Q1

$\varepsilon(x) = \varepsilon_0$

a) $E = -\nabla V = -\text{grad}\, u = \frac{-V_0}{d}\vec{e_z}$

b) $D = \varepsilon_0 E = -\frac{V_0 \varepsilon_0}{d}\vec{e_z}$

c) $\oint D \cdot ds = Q \qquad Q = \frac{V_0 \varepsilon_0 w L}{d} \qquad C = \frac{\varepsilon_0 w L}{d}$

d) $w_e = \frac{1}{2} C V^2 = \frac{1}{2} \varepsilon_0 \frac{wL}{d} V^2$

e) $F_e = \frac{\partial w_e}{\partial e} = 0$

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a) $E = -\nabla V$

b) $D = -\frac{\varepsilon_0 V_0 e^x}{d}\vec{e_z}$

c) $\int \rho_s\, ds = Q_0 = \overbrace{\frac{\varepsilon_0 V_0}{d}(L-l)w}^{\text{in air}} + \overbrace{\frac{\varepsilon_0 V_0 w}{d}\int_0^l e^x dx}^{\text{in slab}}$

$Q_0 = \frac{\varepsilon_0 V_0}{d} w \left[(L-l) + e^l - 1\right]$

$C = \frac{\varepsilon_0 w}{d}\left[(L - l + e^l - 1)\right]$

d) $w_e = \frac{1}{2} CV^2 = \frac{1}{2} \varepsilon_0 \frac{w V_0^2}{d} \left[L - \ell + e^{\ell} - 1 \right]$

e) $F_e = \frac{\partial w_e}{\partial e} = \frac{1}{2} \varepsilon_0 \frac{w \overset{\text{constant}}{V_0^2}}{d} \left[e^{\ell} - 1 \right]$

$\boxed{Q_2}$

$$\oint \vec{E}\, d\vec{s} = \frac{Q}{\varepsilon_0} = \int_0^{\pi}\int_0^{2\pi} E_r\, e_r \cdot e_r\, r^2 \sin\theta\, d\phi\, d\theta = 4\pi E_r$$

$$\boxed{E_r = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

$\underline{r > b}$

$$Q = \int_0^{\pi}\int_0^{2\pi}\int_a^b \rho_0 r^2 \sin\theta\, dr\, d\phi\, d\theta = \rho_0 \frac{4\pi}{3}(b^3 - a^3)$$

$$\vec{E} = \frac{\rho_0 \frac{4}{3}\pi (b^3 - a^3)}{4\pi r^2 \varepsilon_0} \vec{e_r} \Rightarrow E(r) = \frac{\rho_0 (b^3 - a^3)}{3\varepsilon_0 r^2} \vec{e_r}$$

$\underline{b > r > a}$

$$Q = \int_0^{\pi}\int_0^{2\pi}\int_a^r \rho_0 r^2 \sin\theta\, dr\, d\phi\, d\theta = \frac{4}{3}\pi (r^3 - a^3)$$

$$E = \frac{\rho_0 (r^3 - a^3)}{3\varepsilon_0 r^2} \vec{e_r}$$

$\underline{a > r}$

$Q = 0 \quad \vec{E} = 0 \qquad V_2 - V_1 = -\int_1^2 E\, dl$

$\underline{r > b}$

$$V(r) = \int_r^\infty \frac{\rho_0 \vec{e_r} (b^3 - a^3)}{3\varepsilon_0 r'^2} \vec{e_r}\, dr = \boxed{\frac{\rho_0\, b^3 - a^3}{3\varepsilon_0 r}}$$

if $r = b$, $V(r) \Rightarrow V(b)$

$b > r > a$

$$V(b) - V(r) = -\int_r^b \frac{\rho_0 (r'^3 - a^3) \vec{e_r} \cdot \vec{e_r}\, dr'}{3\varepsilon_0 r'^2} = \frac{-\rho_0}{3\varepsilon_0} \int_r^b \left(r' - \frac{a^3}{r'^2} \right) dr'$$

$$V(r) = V(b) + \frac{\rho_0}{3\varepsilon_0} \left[\frac{b^2}{2} + \frac{a^3}{b} - \frac{r^2}{2} - \frac{a^3}{r} \right]$$

$$\boxed{V(r) = \rho_0 \left(\frac{b^2}{2\varepsilon_0} - \frac{r^2}{6\varepsilon_0} - \frac{a^3}{3\varepsilon_0 r} \right)}$$

$\underline{a > r}$

$$V(a) - V(r) = -\int_r^a 0\, dr = 0$$

$$V(r) = V(a) = \boxed{\frac{(b^2 - a^2)}{2\varepsilon_0} \rho_0}$$

I accept and abide the Honor Code of ITU

Sena ERSOY
040200434
Sena