Engineering strain, $e = \frac{l - l_o}{l_o}$

Engineering strain rate, $\dot{e} = \frac{v}{l_o}$

Engineering stress, $\sigma = \frac{P}{A_o}$

True strain, $\epsilon = \ln\left(\frac{l}{l_o}\right)$

True strain rate, $\dot{\epsilon} = \frac{v}{l}$

True stress, $\sigma = \frac{P}{A}$

Modulus of elasticity, $E = \frac{\sigma}{e}$

Shear modulus, $G = \frac{E}{2(1+\nu)}$

Modulus of resilience $= \frac{S_y^2}{2E}$

Elongation $= \frac{l_f - l_o}{l_o} \times 100\%$

Reduction of area $= \frac{A_o - A_f}{A_o} \times 100\%$

79 Shear strain in torsion, $\gamma = \frac{r\phi}{l}$

Hooke's law, $\epsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu(\sigma_2 + \sigma_3)\right]$, etc.

Effective strain (Tresca), $\bar{\epsilon} = \frac{2}{3}(\epsilon_1 - \epsilon_3)$

Effective strain (von Mises), $\bar{\epsilon} = \frac{\sqrt{2}}{3}\left[(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2\right]^{1/2}$

Effective stress (Tresca), $\bar{\sigma} = \sigma_1 - \sigma_3$

Effective stress (von Mises), $\bar{\sigma} = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$

True stress–true strain relationship (power law), $\sigma = K\epsilon^n$ Plain strain $\sigma_2 = \frac{\sigma_1 + \sigma_3}{2}$

True stress–true strain rate relationship, $\sigma = C\dot{\epsilon}^m$

Flow rules, $d\epsilon_1 = \frac{d\bar{\epsilon}}{\bar{\sigma}}\left[\sigma_1 - \frac{1}{2}(\sigma_2 + \sigma_3)\right]$, etc.

Maximum-shear-stress criterion (Tresca), $\sigma_{max} - \sigma_{min} = S_y$

Distortion-energy criterion (von Mises), $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2S_y^2$

Shear yield strength, $k = S_y/2$ for Tresca and $k = S_y/\sqrt{3}$ for von Mises (plane strain)

Volume strain (dilatation), $\Delta = \frac{1-2\nu}{E}(\sigma_1 + \sigma_2 + \sigma_3) = \epsilon_1 + \epsilon_2 + \epsilon_3$

Bulk modulus $= \frac{E}{3(1-2\nu)}$

Test	Indenter	Shape of indentation: Side view	Shape of indentation: Top view	Load, P	Hardness number
Brinell	10-mm steel or tungsten carbide ball	D, d	d	500 kg 1500 kg 3000 kg	$HB = \frac{2P}{(\pi D)(D - \sqrt{D^2 - d^2})}$
Vickers	Diamond pyramid	136°	L	1–120 kg	$HV = \frac{1.854P}{L^2}$
Knoop	Diamond pyramid	$L/b = 7.11$ $b/t = 4.00$ t	b, L	25 g–5 kg	$HK = \frac{14.2P}{L^2}$
Rockwell A, C, D	Diamond cone	120° t = mm		60 kg 150 kg 100 kg	HRA, HRC, HRD = 100 – 500t
B, F, G	$\frac{1}{16}$ - in. diameter steel ball	t = mm		100 kg 60 kg 150 kg	HRB, HRF, HRG = 130 – 500t
E	$\frac{1}{8}$ - in. diameter steel ball			100 kg	HRE

Slip Systems

- a Slip plane in slip direction
- partially governs the Strength + ductility of material
- 5 or more Slip System needed for ductile behavior
- $\frac{b}{a}$ ratio varies for different slip systems → Anisotropy

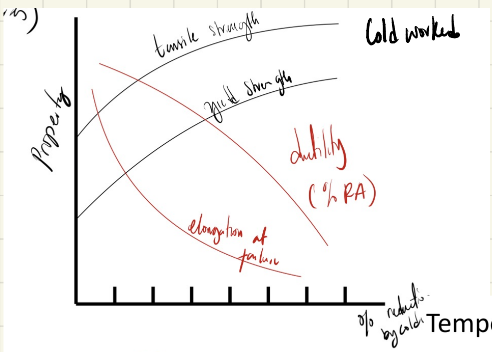
- **BCC** → 48 different Slip System
  In BCC material, many Slip planes with high b/a ratio which increases strength + reduces ductility
  - g strength + moderate ductility

- **FCC** → 12 different slip systems
  - b/a is not as high as bcc
  - moderate strength but good ductility

