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HW#17

10.20 What products have you personally seen that are made of reinforced plastics? How can you tell that they are reinforced?

- Car bumpers and fenders – They have great strength yet with low weight.
- iPhone screen protector – They have high durability, clarity, and high resistance to scratches and impacts.

10.34 Explain how you would go about determining the hardness of the reinforced plastics and composite materials described in this chapter. Are hardness measurements for these types of materials meaningful? Does the size of the indentation make a difference in your answer? Explain.

The hardness of reinforced plastics and composite materials can be determined by measuring the material's resistance to indentation or scratching. Because of the sometimes isotropic, anisotropic or orthotropic nature of reinforced plastics, hardness could vary depending on the material orientation. The size of indentation should not matter since the material hardness is being determined for the whole material and not just a layer of the material.

10.110 Calculate the elastic modulus and load supported by fibers in a composite with an epoxy matrix (E = 100 GPa), made up of 25% fibers made of (a) high-modulus carbon fiber and (b) Kevlar 29 fibers.

$area\ fraction\ of\ fibers, x\ =\ 0.25$

$area\ fraction\ of\ matrix, A_m\ =\ 1\ -\ 0.25\ =\ 0.75$

$Elastic\ modulus\ of\ matrix, E_m\ =\ 100\ GPa$

$Elastic\ modulus\ of\ high-modulus\ carbon\ fiber, E_f\ =\ 415\ GPa$

$Elastic\ modulus\ of\ high-modulus\ carbon\ fiber, E_f\ =\ 70.5\ GPa$

$Elastic\ modulus\ of\ composite, E_c\ =\ x * E_f +\ (1-x) * E_m$

For high-modulus carbon fiber

$E_c\ =\ x * E_f\ +\ (1-x) * E_m$

$E_c\ =\ 0.25 *\ 415\ GPa\ +\ (1-0.25) * 100\ GPa$

$\boldsymbol{E_c\ =\ 178.75\ GPa}$

$Load\ Fraction, \frac{F_f}{F_m}\ = \frac{A_f * E_f}{A_m * E_m}$

$\frac{F_f}{F_m}\ = \frac{(0.25)(415\ GPa)}{(0.75)(100\ GPa)}$

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$$\boldsymbol{\frac{F_f}{F_m} = 1.383}$$

For Kevlar 29 fibers

$$E_c = 0.25 * 70.5\ GPa + (1 - 0.25) * 100\ GPa$$

$$\boldsymbol{E_c = 92.625\ GPa}$$

$$\frac{F_f}{F_m} = \frac{(0.25)(70.5\ GPa)}{(0.75)(100\ GPa)}$$

$$\boldsymbol{\frac{F_f}{F_m} = 0.235}$$

10.111 Calculate the stress in the fibers and in the matrix for Problem 10.110. Assume that the cross-sectional area is 50 mm^2 and Fc = 2000 N

$$from\ eqn(10.12),$$

$$F_c = F_f + F_m$$

$$also, \quad \frac{F_f}{F_m} = ratio$$

$$so, \quad F_c = F_f + \frac{F_f}{ratio}$$

For high-modulus carbon fiber

$$ratio = 1.383$$

$$2000\ N = F_f + \frac{F_f}{1.383}$$

$$2000\ N = 1.72Ff$$

$$F_f = 1162\ N$$

$$from\ eqn(10.12),$$

$$F_m = F_c - F_f$$

$$F_m = 2000\ N - 1162\ N$$

$$F_m = 837\ N$$

$$recall, \quad \sigma = \frac{F}{A}$$

$$\boldsymbol{\sigma_f = \frac{F_f}{A} = \frac{1162\ N}{50\ mm^2} = 23.26\frac{N}{mm^2}}$$

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$$\boldsymbol{\sigma_m = \frac{F_m}{A} = \frac{837\ N}{50\ mm^2} = 16.74\frac{N}{mm^2}}$$

