_4			x_1	x_2	x_3	x_4
✓	1	0	0	0	1	B	1,5	0	-	0	1
✓	4	0	1	0	0	C	4,5	0	1	0	-
✓	8	1	0	0	0	D	4,12	-	1	0	0
✓	5	0	1	0	1	E	8,12	1	-	0	0
✓	12	1	1	0	0	F	12,14	1	1	-	0
A	11	1	0	1	1						
✓	14	1	1	1	0						

Product of Prime Implicants:

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')$$

$$(x_1 + x_3 + x_4')(x_1 + x_2' + x_3)(x_2' + x_3 + x_4)$$

$$(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

Optimization of POS Representation By Quine-McCluskey Method

A	11
B	1,5
C	4,5
D	4,12
E	8,12
F	12,14

Optimization of POS Representation By Quine-McCluskey Method

A 11
B 1,5
C 4,5
D 4,12
E 8,12
F 12,14

	1	4	5	8	11	12	14	
A								
B								
C								
D								
E								
F								

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_2' + x_3)(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

or

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_3 + x_4')(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

Optimization of POS Representation By Quine-McCluskey Method

A 11
B 1,5
C 4,5
D 4,12
E 8,12
F 12,14

	1	4	5	8	11	12	14
A					X		
B	X		X				
C		X	X				
D		X				X	
E				X		X	
F						X	X

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_2' + x_3)(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

or

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_3 + x_4')(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

Optimization of POS Representation By Quine-McCluskey Method

- A 11
- B 1,5
- C 4,5
- D 4,12
- E 8,12
- F 12,14

	1	4	5	8	11	12	14
A					X		
B	X		X				
C		X	X				
D		X				X	
E				X		X	
F						X	X

EPI

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_2' + x_3)(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

or

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_3 + x_4')(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

Optimization of POS Representation By Quine-McCluskey Method

A	11
B	1,5
C	4,5
D	4,12
E	8,12
F	12,14

	1	4	5	8	11	12	14	
A					X			EPI
B	X		X					EPI
C		X	X					
D		X				X		
E				X		X		
F						X	X	

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_2' + x_3)(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

or

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_3 + x_4')(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

Optimization of POS Representation By Quine-McCluskey Method

- A 11
B 1,5
C 4,5
D 4,12
E 8,12
F 12,14

	1	4	5	8	11	12	14	
A					X			EPI
B	X		X					EPI
C		X	X					
D		X				X		
E				X		X		EPI
F						X	X	

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_2' + x_3)(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

or

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_3 + x_4')(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

Optimization of POS Representation By Quine-McCluskey Method

A 11
B 1,5
C 4,5
D 4,12
E 8,12
F 12,14

	1	4	5	8	11	12	14	
A					X			EPI
B	X		X					EPI
C		X	X					
D		X				X		
E				X		X		EPI
F						X	X	EPI

Optimization of POS Representation By Quine-McCluskey Method

A 11
B 1,5
C 4,5
D 4,12
E 8,12
F 12,14

	1	4	5	8	11	12	14	
A					X			EPI
B	X		X					EPI
C		X	X					
D		X				X		
E				X		X		EPI
F						X	X	EPI

Optimal Representation:

Optimization of POS Representation By Quine-McCluskey Method

- A 11
- B 1,5
- C 4,5
- D 4,12
- E 8,12
- F 12,14

	1	4	5	8	11	12	14	
A					X			EPI
B	X		X					EPI
C		X	X					
D		X				X		
E				X		X		EPI
F						X	X	EPI

Optimal Representation:

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_2' + x_3)(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$

or

$$f(x_1, x_2, x_3, x_4) = (x_1' + x_2 + x_3' + x_4')(x_1 + x_3 + x_4')(x_2' + x_3 + x_4)(x_1' + x_3 + x_4)(x_1' + x_2' + x_4)$$