- **HCP** → 3 slip systems
  low probability of slip and generally brittle

Substitutional Impurity

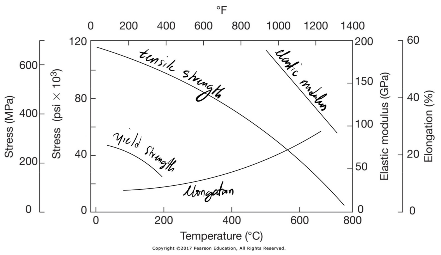
$S_{ut} = 3.5(HB)$ Hardnum $= cS_y$

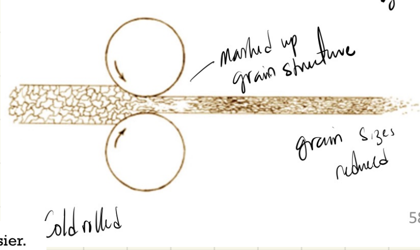


↑ temp ↓ modulus of elasticity, yield strength, ultimate tensile strength

↑ ductility, toughness

Temperature - effect of temp change on various properties





- **Metals: Dislocation motion easier.**
  - non directional bonding
  - lots of slip planes

electron

- **Covalent Ceramics** – Silicon wafers, glass, diamond
  - Strongly directional bond
  - make slip much more difficult.
    ⇓ makes material more brittle

- **Ionic Ceramics** → NaCl
  - Strongest bond type
  - need to overcome charge balance as well as breaking the bond

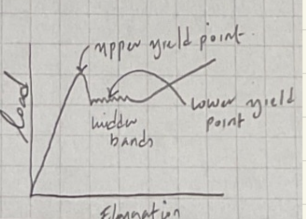
Cannot process with deformation process

Influence of Structure

Dependent on structure
- hardness, yield strength, ductility
  dislocation motion
- electrical conductivity

Independent
Elastic properties
melting point
density
Specific heat
coefficient of thermal expansion
bonding related.

Upper & lower yield point
↑A point at which maximum load or stress required to initial plastic deformation of material - Upper yield point
A point at which minimum load or stress required to maintain the plastic behavior of material - lower yield point.



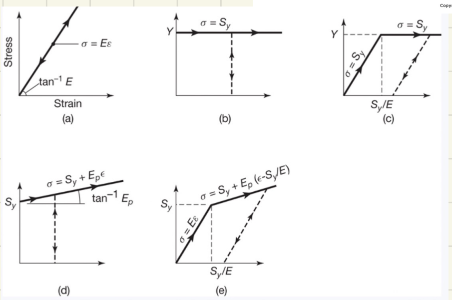


Figure 2.7

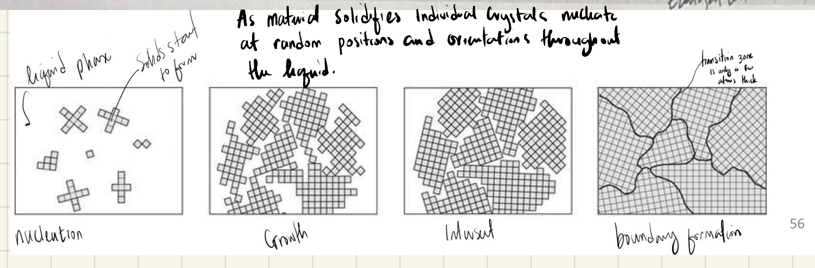
Schematic illustration of various types of idealized stress–strain curves. (a) Perfectly elastic; (b) rigid, perfectly plastic; (c) elastic, perfectly plastic; (d) rigid, linearly strain hardening; and (e) elastic, linearly strain hardening. The broken lines and arrows indicate unloading and reloading during the test.

1. A **perfectly elastic material** displays linear behavior with slope E. The behavior of brittle materials, such as common glass, most ceramics, and some cast irons, may be represented by the curve shown in **Fig. 2.7a**. There is a limit to the stress the material can sustain, after which it fractures. Permanent deformation, if any, is negligible.
2. A **rigid, perfectly plastic** material has, by definition, an infinite value of elastic modulus, E. Once the stress reaches the yield strength, S_y, it continues to undergo deformation at the same stress level; that is, there is no strain hardening. When the load is released, the material has undergone permanent deformation; there is no elastic recovery. (**Fig. 2.7b**).
3. The behavior of an **elastic, perfectly plastic** material is a combination of the first two: It has a finite elastic modulus, no strain hardening, and it undergoes elastic recovery when the load is released (**Fig. 2.7c**).
4. A **rigid, linearly strain-hardening** curve model assumes that the elastic strains are negligible, and that the strain-hardening behavior is defined by a straight line with a slope equal to the **plastic modulus**, E_p. Such materials have no elastic recovery upon unloading (**Fig. 2.7d**).
5. An **elastic, linearly strain-hardening** curve (**Fig. 2.7e**) uses an elastic portion with slope E, and a linear plastic portion with a plastic modulus, E_p.

↗ important for HCP (limited # of slip systems)

twinning refers to the process of producing a mirror image by a grain about the plain of twinning during deformation.

As material solidifies individual crystals nucleate at random positions and orientations throughout the liquid.



Theoretical shear strength of metals, $\tau_{max} = \frac{G}{2\pi}$

Theoretical tensile strength of metals, $\sigma_{max} = \sqrt{\frac{E\gamma}{a}} \simeq \frac{E}{10}$

Hall—Petch equation $S_Y = S_{Y_i} + kd^{-1/2}$

Intrinsit strength

GB blocking constant.

