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ME495 - HW #1

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2.3 Explain why the difference between engineering strain and true strain becomes larger as strain increases. Is this phenomenon true for both tensile and compressive strains? Explain.

Engineering strain, $e = \frac{l - l_0}{l_0}$ ———— eqn (i)

True strain, $\varepsilon = \int_{l_0}^{l} \frac{dl}{l} = \ln\left(\frac{l}{l_0}\right)$ ———— eqn (ii)

where,

l - resulting length
l_0 - original length

The difference between engineering strain and true strain becomes larger as strain increases because based on the formulation of these strains (ie eqn(i) & eqn(ii)), the engineering strain uses the fixed undeformed original length in its calculation, while the true strain accounts for the incremental changes in the length as deformation occurs.

This further means that the engineering strain does not give a true indication of deformation occurring in a specimen because it is based solely on the original length. True strain accounts for the continuous change in dimension occurring during deformation and its thereby a better indication of the deformation occurring in a specimen.

This phenomenon is true for both tensile and compressive strain as both types of strain are defined by the same formula. The only difference is that the compressive strain is expected to have a negative change in length.

2.49 A strip of metal is originally 1.0 m long. It is stretched in three steps: first to a length of 1.5 m, then to 2.5 m, and finally to 3.0 m. Show that the total true strain is the sum of the true strains in each step, that is, that the strains are additive. Show that, using engineering strains, the strain for each step cannot be added to obtain the total strain.

Given:

Original length, $l_0 = 1.0\,m$
first length, $l_1 = 1.5\,m$
Second length, $l_2 = 2.5\,m$
final length, $l_3 = 3.0\,m$

1. Show that the total strain is the sum of the true strains in each step, that is the strains are additive.

2. Show that using engineering strains, the strains for each step cannot be added to obtain the total strain.

#1

Using eqn. (2.9)

$$\text{total strain, } \epsilon = \ln\left(\frac{l}{l_0}\right)$$

final length, $l_3 = 3$

Original length, $l_0 = 1$

$$\epsilon_1 = \ln\left(3/1\right) = 1.0986$$

Finding strain at each step and summing them

$$\epsilon_2 = \ln\left(\frac{1.5}{1}\right) + \ln\left(\frac{2.5}{1.5}\right) + \ln\left(\frac{3}{2.5}\right) = 1.0986$$

$\epsilon_1 = \epsilon_2$, therefore, the total strain is the sum of the true strains in each step

#2

engineering strain, $e = \frac{l - l_0}{l_0}$ eqn(2.1)

$$e_1 = \frac{3-1}{1} = 2$$

Summing strains at each step.

$$e_2 = \frac{1.5-1}{1} + \frac{2.5-1.5}{1.5} + \frac{3-2.5}{2.5} = 1.3667$$

$e_1 \neq e_2$, therefore using engineering strain, the strains for each step cannot be added to obtain the total strain

2.51 A material has the following properties: S_ut = 350MPa and n = 0.20. Calculate its strength coefficient, K.

Given: ultimate tensile strength, $S_{ut} = 350\,MPa$ and strain-hardening exponent, $n = 0.20$

Find: strength coefficient, K

Using, $\sigma = K\epsilon^n$ eqn (2.11) this gives $K = \dfrac{\sigma}{\epsilon^n}$ —— (I)

now finding the true stress σ.

true stress is also defined as, $\sigma = \dfrac{P}{A}$ eqn (2.8) —— (II)

Where A is the actual or instantaneous area supporting the load P

from eqn (2.10), $\epsilon = \ln\left(\dfrac{A_0}{A}\right)$, Recall $\epsilon = 0.2$

So, $\dfrac{A_0}{A} = e^{0.2} = 1.2214$

from figure 2.3, $S_{ut} = \dfrac{P}{A_0}$ —— (III)

Combine (II) & (III)

from (II) $P = \sigma A$ and from (III) $P = S_{ut} A_0$

Therefore $\sigma A = S_{ut} A_0$

$$\sigma = S_{ut}\left(\frac{A_0}{A}\right) = 350\,MPa * 1.2214 = 427.49\,MPa$$

pluggin σ into eqn (1) and at necking $\epsilon = n$

So, $K = \dfrac{427.49}{0.2^{0.2}} = 589.82\,MPa$