For Kevlar 29 fibers

$ratio = 0.235$

$$2000\ N = F_f + \frac{F_f}{0.235}$$

$2000\ N = 5.26F_f$

$F_f = 380\ N$

$from\ eqn(10.12),$

$$F_m = F_c - F_f$$

$$F_m = 2000\ N - 380\ N$$

$$F_m = 1620\ N$$

$$recall, \qquad \sigma = \frac{F}{A}$$

$$\boldsymbol{\sigma_f = \frac{F_f}{A} = \frac{380\ N}{50\ mm^2} = 7.60\frac{N}{mm^2}}$$

$$\boldsymbol{\sigma_m = \frac{F_m}{A} = \frac{837\ N}{50\ mm^2} = 32.40\frac{N}{mm^2}}$$

Composites Web Problem (W5-1)

Your defense firm is working on a stealth plane. Hematite can be added to polymers to absorb radar and you are considering this approach for a landing gear component. You need to determine the coefficient of thermal expansion of an acetal-based composite filled to 45% volume with hematite particles.

$E_{hematite}$=298GPa
E_{acetal}= 2.9GPa
$CTE_{hematite}$= 9.9 x 10-6/°C
CTE_{acetal}=96.3 x 10-6/°C

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The coefficient of thermal expansion of the particulate composite is given by

$$\alpha_{pc} = \tilde{\alpha}\, V_f^{0.67} \frac{\tilde{E}_p}{E_{pc}} + \alpha_b \frac{E_b}{E_{pc}} - \alpha_b\, V_f^{0.67} \frac{E_b}{E_{pc}} \tag{5}$$

where

$$\tilde{\alpha} = \alpha_b - V_f^{0.33}\left(\alpha_b - \alpha_p\right) \tag{6}$$

and

$$\tilde{E}_p = \frac{E_b}{1 - V_f^{0.33}\left(1 - \frac{E_b}{E_p}\right)} \tag{7}$$

$Ehematite = 298\ GPa = E_p$
$Eacetal = 2.9\ GPa = E_b$
$CTEhematite = 9.9 * 10^{-6}/°C = \alpha_p$
$CTEacetal = 96.3 * 10^{-6}/°C = \alpha_b$

$V_f = 0.45$

The normal modulus of the particulate composite is given by

$$E_{pc} = \frac{V_f^{0.67} E_b}{1 - V_f^{0.33} \cdot \left(1 - \frac{E_b}{E_p}\right)} + \left(1 - V_f^{0.67}\right) E_b \tag{2}$$

Using Python (See Appendix A),

$From\ eqn(2),\ E_{pc} = 6.397\ GPa$

$From\ eqn(6),\quad \bar{\alpha} = 2.991 * 10^{-05}\ \ 10^{-6}/°C$

$From\ eqn(7),\quad \overline{E_p} = 12.116\ GPa$

$\boldsymbol{From\ eqn(5),\quad \alpha_{pc} = 5.126 * 10^{-05}\ \ 10^{-6}/°C}$

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Appendix A

```
Eb = 2.9
Ep = 289
alpha_p = 9.9*(10**-6)
alpha_b = 96.3*(10**-6)
Vf = .45

def Epc(Vf, Eb, Ep):
    A = (Vf**0.67)*Eb
    D = 1-(Eb/Ep)
    B = 1-((Vf**0.33)*D)
    C = 1 - (Vf**0.67)*Eb

    return (A/B) + C

def Ep_bar(Eb, Vf, Ep):
    B = 1-(Eb/Ep)
    A = 1-((Vf**0.33)*B)
    return  Eb/A

def alpha_bar(alpha_b, Vf, alpha_p):
    return alpha_b - (Vf**0.33)*(alpha_b-alpha_p)

def alpha_pc(alpha_bar, Vf, Ep_bar, Epc, alpha_b, Eb):
    A = alpha_bar*(Vf**0.67)*(Ep_bar/Epc)
    B = alpha_b*(Eb/Epc)
    C = alpha_b*(Vf**0.67)*(Eb/Epc)
    return A + B - C

print(answer)

Epc = Epc(Vf, Eb, Ep)
print('Epc = '+ str(round(Epc,3)) + ' GPa')

Ep_bar = Ep_bar(Eb, Vf, Ep)
print('Ep_bar = '+ str(round(Ep_bar,3)))

alpha_bar = alpha_bar(alpha_b, Vf, alpha_p)
print('alpha_bar = '+ str(alpha_bar))

alpha_pc = alpha_pc(alpha_bar, Vf, Ep_bar, Epc, alpha_b, Eb)
print('alpha_pc = '+ str(alpha_pc))
```