ASTM grain-size number, $N = 2^{n-1}$

Fracture strength vs. crack length, $S_f \propto \frac{1}{\sqrt{\text{Crack length}}}$

1. A minimum amount of deformation is needed to cause recrystallization.
2. The smaller the degree of deformation, the higher the temperature required to cause recrystallization.
3. Increasing the annealing time decreases the recrystallization temperature. However, temperature is far more important than time. Doubling the annealing time is approximately equivalent to increasing the annealing temperature 10°C.
4. The final grain size depends chiefly on the degree of deformation and to a lesser extent on the annealing temperature. The greater the degree of deformation and the lower the annealing temperature, the smaller the recrystallized grain size.
5. The larger the original grain size, the greater the amount of cold work required to produce an equivalent recrystallization temperature.
6. The recrystallization temperature decreases with increasing purity of the metal. Solid-solution alloying additions always raise the recrystallization temperature.
7. The amount of deformation required to produce equivalent recrystallization behavior increases with increased temperature of working.
8. For a given reduction in cross section, different metalworking processes, such as rolling, drawing, etc., produce somewhat different effective deformations. Therefore, Identical recrystallization behavior may not be obtained.

Six main variables influence recrystallization behavior. Amount of prior deformation ② temperature ③ time ④ initial grain size ⑤ Composition ⑥ amount of recovery or polygonization prior to the start of recrystallization.

What effects does recrystallization have on the properties of metal?

By recrystallizing, the density of crystal deformations, called dislocation, is decreased. This is due to the recrystallization process "repairing the defects in the crystal structure.

A metal undergoing recrystallization will also typically have a lower strength and higher ductility characteristics than its original form.

3.16 Explain why the strength of a polycrystalline metal at room temperature decreases as its grain size increases

Since strength is directly proportional to the entanglements of dislocations that take place with each other and with grain boundaries. Also, more is the grain size less will be the grainboundary area per unit volume. Therefore, for metals with large grains fewer entanglements are generated at grain boundaries and thus have lower strength. Hence, we can say that the strength of a polycrystalline metal at room temperature decreases as its grain size increases

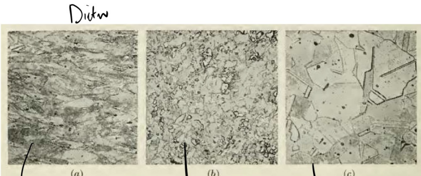
3.9 Describe why different crystal structures exhibit different strengths and ductilities.

Ductility of the material is directly proportional to the number of the slip systems. Along with it, plastic behaviour of the material can be estimated with the help of number of active slip systems. As an example, a hcp material has few grains oriented in reference to a slip system and thus to start plastic deformation high stresses are required because of having few slip systems and are brittle whereas because of having many slip system a fcc material requires lower stresses for the same and thus have moderate strength and good ductility.

- Large grain size is generally associated with low strenght low hardness, high ductility.

Growth

Dictor



(a) (b) (c)

Cartridge brass 70% Cu 30% Zn

400°C for 15 mins

575°C for 15 mins

recrystallization temperature

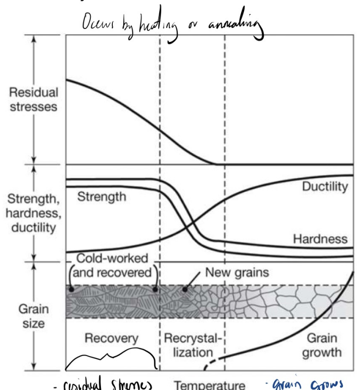
- minimum temperature that recrystallization will occur after 1 hour

- This temperature is normally 0.3 - 0.5 Tmelt.

Homologous temp

$T_H = \frac{T}{T_{melt}}$ (in kelvin)

Occurs by heating or annealing



- residual stresses are relieved
- high energy dislocation move
- no major changes to micro structure

- new grains start to form ↦
- nucleate
- Significantly reduces internal energy

- grain grows
- large grains "eat" small grains
- not desirable for engineering properties

What is the relationship between nucleation rate and the number of grains per unit volume of a metal?

There is a very close relationship between nucleation rate and the number of grains per unit volume. Grain size depends upon the number of sites in which crystal form and the rate at which these grow. Commonly, small grains are formed due to rapid cooling whereas Slow cooling gives larger crystals.

Grain size

$$d = \frac{0.254}{\sqrt{N}}$$

$$N = 2^{n-1}$$

N = # of grains @ 100x

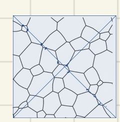
ASTM

$$d = \frac{L}{N}$$

grain diameter

length of line

Number of Intersection



heat & diffusion solid solution strengthening.

4 Strengthening mechanism - precipitation hardening, Strain hardening, grain size strengthening, Solid solution strengthening.