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5.17 Do you think early formation of dendrites in a mold can impede the free flow of molten metal into the mold? Explain.

Yes, early formation of dendrites in a mold can impede the free flow of molten metal into the mold. A long freezing range will result in a large mushy zone. The Solid phase in the mushy zone can be an hinderance to the flow of the liquid.

5.68 A cylindrical casting has a diameter of 300 mm and a length of 1 m. Another casting of the same metal is rectangular in cross section, with a width-to-thickness ratio of 3, and has the same length and cross-sectional area as the round casting. Both pieces are cast under the same conditions. The cylindrical casting solidifies in three minutes. How long does the rectangular cross section casting take to solidify?

Cylindrical casting: diameter $= 300$ mm

length $= 1$ m

$T_s = 3$ minutes

$$\text{Solidification time, } T_s = C\left(\frac{\forall}{A_{surf}}\right)^n \qquad \text{eqn (5.11)}$$

We can Solve for C

$$C = \frac{T_s}{(\forall / A_{surf})^n}$$

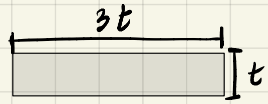
$$A_{surf} = 2\pi \frac{d}{2} h + 2\pi \frac{d^2}{4}$$

$$A_{surf} = \frac{2\pi(.3m)1m}{2} + \frac{2\pi(.3m)^2}{4} = 1.0838m^2$$

$$\forall = \frac{\pi d^2 h}{4} = \frac{\pi(.3m)^2 1m}{4} = 0.0707m^3$$

$$C = \frac{3}{\left(\frac{0.0707m^3}{1.0838m^2}\right)^2} = 705.33 \qquad \text{(same for rectangular cross section)}$$

for rectangular cross-section

width to thickness ratio of 3 $\Rightarrow$ 

$$Area = 3t * t$$

Area = Same as round section

$$\frac{\pi d^2}{4} = 3t^2$$

$$t = \left(\frac{\pi d^2}{12}\right)^{1/2} = 0.1534\,m$$

Surface area of rectangular section, $A_r = 2(w*l + h*l + h*w)$

$$= 2(((3*0.1534)*0.1534) + (1*0.1534) + (1*3*0.1534))$$

$$= 1.369\,m^2$$

Volume of rectangular section, $V_r = 3t*t*h = 0.0707\,m^3$

from eqn (5.11) $$T_R = 705.33\left(\frac{0.0707\,m^3}{1.369\,m^2}\right)^2$$

$$T_R = 1.88\text{ secs}$$ ✱

5.75 A cylinder with a diameter-to-height ratio of 1 solidifies in four minutes in a sand casting operation. What is the solidification time if the cylinder height is tripled? What is the time if the diameter is tripled?

Cylinder diameter-to-height ratio of 1 $\Rightarrow$ $\frac{\text{diameter}}{\text{height}} = 1$

$T_s = 4$ min

From equation 5.11

$$T_s = C\left(\frac{\text{Volume}}{\text{Surface area}}\right)^n$$

Volume of cylinder $= \frac{\pi d^2}{4} h$

Surface area of cylinder $= \pi d h + \frac{\pi d^2}{2}$

$$T_s = C\left(\frac{\frac{\cancel{\pi} d^{\cancel{2}}}{4} h}{\cancel{\pi}\cancel{d}h + \frac{\cancel{\pi} d^{\cancel{2}}}{2}}\right)^2$$

$$T_{s_1} = C\left(\frac{dh}{4h+2d}\right)^2 \Rightarrow 4 = C\left(\frac{dh}{4h+2d}\right)^2$$

when $d/h = 1$

$$4 = C\left(\frac{d^2}{4d+2d}\right)^2 = C\left(\frac{d^2}{6d}\right)^2 = C\frac{d^2}{36}$$

$$C = \frac{144}{d^2} \quad \text{———} \quad (*)$$

If height is tripled

$$T_{s_2} = C\left(\frac{d(3h)}{12h+2d}\right)^2$$

$$T_{s_2} = C\left(\frac{3d}{14}\right)^2$$

$$T_{s_2} = C\left(\frac{9d^2}{196}\right)$$

$$C = \frac{196\,T_{s_2}}{9d^2} \quad \text{———} \quad (*)\,(*)$$

Set eqn ⊛ equal to ⊛⊛

$$\frac{144}{\cancel{d^2}} = \frac{196}{9\cancel{d^2}} T_{s_2}$$

$$T_{s_2} = \frac{9 \times 144}{196} = 6.6122 \text{ mins} \quad *$$

If diameter is tripled

$$T_{s_3} = C\left(\frac{3dh}{4h + 6d}\right)^2$$

$$T_{s_3} = C\left(\frac{3d^{\cancel{2}}}{4\cancel{d} + 6\cancel{d}}\right)^2 = C\,\frac{9d^2}{100}$$

$$C = \frac{100 T_{s_3}}{9d^2} \quad \text{———————} \quad ⊛⊛⊛$$

Set eqn ⊛ equal to ⊛⊛⊛

$$\frac{144}{\cancel{d^2}} = \frac{100 T_{s_3}}{9\cancel{d^2}}$$

$$T_{s_3} = \frac{144 \times 9}{100}$$

$$T_{s_3} = 12.96 \text{ min} \quad *$$

5.80 A sprue is 200 mm long and has a diameter of 125 mm at the top, where the metal is poured. If a flow rate of $60,000\,\text{mm}^3/\text{s}$ is to be achieved, what should be the diameter of the bottom of the sprue?

Sprue height = 200mm

Sprue diameter = 125mm

Flowrate to be achieved = 60,000mm³/s

Find the diameter at the bottom of Sprue.

Mass continuity is given as :

$$Q = A_1 V_1 = A_2 V_2 \qquad \text{eqn (5.4)}$$

$$V_1 = \frac{Q}{A_1}$$

$$V_1 = \frac{60{,}000\ \text{mm}^3/\text{s}}{\frac{\pi (125\text{mm})^2}{4}} = 4.89\ \text{mm/s}$$

from Bernoulli's theorem, we can solve for the velocity at point 2:

$$h_1 + \cancel{\frac{P_1}{\rho g}} + \frac{V_1^2}{2g} = \cancel{h_2} + \cancel{\frac{P_2}{\rho g}} + \frac{V_2^2}{2g} + \cancel{f}^{\,0}$$

Assume . no friction
- $P_1 = P_2$ = atmospheric pressure
- wide shallow pouring basin (velocity @ 1 but no added pressure)

$$h_1 + \frac{V_1^2}{2g} = \frac{V_2^2}{2g}$$

$$V_2 = \left(2gh_1 + V_1^2\right)^{1/2}$$

$$V_2 = \left(\left(2 * 9.81 * 1000\ \text{mm/s}^2 * 200\text{mm} + (4.89\text{mm/s})^2\right)\right)^{1/2} = 1980\ \text{mm/s}$$

At point 2, $A_2 = \frac{Q}{V_2}$ —— $\circledast$

take area at the bottom of sprue to be, $A = \frac{\pi d^2}{4}$

Then eqn (*) becomes

$$\frac{\pi d_2^2}{4} = \frac{Q}{V_2}$$

$$d_2 = \left(\frac{4Q}{\pi V_2}\right)^{1/2}$$

$$d_2 = \left(\frac{4 * 60000 \text{mm}^3/\text{s}}{\pi (1980 \text{mm/s})}\right)^{1/2}$$

$$d_2 = 6.21 \text{mm} \quad *$$